Chapter 1

Write an algebraic expression for each verbal expression.
1. 6 times a number m

**SOLUTION:**
6 → 6
times → ×
a number m → m
6 × m or 6m

2. a number t less twelve

**SOLUTION:**
a number t → t
less 12 → − 12
t − 12

Evaluate each expression if m = 2, t = 6, and z = 5.

3. $2(t - z) + \frac{14}{m}$

**SOLUTION:**
$2(t - z) + \frac{14}{m} = 2(6 - 5) + \frac{14}{2}$
$= 2(1) + \frac{14}{2}$
$= 2 + 7$
$= 9$

4. $(m + 2z)^2 + 12tz$

**SOLUTION:**
$(m + 2z)^2 + 12tz = (2 + 2[5])^2 + 12(6)(5)$
$= (2 + 10)^2 + 12(6)(5)$
$= 12^2 + 12(6)(5)$
$= 144 + 360$
$= 504$

5. **SPORTS** Adam mows lawns at an average rate of 40 minutes per lawn. Write and evaluate an expression to find the number of hours Adam spent mowing last weekend.

<table>
<thead>
<tr>
<th>Lawns Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
</tr>
<tr>
<td>Saturday</td>
</tr>
<tr>
<td>Sunday</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Adam mowed a total of 3 + 11 + 4 lawns. It took him an average of 40 minutes per lawn, so 40 multiplied by the number of lawns will determine the total number of minutes. Divide this number by 60 to get the total number of hours spent.

$$\frac{40(3+11+4)}{60} = \frac{40(18)}{60} = \frac{720}{60} = 12$$

Evaluate each expression. Name the property used in each step.

6. $14\left(5 - \frac{1}{5} \cdot 25\right) + 2 \div (4 \cdot 1)$

**SOLUTION:**
Follow the Order of Operations.

PEMDAS →
Parentheses
Exponents
Multiplication/Division
Addition/Subtraction

$14\left(5 - \frac{1}{5} \cdot 25\right) + 2 \div (4 \cdot 1)$
$= 14\left(5 - 5\right) + 2 \div (4 \cdot 1)$; Substitution
$= 14(0) + 2 \div (4 \cdot 1)$; Additive Inverse
$= 0 + 2 \div (4 \cdot 1)$; Multiplicative Property of Zero
$= 0 + 2 + 4$; Multiplicative Identity
$= 0 + \frac{1}{2}$; Substitution
$= \frac{1}{2}$; Additive Identity
7. \(3(14 + 8 + 6) - 1 \cdot 18\)

**SOLUTION:**
Follow the Order of Operations.

PEMDAS →

Parentheses
Exponents
Multiplication/Division
Addition/Subtraction

\[
3(14 + 8 + 6) - 1 \cdot 18
= 3(20 + 2) - 1 \cdot 18, \text{ Commutative (+)}
= 3(22) - 1 \cdot 18, \text{ Substitution}
= 66 - 1 \cdot 18, \text{ Multiplicative Identity}
= 66, \text{ Substitution}
\]

8. **PETS** Rosa takes two dogs and one cat to the veterinarian during her vacation.

<table>
<thead>
<tr>
<th>Pet</th>
<th>Board</th>
<th>Bath</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>$25/day</td>
<td>$12</td>
</tr>
<tr>
<td>cat</td>
<td>$15/day</td>
<td>$8</td>
</tr>
</tbody>
</table>

a. Use the table to find the total cost of boarding both dogs and the cat for 5 days.

b. If Rosa has the veterinarian give all of the pets a bath while she is on vacation, what is the new total cost?

**SOLUTION:**
**PETS** Rosa takes two dogs and one cat to the veterinarian during her vacation.

<table>
<thead>
<tr>
<th>Pet</th>
<th>Board</th>
<th>Bath</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>cat</td>
<td>$15/day</td>
<td>$8</td>
</tr>
</tbody>
</table>

a. Use the table to find the total cost of boarding both dogs and the cat for 5 days.

b. If Rosa has the veterinarian give all of the pets a bath while she is on vacation, what is the new total cost?

Let \(d\) = the number of days.

Dogs: \(25d\)
Cats: \(15d\)

2 dogs and 1 cat for 5 days:

Cost = \(2(25d) + (15d)\)
\[= 2(25[5]) + 15(5)\]
\[= 2(125) + 75\]
\[= 250 + 75\]
\[= 325\]

b. Include the bath.

Dogs: \(25d + 12\)
Cats: \(15d + 8\)

2 dogs and 1 cat for 5 days:

Cost = \(2(25d + 12) + (15d + 8)\)
\[= 2(125 + 12) + (75 + 8)\]
\[= 2(137) + 83\]
\[= 274 + 83\]
\[= 357\]

Use the Distributive Property to rewrite each expression. Then evaluate.
9. \(14(102)\)

**SOLUTION:**
102 is close to 100, which is an easy number to multiply.

\[14(102) = 14(100 + 2) = 14(100) + 14(2) = 1400 + 28\]
\[= 1428\]
Chapter 1

10. \[ \frac{5}{6} \times (30) \]

**SOLUTION:**
Split the mixed number into a whole number and a fraction, so they can be multiplied one at a time.

\[
\frac{5}{6} \times (30) = \left( 5 + \frac{1}{6} \right) \times (30) \\
= 5 \times (30) + \frac{1}{6} \times (30) \\
= 150 + 5 = 155
\]

11. **ARTS** Logan sells handmade wooden products. He charges $25 per bowl, $14 per picture frame, and $30 per jewelry box. On Friday, he sells 3 bowls, 6 picture frames, and 2 jewelry boxes. On Saturday, he sells 6 bowls, 14 picture frames, and 3 jewelry boxes. Write and evaluate an expression for his total sales.

**SOLUTION:**

**ARTS** Logan sells handmade wooden products. He charges $25 per bowl, $14 per picture frame, and $30 per jewelry box. On Friday, he sells 3 bowls, 6 picture frames, and 2 jewelry boxes. On Saturday, he sells 6 bowls, 14 picture frames, and 3 jewelry boxes. Write and evaluate an expression for his total sales.

Multiply the cost of each item by the total number of items, then find the sum of the products.

Let \( b \) = number of bowls, \( p \) = number of picture frames, and \( j \) = number of jewelry boxes.

\[
25b + 6p + 30j \\
= 3(25) + 6(14) + 2(30) = 75 + 84 + 60 = 219
\]

12. The solution set for each equation if the replacement sets are \( m = \{0, 2, 4, 5, 8\} \) and \( n = \{-2, 0, 2, 7, 9\} \).

13. \( 6(m - 2) = 8 \)

**SOLUTION:**

\[
4(0 - 2) = -8 \neq 8 \\
4(2 - 2) = 0 \neq 8 \\
4(4 - 2) = 8 = 8 \\
4(5 - 2) = 12 \neq 8 \\
4(8 - 2) = 24 \neq 8 \\
\]

The solution is 4.

14. Increasing the amount of fertilizer put on a plant increases the rate at which it grows.

**SOLUTION:**

The independent variable is the value of the variable that determines the output. The domain contains values of the independent variable.

The dependent variable is the variable with a value that is dependent on the value of the independent variable. The range contains the values of the dependent variable.

The rate of growth depends on the amount of fertilizer.
15. Pam babysits to save money. The more kids she babysits, the more money she makes.

**SOLUTION:**

The independent variable is the value of the variable that determines the output. The domain contains values of the independent variable.

The dependent variable is the variable with a value that is dependent on the value of the independent variable. The range contains the values of the dependent variable.

The money Pam makes depends on how much she babysits.

16. **INCOME** Jonathan draws the graph at the right to describe his income throughout his career. Describe what is happening in the graph.

**SOLUTION:**

The graph is near the bottom at the beginning, and is increasing: He started out with a low, but increasing salary.

The graph went up more quickly near the end of his first third of the graph: Then his salary increased even faster as a young adult.

There is a quick decrease in the graph, and then it went up again: He may have changed jobs where his salary dropped, but then it increased very quickly.

The graph increased more slowly in the latter third: His salary level finally slowed down.

The graph had a sharp drop and then went level: He retired and his income dropped to a steady amount.

**Determine whether the relation is a function. Explain.**

17. \{(2, 2), (-5, 2), (6, 6), (9, 4), (4, 9)\}

**SOLUTION:**

In a function, there is exactly one output for each input. Each unique x-value is associated with only one y-value, so this relation is a function.

18. \{(0, 2), (1, 7), (0, -6), (4, 8), (-3, -1)\}

**SOLUTION:**

In a function, there is exactly one output for each input. In this relation, we have (0, 2) and (0, -6). The 0 is associated with more than one y-value, so this relation is not a function.
Chapter 1

19. Identify the function graphed as linear or nonlinear. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the \(x\)-coordinate of any relative extrema, and the end behavior of the graph.

**SOLUTION:**

The graph of a linear function is a straight line, and this graph is curved, so it is nonlinear.

The graph appears to cross the \(y\)-axis at about 10, so the \(y\)-intercept is 10. This means that it took 10 hours to produce the first unit.

The graph may cross the \(x\)-axis as the \(x\)-values increase past 100, but this cannot be determined from the graph. However, after reading the labels, it is understood that hours per unit cannot be negative, so the graph should not cross the \(x\)-axis.

No mirror images can be detected, so there doesn't appear to be any symmetry.

The graph is continuously going down from left to right, so it is decreasing from \(x = 0\) to \(x = 100\). This means that the hours per unit is decreasing as the number of units produced increases.

There are no turning points in the graph, so no extrema can be identified.

The graph appears to be approaching the \(x\)-axis as the \(x\)-values increase, so the graph may be approaching 0 as \(x\) increases.
Chapter 2

Translate each sentence into a formula.

1. The number of seconds is 60 times the number of minutes.
   SOLUTION:
   The number of seconds \( s \) is
   
   \[ s = 60m \]

2. A gallon contains 8 pints.
   SOLUTION:
   A gallon \( g \) contains
   
   \[ g = 8p \]

3. THEATER A theater has 1400 seats. Write and use an equation to find the number of rows if each row has 35 seats.
   SOLUTION:
   There are 1400 total seats \( \rightarrow 1400 \)
   Each row contains 35 seats \( \rightarrow 35r \)
   The seats in all of the rows is equal to the total number of seats \( \rightarrow 35r = 1400 \)
   \[ 35r = 1400 \]
   \[ r = \frac{1400}{35} \]
   \[ r = 40 \]

Solve each equation. Check your solution.

4. \(-18t + 26 = -19\)
   SOLUTION:
   \[ -18t + 26 = -19 \]
   \[ -18t = -45 \]
   \[ t = \frac{-45}{-18} \]
   \[ t = 2.5 \]

5. \(\frac{1}{2}b + \frac{3}{4} = \frac{1}{6} \)
   SOLUTION:
   \[ \frac{1}{2}b + \frac{3}{4} = \frac{1}{6} \]
   \[ \frac{1}{2}b = \frac{1}{6} - \frac{3}{4} \]
   \[ \frac{1}{2}b = \frac{12}{4} - \frac{15}{4} \]
   \[ \frac{1}{2}b = \frac{3}{4} \]
   \[ b = \frac{3}{4} \cdot \frac{2}{1} \]
   \[ b = \frac{3}{2} \]

Write an equation and solve each problem.

6. Find three consecutive even integers with a sum of 78.
   SOLUTION:
   Find three consecutive even integers \( \rightarrow x, x + 2, x + 4 \)
   with a sum of 78 \( \rightarrow x + x + 2 + x + 4 = 78 \)
   \[ x + x + 2 + x + 4 = 78 \]
   \[ 3x + 6 = 78 \]
   \[ 3x = 72 \]
   \[ x = \frac{72}{3} \]
   \[ x = 24 \]
   \[ x = 24, x + 2 = 26, x + 4 = 28 \]
7. Eight more than half a number is negative two.

**SOLUTION:**

Eight more than half a number → $\frac{1}{2}x + 8$

is negative two $\rightarrow = -2$

$\frac{1}{2}x + 8 = -2$

$\frac{1}{2}x + 8 - 8 = -2 - 8$

$\frac{1}{2}x = -10$

$2\left(\frac{1}{2}x\right) = 2(-10)$

$x = -20$

**Solve each equation. Check your solution.**

8. $-3w + 16 = 14.5$

**SOLUTION:**

$-3w + 16 = 14.5$

$-3w + 16 - 16 = 14.5 - 16$

$-3w = -1.5$

$\frac{-3w}{-3} = \frac{-1.5}{-3}$

$w = 0.5$

9. $\frac{4}{7} - \frac{k}{3} = \frac{1}{2}$

**SOLUTION:**

$\frac{4}{7} - \frac{k}{3} = \frac{1}{2}$

$\frac{4}{7} - \frac{k}{3} - \frac{4}{7} = \frac{1}{2} - \frac{4}{7}$

$\frac{-k}{3} = \frac{1(7)}{2(7)} - \frac{4(2)}{7(2)}$

$\frac{-k}{3} = \frac{7}{14} - \frac{8}{14}$

$\frac{-k}{3} = -\frac{1}{14}$

$-3\left(\frac{-k}{3}\right) = -3\left(-\frac{1}{14}\right)$

$k = \frac{3}{14}$

10. **ZOO** The Martin and Smith families went to the zoo. What is the cost of an adult ticket if they spent $75 total?

<table>
<thead>
<tr>
<th>Family</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Martin</td>
<td>2</td>
</tr>
<tr>
<td>Smith</td>
<td>3</td>
</tr>
</tbody>
</table>

**SOLUTION:**

There were $2 + 3 = 5$ adults. The total cost is $75 and we want to find the cost per adult.

$5a = 75$

$\frac{5a}{5} = \frac{75}{5}$

$a = 15$

**Solve each equation. Check your solution.**

11. $16x - 8 = 21 - x$

**SOLUTION:**

$16x - 8 = 21 - x$

$16x - 8 + x = 21 - x + x$

$17x - 8 = 21$

$17x - 8 + 8 = 21 + 8$

$17x = 29$

$\frac{17x}{17} = \frac{29}{17}$

$x = \frac{29}{17}$
12. \( \frac{x}{2} + \frac{1}{5} = 3x \)

**SOLUTION:**

\[
\frac{x}{2} + \frac{1}{5} - \frac{x}{2} = 3x - \frac{x}{2}
\]

\[
\frac{1}{5} = \frac{3x(2)}{2} - \frac{x}{2}
\]

\[
\frac{1}{5} = \frac{6x}{2} - \frac{x}{2}
\]

\[
\frac{1}{5} = \frac{5x}{2}
\]

\[5(5x) = (1)(2)\]

\[25x = 2\]

\[x = \frac{2}{25}\]

13. **GEOMETRY** Find the value of \( x \) so that the figures have the same perimeter.

![Hexagon and triangle with expressions](image)

**SOLUTION:**

Perimeter is the distance around the figures.

The first figure has 6 congruent sides, so the perimeter is \( 6(x + 2) \).

The second figure has a perimeter of \( 2x + 2x + 12 + 3x - 4 \).

\[6(x + 2) = 2x + 2x + 12 + 3x - 4\]

\[6x + 12 = 7x + 8\]

\[6x + 12 - 6x = 7x - 6x + 8\]

\[12 = x + 8\]

\[12 - 8 = x + 8 - 8\]

\[4 = x\]

Evaluate each expression if \( m = 6, n = 15, \) and \( p = \frac{1}{2} \).

14. \( \vert m - 12 \vert - 3p \)

**SOLUTION:**

\[\vert m - 12 \vert - 3p = \vert 6 - 12 \vert - 3\left(\frac{1}{2}\right)\]

\[= \vert -6 \vert - \frac{3}{2}\]

\[= 6 - 1.5\]

\[= 4.5\]

15. \( 18p + \vert n - m \vert \)

**SOLUTION:**

\[18p + \vert n - m \vert = 18\left(\frac{1}{2}\right) + \vert 15 - 6 \vert\]

\[= 9 + 9\]

\[= 18\]

Write an equation involving absolute value for each graph.

16. 

| -12 | -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |

**SOLUTION:**

The midpoint of \(-2\) and \(8\) is \(3\). \(3\) is 5 units away from \(-2\) and \(8\).

\[\vert x - \text{midpoint} \vert = \text{distance midpoint is from endpoints}\]

\[\vert x - 3 \vert = 5\]

17. 

| -10 | -8 | -6 | -4 | -2 | 0 | 2 | 4 | 6 | 8 | 10 |

**SOLUTION:**

The midpoint of \(-9\) and \(5\) is \(-2\).

\(-2\) is 7 units from \(-9\) and \(5\).

\[\vert x - \text{midpoint} \vert = \text{distance midpoint is from endpoints}\]

\[\vert x - (-2) \vert = 7\]

\[\vert x + 2 \vert = 7\]
Chapter 2

Solve each proportion. If necessary, round to the nearest hundredth.

18. \( \frac{1.7}{n} = \frac{16}{30} \)

**SOLUTION:**

\[
\frac{1.7}{n} = \frac{16}{30} \\
16n = 1.7(30) \\
16n = 51 \\
n = \frac{51}{16} \\
n \approx 3.1875 \\
n \approx 3.19
\]

19. \( \frac{418}{83} = \frac{b}{7} \)

**SOLUTION:**

\[
\frac{418}{83} = \frac{b}{7} \\
83b = 7(418) \\
83b = 2926 \\
b = \frac{2926}{83} \\
b \approx 35.25
\]

20. \( \frac{30}{y} = \frac{75}{135} \)

**SOLUTION:**

\[
\frac{30}{y} = \frac{75}{135} \\
\frac{30}{y} = \frac{5}{9} \\
5y = 9(30) \\
5y = 270 \\
y = \frac{270}{5} \\
y = 54
\]

21. **ART** Neil is enlarging a photo to hang on the wall. To keep the pictures proportional, what length is the unknown side?

![Diagram](image)

**SOLUTION:**

For the figures to be proportional, the corresponding sides must be proportional.

\[
\frac{3}{21} = \frac{5}{x} \\
\frac{1}{7} = \frac{5}{x} \\
x = 35
\]

**Find the discounted price of each item.**

22. sofa: $575
discount: 20%

**SOLUTION:**

discounted price = original price – (original price × discount rate)

\[
d = 575 - 575(0.20) \\
= 575 - 115 \\
= 460
\]

23. cell phone: $80
discount: 12%

**SOLUTION:**

discounted price = original price – (original price × discount rate)

\[
d = 80 - 80(0.12) \\
= 80 - 9.6 \\
= 70.4
\]
24. CAMPING Ty’s backpack weighs 38.6 pounds. Roger’s backpack weighs 15% more. How much does Roger’s backpack weigh?

**SOLUTION:**

\[
\frac{2b-a}{c} = -\frac{1}{2}d
\]

\[
\frac{2b-a}{c}(c) = -\frac{1}{2}d(c)
\]

\[
2b - a = -\frac{1}{2}cd
\]

\[
-a = -\frac{1}{2}cd - 2b
\]

\[
a = \frac{1}{2}cd + 2b
\]

Solve each equation or formula for the variable indicated.

