3-1 Graphing Linear Equations

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. \(x = y - 5\)

**SOLUTION:**
Rewrite the equation in standard form.

\[x = y - 5\quad \text{Original equation}\]
\[x - y = y - y - 5\quad \text{Subtract } y \text{ from each side}\]
\[x - y = -5\quad \text{Simplify.}\]

The equation is now in standard form where \(A = 1, B = -1,\) and \(C = -5\). This is a linear equation.

2. \(-2x - 3 = y\)

**SOLUTION:**
Rewrite the equation in standard form.

\[-2x - 3 = y\quad \text{Original equation}\]
\[-1(-2x - 3) = -1 \cdot y\quad \text{Multiply each side by } -1\]
\[2x + 3 = -y\quad \text{Simplify}\]
\[2x + 3 - 3 = -y - 3\quad \text{Subtract } 3 \text{ from each side}\]
\[2x = -y - 3\quad \text{Simplify}\]
\[2x + y = -y + y - 3\quad \text{Add } y \text{ to each side}\]
\[2x + y = -3\quad \text{Simplify}\]

The equation is now in standard form where \(A = 2, B = 1,\) and \(C = -3\). This is a linear equation.

3. \(-4y + 6 = 2\)

**SOLUTION:**
Rewrite the equation in standard form.

\[-4y + 6 = 2\quad \text{Original equation}\]
\[-4y + 6 - 6 = 2 - 6\quad \text{Subtract } 6 \text{ from each side}\]
\[-4y = -4\quad \text{Simplify}\]
\[\frac{-4y}{-4} = \frac{-4}{-4}\quad \text{Divide each side by } -4\]
\[y = 1\quad \text{Simplify}\]

The equation is now in standard form where \(A = 0, B = 1,\) and \(C = 1\). This is a linear equation.

4. \(\frac{2}{3}x - \frac{1}{3}y = 2\)

**SOLUTION:**
Rewrite the equation in standard form.

\[\frac{2}{3}x - \frac{1}{3}y = 2\quad \text{Original equation}\]
\[3\left(\frac{2}{3}x - \frac{1}{3}y\right) = 3 \cdot 2\quad \text{Multiply each side by } 3\]
\[2x - y = 6\quad \text{Simplify}\]

The equation is now in standard form where \(A = 2, B = -1,\) and \(C = 6\). This is a linear equation.

Find the \(x\)- and \(y\)-intercepts of the graph of each linear function. Describe what the intercepts mean.

5. [Graph of a line with increasing temperature shown]

**SOLUTION:**
The \(x\)-intercept is the point at which the \(y\)-coordinate is 0, or the line crosses the \(x\)-axis. So, the \(x\)-intercept is 25.
The \(y\)-intercept is the point at which the \(x\)-coordinate is 0, or the line crosses the \(y\)-axis. So, the \(y\)-intercept is \(-4\).
The \(x\)-intercept 25 means that after 25 minutes, the temperature is \(0^\circ\text{F}\). The \(y\)-intercept \(-4\) means that at time 0, the temperature is \(-4^\circ\text{F}\).
3-1 Graphing Linear Equations

6. **Position of Scuba Diver**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−24</td>
</tr>
<tr>
<td>3</td>
<td>−18</td>
</tr>
<tr>
<td>6</td>
<td>−12</td>
</tr>
<tr>
<td>9</td>
<td>−6</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

**SOLUTION:**

The x-intercept is the point at which the y-coordinate is 0, or the line crosses at the x-axis. So, the x-intercept is 12.

The y-intercept is the point at which the x-coordinate is 0, or the line crosses at the y-axis. So, the y-intercept is −24.

The x-intercept 12 means that after 12 seconds, the scuba diver is at a depth of 0 meters, or at the surface. The y-intercept −24 means that at time 0, the scuba diver is at a depth of −24 meters, or 24 meters below sea level.

---

Graph each equation by using the x- and y-intercepts.

7. \( y = 4 + x \)

**SOLUTION:**

To find the x-intercept, let \( y = 0 \).

\[
\begin{align*}
 y &= 4 + x \\
0 &= 4 + x & \text{Original equation} \\
0 - 4 &= 4 - 4 + x & \text{Subtract 4 from each side} \\
-4 &= x & \text{Simplify.}
\end{align*}
\]

To find the y-intercept, let \( x = 0 \).

\[
\begin{align*}
 y &= 4 + x & \text{Original equation} \\
y &= 4 + 0 & \text{Replace } x \text{ with 0.} \\
y &= 4 & \text{Simplify.}
\end{align*}
\]

So, the x-intercept is −4, and the y-intercept is 4. Plot these two points and then draw a line through them.
3-1 Graphing Linear Equations

8. \(2x - 5y = 1\)

**SOLUTION:**
To find the \(x\)-intercept, let \(y = 0\).

\[
2x - 5y = 1 \quad \text{Original equation}
\]
\[
2x - 5(0) = 1 \quad \text{Replace } y \text{ with } 0.
\]
\[
2x - 0 = 1 \quad \text{Simplify.}
\]
\[
2x = 1 \quad \text{Simplify.}
\]
\[
\frac{2x}{2} = \frac{1}{2} \quad \text{Divide each side by } 2.
\]
\[
x = \frac{1}{2} \quad \text{Simplify.}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
2(0) - 5y = 1 \quad \text{Original equation}
\]
\[
2(0) - 5y = 1 \quad \text{Replace } x \text{ with } 0.
\]
\[
0 - 5y = 1 \quad \text{Simplify.}
\]
\[
-5y = 1 \quad \text{Simplify.}
\]
\[
\frac{-5y}{-5} = \frac{1}{-5} \quad \text{Divide each side by } -5.
\]
\[
y = -\frac{1}{5} \quad \text{Simplify.}
\]

So, the \(x\)-intercept is \(\frac{1}{2}\), and the \(y\)-intercept is \(-\frac{1}{5}\).

Plot these two points, and then draw a line through them.

---

9. \(x + 2y = 4\)

**SOLUTION:**
Solve for \(y\).

\[
x + 2y = 4 \quad \text{Original equation}
\]
\[
2y = 4 - x \quad \text{Subtract } x \text{ from each side.}
\]
\[
y = \frac{4 - x}{2} \quad \text{Divide each side by } 2.
\]
\[
y = 2 - \frac{x}{2} \quad \text{Simplify.}
\]

Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = 2 - \frac{x}{2})</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(y = 2 - \frac{-4}{2})</td>
<td>4</td>
<td>(-4, 4)</td>
</tr>
<tr>
<td>-2</td>
<td>(y = 2 - \frac{-2}{2})</td>
<td>3</td>
<td>(-2, 3)</td>
</tr>
<tr>
<td>0</td>
<td>(y = 2 - \frac{0}{2})</td>
<td>2</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>2</td>
<td>(y = 2 - \frac{2}{2})</td>
<td>1</td>
<td>(2, 1)</td>
</tr>
<tr>
<td>4</td>
<td>(y = 2 - \frac{4}{2})</td>
<td>0</td>
<td>(4, 0)</td>
</tr>
</tbody>
</table>
3-1 Graphing Linear Equations

10. \(-3 + 2y = -5\)

**SOLUTION:**

Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-3 + 2y = -5)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(-3 + 2y = -5)</td>
<td>0</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>1</td>
<td>(-3 + 2y = -5)</td>
<td>1</td>
<td>(1, -1)</td>
</tr>
<tr>
<td>2</td>
<td>(-3 + 2y = -5)</td>
<td>2</td>
<td>(2, -1)</td>
</tr>
</tbody>
</table>

![Graph](image-url)

11. \(y = 3\)

**SOLUTION:**

Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = 3)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3</td>
<td>3</td>
<td>((-2, 3))</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
<td>3</td>
<td>((-1, 3))</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
<td>((0, 3))</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>((1, 3))</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>((2, 3))</td>
</tr>
</tbody>
</table>

![Graph](image-url)

12. CCSS REASONING The equation \(5x + 10y = 60\) represents the number of children \(x\) and adults \(y\) who can attend the rodeo for $60.

![Image](image-url)

\(a.\) Use the \(x\)- and \(y\)-intercepts to graph the equation.

\(b.\) Describe what these values mean.

**SOLUTION:**

\(a.\) To find the \(x\)-intercept, let \(y = 0\).

\[5x + 10y = 60\quad\text{Original equation}\]

\[5x + 10(0) = 60\quad\text{Replace } y \text{ with } 0\]

\[5x = 60\quad\text{Simplify}\]

\[x = 12\quad\text{Simplify}\]

To find the \(y\)-intercept, let \(x = 0\).

\[5x + 10y = 60\quad\text{Original equation}\]

\[5(0) + 10y = 60\quad\text{Replace } x \text{ with } 0\]

\[10y = 60\quad\text{Simplify}\]

\[y = 6\quad\text{Simplify}\]

So the \(x\)-intercept is 12 and the \(y\)-intercept is 6. Plot these points, and then draw a line through them.

\(b.\) The \(x\)-intercept means that 12 children and 0 adults can attend for $60. The \(y\)-intercept means that 0 children and 6 adults can attend for $60.
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

13. \(5x + y^2 = 25\)

**SOLUTION:**
Since \(y\) is squared, the equation cannot be written in standard form. So, the equation is not linear.

14. \(8 + y = 4x\)

**SOLUTION:**
Rewrite the equation in standard form.

\[
8 + y = 4x \\
8 - y = 4x - y \\
8 = 4x - y \\
4x - y = 8
\]

The equation is now in standard form where \(A = 4, \ B = 1,\) and \(C = 8.\)

15. \(9xy - 6x = 7\)

**SOLUTION:**
Since there are two variables in one term, the equation cannot be written in standard form. So, the equation is not linear.

16. \(4y^2 + 9 = -4\)

**SOLUTION:**
Since \(y\) is squared, the equation cannot be written in standard form. So, the equation is not linear.

17. \(12x = 7y - 10y\)

**SOLUTION:**
Rewrite the equation in standard form.

\[
12x = 7y - 10y \\
12x = -3y \\
12x + 3y = -3y + 3y \\
12x + 3y = 0 \\
3(4x + y) = 0 \\
\frac{3(4x + y)}{3} = 0 \\
4x + y = 0
\]

The equation is now in standard form where \(A = 1, \ B = 1,\) and \(C = 0.\)

18. \(y = 4x + x\)

**SOLUTION:**
Rewrite the equation in standard form.

\[
y = 4x + x \\
y = 5x \\
y - y = 5x - y \\
0 = 5x - y \\
5x - y = 0
\]

The equation is now in standard form where \(A = 5, \ B = -1,\) and \(C = 0.\)

Find the \(x-\) and \(y-\)intercepts of the graph of each linear function.

19. **SOLUTION:**
The \(x\)-intercept is the point at which the \(y\)-coordinate is 0, or the line crosses the \(x\)-axis. So, the \(x\)-intercept is 3.
The \(y\)-intercept is the point at which the \(x\)-coordinate is 0, or the line crosses the \(y\)-axis. So, the \(y\)-intercept is 4.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

20. **SOLUTION:**
The \(x\)-intercept is the point at which the \(y\)-coordinate is 0, or the line crosses the \(x\)-axis. So, the \(x\)-intercept is -2.
The \(y\)-intercept is the point at which the \(x\)-coordinate is 0, or the line crosses the \(y\)-axis. So, the \(y\)-intercept is 2.
Find the x- and y-intercepts of each linear function. Describe what the intercepts mean.

### Descent of Eagle

![Graph of Descent of Eagle](image)

**SOLUTION:**
The x-intercept is the point at which the y-coordinate is 0, or the line crosses the x-axis. So, the x-intercept is 6.
The y-intercept is the point at which the x-coordinate is 0, or the line crosses the y-axis. So, the y-intercept is 20.
The x-intercept 6 means that the height of the eagle is 0 ft after 6 seconds, or that it takes 6 seconds for the eagle to land. The y-intercept 20 means that at time 0, the height of the eagle is 20 ft. In other words, the initial height of the eagle is 20 ft.

<table>
<thead>
<tr>
<th>Eva's Distance from Home</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time (min)</strong></td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

22. **SOLUTION:**
The x-intercept is the point at which the y-coordinate is 0, or the line crosses the x-axis. So, the x-intercept is 8.
The y-intercept is the point at which the x-coordinate is 0, or the line crosses the y-axis. So, the y-intercept is 4.
The x-intercept 8 means that after 8 minutes, Eva’s distance from home is 0 mi., or she is home after 8 minutes. The y-intercept 4 means that at time 0, Eva’s distance from home is 4 miles. In other words, she is initially 4 miles from home.

Graph each equation by using the x- and y-intercepts.

23. \( y = 4 + 2x \)

**SOLUTION:**
To find the x-intercept, let \( y = 0 \).

\[
\begin{align*}
y &= 4 + 2x & \text{Original equation} \\
0 &= 4 + 2x & \text{Replace } y \text{ with } 0. \\
0 - 4 &= 4 - 4 + 2x & \text{Subtract } 4 \text{ from each side} \\
-4 &= 2x & \text{Simplify}. \\
\frac{-4}{2} &= \frac{2x}{2} & \text{Divide each side by } 2. \\
-2 &= x & \text{Simplify}. \\
\end{align*}
\]

To find the y-intercept, let \( x = 0 \).

\[
\begin{align*}
y &= 4 + 2x & \text{Original equation} \\
y &= 4 + 2(0) & \text{Replace } x \text{ with } 0. \\
y &= 4 + 0 & \text{Simplify}. \\
y &= 4 & \text{Simplify}. \\
\end{align*}
\]

So, the x-intercept is \(-2\), and the y-intercept is 4. Plot these two points and then draw a line through them.
24. \(5 - y = -3x\)

**SOLUTION:**

To find the \(x\)-intercept, let \(y = 0\).

\[
\begin{align*}
5 - y &= -3x & \text{Original equation} \\
5 - 0 &= -3x & \text{Replace y with 0.} \\
5 &= -3x & \text{Simplify.} \\
\frac{5}{-3} &= \frac{-3x}{-3} & \text{Divide each side by } -3. \\
\frac{5}{-3} &= x & \text{Simplify.}
\end{align*}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
\begin{align*}
5 - y &= -3x & \text{Original equation} \\
5 - y &= -3(0) & \text{Replace x with 0.} \\
5 - y &= 0 & \text{Simplify.} \\
5 - y + y &= 0 + y & \text{Add } y \text{ to each side} \\
5 &= y & \text{Simplify.}
\end{align*}
\]

So, the \(x\)-intercept is \(-\frac{5}{3}\), and the \(y\)-intercept is \(5\).

Plot these two points and then draw a line through them.

25. \(x = 5y + 5\)

**SOLUTION:**

To find the \(x\)-intercept, let \(y = 0\).

\[
\begin{align*}
x &= 5y + 5 & \text{Original equation} \\
x &= 5(0) + 5 & \text{Replace } y \text{ with 0.} \\
x &= 0 + 5 & \text{Simplify.} \\
x &= 5 & \text{Simplify.}
\end{align*}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
\begin{align*}
x &= 5y + 5 & \text{Original equation} \\
0 &= 5y + 5 & \text{Replace } x \text{ with 0.} \\
0 - 5 &= 5y + 5 - 5 & \text{Subtract 5 from each side} \\
-5 &= 5y & \text{Simplify.} \\
\frac{-5}{5} &= \frac{5y}{5} & \text{Divide each side by 5.} \\
-1 &= y & \text{Simplify.}
\end{align*}
\]

So, the \(x\)-intercept is \(5\) and the \(y\)-intercept is \(-1\). Plot these two points and then draw a line through them.
3-1 Graphing Linear Equations

26. \( x + y = 4 \)

**SOLUTION:**
To find the \( x \)-intercept, let \( y = 0 \).

\[ x + y = 4 \quad \text{Original equation} \]
\[ x + 0 = 4 \quad \text{Replace } y \text{ with 0.} \]
\[ x = 4 \quad \text{Simplify.} \]

To find the \( y \)-intercept, let \( x = 0 \).

\[ x + y = 4 \quad \text{Original equation} \]
\[ 0 + y = 4 \quad \text{Replace } x \text{ with 0.} \]
\[ y = 4 \quad \text{Simplify.} \]

So, the \( x \)-intercept is 4 and the \( y \)-intercept is 4. Plot these two points and then draw a line through them.

27. \( x - y = -3 \)

**SOLUTION:**
To find the \( x \)-intercept, let \( y = 0 \).

\[ x - y = -3 \quad \text{Original equation} \]
\[ x - 0 = -3 \quad \text{Replace } y \text{ with 0.} \]
\[ x = -3 \quad \text{Simplify.} \]

To find the \( y \)-intercept, let \( x = 0 \).

\[ x - y = -3 \quad \text{Original equation} \]
\[ 0 - y = -3 \quad \text{Replace } x \text{ with 0.} \]
\[ -y = -3 \quad \text{Simplify.} \]
\[ -1(-y) = -1(-3) \quad \text{Multiply each side by } -1. \]
\[ y = 3 \quad \text{Simplify} \]

So, the \( x \)-intercept is \(-3\) and the \( y \)-intercept is \(3\). Plot these two points and then draw a line through them.
3-1 Graphing Linear Equations

28. \( y = 8 - 6x \)

**SOLUTION:**
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
y &= 8 - 6x \quad \text{Original equation} \\
0 &= 8 - 6x \quad \text{Replace } y \text{ with } 0 \\
0 + 6x &= 8 - 6x + 6x \quad \text{Add } 6x \text{ from each side} \\
6x &= 8 \quad \text{Simplify} \\
\frac{6x}{6} &= \frac{8}{6} \quad \text{Divide each side by } 6 \\
x &= \frac{4}{3} \quad \text{Simplify}
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
y &= 8 - 6x \quad \text{Original equation} \\
y &= 8 - 6(0) \quad \text{Replace } x \text{ with } 0 \\
y &= 8 - 0 \quad \text{Simplify} \\
y &= 8 \quad \text{Simplify}
\end{align*}
\]

So, the \( x \)-intercept is \( \frac{4}{3} \) and the \( y \)-intercept is 8. Plot these two points and then draw a line through them.

---

29. \( x = -2 \)

**SOLUTION:**
Select values from the range and make a table. Create ordered pairs and graph them.

Note that every value in the range is linked to \(-2\) in the domain.

---

30. \( y = -4 \)

**SOLUTION:**
Select values from the domain and make a table. Create ordered pairs and graph them.

---

31. \( y = -8x \)

**SOLUTION:**
Select values from the domain and make a table. Create ordered pairs and graph them.
3.1 Graphing Linear Equations

32. $3x = y$

**SOLUTION:**
Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

![Graph of 3x = y](image)

33. $y - 8 = -x$

**SOLUTION:**
Solve for $y$ in $y - 8 = -x$.

\[
\begin{align*}
y - 8 &= -x & \text{Original equation} \\
y - 8 + 8 &= -x + 8 & \text{Add 8 to each side} \\
y &= -x + 8 & \text{Simplify.}
\end{align*}
\]

Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -x + 8$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$y = (-4) + 8$</td>
<td>12</td>
<td>(-4, 12)</td>
</tr>
<tr>
<td>-2</td>
<td>$y = (-2) + 8$</td>
<td>10</td>
<td>(-2, 10)</td>
</tr>
<tr>
<td>0</td>
<td>$y = 0 + 8$</td>
<td>8</td>
<td>(0, 8)</td>
</tr>
<tr>
<td>1</td>
<td>$y = 1 + 8$</td>
<td>7</td>
<td>(7, 7)</td>
</tr>
<tr>
<td>2</td>
<td>$y = -2 + 8$</td>
<td>6</td>
<td>(2, 6)</td>
</tr>
<tr>
<td>4</td>
<td>$y = -4 + 8$</td>
<td>4</td>
<td>(4, 4)</td>
</tr>
</tbody>
</table>

![Graph of y - 8 = -x](image)

34. $x = 10 - y$

**SOLUTION:**
Solve $x = 10 - y$ for $y$.

\[
\begin{align*}
x &= 10 - y & \text{Original equation} \\
x + y &= 10 - y + y & \text{Add y from each side} \\
- x + y &= - x + 10 & \text{Simplify} \\
- x + y &= - x + 10 & \text{Subtract x from each side} \\
y &= - x + 10 & \text{Simplify.}
\end{align*}
\]

Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -x + 10$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$y = (-4) + 10$</td>
<td>14</td>
<td>(-4, 14)</td>
</tr>
<tr>
<td>-2</td>
<td>$y = (-2) + 10$</td>
<td>12</td>
<td>(-2, 12)</td>
</tr>
<tr>
<td>0</td>
<td>$y = 0 + 10$</td>
<td>10</td>
<td>(0, 10)</td>
</tr>
<tr>
<td>1</td>
<td>$y = -1 + 10$</td>
<td>9</td>
<td>(1, 9)</td>
</tr>
<tr>
<td>2</td>
<td>$y = -2 + 10$</td>
<td>8</td>
<td>(2, 8)</td>
</tr>
<tr>
<td>4</td>
<td>$y = -4 + 10$</td>
<td>6</td>
<td>(4, 6)</td>
</tr>
</tbody>
</table>

![Graph of x = 10 - y](image)
3-1 Graphing Linear Equations

35. TV RATINGS The number of people who watch a singing competition can be given by \( p = 0.15v \), where \( p \) represents the number of people in millions who saw the show and \( v \) is the number of potential viewers in millions.

a. Make a table of values for the points \((v, p)\).

<table>
<thead>
<tr>
<th>( v )</th>
<th>( p = 0.15v )</th>
<th>( p )</th>
<th>((v, p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.15</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>2</td>
<td>(2, 0.3)</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>4</td>
<td>(4, 0.6)</td>
</tr>
<tr>
<td>6</td>
<td>0.9</td>
<td>6</td>
<td>(6, 0.9)</td>
</tr>
<tr>
<td>8</td>
<td>1.2</td>
<td>8</td>
<td>(8, 1.2)</td>
</tr>
<tr>
<td>10</td>
<td>1.5</td>
<td>10</td>
<td>(10, 1.5)</td>
</tr>
</tbody>
</table>

b. Graph the ordered pairs from the table.

c. The table shows that the value of \( p \) increases by 0.3 as \( v \) increases by 2. So at \( v = 14, p \) would be about 2.1 million.

d. \( v < 0 \) does not make sense because there cannot be fewer than 0 viewers.

Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

36. \( x + \frac{1}{y} = 7 \)

**SOLUTION:**

Because \( y \) is in the denominator of a fraction, the equation cannot be written in standard form. Multiplying each side by \( y \) in order to get \( y \) out of the denominator will lead to an \( xy \)-term. So, the equation is not linear.

37. \( \frac{x}{2} = 10 + \frac{2y}{3} \)

**SOLUTION:**

Rewrite the equation in standard form.

\[
\frac{x}{2} = 10 + \frac{2y}{3} \quad \text{Original equation}
\]

\[
2 \left( \frac{x}{2} \right) = 2 \cdot 10 + 2 \left( \frac{2y}{3} \right) \quad \text{Multiply each term by 2.}
\]

\[
x = 20 + \frac{4y}{3} \quad \text{Simplify}
\]

\[
3x = 60 + 4y \quad \text{Multiply each term by 3.}
\]

\[
3x - 4y = 60 \quad \text{Subtract 4y from each side}
\]

\[
3x - 4y = 60 \quad \text{Simplify}
\]

The equation is now in standard form where \( A = 3, B = -4, \) and \( C = 60 \). The equation is linear.

38. \( 7n - 8m = 4 - 2m \)

**SOLUTION:**

Rewrite the equation in standard form.

\[
7n - 8m = 4 - 2m \quad \text{Original equation}
\]

\[
7n - 8m + 2m = 4 - 2m + 2m \quad \text{Add 2m to each side.}
\]

\[
7n - 6m = 4 \quad \text{Reorder terms.}
\]

\[
-6m + 7n = 4 \quad \text{Simplify}
\]

\[
-1( -6m + 7n) = -1 \cdot 4 \quad \text{Multiply each side by -1.}
\]

\[
6m - 7n = -4 \quad \text{Simplify}
\]

The equation is now in standard form where \( A = 6, B = -7, \) and \( C = -4 \). The equation is linear.
3-1 Graphing Linear Equations

39. \(3a + b - 2 = b\)

**SOLUTION:**
Rewrite the equation in standard form.

\[
\begin{align*}
3a + b - 2 &= b \\
3a - 2 &= 0 \\
3a &= 2 \\
\text{Simplify.}
\end{align*}
\]

The equation is now in standard form where \(A = 3\), \(B = 0\), and \(C = 2\). The equation is linear.

40. \(2r - 3rt + 5t = 1\)

**SOLUTION:**
Since there are two variables in one term, the equation cannot be written in standard form. So, the equation is not linear.

41. \(\frac{3m}{4} = \frac{2m}{3} - 5\)

**SOLUTION:**
Rewrite the equation in standard form.

\[
\begin{align*}
\frac{3m}{4} &= \frac{2m}{3} - 5 \\
4 \left( \frac{3m}{4} \right) &= 4 \left( \frac{2m}{3} - 5 \right) \\
3m &= \frac{8m}{3} - 20 \\
\text{Simplify.} \\
3 \cdot 3m &= 3 \left( \frac{8m}{3} - 20 \right) \\
9m &= 8m - 60 \\
\text{Simplify.} \\
9m - 8m &= 8m - 8m - 60 \\
9m - 8m &= -60 \\
\text{Subtract 8m from each side} \\
9m - 8m &= -60 \\
\text{Simplify.}
\end{align*}
\]

The equation is now in standard form where \(A = 9\), \(B = -8\), and \(C = -60\). The equation is linear.

42. **FINANCIAL LITERACY** James earns a monthly salary of $1200 and a commission of $125 for each car he sells.

a. Graph an equation that represents how much James earns in a month in which he sells \(x\) cars.

b. Use the graph to estimate the number of cars James needs to sell in order to earn $5000.

**SOLUTION:**

a. Let the equation \(y = 125x + 1200\) represent James’s monthly salary, where \(y = \) the total monthly salary and \(x = \) the number of cars he sells.

Choose at least two values of \(x\). Solve for \(y\) and plot the points. [Ex. \((0, 1200), (1, 1325), (10, 2450)]\) Draw a line through the points to graph.

b. Let \(y = 5000\). Solve for \(x\).

\[
\begin{align*}
y &= 125x + 1200 & \text{Original equation} \\
5000 &= 125x + 1200 & \text{Subtract} \\
3800 &= 125x & \text{Simplify} \\
3800 &= 125 \times \frac{3800}{125} & \text{Divide} \\
30.4 &= \frac{3800}{125} & \text{Simplify} \\
30.4 &= x
\end{align*}
\]

He needs to sell about 30 cars to earn $5000.
Graph each equation.
43. \(2.5x - 4 = y\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).

\[
2.5x - 4 = y \quad \text{Original equation}
\]
\[
2.5x - 4 = 0 \quad \text{Replace } y \text{ with 0.}
\]
\[
2.5x = 4 \quad \text{Add 4 to each side.}
\]
\[
2.5x = 4 \quad \text{Simplify.}
\]
\[
\frac{2.5x}{2.5} = \frac{4}{2.5} \quad \text{Divide each side by 2.5.}
\]
\[
x = 1.6 \quad \text{Simplify.}
\]
To find the \(y\)-intercept, let \(x = 0\).

\[
y = 2.5x - 4 \quad \text{Original equation}
\]
\[
y = 2.5(0) - 4 \quad \text{Replace } x \text{ with 0.}
\]
\[
y = 0 - 4 \quad \text{Simplify.}
\]
\[
y = -4 \quad \text{Simplify.}
\]
So, the \(x\)-intercept is 1.6 and the \(y\)-intercept is -4.

44. \(1.25x + 7.5 = y\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).

\[
1.25x + 7.5 = y \quad \text{Original equation}
\]
\[
1.25x + 7.5 = 0 \quad \text{Replace } y \text{ with 0.}
\]
\[
1.25x + 7.5 - 7.5 = 0 - 7.5 \quad \text{Subtract 7.5 from each side}
\]
\[
1.25x = -7.5 \quad \text{Simplify.}
\]
\[
\frac{1.25x}{1.25} = \frac{-7.5}{1.25} \quad \text{Divide each side by 1.25.}
\]
\[
x = -6 \quad \text{Simplify.}
\]
To find the \(y\)-intercept, let \(x = 0\).

\[
1.25x + 7.5 = y \quad \text{Original equation}
\]
\[
1.25(0) + 7.5 = y \quad \text{Replace } x \text{ with 0.}
\]
\[
0 + 7.5 = y \quad \text{Simplify.}
\]
\[
7.5 = y \quad \text{Simplify.}
\]
So, the \(x\)-intercept is -6 and the \(y\)-intercept is 7.5.
45. \( y + \frac{1}{2}x = 3 \)

**SOLUTION:**
To graph the equation, find the \( x \)- and \( y \)-intercepts. Plot these two points. Then draw a line through them.
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
y + \frac{1}{2}x &= 3 \\
0 + \frac{1}{2}x &= 3 \\
\frac{1}{2}x &= 3 \\
5\left(\frac{1}{2}x\right) &= 5 \cdot 3 \\
x &= 15
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
y + \frac{1}{2}x &= 3 \\
y + \frac{1}{2}(0) &= 3 \\
y &= 3
\end{align*}
\]

So, the \( x \)-intercept is 15 and the \( y \)-intercept is 3.

46. \( \frac{2}{3}x + y = -7 \)

**SOLUTION:**
To graph the equation, find the \( x \)- and \( y \)-intercepts. Plot these two points. Then draw a line through them.
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
\frac{2}{3}x + y &= -7 \\
\frac{2}{3}x + 0 &= -7 \\
\frac{2}{3}x &= -7 \\
\end{align*}
\]

\[
\begin{align*}
\frac{3}{2}\left(\frac{2}{3}x\right) &= \frac{3}{2}(-7) \\
x &= -\frac{21}{2} \\
x &= -10\frac{1}{2}
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
\frac{2}{3}x + y &= -7 \\
\frac{2}{3}(0) + y &= -7 \\
0 + y &= -7 \\
y &= -7
\end{align*}
\]

So, the \( x \)-intercept is \(-10\frac{1}{2}\) and the \( y \)-intercept is \(-7\).
3-1 Graphing Linear Equations

47. \(2x - 3 = 4y + 6\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).
\[
2x - 3 = 4y + 6 \quad \text{Original equation}
\]
\[
2x - 3 = 4(0) + 6 \quad \text{Replace } y \text{ with } 0.
\]
\[
2x - 3 = 0 + 6 \quad \text{Simplify}
\]
\[
2x - 3 + 3 = 6 + 3 \quad \text{Add } 3 \text{ to each side.}
\]
\[
2x = 9 \quad \text{Simplify}
\]
\[
\frac{2x}{2} = \frac{9}{2} \quad \text{Divide each side by } 2
\]
\[
x = \frac{9}{2} \quad \text{Simplify}
\]
\[
x = 4\frac{1}{2}
\]

To find the \(y\)-intercept, let \(x = 0\).
\[
2x - 3 = 4y + 6 \quad \text{Original equation}
\]
\[
2(0) - 3 = 4y + 6 \quad \text{Replace } x \text{ with } 0.
\]
\[
-3 - 3 = 4y + 6 \quad \text{Simplify}
\]
\[
-6 = 4y + 6 \quad \text{Subtract } 6 \text{ from each side}
\]
\[
-9 = 4y \quad \text{Simplify}
\]
\[
\frac{-9}{4} = \frac{4y}{4} \quad \text{Divide each side by } 4.
\]
\[
-2\frac{1}{4} = y \quad \text{Simplify}
\]

So, the \(x\)-intercept is \(4\frac{1}{2}\) and the \(y\)-intercept is \(-2\frac{1}{4}\).

48. \(3y - 7 = 4x + 1\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).
\[
3y - 7 = 4x + 1 \quad \text{Original equation}
\]
\[
3(0) - 7 = 4x + 1 \quad \text{Replace } y \text{ with } 0.
\]
\[
0 - 7 = 4x + 1 \quad \text{Simplify}
\]
\[
-7 = 4x + 1 \quad \text{Subtract } 1 \text{ from each side}
\]
\[
-8 = 4x \quad \text{Simplify}
\]
\[
-\frac{8}{4} = \frac{4x}{4} \quad \text{Divide each side by } 4.
\]
\[
-2 = x \quad \text{Simplify}
\]

To find the \(y\)-intercept, let \(x = 0\).
\[
3y - 7 = 4x + 1 \quad \text{Original equation}
\]
\[
3y - 7 = 4(0) + 1 \quad \text{Replace } x \text{ with } 0.
\]
\[
3y - 7 = 0 + 1 \quad \text{Simplify}
\]
\[
3y - 7 + 7 = 1 + 7 \quad \text{Add } 7 \text{ to each side.}
\]
\[
3y = 8 \quad \text{Simplify}
\]
\[
\frac{3y}{3} = \frac{8}{3} \quad \text{Divide each side by } 3
\]
\[
y = \frac{8}{3} \quad \text{Simplify}
\]
\[
y = 2\frac{2}{3}
\]

So the \(x\)-intercept is \(-2\) and the \(y\)-intercept is \(2\frac{2}{3}\).
3-1 Graphing Linear Equations

49. CCSS REASONING Mrs. Johnson is renting a car for vacation and plans to drive a total of 800 miles. A rental car company charges $153 for the week including 700 miles and $0.23 for each additional mile. If Mrs. Johnson has only $160 to spend on the rental car, can she afford to rent a car? Explain your reasoning.

**SOLUTION:**
To find the total cost of renting the car, find the sum of the weekly rental fee and the cost per mile times the number of miles she will drive over 700 because 700 miles are included in the weekly rental fee. Graph the function $y = 0.23x + 153$, where $x$ is the number of miles over 700. When $x = 100$, the cost $176.$

![Graph of y = 0.23x + 153]

Because Mrs. Johnson only has $160 to spend, she cannot afford to rent the car.

50. AMUSEMENT PARKS An amusement park charges $50 for admission before 6 p.m. and $20 for admission after 6 p.m. On Saturday, the park took in a total of $20,000.

a. Write an equation that represents the number of admissions that may have been sold. Let $x$ represent the admissions sold before 6 p.m., and let $y$ represent the admissions sold after 6 p.m.
b. Graph the equation.
c. Find the $x$- and $y$-intercepts of the graph. What does each intercept represent?

**SOLUTION:**
a. To find the equation that represents the total amount the park took in on Saturday, find the sum of the amount taken in by admissions sold before 6 p.m. and the amount taken in by admissions sold after 6 p.m. The equation is $20,000 = 50x + 20y$.

b. Solve for $y$.

To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -\frac{5}{2}x + 1000$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = -\frac{5}{2}(0) + 1000$</td>
<td>1000</td>
<td>(0, 100)</td>
</tr>
<tr>
<td>50</td>
<td>$y = -\frac{5}{2}(50) + 1000$</td>
<td>875</td>
<td>(50, 875)</td>
</tr>
<tr>
<td>100</td>
<td>$y = -\frac{5}{2}(100) + 1000$</td>
<td>750</td>
<td>(100, 750)</td>
</tr>
<tr>
<td>200</td>
<td>$y = -\frac{5}{2}(200) + 1000$</td>
<td>500</td>
<td>(200, 500)</td>
</tr>
<tr>
<td>300</td>
<td>$y = -\frac{5}{2}(300) + 1000$</td>
<td>250</td>
<td>(300, 250)</td>
</tr>
<tr>
<td>400</td>
<td>$y = -\frac{5}{2}(400) + 1000$</td>
<td>0</td>
<td>(400, 0)</td>
</tr>
</tbody>
</table>

![Amusement Park Admissions graph]

c. To find the $x$-intercept, let $y = 0$.

\[
20,000 = 50x + 20y \quad \text{Original equation} \\
20,000 = 50x + 20(0) \quad \text{Replace y with 0.} \\
20,000 = 50x + 0 \quad \text{Simplify} \\
\frac{20,000}{50} = \frac{50x}{50} \quad \text{Divide each side by 50} \\
400 = x \quad \text{Simplify.}
\]

To find the $y$-intercept, let $x = 0$.
3-1 Graphing Linear Equations

20,000 = 50x + 20y \quad \text{Original equation}
20,000 = 50(0) + 20y \quad \text{Replace } x \text{ with } 0.
20,000 = 0 + 20y \quad \text{Simplify.}
\frac{20,000}{20} = \frac{20y}{20} \quad \text{Divide each side by } 20
1000 = y \quad \text{Simplify.}

So, the x-intercept of 400 means that if 0 admissions were sold after 6 p.m., 400 admissions must have been sold before 6 p.m. to equal the daily amount taken in. The y-intercept of 1000 means that if 0 admissions were sold before 6 p.m., 1000 admissions must have been sold after 6 p.m. to equal the daily amount taken in.

