12-1 Samples and Studies

Identify each sample, and suggest a population from which it was selected. Then classify the sample as simple, systematic, self-selected, convenience, or stratified. Explain your reasoning.

1. SHOWER At a bridal shower, a sticker was placed on the bottom of three random plates. The guests who receive the stickered plates will win a prize.

SOLUTION:
The sample taken is the guests that received stickers on their plates. The population was all of the guests at the wedding. Since each member of the population has an equal chance of being selected this is a simple sample.

2. BOOK CLUB Mr. Peterson surveys the students in his English classes to gauge the student body’s interest in forming a book club.

SOLUTION:
The sample is the students in Mr. Peterson's classes, and the population is the student body. Since Mr. Peterson is only surveying the students in his own classes, this is a convenience sample.

Identify each sample as biased or unbiased. Explain your reasoning.

3. ELECTION A group of students stands at the door of the school and asks every tenth student who they would vote for in the upcoming class elections and why.

SOLUTION:
Picking every tenth student who walks through the door is a random sample since the students doing the sampling have no control over who every tenth person is, and since the students are not favoring one outcome over another, this is an unbiased sample.

4. SHOPPING Every fifteenth shopper at a clothing store is asked what they would want most for their birthday.

SOLUTION:
The survey is taken at random since every fifteenth shopper cannot be controlled, but shoppers in a clothing store are more likely to want clothes for their birthday, so the sample is biased.

Determine whether each situation describes a survey, an observational study, or an experiment. Explain your reasoning.

5. TELEVISION A television network wants to conduct a cartoon marathon. To choose the episodes, they mail a questionnaire to people selected at random throughout the country.

SOLUTION:
This describes a survey since the television network is collecting responses from its viewers. The network is not observing the behavior of its viewers nor has it split them into experimental or control groups.

6. FOOD A frozen food company is considering creating frozen meals with tofu instead of meat. At a testing, they randomly give half of a group of 100 people the meals with meat and the other half the same meals with tofu and ask the people how they like the meals.

SOLUTION:
The best classification for this would be an experiment. The frozen food company is asking questions and collecting data from the participants, so a survey is close, but since there are two groups that are given different frozen meals, this is an experiment. Also, since the company is actively collecting information from its participants, it is not an observational study.

Identify each survey question as biased or unbiased. If biased, explain your reasoning.

7. What are you planning to do over summer vacation?

SOLUTION:
The survey question does not encourage any particular answer, does not cause a strong reaction, does not address more than one issue, and is not confusing. Therefore, the question is unbiased.

8. Do you think we should serve mouth-watering steak or chicken?

SOLUTION:
This question describes steak as "mouth-watering" and doesn't provide any description of the chicken, which makes it biased towards steak.
9. Don’t you think Suzanne should be the class president?

**SOLUTION:**
This question leads people toward agreeing with the fact that Suzanne should be class president. A better question would be “Who do you think should be class president?” As it stands, this question is biased.

10. What type of music do you listen to?

**SOLUTION:**
This question does not lead people toward a specific answer so it is unbiased.

**Identify the experiment as biased or unbiased. If biased, explain your reasoning.**

**11. POOLS** A national pool manufacturer wants to determine if a new advertising strategy will increase sales. The company continues to use the normal advertising strategy in its stores located in Indiana, Pennsylvania, and Kentucky. The company uses the new advertising strategy in Florida, Louisiana, and Georgia. The company then compares the sales.

**SOLUTION:**
The states that continued to use the old advertising were all in the midwest, while the states that used the new advertising were all in the southeast where it is much warmer. This leads to a biased experiment when pool sales are involved.

12. **EDUCATION** A school district wants to determine if having school in session year-round will improve the performance of students. They select one of their schools to be in session year-round and compare the test scores of those students with the test scores of the other students in the district.

**SOLUTION:**
The school district selects a specific school and then compares the scores from that school with the other schools. This leads to a biased sample since the school wasn't chosen at random, nor was the previous performance of the schools taken into account before the change.

**Identify each sample, and suggest a population from which it was selected. Then classify the sample as simple, systematic, self-selected, convenience, or stratified. Explain your reasoning.**

13. **SPORTS CARDS** Greg divides his baseball cards by teams. Then he randomly selects four cards from each team and records the players’ RBIs.

**SOLUTION:**
The sample taken is the random cards that Greg picks. The population is all of Greg’s baseball cards. Since Greg first divided his cards into teams and then took a random sample from each, this is a stratified sample.

14. **CARS** The service manager at a car dealership inspects every fifth car to make sure that cars are detailed after being serviced.

**SOLUTION:**
The sample is every fifth car that the manager inspects. The population is all the cars at the dealership. Since the cars are selected at specific intervals, this represents a systematic sample.

15. **RAFFLE** The students who attended a prom committee meeting were each given a raffle ticket for a drawing of five prizes.

**SOLUTION:**
The sample is the five students who won the raffle. The population is all of the students at the meeting. Since the sample was drawn at random from a raffle, this is a simple sample.
16. **MUSIC** A music store asks its customers to submit suggestions for local bands that should play on Friday nights.

**SOLUTION:**
The sample is the customers who submit suggestions. The population is all of the customers. Since the customers in the sample volunteer their suggestions, this represents a self-selected sample.

**Identify each sample as biased or unbiased. Explain your reasoning.**

17. **ACTORS** A random sample of ten people is asked to name their favorite actor.

**SOLUTION:**
Only ten people were included in this sample. This is not representative of the population and is a biased survey.

18. **BASKETBALL** Every fifth athlete at a basketball camp is asked to name their favorite brand of basketball shoe.

**SOLUTION:**
The participants are selected randomly, so the only thing else to consider is whether the fact that the kids asked were all at basketball camp affected the data. Since the question was about the specific type of shoe they liked, this is an unbiased survey, and only reflective of kids at basketball camp. If the question had been "Do you like basketball?" then it would have been a biased survey.

19. **TELEVISION** Every tenth person entering a gas station is asked to name their favorite television program.

**SOLUTION:**
The participants are randomly selected and a gas station doesn't have any influence on television, so the survey is unbiased.

20. **MUSIC** Every fifth person entering a play is asked to name their favorite style of music.

**SOLUTION:**
The sample is taken at random, but since the survey was taken at a play, which has a distinct style of music, the survey is biased.

**Determine whether each situation describes a survey, an observational study, or an experiment. Explain your reasoning.**

21. **PARTIES** Federico is throwing a party for one of his friends. He is trying to decide on a theme. He sends a piece of paper in each invitation, asking guests questions to get their opinions.

**SOLUTION:**
Since Federico is gathering information from willing participants, and there are no control groups, this is a survey.

22. **VOLUNTEER** Jaime finds 50 students, half of whom volunteer at a homeless shelter, and compares their grade point averages, extracurricular activities, and involvement in school clubs.

**SOLUTION:**
Since Jaime is observing and comparing students without direct feedback from them, this is an observational study.

23. **SOCCER** A researcher organizes a soccer game in hot weather. One team wears short-sleeved shirts, while the other team wears long sleeves.

**SOLUTION:**
Since the students are broken into two different groups where data is observed, this represents an experiment.

24. **SALONS** A salon emails customers, asking them to rate their experience during their last appointment.

**SOLUTION:**
The salon is collecting direct feedback from customers and has not divided them into groups. This is a survey.

**Identify each survey question as biased or unbiased. If biased, explain your reasoning.**

25. **What outdoor activities do you enjoy?**

**SOLUTION:**
This is a general question that does not lead a person to respond in any specific way so it is unbiased.
12-1 Samples and Studies

26. Do you think the comedian’s stupid antics are funny?
   SOLUTION:
   This is a biased question since it describes the comedian's antics as stupid, which leads toward a negative attitude about the comedian.

27. Do you like to listen to music, read a book, or watch movies?
   SOLUTION:
   This question asks about different issues without asking generalities about them individually.

28. What is your favorite Web site?
   SOLUTION:
   This is a general question about Web sites without leading toward a specific answer, which makes it unbiased.

Identify the experiment as biased or unbiased. If biased, explain your reasoning.

29. EMPLOYMENT The management of a company hopes to increase the morale of their employees. They select some employees at random and move them into two identical office buildings. They build a recreation room in one of the buildings for employees to use. They then compare the morale of the employees in each building.
   SOLUTION:
   Employees are selected at random and the only thing different about the office buildings is the recreation room. This is an unbiased experiment.

30. CONCERTS The manager of a band wants to see if a light show will improve their concerts. For the final date of the band’s tour in their hometown, they perform while doing the light show. The manager then compares the reviews of this concert with the reviews of the rest of the tour.
   SOLUTION:
   The change in the light show was made on the last day for the band's tour and was also made in their hometown. This could lead to better performance reviews even without the light show. This is a biased experiment.

31. GOLF A golf club manufacturer watches a group of randomly selected golfers test a prototype club and notes their performances and reactions.
   SOLUTION:
   This is unbiased because the manufacturer is not influencing the results and the golfers are all randomly selected.

32. MOVIES A production company sets up a test audience of friends and family of the production crew to view the new movie and notes their reactions.
   SOLUTION:
   This is biased because the production company set up a test audience of friends and family of the production crew. They will find it difficult to give unbiased opinions.

Determine whether each situation calls for a survey, an experiment, or an observational study. Explain your reasoning.

33. GYMS A gym owner wants to test whether changing the color of the walls improves member satisfaction.
   SOLUTION:
   This is an experiment because the sample undergoes a change and is affected by that change.

34. GAMING A manufacturer invites twenty randomly selected teens to try out a new gaming system and notes their reactions as they play.
   SOLUTION:
   Observational study
   The members of the study are observed without being affected by the study.

35. SCHOOLS A student asks 100 randomly selected neighbors if they think the school should build a new football field.
   SOLUTION:
   This is a survey because the data are collected from responses given by the sample regarding their opinions about the football field.
36. **SHOES** A shoe company’s Web site allows customers to design their own shoes. This program keeps a count of styles and colors chosen by customers.

a. Identify the sample. From what population was the sample selected?
b. State the method of data collection. observational study
c. Tell whether the sample is biased or unbiased. Explain.
d. If unbiased, classify the sample as simple, stratified, systematic, self-selected, or convenience.

**SOLUTION:**
a. The sample is a part of the population. The sample is the customers that use the Web site. The population is all athletic shoe customers.
b. This is an observational study because the sample is measured but is unaffected by the study.
c. Each person is equally likely to be chosen and the company is not trying to influence the results, so it is unbiased.
d. self-selected (The customers choose to design the shoes on their own.

37. **ERROR ANALYSIS** Amy and Esteban are describing one way to increase the accuracy of a survey. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Both; both Amy and Esteban’s methods can result in an accurate survey.

The closer the sample size gets to the population size, the more accurately the sample will reflect the population.

The more samples are taken, the more that the average of these samples will reflect the population.

38. **CCSS CRITIQUE** Consider the following survey proposal.

**Question:** How do students feel about the new dress code?

**Method:** Take a simple random sample from each of the four classes. Use this sample to conduct the survey.

Discuss the strengths and weaknesses of this survey.

**SOLUTION:**
Sample answer: This method of selecting a sample is valid. Each student has an equally likely chance of being selected for the sample. A weakness may be that this would not reflect that one grade may feel more strongly about the dress code than another.

39. **REASONING** Charlie wants to determine who the most popular athlete is. He conducts three different surveys. For the first survey, he asks 20 random students at school. For the second survey, he asks 50 random people at the mall. For the third survey, he asks 150 random people at a concert. The most popular athlete was different for each survey. Which survey do you think would most likely represent the population? Explain your reasoning.

**SOLUTION:**
Analyze each individual survey.

20 at school

50 at the mall

150 at the concert

all random

Sample answer: The survey of 150 people at the concert would most likely represent the population. While each survey may have regional bias, more people were surveyed at the concert. The larger the sample size, the more likely it will reflect the population.
12-1 Samples and Studies

40. OPEN ENDED Design and conduct a simple experiment.

**SOLUTION:**
Experiment: Test the affects of sunlight on bananas.

The sample of green bananas is randomly divided into two groups of ten. One group is the control group and the other group is the experimental group.

The control group is left in the kitchen under normal conditions: conditions in which sunlight hits the bananas only when it goes through the window.

The experimental group is left directly in front of the window; conditions in which the group’s exposure to sunlight lasts much longer than a few minutes.

The groups are compared to each other to see if more exposure to sunlight causes bananas to go bad sooner. The time in which they go bad is left to interpretation by the experimenter.

| Total Number of Bananas that have Gone Bad |
|-----------------|-----------------|
| **Day** | **Control Group** | **Experimental Group** |
| 1 | 0 | 0 |
| 2 | 0 | 1 |
| 3 | 0 | 5 |
| 4 | 6 | 10 |
| 5 | 8 | 10 |
| 6 | 10 | 10 |

41. WRITING IN MATH Why are accurate studies important to companies?

**SOLUTION:**
Sample answer: They need accurate surveys to make decisions about how to market and sell products that will earn the company the most profit. Inaccurate surveys could lead them to making products that do not sell and lead the company to a loss. They also make decisions about marketing and advertising and how to reach their target audience. If they do not reach their target audience, their product may not sell, even if it is a worthy product. Finally, they make decisions about the types of products they will develop or continue to sell. Inaccurate surveys could lead them to discontinuing a profitable product.

42. GRIDDED RESPONSE The first stage of a rocket burns 28 seconds longer than the second stage. If the total burning time for both stages is 152 seconds, how many seconds does the first stage burn?

**SOLUTION:**

\[ x + (x + 28) = 152 \]
\[ 2x + 28 = 152 \]
\[ 2x = 124 \]
\[ x = 62 \]
\[ x + 28 = 90 \]

The first stage burns for 90 seconds.

43. Ms. Brinkman invested $30,000; part at 5%, and part at 8%. The total interest on the investment was $2100 after one year. How much did she invest at 8%?

A $10,000  
B $15,000  
C $20,000  
D $25,000

**SOLUTION:**

Set up 2 equations: one relating the total amount invested and one relating the interest rates and the interest earned.

Let \( a \) = the amount invested at 5%.
Let \( b \) = the amount invested at 8%.

Amount invested: \( a + b = 30,000 \)
Interest: \( 0.05a + 0.08b = 2100 \)

\[ a + b = 30,000 \]
\[ a = 30,000 - b \]

\[ 0.05a + 0.08b = 2100 \]
\[ 0.05(30,000 - b) + 0.08b = 2100 \]
\[ 1500 - 0.05b + 0.08b = 2100 \]
\[ 0.03b = 600 \]
\[ b = 20,000 \]

She invested $20,000 at 8%.
The correct choice is C.
12-1 Samples and Studies

44. A pair of $25 jeans is on sale for 15% off. What is the sale price?
   F $21.25
   G $22.25
   H $23.25
   J $24.25

   **SOLUTION:**
   The Original Price – (The Original Price × The Percentage Off) = The New Price
   
   \[ x = 25 - (0.15 \times 25) \]
   \[ = 25 - 3.75 \]
   \[ = 21.25 \]

   The sale price is $21.25.

   The correct choice is F.

45. **GEOMETRY** A piece of wire 42 centimeters long is bent into the shape of a rectangle with a width that is twice its length. Find the dimensions of the rectangle.
   A 5 cm, 12 cm
   B 7 cm, 14 cm
   C 9 cm, 16 cm
   D 11 cm, 18 cm

   **SOLUTION:**
   This is a system of 2 equations. Use the perimeter formula and the relation of \( l \) to \( w \).
   
   \[ P = 2l + 2w \]
   \[ 42 = 2l + 2w \]

   and \( w = 2l \)
   \[ 42 = 2l + 2(2l) \]
   \[ 42 = 2l + 4l \]
   \[ 42 = 6l \]
   \[ 7 = l \]
   \[ w = 2l \]
   \[ = 2(7) \]
   \[= 14 \]

   The rectangle is 7 cm by 14 cm.
   The correct choice is B.

Solve each equation. State any extraneous solutions.

46. \[ \frac{3}{c} = \frac{2}{c+2} \]

   **SOLUTION:**
   Cross multiply.
   
   \[ 3(c + 2) = 2c \]
   \[ 3c + 6 = 2c \]
   \[ c = -6 \]

   The solution is \( c = -6 \).

47. \[ \frac{4}{f} = \frac{2}{f-3} \]

   **SOLUTION:**
   Cross multiply.
   
   \[ 4(f - 3) = 2f \]
   \[ 4f - 12 = 2f \]
   \[ 2f = 12 \]
   \[ f = 6 \]

   The solution is \( f = 6 \).

48. \[ \frac{j}{j+2} = \frac{j-6}{j-2} \]

   **SOLUTION:**
   Cross multiply.
   
   \[ j(j - 2) = (j + 2)(j - 6) \]
   \[ j^2 - 2j = j^2 - 4j - 12 \]
   \[ 2j = -12 \]
   \[ j = -6 \]

   The solution is \( j = -6 \).
49. \( \frac{h - 2}{h} = \frac{h - 2}{h - 5} \)

SOLUTION:
Cross multiply.

\[
\frac{h - 2}{h} = \frac{h - 2}{h - 5} \\
(h - 2)(h - 5) = h(h - 2) \\
h^2 - 7h + 10 = h^2 - 2h \\
10 = 5h \\
2 = h 
\]

The solution is \( h = 2 \).

50. \( \frac{3m + 1}{4} + \frac{3m + 4}{3} = \frac{3m + 4}{6} \)

SOLUTION:
The LCD is 12.

\[
12 \left( \frac{3m}{4} + \frac{1}{3} \right) + 12 \left( \frac{3m + 4}{6} \right) = \frac{3m + 4}{6} \\
\frac{3}{4} + \frac{4}{3} \cdot \frac{1}{3} = \frac{2}{3} \cdot \frac{3m + 4}{6} \\
9m + 4 = 6m + 8 \\
3m = 4 \\
m = \frac{4}{3} 
\]

The solution is \( m = \frac{4}{3} \).

51. \( \frac{6}{5} + \frac{4p}{3} = \frac{8p}{5} \)

SOLUTION:
The LCD is 15.

\[
15 \left( \frac{6}{5} + \frac{4p}{3} \right) = 15 \left( \frac{8p}{5} \right) \\
\frac{3}{5} - \frac{6}{8p} + \frac{5}{3} \cdot \frac{4p}{3} = \frac{3}{8} \cdot \frac{8p}{8p} \\
18 + 20p = 24p \\
18 = 4p \\
\frac{18}{4} = p \\
\frac{9}{2} = p 
\]

The solution is \( p = \frac{9}{2} \).

52. \( \frac{r - 2}{r + 2} - \frac{3r}{r - 2} = -2 \)

SOLUTION:
The LCD is \( (r + 2)(r - 2) \).

\[
\frac{r - 2}{r + 2} - \frac{3r}{r - 2} = -2 \\
\frac{(r - 2)(r - 2) - 3r(r + 2)}{(r + 2)(r - 2)} = -2(r + 2)(r - 2) \\
(r - 2)(r - 2) - 3r(r + 2) = -2 \\
r^2 - 4r + 4 - 3r^2 - 6r = -2r^2 + 8 \\
-2r^2 + 2r^2 - 4r - 6r = 8 - 4 \\
-10r = 4 \\
r = -\frac{2}{5} \\
r = -\frac{2}{5} 
\]

The solution is \( r = -\frac{2}{5} \).
55. **SPORTS** When air is pumped into a ball, the pressure required can be computed by using the formula

\[
P = \frac{3412.94}{4\pi r^3}
\]

where \( P \) represents the pressure in pound per square inch (psi), and \( r \) is the radius of the ball in inches.

a. Simplify the complex fraction.

b. Suppose the air pressure inside the ball is 8 psi. Approximate the radius of the ball to the nearest hundredth.

**SOLUTION:**

a. \[
P = \frac{3412.94}{4\pi r^3} \times \frac{3}{3}
\]

\[
= \frac{3412.94}{4\pi r^3} \times \frac{3}{4\pi r^3}
\]

\[
= \frac{3412.94(3)}{\pi r^3}
\]

\[
= \frac{2559.705}{\pi r^3}
\]

b. \[

= \frac{2559.705}{\pi r^3}
\]

\[
8 = \frac{2559.705}{\pi r^3}
\]

\[
8\pi r^3 = 2559.705
\]

\[
r^3 \approx 101.90
\]

\[
r \approx 4.67
\]

The radius of a ball with air pressure of 8 psi is about 4.67 inches.
56. ROLLER COASTERS Suppose a roller coaster climbs 208 feet higher than its starting point, moving horizontally 360 feet. When it comes down, it moves horizontally 44 feet.

![Diagram of roller coaster](image)

a. How far will it travel to get to the top of the ride?

b. How far will it travel on the downhill track?

**SOLUTION:**

a. This is asking for the hypotenuse of the right triangle. We know the lengths of the other two sides, so we can use the Pythagorean theorem.

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 360^2 + 208^2
\]

\[
c^2 = 129,600 + 43,264
\]

\[
c^2 = 172,864
\]

\[
\sqrt{c^2} = \sqrt{172,864}
\]

\[
c \approx 415.8
\]

It will travel about 415.8 ft. to the top.

b. This is asking for the hypotenuse of the smaller right triangle on the right. We know the lengths of the other two sides, so we can use the Pythagorean theorem.

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 208^2 + 44^2
\]

\[
c^2 = 43,264 + 1936
\]

\[
c^2 = 45,200
\]

\[
\sqrt{c^2} = \sqrt{45,200}
\]

\[
c \approx 212.6
\]

It will travel about 212.6 ft. downhill.

57. PHYSICAL SCIENCE Mr. Blackwell’s students recorded the height of an object above the ground after it was dropped from a height of 5 meters.

![Height vs. Time Table](image)

Draw a graph showing the relationship between the height of the object and time.

**SOLUTION:**

Plot the points with time on the horizontal and height on the vertical. The maximum time is 1, so the horizontal should go from 0 to 1 in increments of 0.1. The maximum height is 500, so the vertical should go from 0 to 500 in increments of 50.
1. BOOKS A random sample of 1000 U.S. college students is surveyed about how much money they spend on books per year. Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

**SOLUTION:**
A sample is a portion of the larger group, the population. A statistic is a measure that describes a characteristic of a sample. A parameter is a measure that describes a characteristic of a population. Parameters are usually estimated values based on the statistics of a carefully chosen random sample.

The sample is 1000 U.S. college students. The population is all college students in the United States. The sample statistic is the mean of the money spent on books in a year by the sample. The population parameter is the mean of money spent on books by all college students in the United States.

2. AMUSEMENT PARKS An amusement park manager kept track of how many bags of cotton candy they sold each hour on a Saturday: {16, 24, 15, 17, 22, 16, 18, 24, 17, 13, 25, 21}. Find and interpret the mean absolute deviation.

**SOLUTION:**
To find the mean absolute value, first find the mean:

\[
x = \frac{16 + 24 + 15 + 17 + 22 + 16 + 18 + 24 + 17 + 13 + 25 + 21}{12} = 19
\]

Then take the sum of the absolute differences between each value and the mean, and divide by the number of values.

\[
|16 - 19| = 3 \quad |18 - 19| = 1
\]

\[
|24 - 19| = 5 \quad |24 - 19| = 5
\]

\[
|15 - 19| = 4 \quad |17 - 19| = 2
\]

\[
|17 - 19| = 2 \quad |13 - 19| = 6
\]

\[
|22 - 19| = 3 \quad |25 - 19| = 6
\]

\[
|16 - 19| = 3 \quad |21 - 19| = 2
\]

\[
\text{MAD} = \frac{3 + 5 + 4 + 2 + 3 + 3 + 1 + 5 + 2 + 6 + 6 + 2}{12} = 3.5
\]

The average cotton candy sales per hour were 19 bags. Each hour, on average, the difference from this value was 3.5.

3. PART-TIME JOBS Ms. Johnson asks all of the girls on the tennis team how many hours each week they work at part-time jobs: {10, 12, 0, 6, 9, 15, 12, 10, 11, 20}. Find and interpret the standard deviation of the data set.

**SOLUTION:**
Use a graphing calculator to view statistics for the data set.

Clear all lists. Select STAT, EDIT to enter the data into L1.

Select STAT, CALC, 1-Var Stats.

The standard deviation is approximately 4.98.
12-2 Statistics and Parameters

4. **CCSS MODELING** Mr. Jones recorded the number of pull-ups done by his students. Compare the means and standard deviations of each group.
   - Boys: {5, 16, 3, 8, 4, 12, 2, 15, 0, 1, 9, 3}
   - Girls: {2, 4, 0, 3, 5, 4, 6, 1, 3, 8, 3, 4}

**SOLUTION:**

Use a graphing calculator to view statistics for the two sets of data.

Clear all lists. Select STAT, EDIT and enter the data for the boys into L₁. Do the same thing and enter the data for the girls into L₂.

Select STAT, CALC, 1-Var Stats, L₁. This will give the statistics for the boys.

```
1-Var Stats
x̄=6.5
Sx=28
sx=834
sxx=5.45272254
σx=5.220153254
n=12
```

Select STAT, CALC, 1-Var Stats, L₂. This will give the statistics for the boys.

```
1-Var Stats
x̄=3.583333333
Sx=43
sx=205
sxx=2.1514618
σx=2.059867849
n=12
```

On average the boys did 6.5 pull-ups, whereas the girls only completed 3.6 pull-ups. The standard deviation for the boys was 5.2 whereas the girls had a standard deviation of 2.1. The boys had a broader range of the number of pull-ups they could complete whereas the girls were more consistent.

5. **POLITICS** A random sample of 1003 Mercy County voters is asked if they would vote for the incumbent for governor. The percent responding yes is calculated.

**SOLUTION:**

A sample is a portion of the larger group, the population. A statistic is a measure that describes a characteristic of a sample. A parameter is a measure that describes a characteristic of a population. Parameters are usually estimated values based on the statistics of a carefully chosen random sample.

Sample: 1003 Mercy County voters; Population: All Mercy County voters; Sample statistic: Number of people in the sample who would vote for the incumbent candidate; Population parameter: The number of people in the county who would vote for the incumbent candidate.

6. **ACTIVITIES** A stratified random sample of high school students from each school in the county was polled about the time spent each week on extracurricular activities.

**SOLUTION:**

A sample is a portion of the larger group, the population. A statistic is a measure that describes a characteristic of a sample. A parameter is a measure that describes a characteristic of a population. Parameters are usually estimated values based on the statistics of a carefully chosen random sample.

Sample: stratified random sample from schools in the county; Population: All high school students in the county; Sample statistic: Time spent each week on extracurricular activities by sample; Population parameter: Time spent each week on extracurricular activities by all students in the county.
12-2 Statistics and Parameters

7. **MONEY** A stratified random sample of 2500 high school students across the country was asked how much money they spent each month.

**SOLUTION:**
Sample answer: stratified random sample of 2500 students nationwide; population: high school students in the country; sample statistic: how much money the 2500 students spent individually each month; population parameter: how much money all the students in the country spent individually each month.

8. **DVDS** A math teacher asked all of his students to count the number of DVDs they owned. Find and interpret the mean absolute deviation.

**SOLUTION:**
First find the mean of the data.

The sum of the data is 324. There are 18 data values. So the mean is \( 324 \div 18 = 18 \).

Now find the sum of the absolute value of the difference between each value in the data set and the mean.

\[
|26 - 18| = 8; \quad |0 - 18| = 18; \quad |2 - 18| = 16; \\
|39 - 18| = 21; \quad |3 - 18| = 15; \quad |0 - 18| = 18; \\
|5 - 18| = 13; \quad |15 - 18| = 3; \quad |11 - 18| = 7; \\
|82 - 18| = 64; \quad |19 - 18| = 1; \quad |1 - 18| = 17; \\
|12 - 18| = 6; \quad |41 - 18| = 23; \quad |19 - 18| = 1; \\
|14 - 18| = 4; \quad |6 - 18| = 12; \quad |29 - 18| = 11;
\]

The sum of these values is 258.

\[
258 \div 18 = 14.3
\]

The mean absolute deviation is 14.3.

This means that the average was 18 and that the majority of the students owned \( 18 \pm 14.3 \) DVDs. Since the mean absolute deviation is so close in value to the average, this means that the data was spread out over a large range of values in comparison to the sample size.

9. **SWIMMING** The owner of a public swimming pool tracked the daily attendance. Find and interpret the mean absolute deviation.

**SOLUTION:**
To find the mean absolute deviation, first find the mean:

\[
\text{Mean} = \frac{\text{Sum of all values}}{\text{Number of values}} = \frac{86 + 45 + 91 + 104 + 95 + 86 + 103 + 104 + 163 + 103 + 88 + 80 + 94 + 70 + 102 + 165 + 100 + 88 + 70 + 90 + 70}{19} = 94
\]

Now find the sum of the absolute value of the difference between each value in the data set and the mean.

\[
|26 - 18| = 8; \quad |0 - 18| = 18; \quad |2 - 18| = 16; \\
|39 - 18| = 21; \quad |3 - 18| = 15; \quad |0 - 18| = 18; \\
|5 - 18| = 13; \quad |15 - 18| = 3; \quad |11 - 18| = 7; \\
|82 - 18| = 64; \quad |19 - 18| = 1; \quad |1 - 18| = 17; \\
|12 - 18| = 6; \quad |41 - 18| = 23; \quad |19 - 18| = 1; \\
|14 - 18| = 4; \quad |6 - 18| = 12; \quad |29 - 18| = 11;
\]

The sum of these values is 258.

\[
258 \div 19 = 13.47
\]

The mean absolute deviation is 13.47.

On average the daily attendance is only 14 people away from the mean of 94.
10. **CCSS REASONING** Samantha wants to see if she is getting a fair wage for babysitting at $8.50 per hour. She takes a survey of her friends to see what they charge per hour. The results are {8.00, $8.50, $9.00, $7.50, $15.00, $8.25, $8.75}. Find and interpret the standard deviation of the data.

**SOLUTION:**
To find the standard deviation, first find the mean:

\[ \bar{x} = \frac{8.00 + 8.50 + 9.00 + 7.50 + 15.00 + 8.25 + 8.75}{7} \]

\[ \bar{x} = 9.29 \]

Next calculate the square of the differences and take their sum:

\[ (9.29 - 8.00)^2 = 1.66 \]
\[ (9.29 - 8.50)^2 = 0.62 \]
\[ (9.29 - 9.00)^2 = 0.08 \]
\[ (9.29 - 7.50)^2 = 3.20 \]
\[ (9.29 - 15.00)^2 = 32.60 \]
\[ (9.29 - 8.25)^2 = 1.08 \]
\[ (9.29 - 8.75)^2 = 0.29 \]

The sum of this is 46.15.

Dividing by 7 we get: \[ \frac{46.15}{7} = 6.59 \]

Finally, take the square root: \[ \sqrt{6.59} \approx 2.57 \]

This means that most of Samantha's friends charge $9.29 \pm $2.57 or between $6.72 and $11.86. The outlier of $15.00 skews the data slightly, but Samantha is getting a fair price at $8.50.

11. **ARCHERY** Carla participates in competitive archery. Each competition allows a maximum of 90 points. Carla’s results for the last 8 competitions are {76, 78, 81, 75, 80, 80, 76, 77}. Find and interpret the standard deviation of the data.

**SOLUTION:**
To find the standard deviation, first find the mean:

\[ \bar{x} = \frac{76 + 78 + 81 + 75 + 80 + 80 + 76 + 77}{8} \]

\[ \bar{x} = 77.875 \]

\[ \bar{x} \approx 77.9 \]

Next calculate the square of the differences and take their sum:

\[ (\bar{x} - 76)^2 = 3.52 \]
\[ (\bar{x} - 78)^2 = 0.02 \]
\[ (\bar{x} - 81)^2 = 9.77 \]
\[ (\bar{x} - 75)^2 = 8.27 \]
\[ (\bar{x} - 80)^2 = 4.52 \]
\[ (\bar{x} - 80)^2 = 4.52 \]
\[ (\bar{x} - 76)^2 = 3.52 \]
\[ (\bar{x} - 77)^2 = 0.77 \]

The sum of this is 34.875.

Dividing by 8 we get: \[ \frac{34.875}{8} = 4.36 \]

Finally, take the square root: \[ \sqrt{4.36} \approx 2.09 \]

This means that most of Carla scores are \( \bar{x} \pm 2.09 \) or between 75.8 and 80.0. Carla's scores are fairly consistent.

12. **BASKETBALL** The coach of the Wildcats basketball team is comparing the number of fouls called against his team with the number called against their rivals, the Trojans. He records the number of fouls called against each team for each game of the season. Compare the means and standard deviations of each set of data.
12-2 Statistics and Parameters

<table>
<thead>
<tr>
<th>Wildcats</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>8</td>
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<tr>
<td>11</td>
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<td>12</td>
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<td>12</td>
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<tr>
<td>16</td>
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<tr>
<td>9</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trojans</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>9</td>
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<td>12</td>
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<td>7</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**BASKETBALL** The coach of the Wildcats basketball team is comparing the number of fouls called against his team with the number called against their rivals, the Trojans. He records the number of fouls called against each team for each game of the season. Compare the means and standard deviations of each set of data.

**Use a graphing calculator to view statistics for the two sets of data.**

Clear all lists. Select STAT, EDIT and enter the data for the Wildcats into L1. Do the same thing and enter the data for the Trojans into L2.

Select STAT, CALC, 1-Var Stats, L1. This will give the statistics for the Wildcats.

<table>
<thead>
<tr>
<th>Movie A</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
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<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Movie B</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>6</td>
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<tr>
<td>3</td>
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<tr>
<td>10</td>
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<tr>
<td>9</td>
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<td>9</td>
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<td>2</td>
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<tr>
<td>8</td>
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<tr>
<td>10</td>
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<tr>
<td>3</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Use a graphing calculator to view statistics for the two sets of data.

b. Provide an argument for why Movie A would be preferred. Movie B?

a. Compare the means and standard deviations of each set of data.

b. Provide an argument for why Movie A would be preferred. Movie B?
12-2 Statistics and Parameters

Clear all lists. Select STAT, EDIT and enter the data for the Movies A into L₁. Do the same thing and enter the data for the Movie B into L₂.

Select STAT, CALC, 1-Var Stats, L₁. This will give the statistics for the Movie A.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄ = 7.1875</td>
</tr>
<tr>
<td>σ = 0.837</td>
</tr>
</tbody>
</table>

Select STAT, CALC, 1-Var Stats, L₂. This will give the statistics for the Movie B.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄ = 6.75</td>
</tr>
<tr>
<td>σ = 2.861</td>
</tr>
</tbody>
</table>

The mean for Movie A is 7.2 with a standard deviation of 0.8. The mean for Movie B is 6.75 with a standard deviation of 2.9. On average students liked Movie A better and were more consistent with their ratings. Movie B had on average, lower ratings, but had a greater spread of ratings.

b. Movie A was likely a more pleasing movie to a general audience and received consistently better scores. Movie B had lower scores on average but a larger spread. This means that there were a few people who liked it and a few people who really disliked it. It was probably a genre of movie that only fits a few peoples' taste.

14. PENNIES Mr. Day has another jar of pennies on his desk. There are 30 pennies in this jar. Theo chooses 5 pennies from the jar. Lola chooses 10 pennies, and Peter chooses 20 pennies. Pennies are chosen and replaced.

Given the data, let's calculate the mean absolute deviation for each set of data.

### SOLUTION:


The mean is:

\[
\bar{x} = \frac{1974 + 1975 + 1981 + 1999 + 1992}{5} = \frac{9921}{5} \approx 1984
\]

Now find the sum of the absolute value of the difference between each value in the data set and the mean.

\[|1974 - 1984| = 10\]
\[|1975 - 1984| = 9\]
\[|1981 - 1984| = 3\]
\[|1999 - 1984| = 15\]
\[|1992 - 1984| = 8\]

\[10 + 9 + 3 + 15 + 8 = 45\]

Divide the sum by the number of values: \(45 \div 5 = 9\). The mean absolute deviation is approximately 9.0.