25. \( \frac{2b-a}{c} = -\frac{1}{2}d \), for \( a \)

**SOLUTION:**

\[
\frac{2b-a}{c} = -\frac{1}{2}d
\]

\[
\frac{2b-a}{c}(c) = -\frac{1}{2}d(c)
\]

\[
2b - a = -\frac{1}{2}cd
\]

\[
2b - a - 2b = -\frac{1}{2}cd - 2b
\]

\[
-a = -\frac{1}{2}cd - 2b
\]

\[
a = \left[-\frac{1}{2}cd - 2b\right]
\]

\[
a = \frac{1}{2}cd + 2b
\]

26. \( y = w(h - 5) \), for \( h \)

**SOLUTION:**

\[
y = w(h - 5)
\]

\[
\frac{y}{w} = \frac{w(h - 5)}{w}
\]

\[
\frac{y}{w} = h - 5
\]

\[
\frac{y}{w} + 5 = h
\]

27. \( 3x - 4z = 7 - xy \), for \( x \)

**SOLUTION:**

\[
3x - 4z = 7 - xy
\]

\[
3x - 4z + xy = 7 - xy + xy
\]

\[
3x - 4z + xy = 7
\]

\[
3x + xy = 7 + 4z
\]

\[
x(3 + y) = 7 + 4z
\]

\[
x = \frac{7 + 4z}{3 + y}
\]

28. WEIGHT A watermelon weighs 6.3 pounds. One pound is approximately 0.454 kilogram. How many kilograms does the watermelon weigh?

**SOLUTION:**

\[
\frac{w}{6.3 \, \text{lb}} \cdot \frac{1 \, \text{lb}}{0.454 \, \text{kg}} \approx \frac{w}{2.86 \, \text{kg}}
\]

1 watermelon is approximately 2.86 kg

29. PAINT Jeff painted 400 square feet in 50 minutes. He then painted 700 square feet in 75 minutes. What is Jeff’s average painting speed?

**SOLUTION:**

His average painting speed is the total number of \( \text{ft}^2 \) divided by the total number of minutes.

Total number of \( \text{ft}^2 : 400 + 700 = 1100 \, \text{ft}^2 \)

Total number of minutes: 50 + 75 = 125 minutes

\[
\frac{1100}{125} = 8.8
\]

8.8 \( \text{ft}^2 / \text{min} \)
30. **CHEMISTRY** Sarah has 65 milliliters of a 30% solution. How many millimeters of 75% solution should she add to obtain the required 35% solution?

**SOLUTION:**

<table>
<thead>
<tr>
<th></th>
<th>Amount</th>
<th>Percentage</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Solution</td>
<td>65 mL</td>
<td>30%</td>
<td>0.30(65)</td>
</tr>
<tr>
<td>2nd Solution</td>
<td>x mL</td>
<td>75%</td>
<td>0.75(x)</td>
</tr>
<tr>
<td>Mixture</td>
<td>65 + x mL</td>
<td>35%</td>
<td>0.35(65 + x)</td>
</tr>
</tbody>
</table>

The sum of the 1st and 2nd Solution must equal the mixture.

\[
0.30(65) + 0.75x = 0.35(65 + x)
\]

\[
19.5 + 0.75x = 22.75 + 0.35x
\]

\[
19.5 + 0.40x = 22.75
\]

\[
0.40x = 22.75 - 19.5
\]

\[
0.40x = 3.25
\]

\[
x = \frac{3.25}{0.40}
\]

\[
x = 8.125
\]
Determine whether each equation is a linear equation. Write yes or no. If yes, write the equation in standard form.
1. \(-6xy + 2y = 3x\)

**SOLUTION:**
There is an \(xy\)-term, so this equation is not linear.

2. \(\frac{1}{2}y - 7 = 3x\)

**SOLUTION:**
There are no exponents and no variables are attached via multiplication or division, so this equation is linear.

\[
\frac{1}{2}y - 7 = 3x \\
-7 = 3x - \frac{1}{2}y \\
-14 = 6x - y \\
6x - y = -14
\]

Graph each equation by making a table.
3. \(2y - x = 5\)

**SOLUTION:**
Solve for \(y\).

\[
2y = x + 5 \\
y = \frac{x + 5}{2} \\
y = 0.5x + 2.5
\]

Use even values of \(x\) to offset the one-half.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = 0.5x + 2.5)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.5(-4) + 2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>-2</td>
<td>0.5(-2) + 2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>0</td>
<td>0.5(0) + 2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>0.5(2) + 2.5</td>
<td>3.5</td>
</tr>
<tr>
<td>4</td>
<td>0.5(4) + 2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>6</td>
<td>0.5(6) + 2.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>
4. \( x + y = 6 \)

**SOLUTION:**
Solve for \( y \).
\[ y = 6 - x \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 6 - x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>6 - (-4)</td>
<td>10</td>
</tr>
<tr>
<td>-2</td>
<td>6 - (-2)</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>6 - 0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>6 - 2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6 - 4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>6 - 6</td>
<td>0</td>
</tr>
</tbody>
</table>

6. \( \frac{1}{2}x + 8 = 6x - 12 - \frac{13}{2}x \)

**SOLUTION:**
\[ -\frac{1}{2}x + 8 = 6x - 12 - \frac{13}{2}x \]
\[ -0.5x + 8 = 6x - 12 - 6.5x \]
\[ -0.5x + 8 = -0.5x - 12 \]
\[ 8 = -12 \]

8 does not equal -12, so there is no solution.

7. **NURSERY** The function \( b = 100 - 2.5f \) represents the remaining balance of store credit Louie has at Blooms Nursery. Find the zero and explain what it means in this situation.

**SOLUTION:**
\[ b = 100 - 2.5f \]
\[ 0 = 100 - 2.5f \]
\[ -100 = -2.5f \]
\[ 40 = f \]

Louie will have spent all of his credit when he purchases 40 flowers.

**Determine whether each function is linear. Write yes or no. Explain.**

**8.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Compare the \( x \)-values.

\[ 0 - (-2) = 2 \]
\[ 2 - 0 = 2 \]
\[ 4 - 2 = 2 \]
\[ 6 - 4 = 2 \]

The \( x \)-values increase at a constant rate. Now check the \( y \)-values.

\[ 6 - 0 = 6 \]
\[ 12 - 6 = 6 \]
\[ 18 - 12 = 6 \]
\[ 24 - 18 = 6 \]

The \( y \)-values also increase at a constant rate, so the function is linear.
9. \[ \begin{array}{c|c|c|c|c|c} \hline x & 7 & 4 & 1 & -2 & -5 \\ \hline y & 14 & 2 & 12 & 4 & 10 \\ \hline \end{array} \]

**SOLUTION:**
Compare the \( x \)-values.

\[
\begin{align*}
4 - 7 &= -3 \\
1 - 4 &= -3 \\
-2 - 1 &= -3 \\
-5 - (-2) &= -3
\end{align*}
\]

The \( x \)-values increase at a constant rate. Now check the \( y \)-values.

\[
\begin{align*}
2 - 14 &= -12 \\
12 - 2 &= 10
\end{align*}
\]

The \( y \)-values do not increase at a constant rate, so the function is not linear.

10. **WEATHER** Refer to the graph.

![Weather Graph](image)

a. Find the rate of change in wind speed between 6 a.m. and 8 A.M.

b. Is there a greater change in wind speed during the day? If so, when does it occur?

c. The meteorologist says a storm came through at some point during the day. When do you think this may have happened? Explain your reasoning.

**SOLUTION:**

a. The speed is 5 at 6 A.M and 10 at 8 A.M. It increases by 5 over 2 hours, so it increases 2.5 per hour.

b. The graph is has the steepest increase from 8 A.M to 10 A.M. This indicates the greatest slope which is also the greatest rate of change.

c. The wind was the greatest from 10 A.M to 2 PM. The wind was also pretty high from 8 A.M to 10 A.M and from 2 PM to 4 PM.
Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

11. \[ y = \frac{1}{3}x \]

**SOLUTION:**
The constant of variation is the value of \( k \) in \( y = kx \).

The slope of the graph is the rise over the run. The graph rises 1 for every run of 3, so the slope is \( \frac{1}{3} \).

The constant of variation equals the slope.

12. \[ y = -4x \]

**SOLUTION:**
The constant of variation is the value of \( k \) in \( y = kx \).

The slope of the graph is the rise over the run. The graph rises -4 for every run of 1, so the slope is -4.

The constant of variation equals the slope.

Suppose \( y \) varies directly as \( x \). Write a direct variation equation that relates \( x \) and \( y \). Then solve.

13. If \( y = -6 \) when \( x = 9 \), find \( y \) when \( x = -3 \)

**SOLUTION:**
\[ y = kx \]
\[ -6 = k(9) \]
\[ 0 = \frac{6}{9} = k \]
\[ y = -2 \]

14. If \( y = -7 \) when \( x = -1 \), find \( x \) when \( y = 0 \).

**SOLUTION:**
\[ y = kx \]
\[ -7 = k(-1) \]
\[ 7 = k \]
\[ y = 7x \]
\[ 0 = 7x \]
\[ x = 0 \]

Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.

15. -2, 2, -4, 4, -6, 6 ...

**SOLUTION:**
An arithmetic sequence has a common difference.

\[ 2 - (-2) = 4 \]
\[ -4 - 2 = -6 \]

There is no common difference, so the sequence is not arithmetic.
16. \(-6, -3, 0, 3, 6 \ldots\)

**SOLUTION:**
An arithmetic sequence has a common difference.

\[-3 - (-6) = 3\]
\[0 - (-3) = 3\]
\[3 - 0 = 3\]
\[6 - 3 = 3\]

There is a common difference, so the sequence is arithmetic.

Write an equation for the \(n\)th term of each arithmetic sequence. Then graph the first five terms of the sequence.

17. \(3, 3.5, 4, 4.5 \ldots\)

**SOLUTION:**
Find the common difference, \(d\).

\[3.5 - 3 = 0.5\]
\[4 - 3.5 = 0.5\]
\[4.5 - 4 = 0.5\]

\[d = 0.5\]

The fifth term is \(4.5 + 0.5 = 5\).

\[a_n = a_1 + (n - 1)d\]
\[a_n = 3 + (n - 1)0.5\]
\[a_n = 3 + 0.5n - 0.5\]
\[a_n = 0.5n + 2.5\]

18. \(1, -1.5, -4, -6.5 \ldots\)

**SOLUTION:**
Find the common difference, \(d\).

\[-1.5 - 1 = -2.5\]
\[-4 - (-1.5) = -2.5\]
\[-6.5 - (-4) = -2.5\]

\[d = -2.5\]

The fifth term is \(-6.5 + (-2.5) = -9\).

\[a_n = a_1 + (n - 1)d\]
\[a_n = 1 + (n - 1)(-2.5)\]
\[a_n = 1 - 2.5n + 2.5\]
\[a_n = -2.5n + 3.5\]
19. **NEWSPAPER** The table shows Blocks Papers the number of newspapers Daniel delivers.

<table>
<thead>
<tr>
<th>Blocks</th>
<th>Papers</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>56</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
</tr>
</tbody>
</table>

a. Graph the data.

b. Write an equation to describe the relationship.

c. Find the number of papers delivered if he has 11 blocks.

**SOLUTION:**
a. Plot the points. Use increments of 1 for the $x$-values and increments of 5 for the $y$-values.

b. The $y$-values increase by 8 for every increase in $x$ by 1. The slope is 8. For every block, there is 8 newspapers. If there are 0 block, then there are 0 newspapers, so the $y$-intercept is 0. Therefore, the equation is $y = 8x$.

c. $y = 8(11) = 88$

### Write an equation in function notation for each relation.

20.

**SOLUTION:**
The function is linear. The $y$-intercept is at $(0, -1)$.

The graph rises 2 for every run of 6, so the slope is $\frac{1}{3}$.

$$y = \frac{1}{3}x - 1$$

21.

**SOLUTION:**
The function is linear. The $y$-intercept is at $(0, 1)$.

The graph rises $-2$ for every run of 1, so the slope is $-2$.

$$y = -2x + 1$$
Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

1. slope: 6, y-intercept: -4

**SOLUTION:**
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept. Replace $m$ with 6 and $b$ with -4.

$$y = 6x - 4$$

2. slope: $\frac{1}{3}$, y-intercept: 3

**SOLUTION:**
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept. Replace $m$ with $\frac{1}{3}$ and $b$ with 3.

$$y = \frac{1}{3}x + 3$$

**Write an equation in slope-intercept form for each graph shown.**

3.

![Graph](image)

**SOLUTION:**
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept.

The line intersects the y-axis at (0, -1), so the y-intercept is -1.
The line goes up 2 and over 1 from one point to the next, so the slope is $\frac{2}{1} = 2$.

Replace $m$ with 2 and $b$ with -1.

$$y = 2x - 1$$

4.

![Graph](image)

**SOLUTION:**
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope and $b$ is the y-intercept.

The line intersects the y-axis at (0, 0), so the y-intercept is 0.
The line goes up 1 and over 1 from one point to the next, so the slope is $\frac{1}{1} = 1$.

Replace $m$ with 1 and $b$ with 0.

$$y = 1x + 0$$ or $y = x$
5. **PARTY** Mr. Ramirez paid $60 for 15 specialty balloons for his son’s birthday.

a. Write an equation in slope-intercept form to find the cost \( C \) of \( b \) specialty balloons.

b. How much would 20 of the balloons cost?

**SOLUTION:**

a. The slope represents the cost per balloon.

Find the cost per balloon.

\[
\frac{60}{15} = 4
\]

The \( y \)-intercept represents the cost if 0 balloons are purchased. If 0 balloons are purchased, there is no cost, so the \( y \)-intercept is 0.

\[
C = 4b
\]

b. Input 20 for \( b \)

\[
C = 4(20) = 80
\]

Write an equation in slope-intercept form of the line that passes through each pair of points.

6. \((2, 0), \left(4, \frac{1}{2}\right)\)

**SOLUTION:**

Find the slope between the points.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{\frac{1}{2} - 0}{4 - 2}
\]

\[
= \frac{\frac{1}{2}}{2}
\]

\[
= \frac{1}{4}
\]

Use point-slope form.

\[
y - y_1 = m(x - x_1)
\]

\[
y - 0 = \frac{1}{4}(x - 2)
\]

\[
y = \frac{1}{4}x - \frac{1}{2}
\]

7. \((-5, -2), (5, 2)\)

**SOLUTION:**

Find the slope between the points.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{2 - (-2)}{5 - (-5)}
\]

\[
= \frac{4}{10}
\]

\[
= \frac{2}{5}
\]

Use point-slope form.

\[
y - y_1 = m(x - x_1)
\]

\[
y - 2 = \frac{2}{5}(x - 5)
\]

\[
y - 2 = \frac{2}{5}x - \frac{10}{5}
\]

\[
y = \frac{2}{5}x
\]
Write each equation in standard form.

8. \(\frac{1}{2} (y + 2) = 6x\)

**SOLUTION:**
Standard form is \(Ax + By = C\), where \(A \geq 0\), \(A\) and \(B\) are not both zero, and \(A, B,\) and \(C\) are integers with a greatest common factor of 1.

\[
\frac{1}{2} (y + 2) = 6x
\]

\[
y + 2 = 12x
\]

\[
2 = 12x - y
\]

\[
12 - y = 2
\]

9. \(7y - 13 = 2x + 5\)

**SOLUTION:**
Standard form is \(Ax + By = C\), where \(A \geq 0\), \(A\) and \(B\) are not both zero, and \(A, B,\) and \(C\) are integers with a greatest common factor of 1.

\[
7y - 13 = 2x + 5
\]

\[
-13 = 2x - 7y + 5
\]

\[
-18 = 2x - 7y
\]

\[
2x - 7y = -18
\]

Write each equation in slope-intercept form.

10. \(2y - 8 = \frac{1}{2} (x + 3)\)

**SOLUTION:**
Slope-intercept form is \(y = mx + b\). Isolate the \(y\).

\[
2y - 8 = \frac{1}{2} (x + 3)
\]

\[
2y = \frac{1}{2} x + \frac{3}{2}
\]

\[
2y = \frac{1}{2} x + \frac{10}{2}
\]

\[
y = \frac{1}{4} x + \frac{10}{4}
\]

11. \(y - \frac{1}{2} = 3x - \frac{3}{4}\)

**SOLUTION:**
Slope-intercept form is \(y = mx + b\). Isolate the \(y\).

\[
y - \frac{1}{2} = 3x - \frac{3}{4}
\]

\[
y = 3x - \frac{3}{4} + \frac{1}{2}
\]

\[
y = 3x - \frac{1}{4}
\]

Determine whether the graphs of the following equations are parallel or perpendicular. Explain.

12. \(y = -2x + 6, 2y = x - 3\)

**SOLUTION:**
When two lines are parallel, they have the same slope. When they are perpendicular, their slopes are opposite reciprocals. Find the slope of each line.

\(y = -2x + 6\): \(m = -2\)

\(2y = x - 3\)

\(y = \frac{1}{2}x - 1.5\)

\(m = \frac{1}{2}\)

The slopes are opposite reciprocals, so they are perpendicular.
13. \( y = 3x - 2 \), \(-3y = x + 6\)

**SOLUTION:**
When two lines are parallel, they have the same slope. When they are perpendicular, their slopes are opposite reciprocals. Find the slope of each line.

\( y = 3x - 2; \) \( m = 3 \)

\(-3y = x + 6\)

\( y = -\frac{1}{3}x - 2 \)

\( m: -\frac{1}{3} \)

The slopes are opposite reciprocals, so they are perpendicular.

14. MAPS The director of street repairs wants to first replace curbs on streets that are parallel to each other. Which two streets will get new curbs first?

**SOLUTION:**
Identify the parallel streets. These streets will have the same slope and never intersect. Austin and Travis live on parallel streets.

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the equation.

15. \(( -1, 6 ), y = \frac{1}{4}x - 4\)

**SOLUTION:**
Parallel lines have the same slope, so the slope is \( \frac{1}{4} \). Use the point-slope form.

\( y - y_1 = m(x - x_1) \)

\( y - 6 = \frac{1}{4}(x + 1) \)

\( y - 6 = \frac{1}{4}x + \frac{1}{4} \)

\( y = \frac{1}{4}x + \frac{25}{4} \)

16. \((5, 7), y = -x + 5\)

**SOLUTION:**
Parallel lines have the same slope, so the slope is \(-1\). Use the point-slope form.

\( y - y_1 = m(x - x_1) \)

\( y - 7 = -1(x - 5) \)

\( y - 7 = -x + 5 \)

\( y = -x + 12 \)
Chapter 4

Determine whether each graph shows a positive, negative, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

17. **Weight of Puppy**

   ![Weight of Puppy Graph]

   **SOLUTION:**
   The graph shows a positive correlation. The plots steadily go up from left to right, so as the x-values increase, the y-values also increase at about the same pace.

   As the age increases, the weight also increases.

18. **Carnival Fundraiser**

   ![Carnival Fundraiser Graph]

   **SOLUTION:**
   The graph shows a positive correlation. The plots steadily go up from left to right, so as the x-values increase, the y-values also increase at about the same pace.

   As the number of attendees increases, the money raised also increases.

Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

19. The table shows the numbers of plants in each garden and turnips produced.

<table>
<thead>
<tr>
<th>Plants</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turnips</td>
<td>12</td>
<td>19</td>
<td>24</td>
<td>32</td>
<td>34</td>
<td>45</td>
</tr>
</tbody>
</table>

   **SOLUTION:**
   Enter the data by pressing STAT and selecting the Edit option. Enter the plants into List 1 (L1). These will represent the x-values.

   Enter the turnips into List 2 (L2). These will represent the y-values.

   Perform the regression by pressing STAT and selecting the CALC option. Scroll down to LinReg (ax+b) and press ENTER twice. Make sure Diagnostic is On.

   ![LinReg]

   \[ y = 6.23x - 0.36, \text{ } r^2 = 0.99 \]
20. The table shows the amount of shrimp caught each day and the retail price of the shrimp.

