State the domain and range of each relation. Then determine whether each relation is a function. Write yes or no.

1. \( \{(2, 7), (3, 10), (1, 6)\} \)

**SOLUTION:**
The domain is the set of \( x \)-coordinates.
\( D = \{1, 2, 3\} \)
The range is the set of \( y \)-coordinates.
\( R = \{6, 7, 10\} \)
For each element of the domain, there is only one corresponding element in the range. So, this relation is a function.

**ANSWER:**
\( D = \{1, 2, 3\}, \ R = \{6, 7, 10\}; \) yes

2. \( \{(-6, 0), (5, 5), (9, -2), (-2, -9)\} \)

**SOLUTION:**
The domain is the set of \( x \)-coordinates.
\( D = \{-6, -2, 5, 9\} \)
The range is the set of \( y \)-coordinates.
\( R = \{-9, -2, 0, 5\} \)
For each element of the domain, there is only one corresponding element in the range. So, this relation is a function.

**ANSWER:**
\( D = \{-6, -2, 5, 9\}, \ R = \{-9, -2, 0, 5\}; \) yes

3. \[
\begin{array}{c|c}
\hline
x & y \\
\hline
1 & 5 \\
2 & 7 \\
1 & 9 \\
\hline
\end{array}
\]

**SOLUTION:**
The domain is the set of \( x \)-coordinates.
\( D = \{1, 2\} \)
The range is the set of \( y \)-coordinates.
\( R = \{5, 7, 9\} \)
Because 1 is paired with 5 and 9, this is not a function.

**ANSWER:**
\( D = \{1, 2\}, \ R = \{5, 7, 9\}; \) no

4. \[
\begin{array}{c|c}
\hline
x & y \\
\hline
-12 & 0 \\
-10 & 1 \\
-8 & 2 \\
-6 & 4 \\
\hline
\end{array}
\]

**SOLUTION:**
The domain is the set of \( x \)-coordinates.
\( D = \{-12, -10, -8, -6\} \)
The range is the set of \( y \)-coordinates.
\( R = \{0, 1, 2, 4\} \)
For each element of the domain, there is only one corresponding element in the range. So, this relation is a function.

**ANSWER:**
\( D = \{-12, -10, -8, -6\}, \ R = \{0,1,2,4\}; \) yes
5. **SOLUTION:**
The domain is the set of $x$-coordinates.
$D = \{-2,-1,0,3\}$
The range is the set of $y$-coordinates.
$R = \{-3,-2,2\}$
For each element of the domain, there is only one corresponding element in the range. So, this relation is a function.

**ANSWER:**
$D = \{-2,-1,0,3\}, R = \{-3,-2,2\}; \text{yes}$

6. **SOLUTION:**
The domain is the set of $x$-coordinates.
$D = \{-8,-7,-4\}$
The range is the set of $y$-coordinates.
$R = \{2,3,5,6\}$
Because $-4$ is paired with 2 and 3, this is not a function.

**ANSWER:**
$D = \{-8,-7,-4\}, R = \{2,3,5,6\}; \text{no}$

7. **SOLUTION:**
The domain is the set of $x$-coordinates.
$D = \{-1,0,1,2,3\}$
The range is the set of $y$-coordinates.
$R = \{-3,-2,-1,2,3,4\}$
Because 1 is paired with $-1$ and 2, this is not a function.

**ANSWER:**
$D = \{-1,0,1,2,3\}, R = \{-3,-2,-1,2,3,4\}; \text{no}$

8. **SOLUTION:**
The domain is the set of $x$-coordinates.
$D = \{-3,-2,0,1,2,3,4\}$
The range is the set of $y$-coordinates.
$R = \{-1,0,1,2\}$
For each element of the domain, there is only one corresponding element in the range. So, this relation is a function.

**ANSWER:**
$D = \{-3,-2,0,1,2,3,4\}, R = \{-1,0,1,2\}; \text{yes}$
0-1 Representing Functions

Name the quadrant in which each point is located.

9. (5, 3)

   SOLUTION:
   The point (5, 3) has a positive x-coordinate and a positive y-coordinate. The point is located in Quadrant I.

   ANSWER:
   I

10. (8, –6)

    SOLUTION:
    The point (8, –6) has a positive x-coordinate and a negative y-coordinate. The point is located in Quadrant IV.

    ANSWER:
    IV

11. (2, 0)

    SOLUTION:
    Since the point (2, 0) lies on the x-axis, the point is not in any quadrant.

    ANSWER:
    none

12. (–7, –1)

    SOLUTION:
    The point (–7, –1) has a negative x-coordinate and a negative y-coordinate. The point is located in Quadrant III.

    ANSWER:
    III
Find each product.

1. \((a + 2)(a + 4)\)

SOLUTION:
Use the FOIL method to find the product.

\((a + 2)(a + 4)\)

\[= a \cdot a + a(4) + 2(a) + 2(4)\]

\[= a^2 + 4a + 2a + 8\]

\[= a^2 + 6a + 8\]

ANSWER:
\(a^2 + 6a + 8\)

2. \((v - 7)(v - 1)\)

SOLUTION:
Use the FOIL method to find the product.

\((v - 7)(v - 1)\)

\[= v \cdot v + v \cdot (-1) + (-7)(v) + (-7)(-1)\]

\[= v^2 - v - 7v + 7\]

\[= v^2 - 8v + 7\]

ANSWER:
\(v^2 - 8v + 7\)

3. \((h + 4)(h - 4)\)

SOLUTION:
Use the FOIL method to find the product.

\((h + 4)(h - 4)\)

\[= h(h) + (h)(-4) + (4)(h) + (4)(-4)\]

\[= h^2 - 4h + 4h - 16\]

\[= h^2 - 16\]

ANSWER:
\(h^2 - 16\)

4. \((d - 1)(d + 1)\)

SOLUTION:
Use the FOIL method to find the product.

\((d - 1)(d + 1)\)

\[= (d)(d) + (d)(1) + (-1)(d) + (-1)(1)\]

\[= d^2 + d - d - 1\]

\[= d^2 - 1\]

ANSWER:
\(d^2 - 1\)

5. \((b + 4)(b - 3)\)

SOLUTION:
Use the FOIL method to find the product.

