0-6: Multiplying Probabilities

An Independent Event does not affect the probability of a second event occurring.

### Key Concept: Probability of Independent Events

If two events $A$ and $B$ are independent, then the probability that $A$ and $B$ will occur is

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

### Example:

A coin is tossed and a die is rolled. What is the probability of the coin landing on tails and rolling a 3?

Since the outcome of tossing the coin does not affect the outcome of rolling the die, these events are independent.

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A Dependent Event does affect the probability of a second event occurring.

### Key Concept: Probability of Dependent Events

If two events $A$ and $B$ are dependent, then the probability that $A$ and $B$ will occur is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$

The probability of "B" given "A".

### Example:

A bag contains 12 red, 9 blue, 11 yellow, and 8 green marbles. If two marbles are drawn at random and not replaced, what is the probability that a red and then a blue marble are drawn?

The event of drawing the first marble affects the probability of drawing the second marble, because there is one fewer marble from which to choose. So, the events are dependent.
Conditional Probability: The probability of an event "A" occurring given that event "B" has already occurred.

Key Concept: Conditional Probability

If A and B are dependent events, then the conditional probability of event B occurring, given that event A has already occurred, is

$$ P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} $$

where \( P(A) \neq 0 \).

Example 1

At a restaurant, 25% of customers order chili. If 4% of customers order chili and a baked potato, find the probability that someone who orders chili also orders a baked potato.

$$ P(\text{baked potato} \mid \text{chili}) = \frac{P(\text{chili and baked potato})}{P(\text{chili})} $$

Conditional probability

$$ = \frac{0.04}{0.25} $$

$$ = 0.16 $$

The probability that someone who orders chili also orders a baked potato is 16%.

Example 1

A card is drawn from a standard deck of 52 cards. A spinner with four equal sections of different colors is spun. What is the probability of drawing a jack and the spinner landing on red?
Example 2

On the last day of the grading period, three students with the highest average get to pick a prize from a bag that contains 4 pairs of sunglasses, 5 key chains, and 6 car air fresheners. What is the probability that the first student draws a pair of sunglasses, the second student draws a car air freshener, and the third student draws a keychain?

Example 3

A bag contains 8 red pencils, 6 yellow pencils, and 5 purple pencils. Two pencils are drawn one at a time and not replaced. What is the probability that the second pencil is purple, given that the first pencil is red?
Example 4

ATHLETES The table below shows the number of students who are varsity and junior varsity athletes.

<table>
<thead>
<tr>
<th>Class</th>
<th>Varsity</th>
<th>Junior Varsity</th>
</tr>
</thead>
<tbody>
<tr>
<td>freshman</td>
<td>7</td>
<td>269</td>
</tr>
<tr>
<td>sophomore</td>
<td>22</td>
<td>262</td>
</tr>
<tr>
<td>junior</td>
<td>36</td>
<td>276</td>
</tr>
<tr>
<td>senior</td>
<td>51</td>
<td>257</td>
</tr>
</tbody>
</table>

a. Find the probability that a student is a varsity athlete given that he or she is a junior.

b. Find the probability that a student is a junior varsity athlete given that he or she is a senior.