Find the x–intercept and y–intercept of the graph of each equation.

51. 5x + 3y = 15

SOLUTION:
To find the x-intercept, let y = 0.

5x + 3y = 15 \quad \text{Original equation}
5x + 3(0) = 15 \quad \text{Replace } y \text{ with } 0.
5x + 0 = 15 \quad \text{Simplify.}
5x = 15 \quad \text{Simplify.}
\frac{5x}{5} = \frac{15}{5} \quad \text{Divide each side by } 5
x = 3 \quad \text{Simplify.}

To find the y-intercept, let x = 0.

5x + 3y = 15 \quad \text{Simplify.}
5(0) + 3y = 15 \quad \text{Replace } x \text{ with } 0.
0 + 3y = 15 \quad \text{Simplify.}
3y = 15 \quad \text{Simplify.}
\frac{3y}{3} = \frac{15}{3} \quad \text{Divide each side by } 3
y = 5 \quad \text{Simplify.}

So, the x-intercept is 3 and the y-intercept is 5.

52. 2x – 7y = 14

SOLUTION:
To find the x-intercept, let y = 0.

2x – 7y = 14 \quad \text{Original equation}
2x – 7(0) = 14 \quad \text{Replace } y \text{ with } 0.
2x = 14 \quad \text{Simplify.}
\frac{2x}{2} = \frac{14}{2} \quad \text{Divide each side by } 2
x = 7 \quad \text{Simplify.}

To find the y-intercept, let x = 0.

2x – 7y = 14 \quad \text{Original equation}
2(0) – 7y = 14 \quad \text{Replace } x \text{ with } 0.
0 – 7y = 14 \quad \text{Simplify.}
–7y = 14 \quad \text{Simplify.}
\frac{–7y}{–7} = \frac{14}{–7} \quad \text{Divide each side by } –7
y = –2 \quad \text{Simplify.}

So, the x-intercept is 7 and the y-intercept is –2.
3-1 Graphing Linear Equations

53. \(2x - 3y = 5\)

**SOLUTION:**
To find the \(x\)-intercept, let \(y = 0\).

\[
2x - 3y = 5 \quad \text{Original equation}
\]
\[
2x - 3(0) = 5 \quad \text{Replace } y \text{ with } 0.
\]
\[
2x = 5 \quad \text{Simplify.}
\]
\[
\frac{2x}{2} = \frac{5}{2} \quad \text{Divide each side by } 2
\]
\[
x = \frac{5}{2} \quad \text{Simplify.}
\]
\[
x = 2 \frac{1}{2} \quad \text{Simplify.}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
2x - 3y = 5 \quad \text{Original equation}
\]
\[
2(0) - 3y = 5 \quad \text{Replace } x \text{ with } 0.
\]
\[
-3y = 5 \quad \text{Simplify.}
\]
\[
\frac{-3y}{-3} = \frac{5}{-3} \quad \text{Divide each side by } -3
\]
\[
y = -\frac{5}{3} \quad \text{Simplify.}
\]
\[
y = -1 \frac{2}{3}
\]

So, the \(x\)-intercept is \(2 \frac{1}{2}\), and the \(y\)-intercept is \(-1 \frac{2}{3}\).

54. \(6x + 2y = 8\)

**SOLUTION:**
To find the \(x\)-intercept, let \(y = 0\).

\[
6x + 2y = 8 \quad \text{Original equation}
\]
\[
6x + 2(0) = 8 \quad \text{Replace } y \text{ with } 0.
\]
\[
6x = 8 \quad \text{Simplify.}
\]
\[
\frac{6x}{6} = \frac{8}{6} \quad \text{Divide each side by } 6
\]
\[
x = \frac{4}{3} \quad \text{Simplify.}
\]
\[
x = 1 \frac{1}{3}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
6x + 2y = 8 \quad \text{Original equation}
\]
\[
6(0) + 2y = 8 \quad \text{Replace } x \text{ with } 0.
\]
\[
2y = 8 \quad \text{Simplify.}
\]
\[
\frac{2y}{2} = \frac{8}{2} \quad \text{Divide each side by } 2
\]
\[
y = 4 \quad \text{Simplify.}
\]

So, the \(x\)-intercept is \(1 \frac{1}{3}\) and the \(y\)-intercept is \(4\).
55. \( y = \frac{1}{4}x - 3 \)

**SOLUTION:**
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
  y &= \frac{1}{4}x - 3 & \text{Original equation} \\
  0 &= \frac{1}{4}x - 3 & \text{Replace } y \text{ with } 0. \\
  0 + 3 &= \frac{1}{4}x - 3 + 3 & \text{Add } 3 \text{ to each side} \\
  3 &= \frac{1}{4}x & \text{Simplify.} \\
  3 \cdot 4 &= 4 \cdot \frac{1}{4}x & \text{Multiply each side by } 4 \\
  12 &= x & \text{Simplify.}
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
  y &= \frac{1}{4}(0) - 3 & \text{Original equation} \\
  y &= 0 - 3 & \text{Replace } x \text{ with } 0. \\
  y &= -3 & \text{Simplify.}
\end{align*}
\]

So, the \( x \)-intercept is 12 and the \( y \)-intercept is -3.

56. \( y = \frac{2}{3}x + 1 \)

**SOLUTION:**
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
  y &= \frac{2}{3}x + 1 & \text{Original equation} \\
  0 &= \frac{2}{3}x + 1 & \text{Replace } y \text{ with } 0. \\
  0 - 1 &= \frac{2}{3}x + 1 - 1 & \text{Subtract } 1 \text{ from each side} \\
  -1 &= \frac{2}{3}x & \text{Simplify.} \\
  3(-1) &= 3(\frac{2}{3}x) & \text{Multiply each side by } 3 \\
  -3 &= 2x & \text{Simplify.} \\
  \frac{-3}{2} &= \frac{2x}{2} & \text{Divide each side by } 2. \\
  -\frac{3}{2} &= x & \text{Simplify.} \\
  -1\frac{1}{2} &= x & \text{Simplify.}
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
  y &= \frac{2}{3}x + 1 & \text{Original equation} \\
  y &= \frac{2}{3}(0) + 1 & \text{Replace } x \text{ with } 0. \\
  y &= 0 + 1 & \text{Simplify.} \\
  y &= 1 & \text{Simplify.}
\end{align*}
\]

So, the \( x \)-intercept is \(-1\frac{1}{2}\) and the \( y \)-intercept is 1.

57. **ONLINE GAMES** The percent of teens who play online games can be modeled by \( p = \frac{15}{4}t + 66 \). \( p \) is

the percent of students and \( t \) represents time in years since 2000.

**a.** Graph the equation.

**b.** Use the graph to estimate the percent of students playing the games in 2008.

**SOLUTION:**

**a.** Select values from the domain and make a table. Create ordered pairs and graph them.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( p = \frac{15}{4}t + 66 )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( p = \frac{15}{4}(0) + 66 )</td>
<td>66</td>
</tr>
</tbody>
</table>
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

To find a y-intercept, let x = 0 and solve the equation for y. For example, consider the example $y = 69.75$.

For example, consider the example $A = (2) + 64 = 88$. To find a y-intercept, let x = 0 and solve the equation for y. For example, consider the example $A = (2) + 64 = 88$.

So, the percent of students playing the games in 2008 is 96%.

58. **MULTIPLE REPRESENTATIONS** In this problem, you will explore x– and y–intercepts of graphs of linear equations.

a. **GRAPHICAL** If possible, use a straightedge to draw a line on a coordinate plane with each of the following characteristics.

1.) x- and y- intercepts
2.) x- intercept, no y- intercepts
3.) exactly 2 x- intercepts
4.) no x- intercept, y- intercepts
5.) exactly 2 y- intercepts

b. **ANALYTICAL** For which characteristics were you able to create a line and for which characteristics were you unable to create a line? Explain.

c. **VERBAL** What must be true of the x– and y–intercepts of a line?

**SOLUTION:**

a. 

1.) x- and y- intercepts

2.) x- intercept, no y- intercepts

The percent of students is about 95%
You can solve the equation for $t = 8$ to verify.
3-1 Graphing Linear Equations

3.) exactly 2 \( x \)-intercepts
A straight line can not be drawn that intersects the \( x \)-axis exactly two times.

4.) no \( x \)-intercept, \( y \)-intercepts

5.) exactly 2 \( y \)-intercepts
A straight line can not be drawn that intersects the \( y \)-axis exactly two times.

b. I was able to draw a line with an \( x \)- and a \( y \)-intercept, an \( x \)-intercept and no \( y \)-intercept, and no \( x \)-intercept and a \( y \)-intercept. I was unable to draw a line with 2 \( x \)-intercepts or 2 \( y \)-intercepts. A line that has either 2 \( x \)-intercepts or 2 \( y \)-intercepts would not be a line.

c. Lines that are neither vertical nor horizontal cannot have more than one \( x \)- and/or \( y \)-intercept.

59. CCSS REGULARITY Copy and complete each table. State whether any of the tables show a linear relationship. Explain.

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**

Table 1:
The formula for the perimeter of a square is \( P = 4s \).

<table>
<thead>
<tr>
<th>Side Length</th>
<th>( P = 4s )</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P = 4(1) )</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>( P = 4(2) )</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>( P = 4(3) )</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>( P = 4(4) )</td>
<td>16</td>
</tr>
</tbody>
</table>

The table does show a linear relationship, since \( P = 4s \) is linear.

Table 2:
The formula for the area of a square is \( A = s^2 \).
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

To find a y-intercept, let \( x = 0 \) and solve the equation for \( y \).

For example, consider the example

Table 3:
The formula for the volume of a square is \( V = s^3 \).

<table>
<thead>
<tr>
<th>Side Length</th>
<th>( A = s^2 )</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A = (1)^2 )</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( A = (2)^2 )</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>( A = (3)^2 )</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>( A = (4)^2 )</td>
<td>16</td>
</tr>
</tbody>
</table>

The table does not show a linear relationship since \( A = s^2 \) is not linear.

Table 3:
The formula for the volume of a square is \( V = s^3 \).

<table>
<thead>
<tr>
<th>Side Length</th>
<th>( V = s^3 )</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( V = (1)^3 )</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( V = (2)^3 )</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>( V = (3)^3 )</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>( V = (4)^3 )</td>
<td>64</td>
</tr>
</tbody>
</table>

The table does not show a linear relationship since \( V = s^3 \) is not linear.

60. **REASONING** Compare and contrast the graphs of \( y = 2x + 1 \) with the domain \( \{1, 2, 3, 4\} \) and \( y = 2x + 1 \) with the domain of all real numbers.

**SOLUTION:**
The first graph, \( y = 2x + 1 \) with the domain \( \{1, 2, 3, 4\} \), is a set of points that are not connected.

The second graph, \( y = 2x + 1 \) with the domain of all real numbers, is of a line.

The points of the first graph are points on the line in the second graph.
3-1 Graphing Linear Equations

OPEN ENDED Give an example of a linear equation of the form $Ax + By = C$ for each condition. Then describe the graph of the equation.

61. $A = 0$

**SOLUTION:**
The equation $y = 8$ is in standard form, with $A = 0$. No matter what value of $x$ is chosen, $y$ will always be 8. So, the graph will be a horizontal line.

62. $B = 0$

**SOLUTION:**
The equation $x = 5$ is in standard form, with $B = 0$. No matter what value of $y$ is chosen, $x$ will always be 5. So, the graph will be a vertical line.

63. $C = 0$

**SOLUTION:**
The equation $x – y = 0$ is in standard form, with $A = 1$, $B = -1$, and $C = 0$. The $x$- and $y$-intercepts will both be 0, so the graph will pass through the point (0, 0).

64. WRITING IN MATH Explain how to find the $x$-intercept and $y$-intercept of a graph and summarize how to graph a linear equation.

**SOLUTION:**
To find an $x$-intercept, let $y = 0$ and solve the equation for $x$.

For example, consider the example $y = 2x + 6$.

\[
y = 2x + 6 \quad \text{Original equation}
\]
\[
0 = 2x + 6 \quad \text{Replace $y$ with 0.}
\]
\[
0 - 6 = 2x + 6 - 6 \quad \text{Subtract 6 from each side}
\]
\[
-6 = 2x \quad \text{Simplify.}
\]
\[
-\frac{6}{2} = \frac{2x}{2} \quad \text{Divide each side by 2.}
\]
\[
-3 = x \quad \text{Simplify.}
\]

Verify the $x$-intercept on the graph.

To find a $y$-intercept, let $x = 0$ and solve the equation for $y$.

For example, consider the example $y = 2x + 6$

\[
y = 2x + 6 \quad \text{Original equation}
\]
\[
y = 2(0) + 6 \quad \text{Replace $y$ with 0.}
\]
\[
y = 6 \quad \text{Simplify.}
\]

Verify the $y$-intercept on the graph.

To graph a linear equation, plot the $x$-intercept and $y$-intercept and connect the points to form a line.

Another way to graph an equation is to choose any value in the domain and create ordered pairs. Plot the ordered pairs and connect the points to form a line.
65. Sancho can ride 8 miles on his bicycle in 30 minutes. At this rate, about how long would it take him to ride 30 miles?

A 8 hours
B 6 hours 32 minutes
C 2 hours
D 1 hour 53 minutes

**SOLUTION:**
Understand: Let \( x \) represent the amount of time it would take Sancho to ride 30 miles.
Plan: Write a proportion for the problem.

\[
\frac{8 \text{ miles}}{30 \text{ miles}} = \frac{30 \text{ minutes}}{x} 
\]

Solve:

\[
\frac{8}{30} = \frac{30}{x} \quad \text{Original equation}
\]
\[
8x = 30(30) \quad \text{Find the cross products}
\]
\[
8x = 900 \quad \text{Simplify.}
\]
\[
\frac{8x}{8} = \frac{900}{8} \quad \text{Divide each side by 8.}
\]
\[
x = 112.5 \quad \text{Simplify.}
\]

Divide 112.5 by 60 to convert to hours.

So, at the rate of 8 miles in 30 minutes, Sancho can ride 30 miles in about 1 hour and 53 minutes. The correct choice is D.

66. **GEOMETRY** Which is a true statement about the relation graphed?

![Graph of a cube with surface area plotted against side length.]

F The relation is not a function.
G Surface area is the independent quantity.
H The surface area of a cube is a function of the side length.
J As the side length of a cube increases, the surface area decreases.

**SOLUTION:**

F The relation is not a function.
False: The graph is a function because it passes the vertical line test.

G Surface area is the independent quantity.
False: The surface area is the dependent quantity since it is dependent on the side length.

J As the side length of a cube increases, the surface area decreases.
False: As the side length of a cube increases, then the surface area increases.

H The surface area of a cube is a function of the side length.
Looking at the graph, as the side length increases, the surface area of a cube also increases. The side length is the \( x \)-value, or the domain. The surface area is the \( y \)-value, or the range. So, the surface area of a cube is a function of the side length. The correct choice is H.
3-1 Graphing Linear Equations

67. SHORT RESPONSE Selena deposited $2000 into a savings account that pays 1.5% interest compounded annually. If she does not deposit any more money into her account, how much will she earn in interest at the end of one year?

SOLUTION:
Let \( n \) represent the amount of interest earned at the end of one year. To find the interest earned at the end of one year, multiply the savings by the interest rate. Use the equation \( n = 0.015 \cdot 2000 \). So, the interest earned after one year is $30.

68. A candle burns as shown in the graph. If the height of the candle is 8 centimeters, approximately how long has the candle been burning?

\[ \text{Candle Height} \]

\begin{align*}
\text{Time (h)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\text{Height (cm)} & \quad 24 & 22 & 20 & 18 & 16 & 14 & 12 & 10 & 8 & 6
\end{align*}

A 0 hours  
B 24 minutes  
C 64 minutes  
D 5 \frac{1}{2} hours  

SOLUTION:
According to the graph, when the candle is 8cm high, it has been burning for about 5 \frac{1}{2} hours. The correct choice is D.

69. FUNDRAISING The Madison High School Marching Band sold solid–color gift wrap for $4 per roll and print gift wrap for $6 per roll. The total number of rolls sold was 480, and the total amount of money collected was $2,340. How many rolls of each kind of gift wrap were sold?

SOLUTION:
Let \( x \) represent the number of $4 rolls and let \( y \) represent the number of $6 rolls. 4\(x\) represents the income of the $4 rolls and 6\(y\) represents the income of the $6 rolls. The sum of the incomes of the two different types of rolls should equal the total money collected, $2340. The total amount collected can be represented by the equation \( 2340 = 4x + 6y \). To solve for \( x \), rewrite \( y \) in terms of \( x \). Since 480 rolls have been sold, we can represent the $6 rolls as \( y = 480 - x \).

\[
\begin{aligned}
2340 &= 4x + 6y \\
2340 &= 4x + 6(480 - x) \\
2340 &= 4x + 2880 - 6x \\
2340 &= 2880 - 2x \\
-540 &= -2x \\
270 &= x \\
y &= 480 - 270 \\
y &= 210
\end{aligned}
\]

So, 270 rolls of solid wrap were sold and 210 rolls of print wrap were sold.

Solve each equation or formula for the variable specified.

70. \( S = \frac{n}{2} (A + t) \), for \( A \)

SOLUTION:
\[
\begin{aligned}
S &= \frac{n}{2} (A + t) \quad \text{Original equation} \\
S &= \frac{n(A+t)}{2} \quad \text{Rewrite fraction} \\
2S &= n(A + t) \quad \text{Multiply each side by 2.} \\
\frac{2S}{n} &= A + t \quad \text{Simplify} \\
\frac{2S}{n} &= A + t \quad \text{Divide each side by } n. \\
\frac{2S}{n} - t &= A \quad \text{Subtract } t \text{ from both sides} \\
\frac{2S}{n} - t &= A \quad \text{Simplify}
\end{aligned}
\]
71. \(2g - m = 5 - gh\), for \(g\)

**SOLUTION:**

\[
\begin{align*}
2g - m &= 5 - gh \\
2g + gh - m &= 5 - gh + gh \\
2g + gh - m &= 5 \\
2g + gh - m &= 5 + m \\
g(2 + h) &= 5 + m \\
g(2 + h) &= \frac{5 + m}{2 + h} \\
g &= \frac{5 + m}{2 + h} \\
\end{align*}
\]

72. \(\frac{y + a}{3} = c\) for \(y\)

**SOLUTION:**

\[
\begin{align*}
\frac{y + a}{3} &= c \\
3 \left( \frac{y + a}{3} \right) &= 3 \cdot c \\
y + a &= 3c \\
y &= 3c - a \\
\end{align*}
\]

73. \(4z + b = 2z + c\), for \(z\)

**SOLUTION:**

\[
\begin{align*}
4z + b &= 2z + c \\
4z - 2z + b &= 2z - 2z + c \\
2z + b &= c \\
2z &= c - b \\
\frac{2z}{2} &= \frac{c - b}{2} \\
z &= \frac{c - b}{2} \\
\end{align*}
\]

Evaluate each expression if \(x = 2, y = 5,\) and \(z = 7\).

74. \(3x^2 - 4y\)

**SOLUTION:**

Replace \(x\) with 2, \(y\) with 5, and \(z\) with 7.

\[
3x^2 - 4y = 3(2)^2 - 4(5) \\
= 3(4) - 4(5) \\
= 12 - 20 \\
= -8 \\
\]

75. \(\frac{x - y^2}{2z}\)

**SOLUTION:**

Replace \(x\) with 2, \(y\) with 5, and \(z\) with 7.

\[
\frac{x - y^2}{2z} = \frac{2 - 5^2}{2 \cdot 7} \\
= \frac{2 - 25}{2 \cdot 7} \\
= -\frac{23}{14} \\
\]

76. \((\frac{y}{z})^2 + \frac{xy}{2}\)

**SOLUTION:**

Replace \(x\) with 2, \(y\) with 5, and \(z\) with 7.

\[
\left( \frac{y}{z} \right)^2 + \frac{xy}{2} = \left( \frac{5}{7} \right)^2 + \frac{2 \cdot 5}{2} \\
= \frac{25}{49} + \frac{25}{2} \\
= \frac{25}{49} + \frac{245}{49} \\
= \frac{270}{49} \\
\]

77. \(z^2 - y^3 + 5x^2\)

**SOLUTION:**

Replace \(x\) with 2, \(y\) with 5, and \(z\) with 7.

\[
z^2 - y^3 + 5x^2 = 7^2 - 5^3 + 5 \cdot 2^2 \\
= 49 - 125 + 20 \\
= -56 \\
\]
3-2 Solving Linear Equations by Graphing

Solve each equation by graphing.

1. $-2x + 6 = 0$

**SOLUTION:**
The related function is $f(x) = -2x + 6$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -2x + 6$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = -2(-4) + 6$</td>
<td>14</td>
<td>(-4, 14)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = -2(-2) + 6$</td>
<td>10</td>
<td>(-2, 10)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = -2(0) + 6$</td>
<td>6</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = -2(2) + 6$</td>
<td>2</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>3</td>
<td>$f(3) = -2(3) + 6$</td>
<td>0</td>
<td>(3, 0)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = -2(4) + 6$</td>
<td>-2</td>
<td>(4, -2)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at 3. So the solution is 3.

2. $-x - 3 = 0$

**SOLUTION:**
The related function is $f(x) = -x - 3$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -x - 3$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = -(-4) - 3$</td>
<td>1</td>
<td>(-4, 1)</td>
</tr>
<tr>
<td>-3</td>
<td>$f(-3) = -(-3) - 3$</td>
<td>0</td>
<td>(-3, 0)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = -(-2) - 3$</td>
<td>-1</td>
<td>(-2, -1)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = -(0) - 3$</td>
<td>-3</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = -(2) - 3$</td>
<td>-5</td>
<td>(2, -5)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = -(4) - 3$</td>
<td>-7</td>
<td>(4, -7)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at -3. So the solution is -3.
3-2 Solving Linear Equations by Graphing

3. $4x - 2 = 0$

**SOLUTION:**
Solve Graphically

The related function is $f(x) = 4x - 2$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 4x - 2$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = 4(-4) - 2$</td>
<td>-18</td>
<td>(-4, -18)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = 4(-2) - 2$</td>
<td>-10</td>
<td>(-2, -10)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = 4(0) - 2$</td>
<td>0</td>
<td>(0, -2)</td>
</tr>
<tr>
<td>1/2</td>
<td>$f(1/2) = 4(1/2) - 2$</td>
<td>0</td>
<td>(0.5, 0)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 4(2) - 2$</td>
<td>6</td>
<td>(2, 6)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 4(4) - 2$</td>
<td>14</td>
<td>(4, 14)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $\frac{1}{2}$. So the solution is $\frac{1}{2}$.

Solve algebraically

\[
\begin{align*}
4x - 2 & = 0 \\
4x - 2 + 2 & = 0 + 2 \\
4x & = 2 \\
\frac{4x}{4} & = \frac{2}{4} \\
x & = \frac{1}{2}
\end{align*}
\]

4. $9x + 3 = 0$

**SOLUTION:**
Solve Graphically

The related function is $f(x) = 9x + 3$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -2x + 6$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = 9(-4) + 3$</td>
<td>-33</td>
<td>(-4, -33)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = 9(-2) + 3$</td>
<td>-21</td>
<td>(-2, -15)</td>
</tr>
<tr>
<td>$-\frac{1}{3}$</td>
<td>$f(-\frac{1}{3}) = 9(-\frac{1}{3}) + 3$</td>
<td>0</td>
<td>$(-\frac{1}{3}, 0)$</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = 9(0) + 3$</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 9(2) + 3$</td>
<td>21</td>
<td>(2, 21)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 9(4) + 3$</td>
<td>39</td>
<td>(4, 39)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-\frac{1}{3}$. So the solution is $-\frac{1}{3}$.

Solve algebraically

\[
\begin{align*}
9x + 3 & = 0 \\
9x + 3 - 3 & = 0 - 3 \\
9x & = -3 \\
\frac{9x}{9} & = \frac{-3}{9} \\
x & = \frac{-3}{9} = -\frac{1}{3}
\end{align*}
\]
3-2 Solving Linear Equations by Graphing

5. \(2x - 5 = 2x + 8\)

**SOLUTION:**
Solve Graphically

\[
\begin{align*}
2x-5 &= 2x+8 & \text{Original equation} \\
2x-2x-5 &= 2x-2x+8 \text{ Subtract } 2x \text{ from each side} \\
-5 &= 8 \text{ Simplify.} \\
-5 - 8 &= 8 - 8 \text{ Subtract 8 from each side} \\
-13 &= 0 \text{ Simplify.}
\end{align*}
\]

The related function is \(f(x) = -13\).

The graph does not intersect the \(x\)-axis. Therefore, this equation has no solution.

6. \(4x + 11 = 4x - 24\)

**SOLUTION:**
Solve Graphically

\[
\begin{align*}
4x+11 &= 4x - 24 & \text{Original equation} \\
4x-4x+11 &= 4x-4x - 24 \text{ Subtract } 4x \text{ from each side} \\
11 &= -24 \text{ Simplify.} \\
11+24 &= -24+24 \text{ Add 24 to each side} \\
34 &= 0 \text{ Simplify.}
\end{align*}
\]

Graph the related function, which is \(f(x) = 34\).

The graph does not intersect the \(x\)-axis. Therefore, this equation has no solution.
3-2 Solving Linear Equations by Graphing

7. $3x - 5 = 3x - 10$

**SOLUTION:**
Solve Graphically

- $3x - 5 = 3x - 10$ Original equation
- $3x - 3x - 5 = 3x - 3x - 10$ Subtract $3x$ from each side
- $-5 = -10$ Simplify.
- $-5 + 10 = -10 + 10$ Add 10 to each side.
- $5 = 0$ Simplify.

Graph the related function, which is $f(x) = 5$.

The graph does not intersect the $x$-axis. Therefore, this equation has no solution.

8. $-6x + 3 = -6x + 5$

**SOLUTION:**
Solve Graphically

- $-6x + 3 = -6x + 5$ Original equation
- $-6x + 6x + 3 = -6x + 6x + 5$ Add $6x$ to each side.
- $3 = 5$ Simplify
- $3x - 5 = 5$ Subtract 5 from each side
- $2 = 0$ Simplify

Graph the related function, which is $f(x) = -2$.

The graph does not intersect the $x$-axis. Therefore, this equation has no solution.

9. **NEWSPAPERS** The function $w = 30 - \frac{3}{4}n$ represents the weight $w$ in pounds of the papers in Tyrone’s newspaper delivery bag after he delivers $n$ newspapers. Find the zero and explain what it means in the context of this situation.

**SOLUTION:**
Solve Graphically

The related function is $f(x) = 30 - \frac{3}{4}n$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 30 - \frac{3}{4}n$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$f(0) = 30 - \frac{3}{4}(0)$</td>
<td>30</td>
<td>(0, 30)</td>
</tr>
<tr>
<td>10</td>
<td>$f(10) = 30 - \frac{3}{4}(10)$</td>
<td>22.5</td>
<td>(10, 22.5)</td>
</tr>
<tr>
<td>20</td>
<td>$f(20) = 30 - \frac{3}{4}(20)$</td>
<td>15</td>
<td>(20, 15)</td>
</tr>
<tr>
<td>30</td>
<td>$f(30) = 30 - \frac{3}{4}(30)$</td>
<td>7.5</td>
<td>(30, 7.5)</td>
</tr>
</tbody>
</table>
Solve each equation by graphing.

1. \(-2x + 6 = 0\)

**SOLUTION:**

The related function is \(f(x) = -2x + 6\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = -2x + 6)</th>
<th>(x, f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(-2(-4) + 6 = 2)</td>
<td>(-4, 2)</td>
</tr>
<tr>
<td>-2</td>
<td>(-2(-2) + 6 = 2)</td>
<td>(-2, 2)</td>
</tr>
<tr>
<td>0</td>
<td>(-2(0) + 6 = 6)</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>2</td>
<td>(-2(2) + 6 = 2)</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>4</td>
<td>(-2(4) + 6 = -2)</td>
<td>(4, -2)</td>
</tr>
<tr>
<td>5</td>
<td>(-2(5) + 6 = -4)</td>
<td>(5, -4)</td>
</tr>
</tbody>
</table>

The graph intersects the x-axis at 40. So the solution is 40. Tyrone must deliver 40 newspapers for his bag to weigh 0 pounds.

Solve algebraically

To find the zero of the function, substitute zero in for \(w\):

\[
w = 30 - \frac{3}{4}n
\]

\[
0 = 30 - \frac{3}{4}n
\]

\[
0 - 30 = 30 - 30 - \frac{3}{4}n
\]

\[
-30 = -\frac{3}{4}n
\]

\[
(-30) \left( -\frac{4}{3} \right) = \left( -\frac{4}{3} \right) \left( -\frac{3}{4} n \right)
\]

\[
\frac{-120}{-3} = n
\]

\[
40 = n
\]

This means that Tyrone must deliver 40 newspapers for his bag to weigh 0 pounds.

10. \(0 = x - 5\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = x - 5\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = x - 5)</th>
<th>(x, f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>((-4) - 5 = -9)</td>
<td>(-4, -9)</td>
</tr>
<tr>
<td>-2</td>
<td>((-2) - 5 = -7)</td>
<td>(-2, -7)</td>
</tr>
<tr>
<td>0</td>
<td>((0) - 5 = -5)</td>
<td>(0, -5)</td>
</tr>
<tr>
<td>2</td>
<td>((2) - 5 = -3)</td>
<td>(2, -3)</td>
</tr>
<tr>
<td>4</td>
<td>((4) - 5 = -1)</td>
<td>(4, -1)</td>
</tr>
<tr>
<td>5</td>
<td>((5) - 5 = 0)</td>
<td>(5, 0)</td>
</tr>
</tbody>
</table>

The graph intersects the x-axis at 5. So the solution is 5.

Solve algebraically

\[
0 = x - 5
\]

\[
0 + 5 = x - 5 + 5
\]

\[
5 = x
\]
Solve each equation by graphing.

11. \( 0 = x + 3 \)

**SOLUTION:**
Solve Graphically

The related function is \( f(x) = x + 3 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = x + 3 )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( f(-4) = (-4) + 3 )</td>
<td>-1</td>
<td>(-4, -1)</td>
</tr>
<tr>
<td>-3</td>
<td>( f(-3) = (3) + 3 )</td>
<td>0</td>
<td>(-3, 0)</td>
</tr>
<tr>
<td>-2</td>
<td>( f(-2) = (-2) + 3 )</td>
<td>1</td>
<td>(-2, 1)</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = (0) + 3 )</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = (2) + 3 )</td>
<td>5</td>
<td>(2, 5)</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = (4) + 3 )</td>
<td>7</td>
<td>(4, 7)</td>
</tr>
</tbody>
</table>

The graph intersects the \( x \)-axis at -3. So the solution is -3.

**Solve algebraically**

\[
0 = x + 3 \\
0 - 3 = x + 3 - 3 \\
-3 = x
\]

12. \( 5 - 8x = 16 - 8x \)

**SOLUTION:**
Solve Graphically

\[
5 - 8x = 16 - 8x \quad \text{Original equation} \\
5 - 8x + 8x = 16 - 8x + 8x \quad \text{Add } 8x \text{ to each side} \\
5 = 16 \quad \text{Simplify} \\
5 - 16 = 16 - 16 \quad \text{Subtract 16 from each side} \\
-11 = 0 \quad \text{Simplify}
\]

Graph the related function, which is \( f(x) = -11 \).

The graph does not intersect the \( x \)-axis. Therefore, this equation has no solution.
3-2 Solving Linear Equations by Graphing

13. \(3x - 10 = 21 + 3x\)

**SOLUTION:**

\[
\begin{align*}
3x - 10 &= 21 + 3x \\
3x - 3x - 10 &= 21 + 3x - 3x \\
-10 &= 21 \\
-10 - 21 &= 21 - 21 \\
-31 &= 31 \\
\end{align*}
\]

Simplify.

Graph the related function, which is \(f(x) = -31\).

![Graph of f(x) = -31]

The graph does not intersect the \(x\)-axis. Therefore, this equation has no solution.

14. \(4x - 36 = 0\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = 4x - 36\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 4x - 36)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>(f(-3) = 4(-3) - 36)</td>
<td>-48</td>
<td>(-3, -48)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 4(0) - 36)</td>
<td>-36</td>
<td>(0, -36)</td>
</tr>
<tr>
<td>3</td>
<td>(f(3) = 4(3) - 36)</td>
<td>-24</td>
<td>(3, -24)</td>
</tr>
<tr>
<td>6</td>
<td>(f(6) = 4(6) - 36)</td>
<td>-12</td>
<td>(6, -12)</td>
</tr>
<tr>
<td>9</td>
<td>(f(9) = 4(9) - 36)</td>
<td>0</td>
<td>(9, 0)</td>
</tr>
<tr>
<td>12</td>
<td>(f(12) = 4(12) - 36)</td>
<td>4</td>
<td>(12, 4)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at 9. So the solution is 9.

**Solve Algebraically**

\[
\begin{align*}
4x - 36 &= 0 \\
4x - 36 + 36 &= 0 + 36 \\
4x &= 36 \\
\frac{4x}{4} &= \frac{36}{4} \\
x &= 9
\end{align*}
\]
15. \(0 = 7x + 10\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = 7x + 10\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 7x + 10)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = 7(-4) + 10)</td>
<td>-38</td>
<td>(-4, -28)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 7(-2) + 10)</td>
<td>-24</td>
<td>(-2, -24)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 7(0) + 10)</td>
<td>10</td>
<td>(0, 10)</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = 7(2) + 10)</td>
<td>24</td>
<td>(2, 24)</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = 7(4) + 10)</td>
<td>38</td>
<td>(4, 38)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(-\frac{10}{7}\). So the solution is \(-\frac{10}{7}\).

Solve algebraically

\[
0 = 7x + 10 \\
0 - 10 = 7x + 10 - 10 \\
-10 = 7x \\
-\frac{10}{7} = \frac{7x}{7} \\
x = -\frac{10}{7} \text{ or } -1\frac{3}{7}
\]

16. \(2x + 22 = 0\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = 2x + 22\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 2x + 22)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12</td>
<td>(f(-12) = 2(-12) + 22)</td>
<td>-24</td>
<td>(-12, -22)</td>
</tr>
<tr>
<td>-11</td>
<td>(f(-11) = 2(-11) + 22)</td>
<td>0</td>
<td>(-11, 0)</td>
</tr>
<tr>
<td>-10</td>
<td>(f(-10) = 2(-10) + 22)</td>
<td>2</td>
<td>(-10, 2)</td>
</tr>
<tr>
<td>-8</td>
<td>(f(-8) = 2(-8) + 22)</td>
<td>6</td>
<td>(-8, 6)</td>
</tr>
<tr>
<td>-6</td>
<td>(f(-6) = 2(-6) + 22)</td>
<td>10</td>
<td>(-6, 10)</td>
</tr>
<tr>
<td>-4</td>
<td>(f(-4) = 2(-4) + 22)</td>
<td>14</td>
<td>(-4, 14)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 2(0) + 22)</td>
<td>22</td>
<td>(0, 22)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(-11\). So the solution is \(-11\).

Solve Algebraically

\[
2x + 22 = 0 \\
2x + 22 - 22 = 0 - 22 \\
2x = -22 \\
\frac{2x}{2} = \frac{-22}{2} \\
x = -11
\]
3-2 Solving Linear Equations by Graphing

17. $5x - 5 = 5x + 2$

**SOLUTION:**

\[
\begin{align*}
5x - 5 &= 5x + 2 \\
5x - 5x - 5 &= 5x - 5x + 2 \\
-5 &= 2 \\
-5 - 2 &= 2 - 2 \\
-7 &= 0
\end{align*}
\]

Graph the related function, which is $f(x) = -7$.

The graph does not intersect the $x$-axis. Therefore, this equation has no solution.

18. $-7x + 35 = 20 - 7x$

**SOLUTION:**

\[
\begin{align*}
-7x + 35 &= 20 - 7x \\
-7x + 7x + 35 &= 20 - 7x + 7x \\
35 &= 20 \\
35 - 20 &= 20 - 20 \\
15 &= 0
\end{align*}
\]

Graph the related function, which is $f(x) = 15$.