\((b + 4)(b - 3)\)

\[= (b)(b) + (b)(-3) + (4)(b) + (4)(-3)\]

\[= b^2 - 3b + 4b - 12\]

\[= b^2 + b - 12\]

ANSWER:
\(b^2 + b - 12\)

6. \((t - 9)(t + 11)\)

SOLUTION:
Use the FOIL method to find the product.

\((t - 9)(t + 11)\)

\[= (t)(t) + (t)(11) + (-9)(t) + (-9)(11)\]

\[= t^2 + 11t - 9t - 99\]

\[= t^2 + 2t - 99\]

ANSWER:
\(t^2 + 2t - 99\)
Find each product.

1. \((a + 2)(a + 4)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(a + 2)(a + 4) = (a)(a) + (a)(4) + (2)(a) + (2)(4)
\]

\[
= a^2 + 4a + 2a + 8
\]

\[
= a^2 + 6a + 8
\]

**ANSWER:**
\(a^2 + 6a + 8\)

The area of the current office is 7 square feet larger than that of the new office. Therefore, the area of the new office is 120 square feet.

Write expressions for the dimensions of Monica's new building.

- Let \(x\) be the number greater than the number \(n\).
- Write expressions for the two numbers.

Find each product.

2. \((k - 2)(k + 5)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(k - 2)(k + 5) = (k)(k) + (k)(5) + (-2)(k) + (-2)(5)
\]

\[
= k^2 + 5k - 2k - 10
\]

\[
= k^2 + 3k - 10
\]

**ANSWER:**
k\(^2\) + 3\(k\) - 10

3. \((p + 8)(p + 8)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
\]

\[
= p^2 + 8p + 8p + 64
\]

\[
= p^2 + 16p + 64
\]

**ANSWER:**
p\(^2\) + 16\(p\) + 64

4. \((r + 3)(r - 8)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(r + 3)(r - 8) = (r)(r) + (r)(-8) + (3)(r) + (3)(-8)
\]

\[
= r^2 - 8r + 3r - 24
\]

\[
= r^2 - 5r - 24
\]

**ANSWER:**
r\(^2\) - 5\(r\) - 24

5. \((q + 2)(q + 4)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(q + 2)(q + 4) = (q)(q) + (q)(4) + (2)(q) + (2)(4)
\]

\[
= q^2 + 4q + 2q + 8
\]

\[
= q^2 + 6q + 8
\]

**ANSWER:**
\(q^2 + 6q + 8\)

6. \((b + 3)(b - 2)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(b + 3)(b - 2) = (b)(b) + (b)(-2) + (3)(b) + (3)(-2)
\]

\[
= b^2 - 2b + 3b - 6
\]

\[
= b^2 + b - 6
\]

**ANSWER:**
b\(^2\) + \(b\) - 6

7. \((t + 2)(t + 1)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(t + 2)(t + 1) = (t)(t) + (t)(1) + (2)(t) + (2)(1)
\]

\[
= t^2 + t + 2t + 2
\]

\[
= t^2 + 3t + 2
\]

**ANSWER:**
t\(^2\) + 3\(t\) + 2

8. \((k + 2)(k + 3)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(k + 2)(k + 3) = (k)(k) + (k)(3) + (2)(k) + (2)(3)
\]

\[
= k^2 + 3k + 2k + 6
\]

\[
= k^2 + 5k + 6
\]

**ANSWER:**
k\(^2\) + 5\(k\) + 6

9. \((p + 1)(p + 1)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(p + 1)(p + 1) = (p)(p) + (p)(1) + (1)(p) + (1)(1)
\]

\[
= p^2 + p + p + 1
\]

\[
= p^2 + 2p + 1
\]

**ANSWER:**
p\(^2\) + 2\(p\) + 1

10. \((x - 15)(x - 15)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(x - 15)(x - 15) = (x)(x) + (x)(-15) + (-15)(x) + (-15)(-15)
\]

\[
= x^2 - 15x - 15x + 225
\]

\[
= x^2 - 30x + 225
\]

**ANSWER:**
x\(^2\) - 30\(x\) + 225

11. \((2c + 1)(c - 5)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(2c + 1)(c - 5) = (2c)(c) + (2c)(-5) + (1)(c) + (1)(-5)
\]

\[
= 2c^2 - 10c + c - 5
\]

\[
= 2c^2 - 9c - 5
\]

**ANSWER:**
2\(c^2\) - 9\(c\) - 5

12. \((7n - 2)(n + 3)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(7n - 2)(n + 3) = (7n)(n) + (7n)(3) + (-2)(n) + (-2)(3)
\]

\[
= 7n^2 + 21n - 2n - 6
\]

\[
= 7n^2 + 19n - 6
\]

**ANSWER:**
7\(n^2\) + 19\(n\) - 6
0-2 FOIL

13. \((3m + 4)(2m - 5)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(3m + 4)(2m - 5) = (3m)(2m) + (3m)(-5) + (4)(2m) + (4)(-5)
\]
\[
= 6m^2 - 15m + 8m - 20
\]
\[
= 6m^2 - 7m - 20
\]
**ANSWER:**
\(6m^2 - 7m - 20\)

14. \((5g + 1)(6g + 9)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(5g + 1)(6g + 9) = (5g)(6g) + (5g)(9) + (1)(6g) + (1)(9)
\]
\[
= 30g^2 + 45g + 6g + 9
\]
\[
= 30g^2 + 51g + 9
\]
**ANSWER:**
\(30g^2 + 51g + 9\)

15. \((2q - 17)(q + 2)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(2q - 17)(q + 2) = (2q)(q) + (2q)(2) + (-17)(q) + (-17)(2)
\]
\[
= 2q^2 + 4q - 17q - 34
\]
\[
= 2q^2 - 13q - 34
\]
**ANSWER:**
\(2q^2 - 13q - 34\)

16. \((4t - 7)(3t - 12)\)

**SOLUTION:**
Use the FOIL method to find the product.

\[
(4t - 7)(3t - 12) = (4t)(3t) + (4t)(-12) + (-7)(3t) + (-7)(-12)
\]
\[
= 12t^2 - 48t - 21t + 84
\]
\[
= 12t^2 - 69t + 84
\]
**ANSWER:**
\(12t^2 - 69t + 84\)

17. **NUMBERS** I am thinking of two integers. One is 7 less than a number, and the other is 2 greater than the same number.

a. Write expressions for the two numbers.
b. Write a polynomial expression for the product of the numbers.