#### b. The mean of the years for Lola's pennies is 2001.

Find the sum of the absolute value of the difference between each value in the data set and the mean.

\[|2004 - 2001| = 3\]
\[|1999 - 2001| = 2\]
\[|2004 - 2001| = 3\]
\[|2005 - 2001| = 4\]

The mean absolute deviation is calculated by finding the mean of the absolute differences from the mean. This measure is sensitive to outliers.
12-2 Statistics and Parameters

| 1991 – 2001 | 10 
| 2003 – 2001 | 2 
| 2005 – 2001 | 4 
| 2000 – 2001 | 1 
| 2001 – 2001 | 0 
| 1998 – 2001 | 3 |

Sum = 32

Divide the sum by the number of values: 32 ÷ 10 = 3.2.
The mean absolute deviation is 3.2

c. Enter the data values into L1 in your calculator. Select STAT, CALC, 1-Var-Stats.

<table>
<thead>
<tr>
<th>L1 – Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄=1998.15</td>
</tr>
<tr>
<td>σ=8.229671925</td>
</tr>
<tr>
<td>n=20</td>
</tr>
</tbody>
</table>

The mean of the years for Peter's pennies is 1998.15. Set L2 equal to the absolute value of L1 minus 1998.15. Then select ENTER.

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>3</td>
</tr>
<tr>
<td>2005</td>
<td>1</td>
</tr>
<tr>
<td>1978</td>
<td>9</td>
</tr>
<tr>
<td>2008</td>
<td>1</td>
</tr>
<tr>
<td>2005</td>
<td>1</td>
</tr>
<tr>
<td>1992</td>
<td>6</td>
</tr>
</tbody>
</table>

(L2 = |L1-1998.15|)

L2 should now have the absolute value of the difference between the data value and the mean for every value. The sum of L2 divided by the number of values is the mean absolute deviation.

\[
\text{sum}(L2)/20 = 6.42
\]

The mean absolute deviation is 6.42. If we were to use 1998 instead of 1998.15, the mean absolute deviation would be 6.45, which rounds up to 6.5.

d. Follow the same steps in part c. The mean is approximately 1997 and the mean absolute deviation is approximately 7.4; Peter’s sample was the most accurate. The mean year of his sample was 1 year off from the actual mean year.

The mean absolute deviations for the samples went from 9.0 to 3.2 to 6.5 while the mean absolute deviation of the entire data set was 7.4.

The samples that had more pennies were more accurate. Peter's set had the most pennies of the first three sets, and his deviation of 6.5 was the closest to the true deviation of 7.4, so the closer the sample is in size to the population, the more accurately the deviation of the sample reflects the deviation of the population.
15. **RUNNING** The results of a 5K race are published, but only the times of the top 15 finishers are given. Below is a partial list of times: 15:56, 16:06, 16:11, 16:21, 16:26, 15:56, 16:06, 16:11, 16:21, 16:26.

<table>
<thead>
<tr>
<th>Place</th>
<th>Time (min:s)</th>
<th>Place</th>
<th>Time (min:s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15:56</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>16:06</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>16:11</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>16:21</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>16:26</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Find the mean and standard deviation of the top 15 finishers.

b. Identify the sample and population.

c. Analyze the sample. Classify the data as qualitative or quantitative. Apply the sample be applied to the population? Explain.

**SOLUTION:**

Enter the data in L1 and calculate the mean and standard deviation.

```
L1 | L2 | L3 | 1 | L1         | L2         |
---|----|----|---|------------|------------|
956|    |    |   | 56         |            |
```

The mean is about 16.9 minutes and the standard deviation is approximately 2.5.

b. The sample represents part of a larger group. In this case, the population is all of the people who ran the race. The sample is not random.

c. The data is quantitative because it is a measurement.

The sample needs to be random in order to accurately represent the population. The mean and standard deviation of the running times to the top 15 runners in the race, so it is not random. This will not change the numerator of the standard deviation, but decreasing the denominator by only a few, this will result in a much lower standard deviation.

16. **CCSS CRITIQUE** Jennifer and Megan are determining one way to decrease the size of the standard deviation of a set of data. Is either of them correct? Explain.

**SOLUTION:**

Consider the formula for standard deviation:

\[
\sigma = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_n - \overline{x})^2}{n}}
\]

Jennifer proposes to remove the outliers from the set. This will greatly decrease the numerator, and decrease the denominator by the number of outliers. By decreasing the numerator by a large amount, while decreasing the denominator by only a few, this will result in a much lower standard deviation.

Megan wants to add data values that are equal to the mean. This will not change the numerator since adding values that are equal to the mean will be a difference of 0 from the mean. The denominator, however, will increase by the number of added values. This will result in a lower standard deviation.

Both methods will decrease the standard deviation. However, in practice it is common to look at a set of data without the outliers. Often times equipment can malfunction, or some other event can cause an obscure data point, so it is important to take note of this and look at the data set without this outlier. You never want to simply add data points that are equal to the mean just to get a lower standard deviation.
12-2 Statistics and Parameters

17. REASONING Determine whether the statement
   Two random samples taken from the same
   population will have the same mean and standard
   deviation is sometimes, always, or never true.
   Explain.

   SOLUTION:
   Sometimes; it is possible however, if the samples are
   truly random, they would usually not contain identical
   elements. Therefore the mean and standard deviation
   would differ.

   Consider the population of test scores below.
   90, 95, 85, 90, 100, 100, 95, 90, 80, 100, 85, 90, 75,
   70, 70, 100, 95, 95, 100, 90, 90, 80, 85

   Two random samples of 5 scores could easily contain
   identical scores and thus have the same mean and
   standard deviation.

18. OPEN ENDED Describe a situation in which it
   would be useful to use a sample mean to help
   estimate a population mean. How could you collect a
   random sample?

   SOLUTION:
   While the population mean is more accurate, it will
   often not be feasible to collect every value in the
   population. We should use sample means as
   estimates for populations like these. For example, one
   cannot contact every eligible voter and get their
   opinion on a nationwide election.

   Sample answer: Poll of voters to determine if a
   particular presidential candidate is favored to win the
   election. Use a stratified random sample to call 100
   people throughout the country.

19. CHALLENGE Write a set of data with a standard
   deviation that is equal to the mean absolute deviation.

   SOLUTION:
   Consider the equations for standard deviation and
   mean average deviation:
   
   \[ \sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}} \]
   
   \[ \text{MAD} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \ldots + |x_n - \bar{x}|}{n} \]

   For any given set of data, we know that the number
   of terms, n, will be the same in both of these
   equations. Trying to find further constraints in order
   to make these values equal would be difficult unless
   we consider the case where both of the values are 0.
   This only occurs when every data point is equal to
   the mean, and hence when every data point is
   equal. Consider the data set of 5, 5, 5, 5, 5, 5. Both
   the mean average deviation and standard deviation
   are 0.

   WRITING IN MATH Compare and contrast
   each of the following.

20. statistics and parameters

   SOLUTION:
   A statistic is a characteristic that is computed on a
   sample of the population. A parameter is a
   characteristic of the entire population. Sample
   answer: To determine the average height of a student
   at your high school, you can measure the heights of a
   random sample of students at your school. The mean
   height of the sample is a statistic; the actual mean
   height of the students at your school is a parameter.

21. standard deviation and mean absolute deviation

   SOLUTION:
   Both are calculated statistical values that show how
   each data value deviates from the mean of the data
   set. The mean absolute deviation is calculated by
   taking the mean of the absolute values of the
   differences between each number and the mean of
   the data set. To find the standard deviation, you
   square each difference and then take the square root
   of the mean of the squares.
12-2 Statistics and Parameters

Consider the following set of data: \(\{6, 8, 10, 15, 11, 9, 10, 7, 9, 11\}\).

There are 10 values.
The sum of the values is 96.
The mean is \(96 \div 10 = 9.6\).

Mean Absolute Deviation:
\[
\begin{align*}
|6 - 9.6| &= 3.6 \\
|8 - 9.6| &= 1.6 \\
|10 - 9.6| &= 0.4 \\
|15 - 9.6| &= 5.4 \\
|11 - 9.6| &= 1.4 \\
|9 - 9.6| &= 0.6 \\
|10 - 9.6| &= 0.4 \\
|7 - 9.6| &= 2.6 \\
|9 - 9.6| &= 0.6 \\
|11 - 9.6| &= 1.4 \\
\end{align*}
\]

\[
\text{Sum} = 18 \\
\text{Mean Absolute Deviation} = \frac{18}{10} = 1.8
\]

Standard Deviation:
\[
\begin{align*}
(6 - 9.6)^2 &= 12.96 \\
(8 - 9.6)^2 &= 2.56 \\
(10 - 9.6)^2 &= 0.16 \\
(15 - 9.6)^2 &= 29.16 \\
(11 - 9.6)^2 &= 1.96 \\
(9 - 9.6)^2 &= 0.36 \\
(10 - 9.6)^2 &= 0.16 \\
(7 - 9.6)^2 &= 6.76 \\
(9 - 9.6)^2 &= 0.36 \\
(11 - 9.6)^2 &= 1.96 \\
\end{align*}
\]

\[
\text{Sum} = 56.4 \\
56.4 \div 10 = 5.64 \\
\text{Standard Deviation} = \sqrt{5.64} \approx 2.37
\]

22. Melina bought a shirt that was marked 20% off of for $15.75. What was the original price?
A $16.69 \\
B $17.69 \\
C $18.69 \\
D $19.69

\text{SOLUTION:}
The New Cost = The Original Cost – (The Percentage Off \times The Original Cost)

\[
15.75 = x - 0.2x \\
15.75 = 0.8x \\
19.69 = x
\]
The original price of the shirt was $19.69.
The correct choice is D.

23. SHORT RESPONSE A group of student ambassadors visited the Capitol building. Twenty students met with the local representative. This was 16% of the students. How many student ambassadors were there altogether?

\text{SOLUTION:}
\[
\text{Portion of Students} = \text{Percentage} \times \text{Total Students} \\
20 = 0.16x \\
125 = x
\]
There were 125 student ambassadors altogether.
24. The tallest 7 trees in a park have heights in meters of 19, 24, 17, 26, 24, 20, and 18. Find the mean absolute deviation of their heights.

F 3.0
G 3.2
H 3.4
J 21

**SOLUTION:**
To find the mean absolute deviation, first find the mean:
\[
\bar{x} = \frac{19 + 24 + 17 + 26 + 24 + 20 + 18}{7}
\]
\[\approx 21.14\]

\[|19 - 21.14| = 2.14\]
\[|24 - 21.14| = 2.86\]
\[|17 - 21.14| = 4.14\]
\[|26 - 21.14| = 4.86\]
\[|24 - 21.14| = 2.86\]
\[|20 - 21.14| = 1.14\]
\[|18 - 21.14| = 3.14\]

\[
\text{MAD} = \frac{2.14 + 2.86 + 4.14 + 4.86 + 2.86 + 1.14 + 3.14}{7}
\]
\[= 3.02\]

The closest value is F, 3.0.

25. It takes 3 hours for a boat to travel 27 miles upstream. The same boat travels 30 miles downstream in 2 hours. Find the speed of the boat.

A 3 mph
B 5 mph
C 12 mph
D 14 mph

**SOLUTION:**
Use distance equals rate times time to find the rates for upstream and downstream.

\[r \cdot t = d\]
\[r \cdot 3 = 27\]
\[r = 9\]
\[r \cdot 2 = 30\]
\[r = 15\]

The rate upstream is 9 mph and the rate downstream is 15 mph. The speed of the boat is the average of these rates because the rate upstream is the speed of the boat against the current while the rate downstream is the speed of the boat with the current.

\[r + c = 15\]
\[r - c = 9\]
\[2r = 24\]
\[r = 12\]
The boat travels at 12 mph.

The correct choice is C.

**Identify each sample as biased or unbiased. Explain your reasoning.**

26. SHOPPING Every tenth person walking into the mall is asked to name their favorite store.

**SOLUTION:**
The people are selected randomly and asked about their favorite stores while walking into the mall. This is an unbiased sample. If the people were asked the same question when walking into a particular store, the sample would be biased.
12-2 Statistics and Parameters

27. **MUSIC** Every fifth person at a rock concert is asked to name their favorite radio station.

**SOLUTION:**
The sample is taken at random, but the question is about their favorite radio station at a rock concert. The majority of the people at the concert already prefer rock music and would likely prefer a rock radio station. This is a biased sample.

**Simplify each expression.**

\[ \frac{x^2 - 8x + 15}{x^2 + 3x - 18} \]

28. \[ \frac{x^2 - 8x + 15}{x^2 + 3x - 18} \]

**SOLUTION:**

\[ \frac{x^2 - 8x + 15}{x^2 + 3x - 18} = \frac{(x - 3)(x - 5)}{(x - 3)(x + 6)} = \frac{x - 5}{x + 6} \]

29. \[ \frac{x^2 - x - 12}{x^2 - 6x + 8} \]

**SOLUTION:**

\[ \frac{x^2 - x - 12}{x^2 - 6x + 8} = \frac{(x - 4)(x + 3)}{(x - 4)(x - 2)} = \frac{x + 3}{x - 2} \]

30. \[ \frac{x^2 - x - 30}{x^2 - 4x - 12} \]

**SOLUTION:**

\[ \frac{x^2 - x - 30}{x^2 - 4x - 12} = \frac{(x - 6)(x + 5)}{(x - 6)(x + 2)} = \frac{x + 5}{x + 2} \]

31. **SOCCER** The number of members of the local soccer association has increased by 6% every year. As of the beginning of 2010, there were 880 members.

a. Write an equation for the number of members of the association \( t \) years after 2010.

b. If this trend continues, predict how many members the association will have in 2020.

**SOLUTION:**

**SOCcER** The number of members of the local soccer association has increased by 6% every year. As of the beginning of 2010, there were 880 members.

a. This is a geometric growth. If the number of members increases by 6%, then each successive year the number of members should be multiplied by a factor of 1.06. If the beginning number of members is 880, then the equation should be \( y = 880(1.06)^t \).

b. 2020 is 10 years after 2010. We should use \( t = 10 \) for the equation we found.

\[ y = 880(1.06)^{10} \]

\[ \approx 1576 \]

32. **GEOMETRY** If the side length of a cube is \( s \), the volume is represented by \( s^3 \), and the surface area is represented by \( 6s^2 \).

a. Are the expressions for volume and surface area monomials? Explain.

b. If the side of a cube measures 3 feet, find the volume and surface area.

c. Find a side length \( s \) such that the volume and surface area have the same measure.

d. The volume of a cylinder can be found by \( V = \)
12-2 Statistics and Parameters

\(\pi r^2 h\). Suppose you have two cylinders. Each dimension of the second is twice the measure of the first, so \(V = \pi (2r)^2 (2h)\). What is the ratio of the volume of the first to the second?

**SOLUTION:**
a. Yes they are monomials, because each is the product of variables and/or a real number.
b. 
   
   \[
   V = s^3 \\
   = 3^3 \\
   = 27 \text{ cu.ft.}
   \]
   
   \[
   A = 6s^2 \\
   = 6 \cdot 3^2 \\
   = 6 \cdot 9 \\
   = 54 \text{ sq.ft.}
   \]
c. \(s = 6 \text{ units}\)

   \[
   V = s^3 \\
   = 6^3 \\
   = 216 \text{ cu.ft.}
   \]

   \[
   A = 6s^2 \\
   = 6 \cdot 6^2 \\
   = 6 \cdot 36 \\
   = 216 \text{ sq.ft.}
   \]
d. 

   \[
   \frac{\pi r^2 h}{\pi (2r)^2 (2h)} = \frac{\pi r^2 h}{8 \pi r^2 h} \\
   = \left(\frac{\pi r^2 h}{\pi r^2 h}\right) \\
   = \frac{1}{8}
   \]

   The ratio is 1:8.

**Find the range, median, lower quartile, and upper quartile for each set of data.**

33. \{15, 23, 46, 36, 15, 19\}

**SOLUTION:**

\{15, 23, 46, 36, 15, 19\}

To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

15, 15, 19, 23, 36, 46

The range is the difference between the largest number and the smallest number: \(46 - 15 = 31\).

The median is the middle value, or the average between the two middle values: \(\frac{19 + 23}{2} = \frac{42}{2} = 21\).

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 15, 15, 19 which is 15.

The upper quartile is the middle value of 23, 36, 46 which is 36.
34. \{55, 57, 39, 72, 46, 53, 81\}

**SOLUTION:**
\{55, 57, 39, 72, 46, 53, 81\}

To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

39, 46, 53, 55, 72, 81

The range is the difference between the largest number and the smallest number: 81 – 39 = 42.

The median is the middle value, or the average between the two middle values: 55

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 39, 46, 53 which is 46.

The upper quartile is the middle value of 57, 72, 81 which is 72.

35. \{21, 25, 19, 18, 22, 16, 27\}

**SOLUTION:**
\{21, 25, 19, 18, 22, 16, 27\}

To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

16, 18, 19, 21, 22, 25, 27

The range is the difference between the largest number and the smallest number: 27 – 16 = 9.

The median is the middle value, or the average between the two middle values: 21

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 16, 18, 19 which is 18.

The upper quartile is the middle value of 22, 25, 27 which is 25.

36. \{52, 29, 72, 64, 33, 49, 51, 68\}

**SOLUTION:**
\{52, 29, 72, 64, 33, 49, 51, 68\}

To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

29, 33, 49, 51, 52, 64, 68, 72

The range is the difference between the largest number and the smallest number: 72 – 29 = 43.

The median is the middle value, or the average between the two middle values:

\[
\frac{51+52}{2} = \frac{103}{2} = 51.5
\]

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 33, 49 which is

\[
\frac{33+49}{2} = \frac{82}{2} = 41
\]

The upper quartile is the middle value of 64, 68 which is

\[
\frac{64+68}{2} = \frac{132}{2} = 66
\]
12-2 Statistics and Parameters

37. \{8, 12, 9, 11, 11, 10, 14, 18\}

**SOLUTION:**
\{8, 12, 9, 11, 11, 10, 14, 18\}
To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

8, 9, 10, 11, 11, 12, 14, 18

The range is the difference between the largest number and the smallest number: 18 – 8 = 10.

The median is the middle value, or the average between the two middle values: $\frac{11+11}{2} = 11$.

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 9, 10 which is $\frac{9+10}{2} = \frac{19}{2} = 9.5$.

The upper quartile is the middle value of 12, 14 which is $\frac{12+14}{2} = \frac{26}{2} = 13$.

38. \{133, 119, 147, 94, 141, 106, 118, 149\}

**SOLUTION:**
\{133, 119, 147, 94, 141, 106, 118, 149\}
To find the range, median, lower quartile, and upper quartile, first arrange the data in order.

94, 106, 118, 119, 133, 141, 147, 149

The range is the difference between the largest number and the smallest number: 149 – 94 = 55.

The median is the middle value, or the average between the two middle values: $\frac{119+133}{2} = \frac{252}{2} = 126$.

The lower and upper quartiles are found similarly to the median except they should divide the lower 25% and upper 75% of values.

The lower quartile is the middle of 106, 118 which is $\frac{106+118}{2} = \frac{224}{2} = 112$.

The upper quartile is the middle value of 141, 147 which is $\frac{141+147}{2} = \frac{288}{2} = 144$. 
Use a graphing calculator to construct a histogram and a box-and-whisker plot for the data. Then describe the shape of the distribution.

1. 80, 84, 68, 64, 57, 88, 61, 72, 76, 80, 83, 77, 78, 82, 65, 70, 83, 78, 73, 79, 70, 62, 69, 66, 79, 80, 86, 82, 73, 75, 71, 81, 74, 83, 77, 73

SOLUTION:
First enter the data into L₁ on your graphing calculator. Then hit 2nd, STAT PLOT, hit ENTER on Plot 1, and change the type to histogram. Make sure that the XList is on L₁.

Next, hit WINDOW, change Xmin to the lowest value, 56, and change Xmax to the highest value, 92. Change the scale to 4, and set the y values from 0 to 10 with a scale of 1.

Play with different scales for x to see how this affects the graph. Next hit GRAPH.

To get a box and whisker plot, hit 2nd, STAT PLOT, ENTER on Plot 1, then change the graph to a histogram.

Keep the same window, and hit GRAPH.

This is a negatively skewed data set.

2. 30, 24, 35, 84, 60, 42, 29, 16, 68, 47, 22, 74, 34, 21, 48, 91, 66, 51
33, 29, 18, 31, 54, 75, 23, 45, 25, 32, 57, 40, 23, 32, 47, 67, 62, 23

SOLUTION:
First enter the data into L₁ on your graphing calculator. Then hit 2nd, STAT PLOT, hit ENTER on Plot 1, and change the type to histogram. Make sure that the XList is on L₁.

Next, hit WINDOW, change Xmin to the lowest value, 10, and change Xmax to the highest value, 100. Change the scale to 10, and set the y values from 0 to 10 with a scale of 1.
Play with different scales for $x$ to see how this affects the graph. Next hit GRAPH.

To get a box and whisker plot, hit 2nd, STAT PLOT, ENTER on Plot 1, then change the graph to a histogram.

![Histogram and Box-Whisker Plot]

Keep the same window, and hit GRAPH.

![Box-Whisker Plot]

This is a positively skewed data set.

Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram for the data.

3. 58, 66, 52, 75, 60, 56, 78, 63, 59, 54, 60, 67, 72, 80, 68, 88, 55, 60
   59, 61, 82, 70, 67, 60, 58, 86, 74, 61, 92, 76, 58, 62, 66, 74, 69, 64

**SOLUTION:**

Construct a histogram for the given data.

![Histogram]

The data is positively skewed, so a five number summary would work best.

```
1-Var Stats
\[ n=36 \]
\[ \text{min}=52 \]
\[ Q_1=59.5 \]
\[ \text{Median}=65 \]
\[ Q_3=74 \]
\[ \text{max}=92 \]
```
12-3 Distributions of Data

4. PRESENTATIONS The length of the students’ presentations in Ms. Monroe’s 2nd period class are shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

![Presentations](image)

**SOLUTION:**
Use a graphing calculator to plot a box-and-whisker plot for the data.

![Box and Whisker Plot](image)

We can see that the data is negatively skewed, so a five-number summary would work best.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=20</td>
</tr>
<tr>
<td>minX=6</td>
</tr>
<tr>
<td>Q1=11</td>
</tr>
<tr>
<td>Med=17</td>
</tr>
<tr>
<td>Q3=18.5</td>
</tr>
<tr>
<td>MaxX=21</td>
</tr>
</tbody>
</table>

Use a graphing calculator to construct a histogram and a box-and-whisker plot for the data. Then describe the shape of the distribution.

5. 55, 65, 70, 73, 25, 36, 33, 47, 52, 54, 55, 60, 45, 39, 48, 55, 46, 38
50, 54, 63, 31, 49, 54, 68, 35, 27, 45, 53, 62, 47, 41, 50, 76, 67, 49

**SOLUTION:**
First enter the data into L₁ on your graphing calculator. Then hit 2nd, STAT PLOT, hit ENTER

![Histogram](image)

Keep the same window, and hit GRAPH.
12-3 Distributions of Data

This is a symmetric data set.


**SOLUTION:**
First enter the data into L₁ on your graphing calculator. Then hit 2nd, STAT PLOT, hit ENTER on Plot 1, and change the type to histogram. Make sure that the XList is on L₁.

Next, hit WINDOW, change the values to appropriately fit the graph.

Play with different scales for x to see how this affects the graph. Next hit GRAPH.

To get a box and whisker plot, hit 2nd, STAT PLOT, ENTER on Plot 1, then change the graph to a histogram.

Keep the same window, and hit GRAPH.

This is a negatively skewed data set.
12-3 Distributions of Data

Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram for the data.

7. 32, 44, 50, 49, 21, 12, 27, 41, 48, 30, 50, 23, 37, 16, 49, 53, 33, 25, 35, 40, 48, 39, 50, 24, 15, 29, 37, 50, 36, 43, 49, 44, 46, 27, 42, 47

SOLUTION:
Put the data into L₁ on your calculator, choose a suitable window and create a histogram:

\[
\text{Histogram}
\]

As we can see the data is negatively skewed so a five-number summary would be best to describe the data. Click on STAT, CALC, 1-Var Stats, then hit enter and scroll down.

\[
\begin{array}{c}
\text{1-Var Stats} \\
\text{n}=36 \\
\text{min}=12 \\
\text{Q₁}=28 \\
\text{Med}=39.5 \\
\text{Q₃}=48 \\
\text{Max}=53 \\
\end{array}
\]

The mean is 82 and the standard deviation is about 7.4.
9. **WEATHER** The daily low temperatures for New Carlisle over a 30-day period are shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

<table>
<thead>
<tr>
<th>Temperature [°F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Enter the data into L₁, choose a window between the maximum and minimum values (42 to 62) and create a box-and-whisker plot.

The data is symmetric, so the mean and standard deviation work well in describing the data. Use a calculator to calculate the 1-variable statistics:

1-Var Stats
---
\[ \bar{x} = 52.8 \]
\[ s = 84170 \]
\[ \bar{x} = 4.29434271 \]
\[ s = 4.22163742 \]
\[ n = 30 \]

The mean is 52.8 with a standard deviation of 4.2.

10. **TRACK** While training for the 100-meter dash, Sarah pulled a muscle in her lower back. After being cleared for practice, she continued to train. Sarah’s 100-meter dash times are shown.

<table>
<thead>
<tr>
<th>100-meter dash (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.20</td>
</tr>
<tr>
<td>12.18</td>
</tr>
<tr>
<td>12.87</td>
</tr>
<tr>
<td>12.30</td>
</tr>
<tr>
<td>12.50</td>
</tr>
<tr>
<td>12.46</td>
</tr>
</tbody>
</table>

**a.** Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.

**b.** Sarah’s slowest time prior to pulling a muscle was 12.50 seconds. Use a graphing calculator to create a box-and-whisker plot that does not include the times that she ran after pulling the muscle. Then describe the center and spread of the new data set.

**c.** What effect does removing the times recorded after Sarah pulled a muscle have on the shape of the distribution and on how you should describe the center and spread?

**SOLUTION:**
To create a box-and-whisker plot, choose a suitable window range (11.9 to 13.7). Then create a box-and-whisker plot for the data.

The distribution is positively skewed so a five-number summary works best:

1-Var Stats
---
\[ n = 30 \]
\[ \text{min} = 11.96 \]
\[ Q₁ = 12.18 \]
\[ \text{Med} = 12.34 \]
\[ Q₃ = 12.93 \]
\[ \text{max} = 13.6 \]

The range is $13.6 - 11.9 = 1.7$. The median is 12.34, with the first quartile at 12.18 and the third quartile at 12.93.

**b.** Creating a box and whisker plot without values greater than 12.5 gives:
12-3 Distributions of Data

Without these values the data is symmetric and the mean and standard deviation would work well to describe the data.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} = 12.221 )</td>
</tr>
<tr>
<td>( Sx = 244.42 )</td>
</tr>
<tr>
<td>( 2\bar{x} = 2908.5348 )</td>
</tr>
<tr>
<td>( 5\bar{x} = 15860808.58 )</td>
</tr>
<tr>
<td>( \sigma = 1545930141 )</td>
</tr>
<tr>
<td>( n=20 )</td>
</tr>
</tbody>
</table>

The mean is 12.2 and the standard deviation is 0.15.

c. Removing the times causes the shape of the distribution to go from being skewed to being symmetric. Therefore, the center and spread should be described using the mean and standard deviation.

11. **MENU** The prices for entrees at a restaurant are shown.

a. Use a graphing calculator to create a box-and-whisker plot. Describe the center and spread of the data.

b. The owner of the restaurant decides to eliminate all entrees that cost more than $15. Use a graphing calculator to create a box-and-whisker plot that reflects this change. Then describe the center and spread of the new data set.

<table>
<thead>
<tr>
<th>Entree Prices ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.00</td>
</tr>
<tr>
<td>18.50</td>
</tr>
<tr>
<td>8.00</td>
</tr>
<tr>
<td>13.00</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**MENU** The prices for entrees at a restaurant are shown.
Then delete all of the values greater than $15.00, change the window range, and check the box-and-whisker plot.

The new data is symmetric so the mean and standard deviation work best. Calculate the 1-variable statistics.

The mean is $10.67 and the standard deviation is $1.84.

**CHALLENGE** Identify the box-and-whisker plot that corresponds to each of the following histograms.

**SOLUTION:**
We need to match this histogram with the correct box and whisker plot. This histogram has a peak on the left side of the data, so the median is less than the mean. This is a negatively skewed distribution and corresponds with histogram iii.

**SOLUTION:**
We need to match this histogram with the correct box and whisker plot. This histogram has a peak on the right side of the data, so the median is greater than the mean. This is a positively skewed distribution and corresponds with histogram i.

**SOLUTION:**
We need to match this histogram with the correct box and whisker plot. This histogram has a peak in the middle of the data, so the median is about equal to the mean. This is a symmetric distribution and corresponds with histogram ii.
15. **CCSS ARGUMENTS** Research and write a definition for a bimodal distribution. How can the measures of center and spread of a bimodal distribution be described?

**SOLUTION:**
Data with bimodal distribution should look something like the following:

![Bimodal Distribution](image1)

There are two clusters of data with two peaks. The distribution can be described by summarizing the center and spread of each cluster of data.

16. **OPEN ENDED** Give an example of a set of real-world data with a distribution that is symmetric and one with a distribution that is not symmetric.

**SOLUTION:**
A distribution that is symmetric is likely to be something that is consistent and reoccurring, like the average temperature in a city during the course of the year. This should follow a fairly consistent pattern year after year.

A real world distribution that is skewed could be something like the attendance at a baseball stadium over the course of the season. If a team is doing well then attendance will go up as the season progresses, but if they lose a big game close to the end of the season, then attendance can drop suddenly.

17. **WRITING IN MATH** Explain why the mean and standard deviation are used to describe the center and spread of a symmetrical distribution and the five-number summary is used to describe the center and spread of a skewed distribution.

**SOLUTION:**

![Symmetrical Distribution](image2)

Compare the graphs of the different spreads. In a symmetric distribution the mean and most of the data are located near the center. The standard deviation provides a distance above and below the mean for which most of the data are located. This wouldn't make sense in a skewed graph since within a standard deviation above the mean there can much more data than within standard deviation below the mean. When the data is skewed, the median is less affected than the mean by things such as outliers. A five-number summary works better with skewed data.
12-3 Distributions of Data

18. At the county fair, 1000 tickets were sold. Adult tickets cost $8.50, children’s tickets cost $4.50, and a total of $7300 was collected. How many children’s tickets were sold?

   A 700
   B 600
   C 400
   D 300

   SOLUTION:
   Let \( a \) be the number of adult tickets sold and let \( c \) be number of child’s tickets sold.
   \[ a + c = 1000 \]
   \[ 8.50a + 4.50c = 7300 \]

   Use substitution. Solve the first equation for \( a \).
   \[ a = 1000 - c \]

   Substitute into the second equation.
   \[ 8.50(1000 - c) + 4.50c = 7300 \]
   \[ 8500 - 8.5c + 4.5c = 7300 \]
   \[ 8500 - 4c = 7300 \]
   \[ -4c = -1200 \]
   \[ c = 300 \]

   There were 300 child’s tickets sold. The correct choice is D.

19. Edward has 20 dimes and nickels, which together total $1.40. How many nickels does he have?

   F 12
   G 10
   H 8
   J 6

   SOLUTION:
   Let \( d \) be the number of dimes and let \( n \) be number of nickels.
   \[ d + n = 20 \]
   \[ .10d + .05n = 1.40 \]

   Use substitution. Solve the first equation for \( d \).
   \[ d = 20 - n \]

   Substitute into the second equation.
   \[ .10(20 - n) + .05n = 1.40 \]
   \[ 2 - 0.10n + 0.05n = 1.40 \]
   \[ 2 - 0.5n = 1.40 \]
   \[ -0.5n = -0.60 \]
   \[ n = 12 \]

20. If 4.5 kilometers is about 2.8 miles, about how many miles is 6.1 kilometers?

   A 3.2 miles
   B 3.6 miles
   C 3.8 miles
   D 4.0 miles

   SOLUTION:
   \[
   \frac{4.5 \text{ kilometers}}{2.8 \text{ miles}} = \frac{6.1 \text{ kilometers}}{x \text{ miles}}
   \]
   \[ 4.5 \cdot x = 6.1 \cdot 2.8 \]
   \[ 4.5x = 17.08 \]
   \[ x \approx 3.8 \]

   So, 6.1 kilometers is about 3.8 miles. The correct choice is C.
21. **EXTENDED RESPONSE** Three times the width of a certain rectangle exceeds twice its length by three inches, and four times its length is twelve more than its perimeter.
   a. Translate the sentences into equations.
   b. Find the dimensions of the rectangle.
   c. What is the area of the rectangle?

**SOLUTION:**
   a. Three times the width of the rectangle is 3 more than twice its length, so \(3w = 2l + 3\). Four times the length of the rectangle is 12 more than its perimeter, so \(4l = 12 + P\).

   The perimeter of the rectangle is \(P = 2l + 2w\).

   \[
   \begin{align*}
   4l &= 12 + P \\
   4l &= 12 + 2l + 2w \\
   2l &= 12 + 2w \\
   l &= 6 + w \\
   \\
   b. \text{Substitute} \ 12 + 2w \text{ for} \ l \text{ into the equation} \ 3w = 2l + 3. \\
   3w &= 2l + 3 \\
   3w &= 2(6 + w) + 3 \\
   3w &= 12 + 2w + 3 \\
   3w &= 2w + 15 \\
   w &= 15 \\
   \\
   \text{The width is 15 inches. Substitute this into the equation} \ 2l = 12 + 2w \text{ to find the length.} \\
   2l &= 12 + 2w \\
   2l &= 12 + 2(15) \\
   2l &= 12 + 30 \\
   2l &= 42 \\
   l &= 21 \\
   \text{The length of the rectangle is 21 inches.} \\
   \\
   c. \text{The area of the rectangle is} \ 15 \times 21 = 315 \text{ square inches.}
   \]

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

22. **AMUSEMENT PARK** A systematic sample of 250 guests is asked how much money they spent on concessions inside the park. The median amount of money is calculated.

**SOLUTION:**
The sample is the 250 guests that were selected, and the population would be every guest in the park. The statistic parameter described in the problem is the median amount of money that the 250 guests spend in the park. This is representative of the population parameter, the median amount of money spent by everyone in the park.

23. **PROM** A random sample of 100 high school seniors at North Boyton High School is surveyed, and the mean amount of money spent on prom by a senior is calculated.

**SOLUTION:**
The sample is the 100 high school seniors that were surveyed, and the population is the entire senior student body. The sample statistic is the mean amount of money spent on prom spent by the 100 students, which is representative of the population statistic - the mean amount of money spent on prom by all of the seniors.

Identify each survey question as biased or unbiased. If biased, explain your reasoning.

24. What do you like the most about reality television shows, and which one is your favorite?

**SOLUTION:**
This question assumes that the person being surveyed likes reality television shows. It asks, "What do you like best...?" rather than "Do you like reality television shows?" It then asks which is their favorite, still assuming that they have one. This is a biased question.

25. Are you planning on seeing the school play?

**SOLUTION:**
This is a simple yes or no question that doesn't lead toward a specific answer. It is unbiased.
12-3 Distributions of Data

26. Don’t you agree that the school should renovate the library?

SOLUTION:
This question begins with the phrase "Don't you agree..." which leads the person being surveyed to agree with any opinion that is asked. This is a biased question.

27. GARDENING Trey planted a triangular garden. Write an expression for the perimeter of the triangle.

\[
\frac{2a+6}{a+1} + \frac{3a+1}{a+1} + \frac{6a}{a+1} = \frac{12a+7}{a+1}
\]

SOLUTION:
The perimeter is the sum of the lengths. Add each expression and simplify.

\[
\frac{2a+6}{a+1} + \frac{3a+1}{a+1} + \frac{6a}{a+1} = \frac{12a+7}{a+1}
\]

Find the inverse of each function.