The graph does not intersect the $x$-axis. Therefore, this equation has no solution.
3-2 Solving Linear Equations by Graphing

19. \(-4x - 28 = 3 - 4x\)

**SOLUTION:**

\[
\begin{align*}
-4x - 28 &= 3 - 4x & \text{Original equation} \\
-4x - 28 + 4x &= 3 - 4x + 4x & \text{Add } 4x \text{ to each side} \\
-28 &= 3 & \text{Simplify} \\
-28 - 3 &= 3 - 3 & \text{Subtract } 3 \text{ from each side} \\
-31 &= 0 & \text{Simplify}.
\end{align*}
\]

Graph the related function, which is \(f(x) = -31\).

The graph does not intersect the \(x\)-axis. Therefore, this equation has no solution.

20. \(0 = 6x - 8\)

**SOLUTION:**

**Solve Graphically**

The related function is \(f(x) = 6x - 8\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 6x - 8)</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = 6(-4) - 8)</td>
<td>((-4, 14))</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 6(-2) - 8)</td>
<td>((-2, 10))</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 6(0) - 8)</td>
<td>((0, 6))</td>
</tr>
<tr>
<td>(1\frac{1}{3})</td>
<td>(f(1\frac{1}{3}) = 6(1\frac{1}{3}) - 8)</td>
<td>((1\frac{1}{3}, 0))</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = 6(2) - 8)</td>
<td>((2, 2))</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = 6(4) - 8)</td>
<td>((4, -2))</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(1\frac{1}{3}\). So the solution is \(1\frac{1}{3}\).

**Solve Algebraically**

\[
\begin{align*}
0 &= 6x - 8 \\
0 + 8 &= 6x - 8 + 8 \\
8 &= 6x \\
\frac{8}{6} &= \frac{6x}{6} \\
\frac{4}{3} &= x
\end{align*}
\]

\(x = \frac{4}{3} \text{ or } 1\frac{1}{3}\)
21. \(12x + 132 = 12x - 100\)

**SOLUTION:**

\[
\begin{align*}
12x + 132 &= 12x - 100 & (\text{Original equation}) \\
12x - 12x + 132 &= 12x - 12x - 100 & (\text{Subtract}) \\
132 &= -100 & (\text{Simplify}) \\
132 + 100 &= -100 + 100 & (\text{Add}) \\
232 &= 0 & (\text{Simplify})
\end{align*}
\]

Graph the related function, which is \(f(x) = 232\).

The graph does not intersect the \(x\)-axis. Therefore, this equation has no solution.

22. **TEXT MESSAGING** Sean is sending text messages to his friends. The function \(y = 160 - x\) represents the number of characters \(y\) the message can hold after he has typed \(x\) characters. Find the zero and explain what it means in the context of this situation.

**SOLUTION:**

To find the zero of the function, substitute zero in for \(y\). Then \(y = 160 - x\)

Solve Graphically

The related function is \(f(x) = 160 - x\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = -2x + 6)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(f(0) = 160 - (0))</td>
<td>160</td>
<td>(0, 160)</td>
</tr>
<tr>
<td>20</td>
<td>(f(20) = 160 - (20))</td>
<td>140</td>
<td>(20, 140)</td>
</tr>
<tr>
<td>90</td>
<td>(f(90) = 160 - (90))</td>
<td>70</td>
<td>(90, 70)</td>
</tr>
<tr>
<td>100</td>
<td>(f(100) = 160 - (100))</td>
<td>60</td>
<td>(100, 60)</td>
</tr>
<tr>
<td>130</td>
<td>(f(130) = 160 - (130))</td>
<td>30</td>
<td>(130, 30)</td>
</tr>
<tr>
<td>160</td>
<td>(f(160) = 160 - (160))</td>
<td>0</td>
<td>(160, 0)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at 160. So the solution is 160.

Solve algebraically

\[
\begin{align*}
y &= 160 - x \\
0 &= 160 - x \\
0 - 160 &= 160 - 160 - x \\
-160 &= -x \\
160 &= x
\end{align*}
\]

This means that the text message is full after Sean has typed 160 characters.

23. **GIFT CARDS** For her birthday Kwan receives a \$50 gift card to download songs. The function \(m = -0.50d + 50\) represents the amount of money \(m\) that remains on the card after a number of songs \(d\) are downloaded. Find the zero and explain what it means in the context of this situation.

**SOLUTION:**

To find the zero of the function, substitute zero in for \(m\). Then \(0 = -0.50d + 50\).

Solve Graphically

The related function is \(f(d) = -0.50d + 50\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(d)</th>
<th>(f(d) = -0.50d + 50)</th>
<th>(f(d))</th>
<th>((d, f(d)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(f(0) = -0.50(0) + 50)</td>
<td>50</td>
<td>(0, 50)</td>
</tr>
<tr>
<td>20</td>
<td>(f(20) = -0.50(20) + 50)</td>
<td>40</td>
<td>(20, 40)</td>
</tr>
<tr>
<td>40</td>
<td>(f(40) = -0.50(40) + 50)</td>
<td>30</td>
<td>(40, 30)</td>
</tr>
<tr>
<td>60</td>
<td>(f(60) = -0.50(60) + 50)</td>
<td>20</td>
<td>(60, 20)</td>
</tr>
<tr>
<td>100</td>
<td>(f(100) = -0.50(100) + 50)</td>
<td>0</td>
<td>(100, 0)</td>
</tr>
</tbody>
</table>
3-2 Solving Linear Equations by Graphing

Solve each equation by graphing.

24. $-7 = 4x + 1$

**SOLUTION:**
Rewrite $-7 = 4x + 1$ with zero on the left side
$0 = 4x + 8$

**Solve Graphically**

The related function is $f(x) = 4x + 8$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 4x + 8$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = 4(-4) + 8$</td>
<td>-8</td>
<td>(-4, -8)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = 4(-2) + 8$</td>
<td>0</td>
<td>(-2, 0)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = 4(0) + 8$</td>
<td>8</td>
<td>(0, 8)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 4(2) + 8$</td>
<td>16</td>
<td>(2, 16)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 4(4) + 8$</td>
<td>24</td>
<td>(4, 24)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at -2. So the solution is -2.

**Solve algebraically**

$-7 = 4x + 1$

$-7 - 1 = 4x + 1 - 1$

$-8 = 4x$

$-8 = 4x$

$-2 = x$

----

The graph intersects the $x$-axis at 100. So the solution is 100.

**Solve algebraically**

$m = -0.50d + 50$

$0 = -0.50d + 50$

$0 - 50 = -0.50d + 50 - 50$

$-50 = -0.50d$

$-50 = -0.50d$

$-0.50 = -0.50$

$100 = d$

This means she can download a total of 100 songs before the gift card is completely used.
3-2 Solving Linear Equations by Graphing

25. 4 – 2x = 20

**SOLUTION:**
Rewrite 4 – 2x = 20 with 0 on the right side: −10 – 2x = 0.

**Solve Graphically**

The related function is \( f(x) = −2x − 16 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = −2x − 16 )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−10</td>
<td>( f(−10) = −2(−10) − 16 )</td>
<td>4</td>
<td>(−10, 4)</td>
</tr>
<tr>
<td>−8</td>
<td>( f(−8) = −2(−8) − 16 )</td>
<td>0</td>
<td>(−8, 0)</td>
</tr>
<tr>
<td>−4</td>
<td>( f(−4) = −2(−4) − 16 )</td>
<td>−8</td>
<td>(−4, −8)</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = −2(0) − 16 )</td>
<td>−16</td>
<td>(0, −16)</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = −2(2) − 16 )</td>
<td>−20</td>
<td>(2, −20)</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = −2(4) − 16 )</td>
<td>−24</td>
<td>(4, −24)</td>
</tr>
</tbody>
</table>

The graph intersects the x-axis at −8. So the solution is −8.

**Solve Algebraically**

\[
\begin{align*}
4 − 2x &= 20 \\
4 − 4 − 2x &= 20 − 4 \\
−2x &= 16 \\
−2x &= 16 \\
\frac{−2x}{−2} &= \frac{16}{−2} \\
x &= −8
\end{align*}
\]

26. 2 − 5x = −23

**SOLUTION:**
Rewrite 2 − 5x = −23 with zero on the right side: 25 − 5x = 0.

**Solve Graphically**

The related function is \( f(x) = −5x + 25 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = −5x + 25 )</th>
<th>( f(x) )</th>
<th>( (x, f(x)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( f(0) = −5(0) + 25 )</td>
<td>25</td>
<td>(0, 25)</td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = −5(1) + 25 )</td>
<td>20</td>
<td>(1, 20)</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = −5(2) + 25 )</td>
<td>15</td>
<td>(2, 15)</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = −5(3) + 25 )</td>
<td>10</td>
<td>(3, 10)</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = −5(4) + 25 )</td>
<td>5</td>
<td>(4, 5)</td>
</tr>
<tr>
<td>5</td>
<td>( f(5) = −5(5) + 25 )</td>
<td>0</td>
<td>(5, 0)</td>
</tr>
<tr>
<td>6</td>
<td>( f(6) = −5(6) + 25 )</td>
<td>−5</td>
<td>(6, −5)</td>
</tr>
</tbody>
</table>

The graph intersects the x-axis at 5. So the solution is 5.

**Solve algebraically**

\[
\begin{align*}
2 − 5x &= −23 \\
2 − 2 − 5x &= −23 − 2 \\
−5x &= −25 \\
−5x &= −25 \\
\frac{−5x}{−5} &= \frac{−25}{−5} \\
x &= 5
\end{align*}
\]
3-2 Solving Linear Equations by Graphing

27. $10 - 3x = 0$

**SOLUTION:**
Solve Graphically

The related function is $f(x) = -3x + 10$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -3x + 10$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$f(-4) = -3(-4) + 10$</td>
<td>10</td>
<td>(0, 10)</td>
</tr>
<tr>
<td>1</td>
<td>$f(-2) = -3(-2) + 10$</td>
<td>7</td>
<td>(1, 7)</td>
</tr>
<tr>
<td>2</td>
<td>$f(0) = -3(0) + 10$</td>
<td>6</td>
<td>(2, 6)</td>
</tr>
<tr>
<td>3</td>
<td>$f(2) = -3(2) + 10$</td>
<td>1</td>
<td>(3, 1)</td>
</tr>
<tr>
<td>$3\frac{1}{3}$</td>
<td>$f\left(3\frac{1}{3}\right) = -3\left(3\frac{1}{3}\right) + 10$</td>
<td>0</td>
<td>$\left(3\frac{1}{3}, 0\right)$</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = -3(4) + 10$</td>
<td>-2</td>
<td>(-4, -2)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $3\frac{1}{3}$. So the solution is $3\frac{1}{3}$.

Solve Algebraically

$10 - 3x = 0$
$10 - 10 - 3x = 0 - 10$
$-3x = -10$
$\frac{-3x}{-3} = \frac{-10}{-3}$
$x = \frac{10}{3}$ or $3\frac{1}{3}$

28. $15 + 6x = 0$

**SOLUTION:**
Solve Graphically

The related function is $f(x) = 6x + 15$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 6x + 15$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = 6(-4) + 15$</td>
<td>-9</td>
<td>(-4, -9)</td>
</tr>
<tr>
<td>$-2\frac{1}{2}$</td>
<td>$f\left(-2\frac{1}{2}\right) = 6\left(-2\frac{1}{2}\right) + 15$</td>
<td>0</td>
<td>$\left(-2\frac{1}{2}, 0\right)$</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = 6(-2) + 15$</td>
<td>3</td>
<td>(-2, 3)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = 6(0) + 15$</td>
<td>15</td>
<td>(0, 15)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 6(2) + 15$</td>
<td>27</td>
<td>(2, 27)</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 6(4) + 15$</td>
<td>39</td>
<td>(4, 39)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-2\frac{1}{2}$. So the solution is $-2\frac{1}{2}$.

Solve algebraically

$15 + 6x = 0$
$15 - 15 + 6x = 0 - 15$
$6x = -15$
$\frac{6x}{6} = \frac{-15}{6}$
$x = -\frac{15}{6} = -\frac{5}{2}$ or $-2\frac{1}{2}$
29. $0 = 13x + 34$

SOLUTION:
Solve Graphically

The related function is $f(x) = 13x + 34$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 13x + 34$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$(-4) = 13(-4) + 34$</td>
<td>$-18$</td>
<td>$(-4, -18)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$(-2) = 13(-2) + 34$</td>
<td>$8$</td>
<td>$(-2, 8)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$(0) = 13(0) + 34$</td>
<td>$34$</td>
<td>$(0, 34)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$(2) = 13(2) + 34$</td>
<td>$60$</td>
<td>$(2, 60)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$(4) = 13(4) + 34$</td>
<td>$73$</td>
<td>$(4, 73)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-2\frac{8}{13}$. So the solution is $-2\frac{8}{13}$.

Solve Algebraically

\[
0 = 13x + 34
\]
\[
0 - 13x = 13x - 13x + 34
\]
\[
-13x = 34
\]
\[
\frac{-13x}{-13} = \frac{34}{-13}
\]
\[
x = -\frac{34}{13} \text{ or } -2\frac{8}{13}
\]

30. $0 = 22x - 10$

SOLUTION:
Solve Graphically

The related function is $f(x) = 22x - 10$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 22x - 10$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$(-1) = 22(-1) - 10$</td>
<td>$-32$</td>
<td>$(-1, -32)$</td>
</tr>
<tr>
<td>$-\frac{1}{2}$</td>
<td>$(-\frac{1}{2}) = 22(-\frac{1}{2}) - 10$</td>
<td>$-21$</td>
<td>$(-\frac{1}{2}, -21)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$(0) = 22(0) - 10$</td>
<td>$-10$</td>
<td>$(0, -10)$</td>
</tr>
<tr>
<td>$\frac{5}{11}$</td>
<td>$(\frac{5}{11}) = 22(\frac{5}{11}) - 10$</td>
<td>$0$</td>
<td>$(\frac{5}{11}, 0)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$(1) = 22(1) - 10$</td>
<td>$12$</td>
<td>$(1, 12)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$(2) = 22(2) - 10$</td>
<td>$34$</td>
<td>$(2, 34)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $\frac{5}{11}$. So the solution is $\frac{5}{11}$.

Solve algebraically

\[
0 = 22x - 10
\]
\[
0 - 22x = 22x - 22x - 10
\]
\[
-22x = -10
\]
\[
\frac{-22x}{-22} = \frac{-10}{-22}
\]
\[
x = \frac{10}{22} = \frac{5}{11}
\]
3-2 Solving Linear Equations by Graphing

31. \(25x - 17 = 0\)

**SOLUTION:**
Solve Graphically

The related function is \(f(x) = 25x - 17\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 25x - 17)</th>
<th>(f(x))</th>
<th>(x, f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>(f(-2) = 25(-2) - 17)</td>
<td>(-67)</td>
<td>((-2, -67))</td>
</tr>
<tr>
<td>(-1)</td>
<td>(f(-1) = 25(-1) - 17)</td>
<td>(-42)</td>
<td>((-1, -42))</td>
</tr>
<tr>
<td>(0)</td>
<td>(f(0) = 25(0) - 17)</td>
<td>(-17)</td>
<td>((0, -17))</td>
</tr>
<tr>
<td>(\frac{17}{25})</td>
<td>(f\left(\frac{17}{25}\right) = 25\left(\frac{17}{25}\right) - 17)</td>
<td>(0)</td>
<td>(\left(\frac{17}{25}, 0\right))</td>
</tr>
<tr>
<td>(1)</td>
<td>(f(1) = 25(1) - 17)</td>
<td>(8)</td>
<td>((1, 8))</td>
</tr>
<tr>
<td>(2)</td>
<td>(f(2) = 25(2) - 17)</td>
<td>(33)</td>
<td>((2, 33))</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(\frac{17}{25}\). So the solution is \(\frac{17}{25}\).

**Solve Algebraically**

\[25x - 17 = 0\]
\[25x - 17 + 17 = 0 + 17\]
\[25x = 17\]
\[\frac{25x}{25} = \frac{17}{25}\]
\[x = \frac{17}{25}\]

32. \(0 = \frac{1}{2} + \frac{2}{3}x\)

**SOLUTION:**
Solve Graphically

The related function is \(f(x) = \frac{1}{2} + \frac{2}{3}x\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = \frac{1}{2} + \frac{2}{3}x)</th>
<th>(f(x))</th>
<th>(x, f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-6)</td>
<td>(f(-6) = \frac{1}{2} + \frac{2}{3}(-6))</td>
<td>(-3.5)</td>
<td>((-6, -3.5))</td>
</tr>
<tr>
<td>(-3)</td>
<td>(f(-3) = \frac{1}{2} + \frac{2}{3}(-3))</td>
<td>(-1.5)</td>
<td>((-3, -1.5))</td>
</tr>
<tr>
<td>(-\frac{3}{4})</td>
<td>(f\left(-\frac{3}{4}\right) = \frac{1}{2} + \frac{2}{3}\left(-\frac{3}{4}\right))</td>
<td>(0)</td>
<td>(\left(-\frac{3}{4}, 0\right))</td>
</tr>
<tr>
<td>(0)</td>
<td>(f(0) = \frac{1}{2} + \frac{2}{3}(0))</td>
<td>(0.5)</td>
<td>((0, 0.5))</td>
</tr>
<tr>
<td>(3)</td>
<td>(f(3) = \frac{1}{2} + \frac{2}{3}(3))</td>
<td>(2.5)</td>
<td>((3, 2.5))</td>
</tr>
<tr>
<td>(6)</td>
<td>(f(6) = \frac{1}{2} + \frac{2}{3}(6))</td>
<td>(4.5)</td>
<td>((6, 4.5))</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(-\frac{3}{4}\). So the solution is \(-\frac{3}{4}\).
3-2 Solving Linear Equations by Graphing

Solve each equation by graphing.

1. \(-2x + 6 = 0\)

SOLUTION:
The related function is \(f(x) = -2x + 6\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = \frac{3}{4} - \frac{2}{5}x)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>(\frac{3}{4} - \frac{2}{5}(-2.5))</td>
<td>1.75</td>
<td>(-2.5, 1.75)</td>
</tr>
<tr>
<td>-0.5</td>
<td>(\frac{3}{4} - \frac{2}{5}(-0.5))</td>
<td>0.95</td>
<td>(-0.5, 0.95)</td>
</tr>
<tr>
<td>0</td>
<td>(\frac{3}{4} - \frac{2}{5}(0))</td>
<td>0.75</td>
<td>(0, 0.75)</td>
</tr>
<tr>
<td>0.5</td>
<td>(\frac{3}{4} - \frac{2}{5}(0.5))</td>
<td>0.55</td>
<td>(0.5, 0.55)</td>
</tr>
<tr>
<td>1\frac{7}{8}</td>
<td>(\frac{3}{4} - \frac{2}{5}(1\frac{7}{8}))</td>
<td>0</td>
<td>(1\frac{7}{8}, 0)</td>
</tr>
<tr>
<td>2.5</td>
<td>(\frac{3}{4} - \frac{2}{5}(2.5))</td>
<td>-0.25</td>
<td>(2.5, -0.25)</td>
</tr>
</tbody>
</table>

The related function is \(f(x) = \frac{3}{4} - \frac{2}{5}x\). To graph the function, make a table.

SOLUTION:

33. \(0 = \frac{3}{4} - \frac{2}{5}x\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = \frac{3}{4} - \frac{2}{5}x\). To graph the function, make a table.

Solve Algebraically

The graph intersects the \(x\)-axis at \(1\frac{7}{8}\). So the solution is \(1\frac{7}{8}\).
3-2 Solving Linear Equations by Graphing

34. \(13x + 117 = 0\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = 13x + 117\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 13x + 117)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-12)</td>
<td>(13(-12) + 117)</td>
<td>(-39)</td>
<td>((-12, -39))</td>
</tr>
<tr>
<td>(-9)</td>
<td>(13(-9) + 117)</td>
<td>(0)</td>
<td>((-9, 0))</td>
</tr>
<tr>
<td>(-6)</td>
<td>(13(-6) + 117)</td>
<td>(39)</td>
<td>((-6, 39))</td>
</tr>
<tr>
<td>(-3)</td>
<td>(13(-3) + 117)</td>
<td>(78)</td>
<td>((-3, 78))</td>
</tr>
<tr>
<td>(0)</td>
<td>(13(0) + 117)</td>
<td>(117)</td>
<td>((0, 117))</td>
</tr>
<tr>
<td>(3)</td>
<td>(13(3) + 117)</td>
<td>(156)</td>
<td>((3, 156))</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(-9\). So the solution is \(-9\).

Solve Algebraically

\[
\begin{align*}
13x + 117 &= 0 \\
13x + 117 - 117 &= 0 - 117 \\
13x &= -117 \\
\frac{13x}{13} &= \frac{-117}{13} \\
x &= -9
\end{align*}
\]

35. \(24x - 72 = 0\)

**SOLUTION:**

Solve Graphically

The related function is \(f(x) = 24x - 72\). To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 24x - 72)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4)</td>
<td>(24(-4) - 72)</td>
<td>(-168)</td>
<td>((-4, -168))</td>
</tr>
<tr>
<td>(-2)</td>
<td>(24(-2) - 72)</td>
<td>(-120)</td>
<td>((-2, -120))</td>
</tr>
<tr>
<td>(0)</td>
<td>(24(0) - 72)</td>
<td>(-72)</td>
<td>((0, -72))</td>
</tr>
<tr>
<td>(2)</td>
<td>(24(2) - 72)</td>
<td>(-24)</td>
<td>((2, -24))</td>
</tr>
<tr>
<td>(3)</td>
<td>(24(3) - 72)</td>
<td>(0)</td>
<td>((3, 0))</td>
</tr>
<tr>
<td>(4)</td>
<td>(24(4) - 72)</td>
<td>(24)</td>
<td>((4, 24))</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at \(3\). So the solution is \(3\).

Solve Algebraically

\[
\begin{align*}
24x - 72 &= 0 \\
24x - 72 + 72 &= 0 + 72 \\
24x &= 72 \\
\frac{24x}{24} &= \frac{72}{24} \\
x &= 3
\end{align*}
\]

36. **SEA LEVEL** Parts of New Orleans lie 0.5 meter below sea level. After \(d\) days of rain, the equation \(w = 0.3d - 0.5\) represents the water level \(w\) in meters. Find the zero, and explain what it means in the context of this situation.

**SOLUTION:**

To find the zero of the function, substitute zero in for \(w\): \(0 = 0.3d - 0.5\).
Solve Graphically

The related function is \( f(x) = 0.3d - 0.5 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( d )</th>
<th>( f(d) = 0.3d - 0.5 )</th>
<th>( f(d) )</th>
<th>( (d, f(d)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( f(-3) = 0.3(-4) - 0.5 )</td>
<td>-1.4</td>
<td>((-3, -1.4))</td>
</tr>
<tr>
<td>-1</td>
<td>( f(-1) = 0.3(-2) - 0.5 )</td>
<td>-0.8</td>
<td>((-1, -0.8))</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = 0.3(0) - 0.5 )</td>
<td>-0.5</td>
<td>( (0, -0.5) )</td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = 0.3(2) - 0.5 )</td>
<td>-0.2</td>
<td>( (1, -0.2) )</td>
</tr>
<tr>
<td>( \frac{5}{3} )</td>
<td>( f(\frac{5}{3}) = 0.3\left(\frac{5}{3}\right) - 0.5 )</td>
<td>0</td>
<td>( \left(\frac{5}{3}, 0\right) )</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = 0.3(4) - 0.5 )</td>
<td>0.4</td>
<td>( (3, 0.4) )</td>
</tr>
</tbody>
</table>

The graph intersects the \( x \)-axis at \( \frac{5}{3} \). So the solution is \( \frac{5}{3} \).

Solve Algebraically

\[
\frac{w}{0.3d} - 0.5 = 0
\]
\[
0 = 0.3d - 0.5
\]
\[
0 + 0.5 = 0.3d - 0.5 + 0.5
\]
\[
0.5 = 0.3d
\]
\[
\frac{0.5}{0.3} = \frac{0.3d}{0.3}
\]
\[
1.67 \approx d
\]

This means that the water level in New Orleans has reached sea level after about 1.67 days of rain.

37. **CCSS MODELING** An artist completed an ice sculpture when the temperature was \(-10^\circ\text{C}\). The equation \( t = 1.25h - 10 \) shows the temperature, \( h \) hours after the sculpture’s completion. If the artist completed the sculpture at 8:00 a.m., at what time will it begin to melt?

**SOLUTION:**
To find the time the ice sculpture will melt, substitute 0 in for \( t \) because ice melts at \(0^\circ\text{C}\).
\[
0 = 1.25h - 10
\]

Solve Graphically

The related function is \( f(h) = 1.25h - 10 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( h )</th>
<th>( f(h) = 1.25h - 10 )</th>
<th>( f(h) )</th>
<th>( (h, f(h)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>( f(-8) = 1.25(-8) - 10 )</td>
<td>-20</td>
<td>((-8, -20))</td>
</tr>
<tr>
<td>-4</td>
<td>( f(-4) = 1.25(-4) - 10 )</td>
<td>-15</td>
<td>((-4, -15))</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = 1.25(0) - 10 )</td>
<td>-10</td>
<td>((0, -10))</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = 1.25(4) - 10 )</td>
<td>-5</td>
<td>((4, -5))</td>
</tr>
<tr>
<td>8</td>
<td>( f(8) = 1.25(8) - 10 )</td>
<td>0</td>
<td>((8, 0))</td>
</tr>
<tr>
<td>16</td>
<td>( f(16) = 1.25(16) - 10 )</td>
<td>10</td>
<td>((16, 10))</td>
</tr>
</tbody>
</table>

The graph intersects the \( x \)-axis at 8. So the solution is 8.

Solve Algebraically

\[
t = 1.25h - 10
\]
\[
0 = 1.25h - 10
\]
\[
0 + 10 = 1.25h - 10 + 10
\]
\[
10 = 1.25h
\]
\[
\frac{10}{1.25} = \frac{1.25h}{1.25}
\]
\[
8 = h
\]

So, 8 hours after the sculpture was made it will start to melt. Therefore, it will start to melt at 4:00 P.M. 
Solve each equation by graphing. Verify your answer algebraically.

38. \(7 - 3x = 8 - 4x\)

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.

\[
7 - 3x = 8 - 4x \\
7 - 3x + 3x = 8 - 4x + 3x \\
7 = -x + 8 \\
7 - 7 = -x + 8 - 7 \\
0 = -x + 1
\]

The related function is \(f(x) = -x + 1\).

To graph the function, make a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = -x + 1)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = -(-4) + 1)</td>
<td>5</td>
<td>(-4, 5)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = -(-2) + 1)</td>
<td>3</td>
<td>(-2, 3)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = -(0) + 1)</td>
<td>1</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>1</td>
<td>(f(1) = -(1) + 1)</td>
<td>0</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = -(2) + 1)</td>
<td>-1</td>
<td>(2, -1)</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = -(4) + 1)</td>
<td>-3</td>
<td>(4, -3)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at 1. So, the solution is 1.

Verify by substituting 1 in for \(x\) in the original equation.

\[
7 - 3x = 8 - 4x \\
7 - 3(1) = 8 - 4(1) \\
7 - 3 = -8 - 4 \\
4 = 4\]

39. \(19 + 3x = 13 + x\)

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.

\[
19 + 3x = 13 + x \\
19 + 3x - x = 13 + x - x \\
19 + 2x = 13 \\
19 - 13 + 2x = 13 - 13 \\
2x + 6 = 0
\]

The related function is \(f(x) = 2x + 6\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 2x + 6)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = 2(-4) + 6)</td>
<td>-2</td>
<td>(-4, -2)</td>
</tr>
<tr>
<td>-3</td>
<td>(f(-3) = 2(-3) + 6)</td>
<td>0</td>
<td>(-3, 0)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 2(-2) + 6)</td>
<td>2</td>
<td>(-2, 2)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 2(0) + 6)</td>
<td>6</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = 2(2) + 6)</td>
<td>10</td>
<td>(2, 10)</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = 2(4) + 6)</td>
<td>14</td>
<td>(4, 14)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at -3. So, the solution is -3.

Verify by substituting -3 in for \(x\) in the original equation.

\[
19 + 3x = 13 + x \\
19 + 3(-3) = 13 + (-3) \\
19 - 9 = 13 - 3 \\
10 = 10\]
Solve each equation by graphing.

40. \(16x + 6 = 14x + 10\)

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.

\[
16x + 6 = 14x + 10 \\
16x - 14x + 6 = 14x - 14x + 10 \\
2x + 6 = 10 \\
2x + 6 - 10 = 10 - 10 \\
2x - 4 = 0 \\
The related function is \(f(x) = 2x - 4\).
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 2x - 4)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = 2(-4) - 4)</td>
<td>-12</td>
<td>(-4, -12)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 2(-2) - 4)</td>
<td>-8</td>
<td>(-2, -8)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 2(0) - 4)</td>
<td>-4</td>
<td>(0, -4)</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = 2(2) - 4)</td>
<td>0</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>3</td>
<td>(f(3) = 2(3) - 4)</td>
<td>2</td>
<td>(3, 2)</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = 2(4) - 4)</td>
<td>4</td>
<td>(4, 4)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at 2. So, the solution is 2.

Verify by substituting 2 in for \(x\) in the original equation

\[
16x + 6 = 14x + 10 \\
16(2) + 6 = 14(2) + 10 \\
32 + 6 = 28 + 10 \\
38 = 38
\]

41. \(15x - 30 = 5x - 50\)

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.

\[
15x - 30 = 5x - 50 \\
15x - 5x - 30 = 5x - 5x - 50 \\
10x - 30 = -50 \\
10x - 30 + 50 = -50 + 50 \\
10x + 20 = 0 \\
The related function is \(f(x) = 10x + 20\).
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 10x + 20)</th>
<th>(f(x))</th>
<th>((x, f(x)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>(f(-4) = 10(-4) + 20)</td>
<td>-20</td>
<td>(-4, -20)</td>
</tr>
<tr>
<td>-2</td>
<td>(f(-2) = 10(-2) + 20)</td>
<td>0</td>
<td>(-2, 0)</td>
</tr>
<tr>
<td>0</td>
<td>(f(0) = 10(0) + 20)</td>
<td>20</td>
<td>(0, 20)</td>
</tr>
<tr>
<td>2</td>
<td>(f(2) = 10(2) + 20)</td>
<td>40</td>
<td>(2, 40)</td>
</tr>
<tr>
<td>3</td>
<td>(f(3) = 10(3) + 20)</td>
<td>50</td>
<td>(3, 50)</td>
</tr>
<tr>
<td>4</td>
<td>(f(4) = 10(4) + 20)</td>
<td>60</td>
<td>(4, 60)</td>
</tr>
</tbody>
</table>

The graph intersects the \(x\)-axis at -2. So, the solution is -2.

Verify by substituting -2 in for \(x\) in the original equation.

\[
15x - 30 = 5x - 50 \\
15(-2) - 30 = 5(-2) - 50 \\
-30 - 30 = -10 - 50 \\
-60 = -60
\]

42. \(\frac{1}{2} x - 5 = 3x - 10\)

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.
3-2 Solving Linear Equations by Graphing

\[ \frac{1}{2}x - 5 = 3x - 10 \]
\[ \frac{1}{2}x - \frac{1}{2}x - 5 = 3x - \frac{1}{2}x - 10 \]
\[ -5 = \frac{5}{2}x - 10 \]
\[ -5 + 5 = \frac{5}{2}x - 10 + 5 \]
\[ 0 = \frac{5}{2}x - 5 \]
\[ \frac{5}{2}x - 5 = 0 \]

The related function is \( f(x) = \frac{5}{2}x - 5 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = \frac{5}{2}x - 5 )</th>
<th>( f(\alpha) )</th>
<th>( (\alpha, f(\alpha)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( f(-4) = \frac{5}{2}(-4) - 5 )</td>
<td>-15</td>
<td>(-4, -15)</td>
</tr>
<tr>
<td>-2</td>
<td>( f(-2) = \frac{5}{2}(-2) - 5 )</td>
<td>-10</td>
<td>(-2, -10)</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = \frac{5}{2}(0) - 5 )</td>
<td>-5</td>
<td>(0, -5)</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = \frac{5}{2}(2) - 5 )</td>
<td>0</td>
<td>(2, 0)</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = \frac{5}{2}(3) - 5 )</td>
<td>2.5</td>
<td>(3, 2.5)</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = \frac{5}{2}(4) - 5 )</td>
<td>5</td>
<td>(4, 5)</td>
</tr>
</tbody>
</table>

The graph intersects the \( x \)-axis at 2. So, the solution is 2.

Verify by substituting 2 in for \( x \) in the original equation.

\[ \frac{1}{2} \cdot 2 - 5 = 3 \cdot 2 - 10 \]
\[ \frac{1}{2} \cdot 2 - 5 = \frac{3}{2} \cdot 2 - 10 \]
\[ 1 - 5 = \frac{3}{2} \cdot 2 - 10 \]
\[ -4 = -4 \]

43. \( 3x - 11 = \frac{1}{3}x - 8 \)

\[ \text{SOLUTION:} \]

Manipulate the equation so that there is a zero on either side.

\[ 3x - 11 = \frac{1}{3}x - 8 \]
\[ 3x - \frac{1}{3}x - 11 = \frac{1}{3}x - \frac{1}{3}x - 8 \]
\[ \frac{8}{3}x - 11 = -8 \]
\[ \frac{8}{3}x - 8 + 8 = 0 \]
\[ \frac{8}{3}x - 3 = 0 \]

The related function is \( f(x) = \frac{8}{3}x - 3 \). To graph the function, make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = \frac{8}{3}x - 3 )</th>
<th>( f(\alpha) )</th>
<th>( (\alpha, f(\alpha)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( f(-3) = \frac{8}{3}(-3) - 3 )</td>
<td>-11</td>
<td>(-3, -11)</td>
</tr>
<tr>
<td>-1</td>
<td>( f(-1) = \frac{8}{3}(-1) - 3 )</td>
<td>5.5</td>
<td>(-1, 5.5)</td>
</tr>
<tr>
<td>0</td>
<td>( f(0) = \frac{8}{3}(0) - 3 )</td>
<td>-3</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>1</td>
<td>( f(1) = \frac{8}{3}(1) - 3 )</td>
<td>-1/3</td>
<td>(1, -1/3)</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = \frac{8}{3}(3) - 3 )</td>
<td>5</td>
<td>(3, 5)</td>
</tr>
</tbody>
</table>

From the graph, the \( x \)-intercept appears to be 1. However, from the table above, we can see that it is not.
The solution must remain on the hair for 8 minutes to be completely effective.

d. A possible domain is \( \{t \mid 0 \leq t \leq 8\} \), because the time the solution is left on the hair varies from 0 to 8 minutes. A possible range is \( \{p \mid 0 \leq p \leq 100\} \) because the percentage of the process left to complete varies from 0 to 100.
45. **MUSIC DOWNLOADS** In this problem, you will investigate the change between two quantities.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Number of Songs Downloaded</th>
<th>Total Cost ($)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

b. As the number of songs downloaded increases, how does the total cost change?

c. Interpret the value of the total cost divided by the number of songs downloaded.

**SOLUTION:**

a. Divide the total cost by the number of songs downloaded to complete the last column in the table.

<table>
<thead>
<tr>
<th>Number of Songs Downloaded</th>
<th>Total Cost ($)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

b. As the number of songs downloaded increases by 2 the total cost increases by $4.

c. The total cost divided by the number of songs download is 2 for each line. This costs represents the cost to download a song. The cost to download a song is $2 .

46. **ERROR ANALYSIS** Clarissa and Koko solve $3x + 5 = 2x + 4$ by graphing the related function. Is either of them correct? Explain your reasoning.

**SOLUTION:**

Koko is correct. She solved the equation so that there was a zero on one side. Then wrote the related function and graphed. Clarissa attempted to solve the equations so that there was a zero on one side. However, she added 5 to each side instead of subtracting the 5 from each. Therefore Clarissa’s related function and graph were incorrect.
3-2 Solving Linear Equations by Graphing

47. **CHALLENGE** Find the solution of \( \frac{2}{3} (x + 3) = \frac{1}{2} (x + 5) \) by graphing. Verify your solution algebraically.