**SOLUTION:**
a. Let \(n\) be a number. The words **7 less than a number** represent the expression \((n - 7)\). The words **2 greater than the number** represent the expression \((n + 2)\).

b. Use the FOIL method to find the polynomial expression for the product of \((n - 7)\) and \((n + 2)\).

\[
(n - 7)(n + 2)
\]
\[
= n^2 + 2n - 7n - 14
\]
\[
= n^2 - 5n - 14
\]
**ANSWER:**
a. \(n - 7, n + 2\)
b. \(n^2 - 5n - 14\)
18. **OFFICE SPACE** Monica’s current office is square. Her office in the company’s new building will be 3 feet wider and 5 feet longer.

a. Write expressions for the dimensions of Monica’s new office.

b. Write a polynomial expression for the area of Monica’s new office.

c. Suppose Monica’s current office is 7 feet by 7 feet. How much larger will her new office be?

**SOLUTION:**

a. Let \(x\) (in feet) be the side length of Monica’s current office. The width of the new office is 3 feet wider than the current office. So, the width of the new office is \((x + 3)\). The length of the new office is 5 feet longer than the current office. So, the length of the new office is \((x + 5)\). Therefore, the dimensions of the new office are \((x + 3)\) and \((x + 5)\) feet.

b. To find the area of the new office, find the product of \((x + 3)\) and \((x + 5)\). Use the FOIL method to find the product.

\[
(x + 3)(x + 5) = (x)(x) + (x)(5) + (3)(x) + (3)(5)
\]

\[
= x^2 + 5x + 3x + 15
\]

\[
= x^2 + 8x + 15
\]

The expression for the area of new office is \(x^2 + 8x + 15\) square feet, where \(x\) is the side length of the current office.

c. The area of the current office is \(7^2\) or 49 square feet.

Substitute 7 for \(x\) in the expression \(x^2 + 8x + 15\) to find the area of the new office.

\[
x^2 + 8x + 15 = 7^2 + 8(7) + 15
\]

\[
= 49 + 56 + 15
\]

\[
= 120
\]

The area of the new office is 120 square feet. Therefore, the area of the new office is 71 square feet larger than that of the current office.

**ANSWER:**

a. \(x + 3, x + 5\)

b. \(x^2 + 8x + 15\)

c. \(71 \text{ ft}^2\)
0-3 Factoring Polynomials

Factor each polynomial.

1. \(12x^2 + 4x\)

**SOLUTION:**
Use the Distributive Property to factor.

\[12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x\]
\[4x = 2 \cdot 2 \cdot x\]

The GCF of \(12x^2\) and \(4x\) is \(2 \cdot 2 \cdot x\) or \(4x\).

Rewrite each term using the GCF.

\[12x^2 + 4x = 4x(3x) + 4x(1)\]
\[= 4x(3x + 1)\]

**ANSWER:**

\(4x(3x + 1)\)

2. \(6x^2 y + 2x\)

**SOLUTION:**
Use the Distributive Property to factor.

\[6x^2 y = 2 \cdot 3 \cdot x \cdot x \cdot y\]
\[2x = 2 \cdot x\]

The GCF of \(6x^2 y\) and \(2x\) is \(2 \cdot x\) or \(2x\).

Rewrite each term using the GCF.

\[6x^2 y + 2x = 2x(3xy) + 2x(1)\]
\[= 2x(3xy + 1)\]

**ANSWER:**

\(2x(3xy + 1)\)

3. \(8ab^2 - 12ab\)

**SOLUTION:**
Use the Distributive Property to factor.

\[8ab^2 = 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b\]
\[12ab = 2 \cdot 2 \cdot 3 \cdot a \cdot b\]

The GCF of \(8ab^2\) and \(12ab\) is \(2 \cdot 2 \cdot a \cdot b\) or \(4ab\).

Rewrite each term using the GCF.

\[8ab^2 - 12ab = 4ab(2b) + 4ab(-3)\]
\[= (4ab)(2b - 3)\]

**ANSWER:**

\(4ab(2b - 3)\)

4. \(x^2 + 5x + 4\)

**SOLUTION:**
In this trinomial, \(b\) is 5 and \(c\) is 4. Find two numbers with a product of 4 and a sum of 5.

<table>
<thead>
<tr>
<th>Factors of 4</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,4</td>
<td>5</td>
</tr>
<tr>
<td>2,2</td>
<td>4</td>
</tr>
</tbody>
</table>

The correct factors are 1 and 4.

\[x^2 + 5x + 4 = (x + m)(x + p)\]
\[= (x + 1)(x + 4)\]

**ANSWER:**

\((x + 1)(x + 4)\)

5. \(y^2 + 12y + 27\)

**SOLUTION:**
In this trinomial, \(b\) is 12 and \(c\) is 27. Find two numbers with a product of 27 and a sum of 12.

<table>
<thead>
<tr>
<th>Factors of 27</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,27</td>
<td>28</td>
</tr>
<tr>
<td>3,9</td>
<td>12</td>
</tr>
</tbody>
</table>

The correct factors are 3 and 9.

\[y^2 + 12y + 27 = (y + m)(y + p)\]
\[= (y + 3)(y + 9)\]

**ANSWER:**

\((y + 3)(y + 9)\)

6. \(x^2 + 6x + 8\)

**SOLUTION:**
In this trinomial, \(b\) is 6 and \(c\) is 8. Find two numbers with a product of 8 and a sum of 6.

<table>
<thead>
<tr>
<th>Factors of 8</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,8</td>
<td>9</td>
</tr>
<tr>
<td>2,4</td>
<td>6</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 4.

\[x^2 + 6x + 8 = (x + m)(x + p)\]
\[= (x + 2)(x + 4)\]

**ANSWER:**

\((x + 2)(x + 4)\)
7. \(3y^2 + 13y + 4\)

**SOLUTION:**
In this trinomial, \(a = 3\), \(b = 13\), and \(c = 4\). Find two numbers with a product of \(3(4) = 12\) and a sum of 13.