28. \(f(x) = 2x - 14\)

SOLUTION:
First replace \(f(x)\) with \(y\), then interchange \(y\) and \(x\), and solve for \(y\). Replace \(y\) with \(f^{-1}(x)\) at the end.

\[
f(x) = 2x - 14
\]

\[
f(x) = 2x - 14
\]

\[
y = 2x - 14
\]

\[
x = 2y - 14
\]

\[
x + 14 = 2y
\]

\[
\frac{x + 14}{2} = y
\]

\[
\frac{1}{2}x + 7 = y
\]

\[
\frac{1}{2}x + 7 = f^{-1}(x)
\]

29. \(f(x) = 17 - 5x\)

SOLUTION:
\[
f(x) = 17 - 5x
\]

\[
y = 17 - 5x
\]

\[
x = 17 - 5y
\]

\[
x - 17 = -5y
\]

\[
\frac{x - 17}{-5} = y
\]

\[
-\frac{1}{5}x + \frac{17}{5} = y
\]

\[
-\frac{1}{5}x + \frac{17}{5} = f^{-1}(x)
\]

30. \(f(x) = \frac{1}{4}x + 3\)

SOLUTION:
\[
f(x) = \frac{1}{4}x + 3
\]

\[
y = \frac{1}{4}x + 3
\]

\[
x = \frac{1}{4}y + 3
\]

\[
x - 3 = \frac{1}{4}y
\]

\[
4(x - 3) = y
\]

\[
4x - 12 = y
\]

\[
4x - 12 = f^{-1}(x)
\]
12-3 Distributions of Data

31. \( f(x) = -\frac{1}{7}x - 1 \)

**SOLUTION:**

\[
\begin{align*}
f(x) &= -\frac{1}{7}x - 1 \\
y &= -\frac{1}{7}x - 1 \\
x &= -\frac{1}{7}y - 1 \\
x + 1 &= -\frac{1}{7}y \\
-7(x + 1) &= y \\
-7x - 7 &= y \\
-7x - 7 &= f^{-1}(x)
\end{align*}
\]

32. \( f(x) = \frac{2}{3}x + 6 \)

**SOLUTION:**

\[
\begin{align*}
f(x) &= \frac{2}{3}x + 6 \\
y &= \frac{2}{3}x + 6 \\
x &= \frac{2}{3}y + 6 \\
x - 6 &= \frac{2}{3}y \\
\frac{3}{2}(x - 6) &= y \\
\frac{3}{2}x - 9 &= y \\
\frac{3}{2}x - 9 &= f^{-1}(x)
\end{align*}
\]

33. \( f(x) = 12 - \frac{3}{5}x \)

**SOLUTION:**

\[
\begin{align*}
f(x) &= 12 - \frac{3}{5}x \\
y &= 12 - \frac{3}{5}x \\
x &= 12 - \frac{3}{5}y \\
x - 12 &= -\frac{3}{5}y \\
-\frac{5}{3}(x - 12) &= y \\
-\frac{5}{3}x + 20 &= y \\
-\frac{5}{3}x + 20 &= f^{-1}(x)
\end{align*}
\]

A bowl contains 3 red chips, 6 green chips, 5 yellow chips, and 8 orange chips. A chip is drawn randomly. Find each probability.

34. red

**SOLUTION:**

\[
\frac{\text{number of red}}{\text{total}} = \frac{3}{22}
\]

35. orange

**SOLUTION:**

\[
\frac{\text{number of orange}}{\text{total}} = \frac{8}{22} = \frac{4}{11}
\]

36. yellow or green

**SOLUTION:**

\[
\frac{\text{number of yellow+green}}{\text{total}} = \frac{5+6}{22} = \frac{11}{22} = \frac{1}{2}
\]

37. not orange

**SOLUTION:**

\[
\frac{\text{total- orange}}{\text{total}} = \frac{22-8}{22} = \frac{14}{22} = \frac{7}{11}
\]

38. not green

**SOLUTION:**

\[
\frac{\text{total- green}}{\text{total}} = \frac{22-6}{22} = \frac{16}{22} = \frac{8}{11}
\]
39. red or orange

\[
\text{SOLUTION:} \quad \frac{\text{red + orange}}{\text{total}} = \frac{3 + 8}{22} = \frac{11}{22} = \frac{1}{2}
\]
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

1. 10, 13, 9, 8, 15, 8, 13, 12, 7, 8, 11, 12; + (−7)

**SOLUTION:**
First enter the data into L₁. Then press 2nd, L₁ − 7, S and hit ENTER.

Now L₂ is the list of data modified by subtracting 7 from each value. Next calculate the 1-variable statistics for L₂.

The mean is 3.5. The median is 3.5. The range is 8 – standard deviation is 2.4, and the mode must be found determining which number occurs with the highest frequency. The simplest way to do this is to first sort the data in L₂, then number of occurrences for each piece of data.

As we can see, the number that occurs most frequently is 1, which occurs three times.
## 12-4 Comparing Sets of Data

2. 38, 36, 37, 42, 31, 44, 37, 45, 29, 42, 30, 42; + 23

**SOLUTION:**
First enter the data into L₁. Then press 2nd, L₁ + 23, and hit ENTER.

<table>
<thead>
<tr>
<th>L₁ + 23 → L₂</th>
<th>&lt;161 59 60 65 54...</th>
</tr>
</thead>
</table>

Now L₂ is the list of data modified by adding 23 to each number.

#### Calculate the 1-variable statistics for L₂.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄ = 60.75</td>
<td>x̄ = 60.75</td>
</tr>
<tr>
<td>minX = 729</td>
<td>minX = 729</td>
</tr>
<tr>
<td>maxX = 44619</td>
<td>maxX = 44619</td>
</tr>
<tr>
<td>Sx = 5.495866215</td>
<td>Sx = 5.495866215</td>
</tr>
<tr>
<td>Sx = 5.261891295</td>
<td>Sx = 5.261891295</td>
</tr>
<tr>
<td>n = 12</td>
<td>n = 12</td>
</tr>
</tbody>
</table>

The mean is 60.8. The median is 60.5. The range is 642.

The standard deviation is 5.5, and the mode must be found by determining which number occurs with the highest frequency. The simplest way to do this is to first sort the data in L₂, then calculate the 1-variable statistics for each piece of data.

<table>
<thead>
<tr>
<th>SortA(L₂)</th>
<th>Done</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>L₁</th>
<th>L₂</th>
<th>L₃</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>60</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>46</td>
<td>61</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>48</td>
<td>64</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>52</td>
<td>72</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>58</td>
<td>84</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>96</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>66</td>
<td>15</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>72</td>
<td>18</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>76</td>
<td>22</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>80</td>
<td>27</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>34</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>37</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>L₂(12) = 68</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As we can see, the number that occurs most frequently is 65, which occurs three times.

---

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

3. 6, 10, 3, 7, 4, 9, 3, 8, 5, 11, 2; × 3

**SOLUTION:**
First enter the data into L₁. Then press 2nd, L₁ × 3 , STO, 2nd, L₂, and hit ENTER.

<table>
<thead>
<tr>
<th>L₁ × 3 → L₂</th>
<th>&lt;18 30 9 21 12...</th>
</tr>
</thead>
</table>

Now L₂ is the list of data modified by multiplying each number by 3. Next calculate the 1-variable statistics for L₂.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄ = 17.25</td>
<td>x̄ = 17.25</td>
</tr>
<tr>
<td>minX = 20.7</td>
<td>minX = 20.7</td>
</tr>
<tr>
<td>maxX = 4635</td>
<td>maxX = 4635</td>
</tr>
<tr>
<td>Sx = 9.836157786</td>
<td>Sx = 9.836157786</td>
</tr>
<tr>
<td>Sx = 9.417404101</td>
<td>Sx = 9.417404101</td>
</tr>
<tr>
<td>n = 12</td>
<td>n = 12</td>
</tr>
</tbody>
</table>

The mean is 17.3, the median is 16.5, the mode is the most frequently occurring value which is 9, the range is 33 – 3 = 30, and the standard deviation is 9.4.
4. 42, 39, 45, 44, 37, 42, 38, 37, 41, 49, 42, 36; \times 0.5

**SOLUTION:**
First enter the data into L_1. Then press 2nd, L_1 \times 0.5, STO, 2nd, L_2, and hit ENTER.

Now L_2 is the list of data modified by multiplying each value by 3. Next calculate the 1-variable statistics for L_2.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 20.5 )</td>
</tr>
<tr>
<td>( \bar{x} = 246 )</td>
</tr>
<tr>
<td>( s = 5083.5 )</td>
</tr>
<tr>
<td>( s_x = 1.91806447 )</td>
</tr>
<tr>
<td>( s_x = 1.837117307 )</td>
</tr>
<tr>
<td>( n = 12 )</td>
</tr>
</tbody>
</table>

The mean is 20.5, the median is 20.8, the mode is the most frequently occurring value which is 21, the range is \( 24.5 - 18 = 6.5 \), and the standard deviation is 1.8.

5. **TRACK** Mark and Kyle’s long jump distances are shown.

<table>
<thead>
<tr>
<th>Kyle’s Distances (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.2, 18.28, 19.56, 17.28, 17.36, 18.06, 17.43, 17.71, 17.46, 18.25, 17.51, 17.58, 17.41, 18.21, 17.34, 17.63, 17.55, 17.26, 17.18, 17.78, 17.51, 17.83, 17.92, 18.04, 17.91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mark’s Distances (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.88, 19.24, 17.63, 18.59, 17.74, 19.18, 17.92, 18.96, 18.19, 18.21, 18.46, 17.47, 18.49, 17.86, 18.93, 18.73, 18.34, 18.57, 18.56, 18.79, 18.47, 18.34, 18.87, 17.94, 13.7</td>
</tr>
</tbody>
</table>

**a.** Use a graphing calculator to construct a histogram for each set of data. Then describe the shape of each distribution.

**b.** Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

**SOLUTION:**

**a.** Enter the data for Kyle into L_1, and the data for Mark into L_2. The histogram for Kyle, with a range from 17 to 19.25 and a scale of 0.25 looks like the following:

![Histogram for Kyle](image)

The histogram for Mark, with the same scale looks like:

![Histogram for Mark](image)

Both distributions are skewed. Kyle has a positively skewed distribution and Mark has a negatively skewed distribution.

**b.** Since both sets of data are skewed, we should compare the five-number summaries for each. For Kyle and Mark respectively:
Kyle’s upper quartile is 17.98, while Mark’s lower quartile is 18.065. This means that 75% of Mark’s distances are greater than 75% of Kyle’s distances. Therefore, we can conclude that overall, Mark’s distances are higher than Kyle’s.

6. **TIPS** Miguel and Stephanie are servers at a restaurant. The tips that they earned to the nearest dollar over the past 15 workdays are shown.

<table>
<thead>
<tr>
<th>Miguel’s Tips ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14, 68, 52, 21, 53, 32, 43, 35, 70, 37, 42, 16, 47, 38, 48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stephanie’s Tips ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34, 52, 43, 39, 41, 50, 46, 36, 37, 47, 39, 49, 44, 36, 50</td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

**SOLUTION:**

a. Put the data for Miguel's tips in L1, and the data from Stephanie's tips in L2. Make sure both stat plots are turned on and set to box-and-whisker plots. Choose an appropriate window and graph.

The mean for Miguel's tips is about $41.73 with a standard deviation of about $16.64. The mean for Stephanie's tips is about $42.87 with a standard deviation of about $5.73. On average, they both make about the same amount in tips however Miguel's tips are more varied - some tips he makes a lot more, some tips a lot less.
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

7. 52, 53, 49, 61, 57, 52, 48, 60, 50, 47; + 8

**SOLUTION:**
Place the data into L₁. Then add 8 to each value in L₁ and store this in L₂.

```
L₁+8→L₂
<60 61 57 69 65...
```

Now calculate the statistics for L₂.

```
1-Var Stats
n=10
minX=55
Q₁=57
med=60
Q₃=65
maxX=69
```

The mean is 60.9. The median is 60. The mode is the most frequently occurring value which is 60. The range is 69 – 55 = 14, and the standard deviation is 4.7.

8. 101, 99, 97, 88, 92, 100, 97, 89, 94, 90; + (−13)

**SOLUTION:**
Place the data into L₁. Then subtract 13 from each value and store this in L₂.

```
L₁-13→L₂
<88 86 84 75 79...
```

Now calculate the statistics for L₂.

```
1-Var Stats
n=10
minX=75
Q₁=77
med=82.5
Q₃=86
maxX=88
```

The mean is 81.7. The median is 82.5. The mode is the most frequently occurring value which is 84. The range is 88 – 75 = 13, and the standard deviation is 4.5.
12-4 Comparing Sets of Data

9. 27, 21, 34, 42, 20, 19, 18, 26, 25, 33; + (−4)

**SOLUTION:**
Place the data into L₁. Then subtract 4 from each value in L₁ and store this in L₂.

```
L₁-4→L₂
{23 17 30 38 16...}
```

Now calculate the statistics for L₂.

```
1-Var Stats
x=22.5
Σx=225
Σx²=5605
5x=7.763876466
σx=7.365459931
\downarrow n=10
```

```
1-Var Stats
\downarrow n=10
\text{min} x=14
Q₁=16
\text{Med}=21.5
Q₃=29
\text{max} x=38
```

The mean is 22.5. The median is 21.5. There is no mode since all the values occur with the same frequency. The range is 38 – 14 = 24, and the standard deviation is 7.4.

10. 72, 56, 71, 63, 68, 59, 77, 74, 76, 66; + 16

**SOLUTION:**
Place the data into L₁. Then subtract 4 from each value in L₁ and store this in L₂.

```
L₁+16→L₂
{88 72 87 79 84...}
```

Now calculate the statistics for L₂.

```
1-Var Stats
x=84.2
Σx=842
Σx²=71356
5x=7.146094504
σx=6.779380503
\downarrow n=10
```

```
1-Var Stats
\downarrow n=10
\text{min} x=72
Q₁=79
\text{Med}=85.5
Q₃=90
\text{max} x=93
```

The mean is 84.2. The median is 85.5. There is no mode since all the values occur with the same frequency. The range is 87 – 72 = 15, and the standard deviation is 6.8.
12-4 Comparing Sets of Data

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

11. 11, 7, 3, 13, 16, 8, 3, 11, 17, 3; × 4

**SOLUTION:**
Place the data into L₁. Then multiply each value in L₁ by 4 and store this in L₂.

```
L₁*4→L₂
<44 28 12 52 64...>
```

Now calculate the statistics for L₂.

```
1-Var Stats
\[ \bar{x} = 36.8, \quad \sum x = 368, \quad \sum x^2 = 17536, \quad S_x = 21.06, \quad S_x^2 = 19.93, \quad n = 10 \]
```

```
1-Var Stats
\[ \bar{x} = 12, \quad Q_1 = 12, \quad Med = 38, \quad Q_3 = 52, \quad Max = 68 \]
```

The mean is 36.8. The median is 38. The mode is the most frequently occurring value which is 12. The range is 68 – 12 = 56, and the standard deviation is 20.0.

12. 64, 42, 58, 40, 61, 67, 58, 52, 51, 49; × 0.2

**SOLUTION:**
Place the data into L₁. Then multiply each value in L₁ by 0.2 and store this in L₂.

```
L₁*0.2→L₂
<12.8 8.4 11.6 ...>
```

Now calculate the statistics for L₂.

```
1-Var Stats
\[ \bar{x} = 10.8, \quad \sum x = 108.4, \quad \sum x^2 = 1204.16, \quad S_x = 1.79, \quad S_x^2 = 1.70, \quad n = 10 \]
```

```
1-Var Stats
\[ \bar{x} = 8, \quad Q_1 = 9.8, \quad Med = 11, \quad Q_3 = 12.2, \quad Max = 13.4 \]
```

The mean is 10.8. The median is 11. The mode is the most frequently occurring value which is 11.6. The range is 13.4 – 8 = 5.4, and the standard deviation is 1.7.
13. 33, 37, 38, 29, 35, 37, 27, 40, 28, 31; \times 0.8

**SOLUTION:**
Place the data into L₁. Then multiply each value in L₁ by 0.2 and store this in L₂.

\[ L₁ \times 0.8 \times L₂ \\
\{26.4, 29.6, 30.4\} \]

Now calculate the statistics for L₂.

**1-Var Stats**
\[ \bar{x} = 26.8 \\
\Sigma x = 268 \\
\Sigma x^2 = 7303.04 \\
\Sigma x^3 = 3.661208858 \\
\sigma x = 3.473326935 \\
\downarrow n = 10 \]

**1-Var Stats**
\[ \uparrow n = 10 \]
\[ \text{min} x = 21.6 \\
\text{Q1} = 23.2 \\
\text{Med} = 27.2 \\
\text{Q3} = 29.6 \\
\text{max} x = 32 \]

The mean is 26.8. The median is 27.2. The mode is the most frequently occurring value which is 29.6. The range is 32 – 21.6 = 10.4, and the standard deviation is 3.5.

---

14. 1, 5, 4, 2, 1, 3, 6, 2, 5, 1; \times 6.5

**SOLUTION:**
Place the data into L₁. Then multiply each value in L₁ by 0.2 and store this in L₂.

\[ L₁ \times 6.5 \times L₂ \\
\{6.5, 32.5, 26, 13\} \]

Now calculate the statistics for L₂.

**1-Var Stats**
\[ \bar{x} = 19.5 \\
\Sigma x = 195 \\
\Sigma x^2 = 5154.5 \\
\Sigma x^3 = 12.25651754 \\
\sigma x = 11.62755348 \\
\downarrow n = 10 \]

**1-Var Stats**
\[ \uparrow n = 10 \]
\[ \text{min} x = 6.5 \\
\text{Q1} = 6.5 \\
\text{Med} = 16.25 \\
\text{Q3} = 32.5 \\
\text{max} x = 39 \]

The mean is 19.5. The median is 16.25. The mode is the most frequently occurring value which is 6.5. The range is 39 – 6.5 = 32.5, and the standard deviation is 11.6.

---

15. **BOOKS** The page counts for the books that the student shown.

**1st Period**
388, 439, 206, 436, 413, 253, 311, 427, 258, 511, 283, 578, 291, 358, 297, 303, 325, 506, 531, 482, 343, 372, 456, 267, 484, 227

**5th Period**

**a.** Use a graphing calculator to construct a histogram
12-4 Comparing Sets of Data

data. Then describe the shape of each distribution.
b. Compare the data sets using either the means and deviations or the five-number summaries. Justify you
SOLUTION:
a. Enter the data for first period into L_1 and the data into L_2. Choose a window range from 200 to 600 wit and create histograms for each set of data. For simpli easiest to view each graph individually.

For first period:

![Histogram for first period]

For sixth period:

![Histogram for sixth period]

The data for first period is positively skewed, and the period is symmetric.

b. One distribution is symmetric and the other is skew number summary works best when comparing the da

For The Electronics Superstore:

![Histogram for The Electronics Superstore]

For Game Central:

![Histogram for Game Central]

The data for The Electronics Superstore is symmetri for Game Central is negatively skewed.

Above shows the statistics for first and sixth period respectively. The lower quartile for 1st period is 291 the minimum for 6th period is 294 pages. This means 25% of data for 1st period is lower than any data fro The range for 1st period is 578 – 206 or 372 pages. T 6th period is 506 – 294 or 212 pages. The median for about 351 pages, while the median for 6th period is 3 means that, while the median for 6th period is greater

pages have a greater range and include greater value period.

16. TELEVISIONS The prices for a sample of televisio

<table>
<thead>
<tr>
<th>The Electronics Superstore</th>
</tr>
</thead>
<tbody>
<tr>
<td>46, 25, 62, 45, 30, 43, 40, 46, 33, 53, 35, 38, 39, 40, 52, 42, 44, 48, 50, 35, 32, 55, 28, 58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Game Central</th>
</tr>
</thead>
</table>

a. Use a graphing calculator to construct a histogram data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and deviations or the five-number summaries. Justify you
SOLUTION:
a. Enter the data for The Electronics Superstore into data for Game Central into L_2. Choose a window ran 65 with a scale of 5 and create histograms for each s simplicity, it's easiest to view each graph individually.

For The Electronics Superstore:

![Histogram for The Electronics Superstore]

For Game Central:

![Histogram for Game Central]

b. One distribution is symmetric and the other is skew number summary works best when comparing the da
12-4 Comparing Sets of Data

Above shows the statistics for The Electronics Super Game Central respectively. The minimum and maximum Electronic Superstore are $25 and $62. The minimum for Game Stop Central are $26 and $61. Therefore, they are approximately equal. The upper quartile for The Elec Superstore is $49, while the median for Game Stop C. Since these two values are approximately equal, this about 50% of the data for Game Stop Central is greater than the data from The Electronic Superstore. Overall, stores have similar ranges, Game Stop Central has higher minimums.

17. BRAINTEASERS The time that it took Leon and C complete puzzles is shown.

<table>
<thead>
<tr>
<th>Leon's Times (minutes)</th>
<th>Cassie's Times (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5, 1.8, 3.2, 5.1, 2.0, 2.6, 4.8, 24, 2.2, 28, 1.8, 2.2, 3.9, 2.3, 3.3, 2.4</td>
<td></td>
</tr>
<tr>
<td>2.3, 5.8, 4.8, 3.3, 5.2, 4.6, 3.8, 5.7, 3.8, 4.2, 5.0, 4.3, 5.5, 4.9, 2.4, 5.2</td>
<td></td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and deviations or the five-number summaries. Justify your measurements.

SOLUTION:
a. Put Leon's times in L₁ and put Cassie's times in L₂. Establish a window range from 1.5 to 6 with a scale of 0.5, and use the TRACE button to determine the five-number summary for each set of data.

Leon's times: Min=1.8, Q₁=2.2, Med=2.5, Q₃=3.6, Max=5.1
Cassie's times: Min=2.3, Q₁=3.7, Med=4.7, Q₃=5.2, Max=5.8

b. Since both sets of data are skewed, it's best to use summary to compare the sets of data.

The five-number summaries for Leon and Cassie are given above. The lower quartile for Leon’s times is 2 while the lower quartile for Cassie’s times is 1.5 minutes. The upper quartile for Leon’s times is 3.6 minutes, while the upper quartile for Cassie’s times is 3.7 minutes. This means Leon’s times are less than 75% of Cassie’s time. Over conclude that Leon completed the brainteasers faster than Cassie.

18. DANCE The total amount of money that a sample o spent to attend the homecoming dance is shown.

<table>
<thead>
<tr>
<th>Boys (dollars)</th>
<th>Girls (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>114, 98, 131, 83, 91, 64, 94, 77, 96, 105, 72, 108, 87, 112, 58, 126</td>
<td></td>
</tr>
<tr>
<td>124, 74, 105, 133, 85, 162, 90, 109, 94, 102, 98, 171, 139, 90, 154, 76</td>
<td></td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and deviations or the five-number summaries. Justify your measurements.

SOLUTION:
a. Put the amount the Boys spent in L₁ and put the amount the Girls spent in L₂. Change the window range from 55 to 17 of 10, and plot the box-and-whisker plot. Use the TRACE button to determine which plot belongs to each set of data.
12-4 Comparing Sets of Data

The amount spent by the boys is symmetric, while the girls is positively skewed.

b. Since one set of data is skewed, it's best to use a frequency summary for comparison.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=16</td>
<td>n=16</td>
</tr>
<tr>
<td>min=58</td>
<td>min=74</td>
</tr>
<tr>
<td>Q1=80</td>
<td>Q1=89.5</td>
</tr>
<tr>
<td>Q3=95</td>
<td>Q3=103.5</td>
</tr>
<tr>
<td>Max=131</td>
<td>max=171</td>
</tr>
</tbody>
</table>

The five-number summaries for the boys and the girls are given above. The maximum for the boys is $131, the upper quartile for the girls is $135.50. This means that the data from the girls is greater than all of the data from the boys. When listed from least to greatest, each statistic for the girls is greater than its corresponding statistic for the boys. We can conclude that in general, the girls spent more money than the boys.

19. LANDSCAPING Refer to the beginning of the lesson on another employee that works with Tom, earned the following amounts during the past month.

<table>
<thead>
<tr>
<th>Rhonda’s Pay ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
</tr>
<tr>
<td>47</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>63</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>62</td>
</tr>
</tbody>
</table>

a. Find the mean, median, mode, range, and standard deviation of Rhonda’s earnings.

b. A $5 bonus had been added to each of Rhonda’s earnings. Find the mean, median, mode, range, and standard deviation of Rhonda’s earnings before the $5 bonus.

SOLUTION:

a. Input the data into L₁ and calculate the 1-variable statistics:

<table>
<thead>
<tr>
<th>1-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=23</td>
<td>n=23</td>
</tr>
<tr>
<td>min=39</td>
<td>min=44</td>
</tr>
<tr>
<td>Q₁=42</td>
<td>Q₁=47</td>
</tr>
<tr>
<td>med=48</td>
<td>med=53</td>
</tr>
<tr>
<td>Q₃=54</td>
<td>Q₃=59</td>
</tr>
<tr>
<td>max=58</td>
<td>max=63</td>
</tr>
</tbody>
</table>

The mean is 52.96. The median is 53. The mode is 5 and the standard deviation is 5.63.

b. Subtract 5 from L₁, and store it in L₂.

Next calculate the statistics for L₂.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=23</td>
<td>n=23</td>
</tr>
<tr>
<td>min=34</td>
<td>min=39</td>
</tr>
<tr>
<td>Q₁=41</td>
<td>Q₁=47</td>
</tr>
<tr>
<td>med=47</td>
<td>med=53</td>
</tr>
<tr>
<td>Q₃=55</td>
<td>Q₃=59</td>
</tr>
<tr>
<td>max=58</td>
<td>max=63</td>
</tr>
</tbody>
</table>

The mean is 57.96. The median is 48. The mode is 4 and the standard deviation is 6.08.

20. SHOPPING The items Lorenzo purchased are shown below:

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball hat</td>
<td>$14.98</td>
</tr>
<tr>
<td>Jeans</td>
<td>$24.61</td>
</tr>
<tr>
<td>T-shirt</td>
<td>$12.84</td>
</tr>
<tr>
<td>T-shirt</td>
<td>$16.05</td>
</tr>
<tr>
<td>Backpack</td>
<td>$42.80</td>
</tr>
<tr>
<td>Folders</td>
<td>$2.14</td>
</tr>
<tr>
<td>Sweatshirt</td>
<td>$19.26</td>
</tr>
</tbody>
</table>

a. Find the mean, median, mode, range, and standard deviation of the prices.

SOLUTION:
b. A 7% sales tax was added to the price of each item.

**SOLUTION:**

Enter the data into L₁ and compute the 1-variable statistics.

<table>
<thead>
<tr>
<th>L₁</th>
<th>L₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.95428571</td>
<td>1.07</td>
</tr>
<tr>
<td>132.68</td>
<td>14</td>
</tr>
<tr>
<td>3459.8878</td>
<td>23</td>
</tr>
<tr>
<td>12.55012066</td>
<td>12</td>
</tr>
<tr>
<td>11.61915396</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
</tr>
</tbody>
</table>

The mean is 18.95. The median is 16.05. There is no range, and the standard deviation is 42.8.

b. If a 7% sales tax was added, then the prices from 1.07 times larger than normal. Divide the prices from get the pre-tax prices.

**L₁ ÷ 1.07 → L₂**

Store these values in L₂ and calculate the 1-variable statistics.

The mean is 17.71. The median is 15. There is no mode, and the standard deviation is 10.86.

21. **CHALLENGE** A salesperson has 15 SUVs priced between $33,000 and $37,000 and 5 luxury cars priced between $44,000 and $48,000. The average price for all of the vehicles is $39,250. The salesperson decides to reduce the prices of the SUVs by $2000 per vehicle. What is the new average price for all of the vehicles?

**SOLUTION:**

Let \( x \) be the average cost of the SUVs and \( y \) be the average cost of the luxury cars. The total cost of the SUVs is then \( 15x \), and the total cost of the luxury cars is \( 5y \).

The average cost of all the vehicles is then \( \frac{15x + 5y}{20} \).

If we reduce the price of all of the SUVs by $2000, then the average price will be \( x - 2000 \). We can use this along with the previous equation to determine the new average, \( A \), of all the vehicles.

\[
A = \frac{15(x - 2000) + 5y}{20} = \frac{15x + 5y - 15(2000)}{20} = \frac{15x + 5y}{20} - \frac{15(2000)}{20} = 39,250 - 1500 = 37,750.
\]
22. **REASONING** If every value in a set of data is multiplied by a constant \( k, k < 0 \), then how can the mean, median, mode, range, and standard deviation of the new data set be found?

**SOLUTION:**

The mean is equal to:

\[
\overline{x}_a = \frac{x_1 + x_2 + \ldots + x_n}{n}
\]

If each term is multiplied by a constant \( k, k < 0 \), then the new mean will be:

\[
\overline{x}_b = \frac{kx_1 + kx_2 + \ldots + kx_n}{n} = k\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) = k\overline{x}_a
\]

The new mean is just the old mean multiplied by \( k \).

The median is just the middle number of the set of data. If everything in the set of data is multiplied by a constant, then the term in the middle will still be in the middle when multiplied by \( k \). So the new median is just the old median multiplied by \( k \). The mode is just the most frequently occurring value, so the new value will be \( k \) times the old value.

If \( x \) and \( y \) are the max and min respectively then \( x > y \) and the range is \( x - y \). If all of the values are multiplied by a negative constant \( k \), then the new max will be \( ky \), the min will be \( kx \), and the range will be \( ky - kx = k(x - y) \). Here \(-k\) is the same as \( |k| \), so the range can be found by multiplying the old range by \( |k| \).

Similarly for the standard deviation:

\[
\sigma_b = \sqrt{\frac{\sum (kx_1 - k\overline{x})^2 + \sum (kx_2 - k\overline{x})^2 + \ldots + \sum (kx_n - k\overline{x})^2}{n}}
\]

\[
= \sqrt{\frac{k^2 \sum (x_1 - \overline{x})^2 + k^2 \sum (x_2 - \overline{x})^2 + \ldots + k^2 \sum (x_n - \overline{x})^2}{n}}
\]

\[
= k\sqrt{\frac{\sum (x_1 - \overline{x})^2 + \sum (x_2 - \overline{x})^2 + \ldots + \sum (x_n - \overline{x})^2}{n}} = |k|\sigma_a
\]

The standard deviation is just multiplied by the absolute value of \( k \).

23. **WRITING IN MATH** Compare and contrast the benefits of displaying data using histograms and box-and-whisker plots.

**SOLUTION:**

A histogram and a box-and-whisker plot for the same set of data are given on a graph.

With a box-and-whisker plot, it's easy to determine the range, the quartile values, and the overall spread of the data. With histograms, you can see the frequency of values within each interval. The box-and-whisker plots show the data divided into four sections. This aids when comparing the spread of one set of data to another. However, the box-and-whisker plots are limited because they cannot display the data any more specifically than showing it divided into four sections.
12-4 Comparing Sets of Data

24. **CCSS REGULARITY** If \( k \) is added to every value in a set of data, and then each resulting value is multiplied by a constant \( m \), \( m > 0 \), how can the mean, median, mode, range, and standard deviation of the new data set be found? Explain your reasoning.

**SOLUTION:**

The mean is equal to:

\[
\overline{x}_a = \frac{\sum x_1 + \sum x_2 + \ldots + \sum x_n}{n}
\]

If \( k \) is added to each term and then they're multiplied by \( m \):

\[
\overline{x}_b = \frac{m(x_1 + k) + m(x_2 + k) + \ldots + m(x_n + k)}{n}
\]

\[
= m\left(\frac{x_1 + x_2 + \ldots + x_n + nk}{n}\right)
\]

\[
= m\overline{x}_a + k
\]

The new mean is just the old mean plus \( k \) and then multiplied by \( m \). The median is just the middle number of the set of data. If everything in the set of data increased by \( k \) and multiplied by a constant \( m \), then the term in the middle will still be in the middle when increased by \( k \) and multiplied by \( m \). So the new median is just the old median increased by \( k \) and multiplied by \( m \). The mode is just the most frequently occurring value, so the new value will be increased by \( k \) and multiplied by \( m \).

Since the range and the standard deviation are not affected when a constant is added to a set of data, they can be found by multiplying each original value by the constant \( m \).

25. **WRITING IN MATH** Explain why the mean and standard deviation are used to compare the center and spread of two symmetrical distributions and the five-number summary is used to compare the center and spread of two skewed distributions or a symmetric distribution and a skewed distribution.

**SOLUTION:**

When two distributions are symmetrical, we want to determine if their means are different and how spread out the distribution is. In the histograms above, both have the same mean, but the first graph has a larger distribution. We can compare these by determining the mean and standard deviation.

For skewed distributions, the mean and standard deviation do not provide enough information. If the distributions are skewed, we need a measure of how skewed the distributions are, and in what direction. We determine this by comparing the range, quartiles, and median, provided in the five number summaries.
26. A store manager recorded the number of customers each day for a week: {46, 57, 63, 78, 91, 110, 101}. Find the mean absolute deviation.

A 16.8  
B 18.1  
C 19.4  
D 22.7

**SOLUTION:**
To find the absolute deviation, first calculate the mean:

\[ \bar{x} = \frac{46 + 57 + 63 + 78 + 91 + 110 + 101}{7} \]
\[ = \frac{546}{7} \]
\[ = 78 \]

\[ \mathrm{MAD} = \frac{\sum |x_i - \bar{x}|}{n} = \frac{12 + 21 + 15 + 5 + 13 + 12 + 33}{7} \]
\[ = \frac{126}{7} \]
\[ = 18 \]

The MAD is 19.4, which is answer choice C.

27. **SHORT RESPONSE** Solve the right triangle. Round each side length to the nearest tenth.

**SOLUTION:**
\[ \angle A = 180^\circ - 90^\circ - 54^\circ = 36^\circ. \]
\[ \sin 54^\circ = \frac{8}{c} \]
\[ c \approx 9.9 \]
\[ \tan 54^\circ = \frac{8}{a} \]
\[ a \approx 5.8 \]
\[ \angle A = 36^\circ, \ c \approx 9.9, \ a \approx 5.8 \]

28. A research company divides a group of volunteers by age, and then randomly selects volunteers from each group to complete a survey. What type of sample is this?

F simple  
G systematic  
H self-selected  
J stratified

**SOLUTION:**
Dividing a sample into groups first is an example of a stratified sample - J.
12-4 Comparing Sets of Data

29. Which set of measures can be the measures of the sides of a right triangle?

A 6, 7, 9  
B 9, 12, 19  
C 12, 15, 17  
D 14, 48, 50

**SOLUTION:**
A 6, 7, 9  
B 9, 12, 19  
C 12, 15, 17  
D 14, 48, 50

The measures will define a right triangle only if the Pythagorean Theorem holds.

\[ 6^2 + 7^2 = 9^2 \]
\[ 36 + 49 = 81 \]
\[ 85 \neq 81 \]

\[ 9^2 + 12^2 = 19^2 \]
\[ 81 + 144 = 361 \]
\[ 225 \neq 361 \]

\[ 12^2 + 15^2 = 17^2 \]
\[ 144 + 225 = 289 \]
\[ 369 \neq 289 \]

\[ 14^2 + 48^2 = 50^2 \]
\[ 196 + 2304 = 2500 \]
\[ 2500 = 2500 \]

The sides 14, 48, and 50 define a right triangle. Answer choice D is correct.

30. Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

23, 45, 50, 22, 37, 24, 36, 46, 24, 52, 25, 42, 25, 26, 54, 47, 27, 55  
63, 28, 29, 30, 45, 31, 55, 43, 32, 34, 30, 23, 30, 35, 27, 35, 38, 40

**SOLUTION:**
Enter the data into L_1, on your calculator, choose a suitable window, and create a histogram.

![Histogram](image)

The distribution is positively skewed.

31. **SUBSCRIPTIONS** Ms. Wilson’s students are selling magazine subscriptions. Her students recorded the total number of subscriptions they each sold: {8, 12, 10, 7, 4, 3, 0, 4, 9, 0, 5, 3, 23, 6, 2}. Find and interpret the standard deviation of the data set.

**SOLUTION:**
First enter the data into L_1 and calculate the 1-variable statistics:

![1-Var Stats](image)

The mean is 6.4 and the standard deviation is 5.58. The standard deviation is high compared to the mean. This means that the values of the data are very spread out.
Find the value of $x$ for each figure. Round to the nearest tenth if necessary.