**SOLUTION:**
Manipulate the equation so that there is a zero on either side.

\[
\begin{align*}
\frac{2}{3} (x + 3) &= \frac{1}{2} (x + 5) & \text{Original equation} \\
6 \cdot \frac{2}{3} (x + 3) &= 6 \cdot \frac{1}{2} (x + 5) & \text{Multiply each side by 6} \\
4 (x + 3) &= 3 (x + 5) & \text{Simplify} \\
4x + 12 &= 3x + 15 & \text{Distributive Property} \\
4x - 3x + 12 &= 3x - 3x + 15 & \text{Subtract 3x from each side} \\
x + 12 &= 15 & \text{Simplify} \\
x + 12 - 12 &= 15 - 12 & \text{Subtract 12 from each side} \\
x &= 3 & \text{Simplify} \\
\end{align*}
\]

The related function is \( f(x) = x - 3 \).

Graph the related function on a graphing calculator.

![Graph of the function](image)

The solution is 3.

Verify Algebraically:

\[
\begin{align*}
\frac{2}{3} (x + 3) &= \frac{1}{2} (x + 5) \quad & \text{Original equation} \\
6 \cdot \frac{2}{3} (x + 3) &= 6 \cdot \frac{1}{2} (x + 5) & \text{Multiply each side by 6} \\
4 (x + 3) &= 3 (x + 5) & \text{Simplify} \\
4x + 12 &= 3x + 15 & \text{Distributive Property} \\
4x - 3x + 12 &= 3x - 3x + 15 & \text{Subtract 3x from each side} \\
x + 12 &= 15 & \text{Simplify} \\
x + 12 - 12 &= 15 - 12 & \text{Subtract 12 from each side} \\
x &= 3 & \text{Simplify} \\
\end{align*}
\]

48. **CCSS TOOLS** Explain when it is better to solve an equation using algebraic methods and when it is better to solve by graphing.

**SOLUTION:**
It is better to solve an equation algebraically if an exact answer is needed. It is better to solve an equation graphically if an exact answer is not needed. Also, if the equation has fractions, it is easier to solve algebraically than graphically.

For example, solve \( -7 = 4x + 1 \) graphically and \( 0 = \frac{1}{2} + \frac{2}{3}x \) algebraically.

49. **OPEN ENDED** Write a linear equation that has a root of \( -\frac{3}{4} \). Write its related function.

**SOLUTION:**
If the root is \( -\frac{3}{4} \), then \( x = -\frac{3}{4} \).

Rewrite this equation so that there is an zero on the right side.

\[
\begin{align*}
x &= -\frac{3}{4} & \text{Original equation} \\
4 \cdot x &= -\frac{3}{4} \cdot 4 & \text{Multiply each side by 4} \\
4x &= -3 & \text{Simplify} \\
4x + 3 &= -3 + 3 & \text{Add 3 to each side} \\
4x + 3 &= 0 & \text{Simplify} \\
\end{align*}
\]

Thus, \( 3 + 4x = 0 \), \( y = 3 + 4x \), or \( f(x) = 3 + 4x \).
3-2 Solving Linear Equations by Graphing

50. **WRITING IN MATH** Summarize how to solve a linear equation algebraically and graphically.

**SOLUTION:**
To solve a linear equation algebraically, solve the equation for $x$. To solve a linear equation graphically, find the related function by setting the equation equal to zero. Then, make a table and choose different values for $x$ and find the corresponding $y$–coordinate. Determine where the graph intersects the $x$–axis. This is the solution. If the graph does not intersect the $x$–axis, there is no solution.

Consider the function $f(x) = 3x + 3$. Graphing can be used to find the solution easily.

Consider the function $f(x) = \frac{1}{2} + \frac{2}{3}x$. Solving algebraically will give a more accurate answer.

51. What are the $x$– and $y$–intercepts of the graph of the function?

![Graph of a linear function]

A $-3, 6$
B $6, -3$
C $3, -6$
D $-6, 3$

**SOLUTION:**
The $x$–intercept of the function is where it crosses the $x$–axis and the $y$–value is zero. This point is $(-3, 0)$. This means that choices C and D are not options. The $y$–intercept of the function is where it crosses the $y$–axis and the $x$–value is zero. This point is $(0, 6)$. This means that choice A is the correct option.

52. The table shows the cost $C$ of renting a pontoon boat for $h$ hours.

<table>
<thead>
<tr>
<th>Hours</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>7.25</td>
<td>14.5</td>
<td>21.75</td>
</tr>
</tbody>
</table>

Which equation best represents the data?

F $C = 7.25h$

G $C = h + 7.25$

H $C = 21.75 - 7.25h$

J $C = 7.25h + 21.75$

**SOLUTION:**
The difference in $C$–values is $7.25$ times the difference of $h$–values. This suggests $C = 7.25h$. So, the equation for the relationship in function notation is $f(x) = 7.25h$. The equation for choice G would be $8.25$ for 1 hour, $9.25$ for 2 hours, and $10.25$ for three, which does not match the data. The equation for choice H would be $14.50$ for 1 hour, $7.25$ for 2 hours, and $0$ for 3 hours, which does not match the data. The equation for choice J is $29$ for 1 hour, $36.5$ for 2 hours, and $44$ for three hours. So the correct choice is F.
3-2 Solving Linear Equations by Graphing

53. Which is the best estimate for the $x$–intercept of the graph of the linear function represented in the table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$-1$</td>
</tr>
<tr>
<td>4</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

**A** between 0 and 1  
**B** between 2 and 3  
**C** between 1 and 2  
**D** between 3 and 4

**SOLUTION:**
The $x$–intercept is where the linear function crosses the $x$–axis or when the $y$–value is zero. This will happen between $(2, 1)$ and $(3, −1)$. So the best estimate for the $x$–value is between 2 and 3. Therefore the correct choice is **B**.

54. **EXTENDED RESPONSE** Mr. Kauffmann has the following options for a backyard pool.

If each pool has the same depth, which pool would give the greatest area to swim? Explain your reasoning.

**SOLUTION:**
Find the area of each pool.

The area of the rectangular pool is found by:

$$A = bh$$
$$= (12)(18)$$
$$= 216$$

So, the area of the rectangular pool is $216 \text{ ft}^2$.

The area of the circular pool is found by:

$$A = \pi r^2$$
$$= \pi (9)^2$$
$$= \pi (81)$$
$$\approx 254.5$$

So, the area of the circular pool is about $254.5 \text{ ft}^2$.

Finally, the area of the rounded pool can be found by splitting it into a circle and a rectangle. So, it can be found by:

$$A = \pi r^2 + bh$$
$$= \pi (6^2) + (10)(12)$$
$$= \pi (36) + 120$$
$$\approx 113.1 + 120$$
$$\approx 233.1$$

So the area of the rounded pool is about $233.1 \text{ ft}^2$.

Therefore, the circular pool would have the greatest area.
Find the $x$– and $y$–intercepts of the graph of each linear equation.

55. $y = 2x + 10$

**SOLUTION:**
To find the $x$–intercept, let $y = 0$.

\[
\begin{align*}
y &= 2x + 10 & \text{Original equation} \\
0 &= 2x + 10 & \text{Replace } y \text{ with } 0. \\
0 - 10 &= 2x + 10 - 10 & \text{Subtract } 10 \text{ from each side} \\
-10 &= 2x & \text{Simplify} \\
-10 \div 2 &= 2x \div 2 & \text{Divide each side by } 2. \\
-5 &= x & \text{Simplify}.
\end{align*}
\]

To find the $y$–intercept, let $x = 0$.

\[
\begin{align*}
y &= 2x + 10 & \text{Original equation} \\
y &= 2(0) + 10 & \text{Replace } x \text{ with } 0. \\
y &= 0 + 10 & \text{Simplify.} \\
y &= 10 & \text{Simplify.}
\end{align*}
\]

So, the $x$–intercept is $-5$ and the $y$–intercept is $10$.

56. $3y = 6x - 9$

**SOLUTION:**
To find the $x$–intercept, let $y = 0$.

\[
\begin{align*}
3y &= 6x - 9 & \text{Original equation} \\
3(0) &= 6x - 9 & \text{Replace } y \text{ with } 0. \\
0 &= 6x - 9 & \text{Simplify.} \\
0 + 9 &= 6x - 9 + 9 & \text{Add } 9 \text{ to each side} \\
9 &= 6x & \text{Simplify.} \\
9 \div 6 &= 6x \div 6 & \text{Divide each side by } 6 \\
\frac{3}{2} &= x & \text{Simplify.}
\end{align*}
\]

To find the $y$–intercept, let $x = 0$.

\[
\begin{align*}
3y &= 6x - 9 & \text{Original equation} \\
3y &= 6(0) - 9 & \text{Replace } x \text{ with } 0. \\
3y &= 0 - 9 & \text{Simplify.} \\
3y &= -9 & \text{Simplify.} \\
3y &= -9 & \text{Simplify.} \\
\frac{3y}{3} &= \frac{-9}{3} & \text{Divide each side by } 3 \\
y &= -3 & \text{Simplify.}
\end{align*}
\]

So, the $x$–intercept is $\frac{3}{2}$, and the $y$–intercept is $-3$. 
57. \(4x - 14y = 28\)

**SOLUTION:**

To find the \(x\)-intercept, let \(y = 0\).

\[
\begin{align*}
4x - 14(0) &= 28 & \text{Original equation} \\
4x &= 28 & \text{Replace } y \text{ with 0.} \\
\frac{4x}{4} &= \frac{28}{4} & \text{Simplify.} \\
x &= 7 & \text{Divide each side by 4.} \\
\end{align*}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
\begin{align*}
4(0) - 14y &= 28 & \text{Original equation} \\
-14y &= 28 & \text{Replace } x \text{ with 0.} \\
\frac{-14y}{-14} &= \frac{28}{-14} & \text{Simplify.} \\
y &= -2 & \text{Divide each side by } -14. \\
\end{align*}
\]

So, the \(x\)-intercept is 7, and the \(y\)-intercept is \(-2\).

58. **FOOD** If 2% milk contains 2% butterfat and whipping cream contains 9% butterfat, how much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?

**SOLUTION:**

Let \(m\) represent the gallons of milk needed and let \(w\) represent the gallons of whipping cream needed. \(m\) gallons of milk with 2% butterfat must be combined with \(w\) gallons of whipping cream with 9% butterfat to get a 35-gallon mixture with 4% butterfat.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{FOOD} & \text{amount} & \text{\% \text{butter fat}} & \text{\% \text{butter fat}} \\
\hline
\text{milk} & m & 0.02 & 0.02m \\
\text{whipping cream} & w & 0.09 & 0.09w \\
\text{35} & \text{35} & 0.00 & 0.04(35) \\
\hline
\end{array}
\]

The equation would be, \(0.02m + 0.09w = 0.04(35)\). Substitute 35 \(- w\) for \(m\). Then the equation would be \(0.02(35 - w) + 0.09w = (0.04)(35)\).

\[
\begin{align*}
0.02(35 - w) + 0.09w &= 0.04(35) & \text{Original equation} \\
0.02(35) - 0.02w + 0.09w &= 1.4 & \text{Distributive Property} \\
0.07w &= 1.4 & \text{Simplify.} \\
0.7w &= 1.4 & \text{Divide each side by 0.7.} \\
w &= 2 & \text{Divide each side by 0.07.} \\
\end{align*}
\]

To find the gallons of milk, substitute 10 for \(w\) in the original equation: \(m = 35 - 10 = 25\). So, you need 10 gallons of whipping cream and 25 gallons of 2% milk to make a 35-gallon mixture that has 4% butterfat.

**Translate each sentence into an equation.**

59. The product of 3 and \(m\) plus 2 times \(n\) is the same as the quotient of 4 and \(p\).

**SOLUTION:**

\[
3m + 2n = \frac{4}{p}
\]

60. The sum of \(x\) and five times \(y\) equals twice \(z\) minus 7.

**SOLUTION:**

\[
x + 5y = 2z - 7
\]
Simplify.

61. \( \frac{25}{10} \)

\[ \text{SOLUTION:} \quad \frac{25}{10} = \frac{5}{2} \]

62. \( -\frac{4}{-12} \)

\[ \text{SOLUTION:} \quad \frac{-4}{-12} = \frac{1}{3} \]

63. \( \frac{6}{-12} \)

\[ \text{SOLUTION:} \quad \frac{6}{-12} = -\frac{1}{2} \]

64. \( -\frac{36}{8} \)

\[ \text{SOLUTION:} \quad -\frac{36}{8} = -\frac{9}{2} \]

Evaluate \( \frac{a-b}{c-d} \) for the given values.

65. \( a = 6, b = 2, c = 9, d = 3 \)

\[ \text{SOLUTION:} \quad \frac{a-b}{c-d} = \frac{6-2}{9-3} = \frac{4}{6} = \frac{2}{3} \]

66. \( a = -8, b = 4, c = 5, d = -3 \)

\[ \text{SOLUTION:} \quad \frac{a-b}{c-d} = \frac{-8-4}{5-(-3)} = \frac{-12}{8} = -\frac{3}{2} \]

67. \( a = 4, b = -7, c = -1, d = -2 \)

\[ \text{SOLUTION:} \quad \frac{a-b}{c-d} = \frac{4-(-7)}{-1-(-2)} = \frac{4+7}{1} = 11 \]
Find the rate of change represented in each table or graph.

1. SOLUTION:
To find the rate of change, use the coordinates (3, 6) and (0, 2).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{6 - 2}{3 - 0} = \frac{4}{3}
\]

So, the rate of change is \( \frac{4}{3} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-6</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>11</td>
<td>26</td>
</tr>
</tbody>
</table>

2. SOLUTION:
To find the rate of change, use the coordinates (3, -6) and (5, 2).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{-6 - 2}{3 - 5} = \frac{-8}{-2} = 4
\]

So, the rate of change is 4.

3. CCSS SENSE-MAKING Refer to the graph below.

![Houston Astros Tickets Average Price](chart.png)

a. Find the rate of change of prices from 2006 to 2008. Explain the meaning of the rate of change.

b. Without calculating, find a two-year period that had a greater rate of change than 2006–2008. Explain.

c. Between which years would you guess the new stadium was built? Explain your reasoning.

SOLUTION:

a. To find the rate of change, use the coordinates (2008, 28.73) and (2006, 26.66).

\[
\text{rate of change} = \frac{\text{change in price}}{\text{change in year}} = \frac{28.73 - 26.66}{2008 - 2006} = \frac{2.07}{2} = 1.035
\]

So, the rate of change is 1.035, which means there was an average increase in ticket price of $1.035 per year.

b. Sample answer: The two-year period that had a greater rate of change than 2006–2008 was 1998–2000. There was a steeper segment, which means a greater rate of change.

c. Sample answer: I would guess that the new stadium was built between 1998 and 2000, because the ticket prices show a sharp increase.
3-3 Rate of Change and Slope

Determine whether each function is linear. Write yes or no. Explain.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
<td>(\frac{3}{4})</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
<td>(\frac{3}{4})</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(\frac{1}{3})</td>
<td>(-\frac{1}{3})</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>(\frac{1}{3})</td>
<td>(-\frac{1}{3})</td>
</tr>
</tbody>
</table>

The rate of change, \(\frac{1}{3}\), is constant. So, the function is linear.

Find the slope of the line that passes through each pair of points.

6. (5, 3), (6, 9)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{6 - 5} = \frac{6}{1} = 6
\]

So, the slope is 6.

7. (–4, 3), (–2, 1)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{-2 - (-4)} = \frac{-2}{-2 + 4} = \frac{-2}{2} = -1
\]

So, the slope is −1.

8. (6, −2), (8, 3)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{8 - 6} = \frac{3 + 2}{2} = \frac{5}{2}
\]

So, the slope is \(\frac{5}{2}\).
Find the rate of change represented in each table or graph.

9. (1, 10), (−8, 3)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{3 - 10}{-8 - 1} \]
\[ = \frac{-7}{-9} \]
\[ = \frac{7}{9} \]

So, the slope is \( \frac{7}{9} \).

10. (−3, 7), (−3, 4)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{4 - 7}{-3 - (-3)} \]
\[ = \frac{4 - 7}{0} \]
\[ = \frac{-3}{0} \]

Dividing by 0 is undefined. So, the slope is undefined.

11. (5, 2), (−6, 2)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{2 - 2}{-6 - 5} \]
\[ = \frac{0}{-11} \]
\[ = 0 \]

So, the slope is 0.

Find the value of \( r \) so the line that passes through each pair of points has the given slope.

12. (−4, \( r \)), (−8, 3), \( m = -5 \)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{-5}{-8 - (-4)} \]
\[ = \frac{-5}{-8 + 4} \]
\[ = \frac{-5}{-4} \]
\[ = \frac{5}{4} \]

Multiply each side by \(-4\).

\[ 20 = 3 - r \]
\[ 20 - 3 = 3 - r - 3 \]
\[ 17 = -r \]
\[ 17 = r \]

Simplify.

So, the line goes through (−4, −17).

13. (5, 2), (−7, \( r \)), \( m = \frac{5}{6} \)

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{5}{6} \]
\[ = \frac{r - 2}{-7 - 5} \]
\[ = \frac{r - 2}{-12} \]
\[ = 5(-12) = 6(r - 2) \]
\[ = 5(-12) = 6(r - 2) \]
\[ = -60 = 6r - 12 \]
\[ = -60 + 12 = 6r - 12 + 12 \]
\[ = -48 = 6r \]
\[ = \frac{-48}{6} = \frac{6r}{6} \]
\[ = -8 = r \]

Divide each side by 6.

So, the line goes through (−7, −8).
3-3 Rate of Change and Slope

Find the rate of change represented in each table or graph.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

14. **SOLUTION:**
To find the rate of change, use the coordinates (10, 3) and (5, 2).

rate of change = \( \frac{\text{change in } y}{\text{change in } x} \)
= \( \frac{3 - 2}{10 - 5} \)
= \( \frac{1}{5} \)

So, the rate of change is \( \frac{1}{5} \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
</tr>
</tbody>
</table>

15. **SOLUTION:**
To find the rate of change, use the coordinates (1, 15) and (2, 9).

rate of change = \( \frac{\text{change in } y}{\text{change in } x} \)
= \( \frac{9 - 15}{2 - 1} \)
= \( \frac{-6}{1} \)
= -6

So, the rate of change is -6.
17. SOLUTION: 
To find the rate of change, use the coordinates (−4, −6) and (4, −2).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{-2 - (-6)}{4 - (-4)} = \frac{4}{8} = \frac{1}{2}.
\]

So, the rate of change is \(\frac{1}{2}\).

18. SPORTS What was the annual rate of change from 2004 to 2008 for women participating in college lacrosse? Explain the meaning of the rate of change.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>5545</td>
</tr>
<tr>
<td>2008</td>
<td>6830</td>
</tr>
</tbody>
</table>

SOLUTION: 
To find the rate of change, use the coordinates (2004, 5545) and (2008, 6830).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{6830 - 5545}{2008 - 2004} = \frac{1285}{4} = 321.25.
\]

So, the rate of change is 321.25. This rate of change means there was an average increase of 321.25 women per year competing in triathlons.
19. **RETAIL.** The average retail price in the spring of 2009 for a used car is shown in the table below.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>17,378</td>
</tr>
<tr>
<td>3</td>
<td>16,157</td>
</tr>
</tbody>
</table>

**a.** Write a linear function to model the price of the car with respect to age.  
**b.** Interpret the meaning of the slope of the line.  
**c.** Assuming a constant rate of change, predict the average retail price for a 7–year-old car.

**SOLUTION:**
**a.** To find a linear function, find the y-intercept and slope. Use the coordinates (2, 17,378) and (3, 16,157). Find the slope, or rate of change.

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{16,157 - 17,378}{3 - 2} = \frac{-1221}{1} = -1221
\]

Find the y-intercept.

\[
y = mx + b
\]

\[
17,378 = (-1221)(2) + b \quad 17,378 = -2442 + b \quad 19,820 = b
\]

So, a linear function to model the price of the car with respect to age is \( p = -1221t + 19,820 \).

**b.** The slope of \(-1221\) represents how much the car value depreciates by each year.

**c.**

\[
p = -1221t + 19,820
\]

\[
= -1221(7) + 19,820
\]

\[
= -8547 + 19,820
\]

\[
= 11,273
\]

So, the average retail price for a 7-year-old car is \$11,273.

---

20. **Determine whether each function is linear. Write yes or no. Explain.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( \frac{1-(4)}{2-(-1)} )</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>( \frac{3-1}{0-2} )</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>( \frac{5-3}{-2-0} )</td>
<td>-1</td>
</tr>
<tr>
<td>-4</td>
<td>7</td>
<td>( \frac{7-5}{-4-(-2)} )</td>
<td>-1</td>
</tr>
</tbody>
</table>

The rate of change, \(-1\), is constant. So, the function is linear.

21. **SOLUTION:**

Use the slope formula to determine the rate of change for each pair of consecutive points.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>11</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-5</td>
<td>14</td>
<td>( \frac{14-11}{-5-(-7)} )</td>
<td>( \frac{3}{2} )</td>
</tr>
<tr>
<td>-3</td>
<td>17</td>
<td>( \frac{17-14}{-3-(-5)} )</td>
<td>( \frac{3}{2} )</td>
</tr>
<tr>
<td>-1</td>
<td>20</td>
<td>( \frac{20-17}{-1-(-3)} )</td>
<td>( \frac{3}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>23</td>
<td>( \frac{23-20}{0-(-1)} )</td>
<td>3</td>
</tr>
</tbody>
</table>

The rate of change is not constant. So, the function is not linear.
3-3 Rate of Change and Slope

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-0.2$</th>
<th>$0$</th>
<th>$0.2$</th>
<th>$0.4$</th>
<th>$0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$0.7$</td>
<td>$0.4$</td>
<td>$0.1$</td>
<td>$0.3$</td>
<td>$0.6$</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Use the slope formula to determine the rate of change for each pair of points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.2$</td>
<td>$0.7$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$0$</td>
<td>$0.4$</td>
<td>$\frac{0.4-0.7}{0-(-0.2)}$</td>
<td>$-0.25$</td>
</tr>
<tr>
<td>$0.2$</td>
<td>$0.1$</td>
<td>$\frac{0.2-0.1}{0.2-0}$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0.4$</td>
<td>$0.3$</td>
<td>$\frac{0.4-0.3}{0.4-0.2}$</td>
<td>$2$</td>
</tr>
<tr>
<td>$0.6$</td>
<td>$0.6$</td>
<td>$\frac{0.6-0.6}{0.6-0.4}$</td>
<td>$1.5$</td>
</tr>
</tbody>
</table>

The rate of change is not constant. So, the function is not linear.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{3}{2}$</td>
<td>$1$</td>
<td>$\frac{1-1}{\frac{3}{2}-\frac{1}{2}}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{5}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{\frac{3}{2}-\frac{1}{2}}{\frac{5}{2}-\frac{2}{2}}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{7}{2}$</td>
<td>$2$</td>
<td>$\frac{2-\frac{3}{2}}{\frac{7}{2}-\frac{3}{2}}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\frac{9}{2}$</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{\frac{5}{2}-\frac{2}{2}}{\frac{9}{2}-\frac{7}{2}}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

The rate of change, $\frac{1}{2}$, is constant. So, the function is linear.

**Find the slope of the line that passes through each pair of points.**

24. $(4, 3), (-1, 6)$

**SOLUTION:**

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 3}{-1 - 4}$$

$$= \frac{3}{-5}$$

$$= -\frac{3}{5}$$

So, the slope is $-\frac{3}{5}$. 
25. (8, –2), (1, 1)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{1 - 8} = \frac{1 + 2}{-7} = \frac{3}{-7} = -\frac{3}{7}
\]

So, the slope is \( -\frac{3}{7} \).

26. (2, 2), (–2, –2)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-2 - 2} = \frac{-2 + (-2)}{-2 + (-2)} = \frac{-4}{-4} = 1
\]

So, the slope is 1.

27. (6, –10), (6, 14)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - (-10)}{6 - 6} = \frac{14 + 10}{6 - 6} = \frac{24}{0}
\]

Dividing by 0 is undefined. So, the slope is undefined.

28. (5, –4), (9, –4)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-4)}{9 - 5} = \frac{-4 + 4}{9 - 5} = \frac{0}{4} = 0
\]

So, the slope is 0.

29. (11, 7), (–6, 2)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 7}{-6 - 11} = \frac{-5}{-17} = \frac{5}{17}
\]

So, the slope is \( \frac{5}{17} \).

30. (–3, 5), (3, 6)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 5}{3 - (-3)} = \frac{6 - 5}{3 + 3} = \frac{1}{6}
\]

So, the slope is \( \frac{1}{6} \).
3-3 Rate of Change and Slope

31. \((-3, 2), (7, 2)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{7 - (-3)} = \frac{2 - 2}{7 + 3} = \frac{0}{10} = 0
\]

So, the slope is 0.

32. \((8, 10), (-4, -6)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 10}{-4 - 8} = \frac{-16}{-12} = \frac{4}{3}
\]

So, the slope is \(\frac{4}{3}\).

33. \((-8, 6), (-8, 4)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 6}{-8 - (-8)} = \frac{4 - 6}{-8 + 8} = \frac{-2}{0}
\]

Dividing by 0 is undefined. So, the slope is undefined.

34. \((-12, 15), (18, -13)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - 15}{18 - (-12)} = \frac{-28}{30} = \frac{-14}{15}
\]

So, the slope is \(-\frac{14}{15}\).

35. \((-8, -15), (-2, 5)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-15)}{-2 - (-8)} = \frac{20}{6} = \frac{10}{3}
\]

So, the slope is \(\frac{10}{3}\).
3-3 Rate of Change and Slope

Find the value of \( r \) so the line that passes through each pair of points has the given slope.

36. \((12, 10), (-2, r), m = -4\)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ -4 = \frac{-2 - 10}{-2 - 12} \]

Let \((12, 10) = (x_1, y_1)\) and \((-2, r) = (x_2, y_2)\).

So, the line goes through \((-2, 66)\).

37. \((r, -5), (3, 13), m = 8\)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ 8 = \frac{13 - (-5)}{3 - r} \]

Let \((r, -5) = (x_1, y_1)\) and \((3, 13) = (x_2, y_2)\).

So, the line goes through \((3, 13)\).

38. \((3, 5), (-3, r), m = \frac{3}{4}\)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ \frac{3}{4} = \frac{r - 5}{-3 - 3} \]

Let \((3, 5) = (x_1, y_1)\) and \((-3, r) = (x_2, y_2)\).

So, the line goes through \((-3, \frac{1}{2})\).

39. \((-2, 8), (r, 4), m = -\frac{1}{2}\)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ -\frac{1}{2} = \frac{4 - 8}{r - (-2)} \]

Let \((r, 4) = (x_2, y_2)\) and \((-2, 8) = (x_1, y_1)\).

So, the line goes through \((6, 4)\).

**CCSS TOOLS** Use a ruler to estimate the slope of each object.

40. Refer to photo on Page 178.

**SOLUTION:**

Sample answer: about \(-0.5\)

Use slope = \(\frac{\text{rise}}{\text{run}}\) to calculate the slope. Because it slopes down from left to right, the slope is negative.
3-3 Rate of Change and Slope

41. Refer to photo on Page 178.

**SOLUTION:**
Sample answer: about \(-1\)

Use \(\text{slope} = \frac{\text{rise}}{\text{run}}\) to calculate the slope.

Because it slopes up from right to left, the slope is negative.

42. **DRIVING** When driving up a certain hill, you rise 15 feet for every 1000 feet you drive forward. What is the slope of the road?

**SOLUTION:**
The rise is 15 feet and the run is 1000 feet. Because you are driving up a hill, the slope is positive.

\[
slope = \frac{\text{rise}}{\text{run}} = \frac{15}{1000} = \frac{3}{200}
\]

So, the slope of the road is \(\frac{3}{200}\).

Find the slope of the line that passes through each pair of points.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>-1</td>
</tr>
<tr>
<td>5.3</td>
<td>2</td>
</tr>
</tbody>
</table>

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{5.3 - 4.5} = \frac{3}{0.8} = \frac{15}{4}
\]

So, the slope is \(\frac{15}{4}\).

44. **SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 1}{0.75 - 0.75} = \frac{-2}{0}
\]

Dividing by 0 is undefined. So, the slope is undefined.
3-3 Rate of Change and Slope

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$-1\frac{1}{2}$</td>
</tr>
<tr>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

SOLUTION:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{1}{2} - (-1\frac{1}{2})}{\frac{1}{2} - 2\frac{1}{2}}$$

$$= \frac{\frac{1}{2} + 1\frac{1}{2}}{-\frac{1}{2} - 2\frac{1}{2}}$$

$$= \frac{2}{-\frac{5}{2}}$$

$$= -\frac{4}{5}$$

So, the slope is $-\frac{4}{5}$.

46. MULTIPLE REPRESENTATIONS In this problem, you will investigate why the slope of a line through any two points on that line is constant.

a. VISUAL Sketch a line $\ell$ that contains points $A$, $B$, $A'$, and $B'$ on a coordinate plane.

b. GEOMETRIC Add segments to form right triangles $ABC$ and $A'B'C'$ with right angles $C$ and $C'$. Describe $\overline{AC}$ and $\overline{A'C'}$ and $\overline{BC}$ and $\overline{B'C'}$

c. VERBAL How are triangles $ABC$ and $A'B'C'$ related? What does that imply for the slope between any two distinct points on line $\ell$?

SOLUTION:

a. Draw a slanted line and label it $\ell$. Place four points on the line and label them from left to right $A$, $B,A'$, and $B'$. Draw a line straight down from $B$ and straight across from $A$ to form a right angle and label the vertex of the angle $C$. Repeat the same process from $B'$ and $A'$ to form a right angle at a point $C'$.

b. $\overline{AC}$ and $\overline{A'C'}$ are horizontal, and $\overline{BC}$ and $\overline{B'C'}$ are vertical.

c. Sample answer: Triangles $ABC$ and $A'B'C'$ are similar because line $\ell$ is a transversal of parallel segments $\overline{BC}$ and $\overline{B'C'}$, so $\triangle ABC \cong \triangle A'B'C'$ since corresponding angles of parallel lines are congruent. $\angle C \cong \angle C'$ because all right angles are congruent. Therefore, by AA Similarity, $\triangle ABC \sim \triangle A'B'C'$. The ratios formed by corresponding sides of similar triangles are equal. The slope of a line is defined as $\text{rise over run}$. So $\frac{BC}{AC}$, the slope of line through points $A$ and $B$, is equal to $\frac{B'C'}{A'C'}$, the slope of line through points $A'$ and $B'$.

47. BASKETBALL The table shown below shows the average points per game (PPG) Michael Redd has scored in each of his first 9 seasons with the NBA’s
Milwaukee Bucks.

<table>
<thead>
<tr>
<th>Season</th>
<th>PPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>11.4</td>
</tr>
<tr>
<td>3</td>
<td>15.1</td>
</tr>
<tr>
<td>4</td>
<td>21.7</td>
</tr>
<tr>
<td>5</td>
<td>23.0</td>
</tr>
<tr>
<td>6</td>
<td>25.4</td>
</tr>
<tr>
<td>7</td>
<td>26.7</td>
</tr>
<tr>
<td>8</td>
<td>22.7</td>
</tr>
<tr>
<td>9</td>
<td>21.2</td>
</tr>
</tbody>
</table>

- **a.** Make a graph of the data. Connect each pair of adjacent points with a line.
- **b.** Use the graph to determine in which period Michael Redd’s PPG increased the fastest. Explain your reasoning.
- **c.** Discuss the difference in the rate of change from season 1 through season 4, from season 4 through season 7, from season 7 through season 9.

**SOLUTION:**

- **a.** Graph the season on the x-axis and the points per game on the y-axis. Then, plot the ordered pairs from the table and connect the points with straight lines.

- **b.** To find the period that Michael Redd’s PPG increased the fastest, look for the steepest slope or find the slope and locate the one with the highest rate of change.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>11.4</td>
<td>$\frac{21.7-2.2}{1-1}$</td>
<td>9.2</td>
</tr>
<tr>
<td>3</td>
<td>15.1</td>
<td>$\frac{21.7-11.4}{1-1}$</td>
<td>3.7</td>
</tr>
<tr>
<td>4</td>
<td>21.7</td>
<td>$\frac{26.7-21.7}{1-1}$</td>
<td>6.6</td>
</tr>
<tr>
<td>5</td>
<td>23.0</td>
<td>$\frac{26.7-21.7}{1-1}$</td>
<td>1.3</td>
</tr>
<tr>
<td>6</td>
<td>25.4</td>
<td>$\frac{26.7-23.0}{1-1}$</td>
<td>2.4</td>
</tr>
<tr>
<td>7</td>
<td>26.7</td>
<td>$\frac{22.7-26.7}{1-1}$</td>
<td>1.3</td>
</tr>
<tr>
<td>8</td>
<td>22.7</td>
<td>$\frac{22.7-26.7}{1-1}$</td>
<td>-4</td>
</tr>
<tr>
<td>9</td>
<td>21.2</td>
<td>$\frac{22.7-26.7}{1-1}$</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

From Season 1 to Season 2, the line is the steepest. So, that is the period Michael Redd’s PPG increased the fastest.

- **c.** Use the graph and the table to analyze.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>Slope</th>
<th>Rate of Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>21.7</td>
<td>$\frac{21.7-2.2}{1-1}$</td>
<td>6.5</td>
</tr>
<tr>
<td>7</td>
<td>26.7</td>
<td>$\frac{26.7-21.7}{1-1}$</td>
<td>1.7</td>
</tr>
<tr>
<td>9</td>
<td>21.2</td>
<td>$\frac{21.2-26.7}{1-1}$</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

The rate of change was much more dramatic or steeper in the first four years, it leveled off the next three seasons, and was negative and steeper the last two seasons.
3-3 Rate of Change and Slope

48. **REASONING** Why does the Slope Formula not work for vertical lines? Explain.

**SOLUTION:**
Consider the following graph.

Let \((x_1, y_1) = (-2, -2)\) and \((x_2, y_2) = (4, -2)\), Find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-7)}{-3 - (-3)} = \frac{4 - 7}{-3 + 3} = \frac{-3}{0}
\]

Division by 0 is undefined. Thus, the slope is undefined. This occurs when the difference in the \(x\)–values for vertical lines is always 0, which is in the denominator of the slope formula.

49. **OPEN ENDED** Use what you know about rate of change to describe the function represented by the table.

<table>
<thead>
<tr>
<th>Time (wk)</th>
<th>Height of Plant (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9.0</td>
</tr>
<tr>
<td>6</td>
<td>13.5</td>
</tr>
<tr>
<td>8</td>
<td>18.0</td>
</tr>
</tbody>
</table>

**SOLUTION:**
To find the rate of change, use the coordinates \((4, 9.0)\) and \((6, 13.5)\).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{13.5 - 9}{6 - 4} = \frac{4.5}{2} = 2.25
\]

So, the rate of change is \(2 \frac{1}{4}\) inches of growth per week.
3-3 Rate of Change and Slope

50. **CHALLENGE** Find the value of \( d \) so the line that passes through \((a, b)\) and \((c, d)\) has a slope of \( \frac{1}{2} \).

**SOLUTION:**

Set the slope formula equal to \( \frac{1}{2} \) and solve for \( d \) using the coordinates \((a, b)\) and \((c, d)\).

\[
\begin{align*}
\frac{1}{2} &= \frac{d-b}{c-a} \\
\Rightarrow d &= \frac{1}{2} (c-a) + b
\end{align*}
\]

So, the value of \( d \) so the line that passes through \((a, b)\) and \((c, d)\) has a slope of \( \frac{1}{2} \) is \( d = \frac{c-a+2b}{2} \).

51. **WRITING IN MATH** Explain how the rate of change and slope are related and how to find the slope of a line.

**SOLUTION:**

Students’ answers may vary. Sample answer:

The slope and rate of change are ratios. Rate of change is a ratio that describes how much one quantity changes with respect to a change in another quantity. The slope of a line is the ratio of the change in the \( y \)-coordinates to the change in the \( x \)-coordinates.

52. **CCSS ARGUMENTS** Kyle and Luna are finding the value of \( a \) so the line that passes through \((10, a)\) and \((-2, 8)\) has a slope of \( \frac{1}{4} \). Is either of them correct? Explain.

**SOLUTION:**

Luna is correct because she found the change in \( y \) divided by the change in \( x \). Kyle is incorrect because he found the change in \( x \) divided by the change in \( y \).

Slope is defined as \( m = \frac{y_2-y_1}{x_2-x_1} \).
3-3 Rate of Change and Slope

53. The cost of prints from an online photo processor is given by \( C(p) = 29.99 + 0.13p \). $29.99 is the cost of the membership, and \( p \) is the number of 4-inch by 6-inch prints. What does the slope represent?