<table>
<thead>
<tr>
<th>Factors of 12</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,12</td>
<td>13</td>
</tr>
<tr>
<td>3,4</td>
<td>7</td>
</tr>
</tbody>
</table>

The correct factors are 1 and 12.

\[
3y^2 + 13y + 4 = 3y^2 + (13 - 12)y + 4
\]

\[
= 3y^2 + y + 12y + 4
\]

\[
= (3y^2 + y) + (12y + 4)
\]

\[
y(3y + 1) + 4(3y + 1)
\]

\[
= (3y + 1)(y + 4)
\]

**ANSWER:**
\((3y + 1)(y + 4)\)

8. \(7x^2 + 51x + 14\)

**SOLUTION:**
In this trinomial, \(a = 7\), \(b = 51\), and \(c = 14\). Find two numbers with a product of \(7(14) = 98\) and a sum of 51.

<table>
<thead>
<tr>
<th>Factors of 98</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,98</td>
<td>99</td>
</tr>
<tr>
<td>2,49</td>
<td>51</td>
</tr>
<tr>
<td>7,14</td>
<td>21</td>
</tr>
</tbody>
</table>

The correct factors are 2 and 49.

\[
7x^2 + 51x + 14 = 7x^2 + mx + px + 14
\]

\[
= 7x^2 + 49x + 2x + 14
\]

\[
= (7x^2 + 49x) + (2x + 14)
\]

\[
= 7x(x + 7) + 2(x + 7)
\]

\[
= (x + 7)(7x + 2)
\]

**ANSWER:**
\((7x + 2)(x + 7)\)

9. \(3x^2 + 28x + 32\)

**SOLUTION:**
In this trinomial, \(a = 3\), \(b = 28\), and \(c = 32\). Find two numbers with a product of \(3(32) = 96\) and a sum of 28.

<table>
<thead>
<tr>
<th>Factors of 96</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,96</td>
<td>97</td>
</tr>
<tr>
<td>2,48</td>
<td>50</td>
</tr>
<tr>
<td>3,32</td>
<td>35</td>
</tr>
<tr>
<td>4,24</td>
<td>28</td>
</tr>
<tr>
<td>6,16</td>
<td>22</td>
</tr>
<tr>
<td>8,12</td>
<td>20</td>
</tr>
</tbody>
</table>

The correct factors are 24 and 4.

\[
3x^2 + 28x + 32 = 3x^2 + mx + px + 32
\]

\[
= 3x^2 + 24x + 4x + 32
\]

\[
= (3x^2 + 24x) + (4x + 32)
\]

\[
= 3(x + 8) + 4(x + 8)
\]

\[
= (x + 8)(3x + 4)
\]

**ANSWER:**
\((3x + 4)(x + 8)\)

10. \(x^2 - 5x + 6\)

**SOLUTION:**
In this trinomial, \(b = -5\) and \(c = 6\). This means that \(m + p\) is negative and \(mp\) is positive. So \(m\) and \(p\) must both be negative.

<table>
<thead>
<tr>
<th>Factors of 6</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1,-6</td>
<td>-7</td>
</tr>
<tr>
<td>-2,-3</td>
<td>-5</td>
</tr>
</tbody>
</table>

The correct factors are -2 and -3.

\[
x^2 - 5x + 6 = (x + m)(x + p)
\]

\[
= [x + (-2)][x + (-3)]
\]

\[
= (x - 2)(x - 3)
\]

**ANSWER:**
\((x - 2)(x - 3)\)
11. $y^2 - 5y + 4$

**SOLUTION:**
In this trinomial, $b = -5$ and $c = 4$. This means that $m + p$ is negative and $mp$ is positive. So, $m$ and $p$ must both be negative.

<table>
<thead>
<tr>
<th>Factors of 4</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1, -4</td>
<td>-5</td>
</tr>
<tr>
<td>-2, -2</td>
<td>-4</td>
</tr>
</tbody>
</table>

The correct factors are -1 and -4.

$$y^2 - 5y + 4 = (y + m)(y + p)$$

$$= [y + (-1)][y + (-4)]$$

$$= (y - 1)(y - 4)$$

**ANSWER:**
$(y - 1)(y - 4)$

12. $6x^2 - 13x + 5$

**SOLUTION:**
In this trinomial, $a = 6$, $b = -13$, and $c = 5$. This means that $m + p$ is negative and $mp$ is positive. So, $m$ and $p$ must both be negative.

<table>
<thead>
<tr>
<th>Factors of 30</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30, -1</td>
<td>-31</td>
</tr>
<tr>
<td>-15, -2</td>
<td>-17</td>
</tr>
<tr>
<td>-10, -3</td>
<td>-13</td>
</tr>
<tr>
<td>-6, -5</td>
<td>-11</td>
</tr>
</tbody>
</table>

The correct factors are -10 and -3.

$$6x^2 - 13x + 5 = 6x^2 - 3x - 10x + 5$$

$$= (6x^2 - 3x) + (-10x + 5)$$

$$= 3x(2x - 1) + 5(-2x + 1)$$

$$= 3x(2x - 1) - 5(2x - 1)$$

$$= (2x - 1)(3x - 5)$$

**ANSWER:**
$(3x - 5)(2x - 1)$

13. $6a^2 - 50ab + 16b^2$

**SOLUTION:**

$$6a^2 - 50ab + 16b^2 = 2(3a^2 - 25ab + 8b^2)$$

$$= 2(3a^2 - 24ab - ab + 8b^2)$$

$$= 2[(3a^2 - 24ab) + (-ab + 8b^2)]$$

$$= 2[3a(a - 8b) + b(-a + 8b)]$$

$$= 2[3a(a - 8b) - b(a - 8b)]$$

$$= 2(a - 8b)(3a - b)$$

**ANSWER:**
$2(3a - b)(a - 8b)$

14. $11x^2 - 78x + 7$

**SOLUTION:**
In this trinomial, $a = 11$, $b = -78$, and $c = 7$. This means that $m + p$ is negative and $mp$ is positive. So, $m$ and $p$ must both be negative.