<table>
<thead>
<tr>
<th>Pounds (1000s)</th>
<th>48</th>
<th>62</th>
<th>60</th>
<th>65</th>
<th>73</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost/Tlb ($)</td>
<td>3.30</td>
<td>3.42</td>
<td>3.35</td>
<td>3.15</td>
<td>3.08</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Enter the data by pressing STAT and selecting the Edit option. Enter the pounds into List 1 (L1). These will represent the x-values.

Enter the cost into List 2 (L2). These will represent the y-values.

Perform the regression by pressing STAT and selecting the CALC option. Scroll down to LinReg (ax+b) and press ENTER twice. Make sure Diagnostic is On.

```
LinReg
y=ax+b
a=.0173728914
b=4.335423729
r^2=.9474881303
r=-.9733900196
```

\[ y = -0.02x + 4.34, -0.97 \]

**Find the inverse of each function.**

21. \( f(x) = -3x + 8 \)

**SOLUTION:**
Replace \( f(x) \) with \( y \). Then switch the variables and solve for \( y \).

\[
\begin{align*}
  f(x) &= -3x + 8 \\
  y &= -3x + 8 \\
  x &= -3y + 8 \\
  x - 8 &= -3y \\
  \frac{x - 8}{-3} &= y \\
  -\frac{1}{3}x + \frac{8}{3} &= y
\end{align*}
\]

22. \( f(x) = \frac{1}{2}x + 7 \)

**SOLUTION:**
Replace \( f(x) \) with \( y \). Then switch the variables and solve for \( y \).

\[
\begin{align*}
  f(x) &= \frac{1}{2}x + 7 \\
  y &= \frac{1}{2}x + 7 \\
  x &= \frac{1}{2}y + 7 \\
  x - 7 &= \frac{1}{2}y \\
  2x - 14 &= y
\end{align*}
\]
Solve each inequality. Then graph the solution set on a number line.

1. \(6t < 3\)

\[SOLUTION:\]
\[6t \leq 3\]
\[t \leq \frac{3}{6}\]
\[t \leq \frac{1}{2}\]

The sign is \(\leq\), so draw a closed circle at \(\frac{1}{2}\) and an arrow to the left.

2. \(14 > k + 2\)

\[SOLUTION:\]
\[14 > k + 2\]
\[12 > k\]
\[k < 12\]

The sign is \(<\), so draw an open circle at 12 and an arrow to the left.

3. \(16 < 4n\)

\[SOLUTION:\]
\[16 \leq 4n\]
\[\frac{16}{4} \leq n\]
\[4 \leq n\]
\[n \geq 4\]

The sign is \(\geq\), so draw a closed circle at 4 and an arrow to the right.

4. \(6c < 5c + 3\)

\[SOLUTION:\]
\[6c < 5c + 3\]
\[6c - 5c < 3\]
\[c < 3\]

The sign is \(<\), so draw an open circle at 3 and an arrow to the left.

Define a variable, write an inequality, and solve each problem. Check your solution.

5. The sum of six times a number and three is less than the product of seven and a number.

\[SOLUTION:\]
Let \(n\) = the number.

6 times a number \(\rightarrow 6n\)

The sum of six times a number and three \(\rightarrow 6n + 3\)

is less than the product of seven and a number \(\rightarrow > 7n\)

\[6n + 3 < 7n\]
\[3 < 7n - 6n\]
\[3 < n\]

6. Three times a number is greater than or equal to the sum of twice a number and 12.

\[SOLUTION:\]
Let \(n\) = the number.

Three times a number \(\rightarrow 3n\)

is greater than or equal to \(\rightarrow \geq\)

the sum of twice a number and 12 \(\rightarrow 2n + 12\)

\[3n \geq 2n + 12\]
\[3n - 2n \geq 12\]
\[n \geq 12\]
Chapter 5

Solve each inequality. Graph the solution on a number line.

7. \( \frac{c}{8} > \frac{1}{4} \)

**SOLUTION:**

\( \frac{c}{8} > \frac{1}{4} \)

\( c > \frac{8}{4} \)

\( c > 2 \)

c is greater than, so use an open circle at 2 and an arrow to the right.

\[ -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 \]

8. \(-3 < 4m\)

**SOLUTION:**

\(-3 \leq 4m\)

\(-\frac{3}{4} \leq m\)

m is greater than or equal to, so use a closed circle at \(-\frac{3}{4}\) and an arrow to the right.

\[ -3 -2 -1 0 1 2 3 \]

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

9. **READING** Thomas has a 432-page book to read in 12 days. At least how many pages must he read per day to finish the book on time?

**SOLUTION:**

Let \( x \) = the number of pages per day

12 times the number \( \rightarrow 12x \)

must be at least 432 \( \rightarrow \geq 432 \)

\( 12x \geq 432 \)

\( x \geq \frac{432}{12} \)

\( x \geq 36 \)

Thomas needs to read at least 36 pages a day to finish the book in 12 days.

10. **DOGS** Laura has a maximum of 91 minutes to walk 7 dogs. How much time can she spend walking each dog?

**SOLUTION:**

Let \( m \) = the number of minutes walking each dog

7 times the number \( \rightarrow 7m \)

cannot be more than the maximum of 91 \( \rightarrow \leq 91 \)

\( 7m \leq 91 \)

\( m \leq \frac{91}{7} \)

\( m \leq 13 \)

Laura can spend no more than 13 minutes walking each dog.

Solve each inequality. Graph the solution on a number line.

11. \( 1.2x + 6 < 4.6x - 3 \)

**SOLUTION:**

\( 1.2x + 6 < 4.6x - 3 \)

\( 6 + 3 \leq 4.6x - 1.2x \)

\( 9 \leq 3.4x \)

\( \frac{9}{3.4} \leq x \)

\( 2.647 \leq x \)

\( x \) is greater than or equal to, so use a closed circle at 2.65 and an arrow to the right.

\[ -2 -1 0 1 2 3 4 5 6 \]

12. \( -4\left(\frac{3g + \frac{1}{2}}{2}\right) \leq -6(3 + 2g) \)

**SOLUTION:**

\( -4\left(\frac{3g + \frac{1}{2}}{2}\right) \leq -6(3 + 2g) \)

\( -12g - 2 \leq -18 - 12g \)

\( -2 \leq -18 \)

This statement is never true, so there is no solution.

\[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]
13. **PIZZA** Sam orders 3 large pizzas. Each pizza costs $12, and each topping costs $0.50. Sam has $38 to spend. Write and solve an inequality to find the greatest number of toppings Sam can afford.

**SOLUTION:**
Let \( x \) = the number of toppings

0.5 per topping \( \rightarrow 0.5x \)

3 large pizzas at $12 each \( \rightarrow 3(12) = 36 \)

the sum of the pizzas and the toppings cannot be more than the maximum of 38 \( \rightarrow 0.5x + 36 \leq 38 \)

\[ 0.5x + 36 \leq 38 \]
\[ 0.5x \leq 2 \]
\[ x \leq 4 \]

Sam can buy 4 toppings at the most.

---

Solve each compound inequality. Then graph the solution set.

14. \( 6w + 3 < 9 \) or \( \frac{1}{2}w \geq 2 \)

**SOLUTION:**

\( 6w + 3 < 9 \)

\[ 6w < 6 \]
\[ w < 1 \]

or

\[ \frac{1}{2}w \geq 2 \]
\[ w \geq 4 \]

When we use or, both inequalities are included in the graph.

\( w \) is **less than**, so use an open circle at 1 and an arrow to the left.

\( w \) is **greater than or equal to**, so use a closed circle at 4 and an arrow to the right.
Chapter 5

15. \(16 \geq 3x - 5\) and \(3x - 2 > 2(x - 1)\)

**SOLUTION:**

\[
16 \geq 3x - 5 \\
21 \geq 3x \\
7 \geq x
\]

or

\[
3x - 2 > 2(x - 1) \\
3x - 2 > 2x - 2 \\
x > 0
\]

When we use *and*, the graph contains the intersection of the graphs of both inequalities.

\(x\) is *less than or equal to*, so use a closed circle at 7 and an arrow to the left....

\(x\) is *greater than*, so use an open circle at 0 and an arrow to the right.

The intersection of these graphs is the place where they are both true.

16. **COUPON** Victor has a coupon that is valid only for juice sold in containers between 16 and 32 ounces.

a. Write a compound inequality that describes acceptable juice container sizes.

b. Graph the inequality.

**SOLUTION:**

a. between 16 and 32 \(\rightarrow 16 < x < 32\)

Between asks for values that are greater than one value and less than a larger value.

b.

16 \(< x\) \(\rightarrow\) open circle at 16 and arrow to the right.

x \(< 32\) \(\rightarrow\) open circle at 32 and arrow to the left.

This is an *and* inequality, so the graph is the intersection of these two graphs and includes values that are true for both inequalities.
Solve each inequality. Then graph the solution set.

17. \[ \left| \frac{1}{2}z + 6 \right| \leq 4 \]

**SOLUTION:**
The absolute value inequality can be converted to two inequalities combined by **and**. For the first inequality, drop the absolute value symbols. For the second inequality, flip the symbol and negate one side.

\[ |x| < c \Rightarrow x < c \text{ and } x > -c \]
\[ \left| \frac{1}{2}z + 6 \right| \leq 4 \Rightarrow \frac{1}{2}z + 6 \leq 4 \text{ and } \frac{1}{2}z + 6 \geq -4 \]
\[ \frac{1}{2}z + 6 \leq 4 \]
\[ \frac{1}{2}z \leq -2 \]
\[ z \leq -4 \]

\[ \frac{1}{2}z + 6 \geq -4 \]
\[ \frac{1}{2}z \geq -10 \]
\[ z \geq -20 \]

\(-4 \leq z \) → closed circle at -4 and arrow to the left.
\( z \geq -20 \) → open circle at -20 and arrow to the right.

When an absolute value inequality is of the form \(|x| \geq c\), it is an **or** inequality, so the graph is the union of these two graphs and includes values that are true for either inequality.

---

18. \[ |3p + 3| > 1 \]

**SOLUTION:**
The absolute value inequality can be converted to two inequalities combined by **and**. For the first inequality, drop the absolute value symbols. For the second inequality, flip the symbol and negate one side.

\[ |x| < c \Rightarrow x < c \text{ and } x > -c \]
\[ |3p + 3| > 1 \Rightarrow 3p + 3 > 1 \text{ and } 3p + 3 < -1 \]

\[ 3p + 3 > 1 \]
\[ 3p > -2 \]
\[ p > -\frac{2}{3} \]

\[ 3p + 3 < -1 \]
\[ 3p < -4 \]
\[ p < -\frac{4}{3} \]

\( p > -\frac{2}{3} \) → open circle and arrow to the right.
\( p < -\frac{4}{3} \) → open circle and arrow to the left.

When an absolute value inequality is of the form \(|x| > c\), it is an **or** inequality, so the graph is the union of these two graphs and includes values that are true for either inequality.
19. **SHOP** Kristi is shopping online for gifts for her friends.

a. If prices ranged from $1.50 above and below the average CD price, find the range of prices.

b. Prices for the book varied $2.25 from the average. Write the range of average book prices.

c. Graph the solution set for shirt prices if they varied $6 below to $4 above the average.

**SOLUTION:**

a. $1.50 below the average of $15.50 = $14. $1.50 above the average of $15.50 = $17.

The endpoints are included, so include the equal signs.

$14 ≤ c ≤ 17$

b. $2.25 below the average of $19 = $16.75. $2.25 above the average of $19 = $21.25.

The endpoints are included, so include the equal signs.

$16.75 ≤ b ≤ 21.25$

c. $6 below the average of $32 = $26. $4 above the average of $32 = $36.

The endpoints are included, so include the equal signs.

$26 ≤ s ≤ 36$

$26 ≤ s → Use a closed circle at 26 and an arrow to the right.

$s ≤ 36 → Use a closed circle at 36 and an arrow to the left.

The graph is the intersection of these two graphs, and includes values that are solutions to both inequalities.

---

20. **Graph each inequality.**

\[3(x + y) ≥ 6\]

**SOLUTION:**

Solve for y.

\[3(x + y) ≥ 6\]

\[x + y ≥ 2\]

\[y ≥ -x + 2\]

Graph the line. It is solid because the equal sign is included. y is greater than, so shade above the line.

---

21. \[\frac{1}{2}y < 2(-1-x)\]

**SOLUTION:**

Solve for y.

\[\frac{1}{2}y < -2 - 2x\]

\[y < -4x - 4\]

Graph the line. It is dashed because the equal sign is not included. y is less than, so shade below the line.
Use a graph to solve each inequality.

22. $5x + 3 > -2$

**SOLUTION:**
Solve for the variable.

$5x + 3 > -2$

$5x > -5$

$x > -1$

Graph the line. It is dashed because the equal sign is not included. $x$ is **greater than**, so shade to the right (where the greater $x$-values are).

23. $y - 8 \leq -3$

**SOLUTION:**
Solve for $y$.

$y - 8 \leq -3$

$y \leq 5$

Graph the line. It is solid because the equal sign is included. $y$ is **less than**, so shade below the line.

24. **DOG WASH** It costs Pups and Suds $975 a week to operate their business.

Any combination of small and large dogs that falls within the shaded region will earn a profit.

a. The prices that they charge are shown. Write an inequality to describe how many of each type of dog they need to service to make a profit.

b. How many dogs must they wash to make a profit each week?

**SOLUTION:**

a. Let $x = $ the number of small/medium dogs and let $y = $ the number of large dogs.

The prices that they charge per dog are $20x$ and $30y$.

The total amount of money that they earn is the sum of these $\rightarrow 20x + 30y$

This total must be more than $975 in order to make a profit $\rightarrow 20x + 30y > 975$

\[
20x + 30y > 975
\]

\[
30y > 975 - 20x
\]

\[
y > \frac{975 - 20x}{30}
\]

\[
y > -\frac{2}{3}x + 32.5
\]

Graph this inequality.
Use the graph below to determine whether each system is consistent or inconsistent and if it is independent or dependent.

\[ y = -2x - 2 \]
\[ y = 2x - 2 \]
\[ y = 2x + 2 \]
\[ y = -2x + 2 \]

1. \[ y = -2x - 2 \]

**SOLUTION:**
The equations represent the blue and green lines, which intersect at one point. Therefore, the system is consistent and has exactly one solution (independent).

2. \[ y = -2x + 2 \]

**SOLUTION:**
The equations represent the purple and green lines, which are parallel. Therefore, the system has no intersection or solution and is consistent.

3. **DANCES** Mario and Tanesha are inflating balloons for the school dance. Mario has 12 balloons inflated and is inflating additional balloons at a rate of 3 balloons per minute. Tanesha has 16 balloons inflated and is inflating additional balloons at a rate of 2 balloons per minute.

   a. Write a system of equations to represent the situation.

   b. Graph each equation.

   c. How long will it take Mario to have more balloons filled than Tanesha?

   **SOLUTION:**
   a. Let \( x \) = the number of minutes and Let \( y \) = the number of completed balloons.

   Mario: 12 complete and doing 3 per minute \( \rightarrow y = 3x + 12 \)

   Tanesha: 16 complete and doing 2 per minute \( \rightarrow y = 2x + 16 \)

   b. Graph on the same grid. Part c is asking about when Mario has more balloons than Tanesha, so we will need to find the intersection of the graphs. Use a window large enough to locate this intersection.

   c. The intersection of the graphs is at 4 minutes. After this intersection, Mario’s line is above Tanesha’s.
Use substitution to solve each system of equations.

4. \(3y + 2x = 10\)

**SOLUTION:**

\[
\begin{align*}
3y + 2x &= 10 \\
3y + 2(-y + 3) &= 10 \\
3y - 2y + 6 &= 10 \\
y + 6 &= 10 \\
y &= 4 \\
\end{align*}
\]

Substitute into the other equation.

\[
\begin{align*}
x &= -(y + 3) \\
x &= -(-4) + 3 \\
x &= -1 \\
-x + 2y &= 6 \\
\end{align*}
\]

5. \(4y - 2x = 11\)

**SOLUTION:**

Isolate one of the variables in one of the equations.

\[
\begin{align*}
-x + 2y &= 6 \\
-x &= -2y + 6 \\
x &= 2y - 6 \\
\end{align*}
\]

Substitute into the other equation.

\[
\begin{align*}
4y - 2x &= 11 \\
4y - 2(2y - 6) &= 11 \\
4y - 4y + 12 &= 11 \\
12 &= 11 \\
\end{align*}
\]

12 does not equal 11, so there is no solution.

6. \(y - 7x = 2\)

**SOLUTION:**

Isolate one of the variables in one of the equations.

\[
\begin{align*}
y - 7x &= 2 \\
y &= 7x + 2 \\
\end{align*}
\]

Substitute into the other equation.

\[
\begin{align*}
2x + 3 &= 5y \\
2x + 3 &= 5(7x + 2) \\
2x + 3 &= 35x + 10 \\
-7 &= 33x \\
-\frac{7}{33} &= x \\
\end{align*}
\]

Substitute back into the first equation.

\[
\begin{align*}
y &= 7x + 2 \\
y &= 7\left(-\frac{7}{33}\right) + 2 \\
y &= \frac{17}{33} \\
\end{align*}
\]

7. \(\frac{3}{2} = -\frac{1}{2}x - y\)

**SOLUTION:**

Isolate one of the variables in one of the equations.

\[
\begin{align*}
-2y &= x + 3 \\
\end{align*}
\]

Substitute into the other equation.

\[
\begin{align*}
\frac{3}{2} &= -\frac{1}{2}x - y \\
1.5 &= -0.5(-2y - 3) - y \\
1.5 &= y + 1.5 - y \\
1.5 &= 1.5 \\
\end{align*}
\]

1.5 = 1.5, so there are infinitely many solutions.
8. **FRUIT** Sarah and Toni each bought fruit for a fundraiser. If Toni spent $4.30 and Sarah spent $2.80, how much does each type of fruit cost?

<table>
<thead>
<tr>
<th>Girl</th>
<th>Apples</th>
<th>Oranges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toni</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Sarah</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Set up a system of equations. Let \(a\) = apples and \(o\) = oranges

Toni: \(6a + 5o = 4.3\)
Sarah: \(6a + 2o = 2.8\)

Subtract the bottom equation from the top equation to eliminate the \(6a\)-terms.

\[
\begin{align*}
6a + 5o &= 4.3 \\
6a + 2o &= 2.8 \\
3o &= 1.5 \\
o &= 0.5
\end{align*}
\]

Substitute into one of the original equations.

\[
\begin{align*}
6a + 2o &= 2.8 \\
6a + 2(0.5) &= 2.8 \\
6a &= 2.8 \\
a &= \frac{1.8}{6} \\
a &= 0.3
\end{align*}
\]

**Use elimination to solve each system of equations.**

\(2m + 3n = 16\)
\(-3m - 3n = -4\)

**SOLUTION:**
Add the equations to eliminate the \(3n\)-terms.

\[
\begin{align*}
2m + 3n &= 16 \\
-3m - 3n &= -4 \\
-m &= 12 \\
m &= -12
\end{align*}
\]

Substitute into one of the original equations.

\[
\begin{align*}
2m + 3n &= 16 \\
2(-12) + 3n &= 16 \\
-24 + 3n &= 16 \\
3n &= 40 \\
\frac{n}{3} &= \frac{40}{3}
\end{align*}
\]

\(-5k + 4j = 8\)

**SOLUTION:**
Subtract the bottom equation from the top equation to eliminate the \(-5k\)-terms.

\[
\begin{align*}
-5k + 4j &= 8 \\
-5k - 6j &= -12 \\
10j &= 20 \\
j &= 2
\end{align*}
\]

Substitute into one of the original equations.

\[
\begin{align*}
-5k + 4j &= 8 \\
-5k + 4(2) &= 8 \\
-5k + 8 &= 8 \\
-5k &= 0 \\
k &= 0
\end{align*}
\]
11. The difference of three times a number and a second number is two. The sum of the two numbers is fourteen. What are the two numbers?