A. cost per print  
B. cost of the membership  
C. cost of the membership and 1 print  
D. number of prints

**SOLUTION:**
The slope is 0.13. It represents the cost per print, or choice A. Choice B is the cost of membership or $29.99. Choice C is the cost of the membership and one print or $29.99 + $0.13 = $30.12. Choice D is the number of prints or \( p \). So, the correct choice is A.

54. Danita bought a computer for $1200 and its value depreciated linearly. After 2 years, the value was $250. What was the amount of yearly depreciation?

F. $950  
G. $475  
H. $250  
J. $225

**SOLUTION:**
Since the depreciation is linear, the computer value dropped the same amount every year. Over 2 years it dropped from $1200 to $250, or $950. To find the amount for one year, divide $950 by 2, which is $475. So, the correct choice is G.

55. **SHORT RESPONSE** The graph represents how much the Wright Brothers National Monument charges visitors. How much does the park charge each visitor?

**Wright Brothers National Monument**

**SOLUTION:**
The graph shows a constant rate of change, so the cost per visitor is constant. Look at the cost for one visitor. Find 1 visitor on the x-axis. Move your finger up to the corresponding point, then left to the y-axis. There, you see a 4. So, the cost for 1 visitor is $4.
56. **PROBABILITY** At a gymnastics camp, 1 gymnast is chosen at random from each team. The Flipstars Gymnastics Team consists of 5 eleven–year–olds, 7 twelve–year–olds, 10 thirteen–year–olds, and 8 fourteen–year–olds. What is the probability that the age of the gymnast chosen is an odd number?

   A \[ \frac{1}{30} \]
   B \[ \frac{1}{15} \]
   C \[ \frac{1}{2} \]
   D \[ \frac{3}{5} \]

**SOLUTION:**

\[
\text{probability} = \frac{\text{number of favorable outcomes}}{\text{number of total outcomes}} = \frac{\text{total number of girls with an odd age}}{\text{total number of girls}}
\]

\[
= \frac{5 + 10}{5 + 7 + 10 + 8} = \frac{15}{30} = \frac{1}{2}
\]

The probability of choosing a girl with an age that is an odd number is \( \frac{1}{2} \). So the correct choice is C.

**Solve each equation by graphing.**

57. \( 3x + 18 = 0 \)

**SOLUTION:**

The related function is \( y = 3x + 18 \).

The graph intercepts the \( x \)-axis at \(-6\). So, the solution is \(-6\).

58. \( 8x - 32 = 0 \)

**SOLUTION:**

The related function is \( y = 8x - 32 \).

The graph intercepts the \( x \)-axis at 4. So, the solution is 4.

59. \( 0 = 12x - 48 \)

**SOLUTION:**

The related function is \( y = 12x - 48 \).

The graph intercepts the \( x \)-axis at 4. So, the solution is 4.
Find the rate of change represented in each table or graph.

1. **SOLUTION:**
   To find the rate of change, use the coordinates (1, 26.7) and (3, 9.2).
   
   \[
   \text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9.2 - 26.7}{3 - 1} = \frac{-17.5}{2} = -8.75
   \]

64. **SOLUTION:**
   
   65. **SOLUTION:**
   
   66. **SOLUTION:**
   
   67. **SOLUTION:**
   
   68. **SOLUTION:**

62. **HOMECOMING** Drink tickets are $9 for one person and $15 for two people. If a group of seven students wishes to go to the dance, write and solve an equation that would represent the least expensive price of their tickets.

   **SOLUTION:**
   Let \( p \) be the price, \( x \) be the number of couples’ tickets and \( y \) be the number of single tickets. If seven students are going, then they can form three couples with one student left over.

   \[
   p = 15x + 9y
   \]

   \[
   = 15(3) + 9(1)
   \]

   \[
   = 45 + 9
   \]

   \[
   = 54
   \]

   So, the least expensive price \( p \) of their tickets is $54.

**Find each quotient.**

63. \( \frac{8}{3} \)

   **SOLUTION:**
   
   \[
   8 \div \frac{2}{3} = 8 \cdot \frac{3}{2} = \frac{24}{2} = 12
   \]

64. \( \frac{3}{8} \div \frac{1}{4} \)

   **SOLUTION:**
   
   \[
   \frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \cdot \frac{4}{1} = \frac{12}{8} = \frac{3}{2}
   \]

65. \( \frac{5}{8} \div 2 \)

   **SOLUTION:**
   
   \[
   \frac{5}{8} \div 2 = \frac{5}{8} \div \frac{2}{1} = \frac{5}{8} \cdot \frac{1}{2} = \frac{5}{16}
   \]
3-3 Rate of Change and Slope

66. \( \frac{12 \cdot 6}{9} \)

**SOLUTION:**

\[
\frac{12 \cdot 6}{9} = \frac{72}{9} = 8
\]

67. \( \frac{2 \cdot 15}{6} \)

**SOLUTION:**

\[
\frac{2 \cdot 15}{6} = \frac{30}{6} = 5
\]

68. \( \frac{18 \cdot 5}{15} \)

**SOLUTION:**

\[
\frac{18 \cdot 5}{15} = \frac{90}{15} = 6
\]
3-4 Direct Variation

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

1. \[ y = -\frac{4}{5}x \]

**SOLUTION:**
The constant of variation is \(-\frac{4}{5}\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
= \frac{4 - 0}{-5 - 0}
= \frac{4}{-5}
= \frac{-4}{5}
\]

2. \[ y = 2x \]

**SOLUTION:**
The constant of variation is 2.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
= \frac{2 - 0}{2 - 0}
= \frac{2}{2}
= 2
\]

Graph each equation.

3. \[ y = -x \]

**SOLUTION:**
The slope of \( y = -x \) is \(-1\). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[ -1 = \frac{-1}{1} \]

Graph (0, 0). From there, move down 1 unit and right 1 unit to find another point. Then draw a line containing the points.

4. \[ y = \frac{3}{4}x \]

**SOLUTION:**
The slope of \( y = \frac{3}{4}x \) is \( \frac{3}{4} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[ \frac{\text{rise}}{\text{run}} = \frac{3}{4} \]

Graph (0, 0). From there, move up 3 units and right 4 units to find another point. Then draw a line containing the points.
3-4 Direct Variation

5. $y = -8x$

**SOLUTION:**
The slope of $y = -8x$ is $-8$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$-8 = \frac{-8}{1}$

Graph (0, 0). From there, move down 8 units and right 1 unit to find another point. Then draw a line containing the points.

6. $y = -\frac{8}{5}x$

**SOLUTION:**
The slope of $y = -\frac{8}{5}x$ is $-\frac{8}{5}$. Write the slope as $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{-8}{5}$

Graph (0, 0). From there, move down 8 units and right 5 units to find another point. Then draw a line containing the points.

---

Suppose $y$ varies directly as $x$. Write a direct variation equation that relates $x$ and $y$. Then solve.

7. If $y = 15$ when $x = 12$, find $y$ when $x = 32$.

**SOLUTION:**

\[ y = kx \]
\[ 15 = k(12) \]
\[ \frac{15}{12} = k \]
\[ \frac{5}{4} = k \]

So, the direct variation equation is $y = \frac{5}{4}x$. Substitute 32 for $x$ and find $y$.

\[ y = \frac{5}{4}(32) \]
\[ y = 40 \]

So, $y = 40$ when $x = 32$.

8. If $y = -11$ when $x = 6$, find $x$ when $y = 44$.

**SOLUTION:**

\[ y = kx \quad \text{Direct variation equation} \]
\[ -11 = k(6) \quad \text{Replace } y \text{ with } -11 \text{ and } x \text{ with } 6 \]
\[ -\frac{11}{6} = \frac{k(6)}{6} \quad \text{Divide each side by } 6 \]
\[ -\frac{11}{6} = k \quad \text{Simplify} \]

So, the direct variation equation is $y = -\frac{11}{6}x$.

Substitute 44 for $y$ and find $x$.

\[ y = -\frac{11}{6}x \quad \text{Direct variation formula} \]
\[ 44 = -\frac{11}{6}x \quad \text{Replace } y \text{ with } 44 \]
\[ -\frac{6}{11}(44) = -\frac{6}{11}\left(-\frac{11}{6}x\right) \quad \text{Multiply each side by } -\frac{6}{11} \]
\[ -24 = x \quad \text{Simplify} \]

So, $x = -24$ when $y = 44$.

9. **CCSS REASONING** You find that the number of messages you receive on your message board varies directly as the number of messages you post. When you post 5 messages, you receive 12 messages in
3-4 Direct Variation

return.

a. Write a direct variation equation relating your posts to the messages received. Then graph the equation.

b. Find the number of messages you need to post to receive 96 messages.

**SOLUTION:**

### a.

\[ y = kx \]  
Direct variation formula

\[ 12 = k(5) \]  
Replace \( y \) with 12 and \( x \) with 5

\[ \frac{12}{5} = \frac{k(5)}{5} \]  
Divide each side by 5.

\[ \frac{12}{5} = k \]  
Simplify.

So, the direct variation equation is \( y = \frac{12}{5}x \).

The slope of \( y = \frac{12}{5}x \) is \( \frac{12}{5} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[ \frac{\text{rise}}{\text{run}} = \frac{12}{5} \]

Graph (0, 0). From there, move up 12 units and right 5 units to find another point. Then draw a line containing the points.

### b. Substitute 96 for \( x \) and find \( y \).

\[ y = \frac{12}{5}x \]  
Direct variation formula

\[ 96 = \frac{12}{5}x \]  
Replace \( y \) with 96.

\[ \frac{5}{12}(96) = \frac{5}{12} \left( \frac{12}{5}x \right) \]  
Multiply each side by \( \frac{5}{12} \)

\[ 40 = x \]  
Simplify

So, you need to post 40 messages to receive 96 messages.

**Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.**

**SOLUTION:**

10.

The constant of variation is 4.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ \frac{4 - 0}{1 - 0} = \frac{4}{1} = 4 \]

11.

The constant of variation is −5.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ \frac{5 - 0}{-1 - 0} = \frac{5}{-1} = -5 \]
3-4 Direct Variation

**SOLUTION:**

The constant of variation is \( \frac{2}{3} \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 0} = \frac{4}{6} = \frac{2}{3}
\]

**SOLUTION:**

The constant of variation is \( \frac{4}{3} \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 0}{-6 - 0} = \frac{-8}{-6} = \frac{4}{3}
\]

**SOLUTION:**

The constant of variation is \( \frac{1}{5} \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-10 - 0} = \frac{2}{-10} = \frac{-10}{1} = \frac{-10}{5}
\]

**SOLUTION:**

The constant of variation is \( -12 \).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 0}{1 - 0} = \frac{-12}{1} = -12
\]
3-4 Direct Variation

Graph each equation.

16. \( y = 10x \)

**SOLUTION:**

The slope of \( y = 10x \) is 10. Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\( \frac{10}{1} \)

Graph (0, 0). From there, move up 10 units and right 1 unit to find another point. Then draw a line containing the points.

17. \( y = -7x \)

**SOLUTION:**

The slope of \( y = -7x \) is -7. Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\( \frac{-7}{1} \)

Graph (0, 0). From there, move down 7 units and right 1 unit to find another point. Then draw a line containing the points.

18. \( y = x \)

**SOLUTION:**

The slope of \( y = x \) is 1. Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\( \frac{1}{1} \)

Graph (0, 0). From there, move up 1 unit and right 1 unit to find another point. Then draw a line containing the points.

19. \( y = \frac{7}{6}x \)

**SOLUTION:**

The slope of \( y = \frac{7}{6}x \) is \( \frac{7}{6} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\( \frac{7}{6} \)

Graph (0, 0). From there, move down 7 units and left 6 units to find another point. Then draw a line containing the points.
3-4 Direct Variation

20. \( y = \frac{1}{6} x \)

**SOLUTION:**

The slope of \( y = \frac{1}{6} x \) is \( \frac{1}{6} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[
\frac{\text{rise}}{\text{run}} = \frac{1}{6}
\]

Graph (0, 0). From there, move 1 unit up and 6 units to the right to find another point. Then draw a line containing the points.

21. \( y = \frac{2}{9} x \)

**SOLUTION:**

The slope of \( y = \frac{2}{9} x \) is \( \frac{2}{9} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[
\frac{\text{rise}}{\text{run}} = \frac{2}{9}
\]

Graph (0, 0). From there, move 2 units up and 9 units to the right to find another point. Then draw a line containing the points.

22. \( y = \frac{6}{5} x \)

**SOLUTION:**

The slope of \( y = \frac{6}{5} x \) is \( \frac{6}{5} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[
\frac{\text{rise}}{\text{run}} = \frac{6}{5}
\]

Graph (0, 0). From there, move 6 units up and 5 units to the right to find another point. Then draw a line containing the points.

23. \( y = -\frac{5}{4} x \)

**SOLUTION:**

The slope of \( y = -\frac{5}{4} x \) is \( -\frac{5}{4} \). Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[
\frac{\text{rise}}{\text{run}} = \frac{-5}{-4}
\]

Graph (0, 0). From there, move 5 units up and 4 units to the right to find another point. Then draw a line containing the points.
Suppose \( y \) varies directly as \( x \). Write a direct 
variation equation that relates \( x \) and \( y \). Then 
solve.

24. If \( y = 6 \) when \( x = 10 \), find \( x \) when \( y = 18 \).

**SOLUTION:**

\[
y = kx \\
6 = k(10) \\
\frac{6}{10} = \frac{k(10)}{10} \\
\frac{3}{5} = k \\
\]

So, the direct variation equation is \( y = \frac{3}{5}x \). 
Substitute 18 for \( y \) and find \( x \).

\[
y = \frac{3}{5}x \\
18 = \frac{3}{5}x \\
\frac{5}{3}(18) = \frac{5}{3}\left(\frac{3}{5}x\right) \\
30 = x \\
\]

So, \( x = 30 \) when \( y = 18 \).

25. If \( y = 22 \) when \( x = 8 \), find \( y \) when \( x = -16 \).

**SOLUTION:**

\[
y = kx \\
22 = k(8) \\
\frac{22}{8} = \frac{k(8)}{8} \\
\frac{11}{4} = k \\
\]

So, the direct variation equation is \( y = \frac{11}{4}x \). 
Substitute \(-16\) for \( x \) and find \( y \).

\[
y = \frac{11}{4}x \\
y = \frac{11}{4}(-16) \\
y = -44 \\
\]

So, \( y = -44 \) when \( x = -16 \).

26. If \( y = 4\frac{1}{4} \) when \( x = \frac{3}{4} \), find \( y \) when \( x = 4\frac{1}{2} \).

**SOLUTION:**

\[
y = kx \\
4\frac{1}{4} = k\left(\frac{3}{4}\right) \\
\frac{4\frac{1}{4}}{\frac{3}{4}} = \frac{4\left(\frac{3}{4}\right)}{\frac{3}{4}} \\
\frac{17}{3} = k \\
\frac{5}{3} = k \\
\]

So, the direct variation equation is \( y = \frac{5}{3}x \). 
Substitute \(4\frac{1}{2}\) for \( x \) and find \( y \).

\[
y = \frac{5}{3}x \\
y = \frac{5}{3}\left(4\frac{1}{2}\right) \\
y = \frac{17}{3} \left(\frac{9}{2}\right) \\
y = \frac{51}{2} \\
y = 25\frac{1}{2} \\
\]

So, \( y = 25\frac{1}{2} \) when \( x = 4\frac{1}{2} \).
27. If \( y = 12 \) when \( x = \frac{6}{7} \), find \( x \) when \( y = 16 \).

**SOLUTION:**

\[
\begin{align*}
  y &= kx \\
  12 &= k \left( \frac{6}{7} \right) \\
  \frac{7}{6} (12) &= \frac{7}{6} \left( \frac{6}{7} k \right) \\
  14 &= k
\end{align*}
\]

So, the direct variation equation is \( y = 14x \). Substitute 16 for \( y \) and find \( x \).

\[
\begin{align*}
  y &= 14x \\
  16 &= 14x \\
  \frac{16}{14} &= \frac{14}{14} x \\
  \frac{8}{7} &= x \\
  \frac{1}{7} &= x
\end{align*}
\]

So, \( x = \frac{1}{7} \) when \( y = 16 \).

---

28. **SPORTS** The distance a golf ball travels at an altitude of 7000 feet varies directly with the distance the ball travels at sea level, as shown.

- **a.** Write and graph an equation that relates the distance a golf ball travels at an altitude of 7000 feet \( y \) with the distance at sea level \( x \).
- **b.** What would be a person’s average driving distance at 7000 feet if his average driving distance at sea level is 180 yards?

<table>
<thead>
<tr>
<th>Hitting a Golf Ball</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Altitude (ft)</strong></td>
</tr>
<tr>
<td><strong>Distance (yd)</strong></td>
</tr>
</tbody>
</table>

**SOLUTION:**

**a.**

\[
\begin{align*}
  y &= kx \\
  210 &= k (200) \\
  \frac{210}{200} &= \frac{200k}{200} \\
  1.05 &= k
\end{align*}
\]

So the equation is \( y = 1.05x \).

**b.**

\[
\begin{align*}
  y &= 1.05x \\
  y &= 1.05(180) \\
  y &= 189
\end{align*}
\]

So, the average driving distance at 7000 feet would be 189 yards.
3-4 Direct Variation

29. **FINANCIAL LITERACY** Depreciation is the decline in a car's value over the course of time. The table below shows the values of a car with an average depreciation.

<table>
<thead>
<tr>
<th>Age of Car (years)</th>
<th>Value (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,000</td>
</tr>
<tr>
<td>2</td>
<td>10,200</td>
</tr>
<tr>
<td>3</td>
<td>8,400</td>
</tr>
<tr>
<td>4</td>
<td>6,600</td>
</tr>
<tr>
<td>5</td>
<td>4,800</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. The difference in $y$-values is 1800 times the difference of $x$-values. This suggests $y = 1800x$.

b. First find the values of the car when it was purchased. If after 1 year the car was worth 12,000 and it had decreased in value by 1800, its original worth was 13,800. Next, subtract its current value to find how much its value had decreased by, 13800−300 = 13500. Substitute this value into the equation for $y$.

\[ y = 1800x \]

\[ 13500 = 1800x \]

\[ \frac{13500}{1800} = \frac{1800x}{1800} \]

\[ x = 7.5 \]

So, this means that the car will be worth $300 in 7.5 years or 7 years and 6 months.

Suppose $y$ varies directly as $x$. Write a direct variation equation that relates $x$ and $y$. Then solve.

30. If $y = 3.2$ when $x = 1.6$, find $y$ when $x = 19$.

**SOLUTION:**

\[ y = kx \]

\[ 3.2 = k(1.6) \]

\[ \frac{3.2}{1.6} = \frac{k(1.6)}{1.6} \]

\[ 2 = k \]

So, the direct variation equation is $y = 2x$. Substitute 19 for $x$ and find $y$.

\[ y = 2x \]

\[ y = 2(19) \]

\[ y = 38 \]

So, $y = 38$ when $x = 19$.

31. If $y = 15$ when $x = \frac{3}{4}$, find $x$ when $y = 25$.

**SOLUTION:**

\[ y = kx \]

\[ 15 = k\left(\frac{3}{4}\right) \]

\[ \frac{4}{3}(15) = \frac{4}{3}\left(\frac{3}{4}k\right) \]

\[ 20 = k \]

So, the direct variation equation is $y = 20x$. Substitute 25 for $y$ and find $x$.

\[ y = 20x \]

\[ 25 = 20x \]

\[ \frac{25}{20} = \frac{20x}{20} \]

\[ \frac{5}{4} = x \]

So, $x = \frac{5}{4}$ when $y = 25$. 
32. If \( y = 4.5 \) when \( x = 2.5 \), find \( y \) when \( x = 12 \).

**SOLUTION:**

\[
y = kx \\
4.5 = k(2.5) \\
\frac{4.5}{2.5} = k \\
1.8 = k
\]

So, the direct variation equation is \( y = 1.8x \). Substitute 12 for \( x \) and find \( y \).

\[
y = 1.8x \\
y = 1.8(12) \\
y = 21.6
\]

So, \( y = 21.6 \) when \( x = 12 \).

33. If \( y = -6 \) when \( x = 1.6 \), find \( y \) when \( x = 8 \).

**SOLUTION:**

\[
y = kx \\
-6 = k(1.6) \\
\frac{-6}{1.6} = k \\
-3.75 = k
\]

So, the direct variation equation is \( y = -3.75x \). Substitute 8 for \( x \) and find \( y \).

\[
y = -3.75x \\
y = -3.75(8) \\
y = -30
\]

So, \( y = -30 \) when \( x = 8 \).

CCSS SENSE-MAKING Certain endangered species experience cycles in their populations as shown in this graph. Match each animal to one of the colored lines in the graph.

34. red grouse, 8 years per cycle

**SOLUTION:**

Move right to cycle number 1 and then move up 8 years. The line that intersects with this point is red.

35. voles, 3 years per cycle

**SOLUTION:**

Move right to cycle number 1 and then move up 3 years. The line that intersects with this point is dark green.

36. lemmings, 4 years per cycle

**SOLUTION:**

Move right to cycle number 1 and then move up 4 years. The line that intersects with this point is blue.

37. lynx, 10 years per cycle

**SOLUTION:**

Move right to cycle number 1 and then move up 10 years. The line that intersects with this point is lime green.
3-4 Direct Variation

Write and graph a direct variation equation that relates the variables.

38. PHYSICAL SCIENCE The weight $W$ of an object is $9.8 \, \text{m/s}^2$ times the mass of the object $m$.

**SOLUTION:**
The slope of $W = 9.8m$ is 9.8. Write the slope as \( \frac{\text{rise}}{\text{run}} \)

\[9.8 = \frac{9.8}{1}\]

Graph (0, 0). From there, move up 9.8 units and right 1 unit to find another point. Then draw a line containing the points.

39. MUSIC Music downloads are $0.99$ per song. The total cost of $d$ songs is $T$.

**SOLUTION:**
The slope of $T = 0.99d$ is 0.99. Write the slope as \( \frac{\text{rise}}{\text{run}} \)

\[0.99 = \frac{0.99}{1}\]

Graph (0, 0). From there, move up 0.99 units and right 1 unit to find another point. Then draw a line containing the points.
3-4 Direct Variation

40. GEOMETRY The circumference of a circle \( C \) is approximately 3.14 times the diameter \( d \).

**SOLUTION:**

The slope of \( C = 3.14d \) is 3.14. Write the slope as \( \frac{\text{rise}}{\text{run}} \).

\[ 3.14 = \frac{3.14}{1} \]

Graph (0, 0). From there, move up 3.14 units and right 1 unit to find another point. Then draw a line containing the points.

![Graph](image)

41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the family of direct variation functions.

a. **GRAPHICAL** Graph \( y = x, y = 3x, \) and \( y = 5x \) on the same coordinate plane.

b. **ALGEBRAIC** Describe the relationship among the constant of variation, the slope of the line, and the rate of change of the graph.

c. **VERBAL** Make a conjecture about how you can determine without graphing which of two direct variation equations has steeper graph.

**SOLUTION:**

![Graph](image)

a.

b. The constant of variation, slope, and rate of change of a graph all have the same value.

c. For each equation, find the absolute value of \( k \). The equation with the greater value of \( |k| \) has the steeper graph.

42. **TRAVEL** A map of North Carolina is scaled so that 3 inches represents 93 miles. How far apart are Raleigh and Charlotte if they are 1.8 inches apart on the map?

**SOLUTION:**

\[ y = kx \]

\[ 93 = k(3) \]

\[ \frac{93}{3} = \frac{k(3)}{3} \]

\[ 31 = k \]

So, the direct variation equation is \( y = 31x \). Substitute 1.8 for \( x \) and find \( y \).

\[ y = 31 \times 1.8 \]

\[ y = 55.8 \]

So, \( y = 55.8 \) miles when \( x = 1.8 \) inches.
43. **INTERNET** A company will design and maintain a Web site for your company for $9.95 per month. Write a direct variation equation to find the total cost $C$ for having a Web page for $n$ months.

**SOLUTION:**
The direct variation equation is $C = 9.95n$.

44. **BASEBALL** Before their first game, high school student Todd McCormick warmed all 5200 seats in a new minor league stadium. By literally sitting in every seat. He started at 11:50 a.m. and finished around 3 p.m.

a. Write a direct variation equation relating the number of seats to time. What is the meaning of the constant of variation in this situation?

b. About how many seats had Todd sat in by 1:00 p.m.?

c. How long would you expect it to take Todd to sit in all of the seats at a major league stadium with more than 40,000 seats?

**SOLUTION:**
a. Convert the time to minutes: 3 hours and 10 minutes = 190 minutes.

\[
y = kx
\]

\[
5200 = k(190)
\]

\[
\frac{5200}{190} = \frac{190k}{190}
\]

\[
27.3684 = k
\]

So, the equation is $y = 27.3684t$. This means that Todd warms about 27 seats every minute.

b. $y = 27.3684t$

$y = 27.3684(70)$

$y = 1915.788$

So, Todd had warmed about 1915 seats.

c. $y = 27.3684t$

$40,000 = 27.3684t$

$\frac{40,000}{27.3684} = \frac{27.3684t}{27.3684}$

$1461.54 = t$

It would take Todd about 1461 minutes or 24 hours and 21 minutes.
45. **WHICH ONE DOESN’T BELONG?** Identify the equation that does not belong. Explain.

\[ 9 = rt \quad 9a = 0 \quad z = \frac{1}{9}x \quad w = \frac{9}{t} \]

**SOLUTION:**

\[ z = \frac{1}{9}x \] is a direct variation with the constant of variation of \( \frac{1}{9} \).

\( 9 = rt \), is not a direct variation. When it is rewritten with variables on each side it is \( t = \frac{9}{r} \), which is not linear.

\( 9a = 0 \) only has one variable and thus can not be a direct variation.

\( w = \frac{9}{t} \) is not a direct variation since it is not linear.

Thus \( z = \frac{1}{9}x \) is the only equation that is a direct variation.

46. **REASONING** How are the constant of variation and the slope related in a direct variation equation? Explain your reasoning.

**SOLUTION:**

They are equal; In \( y = kx \), the constant of variation is \( k \). The graph passes through \((0, 0)\) and \((1, k)\), so its slope is \( k \).

47. **OPEN ENDED** Model a real-world situation using a direct variation equation. Graph the equation and describe the rate of change.

**SOLUTION:**

Students’ answers may vary. Sample answer: \( y = 0.50x \) represents the cost of \( x \) apples. The rate of change, 0.50, is the cost per apple.

48. **CCSS STRUCTURE** Suppose \( y \) varies directly as \( x \). If the value of \( x \) is doubled, then the value of \( y \) is also *always, sometimes* or *never* doubled. Explain your reasoning.

**SOLUTION:**

The statement *If the value of \( x \) is doubled, then the value of \( y \) is also doubled* is always true. Given the equation is \( y = kx \) \((k \neq 0)\), then the value of \( y \) when \( x = 2a \) is \( ka \), and the value of \( y \) when \( x = 2a \) is \( k(2a) \) or \( 2(ka) \).

49. **ERROR ANALYSIS** Eddy says the slope between any two points on the graph of a direct variation equation \( y = kx \) is \( \frac{1}{k} \). Adelle says the slope depends on the points chosen. Is either of them correct? Explain.

**SOLUTION:**

Neither of them is correct. The slope is constant, so it does not depend on which points you choose, so Adelle is incorrect. The slope of \( y = kx \) is \( k \), not \( \frac{1}{k} \), so Eddy is incorrect also.

50. **WRITING IN MATH** Describe the graph of a direct variation equation.

**SOLUTION:**

The graph of a direct variation equation \( y = kx \) is a line that always passes through the origin. The graph has a positive slope if \( k \) is positive and the graph has a negative slope if \( k \) is negative.

51. Patricia pays $1.19 each to download songs to her digital media player. If \( n \) is the number of downloaded songs, which equation represents the cost \( C \) in dollars?

A $ C = 1.19n$

B $ n = 1.19C$

C $ C = 1.19 \div n$

D $ C = n + 1.19$

**SOLUTION:**

If Patricia pays $1.19 for each song, this is a direct variation. The equation that represents this is \( C = 1.19n \). So the correct choice is A.
52. Suppose that y varies directly as x, and y = 8 when x = 6. What is the value of y when x = 8?

F 6
G 12
H \( \frac{2}{3} \)
J 16

**SOLUTION:**

\[ y = kx \]
\[ 8 = k(6) \]
\[ \frac{8}{6} = \frac{k(6)}{6} \]
\[ \frac{4}{3} = k \]

So, the direct variation equation is \( y = \frac{4}{3}x \).

Substitute 8 for \( x \) and find \( y \).

\[ y = \frac{4}{3} \times 8 \]
\[ y = \frac{32}{3} \]
\[ y = 10 \frac{2}{3} \]

\( y = 10 \frac{2}{3} \) when \( x = 8 \), so the correct choice is A.

53. What is the relationship between the input \( (x) \) and output \( (y) \)?

A The output is two more than the input.
B The output is two less than the input.
C The output is twice the input.
D The output is half the input.

**SOLUTION:**

Compare some points to check on the relationship. Look at the point (2, 1).

| A | The output is two more than the input. | 2 + 2 = 4 |
| B | The output is two less than the input. | 2 – 2 = 0 |
| C | The output is twice the input. | 2 \times 2 = 4 |
| D | The output is half the input. | 2 \div 2 = 1 |

The only choice that provides the correct choice is D.

54. SHORT RESPONSE A telephone company charges $40 per month plus $0.07 per minute. How much would a month of service cost a customer if the customer talked for 200 minutes?

**SOLUTION:**

Let \( m \) represent the number of minutes a customer talks.

\[ 40 + 0.07m = 40 + 0.07(200) \]
\[ = 40 + 14 \]
\[ = 54 \]

So, a customer would be charged $54.
55. **TELEVISION** The graph shows the average number of television channels American households receive. What was the annual rate of change from 2004 to 2008? Explain the meaning of the rate of change.

![Graph of TV channels at home](image)

**SOLUTION:**

\[
\text{rate of change} = \frac{\text{change in tv channels}}{\text{change in time}} = \frac{118.6 - 92.6}{4} = \frac{26}{4} = 6.5
\]

The average rate of change in the average number of television channels is about 6.5. This means that there was an average increase of 6.5 channels per year.

**Solve each equation by graphing.**

56. \(0 = 18 - 9x\)

**SOLUTION:**

The related function is \(y = 18 - 9x\).

![Graph of y = 18 - 9x](image)

The graph intersects the \(x\)-axis at 2. So, the solution is 2.

57. \(2x + 14 = 0\)

**SOLUTION:**

The related function is \(y = 2x + 14\).

The graph intersects the \(x\)-axis at -7. So, the solution is -7.

58. \(-4x + 16 = 0\)

**SOLUTION:**

The related function is \(y = -4x + 16\).

The graph intersects the \(x\)-axis at 4. So, the solution is 4.
3-4 Direct Variation

59. \(-5x - 20 = 0\)

**SOLUTION:**
The related function is \(y = -5x - 20\).

![Graph](image1.png)
The graph intersects the \(x\)-axis at -4. So, the solution is -4.

60. \(8x - 24 = 0\)

**SOLUTION:**
The related function is \(y = 8x - 24\).

![Graph](image2.png)
The graph intersects the \(x\)-axis at 3. So, the solution is 3.

61. \(12x - 144 = 0\)

**SOLUTION:**
The related function is \(y = 12x - 144\).

![Graph](image3.png)
The graph intersects the \(x\)-axis at 12. So, the solution is 12.

62. \(|2a + c| + 1\)

**SOLUTION:**
Replace \(a\) with 4 and \(c\) with -4.

\[
|2a + c| + 1 = |2(4) + (-4)| + 1 \\
= |0| + 1 \\
= 0 + 1 \\
= 1 \\
= 5
\]

63. \(4a - |3b + 2|\)

**SOLUTION:**
Replace \(a\) with 4 and \(b\) with -2.

\[
4a - |3b + 2| = 4(4) - |3(-2) + 2| \\
= 16 - |6 + 2| \\
= 16 - 8 \\
= 8
\]
3-4 Direct Variation

64. \(-|a + 1| + 3c\)
   **SOLUTION:**
   Replace \(a\) with 4 and \(c\) with \(-4\).
   
   \[-|a + 1| + 3c = -|4 + 1| + 3(-4)|
   \[= -|5| + (-12)|
   \[= -5 + 12
   \[= 7\]

65. \(-a + |2 - a|\)
   **SOLUTION:**
   Replace \(a\) with 4
   
   \[-a + |2 - a| = -4 + |2 - 4|
   \[= -4 + |2|
   \[= -4 + 2
   \[= -2\]

66. \(|c - 2b| - 3\)
   **SOLUTION:**
   Replace \(b\) with \(-2\) and \(c\) with \(-4\)
   
   \[|c - 2b| - 3 = |-4 - 2(-2)| - 3\]
   \[= |-4 + 4| - 3\]
   \[= |0| - 3\]
   \[= 0 - 3\]
   \[= -3\]

67. \(-2|3b - 8|\)
   **SOLUTION:**
   Replace \(b\) with \(-2\).
   
   \[-2|3b - 8| = -2|3(-2) - 8|
   \[= -2|-6 - 8|
   \[= -2|14|
   \[= -2(14)
   \[= -28\]

Find each difference.
68. \(13 - (-1)\)
   **SOLUTION:**
   \(13 - (-1) = 13 + 1\)
   \[= 14\]

69. \(4 - 16\)
   **SOLUTION:**
   \(4 - 16 = -12\)

70. \(-3 - 3\)
   **SOLUTION:**
   \(-3 - 3 = -6\)

71. \(-8 - (-2)\)
   **SOLUTION:**
   \(-8 - (-2) = -8 + 2\)
   \[= -6\]

72. \(16 - (-10)\)
   **SOLUTION:**
   \(16 - (-10) = 16 + 10\)
   \[= 26\]

73. \(-8 - 4\)
   **SOLUTION:**
   \(-8 - 4 = -12\)
3-5 Arithmetic Sequences as Linear Functions

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.
1. 18, 16, 15, 13, ...

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

16 – 18 = –2
15 – 16 = –1
13 – 15 = –2

The difference between terms is not constant. Therefore, it is not an arithmetic sequence.

2. 4, 9, 14, 19, ...

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

9 – 4 = 5
14 – 9 = 5
19 – 14 = 5

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 5.

Find the next three terms of each arithmetic sequence.
3. 12, 9, 6, 3, ...

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

6 – 9 = –3.

The common difference between terms is –3. So, to find the next term, subtract 3 from the last term. To find the next term, subtract 3 from the resulting number, and so on.

3 – 3 = 0
0 – 3 = –3
–3 – 3 = –6

So, the next three terms of this arithmetic sequence are 0, –3, –6.

4. –2, 2, 6, 10, ...

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

2 – (–2) = 4

The common difference between terms is 4. So, to find the next term, add 4 to the last term. To find the next term, add 4 to the resulting number, and so on.

10 + 4 = 14
14 + 4 = 18
18 + 4 = 22

So, the next three terms of this arithmetic sequence are 14, 18, 22.
Write an equation for the $n$th term of each arithmetic sequence. Then graph the first five terms of the sequence.

5. 15, 13, 11, 9, …

**SOLUTION:**
Find the common difference.
$13 - 15 = -2$

Write the equation for the $n$th term of an arithmetic sequence using the first term 15 and common difference $-2$.

$$a_n = a_1 + (n - 1)d$$
$$= 15 + (n - 1)(-2)$$
$$= 15 - 2n + 2$$
$$= 17 - 2n$$

The points to graph are represented by $(n, a_n)$. So, the first four points are (1, 15); (2, 13); (3, 11); (4, 9). To find the fifth point, substitute 5 for $n$ in the equation and evaluate for $a_n$.

$$a_n = 17 - 2n$$
$$= 17 - 2(5)$$
$$= 17 - 10$$
$$= 7$$

The fifth point is (5, 7).

6. –1, –0.5, 0, 0.5, …

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
$-0.5 - (-1) = 0.5$

Write the equation for the $n$th term of an arithmetic sequence using the first term $-1$ and common difference 0.5.

$$a_n = a_1 + (n - 1)d$$
$$= -1 + (n - 1)0.5$$
$$= -1 + 0.5n - 0.5$$
$$= 0.5n - 1.5$$

The points to graph are represented by $(n, a_n)$. So, the first four points are (1, –1); (2, –0.5); (3, 0); (4, 0.5). To find the fifth point, substitute 5 for $n$ in the equation and evaluate for $a_n$.

$$a_n = 0.5n - 1.5$$
$$= 0.5(5) - 1.5$$
$$= 2.5 - 1.5$$
$$= 1$$

The fifth point is (5, 1).
### 3-5 Arithmetic Sequences as Linear Functions

7. **SAVINGS**
   Kaia has $525 in a savings account. After one month she has $580 in the account. The next month the balance is $635. The balance after the third month is $690. Write a function to represent the arithmetic sequence. Then graph the function.