<table>
<thead>
<tr>
<th>Factors of 77</th>
<th>Sum of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-77, -1</td>
<td>-78</td>
</tr>
<tr>
<td>-11, -7</td>
<td>-18</td>
</tr>
</tbody>
</table>

The correct factors are -77 and -1.

$$11x^2 - 78x + 7 = 11x^2 - 77x - x + 7$$

$$= (11x^2 - 77x) + (-x + 7)$$

$$= 11x(x - 7) + (-x + 7)$$

$$= 11x(x - 7) - 1(x - 7)$$

$$= (x - 7)(11x - 1)$$

**ANSWER:**
$(11x - 1)(x - 7)$

15. $18x^2 - 31xy + 6y^2$

**SOLUTION:**
Factor by grouping.

$$18x^2 - 31xy + 6y^2 = 18x^2 - 27xy - 4xy + 6y^2$$

$$= (18x^2 - 27xy) + (-4xy + 6y^2)$$

$$= 9x(2x - 3y) + 2y(-2x + 3y)$$

$$= 9x(2x - 3y) - 2y(2x - 3y)$$

$$= (2x - 3y)(9x - 2y)$$

**ANSWER:**
$(2x - 3y)(9x - 2y)$
0-3 Factoring Polynomials

16. \( x^2 + 4xy + 4y^2 \)
   
   **SOLUTION:**
   
   This is a perfect square trinomial.
   
   \[ x^2 + 4xy + 4y^2 = x^2 + 2(x)(2y) + (2y)^2 \]
   
   \[ = (x + 2y)^2 \]
   
   **ANSWER:**
   
   \((x + 2y)^2\)

17. \( 9x^2 - 24x + 16 \)
   
   **SOLUTION:**
   
   This is a perfect square trinomial.
   
   \[ 9x^2 - 24x + 16 = (3x)^2 - 2(3x)(4) + (4)^2 \]
   
   \[ = (3x - 4)^2 \]
   
   **ANSWER:**
   
   \((3x - 4)^2\)

18. \( 4a^2 + 12ab + 9b^2 \)
   
   **SOLUTION:**
   
   This is a perfect square trinomial.
   
   \[ 4a^2 + 12ab + 9b^2 = (2a)^2 + 2(2a)(3b) + (3b)^2 \]
   
   \[ = (2a + 3b)^2 \]
   
   **ANSWER:**
   
   \((2a + 3b)^2\)

19. \( x^2 - 144 \)
   
   **SOLUTION:**
   
   This polynomial can be factored as a difference of squares.
   
   \[ x^2 - 144 = x^2 - 12^2 \]
   
   \[ = (x + 12)(x - 12) \]
   
   **ANSWER:**
   
   \((x + 12)(x - 12)\)

20. \( 4c^2 - 9 \)
   
   **SOLUTION:**
   
   This polynomial can be factored as a difference of squares.
   
   \[ 4c^2 - 9 = (2c)^2 - 3^2 \]
   
   \[ = (2c + 3)(2c - 3) \]
   
   **ANSWER:**
   
   \((2c + 3)(2c - 3)\)

21. \( 16y^2 - 1 \)
   
   **SOLUTION:**
   
   This polynomial can be factored as a difference of squares.
   
   \[ 16y^2 - 1 = (4y)^2 - 1^2 \]
   
   \[ = (4y + 1)(4y - 1) \]
   
   **ANSWER:**
   
   \((4y + 1)(4y - 1)\)

22. \( 25x^2 - 4y^2 \)
   
   **SOLUTION:**
   
   This polynomial can be factored as a difference of squares.
   
   \[ 25x^2 - 4y^2 = (5x)^2 - (2y)^2 \]
   
   \[ = (5x + 2y)(5x - 2y) \]
   
   **ANSWER:**
   
   \((5x + 2y)(5x - 2y)\)

23. \( 36y^2 - 16 \)
   
   **SOLUTION:**
   
   Factor out a 4. Then factor the rest as a difference of squares.
   
   \[ 36y^2 - 16 = 4(9y^2 - 4) \]
   
   \[ = 4[(3y)^2 - 2^2] \]
   
   \[ = 4(3y + 2)(3y - 2) \]
   
   **ANSWER:**
   
   \(4(3y + 2)(3y - 2)\)
24. $9a^2 - 49b^2$

**SOLUTION:**
This polynomial can be factored as a difference of squares.

$$9a^2 - 49b^2 = (3a)^2 - (7b)^2$$

$$= (3a + 7b)(3a - 7b)$$

**ANSWER:**
$(3a + 7b)(3a - 7b)$
0-4 Counting Techniques

Use the Fundamental Counting Principle to determine the number of outcomes.

1. **FOOD** How many different combinations of sandwich, side, and beverage are possible?

<table>
<thead>
<tr>
<th>Sandwiches</th>
<th>Sides</th>
<th>Beverages</th>
</tr>
</thead>
<tbody>
<tr>
<td>hot dog</td>
<td>chips</td>
<td>bottled water</td>
</tr>
<tr>
<td>hamburger</td>
<td>apple</td>
<td>soda</td>
</tr>
<tr>
<td>veggie burger</td>
<td>pasta salad</td>
<td>juice</td>
</tr>
<tr>
<td>bratwurst</td>
<td></td>
<td>milk</td>
</tr>
<tr>
<td>grilled chicken</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**
There are 5 sandwich choices, 3 sides, and 4 beverages. The number of possible combinations is $5 \times 3 \times 4 = 60$.

**ANSWER:**
60

2. **QUIZZES** Each question on a five question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions?

**SOLUTION:**
There are 4 ways to answer each of the five questions. The number of different ways a students can answer is $4 \times 4 \times 4 \times 4 \times 4 = 1024$.