32. $A = 45$ in$^2$

\[ (x + 7) \text{ in.} \]

\[ (x + 3) \text{ in.} \]

**SOLUTION:**

\[(x + 3)(x + 7) = 45\]

\[x^2 + 10x + 21 = 45\]

\[x^2 + 10x - 24 = 0\]

\[(x + 12)(x - 2) = 0\]

\[x = -12 \text{ or } x = 2\]

If $x = -12$, then the values for the length and width of the rectangle will be negative, so we cannot use this value. Therefore $x = 2$.

33. $A = 20$ ft$^2$

\[ (x + 6) \text{ ft} \]

\[ x \text{ ft} \]

**SOLUTION:**

\[\frac{1}{2}x(x + 6) = 20\]

\[x(x + 6) = 40\]

\[x^2 + 6x - 40 = 0\]

\[(x + 10)(x - 4) = 0\]

\[x = -10 \text{ or } x = 4\]

If $x = -10$, then the values for the base and height of the triangle will be negative, so we cannot use this value. Therefore $x = 4$.

34. $A = 42$ m$^2$

\[ 3x \text{ m} \]

\[ (x + 2) \text{ m} \]

**SOLUTION:**

\[3x(x + 2) = 42\]

\[3x^2 + 6x = 42\]

\[3x^2 + 6x - 42 = 0\]

\[x^2 + 2x - 14 = 0\]

The trinomial in the last line above does not factor, so we must use the quadratic formula:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-2 \pm \sqrt{60}}{2}\]

\[= \frac{-2 \pm 2\sqrt{15}}{2}\]

\[\approx 2.9\]

The result above is from taking the additive term, since subtraction would result in negative values for the length and width of the rectangle. Therefore $x \approx 2.9$.

35. $x^2 - 4x - 21$

**SOLUTION:**

In this trinomial $b = -4$ and $c = -21$, so we need to find one positive and one negative factor of 21 whose sum is $-4$. The negative term should be larger in absolute value since $b$ is negative.

<table>
<thead>
<tr>
<th>Factors of 21</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-21, 1</td>
<td>-20</td>
</tr>
<tr>
<td>-7, 3</td>
<td>-4</td>
</tr>
</tbody>
</table>

\[x^2 - 4x + 21 = (x - 7)(x + 3)\]
12-4 Comparing Sets of Data

36. $11x + x^2 + 30$

**SOLUTION:**
In this trinomial $b = 11$ and $c = 30$, so we need to find two positive factors of 30 whose sum is 11.

<table>
<thead>
<tr>
<th>Factors of 21</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 30</td>
<td>31</td>
</tr>
<tr>
<td>2, 15</td>
<td>17</td>
</tr>
<tr>
<td>3, 10</td>
<td>13</td>
</tr>
<tr>
<td>5, 6</td>
<td>11</td>
</tr>
</tbody>
</table>

$x^2 + 11x + 30 = (x + 6)(x + 5)$

37. $32 + x^2 - 12x$

**SOLUTION:**
$32 + x^2 - 12x$

In this trinomial $b = -12$ and $c = 32$, so we need to find two negative factors of 32 whose sum is $-12$.

<table>
<thead>
<tr>
<th>Factors of 32</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1, -32</td>
<td>-33</td>
</tr>
<tr>
<td>-2, -16</td>
<td>-18</td>
</tr>
<tr>
<td>-4, -8</td>
<td>-12</td>
</tr>
</tbody>
</table>

$x^2 - 12x + 32 = (x - 8)(x - 4)$

38. $-36 - 9x + x^2$

**SOLUTION:**
In this trinomial $b = -9$ and $c = -36$, so we need to find one positive and one negative factor of 21 whose sum is $-9$. The negative term should be larger in absolute value since $b$ is negative.

<table>
<thead>
<tr>
<th>Factors of 36</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, -36</td>
<td>-35</td>
</tr>
<tr>
<td>2, -18</td>
<td>-16</td>
</tr>
<tr>
<td>3, -12</td>
<td>-9</td>
</tr>
<tr>
<td>4, -9</td>
<td>-5</td>
</tr>
<tr>
<td>6, -6</td>
<td>0</td>
</tr>
</tbody>
</table>

$x^2 - 9x - 36 = (x - 12)(x + 3)$

39. $x^2 + 12x + 20$

**SOLUTION:**
In this trinomial $b = 12$ and $c = 20$, so we need to find two positive factors of 20 whose sum is 12.

<table>
<thead>
<tr>
<th>Factors of 20</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 20</td>
<td>21</td>
</tr>
<tr>
<td>2, 10</td>
<td>12</td>
</tr>
<tr>
<td>4, 5</td>
<td>9</td>
</tr>
</tbody>
</table>

$x^2 + 12x + 20 = (x + 4)(x + 5)$

40. $-x + x^2 - 42$

**SOLUTION:**
$-x + x^2 - 42$

In this trinomial $b = -1$ and $c = -42$, so we need to find one positive and one negative factor of 42 whose sum is $-1$. The negative term should be larger in absolute value since $b$ is negative.

<table>
<thead>
<tr>
<th>Factors of 36</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, -42</td>
<td>-41</td>
</tr>
<tr>
<td>2, -21</td>
<td>-19</td>
</tr>
<tr>
<td>3, -14</td>
<td>-11</td>
</tr>
<tr>
<td>6, -7</td>
<td>-1</td>
</tr>
</tbody>
</table>

$x^2 - x - 42 = (x - 7)(x + 6)$
12-4 Comparing Sets of Data

41. MANUFACTURING A company is designing a box for dry pasta in the shape of a rectangular prism. The length is 2 inches more than twice the width, and the height is 3 inches more than the length. Write an expression for the volume of the box.

**SOLUTION:**

A company is designing a box for dry pasta in the shape of a rectangular prism. The length is 2 inches more than twice the width, and the height is 3 inches more than the length. Write an expression for the volume of the box.

The volume of the box is:

\[ V = lwh \]

The second sentence tells us that:

\[ l = 2w + 2 \]

and

\[ h = l + 3 \]

\[ = (2w + 2) + 3 \]

\[ = 2w + 5 \]

Substituting these equations into the volume equation gives:

\[ V = (2w + 2)w(2w + 5) \]

\[ = w(4w^2 + 14w + 10) \]

\[ = 4w^3 + 14w^2 + 10w \]

**Find the degree of each polynomial.**

42. \( 2x^2 + 5y - 21 \)

**SOLUTION:**

For this expression, the term with the highest powers of \( x \), and \( y \) is \( 2x^2 \), which is of degree 2.

43. \( 16xy^3 - 17x^2y - 16x^3 \)

**SOLUTION:**

\( 16xy^3 \) has a degree of \( 1 + 3 = 4 \).

\( 17x^2y \) has a degree is \( 2 + 1 = 3 \).

\( 16x^3 \) has a degree of 3.

So the polynomial has a degree of 4.

44. \( 3ac^3d + 14a^2 \)

**SOLUTION:**

\( 3ac^3d \) has a degree of \( 1 + 3 + 1 = 5 \)

\( 14a^2 \) has a degree of 2.

This is a polynomial of degree 5.

45. 18

**SOLUTION:**

18 is a constant, which is a 0 degree polynomial.

46. \( 3a^2b^3 + 11ab^2c \)

**SOLUTION:**

\( 3a^2b^3 \) has a degree of \( 2 + 3 = 5 \).

\( 11ab^2c \) has a degree of \( 1 + 2 + 1 = 4 \).

This is a polynomial of degree 5.

47. \( 7x + 11 \)

**SOLUTION:**

\( 7x + 11 \) is a linear polynomial and has a degree of 1.
1. MULTIPLE CHOICE A movie theater employee surveyed a sample of customers as they exited the theater. Find the experimental probability of randomly selecting a customer who is older than 12 but younger than 46.

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–7</td>
<td>13</td>
</tr>
<tr>
<td>8–12</td>
<td>28</td>
</tr>
<tr>
<td>13–17</td>
<td>48</td>
</tr>
<tr>
<td>18–23</td>
<td>42</td>
</tr>
<tr>
<td>24–31</td>
<td>27</td>
</tr>
<tr>
<td>32–45</td>
<td>40</td>
</tr>
<tr>
<td>46–64</td>
<td>33</td>
</tr>
<tr>
<td>65+</td>
<td>23</td>
</tr>
</tbody>
</table>

- **A** $\frac{24}{157}$
- **B** $\frac{254}{99}$
- **C** $\frac{127}{213}$
- **D** 254

**SOLUTION:**

The total number of people surveyed is $13 + 28 + 48 + 42 + 27 + 40 + 33 + 23 = 254$

The total number of people that were older than 12 but younger than 46 were $48 + 42 + 27 + 40 = 157$

The probability of randomly selecting a customer who is older than 12 but younger than 46 is then: $\frac{157}{254}$. Answer choice B.

2. FOOTBALL Rico is the kicker on the football team. Last season, he made 94% of his extra points.

   a. Design a simulation that can be used to estimate the probability that Rico will make his next extra point. a. b. See margin.

   b. Conduct the simulation, and report the results.

**SOLUTION:**

a. The simplest simulation to make is through the use of a random number generator. There is a 94% probability of Rico making an extra point and a 6% probability of Rico missing the extra point. Express these percentages as a fraction to determine the range of numbers that is needed.

   \[ 94\% = \frac{47}{50} \]

   \[ 6\% = \frac{3}{50} \]

   The lowest denominator is 50, so we should have a random number generator generate numbers 1 - 50, with 1 - 47 representing an extra point that was made and 48 - 50 representing an extra point that was missed.

   The simulation will consist of 40 trials.

b. Sample answer: $P(\text{made}) = 92.5\%$, $P(\text{missed}) = 7.5\%$
3. **CARDS** Javier is drawing a card from a standard deck of cards, recording the suit, and then replacing the card in the table below shows his results.

<table>
<thead>
<tr>
<th>Suit</th>
<th>clubs</th>
<th>diamonds</th>
<th>hearts</th>
<th>spades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**a.** The total number of cards drawn was $9 + 5 + 4 + 1 = 20$. There were 5 hearts drawn, so the experimental probability of drawing a heart is equal to $\frac{5}{20} = \frac{1}{4} = 20\%$.

**b.** The total number of black cards is equal to $7 + 9 = 16$, so the experimental probability of drawing a black card is equal to $\frac{16}{20} = \frac{4}{5} = 64\%$.

**c.** Combining the results gives

<table>
<thead>
<tr>
<th>Suit</th>
<th>clubs</th>
<th>diamonds</th>
<th>hearts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>

The total number of cards drawn is $12 + 12 + 11 + 1 = 36$.

The experimental probability of drawing a spade is $\frac{15}{36} = \frac{3}{6} = 30\%$.

4. **MUSIC** Shannon’s digital media player has a large collection of songs. She randomly toggles through the songs and then records the genre. The graph shows her results.

**SOLUTION:**

**a.** Find the experimental probability of selecting a country song.

**b.** Find the experimental probability of selecting a song that is not rock.

**SOLUTION:**

**a.** The total number of songs Shannon toggled through is $24 + 33 + 14 + 29 = 100$. The probability of picking a country song is thus: $\frac{24}{100} = \frac{6}{25} = 24\%$.

**b.** There are 29 rock songs, which means that there are $100 - 29 = 71$ songs that are not rock. The probability of choosing a song that is not rock is therefore: $\frac{71}{100} = 71\%$.
12-5 Simulation

5. **BATTING AVERAGE** In a computer baseball game, a player has a batting average of .300. That is, he gets a hit 300 out of 1000, or 30%, of the times he is at bat.

   a. Design a simulation that can be used to estimate the probability that the player will get a hit at his next at bat.

   b. Conduct the simulation, and report the results.

   **SOLUTION:**

   a. The probability of getting a hit is 30% or \( \frac{3}{10} \), while the probability of not getting a hit is 70% or \( \frac{7}{10} \). Use a random number generator to generate numbers 1 - 10, with 1 – 3 representing a player getting a hit and 4 – 10 representing a player not getting a hit. The simulation will consist of 50 trials.

   b. Sample answer: \( P(\text{hit}) = 28\% \), \( P(\text{not a hit}) = 72\% \)

6. **JEANS** Julie examines the stitching on pairs of jeans that are produced at a manufacturing plant. She expects to find defects in 1 out of every 20 pairs.

   a. Design a simulation that can be used to estimate the probability that the next pair of jeans that Julie examines has a defect. See margin.

   b. Conduct the simulation, and report the results.

   **SOLUTION:**

   a. Since 1 out of every 20 pairs of jeans will be defective, use a random number generator to generate integers from 1 – 20. 1 will be the event that the pair of jeans is defective and 2 – 20 will be the event that the pair of jeans is not defective. The simulation will consist of 40 trials.

   b. Sample answer: \( P(\text{defect}) = 2.5\% \), \( P(\text{no defect}) = 97.5\% \)
12-5 Simulation

7. **FOOD** For a promotion, the concession stands at a football stadium are giving away free items. For every tenth customer, a wheel is spun to choose the customer’s prize. Each prize is equally likely.

   a. Design a simulation that can be used to estimate the probability that the next spin is one of the five prizes.
   b. Conduct the simulation, and report the results.

   ![Simulation Diagram]

   **SOLUTION:**
   a. There are five possible outcomes so use a random number generator to generate integers from 1 to 5. The integer 1 will represent a hot pretzel, the integer 2 will represent a burger, the integer 3 will represent a large drink, the integer 4 will represent nachos, and the integer 5 will represent a small popcorn. The simulation will consist of 50 trials.
   b. Sample answer: $P(\text{hot pretzel}) = 20\%$, $P(\text{burger}) = 20\%$, $P(\text{large drink}) = 28\%$, $P(\text{nachos}) = 14\%$, $P(\text{small popcorn}) = 18\%$

8. **CCSS MODELING** For its twentieth anniversary, a store randomly gives each customer a prize from the following choices: a free music download, a free game download, a free bag of popcorn, or a free DVD. The chances of winning each prize are equal.

   a. Design a simulation that can be used to estimate the probability that the next prize given is one of the four prizes.
   b. Conduct the simulation, and report the results.

   **SOLUTION:**
   a. There are four different prizes, each with an equal probability of being given away. Use a random number generator to generate integers 1 to 4. The integer 1 will represent a music download, the integer 2 will represent a game download, the integer 3 will represent a bag of popcorn, and the integer 4 will represent a DVD. The simulation will consist of 40 trials.
   b. Sample answer: $P(\text{music download}) = 20\%$, $P(\text{game download}) = 17.5\%$, $P(\text{popcorn}) = 37.5\%$, $P(\text{DVD}) = 25\%$
12-5 Simulation

9. GAMES Games at the fair require the majority of players to lose in order for game owners to make a profit. New games are tested to make sure they have sufficient difficulty. The results of three test groups are listed in the table. The owners want a maximum of 33% of players to win. There were 50 participants in each test group.

<table>
<thead>
<tr>
<th>Result</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winners</td>
<td>13</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>Losers</td>
<td>37</td>
<td>35</td>
<td>31</td>
</tr>
</tbody>
</table>

a. What is the experimental probability that the participant was a winner in the second group?
b. What is the experimental probability of winning for all three groups?
c. DECISION MAKING Should this game be used? Explain your reasoning.

SOLUTION:
a. The total number of participants in group 2 was 15 + 35 = 50.

The number of winners in Group 2 was 15.

The experimental probability that a participant was a winner in the second group is \( \frac{15}{50} = \frac{3}{10} = 30\% \).
b. The total number of participants was 13 + 15 + 19 + 37 + 35 + 31 = 150.

The total number of winners out of all of the participants was 13 + 15 + 19 = 47.

The experimental probability of winning is therefore \( \frac{47}{150} \approx 31\% \).
c. Yes, the game should be played. The success rate was 31% which is lower than the 33% maximum success rate, but still close. If the success rate of the game was too low no one would ever want to play and it would be a waste of money to implement.

10. TEST Jack forgot to study for his multiple-choice science quiz and is going to guess for each question. There are 20 questions, each with 4 possible answers.

a. Design a simulation that can be used to estimate the number of questions that Jack answers correctly.

b. Conduct the experiment from part a five times, and complete the table.

c. How many should Jack expect to answer correctly?

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Number of Correct Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

SOLUTION:
a. If there are 4 possible answers and Jack randomly guesses, then the probability of Jack getting a correct answer is \( \frac{1}{4} = 25\% \). Use a random number generator to generate numbers 1 to 4. The integer 1 will represent a correct answer, and the integers 2–4 will represent an incorrect answer. The simulation will consist of 20 trials.

b. Sample answer:

c. The average number of correct questions that Jack answered correctly after 5 trials is \( \frac{20}{5} = 4 \) or 5.8 questions. Since a fraction of a question cannot be correct, Jack should expect to answer 5 questions correctly.

11. MULTIPLE REPRESENTATIONS In this problem, you will explore the effect the number of trials has on the experimental probability of an event.

a. Verbal What is the probability of rolling a 1 on a die?
1. **Multiple Choice** A movie theater employee surveyed a sample of customers as they exited the theater. Find the probability that a customer will purchase popcorn by dividing the number of customers who purchase popcorn by the total number of customers. Then multiply by 100, or move the decimal point two places to the right. So, is about 27%.

<table>
<thead>
<tr>
<th>Trials</th>
<th>Frequency</th>
<th>Experimental Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**b. Analytical** Design a simulation that can be used to estimate the probability that the next number rolled on a die is a 1.

<table>
<thead>
<tr>
<th>Rolling a 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Trials</td>
<td>Frequency</td>
<td>Experimental Probability</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>(\frac{1}{6}) or 10%</td>
</tr>
<tr>
<td>50</td>
<td>11</td>
<td>(\frac{11}{50}) or 22%</td>
</tr>
<tr>
<td>100</td>
<td>19</td>
<td>(\frac{19}{100}) or 19%</td>
</tr>
<tr>
<td>200</td>
<td>37</td>
<td>(\frac{37}{200}) or 18.5%</td>
</tr>
</tbody>
</table>

**c. Analytical** Conduct the simulation from part b for 10, 20, 50, and 100 trials, and complete the table.

d. **Analytical** As the number of trials increases, what is happening to the experimental probability?

e. **Verbal** The Law of Large Numbers states that as the number of trials increases, the experimental probability gets closer to the theoretical probability. If you continued the simulation, each time increasing the number of trials, what would you expect the experimental probability for rolling a 1 to approach?

**SOLUTION:**

a. The probability of rolling a 1 on a die is \(\frac{1}{6}\).

b. Since there are 6 possible numbers on a die, use a random number generator to generate integers 1 through 6. The integer 1 will represent a 1. The integers 2–6 will represent the other numbers on the die.

c. Sample answer:

<table>
<thead>
<tr>
<th>Rolling a 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Trials</td>
<td>Frequency</td>
<td>Experimental Probability</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>(\frac{1}{6}) or 10%</td>
</tr>
<tr>
<td>50</td>
<td>11</td>
<td>(\frac{11}{50}) or 22%</td>
</tr>
<tr>
<td>100</td>
<td>19</td>
<td>(\frac{19}{100}) or 19%</td>
</tr>
<tr>
<td>200</td>
<td>37</td>
<td>(\frac{37}{200}) or 18.5%</td>
</tr>
</tbody>
</table>

d. Sample answer: The experimental probability of rolling a 1 is getting closer to the theoretical probability of \(\frac{1}{6}\) or 17%.

e. Sample answer: As the number of trials increases, the experimental probability should approach the theoretical probability of \(\frac{1}{6}\) or 17%.

12. **CCSS ARGUMENTS** The experimental probability of heads when a coin is tossed 15 times is sometimes, never, or always equal to the theoretical probability. Explain your reasoning.

**SOLUTION:**

If the coin is tossed once, then the number of heads you can get is either 1 or 0. The outcomes for the experimental probabilities of a 1 toss event are either 100% or 0%.

If the coin is tossed twice, then the number of heads you can get is either 0, 1, or 2. The outcomes for the experimental probabilities of a 2 toss event are either 0%, 50%, or 100%.

If the coin is tossed three times, then the number of heads you can get is either 0, 1, 2, or 3. The outcomes for the experimental probabilities of a 3 toss event are either 0%, 33%, 67%, or 100%.

If we continue to look at these examples while adding tosses, then we see that when there are an odd number of tosses, then we cannot achieve an experimental probability of 50%, which is the theoretical probability. If we toss a coin 15 times, the closest we can get is either 7 or 8 heads, which would give experimental probabilities of

\[\frac{7}{15} = 47\%\]
\[\frac{8}{15} = 53\%\]

The theoretical and experimental probabilities will never be equal with only 15 tosses.
16. **WRITING IN MATH** What should you consider when using the results of a simulation to make a prediction?

**SOLUTION:**
You should consider the number of trials that were used since a small number of trials likely means that the experimental probabilities are not accurate. You also want to compare the experimental probabilities with the theoretical probabilities to determine whether there might be something faulty with the equipment that you are using (such as a die being weighted toward one side), or even that the theoretical probabilities have been miscalculated. You also want to make sure you take into account the design of the simulation and how well it represents any predictions you are making in the real world. For example if a football kicker makes 94% of the extra points, this may not account for a situation when the team gets penalized and he has to kick from 15 yards back.

17. **GEOMETRY** Suppose a covered water tank in the shape of a right circular cylinder is thirty feet long and eight feet in diameter. What is the surface area of the cylinder?

A $272\pi$ ft$^2$

B $248\pi$ ft$^2$

C $224\pi$ ft$^2$

D $153\pi$ ft$^2$

**SOLUTION:**
$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi (4)^2 + 2\pi (4)(30)$

$= 32\pi + 240\pi$

$= 272\pi$

So, the surface area of the cylinder is $272\pi$ square feet. The correct choice is A.
18. SHORT RESPONSE Two consecutive numbers have a sum of 91. What are the numbers?

**SOLUTION:**
We are looking for two numbers \( n \) and \( n + 1 \) such that
\[
2n + 1 = 91
\]

\[
2n = 90
\]

\[
n = 45
\]

The numbers are 45, and 46.

19. Solve \( \frac{2x}{x-2} + \frac{8}{x} = 6 \).

**SOLUTION:**
\[
\frac{2x}{x-2} + \frac{8}{x} = 6
\]
\[
\frac{2x(x) + 8(x-2)}{x(x-2)} = 6
\]
\[
\frac{2x^2 + 8x - 16}{x^2 - 2x} = 6
\]
\[
2x^2 + 8x - 16 = 6x^2 - 12x
\]
\[
4x^2 - 20x + 16 = 0
\]
\[
x^2 - 5x + 4 = 0
\]
\[
(x - 1)(x - 4) = 0
\]

\[x = 1 \text{ or } x = 4\]

Answer choice G

20. Mr. Bahn has $20,000 to invest. He invests part at 6% and the rest at 7%. He earns $1280 in interest within a year. How much did he invest at 7%?

A $12,000
B $11,275
C $9950
D $8000

**SOLUTION:**
If he invests \( x \) at 7%, then the amount he invests at 6% is \( 20,000 - x \).

\[
0.07x + 0.06(20,000 - x) = 1280
\]
\[
0.07x + 1200 = 1280
\]
\[
0.07x = 80
\]
\[
x = 800 \text{ C}
\]

Answer choice D.
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

21. 12, 16, 4, 8, 7, 11, 9, 4; + 5

**SOLUTION:**
First enter the data into L1, and then add 5 to each value and store it into L2.

Next calculate the 1-variable statistics of L2.

\[
\begin{align*}
\text{1-Var Stats} \\
\hat{n}=8 \\
\text{minX}=9 \\
Q_1=10.5 \\
\text{Med}=13.5 \\
Q_3=16.5 \\
\text{maxX}=21
\end{align*}
\]

The mean is 13.9. The median is 13.5. The mode is 9. The range is 21 – 9 = 12. And the standard deviation is 3.8.

22. 1, 4, 3, 9, 12, 6, 7, 3; + 12

**SOLUTION:**
First enter the data into L1, and then add 12 to each value and store it into L2.

Next calculate the 1-variable statistics of L2.

\[
\begin{align*}
\text{1-Var Stats} \\
\hat{n}=8 \\
\text{minX}=13 \\
Q_1=15 \\
\text{Med}=17 \\
Q_3=20 \\
\text{maxX}=24
\end{align*}
\]

The mean is 17.7. The median is 17. The mode is 15. The range is 24 – 13 = 11. And the standard deviation is 3.4.
23. 18, 12, 8, 13, 7, 15, 8, 6; + (−3)

**SOLUTION:**
First enter the data into L₁, and then subtract 3 from each value and store it into L₂.

Next calculate the 1-variable statistics of L₂.

<table>
<thead>
<tr>
<th>L₁</th>
<th>L₂</th>
<th>L₃</th>
<th>₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The mean is 7.9. The median is 7. The mode is 5. The range is 15 – 3 = 12. And the standard deviation is 4.0.

24. **DANCE RECITAL** The number of dance students in each act of a dance recital is shown. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

<table>
<thead>
<tr>
<th>Number of Dance Recitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 15 1 20 14 4</td>
</tr>
<tr>
<td>18 2 17 10 22 1</td>
</tr>
<tr>
<td>22 15 17 21 10 18</td>
</tr>
<tr>
<td>14 18 2 10 20 15</td>
</tr>
<tr>
<td>18 4 19 12 16 5</td>
</tr>
</tbody>
</table>

**SOLUTION:**
First enter the data into L₁ of your calculator and create a box-and-whisker plot for the data.

The data is skewed so it is best to use the five-number summary. Calculate the 1-variable statistics.

<table>
<thead>
<tr>
<th>L₁</th>
<th>L₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

The median is 15. The range is 22 – 1 = 21. 50% of the data lie between 5 and 18.
25. **PARTIES** Student Council is planning a party for the school volunteers. There are 66 unopened 5-ounce bottles of soda left from a recent dance. When poured over ice, \(5 \frac{1}{2}\) ounces of soda fills a cup. How many servings of soda do they have?

**SOLUTION:**
\[
5 \frac{1}{2} = \frac{11}{2} \text{ ounces of soda per cup. There are } 5 \times 66 = 330 \text{ ounces of soda left over. This gives a total of }
\]

\[
\frac{330 \text{ ounces}}{\frac{11}{2} \text{ ounces per serving}} = 60 \text{ servings}
\]

Write an inverse variation equation that relates \(x\) and \(y\). Assume that \(y\) varies inversely as \(x\). Then solve.

26. If \(y = 8.5\) when \(x = -1\), find \(x\) when \(y = -1\).

**SOLUTION:**
\[
xy = k
\]
\[
(-1)(8.5) = k
\]
\[
-8.5 = k
\]
The constant of variation is \(-8.5\). So, an equation that relates \(x\) and \(y\) is \(xy = -8.5\).
\[
xy = -8.5
\]
\[
x(-1) = -8.5
\]
\[
x = 8.5
\]

27. If \(y = 8\) when \(x = 1.55\), find \(x\) when \(y = -0.62\).

**SOLUTION:**
\[
xy = k
\]
\[
(1.55)(8) = k
\]
\[
12.4 = k
\]
The constant of variation is \(12.4\). So, an equation that relates \(x\) and \(y\) is \(xy = 12.4\).
\[
xy = 12.4
\]
\[
x(-0.62) = 12.4
\]
\[
x = -20
\]

28. If \(y = 6.4\) when \(x = 4.4\), find \(x\) when \(y = 3.2\).

**SOLUTION:**
\[
xy = k
\]
\[
(4.4)(6.4) = k
\]
\[
28.16 = k
\]
The constant of variation is 28.16. So, an equation that relates \(x\) and \(y\) is \(xy = 28.16\).
\[
xy = 28.16
\]
\[
x(3.2) = 28.16
\]
\[
x = 8.8
\]

29. **DELIVERY** Ben and Amado are delivering a freezer. The bank in front of the house is the same height as the back of the truck. They set up their ramp as shown. What is the length of the slanted part of the ramp to the nearest tenth of a foot?

**SOLUTION:**
We can use the Pythagorean Theorem to determine the length of the slanted part of the ramp.
\[
c^2 = 6.2^2 + 3.5^2
\]
\[
c = \sqrt{38.44 + 12.25}
\]
\[
c \approx 7.1
\]
Write each fraction as a percent rounded to the nearest whole number.

30. \[
\frac{26}{58}
\]

**SOLUTION:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{26}{58} \approx 0.448
\]
So, \(\frac{26}{58}\) is about 45%.
12-5 Simulation

31. \[
\frac{55}{125}
\]

**Solution:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{55}{125} = 0.44
\]
So, \[
\frac{55}{125}
\] is about 44%.

32. \[
\frac{14}{128}
\]

**Solution:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{14}{128} \approx 0.109
\]
So, \[
\frac{14}{128}
\] is about 11%.

33. \[
\frac{82}{110}
\]

**Solution:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{82}{110} \approx 0.745
\]
So, \[
\frac{82}{110}
\] is about 75%.

34. \[
\frac{76}{124}
\]

**Solution:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{76}{124} \approx 0.613
\]
So, \[
\frac{76}{124}
\] is about 61%.

35. \[
\frac{23}{86}
\]

**Solution:**
Divide the numerator by the denominator. Then multiply by 100, or move the decimal point two places to the right.
\[
\frac{23}{86} \approx 0.267
\]
So, \[
\frac{23}{86}
\] is about 27%.
1. **CHARITY** A youth charity group is holding a raffle and wants to display a picture of the 6 prizes on a flyer. Many ways can they arrange the prizes in a row?

   **SOLUTION:**
   Once a picture is chosen for the linear arrangement, it is available for the next choice.

   Number of ways to arrange the prizes = \(6 \cdot 5 \cdot 4 \cdot 3\)

   \[= 720\]

   There are 720 ways to arrange the prizes.

2. Identifying each situation as a permutation or a combination.

   2. choosing 3 different pizza toppings from a list of 12

   **SOLUTION:**
   Since the order in which the pizza toppings are chosen does not matter, the situation involves a combination.

3. Choosing team captains for a football team

   **SOLUTION:**
   Since the order in which the team captains are chosen does not matter, the situation involves a combination.

4. Choosing the first-, second-, and third-place winner of an art competition

   **SOLUTION:**
   Since the order in which the winners are chosen is important, the situation involves a permutation.

5. Evaluating each expression.

   5. **P(7, 2)**

   **SOLUTION:**
   \[P(n, r) = \frac{n!}{(n-r)!}\]

   \[P(7, 2) = \frac{7!}{(7-2)!}\]

   \[= \frac{7!}{5!}\]

   \[= \frac{7 \cdot 6 \cdot 5!}{5!}\]

   \[= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1\]

   \[= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1\]

   \[= 42\]

   6. **P(9, 3)**

   **SOLUTION:**
   \[P(n, r) = \frac{n!}{(n-r)!}\]

   \[P(9, 3) = \frac{9!}{(9-3)!}\]

   \[= \frac{9!}{6!}\]

   \[= \frac{9 \cdot 8 \cdot 7 \cdot 6!}{5!}\]

   \[= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\]

   \[= 540\]

7. **C(6, 4)**

   **SOLUTION:**
   \[C(n, r) = \frac{n!}{(n-r)!r!}\]

   \[C(6, 4) = \frac{6!}{(6-4)!4!}\]

   \[= \frac{6!}{2!4!}\]

   \[= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}\]

   \[= \frac{30}{2}\]

   \[= 15\]

8. **C(5, 2)**

   **SOLUTION:**
   \[C(n, r) = \frac{n!}{(n-r)!r!}\]

   \[C(5, 2) = \frac{5!}{(5-2)!2!}\]

   \[= \frac{5!}{3!2!}\]

   \[= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}\]

   \[= \frac{20}{2}\]

   \[= 10\]
9. **CHARITY** A youth charity group is holding a raffle and wants to display a picture of the 6 prizes on a flyer. How many ways can they arrange the prizes in a row?

**SOLUTION:**
The question is asking how many different ways can we choose 6 toppings out of a total set of 14. The order in which the toppings are put on the pizza does not matter, so this is a combination problem. The total number of combinations can be described as:

\[
C(14, 6) = \frac{14!}{(14-6)!6!}
\]

\[
= \frac{14!}{8!6!}
\]

\[
= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9}{6 \times 5 \times 4 \times 3 \times 2 \times 1}
\]

\[
= 3003
\]

10. **PASSWORDS** Students are given 5-digit passwords for their accounts on the school’s computer system. If no numbers can repeat, what is the probability that a student’s password is 93152?

**SOLUTION:**
The digital password consists of 5 digits each with a different value. There are 10 digits to choose from, 0 – 9, so the total number of passwords is:

\[
P(10, 5) = \frac{10!}{(10-5)!}
\]

\[
= \frac{10!}{5!}
\]

\[
= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}
\]

\[
= 30,240
\]

Each password is unique, so the probability of getting a specific password, such as 93152, is \(\frac{1}{30,240}\).

11. **SCIENCE FAIR** There are 8 finalists in a science fair competition. How many ways can they stand in a row on the stage?

**SOLUTION:**
Once a finalist is chosen for the row, the same finalist is not available for the next choice.

Number of ways to arrange the finalists = \(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320\).

There are 40,320 ways that the finalists can stand in a row on the stage.

12. **Evaluate each expression.**

12. \(P(6, 6)\)

**SOLUTION:**

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

\[
P(6, 6) = \frac{6!}{(6-6)!}
\]

\[
= \frac{6!}{0!}
\]

\[
= \frac{6!}{1}
\]

\[
= 720
\]

13. \(P(5, 1)\)

**SOLUTION:**

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

\[
P(5, 1) = \frac{5!}{(5-1)!}
\]

\[
= \frac{5!}{4!}
\]

\[
= \frac{5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}
\]

\[
= 5
\]

14. \(P(4, 1)\)

**SOLUTION:**

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

\[
P(4, 1) = \frac{4!}{(4-1)!}
\]

\[
= \frac{4!}{3!}
\]

\[
= \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}
\]

\[
= 4
\]
12-6 Permutations and Combinations

15. \( P(7, 3) \)

**SOLUTION:**
\[
P(n, r) = \frac{n!}{(n-r)!}
\]
\[
P(7, 3) = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 = 210
\]

16. \( C(7, 6) \)

**SOLUTION:**
\[
C(n, r) = \frac{n!}{(n-r)!r!}
\]
\[
C(7, 6) = \frac{7!}{(7-6)!6!} = \frac{7!}{1!6!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{7}{1} = 7
\]

17. \( C(5, 3) \)

**SOLUTION:**
\[
C(n, r) = \frac{n!}{(n-r)!r!}
\]
\[
C(5, 3) = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{20}{6} = \frac{10}{3}
\]

18. \( C(5, 5) \)

**SOLUTION:**
\[
C(n, r) = \frac{n!}{(n-r)!r!}
\]
\[
C(5, 5) = \frac{5!}{(5-5)!5!} = \frac{5!}{0!5!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1
\]

**Identify each situation as a permutation or a combination.**

20. selecting 5 books to read from a list of 8

**SOLUTION:**
Selecting 5 books to read from a group of 8 implies that the order in which the books are read does not matter. So this is a combination.

21. an arrangement of the letters in the word probability

**SOLUTION:**
An arrangement of letters implies that the order of letters matters, which means that this is a permutation.
22. A list of students by class ranking

**SOLUTION:**
A list of students by class ranking is an ordering of the students by a specific method. This represents a specific permutation. Arranging the list of students by their height would be a different permutation of the list of students.

23. A playlist of songs on a digital media player

**SOLUTION:**
This is assuming that the playlist is a set of songs chosen from the song library on a computer and that the order of the songs doesn’t matter. This is a combination.

24. Selecting 4 different ingredients out of 8 for a salad

**SOLUTION:**
The order in which the ingredients are put in the salad does not matter. We are just choosing 4 from a total list of 8, which makes this a combination.

25. CCSS Tools Abigail works at the jewelry store in the mall. Her manager asks her to place 3 of the 12 birthstone necklaces in the front display case. How many ways can she arrange the necklaces in the display case?

**SOLUTION:**
The order that Abigail displays the necklaces matters, so this involves a permutation.

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

\[
P(12, 3) = \frac{12!}{(12-3)!}
\]

\[
= \frac{12!}{9!}
\]

\[
= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}
\]

\[
= 1,320
\]

Abigail can arrange the necklaces 1,320 different ways.

26. Recycling Juana is setting two recycling bins at the end of her driveway for pick-up. She has four bins from which to choose. How many ways can she pick the bins to set out?