**SOLUTION:**
Set up a system of equations.

The difference of three times a number and a second number is two. → $3x - y = 2$

The sum of the two numbers is fourteen. → $x + y = 14$

Add the equations to eliminate the $y$-terms.

\[
\begin{align*}
3x - y &= 2 \\
x + y &= 14 \\
\hline
4x &= 16 \\
x &= 4
\end{align*}
\]

Substitute into one of the original equations.

\[
\begin{align*}
x + y &= 14 \\
4 + y &= 14 \\
y &= 10
\end{align*}
\]

Use elimination to solve each system of equations.

1. $6x + 2.2y = 5.4$

**SOLUTION:**
1.6 is half of 3.2, so we can multiply the first equation by 2 to get common coefficients for the $x$-terms.

\[
\begin{align*}
2(1.6x + 2.2y) &= 2(5.4) \\
3.2x + 4.4y &= 10.8
\end{align*}
\]

Add the equations to eliminate the $x$-terms.

\[
\begin{align*}
3.2x + 4.4y &= 10.8 \\
-3.2x + 4y &= -2.4 \\
\hline
8.4y &= 8.4 \\
y &= 1
\end{align*}
\]

Substitute into one of the original equations.

\[
\begin{align*}
1.6x &= 3.2 \\
x &= 2
\end{align*}
\]
Chapter 6

2x + 5y = -8
13. 4x - 2y = 0

**SOLUTION:**
2 is half of 4, so we can multiply the first equation by -2 to get common coefficients for the x-terms. (We want the signs to be opposites)

\[-2(2x + 5y = -8) \Rightarrow -4x - 10y = 16\]

Add the equations to eliminate the x-terms.

\[-4x - 10y = 16
4x - 2y = 0\]

\[-12y = 16
y = -\frac{4}{3}\]

Substitute into one of the original equations.

\[4x - 2\left(-\frac{4}{3}\right) = 0
4x + \frac{8}{3} = 0\]

\[4x = -\frac{8}{3}
\]

\[x = -\frac{8}{3} \times \frac{1}{4}
\]

\[x = -\frac{2}{3}\]

14. **CARNIVALS** Scott and Isaac went to the school carnival. Use the table shown to determine how many tickets a ride and game each cost.

<table>
<thead>
<tr>
<th>Rider</th>
<th>Rides</th>
<th>Games</th>
<th>Tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scott</td>
<td>5</td>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>Isaac</td>
<td>7</td>
<td>2</td>
<td>23</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Set up a system of equations. Let \(r\) = rides and \(g\) = games

Scott: \(5r + 4g = 19\)
Isaac: \(7r + 2g = 23\)

Multiply the bottom equation by -2 to get common coefficients.

\[-2(7r + 2g = 23) \Rightarrow -14r - 4g = -46\]

Subtract the bottom equation from the top equation to eliminate the \(g\)-terms.

\[5r + 4g = 19
-14r - 4g = -46\]

\[-9r = -27
r = 3\]

Substitute into one of the original equations.

\[7r + 2g = 23
7(3) + 2g = 23\]

\[21 + 2g = 23
2g = 2
\]

\[g = 1\]
15. **SAVINGS** Caleb made $105 mowing lawns and walking dogs, charging the rates shown. If he mowed half as many lawns as dogs walked, how many lawns did he mow and how many dogs did he walk?

\[ \text{SOLUTION:} \]

Set up a system of equations. Let \( l \) = lawns and \( d \) = dogs

\[ 7.5d + 20l = 105 \]
\[ l = 0.5d \]

Use substitution.

\[ 7.5d + 20(0.5d) = 105 \]
\[ 7.5d + 10d = 105 \]
\[ 17.5d = 105 \]
\[ d = 6 \]
\[ l = 0.5(6) = 3 \]

16. **Determine the best method to solve each system of equations.** Then solve the system.

\[ 4x + 2y = 12 \]
\[ -y - 4x = 2 \]

**SOLUTION:**

Substitution. (One variable has a 1 or -1 as a coefficient.)

Isolate one of the variables in one of the equations.

\[ -y - 4x = 2 \]
\[ -y = 2 + 4x \]
\[ y = -2 - 4x \]

Substitute into the other equation.

\[ 4x + 2y = 12 \]
\[ 4x + 2(2 + 4x) = 12 \]
\[ 4x + 8x = 12 \]
\[ 12x = 12 \]
\[ x = -1 \]

Substitute back into the first equation.

\[ y = -2 - 4x \]
\[ y = -2 + 4(-1) \]
\[ y = -2 - 4 \]
\[ y = -12 \]
Chapter 6

17. \[ -2x + 3y = 11 \]

**SOLUTION:**
Substitution. (One variable has a 1 or –1 as a coefficient.)

Isolate one of the variables in one of the equations.

\[ y + 3x = 11 \]
\[ y = 11 - 3x \]

Substitute into the other equation.

\[-2x + 3y = 11 \]
\[-2x + 3(11 - 3x) = 11 \]
\[-2x + 33 - 9x = 11 \]
\[-11x + 33 = 11 \]
\[-11x = -22 \]
\[ x = 2 \]

11 - 3(2) = 5

18. **BABYSITTING** Kelsey and Emma babysit after school to earn extra money. Kelsey made $52 by charging $10 per hour and $4 per child. Emma made $67.50 by charging $15 per hour and $2.50 per child.

a. Write a system of equations to represent the situation.

b. How many hours and how many children did each babysit?

**SOLUTION:**
a. Let \( x \) = the number of hours and let \( y \) = the number of children.

Kelsey: \( 10x + 4y = 52 \)
Emma: \( 15x + 2.5y = 67.5 \)

Multiply the top equation by –3 and the bottom equation by 2 to get two coefficients of 30. (30 is the LCM of 10 and 15)

\[ 3(10x + 4y = 52) \rightarrow -30x - 12y = -156 \]
\[ 2(15x + 2.5y = 67.5) \rightarrow 30x + 5y = 135 \]

Add the equations.

Add the equations to eliminate the x-terms.

\[-30x - 12y = -156 \]
\[30x + 5y = 135 \]
\[ -7y = -21 \]
\[ y = 3 \]

Substitute into one of the original equations.

\[ 10x + 4y = 52 \]
\[ 10x + 4(3) = 52 \]
\[ 10x + 12 = 52 \]
\[ 10x = 40 \]
\[ x = 4 \]
Solve each system of inequalities by graphing.

19. \( x + y < 3 \)

**SOLUTION:**
Put each equation in slope-intercept form. Then graph the lines.

\[
0.5x - y \geq 3 \\
0.5x \geq y + 3 \\
0.5x - 3 \geq y
\]

The graph of this line will be solid because of \( \geq \).
Shade below this line because \( y \) is **less than**.

\( x + y < 3 \)
\( y < -x + 3 \)

The graph of this line will be dashed because of \( < \).
Shade below this line because \( y \) is **less than**.

\( y < 2x - 1 \)

20. \( y > 4(1 + 0.5x) \)

**SOLUTION:**
Graph each equation.

The graph of the first line will be dashed because of \( \geq \). Shade below this line because \( y \) is **less than**.

The graph of the second line will be dashed because of \( < \). Shade above this line because \( y \) is **greater than**.

There is no intersection of these shaded areas, so there is no solution.
Simplify each expression.

1. \[(2xy^2)^3(2x^2y^3z)^3\]

**SOLUTION:**
\[(2xy^2)^3(2x^2y^3z)^3 = 2^3 \cdot x^3 \cdot y^6(2x^2y^3z) = 2^3 \cdot 2 \cdot x^3 \cdot x^2 \cdot y^6 \cdot y^3 \cdot z = 16x^5y^9z\]

2. \[\left(2ab^2c^3\right)^2\left(3\alpha d^2\right)^2\]

**SOLUTION:**
\[\left(2ab^2c^3\right)^2\left(3\alpha d^2\right)^2 = 2ab^2c^3\left(3\alpha d^2\right) = 2 \cdot 9 \cdot \alpha \cdot \alpha^2 \cdot b^2 \cdot c^3 \cdot d^4 = 18\alpha^2 b^2 c^3 d^4\]

GEOMETRY Express the area of each triangle as a monomial.

3. \[5a^4b^3c^4\]

**SOLUTION:**
\[A = \frac{1}{2}bh = 0.5 \cdot 2a^2b^3c^4 \cdot 5a^4b^2c = 0.5 \cdot 2 \cdot 5 \cdot \alpha^2 + 4 \cdot \beta^3 + 2 \cdot \gamma^4 + 1 = 5\alpha^6 b^5 c^5\]

4. \[5xy^4z^5\]

**SOLUTION:**
\[A = \frac{1}{2}bh = 0.5 \cdot 5xy^4z^5 \cdot 3x^2 = 0.5 \cdot 5 \cdot 3 \cdot \chi^{1+2} \cdot y^4 z^5 = 7.5x^3 y^4 z^5\]

Simplify each expression. Assume that no denominator equals zero.

5. \[\frac{3m^5n^3p^4}{5m^6n^4p^2q^5}\]

**SOLUTION:**
\[\frac{3m^5n^3p^4}{5m^6n^4p^2q^5} = \frac{3m^5n^3p^4 - 6n^3 - 4p^4 - 2q^5}{3m^5n^3p^4} = \frac{3m^5n^3p^4}{5m^6n^3p^2q^5} = \frac{3p^2}{5m^6n^3q^5}\]

6. \[\left(\frac{x^2yz^3}{2xy^2z^3}\right)^3\]

**SOLUTION:**
\[\left(\frac{x^2yz^3}{2xy^2z^3}\right)^3 = \left(\frac{x^2 \cdot 1 \cdot y^{-2} \cdot z^3}{2} \right)^3 = \left(\frac{1 \cdot y^{-2}}{2} \right)^3 = \frac{x^3}{8y^6}\]
7. \( \frac{2a^{-1}b^{-2}c^{-3}}{7a^3b^3c^4d^{-5}} \)

\[
= \frac{2a^{-1-3}b^{-2-3}c^{-3-4}d^{-(-5)}}{7a^3b^3c^4d^{-5}}
= \frac{2a^{-4}b^{-5}c^{-7}d^5}{7a^3b^3c^4d^{-5}}
= \frac{2d^5}{7a^3b^3c^4}
\]

8. \( \left( \frac{4x^{2}y^{2}z^{5}}{3y^{-2}z^{-2}} \right)^{-1} \)

\[
= \left( \frac{4x^{2}y^{2}z^{5}}{3y^{-2}z^{-2}} \right)^{-1}
= \frac{5x^{-2}y^{-2}z^{5}}{4x^{-2}y^{3}z^{3}}
= \frac{5x^{-2-2}y^{-2-3}z^{-2-3}}{4x^{-2}y^{3}z^{3}}
= \frac{5x^{-4}y^{-5}z^{-5}}{4x^{-2}y^{3}z^{3}}
= \frac{5x^{-4}y^{-5}z^{-5}}{4x^{-2}y^{3}z^{3}}
= \frac{5x^{-4}y^{-5}z^{-5}}{4x^{-2}y^{3}z^{3}}
= \frac{5x^{-4}y^{-5}z^{-5}}{4x^{-2}y^{3}z^{3}}
\]

Write each expression in radical form, or write each radical in exponential form.

9. \( 13^{\frac{1}{3}} \)

\[
= \sqrt[3]{13}
\]

10. \( \left( 7k \right)^{\frac{1}{2}} \)

\[
= \sqrt{7k}
\]

11. \( \sqrt[17]{a} \)

SOLUTION:

For any nonnegative real number \( b \), \( b^{\frac{1}{n}} = \sqrt[n]{b} \).

\[
\left( 17a \right)^{\frac{1}{2}} = \sqrt[17]{17a} = \sqrt[17]{a}
\]

12. \( 3\sqrt[2]{2xyz^2} \)

SOLUTION:

For any nonnegative real number \( b \), \( b^{\frac{1}{n}} = \sqrt[n]{b} \).

\[
= 3\sqrt[2]{2xyz^2} = 3\sqrt[2]{2xyz^2} = 3\left(2xyz^2\right)^{\frac{1}{2}}
\]

Simplify.

13. \( \sqrt[4]{\frac{81}{625}} \)

SOLUTION:

\[
\frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}} = \frac{4}{\sqrt[81]{625}}
\]

14. \( \sqrt[5]{0.00001} \)

SOLUTION:

\[
= \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001} = \sqrt[10]{0.00001}
\]

For any nonnegative real number \( b \), \( b^{\frac{1}{n}} = \sqrt[n]{b} \).

\[
= \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001} = \sqrt[5]{0.00001}
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\]
15. \( 4096^{\frac{1}{3}} \)

**Solution:**

\[
4096^{\frac{1}{3}} = \left(2^{12}\right)^{\frac{1}{3}}
\]

\[
= 2^{12 \left(\frac{1}{3}\right)}
\]

\[
= 2^4
\]

\[
= 16
\]

16. \( \left(\frac{125}{343}\right)^{\frac{4}{3}} \)

**Solution:**

\[
\left(\frac{125}{343}\right)^{\frac{4}{3}} = \left(\frac{5^3}{7^3}\right)^{\frac{4}{3}}
\]

\[
= \frac{5^{3 \cdot \frac{4}{3}}}{7^{3 \cdot \frac{4}{3}}}
\]

\[
= \frac{5^4}{7^4}
\]

\[
= \frac{625}{2401}
\]

Solve each equation.

17. \( 8^x = \frac{1}{3} \)

**Solution:**

\[
8^x = \frac{1}{3}
\]

\[
\left(3^4\right)^x = \frac{1}{3}
\]

\[
3^{4x} = \frac{1}{3}
\]

\[
3^{4x} = 3^{-1}
\]

\[
4x = -1
\]

\[
x = -\frac{1}{4}
\]

18. \( 3^{4x} = 3^{x+1} \)

**Solution:**

\[
3^{4x} = 3^{x+1}
\]

\[
4x = x + 1
\]

\[
3x = 1
\]

\[
x = \frac{1}{3}
\]

Express each number in scientific notation.

19. \( 22,100,000,000 \)

**Solution:**

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number \( a \).

\[
a = 2.21
\]

Step 2 Note the number of places \( n \) and the direction that you moved the decimal point.

\[
n = 10
\]

Step 3 If the decimal point is moved left, write the number as \( a \times 10^{-n} \). If the decimal point is moved right, write the number as \( a \times 10^{n} \).

moved left

Step 4 Remove the unnecessary zeros.

\[
2.21 \times 10^{10}
\]
Simplify each expression.

1.

SOLUTION:

2.

SOLUTION:

GEOMETRY
Express the area of each triangle as a product.

We are multiplying by 3 and then subtracting 5.

3 \times 16 = 48 which equals 43 + 5

3 \times 7 = 21 which equals 16 + 5

16 is about 3 \times 7 and 43 is about 3 \times 16.

Some combination of multiplication and addition.

There is no common difference, however there is a common ratio.

a_1 = 729, a_n = a_{n-1}, n \geq 2

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Chapter 7

20. 0.00000003088

SOLUTION:

Step 1 Move the decimal point until it is to the right of the first nonzero digit. The result is a real number a.

a = 3.088

Step 2 Note the number of places n and the direction that you moved the decimal point.

n = 9

Step 3 If the decimal point is moved left, write the number as a \times 10^n. If the decimal point is moved right, write the number as a \times 10^{-n}.

moved right

Step 4 Remove the unnecessary zeros.

3.088 \times 10^{-9}

Evaluate each product. Express the results in both scientific notation and standard form.

21. \left(6.2 \times 10^{-3}\right) \left(1.77 \times 10^{-6}\right)

SOLUTION:

\left(6.2 \times 10^{-3}\right) \left(1.77 \times 10^{-6}\right) = 6.2 \times 1.77 \times 10^{-3+6}

= 10.974 \times 10^9

= 1.0974 \times 10^{10}

= 10,974,000,000

22. \left(4.08 \times 10^{-4}\right)^2

SOLUTION:

\left(4.08 \times 10^{-4}\right)^2 = 4.08^2 \times 10^{-8(2)}

= 16.6464 \times 10^{-8}

= 1.66464 \times 10^{-7}

= 0.00000166464

Graph each equation. Find the y-intercept, and state the domain and range.

23. \( f(x) = -3^x - 1 \)

SOLUTION:

Make a table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>y_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1.037</td>
</tr>
<tr>
<td>-1</td>
<td>1.111</td>
</tr>
<tr>
<td>0</td>
<td>1.333</td>
</tr>
<tr>
<td>1</td>
<td>4.000</td>
</tr>
<tr>
<td>2</td>
<td>10.000</td>
</tr>
<tr>
<td>3</td>
<td>28.000</td>
</tr>
</tbody>
</table>

The graph crosses the y-axis at -2. The graph exists for every possible value of x, so the domain is all real numbers. The graph will never cross y = -1, the the range is \{y | y < -1\}.

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24. \( f(x) = \left( \frac{1}{2} \right)^x + 3 \)

**SOLUTION:**

Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1.5</td>
</tr>
<tr>
<td>-1</td>
<td>3.1</td>
</tr>
<tr>
<td>0</td>
<td>5.5</td>
</tr>
<tr>
<td>1</td>
<td>11.0</td>
</tr>
<tr>
<td>2</td>
<td>21.5</td>
</tr>
<tr>
<td>3</td>
<td>42.0</td>
</tr>
</tbody>
</table>

\( x = -3 \)

The graph crosses the \( y \)-axis at 4. The graph exists for every possible value of \( x \), so the domain is all real numbers. The graph will never cross \( y = 3 \), the range is \( \{ y | y > 3 \} \).

25. Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>1</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{9} )</td>
<td>( \frac{1}{27} )</td>
<td>( \frac{1}{81} )</td>
</tr>
</tbody>
</table>

**SOLUTION:**

The domain intervals occur at regular intervals: increasing by 1.

\[ 1 \div 3 = \frac{1}{3} \]
\[ \frac{1}{3} \div 1 = \frac{1}{3} \]
\[ \frac{1}{9} \div \frac{1}{3} = \frac{1}{3} \]
\[ \frac{1}{27} \div \frac{1}{9} = \frac{1}{3} \]
\[ \frac{1}{81} \div \frac{1}{27} = \frac{1}{3} \]

Yes; the domain values are at regular intervals and the range values have a common factor of \( \frac{1}{3} \).

26. **POPULATION** A neighborhood had 4518 residents in 2006. The number of residents has been declining by 3.5% each year. How many residents will there be in 2012?

**SOLUTION:**

\[ y = a(1 - r)^t \]

\[ = 4518(1 - 0.035)^{2012-2006} \]
\[ = 4518(0.965)^6 \]
\[ \approx 3648 \]
Chapter 7

27. MONEY Sarah put $3000 in an investment that gets 6.2% compounded quarterly for 8 years. What will her investment be worth at the end of the 8 years?

**SOLUTION:**

\[ y = a\left(1 + \frac{r}{n}\right)^{nt} \]

\[ = 3000\left(1 + \frac{0.062}{4}\right)^{4(8)} \]

\[ = 3000(1.0155)^{32} \]

\[ \approx 4907.71 \]

28. SOCCER The Westside Soccer League has 186 players. They expect a 7.5% increase in players for at least the next 4 years. How many players will they have at that point?

**SOLUTION:**

\[ y = a(1 + r)^t \]

\[ = 186(1 + 0.075)^4 \]

\[ \approx 248 \]

Determine whether each sequence is arithmetic, geometric, or neither. Explain.