**SOLUTION:**
The arithmetic sequence is 525, 580, 635, 690, ...

Subtract the 1st term from the 2nd to find the common difference.

\[580 - 525 = 55\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term 580 (note: 525 = \(n_0\)) and common difference 55.

\[f_n = n_0 + (n - 1)d\]

\[= 580 + (n - 1)55\]

\[= 580 + 55n - 55\]

\[= 55n + 525\]

The points to graph are represented by \((n, f_n)\). So, the first four points are (0, 525); (1, 580); (2, 635); (3, 690).

- **Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.**
- **8.** –3, 1, 5, 9, ...

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[1 - (-3) = 4\]

\[5 - 1 = 4\]

\[9 - 5 = 4\]

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 4.

- **9.** \(\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots\)

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[\frac{3}{4} - \frac{1}{2} = \frac{1}{4}\]

\[\frac{5}{8} - \frac{3}{8} = -\frac{1}{8}\]

\[\frac{7}{16} - \frac{5}{8} = -\frac{3}{16}\]

The difference between terms is not constant, so the sequence is not an arithmetic sequence.

- **10.** –10, –7, –4, 1, ...

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[71 - (-10) = 3\]

\[-4 - (-7) = 3\]

\[1 - (-4) = 5\]

The difference between terms is not constant, so the sequence is not an arithmetic sequence.
3-5 Arithmetic Sequences as Linear Functions

11. –12.3, –9.7, –7.1, –4.5, ...

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[-9.7 - (-12.3) = 2.6\]
\[-7.1 - (-9.7) = 2.6\]
\[-4.5 - (-7.1) = 2.6\]

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 2.6.

**Find the next three terms of each arithmetic sequence.**

12. 0.02, 1.08, 2.14, 3.2, ...

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[1.08 - 0.02 = 1.06\]

The common difference between terms is 1.06. So, to find the next term, add 1.06 to the last term. To find the next term, add 1.06 to the resulting number, and so on.

\[3.2 + 1.06 = 4.26\]
\[4.26 + 1.06 = 5.32\]
\[5.32 + 1.06 = 6.38\]

So, the next three terms of this arithmetic sequence are 4.26, 5.32, 6.38.

13. 6, 12, 18, 24, ...

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[12 - 6 = 6\]

The common difference between terms is 6. So, to find the next term, add 6 to the last term. To find the next term, add 6 to the resulting number, and so on.

\[24 + 6 = 30\]
\[30 + 6 = 36\]
\[36 + 6 = 42\]

So, the next three terms of this arithmetic sequence are 30, 36, 42.

14. 21, 19, 17, 15, ...

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[19 - 21 = -2\]

The common difference between terms is –2. So, to find the next term, subtract 2 from the last term. To find the next term, subtract 2 from the resulting number, and so on.

\[15 - 2 = 13\]
\[13 - 2 = 11\]
\[11 - 2 = 9\]

So, the next three terms of this arithmetic sequence are 13, 11, 9.
3-5 Arithmetic Sequences as Linear Functions

15. \(-\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots\)

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[0 - \left(-\frac{1}{2}\right) = \frac{1}{2}\]

The common difference between terms is \(\frac{1}{2}\). So, to find the next term, add \(\frac{1}{2}\) to the last term. To find the next term, add \(\frac{1}{2}\) to the resulting number, and so on.

\[1 + \frac{1}{2} = 1 \frac{1}{2}\]

\[\frac{1}{2} + \frac{1}{2} = 1\]

\[2 + \frac{1}{2} = 2 \frac{1}{2}\]

So, the next three terms of this arithmetic sequence are \(1 \frac{1}{2}, 2, 2 \frac{1}{2}\).

16. \(2 \frac{1}{3}, 2 \frac{2}{3}, 3, 3 \frac{1}{3}, \ldots\)

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[2 \frac{2}{3} - 2 \frac{1}{3} = \frac{1}{3}\]

The common difference between terms is \(\frac{1}{3}\). So, to find the next term, add \(\frac{1}{3}\) to the last term. To find the next term, add \(\frac{1}{3}\) to the resulting number, and so on.

\[3 \frac{1}{3} + \frac{1}{3} = 3 \frac{2}{3}\]

\[\frac{2}{3} + \frac{1}{3} = 4\]

\[\frac{1}{3} + \frac{1}{3} = \frac{4}{3}\]

So, the next three terms of this arithmetic sequence are \(3 \frac{2}{3}, 4, 4 \frac{1}{3}\).

17. \(\frac{7}{12}, 1 \frac{1}{3}, 2 \frac{1}{12}, 2 \frac{5}{6}, \ldots\)

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[1 \frac{1}{3} - \frac{7}{12} = \frac{4}{3} - \frac{7}{12}\]

\[= \frac{16}{12} - \frac{7}{12}\]

\[= \frac{9}{12}\]

\[= \frac{3}{4}\]

The common difference between term is \(\frac{3}{4}\). So, to find the next term, add \(\frac{3}{4}\) to the last term. To find the next term, add \(\frac{3}{4}\) to the resulting number, and so on.
3-5 Arithmetic Sequences as Linear Functions

\[
2 \frac{5}{6} + \frac{3}{4} = \frac{17}{6} + \frac{3}{4} = \frac{34}{12} + \frac{9}{12} = \frac{43}{12} = 3 \frac{7}{12}
\]

\[
3 \frac{7}{12} + \frac{3}{4} = \frac{43}{12} + \frac{9}{12} = \frac{52}{12} = 4 \frac{4}{12} = 4 \frac{1}{3}
\]

\[
4 \frac{1}{3} + \frac{3}{4} = \frac{13}{3} + \frac{3}{4} = \frac{52}{12} + \frac{9}{12} = \frac{61}{12} = 5 \frac{1}{12}
\]

So, the next three terms of this arithmetic sequence are \(3 \frac{7}{12}, 4 \frac{1}{3}, 5 \frac{1}{12}\).

Write an equation for the \(n\)th term of the arithmetic sequence. Then graph the first five terms in the sequence.

18. \(-3, -8, -13, -18, \ldots\)

**SOLUTION:**

Subtract the 1st term from the 2nd to find the common difference.

\(-8 - (-3) = -5\)

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-3\) and common difference \(-5\).

\[
a_n = a_1 + (n - 1)d
\]

\[
= -3 + (n - 1)(-5)
\]

\[
= -3 - 5n + 5
\]

\[
= -5n + 2
\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -3); (2, -8); (3, -13); (4, -18)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[
a_n = -5n + 2
\]

\[
= -5(5) + 2
\]

\[
= -25 + 2
\]

\[
= -23
\]

The fifth point is \((5, -23)\).
19. \(-2, 3, 8, 13, \ldots\)

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
\[3 - (-2) = 5\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-2\) and common difference 5.

\[a_n = a_1 + (n - 1)d\]
\[= -2 + (n - 1)5\]
\[= -2 + 5n - 5\]
\[= 5n - 7\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -2); (2, 3); (3, 8); (4, 13)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[a_n = 5n - 7\]
\[= 5(5) - 7\]
\[= 25 - 7\]
\[= 18\]

The fifth point is \((5, 18)\).

---

20. \(-11, -15, -19, -23, \ldots\)

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
\[-15 - (-11) = -4\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-11\) and common difference \(-4\).

\[a_n = a_1 + (n - 1)d\]
\[= -11 + (n - 1)(-4)\]
\[= -11 - 4n + 4\]
\[= -4n - 7\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -11); (2, -15); (3, -19); (4, -23)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[a_n = -4n - 7\]
\[= -4(5) - 7\]
\[= -20 - 7\]
\[= -27\]

The fifth point is \((5, -27)\).  

---
3-5 Arithmetic Sequences as Linear Functions

21. \(-0.75, -0.5, -0.25, 0, \ldots\)

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
\(-0.5 - (-0.75) = 0.25\)

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-0.75\) and common difference 0.25.

\[ a_n = a_1 + (n - 1)d \]
\[ = -0.75 + (n - 1)0.25 \]
\[ = -0.75 + 0.25n - 0.25 \]
\[ = 0.25n - 1 \]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -0.75); (2, -0.5); (3, -0.25); (4, 0)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[ a_n = 0.25n - 1 \]
\[ = 0.25(5) - 1 \]
\[ = 1.25 - 1 \]
\[ = 0.25 \]

The fifth point is \((5, 0.25)\).

22. **AMUSEMENT PARKS**
Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.

a. Write a function to represent the arithmetic sequence.

b. Graph the function and determine the domain.

**SOLUTION:**
a. The arithmetic sequence is 2, 4, 6,\ldots\ Find the common difference.
\[ 4 - 2 = 2 \]
The sequence is increasing, so the common difference is positive: 2.
Write the equation for the \(n\)th term of an arithmetic sequence using the first term 2 and common difference 2.

\[ f_n = f_1 + (n - 1)d \]
\[ = 2 + (n - 1)2 \]
\[ = 2 + 2n - 2 \]
\[ = 2n \]

So, the function for this arithmetic sequence is \(f(n) = 2n\).

b. The points to graph are represented by \((n, f_n)\). So, the points are \((1, 2); (2, 4); (3, 6)\).

The domain of the function is the number of hours spent at the park. So, the domain is \(\{1, 2, 3, 4, \ldots\}\).
23. **CCSS MODELING** The table shows how Ryan is for cutting 10-foot long 2x4 planks at his lumber yard.

<table>
<thead>
<tr>
<th>Number of 10-ft 2x4 Planks Cut</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount Paid in Commission ($)</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
</tr>
</tbody>
</table>

a. Write a function to represent Ryan’s commission.
b. Graph the function and determine the domain.

**SOLUTION:**
a. The arithmetic sequence is 8, 16, 24, 32, 40, 48, 56. Find the common difference.

16 – 8 = 8

The sequence is increasing, so the common difference is 8.

Write the equation for the n-th term of an arithmetic sequence with first term 8 and common difference 8.

\[ f_n = f_1 + (n - 1)d \]

\[ = 8 + (n - 1)8 \]

\[ = 8 + 8n - 8 \]

\[ = 8n \]

So, the function for this arithmetic sequence is \( f(n) = 8n \).

b. The points to graph are represented by \((n, f_n)\). So, points are (1, 8); (2, 16); (3, 24); (4, 32); (5, 40); (6, 456).

The domain of the function is the number of 10-foot planks cut. So, the domain is \{1, 2, 3, 4, \ldots\}.

24. The graph is a representation of an arithmetic sequence.

a. List the first five terms.
b. Write the formula for the n-th term.
c. Write the function.

**SOLUTION:**
a. The points on the graph are \((1, -3); (2, -1); (3, 1); (4, 3); (5, 5)\). Because the terms are the second coordinate in each coordinate pair, the terms of the arithmetic sequence are -3, -1, 1, 3, and 5.

b. Find the common difference.

\[-1 - (-3) = 2\]

Write the equation for the n-th term of an arithmetic sequence using the first term -3 and common difference 2.

\[ a_n = a_1 + (n - 1)d \]

\[ = -3 + (n - 1)2 \]

\[ = -3 + 2n - 2 \]

\[ = 2n - 5 \]

So, the formula for the n-th term of the arithmetic sequence is \( a_n = 2n - 5 \).

c. \( f(n) = (n - 1)d + a_1 = 2n - 5 \)
3-5 Arithmetic Sequences as Linear Functions

25. **NEWSPAPERS** A local newspaper charges by the number of words for advertising.

![Newspaper Ad Charges](image)

Write a function to represent the advertising costs.

**SOLUTION:**
Find the common difference between sequential terms. Because the terms given skip by fives, divide the difference by five.

\[
8.75 - 7.50 = 1.25 \\
1.25 ÷ 5 = 0.25
\]

Subtract 0.25 nine times from the tenth term to find the first term.

\[
7.50 - 9(0.25) = 5.25
\]

Use the equation for the \(n\)th term of an arithmetic sequence to write a function using first term 5.25 and common difference 0.25.

\[
f(n) = 5.25 + (n - 1)0.25 \\
= 5.25 + 0.25n - 0.25 \\
= 0.25n + 5
\]

So, the function that represents the newspaper charges for advertising is \(f(n) = 0.25n + 5\).

26. The fourth term of an arithmetic sequence is 8. If the common difference is 2, what is the first term?

**SOLUTION:**
To find the first term, subtract 2 three times from the fourth term. \(8 - 2 - 2 - 2 = 2\). Or, subtract \((2 \cdot 3)\) from the fourth term. \(8 - 6 = 2\). So the first term in the sequence is 2.

27. The common difference of an arithmetic sequence is –5. If \(a_{12}\) is 22, what is \(a_1\)?

**SOLUTION:**
To find the first term, subtract –5 eleven times from the twelfth term. \(22 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 77\). Or, subtract \((11 \cdot -5)\) from the twelfth term. \(22 - (-55) = 77\). So the first term is 77.

28. The first four terms of an arithmetic sequence are 28, 20, 12, and 4. Which term of the sequence is \(-36\)?

**SOLUTION:**
Find the common difference.

\(28 - 20 = 8\)

The sequence is decreasing, so the common difference is negative: \(-8\).

Write the equation for the \(n\)th term of an arithmetic sequence using the first term 28 and common difference \(-8\).

\[
a_n = a_1 + (n - 1)d \\
= 28 + (n - 1)(-8) \\
= 28 - 8n + 8 \\
= 36 - 8n
\]

Then, substitute –36 for \(a_n\) and evaluate for \(n\).

\[-36 = 36 - 8n \\
-36 + 36 = 36 + 36 - 8n \\
0 = 72 - 8n \\
0 + 8n = 72 - 8n + 8n \\
8n = 72 \\
\frac{8n}{8} = \frac{72}{8} \\
n = 9
\]

So, it is the \(9^{\text{th}}\) term of the sequence that is \(-36\).
29. **CARS** Jamal’s odometer of his car reads 24,521. If Jamal drives 45 miles every day, what will the odometer reading be after 25 days?

**SOLUTION:**
To find the 25th term, add 45 to $a_0$ 25 times, or add $(45 \cdot 25)$ to $a_0$.

$24,521 + 1125 = 25,646$

So after 25 days, the odometer reading will be 25,646 miles.

30. **YEARBOOKS** The yearbook staff is unpacking a box of school yearbooks. The arithmetic sequence 281, 270, 259, 248 ... represents the total number of ounces that the box weighs as each yearbook is taken out of the box.

**a.** Write a function to represent this sequence.

**b.** Determine the weight of each yearbook.

**c.** If the box weighs at least 17 ounces empty and 292 ounces when it is full, how many yearbooks were in the box?

**SOLUTION:**
**a.** The arithmetic sequence is 281, 270, 259, 248,... Find the common difference.

$270 - 281 = -11$

Write the equation for the $n$th term of an arithmetic sequence using the first term 281 and common difference $-11$.

$$a_n = a_1 + (n - 1)d$$

$$= 281 + (n - 1) - 11$$

$$= 281 - 11n + 11$$

$$= -11n + 292$$

So, the function for this arithmetic sequence is $f(n) = -11n + 292$.

**b.** The weight decreases by 11 oz every time a book is taken out, so the common difference represents the weight of each yearbook, or 11 oz.

**c.** To determine the number of yearbooks in a full box, subtract the weight of the empty box from the weight of the full box.

$292 - 17 = 275$

Now, divide by the weight of each yearbook.

$275 \div 11 = 25$

So there are 25 books in a full box.

31. **SPORTS** To train for an upcoming marathon, Olivia plans to run 3 miles per day for the first week and then increase the daily distance by a half mile each of the following weeks.

**a.** Write an equation to represent the $n$th term of the sequence.

**b.** If the pattern continues, during which week will she run 10 miles per day?

**c.** Is it reasonable to think that this pattern will continue indefinitely? Explain.

**SOLUTION:**
**a.** Write out the first few terms of the sequence.

$3, 3.5, 4, 4.5, 5, 5.5, 6$

The common difference is 0.5.

$$a_n = a_1 + (n - 1)d$$

Replace $a_1$ with 3 and $d$ with 0.5

$$a_n = 3 + 0.5n - 0.5$$

Distribute Property

$$a_n = 2.5 + 0.5n$$

Simplify.

**b.** Solve for $a_n = 10$. week 15

$$2.5 + 0.5n$$

Replace $a_n$ with 10.

$$10 - 2.5 = 2.5 + 0.5n$$

Subtract 2.5 from each side

$$7.5 = 0.5n$$

Simplify

$$\frac{7.5}{0.5} = n$$

Divide each side by 0.5.

$$15 = n$$

Simplify.

In the 15th week she will run 10 miles per day.

**c.** She cannot continue with the daily increase of 0.5 miles each week. Eventually the number of miles ran per day will become unrealistic.

32. **OPEN ENDED**
Create an arithmetic sequence with a common difference of $-10$.

**SOLUTION:**
Students’ answers may vary. Sample answer:

Starting with a first term of 2, subtract 10 to get the next term. Then subtract 10 from the resulting number to get the next term, and so on.

$2 - 10 = -8$

$-8 - 10 = -18$

$-18 - 10 = -28$

So, an arithmetic sequence with a common difference of $-10$ is $2, -8, -18, -28,...$
3-5 Arithmetic Sequences as Linear Functions

33. **CCSS PERSEVERANCE**
Find the value of \( x \) that makes \( x + 8, 4x + 6, \) and \( 3x \) the first three terms of an arithmetic sequence.

**SOLUTION:**
Find the common difference. The difference between the second and first terms should be equal to the difference between the third and second terms.

\[
(4x + 6) - (x + 8) = 3x - (4x + 6)
\]

\[
4x + 6 - x - 8 = 3x - 4x - 6
\]

\[
3x - 2 = -x - 6
\]

\[
x = -1
\]

Substitute \(-1\) for \( x \).

\[
x + 8 = -1 + 8 = 7
\]

\[
4x + 6 = -4 + 6 = 2
\]

\[
3x = -3
\]

Check to see if the terms have a common difference.

\[
2 - 7 = -5
\]

\[
-3 - 2 = -5
\]

The terms have a common difference of \(-5\), so \( x = -1 \) makes these terms an arithmetic sequence.

34. **REASONING**
Compare and contrast the domain and range of the linear functions described by \( Ax + By = C \) and \( a_n = a_1 + (n - 1)d \).

**SOLUTION:**
Sample answer: the domain of the function described by \( Ax + By = C \) is the set of all real numbers, and the range is either the set of all real numbers or a set of just one number when the graph is a horizontal line. For an arithmetic sequence, the domain is the set of all counting numbers. The range will be an infinite discrete set of real numbers if \( d \neq 0 \). If \( d = 0 \), then the range will be \( \{a_1\} \).

35. **CHALLENGE**
Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain. If yes, find the common difference and the next three terms.

- **a.** \( 2x + 1, 3x + 1, 4x + 1 \ldots \)
- **b.** \( 2x, 4x, 8x, \ldots \)

**SOLUTION:**

- **a.** Find the common difference.

\[
(3x + 1) - (2x + 1) = 3x + 1 - 2x - 1
\]

\[
= x
\]

\[
(4x + 1) - (3x + 1) = 4x + 1 - 3x - 1
\]

\[
= x
\]

The common difference between terms is \( x \). So, to find the next term, add \( x \) to the last term. To find the next term, add \( x \) to the resulting number, and so on.

\[
4x + 1 + x = 5x + 1
\]

\[
5x + 1 + x = 6x + 1
\]

\[
6x + 1 + x = 7x + 1
\]

So, the next three terms are \( 5x + 1, 6x + 1, \) and \( 7x + 1 \).

- **b.** Try to find the common difference.

\[
4x - 2x = 2x
\]

\[
8x - 4x = 4x
\]

The difference between terms is not equal, so there is no common difference and the sequence is not an arithmetic sequence.
36. **WRITING IN MATH** How are graphs of arithmetic sequences and linear functions similar? different?

**SOLUTION:**
They are similar in that the graph of the terms of an arithmetic sequence lies on a line. Therefore, an arithmetic sequence can be represented by a linear function. They are different in that the domain of an arithmetic sequence is the set of natural numbers, while the domain of a linear function is all real numbers. Thus, arithmetic sequences are discrete, while linear functions are continuous.

Consider the graph of the arithmetic sequence \( a_n = 4n - 16 \).

For each \( n \), there is a point on the graph.

The graph of the linear function \( y = 4x - 16 \) is given below.

37. **GRIDDED RESPONSE**
The population of Westerville is about 35,000. Each year the population increases by about 400. This can be represented by the following equation, where \( n \) represents the number of years from now and \( p \) represents the population.

\[ p = 35,000 + 400n \]

In how many years will the Westerville population be about 38,200?

**SOLUTION:**
Substitute the new population for \( p \) in the equation and evaluate for \( n \).

\[
\begin{align*}
p & = 35,000 + 400n \\
38,200 & = 35,000 + 400n \\
38,200 - 35,000 & = 35,000 - 35,000 + 400n \\
3200 & = 400n \\
\frac{3200}{400} & = \frac{400n}{400} \\
8 & = n
\end{align*}
\]

So, the population will be about 38,200 in 8 years.

38. Which relation is a function?

A \( \{(-5, 6), (4, -3), (2, -1), (4, 2)\} \)

B \( \{(3, -1), (3, -5), (3, 4), (3, 6)\} \)

C \( \{(-2, 3), (0, 3), (-2, -1), (-1, 2)\} \)

D \( \{(-5, 6), (4, -3), (2, -1), (0, 2)\} \)

**SOLUTION:**
To be a function, then for each member in the domain, there is only one member of the range. You can also graph the points, and use the vertical line test.

In Choice A, the points (4,3) and (4,2) have the same \( x \)-values but different \( y \)-values. Thus the relation in Choice A is not a function.

In Choice B, all four points have 3 as the \( x \)-values but different \( y \)-values. Thus the relation in Choice B is not a function.

In Choice C, \((-2, 3)\) and \((-2, 1)\) have the same \( x \)-values but different \( y \)-values. Thus the relation in Choice C is not a function.

The relation in choice D is a function because none of the \( x \) values are repeated. So, the correct choice is D.
3-5 Arithmetic Sequences as Linear Functions

39. Find the formula for the \( n \)th term of the arithmetic sequence.
   \(-7, -4, -1, 2, \ldots\)
   \( \text{A} \ a_n = 3n - 4 \)
   \( \text{B} \ a_n = -7n + 10 \)
   \( \text{C} \ a_n = 3n - 10 \)
   \( \text{D} \ a_n = -7n + 4 \)

   **SOLUTION:**
   Find the common difference.
   \(-4 - (-7) = 3\)

   Write the equation for the \( n \)th term of an arithmetic sequence using first term \(-7\) and common difference 3.
   \[ a_n = a_1 + (n - 1)d \]
   \[ = -7 + (n - 1)3 \]
   \[ = -7 + 3n - 3 \]
   \[ = 3n - 10 \]
   So, the correct choice is \( \text{C} \).

40. **STATISTICS** A class received the following scores on the ACT. What is the difference between the median and the mode in the scores?
   18, 26, 20, 30, 25, 21, 32, 19, 22, 29, 29, 27, 24
   \( \text{A} \ 1 \)
   \( \text{B} \ 2 \)
   \( \text{C} \ 3 \)
   \( \text{D} \ 4 \)

   **SOLUTION:**
   Find the median by arranging the scores in sequential order and finding the value in the middle. The median is 25. Find the mode by looking for the value that occurs most frequently in the data. The mode is 29. Subtract to find the difference between the median and the mode. \( 29 - 25 = 4 \). So, the correct choice is \( \text{D} \).

Name the constant of variation for each direct variation. Then find the slope of the line that passes through each pair of points.

41. **SOLUTION:**
   Find the constant of variation using the point \((1, 3)\).
   \[ y = kx \]
   \[ 3 = k(1) \]
   \[ 3 = k \]

   The constant of variation is 3. Find the slope of the line through the points \((0, 0)\) and \((1, 3)\).

   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   \[ = \frac{3 - 0}{1 - 0} \]
   \[ = \frac{3}{1} \]
   \[ = 3 \]

   The slope of the line is 3.
3-5 Arithmetic Sequences as Linear Functions

SOLUTION:
Find the constant of variation using the point (−3, 4).

\[ y = kx \]

\[ 4 = k(-3) \]

\[ \frac{4}{-3} = k(-3) \]

\[ \frac{4}{-3} = k \]

The constant of variation is \(-\frac{4}{3}\).

Find the slope of the line through the points (0, 0) and (−3, 4).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{4 - 0}{-3 - 0} \]

\[ = \frac{4}{-3} \]

\[ = -\frac{4}{3} \]

The slope of the line is \(-\frac{4}{3}\).

Find the slope of the line that passes through each pair of points.

43. (5, 3), (−2, 6)

SOLUTION:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{6 - 3}{-2 - 5} \]

\[ = \frac{3}{7} \]

The slope of the line is \(\frac{3}{7}\).

44. (9, 2), (−3, −1)

SOLUTION:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{2 - (-1)}{9 - (-3)} \]

\[ = \frac{3}{12} \]

\[ = \frac{1}{4} \]

The slope of the line is \(\frac{1}{4}\).

45. (2, 8), (−2, −4)

SOLUTION:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{-4 - 8}{-2 - 2} \]

\[ = \frac{-12}{-4} \]

\[ = 3 \]

The slope of the line is 3.

Solve each equation. Check your solution.

46. \(5x + 7 = -8\)

SOLUTION:

Solve.

\[ 5x + 7 = -8 \]

\[ 5x = -15 \]

\[ x = -3 \]

Check.

\[ 5(-3) + 7 = -8 \]

\[ -15 + 7 = -8 \]

\[ -8 = -8 \]
3-5 Arithmetic Sequences as Linear Functions

47. \(8 = 2 + 3n\)

**SOLUTION:**

Solve.

\[
\begin{align*}
8 &= 2 + 3n \\
8 - 2 &= 2 - 2 + 3n \\
6 &= 3n \\
\frac{6}{3} &= \frac{3n}{3} \\
2 &= n
\end{align*}
\]

Check.

\[
\begin{align*}
8 &= 2 + 3n \\
8 &= 2 + 3(2) \\
8 &= 2 + 6 \\
8 &= 8
\end{align*}
\]

48. \(12 = \frac{c - 6}{2}\)

**SOLUTION:**

Solve.

\[
\begin{align*}
12 &= \frac{c - 6}{2} \\
12 \cdot 2 &= 2 \left(\frac{c - 6}{2}\right) \\
24 &= c - 6 \\
24 + 6 &= c - 6 + 6 \\
30 &= c
\end{align*}
\]

Check.

\[
\begin{align*}
12 &= \frac{c - 6}{2} \\
12 &= \frac{30 - 6}{2} \\
12 &= \frac{24}{2} \\
12 &= 12
\end{align*}
\]

49. **SPORTS** The most popular sports for high school girls are basketball and softball. Write and use an equation to find how many more girls play on basketball teams than on softball teams.

**SOLUTION:**

To write the equation, let \(d\) represent the difference between the number of girls on basketball teams and the number of girls on softball teams.

\[
\begin{align*}
453,000 - d &= 369,000 \\
453,000 - d + d &= 369,000 + d \\
453,000 &= 369,000 + d \\
453,000 - 369,000 &= 369,000 - 369,000 + d \\
84,000 &= d
\end{align*}
\]

So, about 84,000 more girls play on basketball teams than on softball teams.

**Graph each point on the same coordinate plane.**

50. \(A(2, 5)\)

**SOLUTION:**

![Graph with points labeled A(2, 5), B(-2, 1), C(-3, -1), D(0, 4), E(5, -3), and F.]
3-5 Arithmetic Sequences as Linear Functions

51. \( B(-2, 1) \)

**SOLUTION:**

52. \( C(-3, -1) \)

**SOLUTION:**

53. \( D(0, 4) \)

**SOLUTION:**

54. \( F(5, -3) \)

**SOLUTION:**

55. \( G(-5, 0) \)

**SOLUTION:**
1. **GEOMETRY** The table shows the perimeter of a square with sides of a given length.

<table>
<thead>
<tr>
<th>Side Length (in.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter (in.)</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Graph the data.
b. Write an equation to describe the relationship.
c. What conclusion can you make regarding the relationship between the side and the perimeter?

**SOLUTION:**

a. [Graph of the data]

b. The difference in y-values is four times the difference in x-values. This suggests $y = 4x$.
c. The perimeter is 4 times the length of the side.

Write an equation in function notation for each relation.

So the equation for the relation in function notation is $f(x) = x - 5$.

2. **SOLUTION:**

Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

The difference in y-values is equal to the difference in x-values. This suggests $y = x$, with no coefficient before the x.

Compare the differences of the y and x values.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>y−x</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>y−x</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
</tr>
</tbody>
</table>
3. **GEOMETRY** The table shows the perimeter of a square with sides of a given length.

   a. Graph the data.
   b. Write an equation to represent the relation in function notation.

   To find the y-intercept, let $x = 0$.

   So, the x-intercept is 4, and the y-intercept is 2.

4. **CCSS STRUCTURE** The table shows the pages of comic books read.

<table>
<thead>
<tr>
<th>Books Read</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pages Read</td>
<td>35</td>
<td>70</td>
<td>105</td>
<td>140</td>
<td>175</td>
</tr>
</tbody>
</table>

   a. Graph the data.
   b. Write an equation to describe the relationship.
   c. Find the number of pages read if 8 comic books were read.

   **SOLUTION:**

   a. The difference in $y$-values is thirty-five times the difference in $x$-values. This suggests $y = 35x$.

   b. $y = 35x$

   $y = 35(8)$

   $y = 280$

   So, 280 pages were read.

   **SOLUTION:**

   Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

   The difference in $y$-values is equal to the negative difference in $x$-values. This suggests $y = -x$ since the ratio of $y$ to $x$ is -1.

   Compare the differences of the $y$ and $-x$ values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$y - (-x)$</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

   So the equation for the relation in function notation is $f(x) = -x + 3$. 
Write an equation in function notation for each relation.

**SOLUTION:**
Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The difference in y-values is double the difference in x-values. This suggests $y = 2x$ since the ratio of y to x is 2.

Compare the differences of the y and x values.

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$y - 2x$</td>
<td>$-2 - 2(-1)$</td>
<td>$0 - 2(0)$</td>
<td>$2 - 2(1)$</td>
<td>$4 - 2(2)$</td>
</tr>
<tr>
<td>$y - 2x$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the difference is 0, there is no constant term. So the equation for the relation in function notation is $f(x) = 2x$.

SOLUTION:
Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

The difference in y-values is twelve times the difference in x-values. This suggests $y = 12x$ since the ratio of y to x is 12.

Find the differences of the y and x values.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>$y - 12x$</td>
<td>$0 - 12(0)$</td>
<td>$12 - 12(1)$</td>
<td>$24 - 12(2)$</td>
</tr>
<tr>
<td>$y - 12x$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the differences are 0, there is no constant term. So the equation for the relation in function notation is $f(x) = 12x$. 

---

**3-6 Proportional and Nonproportional Relationships**

- **SOLUTION:**
  - Let $x = 0$ to find the y-intercept.
  - The x-intercept is 4.
  - The y-intercept is 2.

- **SOLUTION**
  - Solve each equation for $b$.
  - The radius $r$ of the base can be found using the formula for the volume of a cone.

- **SOLUTION**
  - Find the total snowfall each hour of an hour-long snowstorm

- **CHALLENGE**
  - Consider the arithmetic sequence: 4, 7, 10, 13. Find the equation for the relation in function notation.
  - The next term in the sequence is 16.

- **SOLUTION**
  - The difference in the y-values is equal to 4.5 and the common difference is 12.

- **SOLUTION**
  - The ratio of change is 2, since the ratio of change in the y-values is equal to 4.5 and the common difference is 12.

- **SOLUTION**
  - The difference in the y-values is double the difference in x-values. This suggests $y = 2x$ since the ratio of y to x is 2.

- **SOLUTION**
  - The difference in the y-values is twelve times the difference in x-values. This suggests $y = 12x$ since the ratio of y to x is 12.

- **SOLUTION**
  - The difference in the y-values is 4.

- **SOLUTION**
  - The difference in the y-values is 2.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.

- **SOLUTION**
  - The difference in the y-values is 0.
3-6 Proportional and Nonproportional Relationships

**SOLUTION:**
Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$y - 3x$</td>
<td>-2 - 3(0)</td>
<td>1 - 3(1)</td>
<td>4 - 3(2)</td>
</tr>
</tbody>
</table>

The difference in $y$-values is three times the difference in $x$-values. This suggests $y = 3x$ since the ratio of $y$ to $x$ is 3.

Find the differences of the $y$ and $x$ values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>0</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$y - \frac{1}{4}x$</td>
<td>1 - $\frac{1}{4}(-4)$</td>
<td>2 - $\frac{1}{4}(0)$</td>
<td>3 - $\frac{1}{4}(4)$</td>
</tr>
</tbody>
</table>

The difference is 2. So the equation for the relation in function notation is $f(x) = 3x - 2$. 

The difference in $y$-values is one-fourth the difference in $x$-values. This suggests $y = \frac{1}{4}x$ since the ratio of $y$ to $x$ is $\frac{1}{4}$.

Find the difference in $y$- and $x$-values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>0</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$y - \frac{1}{4}x$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The difference is 2. So the equation for the relation in function notation is $f(x) = \frac{1}{4}x + 2$.
For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. Explain.

9. 0, 3, 6, ...

**SOLUTION:**
The nth term of an arithmetic sequence with first term \(a_1\) and common difference \(d\) is given by \(f_n = a_1 + (n - 1)d\), where \(n\) is a positive integer.

\[
\begin{align*}
f_n &= a_1 + (n - 1)d \\
0 &= 3(0) - 3 \\
0 &= 0 - 3 \\
0 &= -3
\end{align*}
\]

So, the line does not contain \((0, 0)\).

10. -4, 0, 4, ...

**SOLUTION:**
The nth term of an arithmetic sequence with first term \(a_1\) and common difference \(d\) is given by \(a_n = a_1 + (n - 1)d\), where \(n\) is a positive integer.

\[
\begin{align*}
a_2 &= a_1 + (n - 1)d \\
a_2 &= -4 + (n - 1)(4) \\
a_n &= -4 + 4n - 4 \\
a_n &= 4n - 8
\end{align*}
\]

This function is nonproportional.

Check to see if \((0, 0)\) is on the graph.

\[
4(0) - 8 = -8
\]

So, the line does not contain \((0, 0)\).

11. BOWLING Marielle is bowling with her friends.
The table shows prices for renting a pair of shoes and bowling. Write an equation to represent the total price \(y\) if Marielle buys \(x\) games.

**SOLUTION:**
Find the difference in the \(x\)- and \(y\)-values.

<table>
<thead>
<tr>
<th>Games Bowled</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Price ($)</td>
<td>7.00</td>
<td>11.50</td>
<td>16.00</td>
<td>20.50</td>
</tr>
</tbody>
</table>

The difference in the \(y\)-values is equal to 4.5 and the difference in the \(x\)-values is 2. This suggest that \(y = \frac{4.5}{2}x\), since the ratio of \(y\) and \(x\) difference is \(\frac{4.5}{2} = 2.25\).

If \(x = 2\), \(y = 2.25(1)\) or 4.5. But the \(y\)-value of \(x = 2\) is 7.

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>7.00</td>
<td>11.50</td>
<td>16.00</td>
<td>20.50</td>
</tr>
</tbody>
</table>

The pattern shows that 2.5 should be added to one side of the equation. Thus, the equation is \(y = 2.25x + 2.50\).
12. SNOWFALL The total snowfall each hour of a winter snowstorm is shown in the table below.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Inches of Snowfall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.65</td>
</tr>
<tr>
<td>2</td>
<td>3.30</td>
</tr>
<tr>
<td>3</td>
<td>4.95</td>
</tr>
<tr>
<td>4</td>
<td>6.60</td>
</tr>
</tbody>
</table>

a. Write an equation to fit the data in the table.
b. Describe the relationship between the hour and inches of snowfall.