**SOLUTION:**
There are 4 different bins and Juana is choosing 2 of them to set out.

\[
C(4, 2) = \frac{4!}{(4-2)!2!}
\]

\[
= \frac{4!}{2!2!}
\]

\[
= 6
\]

There are a total of 6 different ways she can set out two bins.

27. Music What is the probability that the first 8 songs that are played on Kenneth’s playlist are country songs?

**SOLUTION:**
Think about Tino’s song selection as having 8 country songs and 10 non-country songs. We are now choosing 8 songs out of the entire group in which the order does not matter. The total number of combinations is:

\[
C(14, 8) = \frac{14!}{8!10!}
\]

\[
= 43,758
\]

So the probability of choosing any 8 specific songs is \[
\frac{1}{43,758}
\].
12-6 Permutations and Combinations

28. **AMUSEMENT PARKS** Tino is entering an amusement park with 5 of his friends. At the gate they must go in one by one. How many ways can Tino and his friends go through in?

**SOLUTION:**
Once someone goes through the turnstile, the same person is not available to be the next in line.

Number of ways to arrange the friends = $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

There are 720 ways that Tino and his friends can go through the turnstile.

29. **PAGEANTS** The Teen Miss USA pageant has 51 delegates. If the judges choose Teen Miss USA and four runners-up, how many ways can they be chosen?

**SOLUTION:**
For the Miss Teen USA pageant, the order of the 5 winners matters, so this describes a permutation. The number of ways they can be chosen is then:

$$P(51, 4) = \frac{51!}{(51-5)!}$$

$$= \frac{51!}{46!}$$

$$= 281,887,200$$

30. **BASKETBALL** The coach had to select 5 out of 12 players on the team to start the game. How many different groups of players could be selected as starters?

**SOLUTION:**
The order of the players does not matter, so this is a combination.

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

$$C(12,5) = \frac{12!}{(12-5)!5!}$$

$$= \frac{12!}{7!5!}$$

$$= \frac{12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

$$= 95040$$

$$= 792$$

The coach can select 792 different groups of players as starters.

31. **ICE CREAM** How many ways can a customer choose 3 flavors of ice cream at The Dairy Barn?

**SOLUTION:**
There are a total of $5 + 4 + 6 = 15$ flavors of ice cream. If we are choosing 3 flavors, in no particular order, then the total number of choices that can be made is

$$C(15, 3) = \frac{15!}{(15-3)!3!}$$

$$= 455$$
12-6 Permutations and Combinations

32. GAMES Tonisha is playing a game in which you make words to score points. There are 12 letters in the box, and she must choose 4. She cannot see the letters.
   a. Suppose the 12 letters are all different. In how many ways can she choose 4?
   b. She chooses $A$, $T$, $R$, and $E$. How many different arrangements of three letters can she make?
   c. How many of the three-letter arrangements are words? List them.

SOLUTION:
   a. The order of the letters is not important, so this is a combination.

\[
C(n, r) = \frac{n!}{(n-r)!r!}
\]

\[
C(12, 4) = \frac{12!}{(12-4)!4!}
\]

\[
= \frac{12!}{8!4!}
\]

\[
= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}
\]

\[
= \frac{11880}{24}
\]

\[
= 495
\]

There are 495 ways to choose 4 letters.

b. The order of the arrangements of the three letters is important, so this is a permutation.

\[
P(n, r) = \frac{n!}{(n-r)!}
\]

\[
P(4, 3) = \frac{4!}{(4-3)!}
\]

\[
= \frac{4!}{1!}
\]

\[
= 4 \cdot 3 \cdot 2 \cdot 1
\]

\[
= 24
\]

She can make 24 different arrangements of three letters.

c. There are 9 different three-letter arrangements that are words: $ART$, $ATE$, $ARE$, $TAR$, $TEA$, $RAT$, $EAT$, $EAR$, and $ERA$.

33. DANCE At the spring dance, Christy and 7 of her friends sit on one side of a table. How many ways can Christy and her friends fill the 8 empty seats?

SOLUTION:
The order in which Christy and her 7 friends sit matters so this is a permutation problem. The number of ways they can arrange themselves is thus:

\[
P(8, 7) = \frac{8!}{(8-7)!}
\]

\[
= 8!
\]

\[
= 40,320
\]

34. HORSEBACK RIDING Trish and Charliqua entered a horseback riding camp with 22 other people. Six riders are randomly selected to work with the head instructor. How many different groups of people can be placed with the head instructor?

SOLUTION:
There are a total of 24 students (Trish and Charliqua plus 22 others). Since the order in which the students are chosen for the group does not matter, this is a combination problem.

\[
C(24, 6) = \frac{24!}{(24-6)!6!}
\]

\[
= 134,596
\]
35. **BOWLING** Chris and Kelly entered a bowling tournament.
   a. What is the probability that Chris and Kelly are selected to bowl against each other in any lane?
   b. What is the probability that Chris and Kelly are selected to bowl against each other in the last lane?

   **SOLUTION:**
   a. For any lane, 2 players out of all 32 are chosen to bowl against each other. The order in which the players are chosen does not matter, so this is a combination problem. The total number of pairs is:

   \[
   C(32, 2) = \frac{32!}{(32 - 2)!2!} = 496
   \]

   And so the probability that this pair is Chris and Kelly is \(\frac{1}{496}\).

   b. There are 32 players bowling in pairs on each lane, so there are 16 total lanes. The probability that Chris and Kelly are bowling on the last lane is the probability that Chris and Kelly are bowling together times the probability that they are on the last lane:

   \[
   P = \frac{1}{496} \cdot \frac{1}{16} = \frac{1}{7936}
   \]

36. **SECURITY** Banks lock an account after three incorrect PIN entries. Suppose a bank that uses four-digit PINs in which the digits cannot repeat allowed unlimited incorrect entries. What is the maximum time it would take a hacker using a computer program that can enter 100 different codes per second to enter the correct PIN?

   **SOLUTION:**
   The PINs are four digits (0 - 9) in which all are different. Since this is a security code, the order of the digits matters. So the total number of PIN entries is:

   \[
   P(10, 4) = \frac{10!}{(10 - 4)!} = 5040
   \]

   If a computer can enter 100 different codes per second, then the time it would take the computer to reach the last PIN would be:

   \[
   \frac{5040 \text{ codes}}{100 \text{ codes/second}} = 50.4 \text{ seconds}
   \]

37. **DECISION MAKING** Westerville High school is putting on a play. In all, 4 freshmen, 5 sophomores, 6 juniors, and 8 seniors tried out for the 12 open spots.

   a. How many ways can the 12 spots be chosen?

   b. If the students are chosen randomly, what is the probability that at least one senior will be chosen? about 99.97%

   c. What probability model can we use to randomly select the first spot?

   d. How does this model change when selecting the second spot?

   **SOLUTION:**
   a. There are a total of \(4 + 5 + 6 + 8 = 23\) people trying out for 12 spots. The order of the spots does not matter, so the total number of choices for the spots is:

   \[
   C(23, 12) = \frac{23!}{(23 - 12)!12!} = 1,352,078
   \]
12-6 Permutations and Combinations

b. The probability that at least one senior is chosen is the same as 1 minus the probability that no seniors are chosen. We know that there are 1,352,078 different ways to choose the open spots. We then need to determine how many different ways can we choose the spots without any seniors. There are 23 – 8 = 15 students that aren’t seniors trying to fill 12 spots. The number of ways this can be done is

\[ C(15, 12) = \frac{15!}{(15-12)!12!} \]

\[ = 455 \]

Therefore the probability that at least one student chosen is a senior is:

\[ 1 - \frac{455}{1,352,078} = 0.99997 \]

\[ = 99.97\% \]

c. We can use a random number generator to pick the first spot. Have the generator pick numbers between 1 and 23 with 1 – 4 being freshmen, 5 – 9 being sophomores, 10 – 15 being being juniors, and 16 – 23 being seniors.

d. When selecting a second spot, the model needs to be readjusted to account for the students that have been chosen. For example if a freshmen was chosen the first round, then the numbers could be readjusted for the second round so that 1 – 3 represent freshmen, 4 – 8 sophomores, 9 – 14 juniors, and 15 – 22 seniors. Or the numbers could stay the same and any repeated numbers that the generator comes up with could be ignored.

38. COMPpound PROBABILITY There are 7 red marbles, 8 purple marbles, and 6 green marbles in a bag. Fifteen marbles are randomly selected. Use the following steps to determine the probability of selecting 5 of each color.

a. Find the number of possible outcomes.

b. Find the number of successes for selecting 5 of 7 red marbles, for selecting 5 of 8 purple marbles, and for selecting 5 of 6 green marbles.

c. Use the Fundamental Counting Principle to find the total number of successes.

d. Determine the probability of selecting 5 of each color.

SOLUTION:

a. There are a total of 7 + 8 + 6 = 21 marbles. 15 marbles are selected at random, in which the order does not matter, so the total number of outcomes is

\[ C(21, 15) = \frac{21!}{(21-15)!15!} \]

\[ = 54,264 \]

b. The number of ways to choose 5 of 7 red marbles is:

\[ C(7, 5) = \frac{7!}{(7-5)!5!} \]

\[ = 21 \]

The number of ways to choose 5 of 8 purple marbles is

\[ C(8, 5) = \frac{8!}{(8-5)!5!} \]

\[ = 56 \]

The number of ways to choose 5 of 6 green marbles is

\[ C(6, 5) = \frac{6!}{(6-5)!5!} \]

\[ = 6 \]

c. There are 21 possibilities for red marbles, 56 possibilities for purple marbles, and 6 possibilities for green marbles, so by the fundamental counting principle the total number of ways to get 5 of each color of marble is

\[ 21 \times 56 \times 6 = 7056. \]

d. From part c and part a, the probability of getting 5 of each color is

\[ \frac{7056}{54,264} = 0.13 \]

\[ = 13\% \]
12-6 Permutations and Combinations

39. ERROR ANALYSIS Sydney and Ming are determining how many 4-person committees are possible if 10 people are available. Is either of them correct? Explain.

![Sydney and Ming images]

**SOLUTION:**
We are choosing a 4 person committee out of 10 people. The problem does not specify whether or not there are specific ranks or orders to this committee, so we are to assume that all members are equal and that order does not matter. In this case a combination should be used rather than a permutation, and the number of choices should be:

\[
\begin{align*}
C(10, 4) & = \frac{10!}{(10 - 4)!4!} \\
& = 210
\end{align*}
\]

Ming is correct.

40. CCSS perseverence Seven identical mathematics books and 4 identical science books are to be stored on one shelf. In how many different ways can the books be arranged?

**SOLUTION:**
The number of ways to store the books if they are all different is equal to 11!. However, some of the permutations will have identical objects since the 7 math books are identical and the 4 science books are identical. To remove the groups with identical objects, divide the number of ways to store the books by the number of groups with identical objects.

\[
\frac{11!}{7!4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7920
\]

\[
\frac{7920}{24} = 330
\]

There are 330 different ways to arrange the 11 books.

41. WHICH ONE DOESN'T BELONG? Determine which situation does not belong. Explain.

- Choosing 5 players on a quiz team
- Choosing 10 colored marbles from a bag
- Choosing 4 horses from 6 to run a race
- Ranking students in a senior class

**SOLUTION:**
Choosing 5 players to be on a quiz team is a combination because the order in which the players are chosen does not matter. Choosing 10 colored marbles from a bag is also a combination since the order is not specified. Choosing 4 horses to run a race out of 6 is a combination since the order of the horses does not matter. Ranking students in a senior class specifies an order involved in ranking, which is a permutation. Since this is the only permutation, this is the situation that doesn't belong.

42. REASONING Determine whether the statement \( nP_r = nC_r \) is sometimes, always, or never true. Explain your reasoning.

**SOLUTION:**
The statement \( nP_r = nC_r \) is sometimes true. It is true when \( r = 1 \) because order is irrelevant when choosing 1 item.

\[
\frac{n!}{(n - r)!} = \frac{n!}{(n - 1)!} \cdot \frac{1}{1!
\]

43. WRITING IN MATH Write a situation in which order is not important when 3 of 8 objects are being selected.

**SOLUTION:**
A student wants to join 3 clubs and has 8 to choose from. The order in which the student selects the 3 clubs does not matter in this case.
44. In how many ways can 3 of 8 different flowers be planted along one side of a road?
   A 342
   B 338
   C 336
   D 328

   SOLUTION:
The order of the flowers matters, so this involves a permutation.
   \[ P(n,r) = \frac{n!}{(n-r)!} \]
   \[ P(8,3) = \frac{8!}{(8-3)!} \]
   \[ P(8,3) = \frac{8!}{5!} \]
   \[ P(8,3) = \frac{8 \cdot 7 \cdot 6 \cdot \frac{5!}{\cancel{5!} \cdot \cancel{4!} \cdot \cancel{3!} \cdot \cancel{2!} \cdot \cancel{1!}}}{6 \cdot 5 \cdot 4 \cdot 3} \]
   \[ P(8,3) = \frac{8 \cdot 7 \cdot 6}{6 \cdot 5 \cdot 4} \]
   \[ P(8,3) = \frac{8 \cdot 7 \cdot 6}{6 \cdot 5 \cdot 4} = \frac{336}{120} = 336 \]

   There are 336 different ways the flowers can be planted. The correct choice is C.

45. If Jack can eat 21 hard-boiled eggs in 15 minutes, how many can he eat in 25 minutes if he continues eating at the same pace?
   F 18
   G 35
   H 36
   J 37

   SOLUTION:
   \[ \frac{21 \text{ eggs}}{15 \text{ minutes}} = \frac{x \text{ eggs}}{25 \text{ minutes}} \]
   \[ 21 \cdot 25 = 15 \cdot x \]
   \[ 525 = 15x \]
   \[ 35 = x \]

   If Jack continues at the same pace, he can eat 35 eggs. The correct choice is G.

46. Shante has 30 coins, quarters and dimes, that total $5.70. How many quarters does she have?
   A 12
   B 15
   C 18
   D 20

   SOLUTION:
   Let \( q \) be the number of quarters and let \( d \) be the number of dimes.
   \[ q + d = 30 \] or \[ d = 30 - q \]
   \[ .25q + .10d = 5.70 \]

   Use substitution.
   \[ 0.25q + 0.10(30 - q) = 5.70 \]
   \[ 0.25q + 3 - 0.10q = 5.70 \]
   \[ 0.15q = 2.7 \]
   \[ q = \frac{2.7}{0.15} \]
   \[ q = 18 \]

   The correct choice is C.

47. SHORT RESPONSE There are 3 red candies in a bag of 20 candies. What is the probability of selecting a red candy?

   SOLUTION:
   \[ P(\text{red candy}) = \frac{\text{number of red candies}}{\text{total number of candies}} \]
   \[ P(\text{red candy}) = \frac{3}{20} \]

   The probability of drawing a red candy is \( \frac{3}{20} \).
Vacationers were asked how many evenings they spent eating out during their trip. Find each experimental probability.

<table>
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<tr>
<th>Number of Evenings</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
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<td>14</td>
</tr>
<tr>
<td>1</td>
<td>39</td>
</tr>
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<td>21</td>
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<tr>
<td>4</td>
<td>10</td>
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<tr>
<td>5+</td>
<td>13</td>
</tr>
</tbody>
</table>

48. a vacationer ate out at least once

**SOLUTION:**
First determine the total number of vacationers that were asked: $14 + 39 + 28 + 21 + 10 + 13 = 125$.

The number of vacationers that ate out at least once is equal to 125 minus the number of vacationers that ate out 0 times: $125 - 14 = 111$. The frequency of this occurrence is

$$\frac{111}{125} = 0.888 \approx 88.8\%$$

49. a vacationer ate out less than three times

**SOLUTION:**
If a vacationer ate out less than three times, then they ate out either 0, 1, or 2 times. Using the table we can see that a total of $14 + 39 + 28 = 81$ vacationers ate out less than three times. This means that the frequency is

$$\frac{81}{125} = 0.648 \approx 64.8\%$$

50. Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 1.3.

26, 15, 19, 31, 47, 44, 38, 26, 28, 19

**SOLUTION:**
First enter the data into L₁, and then multiply the list by 1.3. Store this in L₂.

```
L₁*1.3→L₂
```

Next, calculate the 1-variable statistics of L₂.

```
1-Var Stats
\[ n=10 \]
\[ \text{mean}=38.09 \]
\[ \text{sum}^2=16279.77 \]
\[ \text{sum}=14.02899479 \]
\[ \text{mean}=13.30997817 \]
\[ \text{max}=61.1 \]
```

The mean is 38.09. The median is 35.1. The mode is 24.7. The range is 61.1 – 19.5 = 41.6. The standard deviation is 13.3.
51. **PET CARE** Kendra takes care of pets while their owners are away. One week she has three dogs that all eat the same dog food. The first dog eats a bag of food every 12 days, the second dog eats a bag every 15 days, and the third dog eats a bag every 16 days. How many same-sized bags of food should Kendra buy for one week?

**SOLUTION:**
The first dog eats one bag of dog food every 12 days, or \( \frac{1}{12} \) bag per day.
The second dog eats one bag of dog food every 15 days, or \( \frac{1}{15} \) bag per day.
The third dog eats one bag of dog food every 16 days, or \( \frac{1}{16} \) bag per day.

Find the total amount of food that the three dogs will eat in one day.

\[
\frac{1}{12} + \frac{1}{15} + \frac{1}{16} = \frac{20}{240} + \frac{16}{240} + \frac{15}{240} = \frac{51}{240}
\]

There are 7 days in one week. So, Kendra will need \( 7 \cdot \frac{51}{240} \) or \( \frac{357}{240} \) bags of dog food. This is about 1.5 bags of food, so in order to have enough Kendra will need to buy 2 bags of dog food for one week.

**Find each product.**

52. \( \frac{8}{x^2} \cdot \frac{1}{4x} \)

**SOLUTION:**

\[
\frac{8}{x^2} \cdot \frac{1}{4x} = \frac{8}{4} \cdot \frac{1}{x^2} \cdot \frac{1}{x} = \frac{2}{8} \cdot \frac{x}{x^3} = \frac{1}{4x^2}
\]

53. \( \frac{10r^3}{6n^3} \cdot \frac{42n^2}{35r^2} \)

**SOLUTION:**

\[
\frac{10r^3}{6n^3} \cdot \frac{42n^2}{35r^2} = \frac{10r^3}{6n^3} \cdot \frac{42n^2}{35r^2} = \frac{20}{21} \cdot \frac{n^2}{n^3} = \frac{20}{21} \cdot \frac{n^2}{n^3}
\]

54. \( \frac{10y^2z^2}{6wx^3} \cdot \frac{12w^2x^2}{25y^2z^4} \)

**SOLUTION:**

\[
\frac{10y^2z^2}{6wx^3} \cdot \frac{12w^2x^2}{25y^2z^4} = \frac{10y^2z^2}{6wx^3} \cdot \frac{12w^2x^2}{25y^2z^4} = \frac{24}{50} \cdot \frac{x^2}{x^3} = \frac{4w^2}{5x} z
\]

55. \( \frac{(n-1)(n+1)}{(n+1)(n-1)(n+4)} \)

**SOLUTION:**

\[
\frac{(n-1)(n+1)}{(n+1)(n-1)(n+4)} = \frac{(n-1)(n+1)}{(n+1)(n-1)(n+4)} = \frac{(n-1)(n+1)}{(n+1)(n-1)(n+4)} = \frac{n-4}{n+4}
\]

56. \( \frac{(x-8)}{(x+8)(x-3)} \cdot \frac{(x+4)(x-3)}{(x-8)} \)

**SOLUTION:**

\[
\frac{(x-8)}{(x+8)(x-3)} \cdot \frac{(x+4)(x-3)}{(x-8)} = \frac{(x-8)}{(x+8)(x-3)} \cdot \frac{(x+4)(x-3)}{(x-8)} = \frac{x+4}{x+8}
\]
57. \[
\frac{3a^2b}{2gh} \cdot \frac{24g^2h}{15ab^2} = \frac{3 \cdot 24}{15} \cdot \frac{a^2 \cdot g^2}{ah \cdot b^2} = \frac{12a}{5b}
\]

**SOLUTION:**

\[
\frac{3a^2b}{2gh} \cdot \frac{24g^2h}{15ab^2} = \frac{3a^2b \cdot 24g^2h}{15ab^2 \cdot 2gh} = \frac{12a}{5b}
\]

58. **COOKING** The formula \( t = \frac{40(25+1.85a)}{50-1.85a} \) relates the time \( t \) in minutes that it takes to cook an average-size potato in an oven to the altitude \( a \) in thousands of feet.

**a.** What is the value of \( a \) for an altitude of 4500 feet?

**b.** Calculate the time it takes to cook a potato at 3500 feet and at 7000 feet. How do your cooking times compare?

**SOLUTION:**

**a.** In the formula, \( a \) is the altitude in thousands of feet. So, the value of \( a \) for an altitude of 4500 ft is 4500 \( \div \) 1000 or 4.5.

**b.** The value of \( a \) for an altitude of 3500 ft is 3500 \( \div \) 1000 or 3.5.

\[
t = \frac{40(25+1.85a)}{50-1.85a}
\]

\[
= \frac{40[25+(1.85)(3.5)]}{50-(1.85)(3.5)}
\]

\[
= \frac{40(25+6.475)}{50-6.475}
\]

\[
= \frac{40(31.475)}{43.525}
\]

\[
\approx 29
\]

So, at an altitude of 3500 feet, it takes about 29 minutes to cook a potato.

So, at an altitude of 7000 feet, it takes about 41 minutes to cook a potato.

The cooking time at the higher altitude, 7000 feet, is longer than the cooking time at the lower altitude of 3500 feet. The difference is 41 \(-\) 29 or about 12 minutes.

**Ten red tiles, 12 blue tiles, 8 green tiles, 4 yellow tiles, and 12 black tiles are placed in a bag and selected at random. Find each probability.**

59. \( P(\text{blue}) \)

**SOLUTION:**

\[
P(\text{blue}) = \frac{\text{number of blue tiles}}{\text{total number of tiles}} = \frac{12}{46} = \frac{6}{23}
\]

60. \( P(\text{red}) \)

**SOLUTION:**

\[
P(\text{red}) = \frac{\text{number of red tiles}}{\text{total number of tiles}} = \frac{10}{46} = \frac{5}{23}
\]
12-6 Permutations and Combinations

61. \( P(\text{black or yellow}) \)

**SOLUTION:**
Since you can’t pick a tile that is both black and yellow, the events are mutually exclusive.

\[
P(\text{black}) = \frac{12}{46}
\]

\[
P(\text{yellow}) = \frac{4}{46}
\]

\[
P(\text{black or yellow}) = P(\text{black}) + P(\text{yellow})
\]

\[
= \frac{12}{46} + \frac{4}{46}
\]

\[
= \frac{16}{46}
\]

\[
= \frac{8}{23}
\]

62. \( P(\text{green or red}) \)

**SOLUTION:**
Since you can’t pick a tile that is both green and red, the events are mutually exclusive.

\[
P(\text{green}) = \frac{8}{46}
\]

\[
P(\text{red}) = \frac{10}{46}
\]

\[
P(\text{green or red}) = P(\text{green}) + P(\text{red})
\]

\[
= \frac{8}{46} + \frac{10}{46}
\]

\[
= \frac{18}{46}
\]

\[
= \frac{9}{23}
\]

63. \( P(\text{not blue}) \)

**SOLUTION:**
There are 12 blue tiles, so there are 46 – 12 or 34 tiles that are not blue.

\[
P(\text{not blue}) = \frac{34}{46}
\]

\[
= \frac{17}{23}
\]

64. \( P(\text{not green}) \)

**SOLUTION:**
There are 8 green tiles, so there are 46 – 8 or 38 tiles that are not green.

\[
P(\text{not green}) = \frac{38}{46}
\]

\[
= \frac{19}{23}
\]
Determine whether the events are independent or dependent. Then find the probability.

1. **BABYSITTING** A toy bin contains 12 toys, 8 stuffed animals, and 3 board games. Marsha randomly chooses 2 items for the child she is babysitting. What is the probability that she chose 2 stuffed animals as the first two choices?

**SOLUTION:**

The events are dependent because the outcome of the first toy choice changes the configuration of toys in the bin. So the second choice will have a different probability based on the first choice.

<table>
<thead>
<tr>
<th>First choice</th>
<th>( P(s) = \frac{\text{# stuffed}}{\text{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = \frac{8}{23} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second choice</th>
<th>( P(s) = \frac{\text{# remaining}}{\text{total remaining}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = \frac{7}{22} )</td>
</tr>
</tbody>
</table>

\( P(s, s) = P(s) \cdot P(s) \)

\( = \frac{8}{23} \cdot \frac{7}{22} \)

\( = \frac{56}{506} \)

\( = \frac{28}{253} \)

\( \approx 0.11 \)

The probability is \( \frac{28}{253} \) or about 11%.

2. **FRUIT** A basket contains 6 apples, 5 bananas, 4 oranges, and 5 peaches. Drew randomly chooses one piece of fruit, eats it, and chooses another. What is the probability that he chose a banana and then an apple?

**SOLUTION:**

The events are dependent because the outcome of the first fruit choice changes the configuration of fruit in the basket. So the second choice will have a different probability based on the first choice.

<table>
<thead>
<tr>
<th>First choice</th>
<th>( P(\text{banana}) = \frac{\text{number of bananas}}{\text{total number of fruit}} )</th>
</tr>
</thead>
</table>
|              | \( = \frac{5}{20} \)
|              | \( = \frac{1}{4} \) |

<table>
<thead>
<tr>
<th>Second choice</th>
<th>( P(\text{apple}) = \frac{\text{number of apples}}{\text{number of fruit remaining}} )</th>
</tr>
</thead>
</table>
|              | \( = \frac{6}{10} \)
|              | \( = \frac{3}{5} \) |

\( P(\text{banana, apple}) = P(\text{banana}) \cdot P(\text{apple}) \)

\( = \frac{1}{4} \cdot \frac{6}{10} \)

\( = \frac{6}{40} \)

\( = \frac{3}{20} \)

\( \approx 0.15 \)

The probability is \( \frac{3}{38} \) or about 8%.
3. **MONEY** Nakos has 4 quarters, 3 dimes, and 2 nickels in his pocket. Nakos randomly picks two coins out of his pocket. What is the probability that he did not choose a dime either time, if he replaced the first coin before choosing a second coin?

**SOLUTION:**
The events are independent. Because he replaces the first coin back in his pocket, the outcome of the first coin choice does not change the configuration of coins in his pocket. So the second choice will have the same probability of not being a dime as the first choice.

First Choice:

\[ P(\text{not a dime}) = \frac{\text{coins} - \text{dimes}}{\text{coins}} \]

\[ = \frac{9 - 3}{9} \]

\[ = \frac{6}{9} \]

\[ = \frac{2}{3} \]

Second Choice:

\[ P(\text{not a dime}) = \frac{\text{coins} - \text{dimes}}{\text{coins}} \]

\[ = \frac{9 - 3}{9} \]

\[ = \frac{6}{9} \]

\[ = \frac{2}{3} \]

\[ P(\text{no d, no d}) = P(\text{no d}) \cdot P(\text{no d}) \]

\[ = \frac{2}{3} \cdot \frac{2}{3} \]

\[ = \frac{4}{9} \]

\[ \approx 0.44 \]

The probability is \(\frac{4}{9}\) or about 44%.

4. **BOOKS** Joanna needs a book to prop up a table leg. She randomly selects a book, puts it back on the shelf, and selects another book. What is the probability that Joanna selected two math books?

**SOLUTION:**
The events are independent. Because she replaces the first book back on the bookshelf, the outcome of the first book choice does not change the configuration of books on the shelf. So the second choice will have the same probability of being a math book as the first choice.

First Choice:

\[ P(\text{math}) = \frac{\text{number (math)}}{\text{total books}} \]

\[ = \frac{8}{17} \]

Second Choice:

\[ P(\text{math}) = \frac{\text{number (math)}}{\text{total books}} \]

\[ = \frac{8}{17} \]

\[ P(\text{math, math}) = P(\text{math}) \cdot P(\text{math}) \]

\[ = \frac{8}{17} \cdot \frac{8}{17} \]

\[ = \frac{64}{289} \]

\[ \approx 0.22 \]

The probability is \(\frac{64}{289}\) or about 22%. 
A card is drawn from a standard deck of playing cards. Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.

5. \(P(\text{two or queen})\)

**SOLUTION:**

Since it is not possible to draw a card that is both a two and a queen, the events are mutually exclusive.

There are four twos in a standard deck.

\[P(\text{two}) = \frac{4}{52}\]

There are 4 queens in a standard deck.

\[P(\text{queen}) = \frac{4}{52}\]

\[P(\text{two or queen}) = P(\text{two}) + P(\text{queen})\]

\[= \frac{4}{52} + \frac{4}{52}\]

\[= \frac{8}{52}\]

\[= \frac{2}{13}\]

\[\approx 0.15\]

The probability is \(\frac{2}{13}\) or about 15%.

6. \(P(\text{diamond or heart})\)

**SOLUTION:**

Since it is not possible to draw a card that is both a diamond and a heart, the events are mutually exclusive.

There are 13 diamonds in a standard deck.

\[P(\text{diamond}) = \frac{13}{52}\]

There are 13 hearts in a standard deck.

\[P(\text{heart}) = \frac{13}{52}\]

\[P(\text{d or h}) = P(\text{d}) + P(\text{h})\]

\[= \frac{13}{52} + \frac{13}{52}\]

\[= \frac{26}{52}\]

\[= \frac{1}{2}\]

\[= 0.50\]

The probability is \(\frac{1}{2}\) or 50%.
7. $P$(seven or club)

**SOLUTION:**
Since there is a seven of clubs, the events are not mutually exclusive.

There are 4 sevens in a standard deck.

$$P(\text{seven}) = \frac{4}{52}$$

There are 13 clubs.

$$P(\text{club}) = \frac{13}{52}$$

There is one seven of clubs.

$$P(\text{seven and club}) = \frac{1}{52}$$

$$P(7 \text{ or c}) = P(7) + P(\text{c}) - P(7 \text{ and c})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

$\approx 0.31$

The probability is $\frac{4}{13}$ or about 31%.

8. $P$(spade or ace)

**SOLUTION:**
Since there is an ace of spades, the events are not mutually exclusive.

There are 4 aces in a standard deck.

$$P(\text{ace}) = \frac{4}{52}$$

There are 13 spades.

$$P(\text{spades}) = \frac{13}{52}$$

There is one ace of spades.

$$P(\text{ace and spade}) = \frac{1}{52}$$

$$P(\text{s or A}) = P(\text{A}) + P(\text{s}) - P(\text{A and s})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

$\approx 0.31$

The probability is $\frac{4}{13}$ or about 31%.
12-7 Probability of Compound Events

**Determine whether the events are independent or dependent. Then find the probability.**

9. **COINS** If a coin is tossed 4 times, what is the probability of getting tails all 4 times?

**SOLUTION:**
The events are independent because tossing the coin does not change the outcome of subsequent tosses. So each flip of the coin will have the same probability of being tails as the first flip.

Each flip:

\[
P(\text{tails}) = \frac{\text{number tails}}{\text{total sides}} = \frac{1}{2}
\]

\[
P(t, t, t, t) = P(t) \cdot P(t) \cdot P(t) \cdot P(t)
\]

\[
= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}
\]

\[
= \frac{1}{16}
\]

\[
\approx 0.06
\]

The probability is \(\frac{1}{16}\) or about 6%.

10. **DICE** A die is rolled twice. What is the probability of rolling two different numbers?

**SOLUTION:**
The events are independent because rolling a die does not change the outcome of subsequent rolls. So each roll of the die will have the same probability of being a certain number as the first roll.

First roll:

\[
P(\text{any}) = \frac{\text{any number}}{\text{total sides}} = \frac{6}{6} = 1
\]

Second roll:

\[
P(\text{not first}) = \frac{\text{any but the first roll}}{\text{total number of sides}} = \frac{5}{6}
\]

\[
P(\text{any, not first}) = P(\text{any}) \cdot P(\text{not first})
\]

\[
= 1 \cdot \frac{5}{6}
\]

\[
= \frac{5}{6}
\]

\[
\approx 0.83
\]

The probability is \(\frac{5}{6}\) or about 83%.
12-7 Probability of Compound Events

11. **CANDY** A box of chocolates contains 10 milk chocolates, 8 dark chocolates, and 6 white chocolates. Sung randomly chooses a chocolate, eats it, and then randomly chooses another. What is the probability that Sung chose a milk chocolate and then a white chocolate?

**SOLUTION:**
The events are dependent because the outcome of the first chocolate choice changes the configuration of chocolates in the box. So the second choice will have a different probability based on the first choice.

First choice:

\[ P(\text{milk}) = \frac{\text{milk}}{\text{total}} = \frac{10}{24} = \frac{5}{12} \]

Second choice:

\[ P(\text{white ch}) = \frac{\text{white}}{\text{total remaining}} = \frac{6}{23} \]

\[ P(\text{milk, white}) = P(\text{milk}) \cdot P(\text{white}) = \frac{5}{12} \cdot \frac{6}{23} = \frac{30}{276} = \frac{5}{46} \approx 0.11 \]

The probability is \( \frac{5}{46} \) or about 11%.

12. **DICE** A die is rolled twice. What is the probability of rolling the same numbers?

**SOLUTION:**
The events are independent because rolling a die does not change the outcome of subsequent rolls. So each roll of the die will have the same probability of being a certain number as the first roll.

First roll:

\[ P(\text{any}) = \frac{\text{any}}{\text{total}} = \frac{6}{6} = 1 \]

Second roll:

\[ P(\text{first roll}) = \frac{\text{same as the first roll}}{\text{total}} = \frac{1}{6} \]

\[ P(\text{any, first}) = P(\text{any}) \cdot P(\text{first}) = 1 \cdot \frac{1}{6} = \frac{1}{6} \approx 0.17 \]

The probability is \( \frac{1}{6} \) or about 17%.
13. **PETS** Chuck and Rashid went to a pet store to buy dog food. They chose from 10 brands of dry food, 6 brands of canned food, and 3 brands of pet snacks. What is the probability that they both chose dry food, if Chuck randomly chose first and liked the first brand he picked up?

**SOLUTION:**
The events are independent because the outcome of Chuck’s choice does not change the configuration of the dog foods available. They will only be dependent if Chuck takes the last remaining food for that particular brand.

Chuck:

\[ P(\text{dry food}) = \frac{\text{dry}}{\text{total}} = \frac{10}{19} \]

Rashid:

\[ P(\text{dry food}) = \frac{\text{dry}}{\text{total}} = \frac{10}{19} \]

\[ P(\text{dry, dry}) = P(\text{dry}) \cdot P(\text{dry}) = \frac{10}{19} \cdot \frac{10}{19} = \frac{100}{361} \approx 0.28 \]

The probability is about 28%.

14. **CCSS MODELING** A rental agency has 12 white sedans, 8 gray sedans, 6 red sedans, and 3 green sedans for rent. Mr. Escobar rents a sedan, returns it because the radio is broken, and gets another sedan. Assuming the returned sedan remains in circulation, what is the probability that Mr. Escobar was given a green sedan and then a gray sedan?

**SOLUTION:**
The events are dependent because Mr. Escobar will not be given the car he just returned.

First rental:

\[ P(\text{green}) = \frac{\text{green}}{\text{total}} = \frac{3}{29} \]

Second rental:

\[ P(\text{gray}) = \frac{\text{gray}}{\text{remaining sedans}} = \frac{8}{28} \]

\[ P(\text{green, grey}) = P(\text{green}) \cdot P(\text{gray}) = \frac{3}{29} \cdot \frac{8}{28} = \frac{24}{812} = \frac{6}{263} \approx 0.03 \]

The probability is \( \frac{6}{263} \) or about 3%.
12-7 Probability of Compound Events

Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.

15. BOWLING Cindy’s bowling records indicate that for any frame, the probability that she will bowl a strike is 30%, a spare 45%, and neither 25%. What is the probability that she will bowl either a spare or a strike for any given frame?

**SOLUTION:**
Since it is not possible to bowl both a spare and a strike in the same frame, the events are mutually exclusive.