**ANSWER:**
1024

3. **DANCES** Dane is renting a tuxedo for prom. Once he has chosen his jacket, he must choose from three types of pants and six colors of vests. How many different ways can he select his attire for prom?

**SOLUTION:**
There are 3 choices of pants and 6 choices of vest for each jacket selection. The number of different ways to select his attire for the prom is $3 \times 6 = 18$.

**ANSWER:**
18

4. **MANUFACTURING** A baseball glove manufacturer makes a glove with the different options shown in the table. How many different gloves are possible?

<table>
<thead>
<tr>
<th>Option</th>
<th>Number of Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>sizes</td>
<td>4</td>
</tr>
<tr>
<td>types by position</td>
<td>3</td>
</tr>
<tr>
<td>materials</td>
<td>2</td>
</tr>
<tr>
<td>levels of quality</td>
<td>2</td>
</tr>
</tbody>
</table>

**SOLUTION:**
There are 4 sizes, 3 position types, 2 types of material, and 2 levels of quality. The number of different possible gloves is $4 \times 3 \times 2 \times 2 = 48$.

**ANSWER:**
48

Evaluate each permutation or combination.

5. **$6^P_3$**

**SOLUTION:**

\[
\begin{align*}
\frac{\not{n}!}{(n-r)!(n-r+1)} & \quad \text{Permutations formula} \\
\frac{6!}{(6-3)!} & \quad \text{Subtract} \\
\frac{6!}{3!} & \quad \text{Divide} \\
\frac{6\cdot5\cdot4}{3\cdot2\cdot1} & \quad \text{Simplify} \\
\frac{6\cdot5\cdot4}{6} & \quad \text{Multiply} \\
6\cdot5 & \quad \text{Multiply} \\
120 & \quad \text{Multiply} \\
\end{align*}
\]

**ANSWER:**
120
0-4 Counting Techniques

6. $7P_5$

**SOLUTION:**

\[ mP_r = \frac{n!}{(n-r)!} \]  
\[ 7P_5 = \frac{7!}{(7-5)!} \]  
\[ = \frac{7!}{2!} \]  
\[ = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot \frac{2!}{2!} \]  
\[ = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \]  
\[ = 2520 \]

**ANSWER:**

2520

7. $4C_2$

**SOLUTION:**

\[ mC_r = \binom{n}{r} \]  
\[ 4C_2 = \frac{4!}{(4-2)!2!} \]  
\[ = \frac{4!}{2!2!} \]  
\[ = 4 \cdot 3 \cdot 2 \cdot 1 \cdot \frac{2!}{2!2!} \]  
\[ = 4 \cdot 3 \cdot \frac{2!}{2!} \]  
\[ = 4 \cdot 3 \]  
\[ = 6 \]

**ANSWER:**

6

8. $12C_7$

**SOLUTION:**

\[ mC_r = \binom{n}{r} \]  
\[ 12C_7 = \frac{12!}{(12-7)!7!} \]  
\[ = \frac{12!}{5!7!} \]  
\[ = \frac{12!}{5!7!} \]  
\[ = 6 \]  
\[ = 792 \]

**ANSWER:**

792

9. $6C_1$

**SOLUTION:**

\[ mC_r = \binom{n}{r} \]  
\[ 6C_1 = \frac{6!}{(6-1)!1!} \]  
\[ = \frac{6!}{5!1!} \]  
\[ = 6 \]  

**ANSWER:**

6

10. $9P_5$

**SOLUTION:**

\[ mP_r = \frac{n!}{(n-r)!} \]  
\[ 9P_5 = \frac{9!}{(9-5)!} \]  
\[ = \frac{9!}{4!} \]  
\[ = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \]  
\[ = 15,120 \]

**ANSWER:**

15,120

---

ANSWER:

2520

ANSWER:

6

ANSWER:

792

ANSWER:

6

ANSWER:

15,120
0-4 Counting Techniques

Determine whether each situation involves permutations or combinations. Then solve the problem.

11. SCHOOL Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?

**SOLUTION:**
Because the classes are different, the order in which the classes are taken is important. This situation involves permutations. Use the permutation formula for 6 things taken 6 at a time.

\[ nPr = \frac{n!}{(n-r)!} \]

**Permutations formula**

\[ 6P_6 = \frac{6!}{(6-6)!} \]

\[ = \frac{6!}{0!} \quad \text{Subtract} \]

\[ = 6! \quad \text{or} \quad 720 \quad 0! = 1 \quad \text{and simplify} \]

There are 720 different possible schedules that Charlita could have next year.

**ANSWER:** permutations, 720

12. BALLOONS How many 4-colored groups can be selected from 13 different colored balloons?

**SOLUTION:**
Because the order in which the balloons are chosen is not important, this situation involves combinations. Use the combination formula for 13 things taken 4 at a time.

\[ nC_r = \frac{n!}{(n-r)!r!} \]

**Combination formula**

\[ 13C_4 = \frac{13!}{(13-4)!4!} \]

\[ = \frac{13!}{9!4!} \quad \text{Subtract} \]

\[ = 715 \quad \text{Use a calculator} \]

There are 715 different groups of 4 colored balloons possible.

**ANSWER:** combinations, 715

13. CONTEST How many ways are there to choose the winner and first, second, and third runners-up in a contest with 10 finalists?

**SOLUTION:**
Because the contest winner and runner ups are different, the order in which they are chosen is important. This situation involves permutations. Use the Permutation formula for 10 things taken 4 at a time.

\[ nPr = \frac{n!}{(n-r)!} \]

**Permutations formula**

\[ 10P_4 = \frac{10!}{(10-4)!} \]

\[ = \frac{10!}{6!} \quad 6! \]

\[ = 5040 \quad \text{Use a calculator} \]

There are 5040 different possible ways to choose the winner and runner ups.

**ANSWER:** permutations, 5040

14. BANDS A band is choosing 3 new backup singers from a group of 18 who try out. How many ways can they choose the new singers?