29. \( -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \ldots \)

**SOLUTION:**

Test for a common difference.

\[ -\frac{1}{4} - (-\frac{1}{2}) = \frac{1}{4} \]

\[ 0 - (-\frac{1}{4}) = \frac{1}{4} \]

\[ \frac{1}{4} - 0 = \frac{1}{4} \]

\[ \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \]

There is a common difference, so the sequence is arithmetic.

30. 100, 90, 85, 75, 60...

**SOLUTION:**

Test for a common difference.

\[ 90 - 100 = -10 \]

\[ 85 - 90 = -5 \]

There is no common difference. Test for a common ratio.

\[ 90 \div 100 = 0.9 \]

\[ 85 \div 90 \approx 0.9444 \]

There is no common ratio. The sequence is neither.

Find the next three terms in each geometric sequence.

31. 48, -96, 192, -384, 768...

**SOLUTION:**

Find the common ratio.

\[ -96 \div 48 = -2 \]

Multiply each term by -2.

\[ 768(-2) = -1536 \]

\[ -1536(-2) = 3072 \]

\[ 3072(-2) = -6144 \]

32. 150, 75, 37.5, 18.75, 9.375...

**SOLUTION:**

Find the common ratio.

\[ 75 \div 150 = 0.5 \]

Multiply each term by 0.5.

\[ (9.375)(0.5) = 4.6875 \]

\[ (4.6875)(0.5) = 2.34375 \]

\[ (2.34375)(0.5) = 1.171875 \]

Find the first five terms of each sequence.

33. \( a_1 = 5, a_n = 3.5a_{n-1} + 1, n \geq 2 \)

**SOLUTION:**

\[ a_1 = 5 \]

\[ a_2 = 3.5(5) + 1 = 17.5 + 1 = 18.5 \]

\[ a_3 = 3.5(18.5) + 1 = 64.75 + 1 = 65.75 \]

\[ a_4 = 3.5(65.75) + 1 = 230.125 + 1 = 231.125 \]

\[ a_5 = 3.5(231.125) + 1 = 809.9375 + 1 = 809.9375 \]
34. \( a_1 = 12, \ a_n = -\frac{1}{2}a_{n-1} + \frac{5}{2}, \ n \geq 2 \)

**SOLUTION:**

\[
\begin{align*}
  a_1 &= 12 \\
  a_2 &= -0.5(12) + 2.5 = -6 + 2.5 = -3.5 \\
  a_3 &= -0.5(-3.5) + 2.5 = 1.75 + 2.5 = 4.25 \\
  a_4 &= -0.5(4.25) + 2.5 = -2.125 + 2.5 = 0.375 \\
  a_5 &= -0.5(0.375) + 2.5 = -0.1875 + 2.5 = 2.3125
\end{align*}
\]

Write a recursive formula for each sequence.

35. 7, 16, 43, 124, ...

**SOLUTION:**

There is no common difference or ratio, so we need to look at these terms more carefully. There will be some combination of multiplication and addition.

16 is about \( 3 \times 7 \) and 43 is about \( 3 \times 16 \).

\[
\begin{align*}
  3 \times 7 &= 21 \text{ which equals } 16 + 5 \\
  3 \times 16 &= 48 \text{ which equals } 43 + 5
\end{align*}
\]

We are multiplying by 3 and then subtracting 5.

\[
a_1 = 7, \ a_n = 3a_{n-1} - 5, \ n \geq 2
\]

36. 729, 243, 81, 27, ...

**SOLUTION:**

There is no common difference, however there is a common ratio.

\[
\begin{align*}
  729(\frac{1}{3}) &= 243 \\
  243(\frac{1}{3}) &= 81 \\
  81(\frac{1}{3}) &= 27
\end{align*}
\]

We are multiplying by \( \frac{1}{3} \).

\[
a_1 = 729, \ a_n = \frac{1}{3}a_{n-1}, \ n \geq 2
\]
Find each sum or difference.
1. \((7g^3 + 2g^2 - 12) - (-2g^3 - 4g)\)

**SOLUTION:**
\((7g^3 + 2g^2 - 12) - (-2g^3 - 4g)\)
\[= 7g^3 + 2g^2 - 12 + 2g^3 + 4g\]
\[= 9g^3 + 2g^2 + 4g - 12\]

2. \((-3h^2 + 3h - 6) + (5h^2 - 3h - 10)\)

**SOLUTION:**
\((-3h^2 + 3h - 6) + (5h^2 - 3h - 10)\)
\[= -3h^2 + 5h^2 + 3h - 3h - 6 - 10\]
\[= 2h^2 - 16\]

Simplify each expression.
3. \(-\frac{1}{2}n^3p^2(5np^3 - 3n^2p^2 + 8n)\)

**SOLUTION:**
\[-\frac{1}{2}n^3p^2(5np^3 - 3n^2p^2 + 8n)\]
\[= -\frac{5}{2}n^4p^5 + \frac{3}{2}n^5p^4 - 4n^4p^2\]

4. \(6j^2(-3j + 3k^2) - 2k^2(2j + 10j^2)\)

**SOLUTION:**
\(6j^2(-3j + 3k^2) - 2k^2(2j + 10j^2)\)
\[= -18j^3 + 18j^2k^2 - 4jk^2 - 20j^2k^2\]
\[= -18j^3 - 4jk^2 - 2j^2k^2\]

Solve each equation.
5. \(-4(b + 3) + b(b - 3) = -b(6 - b) + 2(b - 3)\)

**SOLUTION:**
\[-4(b + 3) + b(b - 3) = -b(6 - b) + 2(b - 3)\]
\[-4b - 12 + b^2 - 3b = -6b + b^2 + 2b - 6\]
\[b^2 - 7b - 12 = b^2 - 4b - 6\]
\[-7b - 12 = -4b - 6\]
\[-12 = 3b - 6\]
\[-6 = 3b\]
\[-2 = b\]

6. \(3(a - 3) + a(a - 1) + 12 = a(a - 2) + 3(a - 2) + 4\)

**SOLUTION:**
\[3(a - 3) + a(a - 1) + 12 = a(a - 2) + 3(a - 2) + 4\]
\[3a - 9 + a^2 - a + 12 = a^2 - 2a + 3a - 6 + 4\]
\[a^2 + 2a + 3 = a^2 + a - 2\]
\[2a + 3 = a - 2\]
\[a + 3 = -2\]
\[a = -5\]

Find each product.
7. \((-3t - 16)(5t + 2)\)

**SOLUTION:**
\((-3t - 16)(5t + 2)\)
\[= -15t^2 - 80t - 6t - 32\]
\[= -15t^2 - 86t - 32\]

8. \(\left(4p + \frac{1}{2}\right)\left(\frac{1}{2}p + 4\right)\)

**SOLUTION:**
\(\left(4p + \frac{1}{2}\right)\left(\frac{1}{2}p + 4\right)\)
\[= 2p^2 + 16p + \frac{1}{4}p + 2\]
\[= 2p^2 + \frac{65}{4}p + 2\]
9. **SIDEWALKS** Reynolds ville is repairing sidewalks.

If the sidewalk is the same width around a city block, write an expression for the area of the block and the sidewalk.

**SOLUTION:**
The sidewalk is the little border around the block.
The sidewalk is the same depth throughout, so let the depth be \(x\). The length of the block including the sidewalk is \(100 + 2x\) and the width is \(80 + 2x\).

\[
(100 + 2x)(80 + 2x) = 8000 + 160x + 200x + 4x^2
= 4x^2 + 360x + 8000
\]

**Find each product.**

10. \((\frac{1}{2}m + 3)^2\)

**SOLUTION:**
\[
\left(\frac{1}{2}m + 3\right)^2 = \left(\frac{1}{2}m + 3\right)\left(\frac{1}{2}m + 3\right)
= \frac{1}{4}m^2 + \frac{3}{2}m + \frac{3}{2}m + 9
= \frac{1}{4}m^2 + 3m + 9
\]

11. \((2n - 6)(2n + 6)\)

**SOLUTION:**
\[
(2n - 6)(2n + 6) = 4n^2 - 12n + 12n - 36
= 4n^2 - 36
\]

12. \((5a - 4)^2\)

**SOLUTION:**
\[
(5a - 4)^2 = (5a - 4)(5a - 4)
= 25a^2 - 20a - 20a + 16
= 25a^2 - 40a + 16
\]

13. \((x - 2y)(x + 2y)\)

**SOLUTION:**
\[
(x - 2y)(x + 2y) = x^2 - 2xy + 2xy - 4y^2
= x^2 - 4y^2
\]

**Use the Distributive Property to factor each polynomial.**

14. \(4m^2n^2 + 16m^2n^3 - 8m^3n^4\)

**SOLUTION:**
Find the GCF of the terms.
The GCF of 4, 16, and 8 is 4.
Each term has at least 2 \(m^2\) and 2\(n\), so \(m^2n^2\) is the GCF of the variable terms.
\[
4m^2n^2 + 16m^2n^3 - 8m^3n^4 = 4m^2n^2(m + 4n - 2mn^2)
\]

15. \(12j^4k^4 + 36j^3k^2 - 3j^2k^5\)

**SOLUTION:**
Find the GCF of the terms.
The GCF of 12, 36, and 3 is 3.
Each term has at least 2\(j\) and 2\(k\) and 3, \(j^2k^2\) is the GCF of the variable terms.
\[
12j^4k^4 + 36j^3k^2 - 3j^2k^5 = 3j^2k^2(4j^2k^2 + 12j + k^3)
\]

**Factor each polynomial.**

16. \(x^2 - 4x + 3xy - 12y\)

**SOLUTION:**
\[
x^2 - 4x + 3xy - 12y = x(x - 4) + 3y(x - 4)
= (x + 3y)(x - 4)
\]

17. \(4a - 10ab + 6b - 15b^2\)

**SOLUTION:**
\[
4a - 10ab + 6b - 15b^2 = 2a(2 - 5b) + 3b(2 - 5b)
= (2a + 3b)(2 - 5b)
\]
Chapter 8

18. **HEIGHT** The height \( h \) of a ball bounced off the ground after \( t \) seconds is modeled by the equation

\[
h = -16t^2 + 28.8t
\]

a. What is the height of the ball at 1.5 seconds?

**SOLUTION:**

\[
h = -16(1.5)^2 + 28.8(1.5)
\]

\[
h = -16(1.5)^2 + 28.8(1.5)
\]

\[
= -36 + 43.2
\]

\[
= 7.2
\]

So, the height of the ball at 1.5 seconds is 7.2 feet.

b. How many seconds before the ball hits the ground again?

**SOLUTION:**

\[
h = -16t^2 + 28.8t
\]

\[
0 = t(-16t + 28.8)
\]

\[
0 = t \times 0 = -16t + 28.8
\]

\[
16t = 28.8
\]

\[
t = 1.8
\]

Thus, the ball will hit the ground again after 1.8 seconds.

**Factor each polynomial.**

19. \( t^2 + 2t - 15 \)

**SOLUTION:**

\[
t^2 + 2t - 15 = t^2 + 5t - 3t - 15
\]

\[
= t(t + 5) - 3(t + 5)
\]

\[
= (t - 3)(t + 5)
\]

20. \( d^2 - 3d - 28 \)

**SOLUTION:**

\[
d^2 - 3d - 28 = d^2 + 4d - 7d - 28
\]

\[
= d(d + 4) - 7(d + 4)
\]

\[
= (d - 7)(d + 4)
\]

21. \( m^2 + 5m - 14 \)

**SOLUTION:**

\[
m^2 + 5m - 14 = m^2 + 7m - 2m - 14
\]

\[
= m(m + 7) - 2(m + 7)
\]

\[
= (m - 2)(m + 7)
\]

22. \( x^2 - 4x - 45 \)

**SOLUTION:**

\[
x^2 - 4x - 45 = x^2 + 5x - 9x - 45
\]

\[
= x(x + 5) - 9(x + 5)
\]

\[
= (x - 9)(x + 5)
\]

**Solve each equation. Check your solution.**

23. \( h^2 + 3h - 4 = 0 \)

**SOLUTION:**

\[
h^2 + 3h - 4 = 0
\]

\[
(h + 4)(h - 1) = 0
\]

\[
h + 4 = 0 \text{ or } h - 1 = 0
\]

\[
h = -4 \text{ or } h = 1
\]

Check.

\[
h^2 + 3h - 4 = 0
\]

\[
( -4)^2 + 3( -4) - 4 = 0
\]

\[
16 - 12 - 4 = 0
\]

\[
0 = 0
\]

\[
h^2 + 3h - 4 = 0
\]

\[
(1)^2 + 3(1) - 4 = 0
\]

\[
1 + 3 - 4 = 0
\]

\[
0 = 0
\]
24. $a^2 + 9a + 18 = 0$

**SOLUTION:**

\[
a^2 + 9a + 18 = 0
\]

\[
(a + 6)(a + 3) = 0
\]

\[
a = -6 \text{ or } a = -3
\]

\[
(-6)^2 + 9(-6) + 18 = 0
\]

\[
36 - 54 + 18 = 0
\]

\[
0 = 0
\]

\[
(-3)^2 + 9(-3) + 18 = 0
\]

\[
9 - 27 + 18 = 0
\]

\[
0 = 0
\]

25. $x^2 - x - 6 = 0$

**SOLUTION:**

\[
x^2 - x - 6 = 0
\]

\[
(x - 3)(x + 2) = 0
\]

\[
x = 3 \text{ or } x = -2
\]

Check.

\[
(3)^2 - (3) - 6 = 0
\]

\[
9 - 3 - 6 = 0
\]

\[
0 = 0
\]

\[
(-2)^2 - (-2) - 6 = 0
\]

\[
4 + 2 - 6 = 0
\]

\[
0 = 0
\]

26. $y^2 + 2y - 15 = 0$

**SOLUTION:**

\[
y^2 + 2y - 15 = 0
\]

\[
(y + 5)(y - 3) = 0
\]

\[
y = -5 \text{ or } y = 3
\]

\[
(-5)^2 + 2(-5) - 15 = 0
\]

\[
25 - 10 - 15 = 0
\]

\[
0 = 0
\]

\[
(3)^2 + 2(3) - 15 = 0
\]

\[
9 + 6 - 15 = 0
\]

\[
0 = 0
\]

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write prime.

27. $6x^2 + 21x - 90$

**SOLUTION:**

\[
6x^2 + 21x - 90 = 3(2x^2 + 7x - 30)
\]

\[
2(-30) = -60
\]

Find 2 numbers whose product is $-60$ and whose sum is 7.

\[
30(-2) = -60
\]

\[
20(-3) = -60
\]

\[
15(-4) = -60
\]

\[
12(-5) = -60
\]

Use 12 and $-5$.

\[
3(2x^2 + 7x - 30) = 3[2x^2 + 12x - 5x - 30]
\]

\[
= 3[2x(x + 6) - 5(x + 6)]
\]

\[
= 3(2x - 5)(x + 6)
\]
28. $3x^2 - 11x - 42$

**SOLUTION:**

$3x^2 - 11x - 42$

$3(-42) = -126$

Find 2 numbers whose product is $-126$ and whose sum is $-11$.

$3(-42) = -39$

$6(-21) = 150$

$7(-18) = -11$

Use 7 and $-18$.

$3x^2 + 7x - 18x - 42 = x(3x + 7) - 6(3x + 7)$

$= (x - 6)(3x + 7)$

29. $6x^2 - 13x - 5$

**SOLUTION:**

$3x^2 - 11x - 42$

$6(-5) = -30$

Find 2 numbers whose product is $-30$ and whose sum is $-13$.

$1(-30) = -29$

$2(-15) = -13$

Use 2 and $-15$.

$6x^2 + 2x - 15x - 5 = 2x(3x + 1) - 5(3x + 1)$

$= (2x - 5)(3x + 1)$

30. $5y^2 - 3y + 11$

**SOLUTION:**

$3x^2 - 11x - 42$

$11(5) = 55$

There are no 2 numbers whose product is $55$ and whose sum is $-3$. The polynomial is prime.

31. **PRIZES** A machine is used to throw T-shirts into the crowd at the Hornets basketball games.

![Image of T-shirt](image)

a. What is the initial height of the T-shirt?

b. If the T-shirt is caught after 2 seconds, what is the height?

**SOLUTION:**

a. The initial height is the value of $c$. It is also the value when $x = 0$.

$-16(0)^2 + 34(0) + 4 = 4$

b. Set $t$ equal to 2.

$-16(2)^2 + 34(2) + 4 = -64 + 68 + 4 = 8$

**Factor each polynomial.**

32. $\frac{1}{2}t^2 - 162$

**SOLUTION:**

$\frac{1}{2}t^2 - 162 = \frac{1}{2}(t^2 - 324)$

$= \frac{1}{2}(t + 18)(t - 18)$

33. $25d^3 - 49d$

**SOLUTION:**

$25d^3 - 49d = d(25d^2 - 49)$

$= d([5d]^2 - [7]^2)$

$= d(5d + 7)(5d - 7)$

34. $196t^2u^3 - 144tu^3$

**SOLUTION:**

$196t^2u^3 - 144tu^3 = 4tu^3(49t^2 - 36)$

$= 4tu^3(7t^2 - 6^2)$

$= 4tu^3(7t + 6)(7t - 6)$
35. \(169a^4b^6 - 12k^8\)

**SOLUTION:**

\[
169a^4b^6 - 12k^8 = (13a^2b^3)^2 - (\sqrt{12}k^4)^2
= (13a^2b^3 + \sqrt{12}k^4)(13a^2b^3 - \sqrt{12}k^4)
\]

36. \(4g^2 - 1296h^2\)

**SOLUTION:**

\[
4g^2 - 1296h^2 = 4(g^2 - 324h^2)
= 4(g + 18h)(g - 18h)
\]

37. \(18a^3 + 27a^2 - 50a - 75\)

**SOLUTION:**

\[
18a^3 + 27a^2 - 50a - 75
= 9a^2(2a + 3) - 25(2a + 3)
= (9a^2 - 25)(2a + 3)
= (3a + 5)(3a - 5)(2a + 3)
\]

38. **FRUIT** An apple fell 25 feet from a tree. The formula \(h = -16t^2 + 25\) can be used to approximate the number of seconds it will take the apple to hit the ground.

a. How long will it take the apple to hit the ground?

b. If you catch it at 4 feet, for how long did the apple drop?

**SOLUTION:**

a. 

\[
h = -16t^2 + 25\]

\[
0 = -16(t^2 - 25)
\]

\[
0 = (t + 5)(t - 5)
\]

\[
t = -5 \text{ or } t = 5
\]

Time is positive, so \(t = 5\).

b. 

\[
h = -16t^2 + 25\]

\[
4 = -16t^2 + 25\]

\[
-21 = -16t^2
\]

\[
\frac{-21}{-16} = t^2
\]

\[
\frac{21}{16} = t^2
\]

\[
\sqrt{\frac{21}{16}} = t
\]

\[
1.15 \approx t
\]

Determine whether each trinomial is a perfect square trinomial. Write yes or no. If so, factor it.