The probability that she will bowl a spare is 45%.

\[ P(\text{spare}) = \frac{45}{100} = \frac{9}{20} \]

The probability that she will bowl a strike is 30%.

\[ P(\text{strike}) = \frac{30}{100} = \frac{3}{10} \]

\[ P(\text{spare or strike}) = P(\text{spare}) + P(\text{strike}) \]

\[ = \frac{9}{20} + \frac{3}{10} \]

\[ = \frac{9}{20} + \frac{6}{20} \]

\[ = \frac{15}{20} \]

\[ = \frac{3}{4} \]

\[ = 0.75 \]

The probability is \( \frac{3}{4} \) or 75%.

16. SPORTS CARDS Dario owns 145 baseball cards, 102 football cards, and 48 basketball cards. What is the probability that he randomly selects a baseball or a football card?

**SOLUTION:**
Since it is not possible for a card to be both a baseball card and a football card, the events are mutually exclusive.

The number of baseball cards is 145.

\[ P(\text{baseball}) = \frac{145}{295} \]

The number of football cards is 102.

\[ P(\text{football}) = \frac{102}{295} \]

\[ P(\text{b or f}) = P(\text{b}) + P(\text{f}) \]

\[ = \frac{145}{295} + \frac{102}{295} \]

\[ = \frac{247}{295} \]

\[ \approx 0.84 \]

The probability is \( \frac{247}{295} \) or about 84%.
12-7 Probability of Compound Events

17. SCHOLARSHIPS 3000 essays were received for a $5000 college scholarship. 2865 essays were the required length, 2577 of the applicants had the minimum required grade-point average, and 2486 had the required length and minimum grade-point average. What is the probability that an essay selected at random will have the required length or the required grade-point average?

SOLUTION:
Since it is possible for an applicant to have both an essay of the required length and the minimum required grade-point average, the events are not mutually exclusive.

There are 2865 essays of the required length.
\[ P(1) = \frac{2865}{3000} \]

There are 2577 applicants with the minimum required grade-point average.
\[ P(gpa) = \frac{2577}{3000} \]

There are 2486 applicants with both an essay of the required length and the minimum required grade-point average.
\[ P(1 \text{ and } gpa) = \frac{2486}{3000} \]

\[ P(1 \text{ or } gpa) = P(1) + P(gpa) - P(1 \text{ and } gpa) \]
\[ = \frac{2865}{3000} + \frac{2577}{3000} - \frac{2486}{3000} \]
\[ = \frac{2956}{3000} \]
\[ = \frac{739}{750} \]
\[ \approx 0.985 \]

The probability is \( \frac{739}{750} \) or about 98.5%.

18. KITTENS Ruby’s cat had 8 kittens. The litter included 2 orange females, 3 mixed-color females, 1 orange male, and 2 mixed-color males. Ruby wants to keep one kitten. What is the probability that she randomly chooses a kitten that is female or orange?

SOLUTION:
Since it is possible for a kitten to be both a female and orange, the events are not mutually exclusive.

There are 5 female kittens.
\[ P(\text{female}) = \frac{5}{8} \]

There are 3 orange kittens.
\[ P(\text{orange}) = \frac{3}{8} \]

There are 2 orange female kittens.
\[ P(\text{f and o}) = \frac{2}{8} \]

\[ P(\text{f or o}) = P(\text{f}) + P(\text{o}) - P(\text{f and o}) \]
\[ = \frac{5}{8} + \frac{3}{8} - \frac{2}{8} \]
\[ = \frac{6}{8} \]
\[ = \frac{3}{4} \]
\[ = 0.75 \]

The probability is \( \frac{3}{4} \) or 75%.
12-7 Probability of Compound Events

CHIPS A restaurant serves red, blue, and yellow tortilla chips. The bowl of chips Gabriel receives has 10 red chips, 8 blue chips, and 12 yellow chips. After Gabriel chooses a chip, he eats it. Find each probability.

19. \( P(\text{red, blue}) \)

**SOLUTION:**

First chip:

\[ P(\text{red}) = \frac{\text{number of red chips}}{\text{total number of chips}} = \frac{10}{30} = \frac{1}{3} \]

Second chip:

\[ P(\text{blue}) = \frac{\text{number of blue chips}}{\text{number of chips remaining}} = \frac{8}{29} \]

\[ P(\text{red, blue}) = \frac{10 \cdot 8}{30 \cdot 29} = \frac{80}{870} = \frac{8}{87} \approx 0.09 \]

The probability is \( \frac{8}{87} \) or about 9%.

20. \( P(\text{blue, yellow}) \)

**SOLUTION:**

First chip:

\[ P(\text{blue}) = \frac{\text{number of blue chips}}{\text{total number of chips}} = \frac{8}{30} \]

Second chip:

\[ P(\text{yellow}) = \frac{\text{number of yellow chips}}{\text{number of chips remaining}} = \frac{12}{29} \]

\[ P(\text{blue, yellow}) = \frac{8 \cdot 12}{30 \cdot 29} = \frac{96}{870} = \frac{16}{145} \approx 0.11 \]

The probability is \( \frac{16}{145} \) or about 11%.
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21. \( P(\text{yellow, not blue}) \)

**SOLUTION:**

First chip:

\[
P(\text{yellow}) = \frac{\text{number of yellow chips}}{\text{total number of chips}} = \frac{12}{30} = \frac{2}{5}
\]

Second chip:

\[
P(\text{not blue}) = \frac{\text{non-blue chips remaining}}{\text{chips remaining}} = \frac{21}{29}
\]

\[
P(\text{yellow, not blue}) = \frac{12}{30} \cdot \frac{21}{29} = \frac{252}{870} = \frac{42}{145} \approx 0.29
\]

The probability is \( \frac{42}{145} \) or about 29%.

22. \( P(\text{red, not yellow}) \)

**SOLUTION:**

First chip:

\[
P(\text{red}) = \frac{\text{number of red chips}}{\text{total number of chips}} = \frac{10}{30} = \frac{1}{3}
\]

Second chip:

\[
P(\text{not yellow}) = \frac{\text{non-yellow chips remaining}}{\text{chips remaining}} = \frac{17}{29}
\]

\[
P(\text{red, not yellow}) = \frac{10}{30} \cdot \frac{17}{29} = \frac{170}{870} = \frac{17}{87} \approx 0.20
\]

The probability is \( \frac{17}{87} \) or about 20%.

23. **SOCKS** Damon has 14 white socks, 6 black socks, and 4 blue socks in his drawer. If he chooses two socks at random, what is the probability that the first two socks are white?

**SOLUTION:**

Since it is possible to have two socks of different colors or two socks of the same color, the events are not mutually exclusive.

First choice:

\[
P(\text{white}) = \frac{\text{white}}{\text{total}} = \frac{14}{24} = \frac{7}{12}
\]

Second choice:

\[
P(\text{white}) = \frac{\text{white remaining}}{\text{total remaining}} = \frac{13}{23}
\]

\[
P(\text{white, white}) = \frac{14}{24} \cdot \frac{13}{23} = \frac{182}{552} = \frac{91}{276} \approx 0.33
\]

The probability is \( \frac{91}{276} \) or about 33%.

**CCSS TOOLS** Cards are being randomly drawn from a standard deck of cards. Find each probability.

24. \( P(\text{heart or spade}) \)

**SOLUTION:**

These are mutually exclusive.

\[
P(\text{heart or spade}) = P(\text{heart}) + P(\text{spade}) = 0.25 + 0.25 = 0.50
\]

25. \( P(\text{spade or club}) \)

**SOLUTION:**

These are mutually exclusive.

\[
P(\text{spade or club}) = P(\text{spade}) + P(\text{club}) = 0.25 + 0.25 = 0.50
\]
26. \( P(\text{queen or heart}) \)

**SOLUTION:**
These are not mutually exclusive.

\[
P(\text{queen or heart}) = P(\text{queen}) + P(\text{heart}) - P(\text{queen of hearts}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \approx 31\%.
\]

27. \( P(\text{jack or spade}) \)

**SOLUTION:**
These are not mutually exclusive.

\[
P(\text{jack or spade}) = P(\text{jack}) + P(\text{spade}) - P(\text{jack of spades}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \approx 31\%.
\]

28. \( P(\text{five or prime number}) \)

**SOLUTION:**
These are mutually exclusive if we take the 5s out of the prime number list. In actuality, since all 5s are prime, we are looking for the probability of any prime number.

\[
P(5 \text{ or prime}) = P(5) + P(2, 3, 7, \ldots) = \frac{4}{52} + \frac{12}{52} = \frac{16}{52} \approx 31\%.
\]

29. \( P(\text{ace or black}) \)

**SOLUTION:**
These are not mutually exclusive.

\[
P(\text{ace or black}) = P(\text{ace}) + P(\text{black}) - P(\text{ace of spades or clubs}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} \approx 54\%.
\]

30. **CANDY** A bag contains 10 red, 6 green, 7 yellow, and 5 orange jelly beans. What is the probability of randomly choosing a red jelly bean, replacing, randomly choosing another red jelly bean, replacing, and then randomly choosing an orange jelly bean?

**SOLUTION:**

\[
P(\text{red}) = \frac{10}{28} = \frac{5}{14} = \frac{5}{28}
\]

\[
P(\text{orange}) = \frac{5}{28}
\]

\[
P(\text{red, red, orange}) = \frac{5}{14} \cdot \frac{5}{14} \cdot \frac{5}{28} = \frac{125}{5488} \approx 0.02
\]

The probability is \( \frac{125}{5488} \) or about 2%. 

---

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- **Problem 26:** \( P(\text{queen or heart}) \)
  - **Solution:**
    - Not mutually exclusive.
    - \( P(\text{queen or heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \approx 31\% \)

- **Problem 27:** \( P(\text{jack or spade}) \)
  - **Solution:**
    - Not mutually exclusive.
    - \( P(\text{jack or spade}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \approx 31\% \)

- **Problem 28:** \( P(\text{five or prime number}) \)
  - **Solution:**
    - Mutually exclusive if 5s taken out, but all 5s are prime.
    - \( P(5 \text{ or prime}) = \frac{4}{52} + \frac{12}{52} = \frac{16}{52} \approx 31\% \)

- **Problem 29:** \( P(\text{ace or black}) \)
  - **Solution:**
    - Not mutually exclusive.
    - \( P(\text{ace or black}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} \approx 54\% \)

- **Problem 30:** A bag contains 10 red, 6 green, 7 yellow, and 5 orange jelly beans. What is the probability of randomly choosing a red jelly bean, replacing, randomly choosing another red jelly bean, replacing, and then randomly choosing an orange jelly bean?
  - **Solution:**
    - \( P(\text{red}) = \frac{10}{28} = \frac{5}{14} = \frac{5}{28} \)
    - \( P(\text{orange}) = \frac{5}{28} \)
    - \( P(\text{red, red, orange}) = \frac{5}{14} \cdot \frac{5}{14} \cdot \frac{5}{28} = \frac{125}{5488} \approx 0.02 \)
    - Probability: \( \frac{125}{5488} \) or about 2%.
31. **SPORTS** The extracurricular activities in which the senior class at Valley View High School participate are shown in the Venn diagram.

![Venn Diagram](image)

**Diagram Description:**
- Drama: 38
- Band: 51
- Athletics: 137
- 67 overlap between Drama and Band
- 10 overlap between Band and Athletics
- 8 overlap between Drama and Athletics
- 4 overlap between Drama, Band, and Athletics

a. How many students are in the senior class?
b. How many students participate in athletics?
c. If a student is randomly chosen, what is the probability that the student participates in athletics or drama?
d. If a student is randomly chosen, what is the probability that the student participates in only drama and band?

**SOLUTION:**

a. There are 38 + 30 + 51 + 10 + 4 + 8 + 137 + 67, or 345 students in the senior class.
b. There are 137 + 4 + 8 + 10, or 159 students who participate in athletics.
c. 

\[
P(a \text{ or } d) = P(a) + P(d) - P(a \text{ and } d)
\]

\[
= \frac{30}{345} + \frac{51}{345} - \frac{14}{345}
\]

\[
= \frac{227}{345}
\]

\[
\approx 0.66
\]

The probability that a student participates in athletics or drama is \(\frac{227}{345}\) or about 66%.
d. 

\[
P(d \text{ and } b) = \frac{10}{345}
\]

\[
= \frac{2}{23}
\]

\[
\approx 0.09
\]

The probability that a student participates in only drama and band is \(\frac{2}{23}\) or about 9%.

32. **TILES** Kirsten and José are playing a game. Kirsten places tiles numbered 1 to 50 in a bag. José selects a tile at random. If he selects a prime number or a number greater than 40, then he wins the game. What is the probability that José will win on his first turn?

**SOLUTION:**

\[
P(pr \text{ or } > 40) = P(pr) + P(> 40) - P(pr \text{ and } > 40)
\]

\[
= \frac{15}{50} + \frac{10}{50} - \frac{3}{50}
\]

\[
= \frac{52}{50}
\]

\[
= \frac{11}{25}
\]

The probability that José will win on his first turn is \(\frac{11}{25}\) or 44%.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore conditional probability. Conditional probability is the probability that event \(B\) occurs given that event \(A\) has already occurred. It is calculated by dividing the probability of the occurrence of both events by the probability of the occurrence of the first event. The notation for conditional probability is \(P(B|A)\), read the probability of \(B\) given \(A\).

a. **GRAPHICAL** Draw a Venn diagram to illustrate \(P(A \text{ and } B)\).
b. **VERBAL** Tell how to find \(P(B|A)\) given the Venn diagram.
c. **ANALYTICAL** A jar contains 12 marbles, of which 8 marbles are red and 4 marbles are green. If marbles are chosen without replacement, find \(P(\text{red})\) and \(P(\text{red, green})\).
d. **ANALYTICAL** Using the probabilities from part c and the Venn diagram in part a, determine the probability of choosing a green marble on the second selection, given that the first marble selected was red.
e. **ANALYTICAL** Write a formula for finding a conditional probability.
f. **ANALYTICAL** Use the formula from part e to answer the following: At a basketball game, 80% of the fans cheered for the home team. In the same
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crowd, 20% of the fans were waving banners and cheering for the home team. What is the probability that a fan waved a banner given that the fan cheered for the home team?

**SOLUTION:**
a. \( P(A \text{ and } B) \) will be represented by the intersection of \( A \) and \( B \).

\[
\begin{align*}
\text{A} & \quad \text{B} \\
P(A \text{ and } B) & \\
& \\
&
\end{align*}
\]

b. Divide the overlap of the two circles by the \( P(A) \) circle. We are given \( P(A) \), so this circle is now our universe. The overlap of the two circles is in the numerator, while the universe is in the denominator.

c. If they are drawn without replacement, then they are dependent and the second drawing is affected by the first drawing.

\[
P(\text{red}) = \frac{8}{12} \text{ or } \frac{2}{3}
\]

\[
P(\text{red, green}) = \frac{2}{3} \cdot \frac{4}{11} = \frac{8}{33}
\]

d.

\[
P(\text{green} | \text{red}) = \frac{\frac{8}{33}}{\frac{2}{3}} \cdot \frac{2}{2} = \frac{4}{11}
\]

e. Using the answer for part b and writing in algebraic form, the probability that event \( B \) occurs given that event \( A \) has occurred is:

\[
P(B | A) = \frac{P(A \text{ and } B)}{P(A)}
\]

\[
f. P(B | A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{20}{80} = \frac{1}{4} = 0.25 \text{ or } 25\%.
\]

The probability that a fan waved a banner given that the fan cheered for the home team is 25%.

34. **ERROR ANALYSIS** George and Aliyah are determining the probability of randomly choosing a blue or red marble from a bag of 8 blue marbles, 6 red marbles, 8 yellow marbles, and 4 white marbles. Is either of them correct? Explain.

George:

\[
P(\text{blue or red}) = P(\text{blue}) \cdot P(\text{red}) = \frac{8}{26} \cdot \frac{6}{26} = \frac{48}{676} \approx 7\%
\]

Aliyah:

\[
P(\text{blue or red}) = P(\text{blue}) + P(\text{red}) = \frac{8}{26} + \frac{6}{26} = \frac{14}{26} \approx 54\%
\]

**SOLUTION:**

Aliyah is correct. The individual probabilities should be added since we want the probability of blue or red. These events are independent because one event does not affect the other. They are also mutually exclusive because they cannot occur at the same time.
35. **CHALLENGE** In some cases, if one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance that the whole string will light?

**SOLUTION:**
These events are independent because one bulb not working does not make another bulb not work (although it does turn off the entire string).

For the entire string to work, every bulb must also work. To find the probability of the string working, we need to multiply the probabilities of each individual bulb together. The probability for each is 99.5% or 0.995.

We need to find the number of bulbs $n$ that we can have on a string in which the probability of the string working $0.995^n = 0.90$.

We can make a table of values to find when $0.995n = 0.90$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$0.995^{10} = .95111$</td>
</tr>
<tr>
<td>20</td>
<td>$0.995^{15} = .92756$</td>
</tr>
<tr>
<td>21</td>
<td>$0.995^{21} = .90008$</td>
</tr>
</tbody>
</table>

Therefore, the maximum number of lights is 21.

36. **CCSS REGULARITY** Suppose there are three events $A$, $B$, and $C$ that are not mutually exclusive. List all of the probabilities you would need to consider in order to calculate $P(A$ or $B$ or $C)$. Then write the formula you would use to calculate it.

**SOLUTION:**
Probabilities: $P(A)$, $P(B)$, $P(C)$, $P(A$ and $B)$, $P(A$ and $C)$, $P(B$ and $C)$, $P(A$ and $B$ and $C)$
Formula: $P(A$ or $B$ or $C) = P(A) + P(B) + P(C) - P(A$ and $B) - P(A$ and $C) - P(B$ and $C) + P(A$ and $B$ and $C)$

37. **OPEN ENDED** Describe a situation in your life that involves dependent and independent events. Explain why the events are dependent or independent.

**SOLUTION:**
When the outcome of one event affects the outcome of another event, those events are dependent.

When the outcome of one event does not affect the outcome of another event, those events are independent.

Sample answer: Choosing a CD to listen to, putting it back, and then choosing another CD to listen to would be an independent event since the CD was placed back before the second CD was chosen. Choosing a pair of jeans to wear would be a dependent event if I did not like the first pair chosen, and I did not put them back.

38. **WRITING IN MATH** Explain why the subtraction occurs when finding the probability of two events that are not mutually exclusive.

**SOLUTION:**
Mutually exclusive events cannot occur at the same time.

Since these events are not mutually exclusive, we need to consider every combination of the events.

If two events, like drawing a king and a spade, are not mutually exclusive, they have item(s) in common. So the subtraction is needed to get rid of the items counted twice. The king of spades is a king and it is a spade, so it needs to be subtracted from the probability so it is not counted twice.
39. In how many ways can a committee of 4 be selected from a group of 12 people?
   A 48
   B 483
   C 495
   D 11,880

   SOLUTION:
   This is a combination because order does not matter. It does not matter if a member is selected 1st, 2nd, 3rd, or 4th.

   \[
   C(12, 4) = \frac{12!}{(12 - 4)4!} = \frac{12!}{8!(4!)} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{1,1880}{24} = 495
   \]

   There are 495 ways to select the committee. The correct choice is C.

40. A total of 925 tickets were sold for $5925. If adult tickets cost $7.50 and children's tickets cost $3.00, how many adult tickets were sold?
   F 700
   G 600
   H 325
   J 225

   SOLUTION:
   Let \( x \) represent the cost of an adult ticket and \( y \) represent the cost of a children’s ticket. Use the total number of tickets and the cost of the tickets to write and solve 2 equations with the 2 variables.

   \[
   \begin{align*}
   x + y &= 925 \\
   y &= 925 - x \\
   x(7.50) + y(3.00) &= 5925 \\
   x(7.50) + (925 - x)(3.00) &= 5925 \\
   7.5x + 2775 - 3x &= 5925 \\
   4.5x &= 3150 \\
   x &= 700
   \end{align*}
   \]

   There were 700 adult tickets sold. The correct choice is F.

41. SHORT RESPONSE A circular swimming pool with a diameter of 28 feet has a deck of uniform width built around it. If the area of the deck is \( 60\pi \) square feet, find its width.

   SOLUTION:
   If the diameter of the pool is 28 feet, then the radius is 14 feet. So the area of the pool is \( A = \pi r^2 = \pi(14)^2 \) or 196\( \pi \). The area of the deck is the combined area of the deck and pool minus the area of the pool. Let \( A_1 \) represent the combined area of the deck and pool.

   \[
   60\pi = A_1 - 196\pi \\
   256\pi = A_1 \\
   256 = \pi r^2 \\
   256 = r^2 \\
   16 = r
   \]

   The radius of the deck and pool is 16 feet, so the total width of the deck is \( 16 - 14 \) or 2.

   Therefore, the width of the deck is 2 ft.

42. The probability of heads landing up when you flip a coin is \( \frac{1}{2} \). What is the probability of getting tails if you flip it again?
   A \( \frac{1}{4} \)
   B \( \frac{1}{3} \)
   C \( \frac{1}{2} \)
   D \( \frac{3}{4} \)

   SOLUTION:
   The probability of a coin flip is always \( \frac{1}{2} \) because the flips are not dependent on the previous flip.

   The correct choice is C.
43. **SHOPPING** The Millers have twelve grandchildren, 5 boys and 7 girls. For their anniversary, the grandchildren decided to pool their money and have three of them shop for the entire group.
   a. Does this situation represent a **combination** or **permutation**?
   b. How many ways are there to choose the three?
   c. What is the probability that all three will be girls?

**SOLUTION:**
   a. This is a combination because the order in which they are chosen does not matter. If Richie was selected first, it would be no different than if was selected third. He is still shopping.

   \[
   C(12,3) = \frac{12!}{(12 - 3)!3!}
   \]

   \[
   = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)}
   \]

   \[
   = \frac{120}{6}
   \]

   \[
   = 220
   \]

   There are 220 ways to choose the three.

   c. 

   \[
   P(\text{girl, girl, girl}) = \frac{7 \cdot 6 \cdot 5}{12 \cdot 11 \cdot 10}
   \]

   \[
   = \frac{210}{1320}
   \]

   \[
   \approx 0.16
   \]

   The probability is \( \frac{210}{1320} \) or about 16%.

44. **HAIR STYLIST** Tia is a hair stylist. Last week, 70% of her clients who called to make appointments made an appointment for a basic haircut.
   a. Design a simulation that can be used to estimate the probability that the next client that makes an appointment will schedule a basic haircut.
   b. Conduct the simulation, and report the results.

**SOLUTION:**
   a. The theoretical probability that the next client that makes an appointment will schedule a basic haircut is 70%, and the theoretical probability that the client will make an appointment for another service is 30%.

   While a spinner could be created to reflect this situation, a random number generator is the best method to use.

   Use a random number generator to generate integers 1 through 10. The integers 1–7 will represent a basic haircut, and the integers 8–10 will represent another service. The simulation will consist of 40 trials.

   b. Tally the results and record them in a graph.

   Sample answer: \( P(\text{haircut}) = 65\% \), \( P(\text{other}) = 35\% \)
Solve each equation. State any extraneous solutions.

45. \( \frac{4}{a} = \frac{3}{a-2} \)

**SOLUTION:**
The LCD is \( a(a - 2) \).

\[ \frac{4}{a} = \frac{3}{a-2} \]
\[ a(a - 2) \left( \frac{4}{a} \right) = a(a - 2) \left( \frac{3}{a-2} \right) \]
\[ 4a - 8 = 3a \]
\[ a = 8 \]

The solution is \( a = 8 \).

46. \( \frac{3}{x} = \frac{1}{x-2} \)

**SOLUTION:**
The LCD is \( x(x - 2) \).

\[ \frac{3}{x} = \frac{1}{x-2} \]
\[ x(x - 2) \left( \frac{3}{x} \right) = x(x - 2) \left( \frac{1}{x-2} \right) \]
\[ 3x - 6 = x \]
\[ 2x = 6 \]
\[ x = 3 \]

The solution is \( x = 3 \).

47. \( \frac{x}{x+1} = \frac{x-6}{x-1} \)

**SOLUTION:**
The LCD is \((x + 1)(x - 1)\).

\[ \frac{x}{x+1} = \frac{x-6}{x-1} \]
\[ (x+1)(x-1) \left( \frac{x}{x+1} \right) = (x+1)(x-1) \left( \frac{x-6}{x-1} \right) \]
\[ (x+1)(x-1) = (x+1)(x-1) \left( \frac{x-6}{x-1} \right) \]
\[ x^2 - x = x^2 - 6x + 6 \]
\[ 4x = -6 \]
\[ x = -\frac{3}{2} \]

The solution is \( x = -\frac{3}{2} \).

48. \( \frac{2n + 1}{3} = \frac{2n - 3}{6} \)

**SOLUTION:**
The LCD is 6.

\[ \frac{2n + 1}{3} = \frac{2n - 3}{6} \]
\[ 6 \left( \frac{2n + 1}{3} \right) = 6 \left( \frac{2n - 3}{6} \right) \]
\[ 2(2n + 1) = 2n - 3 \]
\[ 4n + 3 = 2n - 3 \]
\[ 2n = -6 \]
\[ n = -3 \]

The solution is \( n = -3 \).
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49. **COOKING** Hannah was making candy using a two-quart pan. As she stirred the mixture, she noticed that the pan was about \( \frac{2}{3} \) full. If each piece of candy has a volume of about \( \frac{3}{4} \) ounce, approximately how many pieces of candy will Hannah make? (Hint: There are 32 ounces in a quart.)

**SOLUTION:**
\[
\begin{align*}
\frac{2}{3} \cdot 64 &= \frac{128}{3} \\
\frac{3}{4} x &= \frac{128}{3} \\
x &= \frac{128}{3} \cdot \frac{4}{3} \\
x &= \frac{512}{9} \\
x &= 56.9
\end{align*}
\]
Hannah can make about 57 pieces of candy.

50. **GEOMETRY** A rectangle has a width of \( 3\sqrt{5} \) centimeters and a length of \( 4\sqrt{10} \) centimeters. Find the area of the rectangle. Write as a simplified radical expression.

**SOLUTION:**
\[
A = l \cdot w \\
= 3\sqrt{5} \cdot 4\sqrt{10} \\
= 12\sqrt{50} \\
= 12\sqrt{25 \cdot 2} \\
= 12(5)\sqrt{2} \\
= 60\sqrt{2}
\]
The area is \( 60\sqrt{2} \text{ cm}^2 \).

51. \( \sqrt{-3a} = 6 \)

**SOLUTION:**
\[
\begin{align*}
\sqrt{-3a} &= 6 \\
(-3a)^{2} &= 6^2 \\
-3a &= 36 \\
a &= -12
\end{align*}
\]
Check.
\[
\begin{align*}
\sqrt{-3(-12)} &= 6 \\
\sqrt{36} &= 6 \\
6 &= 6
\end{align*}
\]

52. \( \sqrt{a} = 100 \)

**SOLUTION:**
\[
\begin{align*}
\sqrt{a} &= 100 \\
(a)^{2} &= 100^2 \\
a &= 10,000
\end{align*}
\]
Check.
\[
\begin{align*}
\sqrt{10000} &= 100 \\
100 &= 100
\end{align*}
\]

53. \( \sqrt{-k} = 4 \)

**SOLUTION:**
\[
\begin{align*}
\sqrt{-k} &= 4 \\
(-k)^{2} &= 4^2 \\
-k &= 16 \\
k &= -16
\end{align*}
\]
Check.
\[
\begin{align*}
\sqrt{-(-16)} &= 4 \\
4 &= 4
\end{align*}
\]
54. \(5\sqrt{2} = \sqrt{x}\)

**SOLUTION:**

\[
5\sqrt{2} = \sqrt{x} \\
(5\sqrt{2})^2 = (\sqrt{x})^2 \\
25 \cdot 2 = x \\
x = 50
\]

Check.

\[
5\sqrt{2} = \sqrt{50} \\
5\sqrt{2} = \sqrt{25 \cdot 2} \\
5\sqrt{2} = 5\sqrt{2}
\]

55. \(3\sqrt{7} = \sqrt{-y}\)

**SOLUTION:**

\[
3\sqrt{7} = \sqrt{-y} \\
(3\sqrt{7})^2 = (\sqrt{-y})^2 \\
9 \cdot 7 = -y \\
y = -63
\]

Check.

\[
3\sqrt{7} = \sqrt{-(-63)} \\
3\sqrt{7} = \sqrt{9 \cdot 7} \\
3\sqrt{7} = 3\sqrt{7}
\]

56. \(3\sqrt{4a} - 2 = 10\)

**SOLUTION:**

\[
3\sqrt{4a} - 2 = 10 \\
3\sqrt{4a} = 12 \\
\sqrt{4a} = 4 \\
2\sqrt{a} = 4 \\
\sqrt{a} = 2 \\
(\sqrt{a})^2 = 2^2 \\
a = 4
\]

Check.

\[
3\sqrt{4 \cdot 4} - 2 = 10 \\
3 \cdot 4 - 2 = 10 \\
12 - 2 = 10 \\
10 = 10
\]
1. GPS A car dealership surveys 10,000 of its customers who have a GPS system to ask how often they have used the system within the past year. The results are shown.

![Customers Using the GPS System Table]

a. Find the probability that a randomly chosen customer will have used the GPS system more than 20 times.
b. Find the probability that a randomly chosen customer will have used the GPS system no more than 10 times.

SOLUTION:
a. $X$ is the number of uses.

Divide the occurrences of $X$ being greater than 20 by the total number of occurrences.

$$\frac{X > 20}{\text{total customers}} = \frac{470}{10,000} = 4.7\%$$

b. $X$ is the number of uses.

Divide the occurrences of $X$ being less than 10 by the total number of occurrences.

$$\frac{X < 10}{\text{total customers}} = \frac{1382 + 2350 + 2010}{10,000} = \frac{5742}{10,000} \approx 57.4\%$$

2. JEANS A fashion boutique ordered jeans with different numbers of stripes down the outside seams. The table shows the probability distribution of the number of each type of jean sold in a particular week.

![Types of Jeans Sold Table]

a. Show that the distribution is valid.
b. What is the probability that a randomly chosen pair of jeans has fewer than 3 stripes?
c. Make a probability graph of the data.

SOLUTION:
a. The probabilities of each value of $X$ need to be between 0 and 1. The sum of the probabilities for all values of $X$ must be 1.

All of the values are between 0 and 1; $0.15 + 0.19 + 0.26 + 0.22 + 0.18 = 1$.

b. $X$ is the number of stripes.

We are given the probabilities, so find the sum of the probabilities in which $X$ is less than 3.

$0.26 + 0.19 + 0.15 = 0.60$

c. Use the data from the probability distribution table to draw the graph. Remember to label each axis and give the graph a title.

![Striped Jeans Graph]

3. CCSS ARGUMENTS The producers of a game show provided the probability of winning the prizes for one of the games.
12-8 Probability Distributions

<table>
<thead>
<tr>
<th>Prize Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000</td>
<td>1 in 80</td>
</tr>
<tr>
<td>$5000</td>
<td>1 in 200</td>
</tr>
<tr>
<td>$25,000</td>
<td>1 in 1000</td>
</tr>
<tr>
<td>$100,000</td>
<td>1 in 10,000</td>
</tr>
</tbody>
</table>

a. Create a probability distribution.

b. Calculate the expected value.

c. Interpret your results.

**SOLUTION:**

<table>
<thead>
<tr>
<th>Prize Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000</td>
<td>1 in 80</td>
</tr>
<tr>
<td>$5000</td>
<td>1 in 200</td>
</tr>
<tr>
<td>$25,000</td>
<td>1 in 1000</td>
</tr>
<tr>
<td>$100,000</td>
<td>1 in 10,000</td>
</tr>
</tbody>
</table>

a. Determine the probability of winning each of the prizes.

Let \( X \) be the amount of money won.

\[
P(X = 1000) = \frac{1}{80} = 0.0125
\]

\[
P(X = 5000) = \frac{1}{200} = 0.005
\]

\[
P(X = 25,000) = \frac{1}{1000} = 0.001
\]

\[
P(X = 100,000) = \frac{1}{10,000} = 0.0001
\]

\[
P(X = 0) = 1 - (0.0125 + 0.005 + 0.001 + 0.0001) = 0.9814
\]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( P(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000</td>
<td>0.0125</td>
</tr>
<tr>
<td>$5000</td>
<td>0.005</td>
</tr>
<tr>
<td>$25,000</td>
<td>0.001</td>
</tr>
<tr>
<td>$100,000</td>
<td>0.0001</td>
</tr>
<tr>
<td>$0</td>
<td>0.9814</td>
</tr>
</tbody>
</table>

b. 

\[
E(X) = 0(0.9814) + 1000(0.0125) + 5000(0.005) + 25,000(0.001) + 100,000(0.0001) = 72.50
\]

c. The expected value for the game is $72.50, so the game show producers plan to lose this amount every time the game is played.

4. **HOME THEATER** An electronics store sells the components and speakers for home theaters. The store surveyed its customers to see how many of the 10 components they bought. The results are shown.

<table>
<thead>
<tr>
<th>Home Theater Components Purchased</th>
</tr>
</thead>
<tbody>
<tr>
<td>Components</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>0-2</td>
</tr>
<tr>
<td>3-4</td>
</tr>
<tr>
<td>5-6</td>
</tr>
<tr>
<td>7-8</td>
</tr>
<tr>
<td>9-10</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Find the probability that a randomly chosen customer bought 5 or 6 components.

b. Find the probability that a randomly chosen customer bought fewer than 5 components.

**SOLUTION:**

a. \( X \) is the number of components.

Divide the occurrences of \( X \) being 5 or 6 by the total number of occurrences.

\[
\frac{5 \text{ or } 6}{\text{total customers}} = \frac{33}{165} = 0.20\%
\]

b. \( X \) is the number of components.

Divide the occurrences of \( X \) being less than 5 by the total number of occurrences.

\[
\frac{X < 5}{\text{total customers}} = \frac{26 + 42}{165} = \frac{68}{165} \approx 41.2\%
\]
5. **FOOD DRIVE** Ms. Valdez’s biology class held a food drive. The class kept track of the types of food donated.

<table>
<thead>
<tr>
<th>Product</th>
<th>Count</th>
<th>Packages</th>
</tr>
</thead>
<tbody>
<tr>
<td>boxed dinner</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>pasta</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>juice</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>soup</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the probability that a randomly chosen product will be soup.

\[ P(\text{soup}) = \frac{45}{115} = \frac{9}{23} \approx 0.3913 \]

The probability that a randomly chosen product will be soup is \( \frac{9}{23} \) or about 39.1%.

(b) Find the probability that a randomly chosen product will be a boxed dinner or pasta.

\[ P(\text{boxed dinner or pasta}) = P(\text{boxed dinner}) + P(\text{pasta}) = \frac{36}{115} + \frac{22}{115} = \frac{58}{115} \approx 0.5043 \]

The probability that a randomly chosen product will be a boxed dinner or pasta is \( \frac{58}{115} \) or about 50.4%.

6. **MUSIC** A Web site conducted a survey on the number of different formats on which teens have music. The table shows a probability distribution of the results.

<table>
<thead>
<tr>
<th>Formats</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>0.31</td>
</tr>
<tr>
<td>3</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
</tr>
<tr>
<td>6+</td>
<td>0.02</td>
</tr>
</tbody>
</table>

(a) Show that the distribution is valid.

(b) What is the probability that a student randomly chosen will have music on 2 or more formats?

\[ P(2+) = P(2) + P(3) + P(4) + P(5) + P(6+) = 0.31 + 0.19 + 0.11 + 0.02 + 0.02 = 0.65 \]

(c) Make a probability graph of the data.

\[ 1 - 0.35 = 0.65 \]
12-8 Probability Distributions

7. **GRADES** Mr. Rockwell’s Algebra class took a chapter test last week. The table shows the probability distribution of the results.

<table>
<thead>
<tr>
<th>Score</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.29</td>
</tr>
<tr>
<td>3</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
</tr>
<tr>
<td>1</td>
<td>0.11</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Show that the distribution is valid.
b. What is the probability that a student chosen at random will have no higher than a 3?
c. Make a probability graph of the data.