**SOLUTION:**
Because the order in which the singers are chosen is not important, this situation involves combinations. Use the Combination formula for 18 things taken 3 at a time.

\[ nC_r = \frac{n!}{(n-r)!r!} \]

**Combination formula**

\[ 18C_3 = \frac{18!}{(18-3)!3!} \]

\[ = \frac{18!}{15!3!} \quad 15! \]

\[ = 816 \quad \text{Use a calculator} \]

There are 816 different ways for the band to choose 3 new backup singers.

**ANSWER:** combinations, 816
15. **PIZZA** How many different two-topping pizzas can be made if there are 6 options for toppings?

**SOLUTION:**
Because the order in which the two toppings are chosen is not important, this situation involves combinations. Use the Combinations formula for 6 things taken 2 at a time.

\[ nC_r = \frac{n!}{(n-r)!r!} \quad \text{Combination formula} \]

\[ \begin{align*} 
6C_2 &= \frac{6!}{(6-2)!2!} \\
&= \frac{6!}{4!2!} \\
&= 15 \\
\text{Subtract.} \\
\text{Use a calculator.} \\
\end{align*} \]

There are 15 different two-topping pizzas that can be made.

**ANSWER:** combinations, 15

16. **SOFTBALL** How many ways can the manager of a softball team choose players for the top 4 spots in the lineup if she has 7 possible players in mind?

**SOLUTION:**
Because the player assigned to each position is different, the order in which they are chosen is important. This situation involves permutations. Use the Permutations formula for 7 things taken 4 at a time.

\[ nP_r = \frac{n!}{(n-r)!} \quad \text{Permutations formula} \]

\[ \begin{align*} 
7P_4 &= \frac{7!}{(7-4)!} \\
&= \frac{7!}{3!} \\
&= 840 \\
\text{Subtract.} \\
\text{Use a calculator.} \\
\end{align*} \]

There are 840 different possible ways to choose players for the top four spots in the lineup.

**ANSWER:** permutations, 840

17. **NEWSPAPERS** A newspaper has 9 reporters available to cover 4 different stories. How many ways can the reporters be assigned?

**SOLUTION:**
Because the order in which reporters are assigned to stories is different, the order is important. This situation involves permutations. Use the Permutations formula for 9 things taken 4 at a time.

\[ nP_r = \frac{n!}{(n-r)!} \quad \text{Permutations formula} \]

\[ \begin{align*} 
9P_4 &= \frac{9!}{(9-4)!} \\
&= \frac{9!}{5!} \\
&= 3024 \\
\text{Subtract.} \\
\text{Use a calculator.} \\
\end{align*} \]

There are 3024 ways to assign the reporters to the stories.

**ANSWER:** permutations, 3024

18. **READING** Jack has a reading list of 12 books. How many ways can he select 9 books from the list to check out of the library?

**SOLUTION:**
The order in which the 9 books are selected is not important. This situation involves combinations. Use the Combination formula for 12 things taken 9 at a time.

\[ nC_r = \frac{n!}{(n-r)!r!} \quad \text{Combination formula} \]

\[ \begin{align*} 
12C_9 &= \frac{12!}{(12-9)!9!} \\
&= \frac{12!}{3!} \\
&= 220 \\
\text{Subtract.} \\
\text{Use a calculator.} \\
\end{align*} \]

There are 220 ways Jack can select the 9 books to check out of the library.

**ANSWER:** combinations, 220

19. **CHALLENGE** Abby is registering at a Web site and must select a six-character password. The password can contain either letters or digits.

a. How many passwords are possible if characters can be repeated? if no characters can be repeated?
b. How many passwords are possible if all characters are letters that can be repeated? If the password must contain exactly one digit? Which type of password is more secure? Explain.

**SOLUTION:**

a. There are 26 letters and 10 digits from which to choose, or 36 choices for each character in the password. If the characters can be repeated then there are 36 choices for each of the 6 characters or $36^6$. This would equal 2,176,782,336 possible passwords.

If no characters can be repeated, then there are 36 choices for the first, 35 for the second, 34 for the third, 33 for the fourth, 32 for the fifth, and 31 for the sixth. The total number of passwords would be $36 \times 35 \times 34 \times 33 \times 32 \times 31$ or 1,402,410,240.

b. If the characters can be repeated, then there are 26 possible letters for each of the 6 characters. This would give a total of $26^6$ or 308,915,776 possible passwords.

If the password must contain exactly one digit and it is placed first, then the password would have 10 choices for the first character and 26 for each of the next five.

If the digit is the second character, then there are 10 choices for the second and 26 choices for all the others. Likewise the digit could be placed in the third, fourth, fifth, or sixth spot with similar results. The total number of passwords would be six times the product of each way or $6(10 \times 26 \times 26 \times 26 \times 26 \times 26)$ or 712,882,560.

The password with one digit is more secure, because the chance of someone guessing this password at random is $\frac{1}{712,882,560}$, which is less than guessing a 6-character password that contains only letters, $\frac{1}{308,915,776}$.

**ANSWER:**

a. 2,176,782,336; 1,402,410,240

b. 308,915,776; 712,882,560; The password with one digit is more secure, because the chance of someone guessing this password at random is $\frac{1}{712,882,560}$, which is less than the chance of someone guessing a 6-character password that contains only letters,
1. **CARNIVAL GAMES** A spinner has sections of equal size. The table shows the results of several spins.

<table>
<thead>
<tr>
<th>Color</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>6</td>
</tr>
<tr>
<td>blue</td>
<td>7</td>
</tr>
<tr>
<td>yellow</td>
<td>9</td>
</tr>
<tr>
<td>orange</td>
<td>12</td>
</tr>
<tr>
<td>purple</td>
<td>5</td>
</tr>
<tr>
<td>green</td>
<td>11</td>
</tr>
</tbody>
</table>

a. Copy the table and add a column to show the experimental probability of the spinner landing on each of the colors with the next spin.

b. Create a bar graph that shows these experimental probabilities.

c. Add a column to your table that shows the theoretical probability of the spinner landing on each of the colors with the next spin.

d. Create a bar graph that shows these theoretical probabilities.

e. Interpret and compare the graphs you created in parts b and d.