**SOLUTION:**
a. \(a_n = a_1 + (n - 1)d\)
   \[a_n = 1.65 + (n - 1)1.65\]
   \[a_n = 1.65 + 1.65n - 1.65\]
   \[a_n = 1.65n\]
b. As the number of hours increases from hour to hour, the inches of snowfall also increases at a constant rate. This means that the relation is proportional.

13. FUNDRAISER The Cougar Pep Squad wants to sell T-shirts in the bookstore for the spring dance. The cost in dollars to order T-shirts in their school colors is represented by the equation \(C = 2t + 3\).

a. Make a table of values that represents this relationship.
b. Rewrite the equation in function notation.
c. Graph the function.
d. Describe the relationship between the number of T-shirts and the cost.

**SOLUTION:**
a. Students’ answers may vary.

<table>
<thead>
<tr>
<th>Number of T-Shirts Ordered</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($5)</td>
<td>13</td>
<td>23</td>
<td>33</td>
<td>43</td>
<td>53</td>
</tr>
</tbody>
</table>

b. \(C(t) = 2t + 3\)
c. To graph the function, use the table of values from part a.
d. This relation is nonproportional. \(C(0) = 2(0) + 3 = 3\).
14. **CCSS CRITIQUE** Quentin thinks that \( f(x) \) and \( g(x) \) are both proportional. Claudia thinks they are not proportional. Is either of them correct? Explain your reasoning.

**SOLUTION:**
A proportional equation is of the form \( f(x) = kx \). Proportional equation, pass through the origin. The graph represents \( f(x) = 3x \) is proportional. However, the table represents \( g(x) = 3x - 1 \). Since \( g(x) \) does not pass through the origin, it is not proportional. Therefore there is one proportional equation and one nonproportional equation. Thus, neither Quentin or Claudia is correct.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-7</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

15. **OPEN ENDED** Create an arithmetic sequence in which the first term is 4. Explain the pattern that you used. Write an equation that represents your sequence.

**SOLUTION:**
Consider the arithmetic sequence: 4, 7, 10, 13.

To find the common difference, subtract two consecutive numbers in the sequence.

\[ 7 - 4 = 3 \]
\[ 10 - 7 = 3 \]
\[ 13 - 10 = 3 \]

The difference between terms is constant, so the sequence is an arithmetic sequence and the common difference is 3.

Compare \( 3x \) to the series values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 3x )</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Actual</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Difference</td>
<td>4-3</td>
<td>7-6</td>
<td>10-9</td>
<td>13-12</td>
</tr>
<tr>
<td>Difference</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The difference is on. Thus \( a_n = 3n + 1 \).

16. **CHALLENGE** Describe how inductive reasoning can be used to write an equation from a pattern.

**SOLUTION:**
Once you recognize a pattern, you can find a general rule that can be written as an algebraic expression.

For example, the sequence 12, 19, 26, 33, ... follows a pattern of adding 7 to each term. Therefore, our common difference is 7. The first term is 12, and \( 5 + 7 = 12 \), so \( a_0 = 5 \). The algebraic expression is \( a_n = 5 + 7n \) where \( n \) is the term number.
17. **REASONING** A **counterexample** is a specific case that shows that a statement is false. Provide a counterexample to the following statement. *The related function of an arithmetic sequence is always proportional.* Explain why the counterexample is true.

**SOLUTION:**

![Graph of y = 3x + 2](image)

\[ f(n) = 3n + 2 \] is the related function for the arithmetic sequence 5, 8, 11, 14, ..., but it is not proportional. The line through (1, 5) and (2, 8) does not pass through (0, 0). Any linear equation that has a constant term is not proportional. However \( f(n) = 3n \) is proportion.

---

18. **WRITING IN MATH** Compare and contrast proportional relationships with nonproportional relationships.

**SOLUTION:**

In a proportional relationship, the ratio of \( \frac{y}{x} \) is the same for each ordered pair in the line for which \( x \neq 0 \). However, this is not the case in a nonproportional relationship. Both can be represented by a linear equation.

This is an example of a proportional relation. The equation \( y = 3x \) is represented in the table. The difference in the \( x \)-values is 1, the difference in the \( y \)-values, so the ratio of the \( y \) to \( x \) is 3.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

Consider the difference of \( y \) and \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>( y - 3x )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the difference is 0, it is a proportion.

This is an example of a nonproportional relation. The equation \( y = 3x + 1 \) is represented in the table. The difference in the \( x \)-values is 1, the difference in the \( y \)-values, so the ratio of the \( y \) to \( x \) is 3.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( y - 3x )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Since the difference is 0, it is a nonproportional.
19. What is the slope of a line that contains the point (1, −5) and has the same \( y \)-intercept as \( 2x - y = 9 \)?

A  −9
B  −7
C  2
D  4

**SOLUTION:**
First, find the \( y \)-intercept of \( 2x - y = 9 \).

\[ 2x - y = 9 \]  \hspace{1cm} \text{Original equation}

\[ 2(0) - y = 9 \]  \hspace{1cm} \text{Replace } x \text{ with 0.}

\[ -y = 9 \]  \hspace{1cm} \text{Simplify.}

\[ \frac{-y}{-1} = \frac{-9}{-1} \]  \hspace{1cm} \text{Divide each side by } -1.

\[ y = -9 \]  \hspace{1cm} \text{Simplify.}

If the equation has the same intercept, then it has the point \((0, -9)\). Use points \((0, -9)\) and \((1, -5)\) to calculate the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{-5 - (-9)}{1 - 0} \]

\[ = \frac{-5 + 9}{1} \]

\[ = 4 \]

The slope is 4, so the correct choice is D.

20. **SHORT RESPONSE** \( \triangle FGR \) is an isosceles triangle. What is the measure of \( \angle G \)?

**SOLUTION:**
If \( \triangle FGR \) is an isosceles triangle, then \( \angle G \) is equal to \( \angle R \). The three angles in a triangle add up to 180°.

\[ 98 + x + x = 180 \]
\[ 98 + 2x = 180 \]
\[ 98 - 98 + 2x = 180 - 98 \]
\[ 2x = 82 \]
\[ 2x = \frac{82}{2} \]
\[ x = 41 \]

So \( \angle G = 41° \).

21. Luis deposits $25 each week into a savings account from his part–time job. If he has $350 in savings now, how much will he have in 12 weeks?

F $600
G $625
H $650
J $675

**SOLUTION:**
Let \( w \) represent the number of weeks Luis deposits money.

\[ 25w + 350 = 25(12) + 350 \]
\[ = 300 + 350 \]
\[ = 650 \]

Luis will have $650 after 12 weeks, so the correct choice is H.
3-6 Proportional and Nonproportional Relationships

22. **GEOMETRY** Omar and Mackenzie want to build a pulley system by attaching one end of a rope to their 8-foot–tall tree house and anchoring the other end to the ground 28 feet away from the base of the tree house. How long, to the nearest foot, does the piece of rope need to be?

A 26 ft
B 27 ft
C 28 ft
D 29 ft

**SOLUTION:**
This creates a right triangle, so use the Pythagorean Theorem.

\[ a^2 + b^2 = c^2 \]

\[ 8^2 + 28^2 = c^2 \]

\[ 64 + 784 = c^2 \]

\[ 848 = c^2 \]

\[ \sqrt{848} = \sqrt{c^2} \]

\[ 29.12 = c \]

The piece of rope needs to be about 29 feet long, so the correct choice is D.

**Find the next three terms in each sequence.**
23. 3, 13, 23, 33, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[ 13 - 3 = 10 \]

The common difference is 10. Add 10 to the last term of the sequence until three terms are found; the next three terms are 43, 53, and 63.

24. –2, –1.4, –0.8, –0.2, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[ -1.4 - (-2) = 0.6 \]

The common difference is 0.6. Adding 0.6 to the last term of the sequence until three terms are found; the next three terms are 0.4, 1, and 1.6.

25. \[ \frac{3}{4}, \frac{7}{8}, 1, \frac{9}{8} \ldots \]

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[ 1 - \frac{7}{8} = \frac{1}{8} \]

The common difference is \( \frac{1}{8} \). Add \( \frac{1}{8} \) to the last term of the sequence until three terms are found; the next three terms are \( \frac{5}{8}, \frac{11}{8}, \) and \( \frac{3}{2} \).

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

26. If \( y = 45 \) when \( x = 9 \), find \( y \) when \( x = 7 \).

**SOLUTION:**
\[ y = kx \]
\[ 45 = k(9) \]
\[ 45 = \frac{k(9)}{9} \]
\[ 5 = k \]

So, the direct variation equation is \( y = 5x \). Substitute 7 for \( x \) and find \( y \).

\[ y = 5x \]
\[ y = 5(7) \]
\[ y = 35 \]

So, \( y = 35 \) when \( x = 7 \).
27. If \( y = -7 \) when \( x = -1 \), find \( x \) when \( y = -84 \).

**SOLUTION:**

\[
y = kx \\
-7 = k(-1) \\
-7 = k \cdot (-1) \\
-1 = -1 \\
7 = k
\]

So, the direct variation equation is \( y = 7x \). Substitute \(-84\) for \( y \) and find \( x \).

\[
y = 7x \\
-84 = 7x \\
-84 = 7x \\
7 = 7 \\
-12 = x
\]

So, \( x = -12 \) when \( y = -84 \).

28. **GENETICS** About \( \frac{2}{25} \) of the male population in the world cannot distinguish red from green. If there are 14 boys in the ninth grade who cannot distinguish red from green, about how many ninth-grade boys are there in all? Write and solve an equation to find the answer.

**SOLUTION:**

Let \( b \) represent the number of boys.

\[
\frac{14}{25} = \frac{2}{25} \cdot b \\
14 = \frac{2}{25} \cdot b \\
\frac{25 \cdot 14}{2} = \frac{25 \cdot 2}{25} \cdot b \\
175 = b
\]

So, there are 175 boys.

29. **GEOMETRY** The volume \( V \) of a cone equals one-third times the product of \( \pi \), the square of the radius \( r \) of the base, and the height \( h \).

a. Write the formula for the volume of a cone.

b. Find the volume of a cone if \( r \) is 10 centimeters and \( h \) is 30 centimeters.

**SOLUTION:**

\[
V = \frac{1}{3} \pi r^2 h
\]

b. \( V = \frac{1}{3} \pi r^2 h \\
= \frac{1}{3} \pi (10)^2 (30) \\
= \frac{1}{3} \pi (100)(30) \\
= \frac{1}{3} \pi (3000) \\
= 1000\pi \\
\approx 3141.59265
\]

So the volume is about 3142 cm\(^3\).

Solve each equation for \( y \).

30. \( 3x = y + 7 \)

**SOLUTION:**

\[
\begin{align*}
5x &= y + 7 & \text{Original equation} \\
3x - 7 &= y + 7 - 7 & \text{Subtract 7 from each side} \\
3x - 7 &= y & \text{Simplify.}
\end{align*}
\]
31. \(2y = 6x - 10\)

**SOLUTION:**

\[
\begin{align*}
2y &= 6x - 10 & \text{Original equation} \\
\frac{2y}{2} &= \frac{6x - 10}{2} & \text{Divide each side by 2} \\
y &= \frac{6x - 10}{2} & \text{Rewrite fractions} \\
y &= 3x - 5 & \text{Simplify.}
\end{align*}
\]

32. \(9y + 2x = 12\)

**SOLUTION:**

\[
\begin{align*}
9y + 2x &= 12 & \text{Original equation} \\
9y + 2x - 2x &= 12 - 2x & \text{Subtract } x \text{ from each side} \\
9y &= -2x + 12 & \text{Simplify.} \\
\frac{9y}{9} &= \frac{-2x + 12}{9} & \text{Divide each side by 9} \\
y &= -\frac{2}{9}x + \frac{12}{9} & \text{Simplify.} \\
y &= -\frac{2}{9}x + \frac{4}{3} & \text{Simplify.}
\end{align*}
\]

33. \(y = x - 8\)

**SOLUTION:**

To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.

To find the \(x\)-intercept, let \(y = 0\).

\[
\begin{align*}
y &= x - 8 \\
0 &= x - 8 \\
0 + 8 &= x - 8 + 8 \\
8 &= x
\end{align*}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
\begin{align*}
y &= x - 8 \\
y &= 0 - 8 \\
y &= -8
\end{align*}
\]

So, the \(x\)-intercept is 8 and the \(y\)-intercept is -8.
3-6 Proportional and Nonproportional Relationships

34. \( x - y = -4 \)

**SOLUTION:**

To graph the equation, find the \( x \)- and \( y \)-intercepts. Plot these two points. Then draw a line through them.

To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
  x - y &= -4 \\
  x - 0 &= -4 \\
  x &= -4 \\

  \text{To find the } y \text{-intercept, let } x &= 0. \\
  x - y &= -4 \\
  0 - y &= -4 \\
  -y &= -4 \\
  -1 \\
  y &= 4
\end{align*}
\]

So, the \( x \)-intercept is \(-4\), and the \( y \)-intercept is \( 4 \).

![Graph of the line](image)

35. \( 2x + 4y = 8 \)

**SOLUTION:**

To graph the equation, find the \( x \)- and \( y \)-intercepts. Plot these two points. Then draw a line through them.

To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
  2x + 4y &= 8 \\
  2x + 4(0) &= 8 \\
  2x &= 8 \\
  2x &= 8 \\
  2 &= 2 \\
  x &= 4
\end{align*}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
\begin{align*}
  2x + 4y &= 8 \\
  2(0) + 4y &= 8 \\
  0 + 4y &= 8 \\
  4y &= 8 \\
  \frac{4y}{4} &= \frac{8}{4} \\
  y &= 2
\end{align*}
\]

So, the \( x \)-intercept is \( 4 \), and the \( y \)-intercept is \( 2 \).
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.

1. \( y = -4x + 3 \)

**SOLUTION:**
Rewrite the equation in standard form.

\[
y = -4x + 3
\]

\[
y + 4x = -4x + 4x + 3
\]

\[
4x + y = 3
\]

The equation is now in standard form where \( A = 4 \), \( B = 1 \), and \( C = 3 \). The equation is linear.

2. \( x^2 + 3y = 8 \)

**SOLUTION:**
Because \( x \) is squared, the equation cannot be written in standard form. The equation is not linear.

3. \( \frac{1}{4}x - \frac{3}{4}y = -1 \)

**SOLUTION:**
Rewrite the equation in standard form.

\[
\frac{1}{4}x - \frac{3}{4}y = -1
\]

\[
4 \left( \frac{1}{4}x - \frac{3}{4}y \right) = 4(-1)
\]

\[
x - 3y = -4
\]

The equation is now in standard form where \( A = 1 \), \( B = -3 \), and \( C = -4 \). The equation is linear.

Graph each equation using the \( x \)- and \( y \)-intercepts.

4. \( y = 3x - 6 \)

**SOLUTION:**
To graph the equation, find the \( x \)- and \( y \)-intercepts. Plot these two points. Then draw a line through them.

To find the \( x \)-intercept, let \( y = 0 \).

\[
y = 3x - 6 \quad \text{Original equation}
\]

\[
0 = 3x - 6 \quad \text{Replace } y \text{ with 0.}
\]

\[
0 + 6 = 3x - 6 + 6 \quad \text{Add 6 to each side.}
\]

\[
6 = 3x \quad \text{Simplify.}
\]

\[
\frac{6}{3} = \frac{3x}{3} \quad \text{Divide each side by 3}
\]

\[
2 = x \quad \text{Simplify.}
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
y = 3x - 6 \quad \text{Original equation}
\]

\[
y = 3(0) - 6 \quad \text{Replace } y \text{ with 0.}
\]

\[
y = -6 \quad \text{Simplify.}
\]

So, the \( x \)-intercept is 2 and the \( y \)-intercept is \(-6\).
Mid-Chapter Quiz

5. \(2x + 5y = 10\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).
\[
2x + 5y = 10 \quad \text{Original equation}
\]
\[
2x + 5(0) = 10 \quad \text{Replace } y \text{ with } 0.
\]
\[
x = 5 \quad \text{Simplify.}
\]
To find the \(y\)-intercept, let \(x = 0\).
\[
2x + 5y = 10 \quad \text{Original equation}
\]
\[
2(0) + 5y = 10 \quad \text{Replace } x \text{ with } 0.
\]
\[
y = 2 \quad \text{Simplify.}
\]
So, the \(x\)-intercept is 5 and the \(y\)-intercept is 2.

6. \(y = -2x\)

**SOLUTION:**
To make the table of values, start at \(x = 0\) and evaluate \(y\). Continue this process for other values of \(x\). Plot the points and draw a line through them.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
<td>(-2, 4)</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>(-1, 2)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>(1, -2)</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
<td>(2, -4)</td>
</tr>
</tbody>
</table>

7. \(x = 8 - y\)

**SOLUTION:**
To make the table of values, start at \(x = 0\) and evaluate \(y\). Continue this process for other values of \(x\). Plot the points and draw a line through them.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>10</td>
<td>(-2, 10)</td>
</tr>
<tr>
<td>-1</td>
<td>9</td>
<td>(-1, 9)</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>(0, 8)</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>(1, 7)</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>(2, 6)</td>
</tr>
</tbody>
</table>
Find the root of each equation.

9. \( x + 8 = 0 \)

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
x + 8 = 0 \quad \text{Original equation}
\]
\[
x + 8 - 8 = 0 - 8 \quad \text{Subtract 8 from each side}
\]
\[
x = -8 \quad \text{Simplify}
\]

So, the root is \(-8\).

10. \( 4x - 24 = 0 \)

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
4x - 24 = 0 \quad \text{Original equation}
\]
\[
4x - 24 + 24 = 24 \quad \text{Add 24 to each side}
\]
\[
4x = 24 \quad \text{Simplify}
\]
\[
\frac{4x}{4} = \frac{24}{4} \quad \text{Divide each side by 4}
\]
\[
x = 6 \quad \text{Simplify}
\]

So, the root is \(6\).

11. \( 18 + 8x = 0 \)

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
18 + 8x = 0 \quad \text{Original equation}
\]
\[
18 + 8 - 18 = 0 - 18 \quad \text{Subtract 18 from each side}
\]
\[
8x = -18 \quad \text{Simplify}
\]
\[
\frac{8x}{8} = \frac{-18}{8} \quad \text{Divide each side by 8}
\]
\[
x = -\frac{9}{4} \quad \text{Simplify}
\]

So, the root is \(-\frac{9}{4}\).
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form. This means that there was an average increase of about 630 people per year in Heckertsville between 2003 and 2009.

12. \( \frac{3}{5}x - \frac{1}{2} = 0 \)

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
\begin{align*}
\frac{3}{5}x - \frac{1}{2} &= 0 & \text{Original equation} \\
\frac{3}{5}x - \frac{1}{2} + \frac{1}{2} &= 0 + \frac{1}{2} & \text{Add } \frac{1}{2} \text{ to each side} \\
\frac{3}{5}x &= \frac{1}{2} & \text{Simplify} \\
\frac{5}{3}\left(\frac{3}{5}x\right) &= \frac{5}{3}\left(\frac{1}{2}\right) & \text{Multiply each side by } \frac{5}{3} \\
x &= \frac{5}{6} & \text{Simplify}
\end{align*}
\]

So, the root is \( \frac{5}{6} \).

**Solve each equation by graphing.**

13. \(-5x + 35 = 0\)

**SOLUTION:**
The related function is \( y = -5x + 35 \).

![Graph of \( y = -5x + 35 \)](image)

The graph intersects the \( x \)-axis at 7. So, the solution is 7.

14. \( 14x - 84 = 0 \)

**SOLUTION:**
The related function is \( y = 14x - 84 \).

![Graph of \( y = 14x - 84 \)](image)

The graph intersects the \( x \)-axis at 6. So, the solution is 6.

15. \( 118 + 11x = -3 \)

**SOLUTION:**
The related function is \( y = 11x + 121 \).

![Graph of \( y = 11x + 121 \)](image)

The graph intersects the \( x \)-axis at -11. So, the solution is -11.
16. MULTIPLE CHOICE
The function \( y = -15 + 3x \) represents the outside temperature, in degrees Fahrenheit, in a small Alaskan town where \( x \) represents the number of hours after midnight. The function is accurate for \( x \) values representing midnight through 4:00 p.m. Find the zero of this function.
A 0
B 3
C 5
D -15

SOLUTION:
The zero is located at the \( x \)-intercept of the function.
To find the \( x \)-intercept, let \( y = 0 \).

\[
\begin{align*}
y &= -15 + 3x & \text{Original equation} \\
0 &= -15 + 3x & \text{Replace } y \text{ with } 0. \\
0 + 15 &= -15 + 15 + 3x & \text{Add 15 to each side.} \\
15 &= 3x & \text{Simplify.} \\
\frac{15}{3} &= \frac{3x}{3} & \text{Divide each side by } 3 \\
5 &= x & \text{Simplify.}
\end{align*}
\]

The zero is 5. The correct choice is C.

17. Find the rate of change represented in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

SOLUTION:
To find the rate of change, use the coordinates (1, 2) and (4, 6).

\[
\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{6 - 2}{4 - 1} = \frac{4}{3}
\]

So, the rate of change is \( \frac{4}{3} \).
23. Find the slope of the line that passes through the pair of points.

\[
\begin{array}{cc}
\text{x} & \text{y} \\
2.6 & -2 \\
3.1 & 4
\end{array}
\]

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
m = \frac{4 - (-2)}{3.1 - 2.6} = \frac{6}{0.5} = 12
\]

So, the slope is 12.
24. POPULATION GROWTH

The graph shows the population growth in Heckertsville since 2003.

![Population Growth Graph]

a. For which time period is the rate of change the greatest?
b. Explain the meaning of the slope from 2003 to 2009.

**SOLUTION:**
a. Looking at the graph, the slope is the steepest from 2006–2007, so the rate of change for that period is the greatest.

b. The population changed from about 16,000 to about 19,700 from 2003 to 2009. Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{19,780 - 16,000}{2006 - 2000} \]
\[ = \frac{3780}{6} \]
\[ = 630 \]

So, the average rate of change in population is about 630. This means that there was an average increase of about 630 people per year in Heckertsville between 2003 and 2009.
1. **TEMPERATURE** The equation to convert Celsius temperature $C$ to Kelvin temperature $K$ is shown.

![Graph of the equation $K = C + 273$.](image)

**Graph each equation.**

2. $y = x + 2$

**SOLUTION:**
To graph the equation, find the $x$- and $y$-intercepts. Plot these two points. Then draw a line through them.
To find the $x$-intercept, let $y = 0$.

\[
\begin{align*}
y &= x + 2 & \text{Original equation} \\
0 &= x + 2 & \text{Replace } y \text{ with } 0. \\
0 - 2 &= x + 2 - 2 & \text{Subtract 2 from each side} \\
-2 &= x & \text{Simplify.}
\end{align*}
\]

To find the $y$-intercept, let $x = 0$.

\[
\begin{align*}
y &= x + 2 & \text{Original equation} \\
y &= 0 + 2 & \text{Replace } x \text{ with } 0. \\
y &= 2 & \text{Simplify.}
\end{align*}
\]

So, the $x$-intercept is $-2$ and the $y$-intercept is 2.

3. $y = 4x$

**SOLUTION:**
The slope of $y = 4x$ is 4. Graph $(0, 0)$. From there, move up 4 units and right 1 unit to find another point. Then draw a line containing the points.

---

a. State the independent and dependent variables. Explain.
b. Determine the $C$- and $K$-intercepts and describe what the intercepts mean in this situation.

**SOLUTION:**
a. The Kelvin temperature is dependent on the Celsius temperature. So, the Celsius temperature is the independent variable and the Kelvin temperature is the dependent variable.

b. The $C$-intercept is $(-273, 0)$ and it means that a Celsius temperature of $-273$ degrees is equal to a Kelvin temperature of 0 degrees. The $K$-intercept is $(0, 273)$ and it means that a Celsius temperature of 0 degrees is equal to a Kelvin temperature of 273 degrees.
4. \( x + 2y = -1 \)

**SOLUTION:**
To graph the equation, find the \( x \)- and \( y \)-intercepts.
Plot these two points. Then draw a line through them.
To find the \( x \)-intercept, let \( y = 0 \).

\[
x + 2y = -1 \quad \text{Original equation}
\]

\[
x + 2(0) = -1 \quad \text{Replace} \ y \text{ with} \ 0.
\]

\[
x + 0 = -1 \quad \text{Simplify}.
\]

\[
x = -1 \quad \text{Simplify}.
\]

To find the \( y \)-intercept, let \( x = 0 \).

\[
x + 2y = -1 \quad \text{Original equation}
\]

\[
0 + 2y = -1 \quad \text{Replace} \ x \text{ with} \ 0.
\]

\[
2y = -1 \quad \text{Simplify}.
\]

\[
\frac{2y}{2} = \frac{-1}{2} \quad \text{Divide each side by} \ 2
\]

\[
y = -\frac{1}{2} \quad \text{Simplify}.
\]

So, the \( x \)-intercept is \(-1\) and the \( y \)-intercept is \(-\frac{1}{2}\).

![Graph of the equation](image)

5. \( -3x = 5 - y \)

**SOLUTION:**
To graph the equation, find the \( x \)- and \( y \)-intercepts.
Plot these two points. Then draw a line through them.
To find the \( x \)-intercept, let \( y = 0 \).

\[
-3x = 5 - y \quad \text{Original equation}
\]

\[
-3(0) = 5 - y \quad \text{Replace} \ x \text{ with} \ 0.
\]

\[
0 = 5 - y \quad \text{Simplify}.
\]

\[
0 - 5 = 5 - 5 - y \quad \text{Subtract} \ 5 \text{ from each side}
\]

\[
-5 = -y \quad \text{Simplify}.
\]

\[
\frac{-5}{-1} = \frac{-y}{-1} \quad \text{Divide each side by} \ -1.
\]

\[
y = 5 \quad \text{Simplify}
\]

So, the \( x \)-intercept is \(-1\frac{2}{3}\) and the \( y \)-intercept is \(5\).

![Graph of the equation](image)
Solve each equation by graphing.

6. $4x + 2 = 0$

**SOLUTION:**
The related function is $y = 4x + 2$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 4x + 2$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$f(-4) = 4(-4) + 2$</td>
<td>$-14$</td>
<td>$(-4, -14)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$f(-2) = 4(-2) + 2$</td>
<td>$-6$</td>
<td>$(-2, -6)$</td>
</tr>
<tr>
<td>$-0.5$</td>
<td>$f(-0.5) = 4(-0.5) + 2$</td>
<td>$0$</td>
<td>$(-0.5, 0)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = 4(0) + 2$</td>
<td>$2$</td>
<td>$(0, 2)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$f(2) = 4(2) + 2$</td>
<td>$10$</td>
<td>$(2, 10)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$f(4) = 4(4) + 2$</td>
<td>$18$</td>
<td>$(4, 18)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-\frac{1}{2}$. So, the solution is $x = -\frac{1}{2}$.

7. $0 = 6 - 3x$

**SOLUTION:**
The related function is $y = -3x + 6$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -3x + 6$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$f(-4) = -3(-4) + 6$</td>
<td>$18$</td>
<td>$(-4, 18)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$f(-2) = -3(-2) + 6$</td>
<td>$12$</td>
<td>$(-2, 12)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = -3(0) + 6$</td>
<td>$6$</td>
<td>$(0, 6)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$f(2) = -3(2) + 6$</td>
<td>$0$</td>
<td>$(2, 0)$</td>
</tr>
<tr>
<td>$3$</td>
<td>$f(3) = -3(3) + 6$</td>
<td>$-3$</td>
<td>$(3, -3)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$f(4) = -3(4) + 6$</td>
<td>$-6$</td>
<td>$(4, -6)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at 2. So, the solution is $x = 2$. 
8. $5x + 2 = -3$

**SOLUTION:**
The related function is $y = 5x + 5$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 5x + 5$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$f(-4) = 5(-4) + 5$</td>
<td>$-15$</td>
<td>$(-4, -15)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$f(-2) = 5(-2) + 5$</td>
<td>$-5$</td>
<td>$(-2, -5)$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$f(-1) = 5(-1) + 5$</td>
<td>$0$</td>
<td>$(-1, 0)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = 5(0) + 5$</td>
<td>$5$</td>
<td>$(0, 5)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$f(2) = 5(2) + 5$</td>
<td>$15$</td>
<td>$(2, 15)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$f(4) = 5(4) + 5$</td>
<td>$25$</td>
<td>$(4, 25)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-1$. So, the solution is $-1$.

9. $12x = 4x + 16$

**SOLUTION:**
The related function is $y = -8x + 16$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -8x + 16$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td>$f(-4) = -8(-4) + 16$</td>
<td>$48$</td>
<td>$(-4, 48)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$f(-2) = -8(-2) + 16$</td>
<td>$32$</td>
<td>$(-2, 32)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = -8(0) + 16$</td>
<td>$16$</td>
<td>$(0, 16)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$f(2) = -8(2) + 16$</td>
<td>$0$</td>
<td>$(2, 0)$</td>
</tr>
<tr>
<td>$3$</td>
<td>$f(3) = -8(3) + 16$</td>
<td>$-8$</td>
<td>$(3, -8)$</td>
</tr>
<tr>
<td>$4$</td>
<td>$f(4) = -8(4) + 16$</td>
<td>$-16$</td>
<td>$(4, -16)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $2$. So, the solution is $2$.

Find the slope of the line that passes through each pair of points.

10. $(5, 8), (-3, 7)$

**SOLUTION:**

To find the slope, use the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - 8}{-3 - 5}$$
$$m = \frac{-1}{-8}$$
$$m = \frac{1}{8}$$

So, the slope is $\frac{1}{8}$.
11. (5, -2), (3, -2)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{-2 - (-2)}{3 - 5}
\]

\[
= \frac{0}{-2}
\]

\[
= 0
\]

So, the slope is 0.

12. (4, 7), (8, -1)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{-1 - 7}{8 - (-4)}
\]

\[
= \frac{-8}{12}
\]

\[
= -\frac{2}{3}
\]

So, the slope is \(-\frac{2}{3}\).

13. (6, -3), (6, 4)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{4 - (-3)}{6 - 6}
\]

\[
= \frac{7}{0}
\]

So, the slope is undefined.

14. **MULTIPLE CHOICE** Which is the slope of the linear function shown in the graph?

\[
\text{y} = mx + b
\]

\[
A - \frac{5}{2}
\]

\[
B - \frac{2}{5}
\]

\[
C \frac{5}{2}
\]

\[
D \frac{2}{5}
\]

**SOLUTION:**

The line passes through the points (-2, 3) and (3, 1).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{1 - 3}{3 - (-2)}
\]

\[
= \frac{-2}{5}
\]

The slope is \(-\frac{2}{5}\), so the correct choice is B.
Suppose $y$ varies directly as $x$. Write a direct variation equation that relates $x$ and $y$. Then solve.

15. If $y = 6$ when $x = 9$, find $x$ when $y = 12$.

**SOLUTION:**

\[
y = kx \\
6 = k(9) \\
\frac{6}{9} = \frac{k(9)}{9} \\
\frac{2}{3} = k
\]

So, the direct variation equation is $y = \frac{2}{3}x$.

Substitute 12 for $y$ and find $x$.

\[
y = \frac{2}{3}x \\
12 = \frac{2}{3}x \\
18 = x
\]

So, $x = 18$ when $y = 12$.

16. When $y = -8$, $x = 8$. What is $x$ when $y = -6$?

**SOLUTION:**

\[
y = kx \\
-8 = k(8) \\
\frac{-8}{8} = \frac{k(8)}{8} \\
-1 = k
\]

So, the direct variation equation is $y = -x$. Substitute $-6$ for $y$ and find $x$.

\[
y = -x \\
-6 = -x \\
-6 = -x \\
-1 = -1 \\
6 = x
\]

So, $x = 6$ when $y = -6$.

17. If $y = -5$ when $x = -2$, what is $y$ when $x = 14$?

**SOLUTION:**

\[
y = kx \\
-5 = k(-2) \\
\frac{-5}{-2} = \frac{k(-2)}{-2} \\
\frac{5}{2} = k
\]

So, the direct variation equation is $y = \frac{5}{2}x$. Substitute 14 for $x$ and find $y$.

\[
y = \frac{5}{2}x \\
y = \frac{5}{2}(14) \\
y = 35
\]

So, $y = 35$ when $x = 14$.

18. If $y = 2$ when $x = -12$, find $y$ when $x = -4$.

**SOLUTION:**

\[
y = kx \\
2 = k(-12) \\
\frac{2}{-12} = \frac{k(-12)}{-12} \\
\frac{1}{6} = k
\]

So, the direct variation equation is $y = -\frac{1}{6}x$.

Substitute $-4$ for $x$ and find $y$.

\[
y = -\frac{1}{6}x \\
y = -\frac{1}{6}(-4) \\
y = \frac{2}{3}
\]

So, $y = \frac{2}{3}$ when $x = -4$. 
19. **BIOLOGY** The number of pints of blood in a human body varies directly with the person’s weight. A person who weighs 120 pounds has about 8.4 pints of blood in his or her body.

a. Write and graph an equation relating weight and amount of blood in a person’s body.

b. Predict the weight of a person whose body holds 12 pints of blood.

**SOLUTION:**

a. To write a direct variation equation, find the constant of variation $k$. Let $x = 120$ and $y = 8.4$.

\[
y = kx
\]

\[
8.4 = k (120)
\]

\[
8.4 = k (120)
\]

\[
120 \quad 120
\]

\[
0.07 = k
\]

So, the direct variation equation is $y = 0.07x$.

b. Using the direct variation equation from part a, let $y = 12$.

\[
y = 0.07x
\]

\[
12 = 0.07x
\]

\[
0.07 \quad 0.07
\]

\[
171 \approx x
\]

So, a person who holds 12 pints of blood would weigh about 171 pounds.

---

**Find the next three terms of each arithmetic sequence.**

20. 0, –15, –30, –45, –60, …

**SOLUTION:**

Find the common difference by subtracting two consecutive terms.

\[-15 - 0 = -15.\]

The common difference between terms is –15. So, to find the next term, subtract 15 from the last term. To find the next term, subtract 15 from the resulting number, and so on.

\[-60 - 15 = -75\]

\[-75 - 15 = -90\]

\[-90 - 15 = -105\]

So, the next three terms of this arithmetic sequence are –75, –90, –105.

21. 5, 8, 11, 14, …

**SOLUTION:**

Find the common difference by subtracting two consecutive terms.

\[8 - 5 = 3.\]

The common difference between terms is 3. So, to find the next term, add 3 from the last term. To find the next term, add 3 from the resulting number, and so on.

\[14 + 3 = 17\]

\[17 + 3 = 20\]

\[20 + 3 = 23\]

So, the next three terms of this arithmetic sequence are 17, 20, 23.
Determine whether each sequence is an arithmetic sequence. If it is, state the common difference.

22. $-40, -32, -24, -16, \ldots$

**SOLUTION:**

$-40, -32, -24, -16, \ldots$

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

$-32 - (-40) = 8$
$-24 - (-32) = 8$
$-16 - (-24) = 7$

The difference between terms is constant, so the sequence is an arithmetic sequence. The common difference is 4.

23. $0.75, 1.5, 3, 6, 12, \ldots$

**SOLUTION:**

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

$1.5 - 0.75 = 0.75$
$3 - 1.5 = 1.5$
$6 - 3 = 3$
$12 - 6 = 6$

The difference between terms is not constant, so the sequence is not an arithmetic sequence and does not have a common difference.

24. $5, 17, 29, 41, \ldots$

**SOLUTION:**

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

$17 - 5 = 12$
$29 - 17 = 12$
$41 - 29 = 12$

The difference between terms is constant, so the sequence is an arithmetic sequence. The common difference is 4.

25. **MULTIPLE CHOICE** In each figure, only one side of each regular pentagon is shared with another pentagon. Each side of each pentagon is 1 centimeter. If the pattern continues, what is the perimeter of a figure that has 6 pentagons?

\[ \text{F 15 cm} \]
\[ \text{H 20 cm} \]
\[ \text{G 25 cm} \]
\[ \text{J 30 cm} \]

**SOLUTION:**

The perimeter increases by 3 as the number of polygons increases by 1. Continue the sequence to find the perimeter of a figure that has 6 pentagons.

<table>
<thead>
<tr>
<th>Number of Pentagons</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

The perimeter increases by 3 as the number of polygons increases by 1. Continue the sequence to find the perimeter of a figure that has 6 pentagons.

<table>
<thead>
<tr>
<th>Number of Pentagons</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

So, a figure that has 6 pentagons has a perimeter of 20 centimeters. The correct choice is H.
Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. What does the $x$-intercept mean in the context of the situation given below?