39. \(64x^2 - 32x + 4\)

**SOLUTION:**

\[
64x^2 - 32x + 4 = (8x)^2 - 2(8x)(2) + (2)^2
= (8x - 2)^2
\]

40. \(4a^2 - 12a + 16\)

**SOLUTION:**

\[
4 = 2^2 \text{ and } 16 = 4^2, \text{ but } 12 \neq 2(2)(4)
\]
41. \(12y^2 - 36y + 27\)

**SOLUTION:**
\[
12y^2 - 36y + 27 = 3(4y^2 - 12y + 9)
\]
\[
4y^2 - 12y + 9 = (2y)^2 - 2(2y)(3) + (3)^2
\]
\[
= (2y - 3)^2
\]
\[
3(2y - 3)^2
\]

42. \(75b^3 - 60ab^2 + 12a^2b\)

**SOLUTION:**
\[
75b^3 - 60ab^2 + 12a^2b = 3b[25b^2 - 20ab + 4a^2]
\]
\[
= 3b[(5b)^2 - 2(5b)(2a) + (2a)^2]
\]
\[
= 3b(5b - 2a)^2
\]
Chapter 9

Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph of each equation.

1. \( y = 4x^2 + 8x - 5 \)

**SOLUTION:**
For a quadratic of the form \( y = ax^2 + bx + c \), the equation for the axis of symmetry is \( x = \frac{-b}{2a} \).

For this equation, \( a = 4 \) and \( b = 8 \). Find the equation for the axis of symmetry.

\[
x = \frac{-b}{2a} = \frac{-8}{2(4)} = -1
\]

The \( x \)-coordinate of the vertex is 1. Substitute 1 for \( x \) in the original equation to find the \( y \)-coordinate.

\[
y = 4x^2 + 8x - 5 = 4(1)^2 + 8(1) - 5 = 4 + 8 - 5 = 7
\]

The \( y \)-intercept is the value of \( c \), which is \(-5\).

2. \( y = -2x^2 + 8x + 5 \)

**SOLUTION:**
For a quadratic of the form \( y = ax^2 + bx + c \), the equation for the axis of symmetry is \( x = \frac{-b}{2a} \).

For this equation, \( a = -2 \) and \( b = 8 \). Find the equation for the axis of symmetry.

\[
x = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2
\]

The \( x \)-coordinate of the vertex is 2. Substitute 2 for \( x \) in the original equation to find the \( y \)-coordinate.

\[
y = -2x^2 + 8x + 5 = -2(2)^2 + 8(2) + 5 = -8 + 16 + 5 = 13
\]

The \( y \)-intercept is the value of \( c \), which is 5.
3. \( y = x^2 - 8x + 9 \)

**SOLUTION:**

For a quadratic of the form \( y = ax^2 + bx + c \), the equation for the axis of symmetry is \( x = \frac{-b}{2a} \).

For this equation, \( a = 1 \) and \( b = -8 \). Find the equation for the axis of symmetry.

\[
\begin{align*}
  x &= \frac{-b}{2a} \\
  x &= \frac{-(-8)}{2(1)} \\
  x &= \frac{8}{2} \\
  x &= 4
\end{align*}
\]

The \( x \)-coordinate of the vertex is 4. Substitute 4 for \( x \) in the original equation to find the \( y \)-coordinate.

\[
\begin{align*}
  y &= x^2 - 8x + 9 \\
  &= (4)^2 - 8(4) + 9 \\
  &= 16 - 32 + 9 \\
  &= -7
\end{align*}
\]

The \( y \)-intercept is the value of \( c \), which is 9.

4. \( y = 4x^2 + 16x - 6 \)

**SOLUTION:**

For a quadratic of the form \( y = ax^2 + bx + c \), the equation for the axis of symmetry is \( x = \frac{-b}{2a} \).

For this equation, \( a = 4 \) and \( b = 16 \). Find the equation for the axis of symmetry.

\[
\begin{align*}
  x &= \frac{-b}{2a} \\
  x &= \frac{-(16)}{2(4)} \\
  x &= \frac{-16}{8} \\
  x &= -2
\end{align*}
\]

The \( x \)-coordinate of the vertex is –2. Substitute –2 for \( x \) in the original equation to find the \( y \)-coordinate.

\[
\begin{align*}
  y &= 4x^2 + 16x - 6 \\
  &= 4(-2)^2 + 16(-2) - 6 \\
  &= 4(4) - 32 - 6 \\
  &= 16 - 32 - 6 \\
  &= -22
\end{align*}
\]

The \( y \)-intercept is the value of \( c \), which is –6.

5. **KICKBALL** A kickball is kicked in the air. The equation \( h = -16t^2 + 60t \) gives the height \( h \) of the ball in feet after \( t \) seconds.

a. What is the height of the ball after one second?

b. When will the ball reach its maximum height?

c. When will the ball hit the ground?

**SOLUTION:**

a. Substitute 1 into the equation.

\[
\begin{align*}
  h &= -16t^2 + 60t \\
  &= -16(1)^2 + 60(1) \\
  &= -16 + 60 \\
  &= 44
\end{align*}
\]

b. The maximum height is the \( y \)-coordinate of the vertex. The time in which the ball reaches the maximum height is the \( x \)-coordinate.
For a quadratic of the form \( y = ax^2 + bx + c \), the equation for the axis of symmetry is \( x = \frac{-b}{2a} \).

For this equation, \( a = -16 \) and \( b = 60 \). Find the equation for the axis of symmetry.

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-(60)}{2(-16)}
\]

\[
x = \frac{60}{32}
\]

\[
x = 1.875
\]

The \( x \)-coordinate of the vertex is 1.875, so the maximum height is reached at 1.875 seconds.

c. The ball reaches the ground when \( h = 0 \).

\[
0 = -16t^2 + 60t
\]

\[
0 = -4t(4t - 15)
\]

\[-4t = 0 \text{ or } 4t - 15 = 0
\]

\[
t = 0 \text{ or } t = \frac{15}{4}
\]

\[
t = 0 \text{ or } t = 3.75
\]

The ball is at the ground before it is kicked (at \( t = 0 \)) and it hits the ground after 3.75 seconds.

Solve each equation by graphing.

6. \(-2x^2 - 2x + 4 = 0\)

**SOLUTION:**

Graph the corresponding function: \( f(x) = -2x^2 - 2x + 4 \).

Find the axis of symmetry.

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-(2)}{2(-2)}
\]

\[
x = -0.5
\]

\[
y = -2x^2 - 2x + 4
\]

\[
y = -2(-0.5)^2 - 2(-0.5) + 4
\]

\[
y = 4.5
\]

The vertex is at \((-0.5, 4.5)\). Plot additional points on either side of the axis of symmetry.

The solutions are the \( x \)-coordinates of the locations where the graph intersects the \( x \)-axis.

\(-2, 1\)

7. \(x^2 = -2x + 3\)

**SOLUTION:**

Graph the corresponding function: \( f(x) = x^2 + 2x - 3 \).

Find the axis of symmetry.
Chapter 9

\[ x = \frac{-b}{2a} \]
\[ = \frac{-(2)}{2(1)} \]
\[ = -1 \]

\[ y = x^2 + 2x + 3 \]
\[ = (-1)^2 + 2(-1) - 3 \]
\[ = 1 - 2 - 3 \]
\[ = -4 \]

The vertex is at \((-1, -4)\). Plot additional points on either side of the axis of symmetry.

\[ y = x^2 + 2x - 3 \]
\[ = (-3)^2 + 2(-3) - 3 \]
\[ = 9 - 6 - 3 \]
\[ = 0 \]

\[ y = x^2 + 2x - 3 \]
\[ = (1)^2 + 2(1) - 3 \]
\[ = 1 + 2 - 3 \]
\[ = 0 \]

The solutions are the x-coordinates of the locations where the graph intersects the x-axis.

8. MARBLES Jason shot a marble straight up using a slingshot. The equation \( h = -16t^2 + 42t + 5.5 \) models the height \( h \), in feet, of the marble after \( t \) seconds. After how long will the marble hit the ground?

**SOLUTION:**

Graph the equation.

![Graph of the equation](image)

The graph intersects the x-axis at about 2.7.

**Describe how the graph of each function is related to the graph of** \( f(x) = x^2 \).

9. \( g(x) = -x^2 - 4 \)

**SOLUTION:**

The value of \( a \) is negative, so the graph is reflected across the x-axis. The value of \( c \) is \(-4\), so the graph is translated 4 units down.

10. \( h(x) = 7x^2 + 2 \)

**SOLUTION:**

The value of \( c \) is 2, so the graph is translated 2 units up. The value of \( a \) is 7, so the graph is stretched vertically.
11. \( y = 3x^2 \)

**SOLUTION:**
This graph is stretched vertically due to \( a \) being 3. The vertex is at (0, 0) because \( c = 0 \). This is graph A.

12. \( y = \frac{1}{4}x^2 \)

**SOLUTION:**
This graph is compressed vertically due to \( a \) being less than 1. The vertex is at (0, 0) because \( c = 0 \). This is graph B.

13. \( y = -(x + 4)^2 \)

**SOLUTION:**
This graph is reflected across the \( x \)-axis due to \( a \) being negative. The vertex is at \((-4, 0)\) because the equation is in vertex form and \( h = -4 \). This is graph D.

14. \( y = 2(x - 3)^2 \)

**SOLUTION:**
This graph is compressed vertically due to \( a \) being 2. The vertex is at \((3, 0)\) because the equation is in vertex form and \( h = 3 \). This is graph C.
15. **BIOLOGY** The number of cells in a Petri dish can be modeled by the quadratic equation
\[ n = 6t^2 - 4.5t + 74 \] where \( t \) is the number of hours the cells have been in the dish. When will there be 200 cells in the Petri dish?

**SOLUTION:**
Find \( t \) when \( n = 200 \).

\[ n = 6t^2 - 4.5t + 74 \]
\[ 200 = 6t^2 - 4.5t + 74 \]
\[ 0 = 6t^2 - 4.5t - 126 \]
\[ 0 = t^2 - \frac{4.5}{6}t - 21 \]
\[ 0 = t^2 - \frac{3}{2}t - 21 \]
\[ 0 = t^2 - \left( \frac{3}{2} \right)^2 + \frac{3}{2} \left( \frac{3}{2} \right)^2 - 21 - \left( \frac{3}{2} \right)^2 \]
\[ 0 = \left( t - \frac{3}{2} \right)^2 - 21 + \frac{9}{4} \]
\[ 0 = \left( t - \frac{3}{2} \right)^2 - 23.25 \]
\[ 23.25 = \left( t - \frac{3}{2} \right)^2 \]
\[ \pm \sqrt{23.25} = t - \frac{3}{2} \]
\[ \frac{3}{2} \pm \sqrt{23.25} = t \]
\[ t = 6.3 \text{ or } -3.3 \]

There will not be 200 cells in the dish after 6 hours, but there will be 200 after 7 hours.

**Solve each equation by completing the square. Round to the nearest tenth if necessary.**

16. \( x^2 + 4x - 8 = 5 \)

**SOLUTION:**
\( x^2 + 4x - 8 = 5 \)
\( x^2 + 4x = 13 \)
\( x^2 + 4x + 4 = 13 + 4 \)
\( (x + 2)^2 = 17 \)
\( x + 2 = \pm \sqrt{17} \)
\( x = -2 \pm \sqrt{17} \)
\( x \approx 2.1 \text{ or } -6.1 \)

17. \( 3x^2 + 5x = 18 \)

**SOLUTION:**
\( 3x^2 + 5x = 18 \)
\( x^2 + \frac{5}{3}x = 6 \)
\( x^2 + 2 \left( \frac{5}{6} \right) x + \left( \frac{5}{6} \right)^2 = 6 + \left( \frac{5}{6} \right)^2 \)
\( \left( x + \frac{5}{6} \right)^2 = 6 + \frac{25}{36} \)
\( x + \frac{5}{6} = \pm \sqrt{6.69444} \)
\( x = -\frac{5}{6} \pm \sqrt{6.69444} \)
\( x \approx 1.8 \text{ or } -3.4 \)

18. Find the value of \( x \) in the figure if the area is 36 square inches.

**SOLUTION:**
\[ A = 0.5bh \]
\[ 36 = 0.5(x + 8)(x + 2) \]
\[ 72 = x^2 + 10x + 16 \]
\[ 56 = x^2 + 10x \]
\[ 56 + 25 = x^2 + 10x + 25 \]
\[ 81 = (x + 5)^2 \]
\[ \pm \sqrt{81} = x + 5 \]
\[ -5 \pm 9 = x \]
\[ 4 \text{ or } -14 = x \]

Length is positive, so \( x = 4 \).
Chapter 9

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

19. \[3x^2 + 10x = 15\]

**SOLUTION:**

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-10 \pm \sqrt{(10)^2 - 4(3)(-15)}}{2(3)}
\]

\[
= \frac{-10 \pm \sqrt{100 + 180}}{6}
\]

\[
= \frac{-10 \pm \sqrt{280}}{6}
\]

\[
= \frac{-10 \pm 13.97}{6}
\]

\[x = -4.5 \text{ or } 1.1
\]

20. \[\frac{1}{2}x^2 - 8x + 6 = 0\]

**SOLUTION:**

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(0.5)(6)}}{2(0.5)}
\]

\[
= \frac{8 \pm \sqrt{64 - 12}}{1}
\]

\[
= \frac{8 \pm \sqrt{52}}{1}
\]

\[x = 15.2 \text{ or } 0.8
\]

State the value of the discriminant. Then determine the number of solutions of the equation.

21. \[4x^2 - 12x = -9\]

**SOLUTION:**

Write the equation in \(ax^2 + bx + c\) form.

\[4x^2 - 12x + 9 = 0\]

\[a = 4, \ b = -12 \text{ and } c = 9
\]

\[( -12)^2 - 4(4)(9) = 144 - 144 = 0
\]

When the discriminant is 0, there is one real solution.

22. \[3x^2 + 8 = 9x\]

**SOLUTION:**

Write the equation in \(ax^2 + bx + c\) form.

\[3x^2 - 9x + 8 = 0\]

\[a = 3, \ b = -9 \text{ and } c = 8
\]

\[( -9)^2 - 4(3)(8) = 81 - 96 = -15
\]

When the discriminant is negative, there are no real solutions.
23. | x  | 2  | 3  | 4  | 5  | 6  |
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<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>9/4</td>
<td>27/8</td>
<td>81/16</td>
<td>243/32</td>
<td>729/64</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Find the first differences.
\[
\begin{align*}
\frac{27}{8} - \frac{9}{4} &= \frac{0}{8} \\
\frac{81}{16} - \frac{27}{8} &= \frac{27}{16} \\
\frac{243}{32} - \frac{81}{16} &= \frac{81}{32} \\
\frac{729}{64} - \frac{243}{32} &= \frac{243}{64}
\end{align*}
\]

The first differences are not equal. Find the second differences.
\[
\begin{align*}
\frac{27}{8} - \frac{9}{4} &= \frac{0}{8} \\
\frac{81}{16} - \frac{27}{8} &= \frac{27}{16} \\
\frac{243}{32} - \frac{81}{16} &= \frac{81}{32} \\
\frac{729}{64} - \frac{243}{32} &= \frac{243}{64}
\end{align*}
\]

The first differences are not equal. Find the ratios.
\[
\begin{align*}
\frac{27}{8} - \frac{9}{4} &= \frac{27}{8} \times \frac{4}{9} = \frac{3}{2} \\
\frac{81}{16} - \frac{27}{8} &= \frac{81}{16} \times \frac{8}{27} = \frac{3}{2} \\
\frac{243}{32} - \frac{81}{16} &= \frac{243}{32} \times \frac{16}{81} = \frac{3}{2}
\end{align*}
\]

The ratios of successive y-values are equal, so the function is exponential.

24. | x  | -2 | -1 | 0  | 1  | 2  |
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-13</td>
<td>-6.25</td>
<td>0</td>
<td>5.75</td>
<td>11</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Find the first differences.
\[
\begin{align*}
-6.25 - (-13) &= 6.75 \\
0 - (-6.25) &= 6.25 \\
5.75 - 0 &= 5.75 \\
11 - 5.75 &= 5.25
\end{align*}
\]

The first differences are not equal. Find the second differences.
\[
\begin{align*}
6.25 - 6.75 &= -0.50 \\
5.75 - 6.25 &= -0.50 \\
5.25 - 5.75 &= -0.50
\end{align*}
\]

The second differences are equal, so the function is quadratic.

Graph each function. State the domain and range.
25. \(f(x) = |x + 5|\)

**SOLUTION:**
The vertex is located where \(x + 5 = 0\), or at \(x = -5\).

Graph \(y = x + 5\) for \(x > -5\), and graph \(y = -(x + 5)\) for \(x < -5\).

The domain is all real numbers and the range is \(|y| y \geq 0\).
26. \( f(x) = 2 \lfloor x \rfloor \)

**SOLUTION:**
This is a greatest integer function.

For \( 0 \leq x < 1 \), \( y = 0 \)
For \( 1 \leq x < 2 \), \( y = 2 \)
For \( 2 \leq x < 3 \), \( y = 4 \)

Repeat the process, with a solid circle on the left and an open circle on the right.

The domain is all real numbers and the range is all multiples of 2.
Graph each function. Compare to the parent graph. State the domain and range.

1. \( y = 4\sqrt{x + 2} \)

**SOLUTION:**
The radicand must be nonnegative, so \( x + 2 \geq 0 \). Therefore, the domain is \( x \geq -2 \).

Make a table of values, starting with \( x = -2 \). Then graph. Some good points to use for \( x \) are 2, 3, 6, and 11. These values make the radicand a perfect square, eliminating the square root.

![Graph of \( y = 4\sqrt{x + 2} \)](image)

The graph is translated 2 units to the left, since it begins at \( x = -2 \).

The graph is compressed vertically because of the coefficient of 4.

2. \( y = -3\sqrt{x} \)

**SOLUTION:**
The radicand must be nonnegative, so \( x \geq 0 \). Therefore, the domain is \( x \geq 0 \).

Make a table of values, starting with \( x = 0 \). Then graph. Some good points to use for \( x \) are 0, 1, 4, and 9. These values make the radicand a perfect square, eliminating the square root.

![Graph of \( y = -3\sqrt{x} \)](image)

The graph is translated reflected across the \( x \)-axis and is compressed vertically because of the coefficient of \(-3\). The range is all negative values of \( y \).
Chapter 10

3. \( y = -2\sqrt{x - 2} \)

**SOLUTION:**
The radicand must be nonnegative, so \( x - 2 \geq 0 \).
Therefore, the domain is \( x \geq 2 \).

Make a table of values, starting with \( x = 2 \). Then graph. Some good points to use for \( x \) are 2, 3, 6, and 11. These values make the radicand a perfect square, eliminating the square root.

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
x & 2 & 3 & 6 & 11 & 12 & 14 & 16 & 18 \\
\hline
y & 0 & 2 & 6 & 10 & 12 & 14 & 16 & 18 \\
\hline
\end{array}
\]

The graph is translated reflected across the \( x \)-axis and is compressed vertically because of the coefficient of \(-2\). The graph is also translated 2 units right because of the radicand.

4. \( y = \sqrt{x^2 + 2 + 2} \)

**SOLUTION:**
The radicand must be nonnegative, so \( x + 2 \geq 0 \).
Therefore, the domain is \( x \geq -2 \).

Make a table of values, starting with \( x = -2 \). Then graph. Some good points to use for \( x \) are \(-2\), \(-1\), 2, and 7. These values make the radicand a perfect square, eliminating the square root.

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
x & -2 & -1 & 2 & 7 & 8 & 10 & 12 & 14 \\
\hline
y & 0 & 2 & 6 & 10 & 12 & 14 & 16 & 18 \\
\hline
\end{array}
\]

The graph is also translated 2 units left because of the radicand. It is translated 2 units up because of the +2 after the radicand.

5. **PENDULUMS** Find the period \( T \) of a pendulum, the time in seconds it takes to swing from one side to the other and back, if the length of the pendulum \( L \) is 50 meters and \( T = 2\pi \sqrt{\frac{L}{g}} \), where \( g \) is the gravitational constant, 9.8 meters per second squared.