**SOLUTION:**

a. For each $X$, the probability is greater than or equal to 0 and less than or equal to 1.

$$0.29 + 0.43 + 0.17 + 0.11 + 0 = 1$$

The sum of the probabilities is 1, so the distribution is valid.

b. 

$$P(X \leq 3) = P(3) + P(2) + P(1) + P(0)$$

$$= 0.43 + 0.17 + 0.11 + 0$$

$$= 0.71$$

The probability that a randomly chosen student will have no higher than a 3 is 0.71 or 71%.

c. Place the probabilities on the vertical scale and the scores on the horizontal scale.

---

8. **CCSS ARGUMENTS** Kylie entered a drawing at the county fair. The table shows the value and probability of winning each prize.

<table>
<thead>
<tr>
<th>Prize Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20</td>
<td>1 in 50</td>
</tr>
<tr>
<td>$50</td>
<td>1 in 100</td>
</tr>
<tr>
<td>$100</td>
<td>1 in 250</td>
</tr>
<tr>
<td>$250</td>
<td>1 in 1000</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Create a probability distribution. See margin.
b. Calculate the expected value. $1.55
c. Interpret your results.

**SOLUTION:**

Divide 1 by 50 to find the probability for $20. Do the same for each remaining value.

<table>
<thead>
<tr>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20</td>
<td>0.02</td>
</tr>
<tr>
<td>$50</td>
<td>0.01</td>
</tr>
<tr>
<td>$100</td>
<td>0.004</td>
</tr>
<tr>
<td>$250</td>
<td>0.001</td>
</tr>
<tr>
<td>$0</td>
<td>0.965</td>
</tr>
</tbody>
</table>

b. 

$$20(0.02) + 50(0.01) + 100(0.004) + 250(0.001) = 1.55$$

c. Sample answer: Kylie’s ticket is worth about $1.55. If Kyle paid $1 for the ticket, then he will average a profit of $0.55.

If the organizers of the fair expect to make any money off of the drawing, they will need to charge more than $1.55 per ticket.
9. CONTESTS Nikia entered a contest where each ticket cost $1. The table shows the value and probability of each prize.

<table>
<thead>
<tr>
<th>Prize Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$200</td>
<td>1 in 500</td>
</tr>
<tr>
<td>$1000</td>
<td>1 in 5000</td>
</tr>
<tr>
<td>$5000</td>
<td>1 in 25,000</td>
</tr>
<tr>
<td>$25,000</td>
<td>1 in 100,000</td>
</tr>
</tbody>
</table>

a. Create a probability distribution.

b. Calculate the expected value. $1.05

c. Interpret your results.

**SOLUTION:**

a. Divide 1 by 500 to get the probability of $200. Do the same for each remaining prize value.

<table>
<thead>
<tr>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$200</td>
<td>0.002</td>
</tr>
<tr>
<td>$1000</td>
<td>0.0002</td>
</tr>
<tr>
<td>$5000</td>
<td>0.00004</td>
</tr>
<tr>
<td>$25,000</td>
<td>0.00001</td>
</tr>
<tr>
<td>$0</td>
<td>0.99775</td>
</tr>
</tbody>
</table>

b. 200(0.02) + 1000(0.0002) + 5000(0.00004) + 25,000(0.00001) = 1.05

c. Sample answer: The makers of the contest expect to lose $1.05 for each ticket valued at $1.00. Therefore, they can expect to lose money by doing this contest. If a contest had this expected value, then the majority of people will want to enter into it because in the long run they will expect to win $1.05 for every $1 they spend.

10. MARKETING A retail marketing group conducted a survey on teen shopping habits and asked the teens for the number of stores they visited to complete their holiday shopping. The table shows the probability distribution of the results.

<table>
<thead>
<tr>
<th>Number of Stores</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>0.35</td>
</tr>
<tr>
<td>3–5</td>
<td>0.32</td>
</tr>
<tr>
<td>6–8</td>
<td>0.17</td>
</tr>
<tr>
<td>9–11</td>
<td>0.11</td>
</tr>
<tr>
<td>12+</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a. Show that the distribution is valid.

b. What is the probability that a shopper chosen at random will shop at more than 5 stores but fewer than 12? 28%

c. Make a probability graph of the data.

**SOLUTION:**

a. All of the values are between 0 and 1; 0.35 + 0.32 + 0.17 + 0.11 + 0.05 = 1.

b. 0.17 + 0.11 = 28%

c. **Shopping Places**

**Number of Stores**

<table>
<thead>
<tr>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
</tbody>
</table>

**Number of Stores**

<table>
<thead>
<tr>
<th>0–2</th>
<th>3–5</th>
<th>6–8</th>
<th>9–11</th>
<th>12+</th>
</tr>
</thead>
</table>
12-8 Probability Distributions

11. DECISION MAKING An auto insurance company uses many variables to calculate each driver’s six-month payment. The table at the right shows the probability of a specific driver getting into an accident that will cost the company $10,000.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>driving record</td>
<td>1 in 50</td>
</tr>
<tr>
<td>vehicle type</td>
<td>1 in 500</td>
</tr>
<tr>
<td>age</td>
<td>1 in 250</td>
</tr>
<tr>
<td>gender</td>
<td>1 in 200</td>
</tr>
<tr>
<td>residence</td>
<td>1 in 1000</td>
</tr>
</tbody>
</table>

**a.** Taking all of the probabilities into account, what is the probability of the driver having an accident?
**b.** What is the company’s expected payout for this driver?
**c.** What should the company charge the driver for a six-month policy? Explain your reasoning.

**SOLUTION:**

**a.** Find the sum of the probabilities.

\[
\frac{1}{50} + \frac{1}{500} + \frac{1}{250} + \frac{1}{200} + \frac{1}{1000} = 0.02 + 0.002 + 0.004 + 0.005 + 0.001 = 0.032
\]

3.2%

**b.** 0.032 \times 10,000 = $320

**c.** Sample answer: The company should charge at least $320 to cover their expected costs. However, in order to cover other costs such as paying employees and other business expenses and still earn a profit, the company should charge around $600 for the policy.

12. DECISION MAKING Amber is investing in stocks for her math class. After analyzing five stocks, she has determined the following probabilities.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Cost Per Share</th>
<th>Probability of $100 Gain</th>
<th>Probability of $50 Gain</th>
<th>Probability of $20 Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$100</td>
<td>47%</td>
<td>25%</td>
<td>28%</td>
</tr>
<tr>
<td>B</td>
<td>$200</td>
<td>40%</td>
<td>40%</td>
<td>20%</td>
</tr>
<tr>
<td>C</td>
<td>$200</td>
<td>50%</td>
<td>30%</td>
<td>20%</td>
</tr>
<tr>
<td>D</td>
<td>$200</td>
<td>60%</td>
<td>0%</td>
<td>40%</td>
</tr>
<tr>
<td>E</td>
<td>$500</td>
<td>30%</td>
<td>60%</td>
<td>10%</td>
</tr>
</tbody>
</table>

**a.** Calculate the expected gain or loss for one share of each stock.
**b.** If Amber is allowed to spend $1000, what combination of stocks should she purchase to ensure the greatest expected value? Explain your reasoning.

**SOLUTION:**

**a.**

\[A: (0.47)(100) - (0.28)(100) = 9.4 - 28 = -18.6\]
\[B: (0.40)(200) - (0.40)(100) = 8 - 40 = -32\]
\[C: (0.50)(200) - (0.30)(100) = 10 - 30 = -20\]
\[D: (0.60)(200) - (0.40)(100) = 12 - 40 = -28\]
\[E: (0.30)(500) - (0.10)(500) = 150 - 50 = 100\]

$1; $2; $1; $0; $3

**b.** Sample answer: Of the stocks that cost $200 per share, Stock B has the greatest expected gain, so it is the best choice. If Amber selects five shares of Stock B, she will spend the entire $1000 and have an expected gain of $10. Amber can also select three shares of Stock E and one share of Stock A and have an expected gain of $10. A third option would be to select 10 shares of Stock A.

13. MULTIPLE REPRESENTATIONS In this problem, you will investigate probability distributions and simulations.
**a.** Tabular Construct a relative-frequency table showing the theoretical probability for the sum obtained from rolling a die and spinning the spinner.

**b.** Graphical Make a probability graph of the data.
**c.** Analytical Calculate the expected value of one spin.
**d.** Concrete Design a simulation for 50 trials. Explain your reasoning. Conduct the simulation and
12-8 Probability Distributions

tally your results.

**e. Graphical** Make a probability graph of the data in the simulation. Compare and contrast the two graphs.

**SOLUTION:**

- **a.** The sections labeled 1 and 4 are each one-fourth of the entire spinner. The rest are one-eighths.

  Calculate probability of 6:

  - 1 then 5: \( \frac{1}{6} \times \frac{1}{8} = \frac{1}{48} \)
  - 2 then 4: \( \frac{1}{6} \times \frac{1}{8} = \frac{1}{24} \)
  - 3 then 3: \( \frac{1}{6} \times \frac{1}{8} = \frac{1}{48} \)
  - 4 then 2: \( \frac{1}{6} \times \frac{1}{8} = \frac{1}{48} \)
  - 5 then 1: \( \frac{1}{6} \times \frac{1}{4} = \frac{1}{24} \)

  Now find the sum.

  \[
  \frac{1}{48} + \frac{1}{24} + \frac{1}{48} + \frac{1}{48} + \frac{1}{24} = \frac{7}{48}
  \]

  Do the same for the rest of the sums.

<table>
<thead>
<tr>
<th>X = Sum</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

- **b.** List all of the sums on the x-axis and the probabilities on the y-axis.

- **c.** Multiply each sum by the corresponding probability and find the sum.

  (Or, multiply each sum by the corresponding frequency, find the sum, then divide by 48)

  - \( 2 \times 2 = 4 \)
  - \( 3 \times 3 = 9 \)
  - \( 4 \times 4 = 16 \)
  - \( 5 \times 6 = 30 \)

- **d.** Sample answer: Use values 1–48 in a random number generator. Set up the following representations. This way, the exact probabilities of each sum are accurately represented.

<table>
<thead>
<tr>
<th>X = Sum</th>
<th>Range</th>
<th>Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1-2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3-5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6-9</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>10-15</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>16-22</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>23-30</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>31-36</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>37-41</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>42-45</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>46-47</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>48</td>
<td>0</td>
</tr>
</tbody>
</table>

- **e.** List all of the sums on the x-axis and the probabilities on the y-axis.

Sample answer: The graph in part b is the theoretical probability, while the graph in part e is the experimental probability. The graphs will be similar in shape, but the experimental probability graph will vary and more than likely will never completely mirror the theoretical probability graph.
14. **CHALLENGE** What is wrong with the probability distribution shown? Explain your reasoning.

**SOLUTION:**
The problem with this probability distribution is that shooting foul shots is not a random event. Events involving athletic ability should not be considered when making probability distributions.

15. **REASONING** Suppose two dice are rolled twelve times to show the probability distribution. Then make a probability graph of the data.

**SOLUTION:**
First find the number of ways to get each sum.

<table>
<thead>
<tr>
<th>Sum</th>
<th>Dice</th>
<th>Number of Ways</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1,1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2,1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1,2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1,1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2,1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1,3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2,2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3,1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1,4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2,3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3,2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1,5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2,4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3,3</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1,6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2,5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3,4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4,3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2,6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3,5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5,3</td>
<td></td>
</tr>
</tbody>
</table>

There are 36 possible sums from rolling two dice.

<table>
<thead>
<tr>
<th>Sum of Dice</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(\frac{1}{36})</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{2}{36} = \frac{1}{18})</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{3}{36} = \frac{1}{12})</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{4}{36} = \frac{1}{9})</td>
</tr>
</tbody>
</table>

The sum of 7 is most likely to occur because the prob
12-8 Probability Distributions

16. **CCSS STRUCTURE** Explain why the sum of the probabilities in a probability distribution should always be 1. Include an example.

**SOLUTION:**
Since the probability is found by dividing the number of desired outcomes by the number of possible outcomes, the sum of the probabilities of each outcome is 1. For example, if a jar contains 2 red marbles, 1 green marble, and 3 blue marbles the probability of drawing a red marble is \( \frac{2}{2+1+3} \) or \( \frac{1}{3} \),
the probability of drawing a green marble is \( \frac{1}{2+1+3} \) or \( \frac{1}{6} \), and the probability of drawing a blue marble is \( \frac{3}{2+1+3} \) or \( \frac{1}{2} \). The sum of the probabilities in the distribution is 1.

<table>
<thead>
<tr>
<th>Marble</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>Green</td>
<td>( \frac{1}{6} )</td>
</tr>
<tr>
<td>Blue</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>Sum of Probabilities</td>
<td>( \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{2}{6} + \frac{1}{6} + \frac{3}{6} = \frac{6}{6} = 1 )</td>
</tr>
</tbody>
</table>

17. **REASONING** Determine whether the following statement is true or false. Explain your reasoning.

*Discrete random variables can take on an infinite number of values.*

**SOLUTION:**
Discrete random variables are random variables with a countable number of possibilities.

True; sample answer: While a discrete random variable is a countable variable, it can still take on an infinite number of values. For example, the number of hits a Web site receives is countable, but can be infinite.

18. **OPEN ENDED** Write a real-world problem in which you could find a probability distribution. Create a probability graph for your data.

**SOLUTION:**
Sample answer: There are 870 students in a school: 179 freshmen, 215 sophomores, 211 juniors, and 265 seniors. Find the probability distribution for each class. What is the probability that a randomly chosen student is a sophomore?

<table>
<thead>
<tr>
<th>Class</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>( \frac{179}{870} ) ≈ 0.21</td>
</tr>
<tr>
<td>Sophomore</td>
<td>( \frac{215}{870} ) ≈ 0.25</td>
</tr>
<tr>
<td>Junior</td>
<td>( \frac{211}{870} ) ≈ 0.24</td>
</tr>
<tr>
<td>Senior</td>
<td>( \frac{265}{870} ) ≈ 0.30</td>
</tr>
</tbody>
</table>

19. **REASONING** Determine whether the following statement is true or false. Explain your reasoning.

*The expected value of a random variable is the value for the random variable most likely to occur.*

**SOLUTION:**
False; sample answer: The expected value of a random variable is the weighted average of the variable. The expected value of a random variable does not have to be a possible value. For example, the expected value when a die is rolled is 3.5. This value cannot occur.
12-8 Probability Distributions

20. **WRITING IN MATH** Write a real-word story in which you are the owner of a business. Explain how you could use a probability distribution to help you make a business decision.

**SOLUTION:**
Sample answer: I own a cosmetics store. As part of my inventory, I keep track of each type of cosmetic that I sell. A probability distribution would help me to decide how much of each type of item to keep in stock. If products have a high probability of being sold at certain times, then I will make sure that I have enough of that product in stock. If they have a low probability of being sold at another time, I will make sure that I don't have too much of the product in inventory.

21. A coin is flipped and a die is rolled. What is the probability of the coin landing heads up and rolling a 3?

A \( \frac{1}{12} \)
B \( \frac{1}{6} \)
C \( \frac{1}{8} \)
D \( \frac{2}{3} \)

**SOLUTION:**
\[ P(\text{heads and 3}) = P(\text{heads}) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \]

22. **SHORT RESPONSE** How many different ways can the letters P, Q, R, S be arranged?

**SOLUTION:**
Order matters, so this is a permutation. We have 4 objects taken 4 at a time.

We can list them: PQRS, PQRS, ... or we can calculate \( P(4, 4) = 24 \).

23. Suppose there are 10 tickets in a box for a drawing numbered as follows: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. A single ticket is randomly chosen from the box. What is the probability of drawing a ticket with a number less than 10?

F \( \frac{1}{5} \)
G \( \frac{3}{10} \)
H 1
J 0

**SOLUTION:**
All of the numbers in the box are less than 10. Therefore, the probability of drawing a ticket with a number less than 10 is 1. The correct choice is H.

24. **GEOMETRY** The height of a triangle is 5 inches less than the length of its base. If the area of the triangle is 52 square inches, find the base and the height.

A 15 in., 9 in.
B 11 in., 7 in.
C 13 in., 8 in.
D 17 in., 11 in.

**SOLUTION:**
Let \( b \) = the length of the base of the triangle. Then the height of the triangle is \( b - 5 \).

\[
\text{Area} = \frac{1}{2} \text{base} \cdot \text{height} \\
52 = \frac{1}{2} b(b - 5) \\
2 \cdot 52 = 2 \cdot \frac{1}{2} b(b - 5) \\
104 = b(b - 5) \\
104 = b^2 - 5b \\
0 = b^2 - 5b - 104 \\
0 = (b - 13)(b + 8) \\
0 = b - 13 \quad \text{or} \quad 0 = b + 8 \\
13 = b \quad -8 = b
\]

Since length can’t be negative, the length of the base of the triangle is 13 inches. So, the height of the triangle is 13 – 5 or 8 inches. The correct choice is C.
25. **PET TOYS** A pet store has a bin of clearance items that contains 6 balls, 5 tug toys, 8 rawhide chews, and 4 chew toys, all in equal-sized boxes. If Johnda reaches in the bin and pulls out two items, what is the probability that she will pull out a tug toy each time?

**SOLUTION:**

First item:

\[
P(tug\ toy) = \frac{\text{number of tug toys}}{\text{total number of items}} = \frac{5}{23}
\]

Second item:

\[
P(tug\ toy) = \frac{\text{number of tug toys remaining}}{\text{number of items remaining}} = \frac{4}{22}
\]

\[
P(tug\ toy, tug\ toy) = P(tug\ toy) \cdot P(tug\ toy) = \frac{5}{23} \cdot \frac{4}{22} = \frac{20}{506} = \frac{10}{253} \approx 0.04
\]

The probability is \(\frac{10}{253}\) or about 4%.

26. **GAMES** For a certain game, each player rolls four dice at the same time.

a. Do the outcomes of rolling the four dice represent permutations or combinations? Explain.

b. How many outcomes are possible?

c. What is the probability that four dice show the same number on a single roll?

**SOLUTION:**

a. The order of the dice is not important, so this situation involves a combination.

b. Use the Fundamental Counting Principle and the fact that each die has 6 possible outcomes.

Number of outcomes = \(6 \cdot 6 \cdot 6 \cdot 6 = 1296\)

There are 1296 possible outcomes.

c. There are 6 different ways that all four dice show the same number on a single roll – they can all be 1’s, 2’s, 3’s, 4’s, 5’s, or 6’s.

\[
P(\text{all the same number}) = \frac{6}{1296} = \frac{1}{216}
\]

The probability that all four dice show the same number on a single roll is \(\frac{1}{216}\).

Find each sum.

27. \[\frac{4}{a^2} + \frac{6}{a}\]

**SOLUTION:**

\[
\frac{4}{a^2} + \frac{6}{a} = \frac{4}{a^2} + \frac{6 \cdot a}{a^2} = \frac{4 + 6a}{a^2} = \frac{6a + 4}{a^2}
\]
12-8 Probability Distributions

28. \( \frac{3}{b^3} + \frac{7}{b^2} \)

**SOLUTION:**
\[
\frac{3}{b^3} + \frac{7}{b^2} = \frac{3}{b^3} \cdot \frac{b}{b} + \frac{7}{b^2} \cdot \frac{b}{b} = \frac{3b + 7b}{b^3} = \frac{10b}{b^3} = \frac{10}{b^2}
\]

29. \( \frac{4}{d + 6} + \frac{5}{d - 5} \)

**SOLUTION:**
\[
\frac{4}{d + 6} + \frac{5}{d - 5} = \frac{4(d - 5) + 5(d + 6)}{(d + 6)(d - 5)} = \frac{4d - 20 + 5d + 30}{(d + 6)(d - 5)} = \frac{9d + 10}{(d + 6)(d - 5)}
\]

30. \( \frac{f}{f + 5} + \frac{4}{f - 4} \)

**SOLUTION:**
\[
\frac{f}{f + 5} + \frac{4}{f - 4} = \frac{f(f - 4) + 4(f + 5)}{(f + 5)(f - 4)} = \frac{f^2 - 4f + 4f + 20}{(f + 5)(f - 4)} = \frac{f^2 + 20}{(f + 5)(f - 4)}
\]

31. \( \frac{8h + h}{h + 6} + \frac{h}{h - 3} \)

**SOLUTION:**
\[
\frac{8h + h}{h + 6} + \frac{h}{h - 3} = \frac{8h}{h + 6} \cdot \frac{h - 3}{h - 3} + \frac{h}{h - 3} \cdot \frac{h + 6}{h + 6} = \frac{8h(h - 3) + h(h + 6)}{(h + 6)(h - 3)} = \frac{8h^2 - 24h + h^2 + 6h}{(h + 6)(h - 3)} = \frac{9h^2 - 18h}{(h + 6)(h - 3)} = \frac{9h(h - 2)}{(h + 6)(h - 3)}
\]

32. \( \frac{7k + k}{k - 3} + \frac{k}{k + 2} \)

**SOLUTION:**
\[
\frac{7k + k}{k - 3} + \frac{k}{k + 2} = \frac{7k}{k - 3} \cdot \frac{k + 2}{k + 2} + \frac{k}{k + 2} \cdot \frac{k - 3}{k - 3} = \frac{7k(k + 2) + k(k - 3)}{(k - 3)(k + 2)} = \frac{7k^2 + 14k + k^2 - 3k}{(k - 3)(k + 2)} = \frac{8k^2 + 11k}{(k - 3)(k + 2)}
\]
Find the values of the three trigonometric ratios for angle $A$.

33. **SOLUTION:**

\[
\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \quad \cos A = \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} \quad \tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A}
\]

\[
\begin{align*}
\sin A &= \frac{8}{10} = \frac{4}{5} \\
\cos A &= \frac{6}{10} = \frac{3}{5} \\
\tan A &= \frac{8}{6} = \frac{4}{3}
\end{align*}
\]

34. **SOLUTION:**

\[
\begin{align*}
\sin A &= \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \\
&= \frac{5}{13} \\
\cos A &= \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} \\
&= \frac{12}{13} \\
\tan A &= \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A} \\
&= \frac{5}{12}
\end{align*}
\]

35. **SOLUTION:**

\[
\begin{align*}
\sin A &= \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \\
&= \frac{16}{20} = \frac{4}{5} \\
\cos A &= \frac{\text{leg adjacent } \angle A}{\text{hypotenuse}} \\
&= \frac{12}{20} = \frac{3}{5} \\
\tan A &= \frac{\text{leg opposite } \angle A}{\text{leg adjacent } \angle A} \\
&= \frac{16}{12} = \frac{4}{3}
\end{align*}
\]
Simplify each expression.

36. \( \sqrt[5]{\frac{50}{x^4}} \)

**SOLUTION:**

\[
\sqrt[5]{\frac{50}{x^4}} = \sqrt[5]{\frac{25 \cdot 2}{x^4}} = \frac{5\sqrt[5]{2}}{x^2}
\]

37. \( \sqrt[3]{\frac{t}{18}} \)

**SOLUTION:**

\[
\sqrt[3]{\frac{t}{18}} = \frac{\sqrt[3]{t^2 \cdot 2}}{\sqrt[3]{2^2 \cdot 2}} = \frac{t\sqrt[3]{2}}{3\sqrt[3]{2}}
\]

38. \( \sqrt[4]{\frac{15}{14}} \cdot \sqrt[5]{\frac{21}{10}} \)

**SOLUTION:**

\[
\sqrt[4]{\frac{15}{14}} \cdot \sqrt[5]{\frac{21}{10}} = \sqrt[4]{\frac{3^2 \cdot 5 \cdot 7}{2^2 \cdot 5 \cdot 7}} = \frac{3\sqrt[4]{35}}{2\sqrt[4]{35}} = \frac{3}{2}
\]

39. \( \frac{6}{3-\sqrt{5}} \)

**SOLUTION:**

\[
\frac{6}{3-\sqrt{5}} = \frac{6(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{18+6\sqrt{5}}{9-5} = \frac{9+3\sqrt{5}}{2}
\]

40. \( \frac{3}{\sqrt{7}+\sqrt{6}} \)

**SOLUTION:**

\[
\frac{3}{\sqrt{7}+\sqrt{6}} = \frac{3(\sqrt{7}-\sqrt{6})}{(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})} = \frac{3\sqrt{7}-3\sqrt{6}}{7-6} = 3\sqrt{7} - 3\sqrt{6}
\]

41. \( \sqrt[2]{\frac{\sqrt{2}}{\sqrt{8} - \sqrt{6}}} \)

**SOLUTION:**

\[
\sqrt[2]{\frac{\sqrt{2}}{\sqrt{8} - \sqrt{6}}} = \frac{\sqrt{2}(\sqrt{8}+\sqrt{6})}{(\sqrt{8}-\sqrt{6})(\sqrt{8}+\sqrt{6})} = \frac{\sqrt{16}+\sqrt{12}}{8-6} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}
\]
Identify each sample, and suggest a population from which it was selected. Then classify the sample as simple, systematic, self-selected, convenience, or stratified. Explain your reasoning.

1. CEREAL A cereal company invites 100 random children and parents to test a new cereal and records the reactions.

   **SOLUTION:**
   The sample is the 100 children and parents, which represent the population of all children and parents. This is a simple sample since participants were chosen randomly and they were all given the same cereal.

2. SCHOOL LUNCH A school is creating a new lunch menu. They send out a questionnaire to all students with odd homeroom numbers to determine what items should be on the new menu.

   **SOLUTION:**
   The sample is all of the students with odd homeroom numbers, which represents the population of all the students in the school. Since the sample was taken of students with odd homeroom numbers, this is a systematic sample.

3. MASCOTS The cheerleaders send out a flyer with pictures of possible options for the new mascot to all the girls in the school. The girls mark their favorite mascot and send it back. The new mascot is chosen from the survey.

   **SOLUTION:**
   The sample is all the girls in the school, which is supposed to represent the population of the entire student body. This is a convenience sample since only the girls in the school are being surveyed.

   **Identify each sample as biased or unbiased. Explain your reasoning.**

4. ART Every fifth person leaving the art museum is asked to name their favorite piece.

   **SOLUTION:**
   The survey is taken systematically, but the people are chosen at random and the question does not lead toward a specific answer. This is an unbiased sample.

5. SHOPPING Each person leaving the Earring Pagoda is asked to name their favorite store in the mall.

   **SOLUTION:**
   This is a biased sample since the people leaving the Earring Pagoda have a bias toward that store.

6. FOOTBALL Every 10th student leaving the student union at Ohio State is asked to name their favorite college football team.

   **SOLUTION:**
   The sample is taken systematically, but the students chosen are random. The question is biased since students at Ohio State already have a preference toward Ohio State football. This is a biased sample.

   **Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.**

7. DINING At a restaurant, a random sample of 15 diners is selected. The amount of money spent on each meal is recorded.

   **SOLUTION:**
   The sample is the 15 selected diners. The population is all of the diners. The sample statistic is the average amount of money spent on each meal, and the population parameter is the average amount of money spent on meals for each diner.

8. POOLS A random sample of 25 children at a community pool is asked if they visit the pool at least once each week. The percent responding yes is calculated.

   **SOLUTION:**
   The sample is the 25 children that were selected, which is representative of the population of all of the children at the pool. The sample statistic is the percentage of children in the sample that visited the pool at least once a week. The population parameter is the percentage of children in the community who visit the pool at least once a week.
9. PLAY AREA Ian listed the ages of the children playing at the play area at the mall. Find and interpret the standard deviation of the data set.

\{2, 3, 2, 4, 2, 3, 2, 8, 3, 4, 2\}

**SOLUTION:**
Put the data into L₁ and calculate the 1 variable statistics:

```
1-Var Stats
\[\bar{x}=3.083333333\]
\[\sum x=37\]
\[\sum x^2=147\]
\[s_x=\sqrt{1.72962492}\]
\[\bar{x}=1.656217243\]
\[\bar{n}=12\]
```

The standard deviation is 1.66, which is about half of the mean of 3.08. This is a fairly normal amount of spread in the data, however if we remove the outlier of 8, the data looks different:

```
1-Var Stats
\[\bar{x}=2.636363636\]
\[\sum x=29\]
\[\sum x^2=83\]
\[s_x=\sqrt{8.090390835}\]
\[\bar{x}=1.7713892158\]
\[\bar{n}=11\]
```

Here the average is 2.6 with a deviation of only 0.77. This means that there isn't much distribution and almost all of the children in the play area are about the same age.

10. MULTIPLE CHOICE Several friends are chipping in to buy a gift for their teacher. Indigo is keeping track of how much each friend spends. Find the mean absolute deviation.

\{10, 5, 3, 6, 7, 8\}

**SOLUTION:**
To find the mean standard deviation, first calculate the mean:
\[\bar{x} = \frac{10 + 5 + 3 + 6 + 7 + 8}{6}\]
\[= \frac{39}{6}\]
\[= 6.5\]

Next calculate the sum of the absolute differences between the mean and each term. Then divide by the number of terms.

\[\text{MAD} = \frac{|10 - 6.5| + |5 - 6.5| + |3 - 6.5| + |6 - 6.5| + |7 - 6.5| + |8 - 6.5|}{6}\]
\[= \frac{3.5 + 1.5 + 3.5 + 0.5 + 0.5 + 1.5}{6}\]
\[= 1.83\]

11. Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

19, 36, 26, 36, 40, 31, 30, 33, 23, 38, 23, 46

**SOLUTION:**
First enter the data into L₁ on your graphing calculator. Change the x range to fit the highest and lowest values, and plot the graph.

The distribution is symmetric.
Identify each sample, and suggest a population from which it was selected. Then classify the sample as simple, stratified, or cluster. What is the mean and standard deviation? The data is symmetric, so use the mean and standard deviation to describe the data.

12. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

9, 11, 2, 6, 8, 10, 6, 3, 10, 11, 9, 8, 3, 8, 5, 11, 14, 6, 8, 6, 11, 5, 9, 10, 8

**SOLUTION:**
First enter the data into L₁ on your graphing calculator, choose a suitable window, and create a box-and-whisker plot.

The data is symmetric, so use the mean and standard deviation to describe the data.

![Box-and-Whisker Plot](image)

The mean is 7.88 and the standard deviation is 2.88.

13. **MULTIPLE CHOICE** Which pair of box-and-whisker plots depicts two positively skewed sets of data in which 75% of one set of data is larger than 75% of the other set of data?

**F**

![Box-and-Whisker Plot](image)

**G**

![Box-and-Whisker Plot](image)

**SOLUTION:**
Take a look at the choices individually.

In F, the upper quartile of the top plot is less than the lower quartile of the bottom plot. This means that the 0 – 75% of values from the top plot are less than the 25 – 100% of values from the second plot. So 75% of the data on the bottom plot is larger than 75% of the data on the top plot. This is correct, but check the other answers to be sure.

In G both of the plots have means that are about equal, but the bottom plot has a smaller quartile range. These sets of data are roughly equal in value, but the bottom plot is just less spread out.

In H, the lower quartiles are about equal, which means that the lower 25% of each set of data is about the same. So 75% of the values from one set of data cannot be larger than 75% of values from the other set of data.

In J the median of the top plot is roughly equal to the lower quartile of the bottom plot. This means that 75% of the values from the bottom set of data are larger than 50% of the values from the top set of data, which is not what we are looking for.

The best answer is F.
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

14. 6, 9, 0, 15, 9, 14, 11, 13, 9, 5, 8, 6; + (−3)

**SOLUTION:**
First enter the data into L₁ on your graphing calculator.

L₁−3→L₂
{3 6 -3 12 6 11...}

Then calculate the 1-variable statistics of L₂.

<table>
<thead>
<tr>
<th>L₁-Var Stats</th>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄=5.75</td>
<td>x̄=26.7</td>
</tr>
<tr>
<td>m=69</td>
<td>m=267</td>
</tr>
<tr>
<td>Sx=4.22350032</td>
<td>Sx²=7359</td>
</tr>
<tr>
<td>σx=4.04402852</td>
<td>σx=5.056349144</td>
</tr>
<tr>
<td>n=12</td>
<td>n=10</td>
</tr>
</tbody>
</table>

The mean is 5.75. The median is 6. The mode is 6. T

15. 19, 22, 10, 17, 26, 24, 12, 22, 18, 17; + 8

**SOLUTION:**
First enter the data into L₁ on your graphing calculator. Add 8 to L₁ and store this in L₂.

L₁+8→L₂
{27 30 18 25 34...}

Then calculate the 1-variable statistics of L₂.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄=26.7</td>
</tr>
<tr>
<td>m=267</td>
</tr>
<tr>
<td>Sx²=7359</td>
</tr>
<tr>
<td>σx=5.056349144</td>
</tr>
<tr>
<td>n=10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>x̄=26.5</td>
</tr>
<tr>
<td>m=25</td>
</tr>
<tr>
<td>Sx=30</td>
</tr>
<tr>
<td>n=10</td>
</tr>
</tbody>
</table>

The mean is 26.7. The median is 26.5. The mode is 30. The range is 34 – 18 = 16. And the standard deviation is 4.80.
1. **CHOCOLATE** Rico is selling candy. If Marisa randomly selects two candy bars to purchase, what is the probability that she buys a milk chocolate bar followed by a caramel bar?

**SOLUTION:**
\[
\frac{8}{25} \cdot \frac{6}{25} \approx 9.6
\]

2. A die is rolled 200 times. What is the experimental probability of rolling less than 3?

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
</tr>
</tbody>
</table>

**SOLUTION:**
\[
P(X < 3) = P(1) + P(2) = \frac{56}{200} = 28\%
\]

3. **MULTIPLE CHOICE** Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

16, 18, 14, 31, 19, 18, 10, 29, 12, 28, 19, 17, 26, 15, 20

A positively skewed  
B negatively skewed  
C symmetric  
D none of the above

**SOLUTION:**

The data are bunched to the left with a tail trailing to the right, so the distribution is positively skewed.
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

4. 9, 17, 31, 21, 17, 25, 13, 9, 12, 9; × 3

SOLUTION:
Enter the data into L1 of your calculator.

Set L2 = 3 × L1.

<table>
<thead>
<tr>
<th>L1</th>
<th>L3</th>
<th>L2 =3L1</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>31</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>-----</td>
</tr>
</tbody>
</table>

Calculate the 1-variable stats for L2.

1-Var Stats
- \( \bar{x} = 48.9 \)
- \( \text{Sx} = 28449 \)
- \( \text{Sx} = 22.45217139 \)
- \( \text{Sx} = 21.3 \)
- \( \text{Sx} = 10 \)

1-Var Stats
- \( \text{Sx} = 48.9 \)
- \( \text{Sx} = 28449 \)
- \( \text{Sx} = 22.45217139 \)
- \( \text{Sx} = 21.3 \)
- \( \text{Sx} = 10 \)

5. 16, 14, 23, 41, 38, 29, 18, 13, 16; × 0.25

SOLUTION:
Enter the data into L1 of your calculator.

Set L2 = 0.25 × L1.

<table>
<thead>
<tr>
<th>L1</th>
<th>L3</th>
<th>L2 = .25L1</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>41</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>38</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>29</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>-----</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>-----</td>
</tr>
</tbody>
</table>

Calculate the 1-variable stats for L2.

1-Var Stats
- \( \bar{x} = 5.777777778 \)
- \( \text{Sx} = 52 \)
- \( \text{Sx} = 356 \)
- \( \text{Sx} = 2.635231383 \)
- \( \text{Sx} = 2.484519975 \)
- \( \text{Sx} = 9 \)

1-Var Stats
- \( \text{Sx} = 5.777777778 \)
- \( \text{Sx} = 52 \)
- \( \text{Sx} = 356 \)
- \( \text{Sx} = 2.635231383 \)
- \( \text{Sx} = 2.484519975 \)
- \( \text{Sx} = 9 \)

5.78, 4.5, 4, 7, 2.48

Identify each sample as biased or unbiased. Explain your reasoning.

6. NEWSPAPERS A survey is sent to all people who subscribe to The Dispatch to determine what newspaper people prefer to read.

SOLUTION:
If a sample favors one group over another, then the data are invalid because it is a biased sample. A sample is unbiased if it is random. Members of a random sample have an equal probability of being chosen.

This sample is biased because the survey is done from subscribers to a particular newspaper, and the respondents are more likely to choose The Dispatch.
7. **SHOPPING** Each person leaving the Maxtowne Mall is asked to name their favorite clothing store in the mall.