A amount of time needed to drain the bathtub  
B number of gallons in the tub when the drain plug is pulled  
C number of gallons in the tub after $x$ minutes  
D amount of water drained each minute

**SOLUTION:**
The $x$-intercept, $(8, 0)$, means that at 8 minutes elapsed time, the number of gallons of water in the bathtub is 0. So, the $x$-intercept represents the amount of time needed to drain the bathtub.
The correct choice is A.

2. The amount of money raised by a charity carwash varies directly as the number of cars washed. When 11 cars are washed, $79.75 is raised. How many cars must be washed to raise $174.00?
   
   **F** 10 cars  
   **G** 16 cars  
   **H** 22 cars  
   **J** 24 cars

   **SOLUTION:**
   Read the problem carefully. You need to find out how many cars need to be washed to raise $174.00. You know that 11 cars need to be washed to raise $79.75. Because the money raised varies directly with the number of cars washed, write a direct variation to solve the problem.

   $$y = kx$$

   $$79.75 = 11k$$

   $$\frac{79.75}{11} = k$$

   $$7.25 = k$$

   So, the direct variation is $y = 7.25x$. Use the direct variation equation to solve for $x$ when $y = 174.00$.

   $$y = 7.25x$$

   $$174.00 = 7.25x$$

   $$\frac{174.00}{7.25} = \frac{7.25x}{7.25}$$

   $$24 = x$$

   So, 24 cars need to be washed to raise $174.00 in profit. The correct choice is J.
3. The function $C = 25 + 0.45(x - 450)$ represents the cost of a monthly cell phone bill, when $x$ minutes are used. Which statement best represents the formula for the cost of the bill?

A The cost consists of a flat fee of $0.45 and $25 for each minute used over 450.
B The cost consists of a flat fee of $450 and $0.45 for each minute used over 25.
C The cost consists of a flat fee of $25 and $0.45 for each minute used over 450.
D The cost consists of a flat fee of $25 and $0.45 for each minute used.

**SOLUTION:**

Because the term 25 does not include a variable, it would be considered a flat fee, or a fee that is the same every month. Choices A and B do not have the correct amount listed for the flat fee, so they can be eliminated.

Because $x$ represents the number of minutes used and the equation shows $x - 450$, this would equal the number of minutes over 450. So, the answer that states that $0.45 is charged for each minute used over 450 is correct.

The correct choice is C.
1. Horatio is purchasing a computer cable for $15.49. If the sales tax rate in his state is 5.25%, what is the total cost of the purchase?
   A $16.42
   B $16.30
   C $15.73
   D $15.62

   SOLUTION:
The tax is 5.25% of the price of the cable.

   \[ \text{Tax} = 0.0525 \times 15.49 = 0.813225 \]

   To find the total cost, round 0.813225 to $0.81 since tax is always rounded to the nearest cent. Add this amount to the original price: $15.49 + $0.81 = $16.30

2. What is the value of the expression below?
   \[ 3^2 + 5^3 - 2^4 \]
   F 14
   G 34
   H 102
   J 166

   SOLUTION:
   \[ 3^2 + 5^3 - 2^4 = 9 + 125 - 32 = 102 \]

3. What is the slope of the linear function graphed below?

   SOLUTION:
The line passes through the points \((0, -1)\) and \((3, 1)\).

   Let \((0, -1) = (x_1, y_1)\) and \((3, 1) = (x_2, y_2)\).

   \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - 0} = \frac{2}{3} \]

   Therefore, C is the correct answer.
4. Find the rate of change for the linear function represented in the table.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Earned ($)</td>
<td>5.50</td>
<td>11.00</td>
<td>16.50</td>
<td>22.00</td>
</tr>
</tbody>
</table>

F increase $6.50/h  
G increase $5.50/h  
H decrease $5.50/h  
J decrease $6.50/h

**SOLUTION:**

\[
\text{rate of change} = \frac{\text{change in money earned}}{\text{change in hours worked}} = \frac{11.00 - 5.50}{2-1} = 5.50
\]

Therefore, the correct answer is G.

5. Suppose that y varies directly as x, and \( y = 14 \) when \( x = 4 \). What is the value of y when \( x = 9 \)?

A 25.5  
B 27.5  
C 29.5  
D 31.5

**SOLUTION:**

\[ y = kx \quad \text{Direct variation formula} \]
\[ 14 = k(4) \quad \text{Replace x with 4 and y with 4} \]
\[ \frac{14}{4} = \frac{k(4)}{4} \quad \text{Divide each side by 4.} \]
\[ 3.5 = k \quad \text{Simplify.} \]

Substitute 3.5 for \( k \) into the direct variation formula.

\[ y = 3.5x \quad \text{Direct variation formula} \]
\[ y = 3.5(9) \quad \text{Replace x with 9.} \]
\[ y = 31.5 \quad \text{Simplify.} \]

Therefore, the correct answer is 31.5.

6. Write an equation for the \( n \)th term of the arithmetic sequence shown below.
-2, 1, 4, 7, 10, 13

F \( a_n = 2n - 1 \)  
G \( a_n = 2n + 4 \)  
H \( a_n = 3n + 2 \)  
J \( a_n = 3n - 5 \)

**SOLUTION:**

First find the common difference in the sequence -2, 1, 4, 7, 10, and 13. Each time 3 is added to the number so the difference is 3.

Next, use the formula for the \( n \)th term, \( a_n = a_1 + (n - 1)d \) when \( a_1 = -2 \) and \( d = 3 \).

\[ a_n = -2 + (n - 1)3 \]
\[ a_n = -2 + 3n - 3 \]
\[ a_n = 3n - 5 \]

Therefore, the answer is J.
7. The table shows the labor charges of an electrician for jobs of different lengths.

<table>
<thead>
<tr>
<th>Number of Hours ( n )</th>
<th>Labor Charges ( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$60</td>
</tr>
<tr>
<td>2</td>
<td>$85</td>
</tr>
<tr>
<td>3</td>
<td>$110</td>
</tr>
<tr>
<td>4</td>
<td>$135</td>
</tr>
</tbody>
</table>

Which function represents the situation?

A \( C(n) = 25n + 35 \)

B \( C(n) = 25n + 30 \)

C \( C(n) = 35n + 25 \)

D \( C(n) = 35n + 40 \)

**SOLUTION:**
Find the difference of the \( n \)-values and of the \( C(n) \)-values.

\[
\begin{array}{c|c|c|c|c}
 n & 1 & 2 & 3 & 4 \\
\hline
 C(n) & 60 & 85 & 110 & 135 \\
\hline
 C(n) - 25n & 10 & 20 & 30 & 40 \\
\hline
 C(n) - 25n & -5 & -15 & -25 & -35 \\
\end{array}
\]

The difference between the \( n \)-values is 1, while the difference between the \( C(n) \)-values is 25. This suggests that \( C(n) = 25n \) since the ratio of \( C(n) \) to \( n \) is 25.

Find the difference of \( y \) and \( x \).

8. Find the value of \( x \) so that the figures have the same area.

**SOLUTION:**
For both shapes the area formula is \( A = \ell w \). The first figure would be \( A = 6(x - 1) \) and the second would be \( A = 4x \). Since they are to be equal, then their areas can be set equal to each other.

\[
\begin{align*}
5(x - 1) &= 4x & \text{Original equation} \\
6x - 6 &= 4x & \text{Distributive Property} \\
6x - 6 - 4x &= 4x - 4x & \text{Subtract } x \text{ from each side} \\
2x - 6 &= 0 & \text{Simplify} \\
2x - 6 + 6 &= 0 + 6 & \text{Add } 6 \text{ to each side} \\
2x &= 6 & \text{Simplify} \\
\frac{2x}{2} &= \frac{6}{2} & \text{Divide by } 2 \\
x &= 3 & \text{Simplify} \\
\end{align*}
\]

Therefore, the correct answer is F.
9. The table shows the total amount of rain during a storm. Write a formula to find about how much rain will fall after a given hour.

<table>
<thead>
<tr>
<th>Hours (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches (n)</td>
<td>0.45</td>
<td>0.9</td>
<td>1.35</td>
<td>1.8</td>
</tr>
</tbody>
</table>

A  \( h = 0.45n \)
B  \( n = 0.45h \)
C  \( h = 0.9n \)
D  \( h = 1.8n \)

**SOLUTION:**
Find the difference of the \( h \)-values and of the \( n \)-values.

The difference between \( h \)-values is 1, while the difference between \( n \)-values is 0.45. This suggests that \( n = 0.45h \) since the ratio of \( n \) to \( h \) is 0.45.

Find the difference of \( n \) and \( h \).

<table>
<thead>
<tr>
<th>( h )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>0.45</td>
<td>0.9</td>
<td>1.35</td>
<td>1.8</td>
</tr>
<tr>
<td>( n - 0.45 )</td>
<td>0.45</td>
<td>0.90</td>
<td>1.35</td>
<td>1.80</td>
</tr>
<tr>
<td>( n - 0.45h )</td>
<td>0.45</td>
<td>0.90</td>
<td>1.35</td>
<td>1.80</td>
</tr>
</tbody>
</table>

The difference between \( n \) and 0.45\( h \) is 0. Thus the function is \( n = 0.45h \). Therefore, B is the correct choice.

10. The scale on a map is 1.5 inches = 6 miles. If two cities are 4 inches apart on the map, what is the actual distance between the cities?

**SOLUTION:**
Set up a proportion in which the same type of units are used in the numerator of both fractions.

\[
\frac{1.5 \text{ in}}{6 \text{ mi}} = \frac{4 \text{ in}}{x}
\]

- **Original proportion**
- **Find the cross products.**
- **Simplify.**
- **Divide each side by 1.5**
- **Simplify.**

Therefore, the actual distance between the two cities is 16 miles.

11. Write a direct variation equation to represent the graph below.

**SOLUTION:**
The standard form for a direct variation equation is \( y = mx \) where \( m \) is the slope of the line. In the graph, the \( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2 \). Replacing \( m \) with 2, the equation would be \( y = 2x \).
12. Justine bought a car for $18,500 and its value depreciated linearly. After 3 years, the value was $14,150. What is the amount of yearly depreciation?

**SOLUTION:**
If the car’s value depreciates linearly, then each year’s depreciation is the same. The amount that the car has depreciated in 3 years can be found by subtracting the car’s value after 3 years from the original cost, $18,500 – $14,150 = $4,350. Since each year’s depreciation is the same and 3 years have passed, $4,350 ÷ 3 = $1,450. Therefore, the amount of yearly depreciation is $1,450.

13. **GRIDDED RESPONSE** Use the graph to determine the solution to the equation $\frac{1}{3}x + 1 = 0$?

**SOLUTION:**
The solution of the equation $\frac{1}{3}x + 1 = 0$ is determined by the x-intercept. The graph shows the x-intercept to be the point (3, 0). Therefore, the solution to the equation is the x-value which is 3.

14. Write an expression that represents the total surface area (including the top and bottom) of a tower of \( n \) cubes each having a side length of \( s \). (Do not include faces that cover each other.)

**SOLUTION:**
As cubes are stacked upon each other, a pattern develops. Each end cube contributes 5 faces. (One face is covered by other blocks). Each inside cube contributes 4 faces. (2 faces are covered by other cubes.) The area of one face is \( A = s^2 \) where \( s \) = the length of a side. The surface area of each tower is equal to the number of outside faces - the area of one face \( (s^2) \)

<table>
<thead>
<tr>
<th># of cubes ( n )</th>
<th>Description</th>
<th># of outside faces</th>
<th>Surface area = # of faces ( \times s^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6s^2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 end cubes</td>
<td>( 2 \times 5 = 10 )</td>
<td>10s^2</td>
</tr>
<tr>
<td>3</td>
<td>2 end cubes + 1 inside cube</td>
<td>( 2 \times 5 + 4 = 14 )</td>
<td>14s^2</td>
</tr>
<tr>
<td>4</td>
<td>2 end cubes + 2 inside cubes</td>
<td>( 2 \times 5 + 2 \times 4 = 18 )</td>
<td>18s^2</td>
</tr>
<tr>
<td>5</td>
<td>2 end cubes + 3 inside cubes</td>
<td>( 2 \times 5 + 3 \times 4 = 22 )</td>
<td>22s^2</td>
</tr>
<tr>
<td>( n )</td>
<td>2 end cubes + ((n-2)) inside cubes</td>
<td>( 2 \times 5 + (n-2) \times 4 ) (= 4n + 2 )</td>
<td>((4n + 2)s^2 )</td>
</tr>
</tbody>
</table>

Therefore, the total surface area is \((4n + 2)s^2\).
15. **GRIDDED RESPONSE** There are 120 members in the North Carolina House of Representatives. This is 70 more than the number of members in the North Carolina Senate. How many members are in the North Carolina Senate?

**SOLUTION:**

Let \( s \) = the number of members in the North Carolina Senate. Since there are 70 more members in the House of Representatives than the Senate,

\[ s + 70 = \text{Representatives}. \]

Because there are 120 members in the House,

\[ 120 = s + 70 \]
\[ 120 - 70 = s + 70 - 70 \]
\[ 50 = s \]

Therefore, there are 50 members in the North Carolina Senate.

16. A hot air balloon was at a height of 60 feet above the ground when it began to ascend. The balloon climbed at a rate of 15 feet per minute.

a. Make a table that shows the height of the hot air balloon after climbing for 1, 2, 3, and 4 minutes.

b. Let \( t \) represent the time in minutes since the balloon began climbing. Write an algebraic equation for a sequence that can be used to find the height, \( h \), of the balloon after \( t \) minutes.

c. Use your equation from part b to find the height, in feet, of the hot air balloon after climbing for 8 minutes.

**SOLUTION:**

a. For each minute, add 15 feet to the previous height.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
</tbody>
</table>

b. For each minute the balloon rises 15 feet, so \( 15t \) represents how far the balloon has risen after \( t \) minutes. 60 must be added to this because this is the height that the balloon started at. The equation is \( h = 15t + 60 \)

c. Let \( t = 8 \)

\[ h = 15t + 60 \]
\[ h = 15(8) + 60 \]
\[ h = 120 + 60 \]
\[ h = 180 \]

The balloon will climb 180 ft after 8 minutes.
State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. The x-coordinate of the point at which the graph of an equation crosses the x-axis is an x-intercept.
   
   SOLUTION:
   The x-coordinate of the point at which a graph of an equation crosses the x-axis is an x-intercept. So, the statement is true.

2. A linear equation is an equation of a line.
   
   SOLUTION:
   A linear equation is the equation of a line because its graph forms a line. So, the statement is true.

3. The difference between successive terms of an arithmetic sequence is the constant of variation.
   
   SOLUTION:
   The statement is false. The difference between the terms of an arithmetic sequence is called the common difference.

4. The regular form of a linear equation is $Ax + By = C$.
   
   SOLUTION:
   The statement is false. $Ax + By = C$ represents the standard form of a linear equation.

5. Values of $x$ for which $f(x) = 0$ are called zeros of the function $f$.
   
   SOLUTION:
   Values of $x$ for which $f(x) = 0$ are called zeros of the function $f$. So, the statement is true.

6. Any two points on a nonvertical line can be used to determine the slope.
   
   SOLUTION:
   The slope of a line is the ratio of the change in the y-coordinates to the change in the x-coordinates. Any two points on a line can be used to determine the slope. So, the statement is true.

7. The slope of the line $y = 5$ is $5$.
   
   SOLUTION:
   This statement is false. A slope of 5 would be very steep, and $y = 5$ is a horizontal line. The slope of the line $y = 5$ is 0.

8. The graph of any direct variation equation passes through $(0, 1)$.
   
   SOLUTION:
   This statement is false. The graph of any direct variation equation passes through $(0, 0)$.

9. A ratio that describes, on average, how much one quantity changes with respect to a change in another quantity is a rate of change.
   
   SOLUTION:
   A ratio that describes, on average, how much one quantity changes with respect to a change in another quantity is a rate of change. So, this statement is true.

10. In the linear equation $4x + 3y = 12$, the constant term is 12.
    
    SOLUTION:
    A constant includes no variables. It is just a number. In the linear equation $4x + 3y = 12$, the constant is 12. So, this statement is true.

11. Find the x-intercept and y-intercept of the graph of each linear function.

    | x  | y  |
    |----|----|
    | −8 | 0  |
    | −4 | 3  |
    | 0  | 6  |
    | 4  | 9  |
    | 8  | 12 |

   SOLUTION:
   The x-intercept is the point at which the y-coordinate is 0, or the line crosses the x-axis. So, the x-intercept is $−8$. The y-intercept is the point at which the x-coordinate is 0, or the line crosses the y-axis. So, the y-intercept is 6.
Graph each equation.

13. $y = -x + 2$

**SOLUTION:**
To graph the equation, find the $x$- and $y$-intercepts. Plot these two points. Then draw a line through them.

To find the $x$-intercept, let $y = 0$.

\[
\begin{align*}
  y &= -x + 2 \\
  0 &= -x + 2 \\
  0 - 2 &= -x + 2 - 2 \\
  -2 &= -x \\
  \frac{-2}{-1} &= \frac{-x}{-1} \\
  2 &= x
\end{align*}
\]

To find the $y$-intercept, let $x = 0$.

\[
\begin{align*}
  y &= -x + 2 \\
  y &= -(0) + 2 \\
  y &= 0 + 2 \\
  y &= 2
\end{align*}
\]

So, the $x$-intercept is 2 and the $y$-intercept is 2.
14. $x + 5y = 4$

**SOLUTION:**
To graph the equation, find the $x$- and $y$-intercepts.
Plot these two points. Then draw a line through them.
To find the $x$-intercept, let $y = 0$.

$$x + 5y = 4$$
$$x + 5(0) = 4$$
$$x + 0 = 4$$
$$x = 4$$

To find the $y$-intercept, let $x = 0$.

$$x + 5y = 4$$
$$0 + 5y = 4$$
$$5y = 4$$
$$\frac{5y}{5} = \frac{4}{5}$$
$$y = \frac{4}{5}$$

So, the $x$-intercept is 4 and the $y$-intercept is $\frac{4}{5}$.

15. $2x - 3y = 6$

**SOLUTION:**
To graph the equation, find the $x$- and $y$-intercepts.
Plot these two points. Then draw a line through them.
To find the $x$-intercept, let $y = 0$.

$$2x - 3y = 6$$
$$2x - 3(0) = 6$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

To find the $y$-intercept, let $x = 0$.

$$2x - 3y = 6$$
$$2(0) - 3y = 6$$
$$-3y = 6$$
$$\frac{-3y}{-3} = \frac{6}{-3}$$
$$y = -2$$

So, the $x$-intercept is 3 and the $y$-intercept is $-2$. 
16. \(5x + 2y = 10\)

**SOLUTION:**
To graph the equation, find the \(x\)- and \(y\)-intercepts. Plot these two points. Then draw a line through them.
To find the \(x\)-intercept, let \(y = 0\).

\[
\begin{align*}
5x + 2y &= 10 \\
5x + 2(0) &= 10 \\
5x &= 10 \\
x &= \frac{10}{5} \\
x &= 2
\end{align*}
\]

To find the \(y\)-intercept, let \(x = 0\).

\[
\begin{align*}
5x + 2y &= 10 \\
5(0) + 2y &= 10 \\
2y &= 10 \\
y &= \frac{10}{2} \\
y &= 5
\end{align*}
\]

So, the \(x\)-intercept is 2 and the \(y\)-intercept is 5.

17. **SOUND** The distance \(d\) in kilometers that sound waves travel through water is given by \(d = 1.6t\), where \(t\) is the time in seconds.

a. Make a table of values and graph the equation.
b. Use the graph to estimate how far sound can travel through water in 7 seconds.

**SOLUTION:**
a. To make the table of values, start at \(t = 0\) seconds and evaluate the expression \(1.6t\). Continue this process for 1 through 5 seconds.

\[
\begin{array}{c|c}
\text{t} & \text{d} \\
\hline
0 & 0 \\
1 & 1.6 \\
2 & 3.2 \\
3 & 4.8 \\
4 & 6.4 \\
5 & 8 \\
\end{array}
\]

b. To estimate how far sound can travel through water in 7 seconds, use the graph in part a. Go to 7 on the \(t\)-axis, then move up to the line to find the \(d\) value. The distance sound can travel through water in 7 seconds is about 11 kilometers.
Find the root of each equation.

18. $0 = 2x + 8$

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
\begin{align*}
0 &= 2x + 8 \quad \text{Original equation} \\
0 - 8 &= 2x + 8 - 8 \quad \text{Subtract 8 from each side} \\
-8 &= 2x \quad \text{Simplify} \\
\frac{-8}{2} &= 2 \times \frac{-8}{2} \quad \text{Divide each side by 2} \\
-4 &= x \quad \text{Simplify}
\end{align*}
\]

So, the root is $-4$.

19. $0 = 4x - 24$

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
\begin{align*}
0 &= 4x - 24 \quad \text{Original equation} \\
0 + 24 &= 4x - 24 + 24 \quad \text{Add 24 to each side} \\
24 &= 4x \quad \text{Simplify} \\
\frac{24}{4} &= \frac{4x}{4} \quad \text{Divide each side by 4} \\
6 &= x \quad \text{Simplify}
\end{align*}
\]

So, the root is $6$.

20. $3x - 5 = 0$

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
\begin{align*}
3x - 5 &= 0 \quad \text{Original equation} \\
3x - 5 + 5 &= 0 + 5 \quad \text{Add 5 to each side} \\
3x &= 5 \quad \text{Simplify} \\
\frac{3x}{3} &= \frac{5}{3} \quad \text{Divide each side by 3} \\
x &= \frac{5}{3} \quad \text{Simplify}
\end{align*}
\]

So, the root is $\frac{5}{3}$ or $1 \frac{2}{3}$.

21. $6x + 3 = 0$

**SOLUTION:**
The root, or solution, of an equation is any value that makes the equation true.

\[
\begin{align*}
6x + 3 &= 0 \quad \text{Original equation} \\
6x + 3 - 3 &= 0 - 3 \quad \text{Subtract 3 from each side} \\
6x &= -3 \quad \text{Simplify} \\
\frac{6x}{6} &= \frac{-3}{6} \quad \text{Divide each side by 6} \\
x &= -\frac{1}{2} \quad \text{Simplify}
\end{align*}
\]

So, the root is $-\frac{1}{2}$.

Solve each equation by graphing.

22. $0 = 16 - 8x$

**SOLUTION:**
The related function is $y = -8x + 16$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>$f(-4) = -8(-4) + 16$</td>
<td>48 (-4, 48)</td>
</tr>
<tr>
<td>-2</td>
<td>$f(-2) = -8(-2) + 16$</td>
<td>32 (-2, 32)</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = -8(0) + 16$</td>
<td>16 (0, 16)</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = -8(2) + 16$</td>
<td>0 (2, 0)</td>
</tr>
<tr>
<td>4</td>
<td>$f(7) = -8(4) + 16$</td>
<td>-16 (4, -16)</td>
</tr>
<tr>
<td>8</td>
<td>$f(8) = -8(8) + 16$</td>
<td>-32 (8, -32)</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at 2. So, the solution is 2.
23. $0 = 21 + 3x$

**SOLUTION:**
The related function is $y = 3x + 21$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 3x + 21$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10$</td>
<td>$f(-10) = 3(-10) + 21$</td>
<td>$-9$</td>
<td>$(-10, -9)$</td>
</tr>
<tr>
<td>$-7$</td>
<td>$f(-7) = 3(-7) + 21$</td>
<td>$0$</td>
<td>$(-7, 0)$</td>
</tr>
<tr>
<td>$-5$</td>
<td>$f(-5) = 3(-5) + 21$</td>
<td>$6$</td>
<td>$(-5, 6)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = 3(0) + 21$</td>
<td>$21$</td>
<td>$(0, 21)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$f(1) = 3(1) + 21$</td>
<td>$24$</td>
<td>$(1, 24)$</td>
</tr>
<tr>
<td>$3$</td>
<td>$f(3) = 3(3) + 21$</td>
<td>$27$</td>
<td>$(3, 27)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-7$. So, the solution is $-7$.

24. $-4x - 28 = 0$

**SOLUTION:**
The related function is $y = -4x - 28$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = -4x - 28$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10$</td>
<td>$f(-10) = -4(-10) - 28$</td>
<td>$-12$</td>
<td>$(-10, 12)$</td>
</tr>
<tr>
<td>$-7$</td>
<td>$f(-7) = -4(-7) - 28$</td>
<td>$-8$</td>
<td>$(-7, 8)$</td>
</tr>
<tr>
<td>$-5$</td>
<td>$f(-5) = -4(-5) - 28$</td>
<td>$-8$</td>
<td>$(-5, 8)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$f(0) = -4(0) - 28$</td>
<td>$-28$</td>
<td>$(0, -28)$</td>
</tr>
<tr>
<td>$5$</td>
<td>$f(5) = -4(5) - 28$</td>
<td>$-48$</td>
<td>$(5, -48)$</td>
</tr>
<tr>
<td>$10$</td>
<td>$f(10) = -4(10) - 28$</td>
<td>$-68$</td>
<td>$(10, -68)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at $-7$. So, the solution is $-7$. 
25. $25x - 225 = 0$

**SOLUTION:**
The related function is $y = 25x - 225$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 25x - 225$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>$f(-10) = 25(-10) - 225$</td>
<td>-475</td>
<td>$(-10, -475)$</td>
</tr>
<tr>
<td>-5</td>
<td>$f(-5) = 25(-5) - 225$</td>
<td>-350</td>
<td>$(-5, -350)$</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = 25(0) - 225$</td>
<td>-225</td>
<td>$(0, -225)$</td>
</tr>
<tr>
<td>5</td>
<td>$f(5) = 25(5) - 225$</td>
<td>-100</td>
<td>$(5, -100)$</td>
</tr>
<tr>
<td>9</td>
<td>$f(9) = 25(9) - 225$</td>
<td>0</td>
<td>$(9, 0)$</td>
</tr>
<tr>
<td>10</td>
<td>$f(10) = 25(10) - 225$</td>
<td>25</td>
<td>$(10, 25)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at 9. So, the solution is 9.

26. **FUNDRAISING** Sean’s class is selling boxes of popcorn to raise money for a class trip. Sean’s class paid $85 for the popcorn and they are selling each box for $1. The function $y = x - 85$ represents their profit $y$ for each box of popcorn sold $x$. Find the zero and describe what it means in this situation.

**SOLUTION:**
The related function is $f(x) = x - 85$. To graph the function, make a table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = x - 85$</th>
<th>$f(x)$</th>
<th>$(x, f(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>$f(-100) = (-100) - 85$</td>
<td>-185</td>
<td>$(-100, -185)$</td>
</tr>
<tr>
<td>-50</td>
<td>$f(-50) = (-50) - 85$</td>
<td>-135</td>
<td>$(-50, -135)$</td>
</tr>
<tr>
<td>0</td>
<td>$f(0) = (0) - 85$</td>
<td>-85</td>
<td>$(0, -85)$</td>
</tr>
<tr>
<td>50</td>
<td>$f(50) = (50) - 85$</td>
<td>-35</td>
<td>$(50, -35)$</td>
</tr>
<tr>
<td>85</td>
<td>$f(85) = (85) - 85$</td>
<td>0</td>
<td>$(85, 0)$</td>
</tr>
<tr>
<td>100</td>
<td>$f(100) = (100) - 85$</td>
<td>15</td>
<td>$(100, 15)$</td>
</tr>
</tbody>
</table>

The graph intersects the $x$-axis at 85. So the zero is 85. Once they have sold 85 boxes of popcorn they will have earned back their initial investment.
Find the rate of change represented in each table or graph.

27. \begin{align*}
\text{(1, 3)} & \quad \text{Graph the data.} \\
\text{rate of change} & = \frac{\text{change in } y}{\text{change in } x} \\
 & = \frac{3 - (-6)}{1 - (-2)} \\
 & = \frac{9}{3} \\
& = 3
\end{align*}

So, the rate of change is 3.

28. \begin{tabular}{|c|c|} 
\hline
x & y \\
\hline
-2 & -3 \\
0 & -3 \\
4 & -3 \\
12 & -3 \\
\hline
\end{tabular}

\text{SOLUTION:}
To find the rate of change, use the coordinates (-2, -3) and (0, -3).

\begin{align*}
\text{rate of change} & = \frac{\text{change in } y}{\text{change in } x} \\
 & = \frac{-3 - (-3)}{0 - (-2)} \\
 & = \frac{3}{2} \\
 & = 0
\end{align*}

So, the rate of change is 0.

Find the slope of the line that passes through each pair of points.

29. (0, 5), (6, 2)

\text{SOLUTION:}
\begin{align*}
m & = \frac{y_2 - y_1}{x_2 - x_1} \\
& = \frac{2 - 5}{6 - 0} \\
& = \frac{-3}{6} \\
& = -\frac{1}{2}
\end{align*}

So, the slope is $-\frac{1}{2}$.

30. (-6, 4), (-6, -2)

\text{SOLUTION:}
\begin{align*}
m & = \frac{y_2 - y_1}{x_2 - x_1} \\
& = \frac{-2 - 4}{-6 - (-6)} \\
& = \frac{-6}{0}
\end{align*}

Division by zero is undefined, so the slope is undefined.

31. PHOTOS The average cost of online photos decreased from $0.50 per print to $0.15 per print between 2002 and 2009. Find the average rate of change in the cost. Explain what it means.

\text{SOLUTION:}
\begin{align*}
\text{rate of change} & = \frac{\text{change in cost}}{\text{change in time}} \\
& = \frac{0.15 - 0.50}{2009 - 2002} \\
& = \frac{-0.35}{7} \\
& = -0.05
\end{align*}

The average rate of change in the cost is about $0.05. This means that there was an average decrease in cost of about $0.05 per year.
Graph each equation.
32. \( y = x \)

**SOLUTION:**
The slope of \( y = x \) is 1. Graph (0, 0). From there, move up 1 unit and right 1 unit to find another point. Then draw a line containing the points.

33. \( y = \frac{4}{3}x \)

**SOLUTION:**
The slope of \( y = \frac{4}{3}x \) is \( \frac{4}{3} \). Graph (0, 0). From there, move up 4 units and right 3 units to find another point. Then draw a line containing the points.

34. \( y = -2x \)

**SOLUTION:**
The slope of \( y = -2x \) is \( -2 \). Graph (0, 0). From there, move down 2 units and right 1 unit to find another point. Then draw a line containing the points.

35. If \( y = 15 \) when \( x = 2 \), find \( y \) when \( x = 8 \).

**SOLUTION:**
\[
\begin{align*}
y &= kx \\
15 &= k(2) \\
\frac{15}{2} &= k \\
7.5 &= k
\end{align*}
\]
So, the direct variation equation is \( y = 7.5x \).
Substitute 8 for \( x \) and find \( y \).
\[
\begin{align*}
y &= 7.5(8) \\
y &= 60
\end{align*}
\]
So, \( y = 60 \) when \( x = 8 \).

36. If \( y = -6 \) when \( x = 9 \), find \( x \) when \( y = -3 \).

**SOLUTION:**
\[
\begin{align*}
y &= kx \\
-6 &= k(9) \\
\frac{-6}{9} &= k \\
\frac{-2}{3} &= k
\end{align*}
\]
So, the direct variation equation is \( y = -\frac{2}{3}x \).
Substitute \(-3\) for \( y \) and find \( x \).
\[
\begin{align*}
y &= -\frac{2}{3}x \\
-3 &= -\frac{2}{3}x \\
-3\left(-\frac{3}{2}\right) &= \left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right)x \\
\frac{9}{2} &= x
\end{align*}
\]
So, \( x = \frac{9}{2} \) or \( 4\frac{1}{2} \) when \( y = -3 \).
37. If \( y = 4 \) when \( x = -4 \), find \( y \) when \( x = 7 \).

**SOLUTION:**

\[
y = kx
\]

\[
4 = k(-4)
\]

\[
\frac{4}{-4} = \frac{k(-4)}{-4}
\]

\[
-1 = k
\]

So, the direct variation equation is \( y = -x \). Substitute 7 for \( x \) and find \( y \).

\[
y = -x
\]

\[
y = -(7)
\]

\[
y = -7
\]

So, \( y = -7 \) when \( x = 7 \).

38. **JOBS** Suppose you earn $127 for working 20 hours.

a. Write a direct variation equation relating your earnings to the number of hours worked.

b. How much would you earn for working 35 hours?

**SOLUTION:**

a. To write a direct variation equation, find the constant of variation \( k \). Let \( x = 20 \) and \( y = 127 \).

\[
y = kx
\]

\[
127 = k(20)
\]

\[
\frac{127}{20} = \frac{k(20)}{20}
\]

\[
6.35 = k
\]

So, the direct variation equation is \( y = 6.35x \).

b. Using the direct variation equation from part a, let \( x = 35 \).

\[
y = 6.35x
\]

\[
= 6.35(35)
\]

\[
= 222.25
\]

So, you would earn $222.25 for working 35 hours.

Find the next three terms of each arithmetic sequence.

39. 6, 11, 16, 21, …

**SOLUTION:**

The common difference is 5. Add 5 to the last term of the sequence until three terms are found. The next three terms are 26, 31, and 36.

40. 1.4, 1.2, 1.0, …

**SOLUTION:**

The common difference is –0.2. Add –0.2 to the last term of the sequence until three terms are found. The next three terms are 0.8, 0.6, and 0.4.

Write an equation for the  \( n \)th term of each arithmetic sequence.

41. \( a_1 = 6, d = 5 \)

**SOLUTION:**

The \( n \)th term of an arithmetic sequence with first term \( a_1 \) and common difference \( d \) is given by \( a_n = a_1 + (n - 1)d \), where \( n \) is a positive integer.

\[
a_n = a_1 + (n - 1)d
\]

\[
a_n = 6 + (n - 1)5
\]

\[
a_n = 6 + 5n - 5
\]

\[
a_n = 5n + 1
\]

42. 28, 25, 22, 19, …

**SOLUTION:**

The \( n \)th term of an arithmetic sequence with first term \( a_1 \) and common difference \( d \) is given by \( a_n = a_1 + (n - 1)d \), where \( n \) is a positive integer. The common difference is –3.

\[
a_n = a_1 + (n - 1)d
\]

\[
a_n = 28 + (n - 1)(-3)
\]

\[
a_n = 28 - 3n + 3
\]

\[
a_n = -3n + 31
43. **SCIENCE** The table shows the distance traveled by sound in water. Write an equation for this sequence. Then find the time for sound to travel 72,300 feet.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>4820</td>
<td>9640</td>
<td>14,460</td>
<td>19,280</td>
</tr>
</tbody>
</table>

**SOLUTION:**
The \( n \)th term of an arithmetic sequence with first term \( a_1 \) and common difference \( d \) is given by \( a_n = a_1 + (n - 1)d \), where \( n \) is a positive integer. The common difference is 4820.

\[
a_n = a_1 + (n - 1)d \\
a_n = 4820 + (n - 1)(4820) \\
a_n = 4820 + 4820n - 4820 \\
a_n = 4820n
\]

To find the time for sound to travel 72,300 feet, let \( a_n = 72,300 \) in the equation above and solve for \( n \).

\[
72,300 = 4820n \\
\frac{72,300}{4820} = \frac{4820n}{4820} \\
15 = n
\]

So, it takes sound 15 seconds to travel 72,300 feet.

44. **Write an equation in function notation for this relation.**

![Graph](image)

**SOLUTION:**
Make a table of ordered pairs for several points on the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-6</td>
<td>-3</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

The difference in \( y \)-values is three times the difference of \( x \)-values. This suggests \( y = 3x \). So the equation for the relation in function notation is \( f(x) = 3x \).
45. **ANALYZE TABLES** The table shows the cost of picking your own strawberries at a farm.

<table>
<thead>
<tr>
<th>Number of Pounds</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>1.25</td>
<td>2.50</td>
<td>3.75</td>
<td>5.00</td>
</tr>
</tbody>
</table>

**a.** Graph the data.

**b.** Write an equation in function notation to describe this relationship.

**c.** How much would it cost to pick 6 pounds of strawberries?

**SOLUTION:**

**a.**

![Graph](image)

**b.** The difference in y-values is 1.25 times the difference of x-values. This suggests \( y = 1.25x \). So, the equation for the relationship in function notation is \( f(x) = 1.25x \).

**c.** To find the cost of picking 6 pounds of strawberries, let \( x = 6 \) in the equation from part b.

\[
f(x) = 1.25(6) = 7.5
\]

So, it would cost $7.50 to pick 6 pounds of strawberries.