**SOLUTION:**
Substitute 9.8 for \( g \) and 50 for \( L \).

\[
T = 2\pi \sqrt{\frac{L}{g}} \\
= 2\pi \sqrt{\frac{50}{9.8}} \\
\approx 14.19
\]
Chapter 10

Simplify each expression.
6. \( \sqrt{24 \cdot 3\sqrt{14}} \)
   
   **SOLUTION:**
   
   \[ \sqrt{24 \cdot 3\sqrt{14}} = 3\sqrt{24 \cdot 14} = 3\sqrt{2^3 \cdot 3 \cdot 2 \cdot 7} = 3\sqrt{2^4 \cdot 21} = 3 \cdot 4\sqrt{21} = 12\sqrt{21} \]

7. \( \sqrt{45x^4y^3z^6} \)
   
   **SOLUTION:**
   
   \[ \sqrt{45x^4y^3z^6} = \sqrt{9 \cdot 5 \cdot x^4 \cdot y^2 \cdot y \cdot z^6} = \sqrt{9x^2y^2z^6 \cdot 5y} = 3x^2y^2z^3\sqrt{5y} \]

8. \( \frac{7}{6-\sqrt{10}} \)
   
   **SOLUTION:**
   
   \[ \frac{7}{6-\sqrt{10}} = \frac{7(6+\sqrt{10})}{(6-\sqrt{10})(6+\sqrt{10})} = \frac{42+7\sqrt{10}}{6^2-6\sqrt{10}+6\sqrt{10}-10} = \frac{42+7\sqrt{10}}{36-10} = \frac{42+7\sqrt{10}}{26} \]

9. \( \frac{3}{8+\sqrt{14}} \)
   
   **SOLUTION:**
   
   \[ \frac{3}{8+\sqrt{14}} = \frac{3(8-\sqrt{14})}{(8+\sqrt{14})(8-\sqrt{14})} = \frac{24-3\sqrt{14}}{64-14} = \frac{24-3\sqrt{14}}{50} \]

10. \( 6\sqrt{18} + 3\sqrt{2} \)
    
    **SOLUTION:**
    
    \[ 6\sqrt{18} + 3\sqrt{2} = 6\sqrt{9 \cdot 2} + 3\sqrt{2} = 6 \cdot 3\sqrt{2} + 3\sqrt{2} = 18\sqrt{2} + 3\sqrt{2} = 21\sqrt{2} \]

11. \( \sqrt{6} (\sqrt{24} + 3\sqrt{3}) \)
    
    **SOLUTION:**
    
    \[ \sqrt{6} (\sqrt{24} + 3\sqrt{3}) = \sqrt{6} (\sqrt{4 \cdot 6} + 3\sqrt{3}) = \sqrt{6} (2\sqrt{6} + 3\sqrt{3}) = 2 \cdot 6 + 3\sqrt{18} = 12 + 3\sqrt{9 \cdot 2} = 12 + 9\sqrt{2} \]

12. \( 4\sqrt{7} (3\sqrt{63}) \)
    
    **SOLUTION:**
    
    \[ 4\sqrt{7} (3\sqrt{63}) = 12\sqrt{7 \cdot 63} = 12\sqrt{7 \cdot 3 \cdot 3 \cdot 3} = 12 \cdot 7 \cdot 3 \cdot 3 = 252 \]

13. \( \sqrt{12} + 5\sqrt{48} - 2\sqrt{3} \)
    
    **SOLUTION:**
    
    \[ \sqrt{12} + 5\sqrt{48} - 2\sqrt{3} = \sqrt{4 \cdot 3} + 5\sqrt{16 \cdot 3} - 2\sqrt{3} = 2\sqrt{3} + 5 \cdot 4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3} + 20\sqrt{3} - 2\sqrt{3} = 20\sqrt{3} \]
Graph each function. Compare to the parent graph. State the domain and range.

1. **SOLUTION:**
   The radicand must be nonnegative, so \( \sqrt{\frac{1}{3} - \sqrt{3}} = \frac{1}{\sqrt[3]{3}} - \sqrt{3} \)
   
   \[
   \begin{align*}
   &= \frac{1}{\sqrt[3]{3}} \left( \sqrt[3]{3} \right) - \sqrt{3} \\
   &= \sqrt[3]{3} - 3 \sqrt[3]{3} \\
   &= - \frac{2\sqrt[3]{3}}{3} 
   \end{align*}
   \]

15. **SOLUTION:**
   \[
   (2\sqrt[3]{3} - 2\sqrt[5]{5})(2\sqrt[15]{15} - 4)
   \]
   
   \[
   \begin{align*}
   &= 4\sqrt[3]{45} - 8\sqrt[3]{3} - 4\sqrt[3]{225} + 8\sqrt[3]{5} \\
   &= 4\cdot 3\sqrt[3]{5} - 8\sqrt[3]{3} - 4\cdot 5\sqrt[3]{3} + 8\sqrt[3]{5} \\
   &= 12\sqrt[3]{5} - 8\sqrt[3]{3} - 20\sqrt[3]{3} + 8\sqrt[3]{5} \\
   &= 20\sqrt[3]{5} - 24\sqrt[3]{3}
   \end{align*}
   \]

16. **PARKS** Crandall Lake has a swimming area shaped like a trapezoid. The area can be found using the formula \( A = \frac{1}{2} h(b_1 + b_2) \), where \( h \) represents the height and \( b_1 \) and \( b_2 \) are the lengths of the two bases. What is the area available for swimming?

   \[
   \begin{align*}
   A &= 0.5(b_1 + b_2)h \\
   &= 0.5(3\sqrt{8} + 24 + 10 + 2\sqrt{3}) \cdot 3\sqrt{5} \\
   &= 1.5\sqrt{5}(5\sqrt{8} + 34) \\
   &= 7.5\sqrt{40} + 51\sqrt{5} \\
   &= 7.5\sqrt{4\cdot 10} + 51\sqrt{5} \\
   &= 15\sqrt{10} + 51\sqrt{5}
   \end{align*}
   \]

Solve each equation. Check your solution.

17. \( \sqrt{24} - n = \frac{n}{2} \)
   **SOLUTION:**
   \[
   \begin{align*}
   \sqrt{24} - n &= \frac{n}{2} \\
   24 - n &= 0.5n^2 \\
   0 &= 0.5n^2 + n - 24 \\
   0 &= n^2 + 4n - 96 \\
   0 &= (n - 8)(n + 12)
   \end{align*}
   \]
   \( n = 8 \) or \( n = -12 \).

   Check.
   \[
   \begin{align*}
   \sqrt{24} - n &= \frac{n}{2} \\
   \sqrt{24} - 8 &= \frac{8}{2} \\
   \sqrt{16} &= 4 \\
   4 &= 4 \\
   \sqrt{24} - (-12) &= \frac{-12}{2} \\
   \sqrt{36} &= -6 \\
   6 &\neq -6
   \end{align*}
   \]

   Therefore, the solution is 8.
Chapter 10

18. \( \sqrt{b + 6} = 3\sqrt{15} \)

**SOLUTION:**

\[
\sqrt{b + 6} = 3\sqrt{15} \\
b + 6 = 9 \cdot 15 \\
b + 6 = 135 \\
b = 129
\]

Check.

\[
\sqrt{129 + 6} = 3\sqrt{15} \\
\sqrt{135} = 3\sqrt{15} \\
3\sqrt{15} = 3\sqrt{15}
\]

Therefore, the solution is 129.

---

19. \( f = 2\sqrt{3f + 6} \)

**SOLUTION:**

\[
f = 2\sqrt{3f + 6} \\
f^2 = 4(3f + 6) \\
f^2 = 12f + 24 \\
f^2 - 12f - 24 = 0 \\
f = \frac{-(12) \pm \sqrt{(-12)^2 - 4(3)(-24)}}{2(3)} \\
f = \frac{12 \pm \sqrt{144 + 36}}{6} \\
f = \frac{12 \pm \sqrt{180}}{6} \\
f = \frac{12 \pm 6\sqrt{5}}{6} \\
f = 2 \pm \sqrt{5}
\]

Check by using \( 6 + 2\sqrt{15} \approx 13.75 \) and \( 6 - 2\sqrt{15} \approx -1.75 \).

\[
f = 2\sqrt{3f + 6} \\
13.75 = 2\sqrt{3(13.75) + 6} \\
13.75 = 2\sqrt{47.25} \\
13.75 = 13.75 \checkmark \\
-1.75 = 2\sqrt{3(-1.75) + 6} \\
-1.75 = 2\sqrt{0.75} \\
-1.75 \neq 1.73
\]

Therefore, the solution is \( 6 + 2\sqrt{15} \).
20. \(\sqrt{6 - m} = m + 6\)

**SOLUTION:**
\[
\sqrt{6 - m} = m + 6
\]
\[
6 - m = (m + 6)^2
\]
\[
6 - m = m^2 + 12m + 36
\]
\[
0 = m^2 + 13m + 30
\]
\[
0 = (m + 3)(m + 10)
\]
\[
m = -3 \text{ or } -10
\]

Check.
\[
\sqrt{6 - (-3)} = (-3) + 6
\]
\[
\sqrt{9} = 3
\]
\[
3 = 3
\]
\[
\sqrt{6 - (-10)} = (-10) + 6
\]
\[
\sqrt{16} = -4
\]
\[
4 \neq -4
\]

Therefore, the solution is -3.

21. \(\sqrt{t} = \sqrt{4t - 6}\)

**SOLUTION:**
\[
\sqrt{t} = \sqrt{4t - 6}
\]
\[
t = 4t - 6
\]
\[
6 = 3t
\]
\[
2 = t
\]

Check.
\[
\sqrt{2} = \sqrt{4(2) - 6}
\]
\[
\sqrt{2} = \sqrt{2}
\]

Therefore, the solution is 2.

22. \(5 + \sqrt{17 - m} = m\)

**SOLUTION:**
\[
5 + \sqrt{17 - m} = m
\]
\[
\sqrt{17 - m} = m - 5
\]
\[
17 - m = (m - 5)^2
\]
\[
17 - m = m^2 - 10m + 25
\]
\[
0 = m^2 - 9m + 8
\]
\[
0 = (m - 8)(m - 1)
\]
\[
m = 8 \text{ or } m = 1
\]

Check.
\[
5 + \sqrt{17 - 8} = 8 - 5
\]
\[
\sqrt{9} = 3
\]
\[
3 = 3
\]
\[
5 + \sqrt{17 - 1} = 1 - 5
\]
\[
\sqrt{16} = -4
\]
\[
4 \neq -4
\]

Therefore, the solution is 8.
Find each missing length. If necessary, round to the nearest hundredth.

23. \[ \begin{align*}
7 & \quad d \\
12 & \\
\end{align*} \]

**SOLUTION:**
\[ \begin{align*}
\alpha^2 + b^2 &= c^2 \\
7^2 + 12^2 &= c^2 \\
49 + 144 &= c^2 \\
\sqrt{193} &= c \\
13.89 &\approx c \\
\end{align*} \]

24. \[ \begin{align*}
5 & \\
13 & \quad b \\
\end{align*} \]

**SOLUTION:**
\[ \begin{align*}
\alpha^2 + b^2 &= c^2 \\
5^2 + b^2 &= 13^2 \\
b^2 &= 13^2 - 5^2 \\
b^2 &= 169 - 25 \\
b &= \sqrt{144} \\
b &= 12 \\
\end{align*} \]

25. **BAKING** Jasmen is cutting triangles from dough to bake for her math class. If she wants the base to be 1.5 inches and the height to be 3 inches, how long is the hypotenuse?

**SOLUTION:**
\[ \begin{align*}
\alpha^2 + b^2 &= c^2 \\
1.5^2 + 3^2 &= c^2 \\
2.25 + 9 &= c^2 \\
\sqrt{11.25} &= c \\
3.35 &\approx c \\
\end{align*} \]

Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

26. \(\cos 52^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\cos 52\) into the calculator.

27. \(\tan 28^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\tan 28\) into the calculator.

28. \(\sin 17^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\sin 17\) into the calculator.

29. \(\cos 75^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\cos 75\) into the calculator.

30. \(\tan 55^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\tan 55\) into the calculator.

31. \(\sin 65^\circ\)

**SOLUTION:**
Confirm the mode is in degrees before entering \(\sin 65\) into the calculator.
Solve each right triangle. Round each side length to the nearest tenth.

32. \[ \begin{align*} \triangle ABC & \quad \text{SOLUTION:} \quad B = 180 - (90 + 41) = 49 \\ & \quad \text{Use the Law of Sines.} \\ & \quad \frac{\sin 41}{c} = \frac{\sin 90}{18} \\ & \quad 18 \sin 41 = c \\ & \quad 11.8 \approx c \\ & \quad \frac{\sin 49}{b} = \frac{\sin 90}{18} \\ & \quad 18 \sin 49 = b \\ & \quad 13.6 \approx b \end{align*} \]

33. \[ \begin{align*} \triangle PQR & \quad \text{SOLUTION:} \\ & \quad P = 180 - (90 + 32) = 58 \\ & \quad \text{Use the Law of Sines.} \\ & \quad \frac{\sin 32}{6} = \frac{\sin 90}{q} \\ & \quad q \sin 32 = 6 \\ & \quad q = \frac{6}{\sin 32} \\ & \quad q \approx 11.3 \\ & \quad \frac{\sin 32}{6} = \frac{\sin 58}{p} \\ & \quad p \sin 32 = 6 \sin 58 \\ & \quad p = \frac{6 \sin 58}{\sin 32} \\ & \quad p \approx 9.6 \end{align*} \]

34. **ROLLERBLADING** The path for rollerblading in the park has a vertical rise of 35 feet. The angle the rise makes with the path is 75°. How long is the incline?

\[ \begin{align*} \text{SOLUTION:} \\ & \quad \text{Use trigonometry.} \\ & \quad \cos x = \frac{\text{adjacent}}{\text{hypotenuse}} \\ & \quad \cos 75 = \frac{35}{d} \\ & \quad d \cos 75 = 35 \\ & \quad d = \frac{35}{\cos 75} \\ & \quad d \approx 135.2 \end{align*} \]
Determine whether each table or equation represents an inverse or direct variation. Explain.

1. 

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-3</td>
<td>-6</td>
<td>-9</td>
<td>-12</td>
<td>-15</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Find \( xy \).

\[ 1(-3) = -3 \]
\[ 2(-6) = -12 \]

\( xy \) is not constant, so there is no inverse relation.

Test for a common ratio by dividing the \( y \)-terms by the \( x \)-terms.

\[ -3 \div 1 = -3 \]
\[ -6 \div 2 = -3 \]
\[ -9 \div 3 = -3 \]
\[ -12 \div 4 = -3 \]
\[ -15 \div 5 = -3 \]

There is a common ratio, so there is direct variation.

2. \( y = \frac{-2}{x} \)

**SOLUTION:**
Multiply both sides by \( x \) and we have \( xy = -2 \), which is an inverse relationship.

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>(-\frac{1}{2})</th>
<th>(\frac{1}{2})</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-3</td>
<td>-12</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Find \( xy \).

\[ -2(-3) = 6 \]
\[ -0.5(-12) = 6 \]
\[ 0.5(12) = 6 \]
\[ 2(3) = 6 \]

\( xy \) is constant, so there is an inverse relationship.

State the excluded value for each function.

4. \( y = \frac{2x - 3}{x - 5} \)

**SOLUTION:**
The excluded values are values which make the denominator equal to 0.

If \( x = 5 \), then \( x - 5 = 0 \)

5. \( y = \frac{x + 3}{2x - 3} \)

**SOLUTION:**
The excluded values are values which make the denominator equal to 0.

If \( x = 1 \), then \( 3x - 3 = 0 \)

Identify the asymptotes of each function. Then graph the function.

6. \( y = \frac{-3}{2x - 2} \)

**SOLUTION:**
The vertical asymptotes occur at the excluded values of the function. These values make the denominator equal to 0.

If \( x = 1 \), then \( 2x - 2 = 0 \). So, \( x = 1 \) is a vertical asymptote.

The horizontal asymptote occurs at \( c \) for \( y = \frac{a}{x - b} + c \), and \( c = 0 \), so the asymptote is at \( y = 0 \).

\[ x = 1; y = 0 \]
7. \( y = \frac{2}{3x} - 4 \)

**SOLUTION:**
The vertical asymptotes occur at the excluded values of the function. These values make the denominator equal to 0.

If \( x = 0 \), then \( 3x = 0 \). So, \( x = 0 \) is a vertical asymptote.

The horizontal asymptote occurs at \( c \) for \( y = \frac{a}{x - b} + c \), and \( c = -4 \), so the asymptote is at \( y = -4 \).

\[ x = 0; \, y = -4 \]

---

8. **FUNDRAISER** The Sophomore class has committed to walking 200 miles for a fundraiser. The equation \( y = \frac{200}{x} \) models the number of miles each person will walk depending on the number of volunteers. Graph the function.

**SOLUTION:**
Make a table of values. Note that \( xy = 200 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
</tr>
</tbody>
</table>

The number of miles walked and number of volunteers are both values that must be positive.
State the excluded value(s) for each rational expression.

9. \( \frac{2t^2 - 18}{6t} \)

**SOLUTION:**
The excluded values are values that make the denominator equal 0.

\[
0 = 2t^2 - 18 \\
18 = 2t^2 \\
9 = t^2 \\
\pm 3 = t
\]

10. \( \frac{2x^2 - x - 28}{x - 2} \)

**SOLUTION:**
The excluded values are values that make the denominator equal 0.

\[
0 = 2x^2 - x - 28 \\
0 = (2x + 7)(x - 4) \\
2x + 7 = 0 \quad \text{or} \quad x - 4 = 0 \\
x = \frac{-7}{2} \quad \text{or} \quad x = 4
\]

11. \( \frac{2x + 1}{x - 4} \)

**SOLUTION:**
The excluded values are values that make the denominator equal 0.

\( x - 4 = 0 \) when \( x = 4 \)

12. \( \frac{7y^2}{2y^2 - 5y - 12} \)

**SOLUTION:**
The excluded values are values that make the denominator equal 0.

\[
0 = 2y^2 - 5y - 12 \\
0 = (2y + 3)(y - 4) \\
2y + 3 = 0 \quad \text{or} \quad y - 4 = 0 \\
y = -\frac{3}{2} \quad \text{or} \quad y = 4
\]

Find the zeros of each function.

13. \( f(x) = \frac{x^2 - 2x - 8}{2x^2 + 8x + 6} \)

**SOLUTION:**
Factor the numerator and denominator.

\[
f(x) = \frac{x^2 - 2x - 8}{2x^2 + 8x + 6} = \frac{(x - 4)(x + 2)}{2(x^2 + 4x + 3)} = \frac{(x - 4)(x + 2)}{2(x + 1)(x + 3)}
\]

The zeros occur at \( x \)-values which make the numerator equal 0.

\( x - 4 = 0 \) when \( x = 4 \)

\( x + 2 = 0 \) when \( x = -2 \)

Therefore, the zeros of \( f(x) \) are 4, -2.

14. \( f(x) = \frac{x^2 + 7x + 10}{x + 5} \)

**SOLUTION:**
Factor the numerator and denominator.

\[
f(x) = \frac{x^2 + 7x + 10}{x + 5} = \frac{(x + 5)(x + 2)}{x + 5} = x + 2
\]

The zeros occur at \( x \)-values which make the numerator equal 0.