**SOLUTION:**
If a sample favors one group over another, then the data are invalid because it is a biased sample. A sample is unbiased if it is random. Members of a random sample have an equal probability of being chosen.

This sample is unbiased because all people leaving the mall were asked.

8. **SALES** Nate is keeping track of how much people spent at the school bookstore in one day. Find the mean absolute deviation for the data to the nearest tenth: 1, 1, 2, 3, 4, 5, 12.

**SOLUTION:**
First find the mean of the data.
\[
\bar{x} = \frac{1+1+2+3+4+5+12}{7} = \frac{28}{7} = 4
\]
Now find the sum of the absolute value of the difference between each value in the data set and the mean.
\[
\sum |x_i - \bar{x}| = |1-4|+|1-4|+|2-4|+|3-4|+|4-4|+|5-4|+|12-4| = 3+3+2+1+0+1+8 = 18
\]
Divide the sum by the number of values: 18 ÷ 7 ≈ 2.6.
The mean absolute deviation is approximately 2.6.

9. **PIZZA** How many ways can 3 different toppings be chosen from a list of 10 toppings?

**SOLUTION:**
Because the order does not matter, this is a combination of 10 things taken 3 at a time.
\[
C(10,3) = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!} = \frac{720}{6} = 120
\]
There are 120 ways that 3 different toppings can be chosen from a list of 10 toppings.

10. **EDUCATION** Kristin surveys 200 people in her school to determine how many nights a week students do homework. The results are shown.

<table>
<thead>
<tr>
<th>Number of Nights</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5 or more</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Find the probability that a randomly chosen student will have studied more than 4 nights.
b. Find the probability that a randomly chosen student will have studied no more than 3 nights.

**SOLUTION:**
\[
P(X > 4) = \frac{10}{200} = 0.05
\]
The probability that a randomly chosen student will have studied 5 or more nights is 0.05 or 5%.

\[
P(X \leq 3) = \frac{10}{200} + \frac{30}{200} + \frac{50}{200} + \frac{90}{200} = 0.9
\]
The probability that a randomly chosen student will have studied no more than 3 nights is 0.9 or 90%.

11. **MULTIPLE CHOICE** The second graders are divided into boys and girls. Then 2 girls and 2 boys are chosen at random to represent the class at the Pride Assembly. Which of the following best describes the sample?

F simple
G stratified
H systematic
J self-selected

**SOLUTION:**
Because the class is divided into categories before there is a random sample, the sample is stratified. The correct choice is B.
Practice Test - Chapter 12

Identify each situation as a permutation or a combination.
12. a student’s daily class schedule

SOLUTION:
Order is important with a class schedule, so it is a permutation.

13. a list of teachers’ names at school

SOLUTION:
Order is not important with a list, so it is a combination. If the list were a ranking in some way, then it would be a permutation.

14. gold, silver, and bronze medalists

SOLUTION:
Order is important with gold, silver, and bronze medals, so it is a permutation.

15. RAFFLE Carmen is considering paying $1 for a raffle ticket. What is the expected value of this ticket?

<table>
<thead>
<tr>
<th>Price Value</th>
<th>$10</th>
<th>$50</th>
<th>$500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>1 in 100</td>
<td>1 in 500</td>
<td>1 in 5000</td>
</tr>
</tbody>
</table>

SOLUTION:
$10(0.01) + 50(0.002) + 500(0.0002) = 0.1 + 0.1 + 0 = 0.3$

The ticket costs $1, so the expected value is $0.30 – 1.00 = –$0.70.
Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. There are 40 students, 9 camp counselors, and 5 teach Camp Kern. Each person is assigned to one activity during the afternoon. There are 9 students going hiking and 17 going horseback riding. Of the camp counselors, 2 will supervise the hike and 3 will help with the canoe trip. 2 teachers helping with the canoe trip and 2 going horseback riding. Suppose a person is selected at random during the afternoon activities. What is the probability that the one selected is a student on the canoe trip or a camp counselor on a horseback ride?

**SOLUTION:**

Use a table to organize the data.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Students</th>
<th>Counselors</th>
<th>Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hiking</td>
<td>9</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Horseback</td>
<td>17</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Horseback</td>
<td>17</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Canoe trip</td>
<td>14</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

Let \( s/c \) represent student on a canoe trip and let \( c/h \) represent counselor horseback riding.

\[
P(s/c \text{ or } c/h) = P(s/c) + P(c/h) \\
= \frac{14}{54} + \frac{4}{54} \\
= \frac{18}{54} \\
= \frac{1}{3}
\]

The probability that the person is a student on the canoe trip or a camp counselor on a horseback ride is \( \frac{1}{3} \).

2. The table shows the number of coins in a piggy bank.

<table>
<thead>
<tr>
<th>Coin</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny</td>
<td>16</td>
</tr>
<tr>
<td>Nickel</td>
<td>18</td>
</tr>
<tr>
<td>Dime</td>
<td>20</td>
</tr>
<tr>
<td>Quarter</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Find the probability that a randomly selected coin will be a dime.

b. Find the probability that a randomly selected coin will be either a nickel or a quarter.

**SOLUTION:**

a. The data is already organized in a table. There are 20 dimes and a total of 64 coins. The probability that a randomly selected coin will be a dime is \( \frac{20}{64} = \frac{5}{16} \).

b. There are 18 nickels and 10 quarters.

\[
P(\text{nickel or quarter}) = P(\text{nickel}) + P(\text{quarter}) \\
= \frac{18}{64} + \frac{10}{64} \\
= \frac{28}{64} \\
= \frac{7}{16}
\]

The probability that a randomly selected coin will be either a nickel or a quarter is \( \frac{7}{16} \).
3. It takes Craig 40 minutes to mow his family’s lawn. His brother Jacob can do the same job in 50 minutes. How long would it take them to mow the lawn together? Round your answer to the nearest tenth of a minute.

**SOLUTION:**
Because Craig can mow one lawn in 40 minutes, his lawn mowing rate is \( \frac{1}{40} \). Because Jacob can mow one lawn in 50 minutes, his lawn mowing rate is \( \frac{1}{50} \).

\[
d = rt
\]
\[
1 = \left( \frac{1}{40} + \frac{1}{50} \right) t
\]
\[
1 = \left( \frac{5}{200} + \frac{4}{200} \right) t
\]
\[
1 = \frac{9}{200} t
\]
\[
\frac{200}{9} = t
\]

It will take Craig and Jacob about 22.2 minutes to mow the lawn together.
1. What are the excluded values of the variable in the expression below?

\[ x^2 - x - 12 \]
\[ x^2 - x - 2 \]

A. −1, 2
B. −2, 2
C. −2, 1
D. −3, 4

**SOLUTION:**
The excluded values are the values of \( x \) such that \( x^2 - x - 2 = 0 \).

\[(x - 2)(x + 1) = 0\]
\[x - 2 = 0 \quad \text{or} \quad x + 1 = 0\]
\[x = 2 \quad \text{or} \quad x = -1\]

The excluded values are −1 and 2. Choice A is the correct answer.

2. The table shows the number of Calories in twelve different snacks. Which measure of central tendency would be most affected by the outlier 342 Calories?

<table>
<thead>
<tr>
<th>Number of Calories in Snacks</th>
<th>122</th>
<th>87</th>
<th>149</th>
<th>121</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>64</td>
<td>138</td>
<td>342</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>179</td>
<td>105</td>
<td>99</td>
<td>114</td>
</tr>
</tbody>
</table>

F. mean
G. median
H. mode
J. range

**SOLUTION:**
Range is not a measure of central tendency.

The median is the middle term, so it is not affected by outliers (terms on the outer parts of the data set).

The mode is the term that occurs the most often. If an outlier occurs enough to be considered a mode, then it is no longer an outlier. Therefore, the mode is not affected by outliers.

The answer is F.

3. Which of the following is not a factor of \( x^4 - 6x^2 - 27? \)

A. \( x^2 + 3 \)
B. \( x - 3 \)
C. \( x + 3 \)
D. \( x^2 - 3 \)

**SOLUTION:**
Find the factors of \( x^4 - 6x^2 - 27 \).

This expression resembles a basic quadratic, with \( x^4 \) replacing \( x^2 \) and \( x^2 \) replacing \( x \). Therefore, we can factor it just like \( x^2 - 6x - 27 \).

\[(x^2 + 3)(x^2 - 9) = 0\]

\( x^2 - 9 \) is a difference of squares.

\[(x^2 + 3)(x + 3)(x - 3) = 0\]

\( x^2 + 3, x + 3, \) and \( x - 3 \) are all factors of \( x^4 - 6x^2 - 27 \), so choice D is the correct answer.

4. Eduardo has 20 CDs. He wants to choose 3 of them at random to take on a road trip. How many different ways can he do this if the order is not important?

F. 60
G. 84
H. 1,140
J. 6,840

**SOLUTION:**
\[ C(n, r) = \frac{n!}{(n - r)!r!} \]
\[ C(20, 3) = \frac{20!}{(20 - 3)!3!} \]
\[ = \frac{20!}{17!3!} \]
\[ = \frac{6840}{6} \]
\[ = 1140 \]

Choice H is the correct answer.
5. Which of the following does not accurately describe the graph \( y = -2x^2 + 4 \)?

A. The parabola is symmetric about the y-axis.
B. The parabola opens downward.
C. The parabola has the origin as its vertex.
D. The parabola crosses the x-axis in two different places.

**SOLUTION:**

Compare \( y = -2x^2 + 4 \) to the standard form \( y = ax^2 + bx + c \).

A. Find the axis of symmetry:

\[
x = \frac{-b}{2a}
\]

\[
= \frac{0}{2(-2)}
\]

\[
= 0
\]

Therefore, the parabola is symmetric about the y-axis.

B. \( a < 0 \), so the parabola opens downward.

C. Use the axis of symmetry to find the vertex

\[
y = -2x^2 + 4
\]

\[
= -2(0)^2 + 4
\]

\[
= 4
\]

The vertex lies at \((0, 4)\), not at the origin \((0, 0)\).

D. Because the vertex is \((0, 4)\) (which is above the x-axis) and we know that it opens downward, we know that the parabola will cross the x-axis in two different places.

Therefore, choice C is the only answer that does not accurately describe the graph.

6. The highest point in North Carolina is Mt. Mitchell at an elevation of 2,037 meters above sea level.
Suppose the position of a hiker is given by the function \( p(t) = -2.5t + 2,037 \), where \( t \) is the number of minutes. Which of the following is the best interpretation of the slope of the function?

F. The hiker’s initial position was 2,037 feet below sea level.
G. The hiker’s initial position was 2,037 feet above sea level.
H. The hiker is descending at a rate of 2.5 meters per minute.
J. The hiker is ascending at a rate of 2.5 meters per minute

**SOLUTION:**

Compare \( p(t) = -2.5t + 2,037 \) to the slope-intercept form \( y = mx + b \). The slope is \( m \) or \(-2.5\) and because it is negative, it is reflecting the hiker’s descent from the highest point of the mountain. Choice H is the correct answer.

7. Jorge has made 39 out of 52 free throw attempts this season. What is the experimental probability that he makes a free throw?

A. 54%
B. 68%
C. 75%
D. 79%

**SOLUTION:**

\[
P(\text{making a free throw}) = \frac{39}{52}
\]

\[
= 0.75
\]

The probability is 75% that Jorge makes a free throw. Choice C is the correct answer.
8. Which equation passes through the points \((-1, -3)\) and \((-2, 3)\).

\[ F \ y = -6x - 9 \]
\[ G \ y = -\frac{1}{4}x + 3 \]
\[ H \ y = 4x - 5 \]
\[ J \ y = \frac{2}{3}x + 1 \]

**SOLUTION:**
Find the slope of the line containing the given points. Use a proportion to find the amount of sales tax.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{-2 - (-1)} = \frac{6}{-1} = -6
\]

Use the slope and either of the two points to find the \(y\)-intercept.

\[ y = mx + b \]
\[ 3 = -6(-2) + b \]
\[ 3 = 12 + b \]
\[ -9 = b \]

Write the equation in slope-intercept form.

\[ y = -6x - 9 \]
Choice F is the correct answer.

9. At a museum, each child admission costs $5.75 and each adult costs $8.25. How much does it cost a family that consists of 2 adults and 4 children?

\[ A \ \text{ $34.50} \]
\[ B \ \text{ $39.50} \]
\[ C \ \text{ $44.50} \]
\[ D \ \text{ $49.50} \]

**SOLUTION:**
The cost of admission for 2 adults and 4 children is 2 \((8.25) + 4(5.75)\) or $39.50. Choice B is the correct answer.

10. **GRIDDED RESPONSE** Suppose Colleen spins the spinner below 80 times and records the results in a frequency table. How many times should she expect to spin a vowel?

\[
\text{SOLUTION:}
\]
There are 2 vowels and 3 consonants on the spinner. The probability of spinning a vowel in one spin is \(\frac{2}{5}\). To find the probability of spinning a consonant in 80 spins, find \(\frac{2}{5}\) of 80.

\[ \frac{2}{5} \times 80 = \frac{160}{5} = 32 \]
Malcolm should expect to spin a consonant 32 times.

11. What is the value of \(\sin B\)? Express your answer as a fraction.

\[
\text{SOLUTION:}
\]
\[ \sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13} \]
12. Graph \( f(x) \geq |x - 2| \) on a coordinate grid.

**SOLUTION:**
One method is to set up a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Enter enough values to see the pattern in the graph. Remember that the graph of an absolute value function is shaped like a V.

13. **GRIDDED RESPONSE** Find the standard deviation of the set of data below. Show your work. Round to the nearest tenth if necessary.

<table>
<thead>
<tr>
<th>14</th>
<th>11</th>
<th>9</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>19</td>
<td>10</td>
</tr>
</tbody>
</table>

**SOLUTION:**
First, find the mean by adding the numbers then dividing by how many numbers are in the data set.

The sum is 144 and the mean is 12.

Now find the variance by squaring the difference between each number and the mean. Then sum the squares and divide by the number of values.

\[
\begin{align*}
(14 - 12)^2 &= 4 \\
(11 - 12)^2 &= 1 \\
(9 - 12)^2 &= 9 \\
(6 - 12)^2 &= 36 \\
(10 - 12)^2 &= 4 \\
(16 - 12)^2 &= 16 \\
(15 - 12)^2 &= 9 \\
(13 - 12)^2 &= 1 \\
(9 - 12)^2 &= 9 \\
(12 - 12)^2 &= 0 \\
(19 - 12)^2 &= 49 \\
(10 - 12)^2 &= 4 \\
\text{Sum} &= 142 \\
\sigma^2 &= \frac{142}{12} \\
&= \frac{71}{6} \\
\text{The standard deviation is the square root of the variance.} \\
\sigma^2 &= \frac{71}{6} \\
\sqrt{\sigma^2} &= \sqrt{\frac{71}{6}} \\
\sigma &= 3.4
\end{align*}
\]
14. Larissa has 5 peanut butter cookies, 7 chocolate chip cookies, 4 sugar cookies, and 9 oatmeal raisin cookies in a jar. If she picks two cookies at random without replacing them, what is the probability that she will choose a peanut butter cookie then a sugar cookie? Express your answer as a fraction.

**SOLUTION:**
The events are dependent because the outcome of the first choice changes the number of cookies in the jar. So the second choice will have a different probability based on the first choice.

<table>
<thead>
<tr>
<th>First choice</th>
<th>( P(\text{peanut butter}) = \frac{\text{number of peanut butter cookies}}{\text{total number of cookies}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = \frac{5}{25} = \frac{1}{5} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second choice</th>
<th>( P(\text{sugar}) = \frac{\text{number of sugar cookies}}{\text{number of cookies remaining}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( = \frac{4}{24} = \frac{1}{6} )</td>
</tr>
</tbody>
</table>

\( P(\text{peanut butter, sugar}) = P(\text{peanut butter}) \cdot P(\text{sugar}) \)

\( = \frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30} \)

The probability is \( \frac{1}{30} \).

15. Write an expression that describes the area in square units of a triangle with a height of \( 4c^3d^2 \) and a base of \( 3cd^4 \).

**SOLUTION:**
The area of a triangle \( = \frac{1}{2}bh \).

\( = \frac{1}{2}(3cd^4)(4c^3d^2) \)

\( = 12c^4d^6 \)

\( = \frac{6c^4d^6}{2} \)

16. Casey made 84 field goals during the basketball season for a total of 183 points. Each field goal was worth either 2 or 3 points. How many 2-point and 3-point field goals did Casey make during the season?

**SOLUTION:**
There are a total of 84 field goals combining the 2-point, \( x \) with the 3-point, \( y \) field goals. The total number of points for both types of field goals is 183. Therefore we can set up two different equations to solve for \( x \) and \( y \).

\[ 84 = x + y \]
\[ 183 = 2x + 3y \]

Multiply the first equation by \(-2\) to solve for \( y \).

\[ -168 = -2x + 2y \]
\[ 183 = 2x + 3y \]

Enter 15 for \( y \) in either equation to find \( x \)

\[ 84 = x + 15 \]
\[ 69 = x \]

Therefore, Casey made 15 3-point field goals and 69 2-point field goals.

17. **GRIDDED RESPONSE** The booster club pays $180 to rent a concession stand at a football game. They purchase cans of soda for $0.25 and sell them at the game for $1.15. How many cans of soda must they sell to break even?

**SOLUTION:**
The booster club needs to make $180 to break even. If they bought the soda for $0.25 each and sold them for $1.15 each, they will make a profit of $1.15 – $0.25 or $0.90 for each can.

Now find how many cans need to be sold to make a profit of $180.

180 = 0.9x

200 = x
18. To predict whether or not an issue on a ballot will pass or fail, a committee randomly calls 250 houses with area codes that are inside the voting district and asks the opinions of registered voters. Based on these efforts, the committee determines that 71% (±2.5%) of the voting population supports the issue. The committee concludes that the issue will pass.

a. Identify the sample.
b. Describe the population.
c. What method of data collection did the committee use: survey, experiment, or observational survey? Explain.
d. Is the sample biased or unbiased. Explain.
e. If unbiased, classify the sample as simple, stratified, or systematic. Explain.

**SOLUTION:**

a. / b. The sample is the data that is collected and measured and used to describe the population. In this case, the random 250 registered voters in the voting district that are contacted is the sample that will be measured and used to describe the population: all of the registered voters in the district.

c. A survey is used when data is collected via calling people and tallying responses to questions.
d. The voters are chosen randomly and then contacted, so there is no bias.
e. Each voter in the voting district is equally likely to be called, so this is a simple random sample.
Choose the term that best completes each sentence.

1. An arrangement in which order is important is called a (combination, permutation).

   SOLUTION: permutation

   An arrangement in which order is not important is called a combination.

2. A (parameter, statistic) is a measure that describes the characteristic of a sample.

   SOLUTION: statistic

   A parameter is a measure that describes the characteristic of a population.

3. A (sample, population) consists of all of the members of a group.

   SOLUTION: population

   A sample consists of some of the members of a group.

4. (Experimental probability, Theoretical probability) is the ratio of the number of favorable outcomes to the total number of outcomes.

   SOLUTION: Theoretical probability

   The experimental probability is determined using data from experiments.

5. A variable with a value that is the numerical outcome of a random event is called a (discrete random variable, random variable).

   SOLUTION: random variable

   A random variable with a countable number of possibilities is a discrete random variable.

Identify the sample as biased or unbiased. Explain your reasoning.

6. GOVERNMENT To determine whether voters support a new trade agreement, 5 people from the list of registered voters in each state are selected at random.

   SOLUTION: A bias is an error that results in a misrepresentation of a population. If a sample favors one conclusion over another, the sample is biased. There is no misrepresentation or favoritism, so this sample is unbiased.

Determine whether each situation describes a survey, an observational study, or an experiment. Explain your reasoning.

7. SCHOOL DANCE The homecoming dance committee sends out a questionnaire to all the girls in the school to decide on a theme for the homecoming dance.

   SOLUTION: In a survey, data are collected from responses given by a sample regarding their characteristics, behaviors, or opinions.

   The data are obtained from opinions given by the girls that return the questionnaire, so this is a survey.

8. MILKSHAKE Mary wants to test her milkshake recipe using honey instead of sugar. She randomly gives half of her 6 friends milkshakes sweetened with honey and the other half the same milkshakes sweetened with sugar. Then she asks them how they like the milkshakes.

   SOLUTION: In an experiment, the sample is divided into two groups:

   • an experimental group that undergoes a change, and
   • a control group that does not undergo the change.

   The effect on the experimental group is then compared to the control group.

   The people are divided into two groups, one of which undergoes a change, so this is an experiment.
9. SHOVELING Ben shovels driveways to raise money. The number of driveways he shovels each day is {2, 4, 3, 5, 3}. Find and interpret the mean absolute deviation.

**SOLUTION:**
Enter the values into L1. Calculate the mean. The mean is 3.4.

For L2, enter \( \text{abs}(L1 - 3.4) \).

Then divide the sum of L2 by \( n \).

\[
\frac{\text{sum}(L2)}{5} = 0.88
\]

Sample answer: On average, the number of driveways that Ben shovels each day is 0.88 away from the mean of 3.4.

10. CANDY BARS Luci is keeping track of the number of candy bars each member of the drill team sold. The results are \{20, 25, 30, 50, 40, 60, 20, 10, 42\}. Find and interpret the mean absolute deviation.

**SOLUTION:**
Enter the values into L1. Calculate the mean. The mean is 33.

For L2, enter \( \text{abs}(L1 - 33) \).

Then divide the sum of L2 by \( n \).

\[
\frac{\text{sum}(L2)}{9} = 13.33333333
\]

Sample answer: On average, the number of candy bars sold is 13.33 away from the mean of 33. The mean absolute deviation is affected by outliers 10 and 60.

---

**Study Guide and Review - Chapter 12**

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Then divide the sum of L2 by \( n \).

\[
\frac{\text{sum}(L2)}{9} = 13.33333333
\]

Sample answer: On average, the number of candy bars sold is 13.33 away from the mean of 33. The mean absolute deviation is affected by outliers 10 and 60.
11. FOOD A fast food company polls a random sample of its day and night customers to find how many times a month they eat out. Compare the means and standard deviations of each data set.

<table>
<thead>
<tr>
<th>Day Customers</th>
<th>Night Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>10, 3, 12, 15, 7, 8, 4, 12, 9, 14, 12, 9</td>
<td>15, 12, 13, 9, 11, 12, 14, 12, 8, 16, 9, 9</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Enter the data in L1 and L2. Calculate the mean and standard deviation for each.

\[ \begin{array}{c|c|c}
\text{1-Var Stats} & \text{L1} & \text{L2} \\
\hline
\bar{x} & 9.583333333 & 11.666666667 \\
\sum{x} & 115 & 149 \\
\sum{x^2} & 1253 & 1706 \\
\sigma_x & 3.70401093 & 2.570225789 \\
\downarrow n & 12 & 12 \\
\end{array} \]

The day customers had a mean of about 9.6 times per month with a standard deviation of about 3.5. The night customers had a mean of about 11.7 times per month with a standard deviation of about 2.5. The night customers had a higher average and their data values were more consistent.

Use a graphing calculator to construct a histogram for the data. Then describe the shape of the distribution.

12. 55, 62, 32, 56, 31, 59, 19, 61, 8, 48, 41, 69, 32, 63, 48, 60, 43, 66, 71, 70, 49, 56, 21, 67

**SOLUTION:**
Enter the data into L1, turn on the stat plot, and choose a suitable window before creating the graph.

The data is negatively skewed.
13. 4, 19, 62, 28, 26, 59, 33, 39, 36, 72, 46, 48, 49, 44, 72, 76, 55, 53, 55, 62, 66, 69, 71, 74

**SOLUTION:**
Enter the data in L1. Then select STAT PLOT. Then select the histogram icon. Select ZOOMSTAT.

[Graph showing histogram]

The graph is clustered to the right and trails off to the left, so the distribution is negatively skewed.

14. **MILK** A grocery store manager tracked the amount of milk in gallons sold each day. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

<table>
<thead>
<tr>
<th>Gallons of Milk Sold Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>383</td>
</tr>
<tr>
<td>296</td>
</tr>
<tr>
<td>364</td>
</tr>
<tr>
<td>238</td>
</tr>
<tr>
<td>195</td>
</tr>
<tr>
<td>372</td>
</tr>
<tr>
<td>421</td>
</tr>
<tr>
<td>367</td>
</tr>
<tr>
<td>411</td>
</tr>
<tr>
<td>355</td>
</tr>
<tr>
<td>296</td>
</tr>
<tr>
<td>221</td>
</tr>
<tr>
<td>403</td>
</tr>
<tr>
<td>357</td>
</tr>
<tr>
<td>432</td>
</tr>
<tr>
<td>220</td>
</tr>
<tr>
<td>180</td>
</tr>
<tr>
<td>256</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Enter the data in L1. Then select STAT PLOT. Then select the box-and-whisker plot icon. Select ZOOMSTAT.

[Box-and-whisker plot]

Sample answer: The data are negatively skewed because the left whisker is longer than the right and the median is closer to the right whisker. Use the five-number summary. The range is 252 gallons. The median is 354.5 gallons. Half of the data are between 288 and 383 gallons.
Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

15. 27, 21, 34, 42, 20, 19, 18, 26, 25, 33; + (−4)

**SOLUTION:**
Enter the data in L1.
Set L2 = L1 – 4.
Find the statistics for L2.

```
1-Var Stats
\[ n = 10 \]
\[ \text{min} = 14 \]
\[ Q_1 = 16 \]
\[ \text{Median} = 21.5 \]
\[ Q_3 = 29 \]
\[ \text{max} = 38 \]
```

22.5, 21.5, no mode, 24, 7.4

16. 72, 56, 71, 63, 68, 59, 77, 74, 76, 66; +16

**SOLUTION:**
Enter the data in L1.

```
1-Var Stats
\[ n = 10 \]
\[ \text{min} = 72 \]
\[ Q_1 = 79 \]
\[ \text{Median} = 85.5 \]
\[ Q_3 = 90 \]
\[ \text{max} = 93 \]
```

84.2, 85.5, no mode, 21, 6.8

17. **SCHOOL** Principal Andrews tracked the number of disciplinary actions given by Ms. Miller and Ms. Anderson to their students each week.

<table>
<thead>
<tr>
<th>Ms. Miller</th>
</tr>
</thead>
<tbody>
<tr>
<td>9, 16, 12, 11, 12, 9,</td>
</tr>
<tr>
<td>10, 14, 13, 10, 9, 10,</td>
</tr>
<tr>
<td>11, 9, 12, 10, 11, 12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ms. Anderson</th>
</tr>
</thead>
<tbody>
<tr>
<td>7, 1, 0, 4, 2, 1,</td>
</tr>
<tr>
<td>6, 2, 2, 1, 4, 3,</td>
</tr>
<tr>
<td>0, 7, 0, 2, 5, 0</td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to construct a histogram
for each set of data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

**SOLUTION:**

a. Enter the data in L1 and L2 of your calculator. Select STATPLOT and the Histogram icon. Select ZOOMSTAT. Do this for L1 and L2.

18. **GAMES** While watching a game at a carnival where participants guess which of three shells is covering a ball, Jeremy tallies the following results.

<table>
<thead>
<tr>
<th>Ball Location</th>
<th>Left</th>
<th>Middle</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>16</td>
<td>18</td>
<td>33</td>
</tr>
</tbody>
</table>

a. Find the experimental probability of the ball being under the right shell.

SOLUTION:

\[
\frac{33}{67} \approx 50\%
\]

b. Find the experimental probability of the ball not being under the middle shell.

SOLUTION:

\[
\frac{33+16}{67} = \frac{49}{67} \approx 73\%
\]

19. **SCHOOL BUS** Christy has determined that the school bus is late 60% of the time.

a. Design a simulation that can be used to estimate the probability that the school bus is late today.

b. Conduct the simulation, and report the results.

**SOLUTION:**

a. Sample answer: The theoretical probability that the bus is late is 60%, and the theoretical probability that the bus is not late is 40%. A spinner can be created, with 3/5 representing late and 2/5 representing not late. A six-sided die can be rolled, with 1-3 representing late, 4-5 representing not late and 6 being a re-roll. We can also use a number generator.

Use a random number generator to generate integers 1 through 5. The integers 1–3 will represent the bus being late, and the integers 4–5 will represent the bus not being late. The simulation will consist of 50 trials.

b. Sample answer: \(P(\text{late}) = 58\%\)
Choose the term that best completes each sentence.

1. An arrangement in which order is important is called a _______.

20. selecting 3 different toppings for pizza from a list of 15

SOLUTION:
The order of the toppings is not important, so it is a combination.

21. an arrangement of textbooks on a bookshelf

SOLUTION:
The order of the books in the arrangement is important, so it is a permutation.

22. a list of teams participating in a tournament

SOLUTION:
The order of the teams is not important because it is just a list, and not a ranking, so it is a combination.

23. a ranking of students by scores

SOLUTION:
The scores are listed by ranking, so the order is important and it is a permutation.

24. CLASS PHOTO The Spanish teacher at South High School wants to arrange 7 students who traveled to Mexico for a yearbook photo. In how many ways can the students be arranged?

SOLUTION:
Order is important, so it is a permutation. We are arranging 7 objects 7 at a time.

\[ P(7, 7) = 5040 \]

25. GOLF BALLS A golf bag contains 5 white golf balls, 6 yellow golf balls, and 4 orange golf balls. Two balls are pulled from the bag at random. What is the probability that both balls are orange golf balls?

SOLUTION:

\[ P(\text{orange, orange}) = P(\text{orange}) \cdot P(\text{orange}) \]

\[ = \frac{4}{15} \cdot \frac{3}{14} \]

\[ = \frac{12}{210} \]

\[ = \frac{2}{35} \]

26. MUSIC Tracie is playing a mix playlist with 6 classic rock, 8 pop, and 4 dance songs.

a. If she selects random play and the songs can repeat, what is the probability that she hears 2 classic rock songs and then a dance song?

b. If the songs cannot repeat, what is the probability that she hears 3 dance songs in a row?

SOLUTION:
a. The outcome of one event does not affect the outcome of the other, so they are independent.

\[ P(\text{classic, classic, dance}) = \frac{6}{18} \cdot \frac{6}{18} \cdot \frac{4}{18} \]

\[ = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{9} \]

\[ = \frac{2}{81} \]

b. The songs can repeat, so the outcome of one event does affect the outcome of another, so they are dependent.

\[ P(\text{dance, dance, dance}) = \frac{4}{18} \cdot \frac{3}{17} \cdot \frac{2}{16} \]

\[ = \frac{2}{9} \cdot \frac{3}{17} \cdot \frac{1}{8} \]

\[ = \frac{1}{204} \]
A box contains 8 red chips, 6 blue chips, and 12 white chips. Chips are randomly drawn from the box and are not replaced.

27. \(P(\text{red, white, blue})\)

**SOLUTION:**

\[
P(\text{red}) = \frac{\text{number of red chips}}{\text{total number of chips}} = \frac{8}{26} = \frac{4}{13}
\]

\[
P(\text{white}) = \frac{\text{number of white chips}}{\text{number of chips remaining}} = \frac{12}{25}
\]

\[
P(\text{blue}) = \frac{\text{number of blue chips}}{\text{number of chips remaining}} = \frac{6}{24} = \frac{1}{4}
\]

\[
P(\text{red, white, blue}) = P(\text{red}) \cdot P(\text{white}) \cdot P(\text{blue})
\]

\[
= \frac{4}{13} \cdot \frac{1}{25} \cdot \frac{1}{4} = \frac{12}{325}
\]

The probability is \(\frac{12}{325}\) or about 3.6%.

28. \(P(\text{red, red, red})\)

**SOLUTION:**

\[
P(\text{red}) = \frac{\text{number of red chips}}{\text{total number of chips}} = \frac{8}{26} = \frac{4}{13}
\]

\[
P(\text{red}) = \frac{\text{number of red chips remaining}}{\text{number of chips remaining}} = \frac{7}{25}
\]

\[
P(\text{red}) = \frac{\text{number of red chips remaining}}{\text{number of chips remaining}} = \frac{6}{24} = \frac{1}{4}
\]

\[
P(\text{red, red, red}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{red})
\]

\[
= \frac{4}{13} \cdot \frac{7}{25} \cdot \frac{1}{4} = \frac{7}{325}
\]

The probability is \(\frac{7}{325}\) or about 2.2%.
29. \( P(\text{red, white, white}) \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>First chip</th>
<th>( P(\text{red}) = \frac{\text{number of red chips}}{\text{total number of chips}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{8}{26} = \frac{4}{13} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second chip</th>
<th>( P(\text{white}) = \frac{\text{number of white chips}}{\text{number of chips remaining}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{12}{25} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Third chip</th>
<th>( P(\text{white}) = \frac{\text{number of white chips remaining}}{\text{number of chips remaining}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{11}{24} )</td>
</tr>
</tbody>
</table>

\[ P(\text{red, white, white}) = P(\text{red}) \cdot P(\text{white}) \cdot P(\text{white}) \]

\[ = \frac{2}{13} \cdot \frac{11}{25} \cdot \frac{11}{24} \]

\[ = \frac{22}{325} \]

The probability is \( \frac{22}{325} \) or about 6.8%.

30. \( P(\text{blue, blue}) \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>First chip</th>
<th>( P(\text{blue}) = \frac{\text{number of blue chips}}{\text{total number of chips}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{6}{26} = \frac{3}{13} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second chip</th>
<th>( P(\text{blue}) = \frac{\text{number of blue chips remaining}}{\text{number of chips remaining}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{5}{25} = \frac{1}{5} )</td>
</tr>
</tbody>
</table>

\[ P(\text{blue, blue}) = P(\text{blue}) \cdot P(\text{blue}) \]

\[ = \frac{3}{13} \cdot \frac{1}{5} \]

\[ = \frac{3}{65} \]

The probability is \( \frac{3}{65} \) or about 4.6%.
One card is randomly drawn from a standard deck of 52 cards. Find each probability.

31. \(P(\text{heart or red})\)

**SOLUTION:**
Since hearts are red, the events are not mutually exclusive.

There are 13 hearts in a standard deck.

\[ P(h) = \frac{13}{52} \]

Half of the deck is red. So, there are 26 red cards.

\[ P(r) = \frac{26}{52} \]

Of the 13 hearts in the deck, all 13 are red.

\[ P(h \text{ and } r) = \frac{13}{52} \]

\[ P(h \text{ or } r) = P(h) + P(r) - P(h \text{ and } r) \]

\[ = \frac{13}{52} + \frac{26}{52} - \frac{13}{52} \]

\[ = \frac{26}{52} \]

\[ = \frac{1}{2} \]

The probability is \(\frac{1}{2}\) or about 50%.

32. \(P(10 \text{ or spade})\)

**SOLUTION:**
Since there is a 10 of spades, the events are not mutually exclusive.

There are four 10s in a standard deck.

\[ P(10) = \frac{4}{52} \]

There are 13 spades.

\[ P(s) = \frac{13}{52} \]

There is one 10 of spades.

\[ P(10 \text{ and } s) = \frac{1}{52} \]

\[ P(10 \text{ or } s) = P(10) + P(s) - P(10 \text{ and } s) \]

\[ = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \]

\[ = \frac{16}{52} \]

\[ = \frac{4}{13} \]

The probability is \(\frac{4}{13}\) or about 30.8%.

A local cable provider asked its subscribers how many television sets they had in their homes. The results of their survey are shown in the probability distribution.

<table>
<thead>
<tr>
<th>(X)</th>
<th>Number of Televisions</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.36</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.34</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td>5+</td>
<td></td>
<td>0.04</td>
</tr>
</tbody>
</table>

33. Show that the distribution is valid.

**SOLUTION:**
For each \(X\), the probability is greater than or equal to 0 and less than or equal to 1, and \(0.18 + 0.36 + 0.34 + 0.08 + 0.04 = 1\)

So, the sum of the probabilities is 1.
34. If a household is selected at random, what is the probability that it has fewer than 4 televisions?

**SOLUTION:**

\[
P(X < 4) = P(X = 3) + P(X = 2) + P(X = 1) \\
= 0.34 + 0.36 + 0.18 \\
= 0.88
\]

The probability that the household has fewer than 4 televisions is 0.88.