\( x + 2 = 0 \) when \( x = -2 \)

Therefore, the zero of \( f(x) \) is -2.
Chapter 11

15. \( f(x) = \frac{x^3 + x^2 - 2x}{x^2 - 2x + 1} \)

**SOLUTION:**
Factor the numerator and denominator.

\[
f(x) = \frac{x^3 + x^2 - 2x}{x^2 - 2x + 1} = \frac{x(x^2 + x - 2)}{(x-1)(x-1)} = \frac{x(x+2)(x-1)}{(x-1)(x-1)} = \frac{x(x+2)}{(x-1)}
\]

The zeros occur at \( x \)-values which make the numerator equal 0.

\[ x + 2 = 0 \text{ when } x = -2 \]
\[ x = 0 \text{ when } x = 0 \]

Therefore, the zeros of \( f(x) \) are \(-2, 0\).

16. \( \frac{3a^3}{6b^2} \div \frac{9a^6}{4b^3} \)

**SOLUTION:**

\[
\frac{3a^3}{6b^2} \div \frac{9a^6}{4b^3} = \frac{3a^3}{6b^2} \cdot \frac{4b^3}{9a^6} = \frac{1}{3} \cdot \frac{2b}{3a^3} = \frac{2b}{9a^3}
\]

17. \( \frac{2t-1}{t+2} = \frac{6t^2-2tr+9}{3t^2+2t-8} \)

**SOLUTION:**

\[
\frac{2t-1}{t+2} = \frac{6t^2-2tr+9}{3t^2+2t-8} = \frac{2t-1}{t+2} \cdot \frac{3t^2-2t+9}{3t^2-2t+9} = \frac{2t-1}{t+2} \cdot \frac{3(t-3)(t+3)}{3(t-3)(t+3)} = \frac{3(t-3)}{(t+3)}
\]

18. \( \frac{4x^2yz}{5x^3} + \frac{2yz^4}{7xy^2z^3} \)

**SOLUTION:**

\[
\frac{4x^2yz}{5x^3} + \frac{2yz^4}{7xy^2z^3} = \frac{4x^2yz}{5x^3} \cdot \frac{7xy^2z^3}{2yz^4} = \frac{2x^2}{5} \cdot \frac{7xy^2}{z^3} = \frac{14x^2y^2}{5z^3}
\]

19. \( \frac{4g^2+4g-8}{6n^2+6n-12} \div \frac{5g+10}{5n-5} \)

**SOLUTION:**

\[
\frac{4g^2+4g-8}{6n^2+6n-12} \div \frac{5g+10}{5n-5} = \frac{4(g^2+g-2)}{6(n^2+n-2)} \cdot \frac{5(n-1)}{5(g+2)} = \frac{2(g+2)(g-1)}{3(n+1)(n-1)} \cdot \frac{5(n-1)}{5(g+2)} = \frac{2(g+2)}{3(n+1)}
\]

20. **BOOKS** John has read 6 books recently. If he reads 40 words per minute, each page has approximately 500 words, and each book has an average of 225 pages, how many hours did John spend reading?

**SOLUTION:**

- Books: \( \frac{225 \text{ pages}}{\text{book}} \)
- Pages: \( \frac{500 \text{ words}}{\text{page}} \)
- Words: \( \frac{40 \text{ words}}{\text{minute}} \)

\[
\frac{16,875 \text{ minutes}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = 281.2 \text{ hours}
\]
Chapter 11

21. **AREA** The area \( A \) of a rectangle is given by \( lw \). Find the length of the unknown side.

\[
A = 2x^2 + x - 3
\]

\[
2x + 3
\]

**SOLUTION:**

\[
\frac{2x^2 + x - 3}{2x + 3} = \frac{2x^2 + 3x - 2x - 3}{2x + 3}
\]

\[
= \frac{x(2x + 3) - 1(2x + 3)}{2x + 3}
\]

\[
= \frac{(x - 1)(2x + 3)}{2x + 3}
\]

\[
= x - 1
\]

Find each quotient.

22. \[
\frac{m^2 - m - 6}{m} \div (m - 3)
\]

**SOLUTION:**

\[
\frac{m^2 - m - 6}{m} \div (m - 3) = \frac{m^2 - m - 6}{m} \cdot \frac{1}{m - 3}
\]

\[
= \frac{(m - 3)(m + 2)}{m} \cdot \frac{1}{m - 3}
\]

\[
= \frac{m + 2}{m}
\]

23. \[
(2g^2 - 3g - 2) + (g - 2)
\]

**SOLUTION:**

\[
(2g^2 - 3g - 2) + (g - 2) = \frac{2g^2 - 3g - 2}{g^2 - 2}
\]

\[
= \frac{2g^2 - 4g + g - 2}{g^2 - 2}
\]

\[
= \frac{2g(g - 2) + 1(g - 2)}{g^2 - 2}
\]

\[
= \frac{(2g + 1)(g - 2)}{g^2 - 2}
\]

\[
= 2g + 1
\]

Find each sum or difference.

24. \[
\frac{7}{g} - \frac{2}{3g}
\]

**SOLUTION:**

\[
\frac{7}{g} - \frac{2}{3g} = \frac{21}{3g} - \frac{2}{3g}
\]

\[
= \frac{19}{3g}
\]

25. \[
\frac{3n}{2n + 4} + \frac{n - 2}{2n + 4}
\]

**SOLUTION:**

\[
\frac{3n}{2n + 4} + \frac{n - 2}{2n + 4} = \frac{4n - 2}{2n + 4}
\]

\[
= \frac{2(2n - 1)}{2(n + 2)}
\]

\[
= \frac{2n - 1}{n + 2}
\]

26. \[
\frac{-6}{k + 2} + \frac{k}{k - 2}
\]

**SOLUTION:**

\[
\frac{-6}{k + 2} + \frac{k}{k - 2} = \frac{-6(k - 2)}{(k + 2)(k - 2)} + \frac{k(k + 2)}{(k - 2)(k + 2)}
\]

\[
= \frac{-6k + 12}{(k + 2)(k - 2)} + \frac{k^2 + 2k}{(k - 2)(k + 2)}
\]

\[
= \frac{k^2 - 4k + 12}{(k - 2)(k + 2)}
\]

27. \[
\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}
\]

**SOLUTION:**

\[
\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9} = \frac{2p - 3(p^2 - 9)}{(p - 3)(p + 3)} - \frac{5(p^2 - 2)}{(p - 3)(p + 3)}
\]

\[
= \frac{(2p - 3)p^2 - (2p - 3)(-9) - 5p^2 + 10}{(p - 3)(p + 3)}
\]

\[
= \frac{2p^3 - 2p + 1}{(p - 3)(p + 3)}
\]
28. **TRAINING** Miguel is training for a triathlon. He runs 5 miles at \( x \) miles per hour and 7 miles at 1.5 \( x \) miles per hour. Write and simplify an expression for the time it takes him to run the 12 miles.

**SOLUTION:**
\[
t = t_1 + t_2 = \frac{d_1}{r_1} + \frac{d_2}{r_2} = \frac{5}{x} + \frac{7}{1.5x} = \frac{5(1.5)}{1.5x} + \frac{7}{1.5x} = \frac{14.5}{1.5x}
\]

Simplify each expression.

\[
\begin{align*}
4 \frac{5}{2} & = 4 \frac{5}{9} \\
2 \frac{5}{9} & = 2 \frac{5}{9}
\end{align*}
\]

29. **SOLUTION:**
\[
\begin{align*}
4 \frac{5}{2} & = 4 \frac{5}{9} = 2 \frac{5}{9} \\
& = \frac{22}{5} - \frac{23}{9} \\
& = \frac{22 \cdot 9}{45} - \frac{23 \cdot 5}{45} \\
& = \frac{198}{45} - \frac{115}{45} \\
& = \frac{83}{45}
\end{align*}
\]

\[
\frac{3}{2a + c} = \frac{1 + a}{4a + 8}
\]

**SOLUTION:**
\[
\begin{align*}
\frac{3}{2a + c} & = \frac{1 + a}{4a + 8} \\
& = \frac{3}{2(a+1)} \cdot \frac{1 + a}{4a + 8} \\
& = \frac{12(a+2)}{2(a+1)^2} \\
& = \frac{6(a+2)}{(a+1)^2}
\end{align*}
\]

**Solve each equation. State any extraneous solutions.**

31. \( \frac{a}{a+1} - 1 = \frac{a}{2} \)

**SOLUTION:**
\[
\begin{align*}
\frac{a}{a+1} - 1 & = \frac{a}{2} \\
\frac{a(2) - (2)(a-1)}{2(a-1)} & = \frac{a(a-1)}{2(a-1)} \\
2a - 2a + 2 & = a^2 - a \\
0 & = a^2 - a - 2 \\
0 & = (a - 2)(a + 1) \\
a & = 2 \text{ or } a = -1
\end{align*}
\]

33. \( \frac{4}{w - 2} = \frac{-1}{w + 3} \)

**SOLUTION:**
\[
\begin{align*}
\frac{4}{w - 2} & = \frac{-1}{w + 3} \\
4(w + 3) & = -1(w - 2) \\
4w + 12 & = -w + 2 \\
5w & = -10 \\
w & = -2
\end{align*}
\]
34. \[
\frac{1}{y+2} + \frac{1}{y-2} = \frac{3}{y^2-4}
\]

\textbf{SOLUTION:}

\[
\frac{y-2}{(y+2)(y-2)} + \frac{y+2}{(y+2)(y-2)} = \frac{3}{(y+2)(y-2)}
\]

\[
y-2 + y+2 = 3
\]

\[
2y = 3
\]

\[
y = \frac{3}{2}
\]

35. \textbf{RECYCLING} Jamie and Adam volunteer at a recycle sorting center. Jamie sorts 3 bins in 2 hours. Adam sorts 3 bins in 3 hours. If they work together, how long would it take them to sort the 3 bins?

\textbf{SOLUTION:}

Jamie's rate is 1.5 bins per hour.

Adam's rate is 1 bin per hour.

Combined, they can do 2.5 bins per hour.

Together, they can do 3 bins in \(3 ÷ 2.5 = 1.2\) hours.
Determine whether each situation calls for a survey, an experiment, or an observational study. Explain your reasoning.

1. **AUTO REPAIR** An auto repair company sends each customer a letter asking them to rate their experience during their last service appointment.

   **SOLUTION:**
   Data are collected from responses given by a sample regarding their characteristics, behaviors, or opinions. This is a survey.

2. **ICE CREAM** An ice cream manufacturer is testing a new formula for their chocolate cherry flavor. They randomly give half of a group of 150 people the original formula, and the other half gets the new formula. Both are asked how they like their ice cream.

   **SOLUTION:**
   The sample is divided into two groups:
   - an experimental group that undergoes a change, and
   - a control group that does not undergo the change.
   The effect on the experimental group is then compared to the control group.

   This is an experiment.

3. **GAS** A random sample of 1000 car owners in Los Angeles were surveyed about the price they last paid for a gallon of gasoline. Identify the sample and the population for the situation. Then describe the sample statistic and the population parameter.

   **SOLUTION:**
   The sample: 1000 car owners in Los Angeles

   The sample is a part of the population, so the population is all Los Angeles drivers, and not all drivers in the country. Every part of the population must have an equal chance to be in the sample.

   The sample statistic is the average price of the sample.

   The population parameter is the average price in Los Angeles.

4. **BOWLING** Tina’s results for 8 bowling games are \{110, 123, 147, 119, 153, 142, 113, 143\}. Find and interpret the standard deviation of the data.

   **SOLUTION:**
   Find the mean.

   \[ \frac{110 + 123 + 147 + 119 + 153 + 142 + 113 + 143}{8} = 131.25 \]

   Find the square of the difference of each term and the mean.

   \[
   \begin{align*}
   (110 – 131.25)^2 &= 451.5625 \\
   (123 – 131.25)^2 &= 68.0625 \\
   (147 – 131.25)^2 &= 248.0625 \\
   (119 – 131.25)^2 &= 150.0625 \\
   (153 – 131.25)^2 &= 473.0625 \\
   (142 – 131.25)^2 &= 115.5625 \\
   (113 – 131.25)^2 &= 333.0625 \\
   (143 – 131.25)^2 &= 138.0625
   \end{align*}
   \]

   Find the square root of the sum of these values.

   The standard deviation is about 16.8.
For Exercises 5 and 6, use these data. 
\{12, 18, 21, 18, 19, 18, 16, 23, 20, 15, 17, 18\}

5. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a histogram.

SOLUTION:
Enter the data into your calculator and create a histogram.

\[ \text{Histogram} \]

[12, 25] scl: 2 by [-1, 6] scl: 1

The data is fairly evenly distributed, so use the mean and standard deviation.

\[ \text{1-Var Stats} \]

\[
\begin{align*}
\bar{x} &= 17.91666667 \\
\sum x &= 215 \\
\sum x^2 &= 3941 \\
5x &= 2.843120352 \\
\sigma x &= 2.722080495 \\
\downarrow n &= 12
\end{align*}
\]

The mean is about 18 and the standard deviation is 2.8.

6. Find the mean, median, mode, range, and standard deviation of the data after multiplying each value by 3.

SOLUTION:
Multiply the data by 3.

\{12, 18, 21, 18, 19, 18, 16, 23, 20, 15, 17, 18\}

\{36, 54, 63, 54, 57, 54, 48, 69, 60, 45, 51, 54\}

Mode: 54 occurs the most often.

Median: Arrange the data in increasing order.

\{36, 45, 48, 51, 54, 54, 54, 54, 57, 60, 63, 69\}

54 is the median.

Enter the values into your calculator and find the mean and standard deviation.

\[ \text{1-Var Stats} \]

\[
\begin{align*}
\bar{x} &= 53.75 \\
\sum x &= 645 \\
\sum x^2 &= 35469 \\
5x &= 8.529361055 \\
\sigma x &= 8.166241486 \\
\downarrow n &= 12
\end{align*}
\]

The mean is 53.75 and the standard deviation is 8.5.

7. BASKETBALL Keisha has a 75\% free throw average. Describe how to simulate her next 10 free throws.

SOLUTION:
A few options are possible.

A 6-sided die can be rolled, with rolls of 5 or 6 being re-rolls.

A spinner with 4 equal sections can be spun.

A random number generator can be used and set up for digits 1-4.
8. FLIGHTS Three airlines have the departure results shown.
a. What is the experimental probability of each airline having an on-time departure?
b. What is the experimental probability for an on-time departure for all the airlines?

<table>
<thead>
<tr>
<th>Status</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>on-time</td>
<td>21</td>
<td>19</td>
<td>24</td>
</tr>
<tr>
<td>late</td>
<td>6</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

SOLUTION:
a. \( \frac{21}{27} \approx 78\% \)
b. \( \frac{10}{22} \approx 45\% \)
c. \( \frac{24}{33} \approx 73\% \)

b. \( \frac{21+19+24}{27+22+33} = \frac{64}{82} \approx 78\% \)

Evaluate each expression.
9. \( P(8, 5) \)

SOLUTION:
\[ P(8, 5) = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 8 \times 7 \times 6 \times 5 \times 4 = 6720 \]

10. \( P(7, 3) \)

SOLUTION:
\[ P(7, 3) = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \times 6 \times 5 = 210 \]

11. \( C(8, 5) \)

SOLUTION:
\[ C(8, 5) = \frac{8!}{(8-5)!5!} = \frac{8!}{3!5!} = 56 \]

12. \( C(7, 3) \)

SOLUTION:
\[ C(7, 3) = \frac{7!}{(7-3)!3!} = \frac{7!}{4!3!} = 35 \]

13. DOG SHOW There are 12 dogs in the finals for best in show. If 4 dogs will get ribbons, how many can the ribbons be awarded?

SOLUTION:
Order matters because the ribbons are distinguished between 1st and 4th. We have 12 objects, taken 4 at a time.

This is a permutation. \( P(12, 4) = 11,880 \)

14. SOCKS A drawer contains 4 blue socks, 8 red socks, and 12 white socks. What is the probability of drawing two white socks in successive draws?

SOLUTION:
\[ \frac{12\text{ white}}{24\text{ total}} \times \frac{11\text{ remaining white}}{23\text{ remaining socks}} = \frac{12 \times 11}{24 \times 23} = \frac{11}{46} \]

A card is drawn from a standard deck of playing cards. Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.

15. \( P(\text{eight or king}) \)

SOLUTION:
A card cannot be both an 8 and a king, so they are mutually exclusive.
\[ P(\text{8 or king}) = P(\text{8}) + P(\text{king}) \]
\[ = \frac{1}{13} + \frac{1}{13} \]
\[ = \frac{2}{13} \]

16. \( P(\text{heart or club}) \)

SOLUTION:
A card cannot be both a heart and a club, so the events are mutually exclusive.
\[ P(\text{heart or club}) = P(\text{heart}) + P(\text{club}) \]
\[ = \frac{1}{4} + \frac{1}{4} \]
\[ = \frac{1}{2} \]
Chapter 12

17. \( P(\text{two or red card}) \)

\text{SOLUTION:} 

The 2 of hearts and 2 of diamonds are both 2s and red, so the events are not mutually exclusive.

\[ P(2 \text{ or red}) = P(2) + P(\text{red}) - P(2 \text{ & red}) \]

\[ = \frac{1}{13} + \frac{1}{2} - \frac{2}{52} \]

\[ = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} \]

\[ = \frac{28}{52} \]

\[ = \frac{7}{13} \]

18. \( P(\text{even number or ace}) \)

\text{SOLUTION:} 

A card cannot be both an even number and an ace, so the events are mutually exclusive.

\[ P(\text{even or ace}) = P(\text{even}) + P(\text{ace}) \]

\[ = \frac{5}{13} + \frac{1}{13} \]

\[ = \frac{6}{13} \]

19. \textbf{SWEATERS} A local pet store sells five different sizes of dog sweaters. The table shows the probability distribution of each size sold in a month.

a. Show that the distribution is valid.

b. What is the probability that a randomly chosen purchase is XL or larger?

c. Find the probability that a customer purchased a medium sweater.

\begin{center}
\textbf{Dog Sweater Sizes}
\begin{tabular}{|c|c|}
\hline
Size & Probability \\
\hline
S & 0.38 \\
M & 0.29 \\
L & 0.13 \\
XL & 0.11 \\
XL+ & 0.09 \\
\hline
\end{tabular}
\end{center}

\text{SOLUTION:} 

a. The sum of the probabilities is 1. 0.38 + 0.29 + 0.13 + 0.11 + 0.09 = 1

The probability of each value of \( X \) is greater than or equal to 0 and less than or equal to 1.

b. 0.11 + 0.09 = 0.20

c. 0.29
20. **MOVIES** A movie theater surveyed all attendees to a movie on Sunday afternoon to determine the average age. The table shows the results.

<table>
<thead>
<tr>
<th>Movie Attendee Age</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–10</td>
<td>11</td>
</tr>
<tr>
<td>11–20</td>
<td>115</td>
</tr>
<tr>
<td>21–30</td>
<td>82</td>
</tr>
<tr>
<td>31–40</td>
<td>15</td>
</tr>
<tr>
<td>41–50</td>
<td>37</td>
</tr>
<tr>
<td>51+</td>
<td>28</td>
</tr>
</tbody>
</table>

a. Which category will have the highest probability? Explain.
b. Find the probability of a person being between 21 and 40 years of age.
c. Find the probability of someone being over 40.

**SOLUTION:**
a. The 11–20 age group because it has the highest tally of customers.

\[
\frac{32 + 15}{11 + 115 + 82 + 15 + 37 + 28} = \frac{97}{288} \approx 33.7\% 
\]
b. 

\[
\frac{37 + 28}{11 + 115 + 82 + 15 + 37 + 28} = \frac{65}{288} \approx 2.3\% 
\]