**SOLUTION:**

a. Total spins = 6 + 7 + 9 + 12 + 5 + 11 or 50

<table>
<thead>
<tr>
<th>Color</th>
<th>Frequency</th>
<th>Experimental Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>blue</td>
<td>7</td>
<td>0.14</td>
</tr>
<tr>
<td>yellow</td>
<td>9</td>
<td>0.18</td>
</tr>
<tr>
<td>orange</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>purple</td>
<td>6</td>
<td>0.10</td>
</tr>
<tr>
<td>green</td>
<td>11</td>
<td>0.22</td>
</tr>
</tbody>
</table>

b. The spinner has 6 equal outcomes.

d. Sample answer: Since all the bars in the graph of the theoretical probabilities are the same height, the graph represents a uniform distribution. This means that in theory, the chance of landing on any one of the colors is equally likely. The graph of the experimental probabilities indicates that in practice, it is more likely that the spinner will land on orange or green than on any of the other colors, since the heights of those bars are taller than any others in the graph.

**ANSWER:**
### CARNIVAL GAMES

<table>
<thead>
<tr>
<th>Color</th>
<th>Frequency</th>
<th>Experimental Probability</th>
<th>Theoretical Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>6</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>blue</td>
<td>7</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>yellow</td>
<td>9</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>orange</td>
<td>12</td>
<td>0.24</td>
<td>0.15</td>
</tr>
<tr>
<td>purple</td>
<td>6</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>green</td>
<td>11</td>
<td>0.22</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**SOLUTION:**

The spinner is fair, since all parts are equal, except for the green section, which is larger. The experimental probability is calculated based on the number of spins each color was landed on. The theoretical probability is calculated using the formula for the probability of landing on each color.

- **a.** Copy the table.
  
  The experimental probability of the spinner landing on blue or orange is 0.24 + 0.24 = 0.48 or 12 to 1.
  
  The odds of rolling a number less than 3 are 40 to 80 or 1 to 2.
  
  **ANSWER:** 96 to 24 or 4 to 1; 40 to 80 or 1 to 2

- **b.** For rolling a number less than 3, find the experimental odds against rolling a 1 or a 6.
  
  The odds against rolling a 1 or a 6 are 48 to 4 or 12 to 1.
  
  **ANSWER:** 48 to 4 or 12 to 1

- **c.** For exactly one landing on tails, both landing on heads are 1 to 3.
  
  The events are mutually exclusive, because the sum cannot be both 6 and 7.
  
  The probability of rolling a sum of 6 or 7 is 11 to 36.
  
  **ANSWER:** 11 to 36

- **d.** The events are mutually exclusive, because the sum cannot be both less than 3 and greater than 10.
  
  The probability of rolling a sum less than 3 or greater than 10 is 2 to 9.
  
  **ANSWER:** 2 to 9

- **e.** Sample answer: Since all the bars in the graph of the theoretical probabilities are the same height, the graph represents a uniform distribution. This means that in theory, the chance of landing on any one of the colors is equally likely. The graph of the experimental probabilities indicates that in practice, it is more likely that the spinner will land on orange or green than on any of the other colors, since the heights of those bars are taller than any others in the graph.

---

**Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.**

2. Two dice are rolled.

   a. **P(sum of 10 or doubles)**
   
   **b. P(sum of 6 or 7)**
   
   **c. P(sum < 3 or sum > 10)**

   **SOLUTION:**
   
   a. Because 5 + 5 = 10, these events are not mutually exclusive.
   
   \[ P(\text{sum of 10 or doubles}) = P(10) + P(\text{doubles}) - P(10) \]
   
   \[ = \frac{3}{36} + \frac{6}{36} - \frac{1}{36} = \frac{2}{9}. \]
   
   **b.** The events are mutually exclusive, because the sum cannot be both 6 and 7.
   
   \[ P(6 \text{ or } 7) = P(6) + P(7) \]
   
   \[ = \frac{5}{36} + \frac{6}{36} = \frac{11}{36}. \]
   
   **c.** The events are mutually exclusive, because the sum cannot be both less than 3 and greater than 10.
   
   \[ P(\text{sum < 3 or sum > 10}) = P(\text{sum < 3}) + P(\text{sum > 10}) \]
   
   \[ = \frac{1}{36} + \frac{3}{36} = \frac{4}{9}. \]

   **ANSWER:**
   
   a. not mutually exclusive, \( \frac{2}{9} \)
   
   b. mutually exclusive, \( \frac{11}{36} \)
   
   c. mutually exclusive, \( \frac{4}{9} \)
3. A card is drawn at random from a standard deck of cards.
   a. \( P(\text{club or diamond}) \)
   b. \( P(\text{ace or spade}) \)
   c. \( P(\text{jack or red card}) \)

   **SOLUTION:**
   a. The events are mutually exclusive, because a card cannot be both a club (c) and a diamond (d).
   \[ P(\text{club or diamond}) = P(c) + P(d) \]
   \[ P(\text{club or diamond}) = \frac{13}{52} + \frac{13}{52} = \frac{1}{2}. \]
   b. Because there is an ace (A) of spades (S), these events are not mutually exclusive.
   \[ P(\text{ace or spade}) = P(A) + P(S) - P(A \text{ and } S) \]
   \[ P(\text{ace or spade}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}. \]
   c. Because 2 jacks (J) are red (R), these events are not mutually exclusive.
   \[ P(\text{jack or red}) = P(J) + P(R) - P(J \text{ and } R) \]
   \[ P(\text{jack or red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{7}{13}. \]

   **ANSWER:**
   a. mutually exclusive, \( \frac{1}{2} \)
   b. not mutually exclusive, \( \frac{4}{13} \)
   c. not mutually exclusive, \( \frac{7}{13} \)

4. In a French class, there are 10 freshmen, 8 sophomores, and 2 juniors. Of these students, 9 freshmen, 2 sophomores, and 1 junior are female. A student is selected at random.
   a. \( P(\text{freshman or female}) \)
   b. \( P(\text{sophomore or male}) \)
   c. \( P(\text{freshman or sophomore}) \)

   **SOLUTION:**
   The class contains 10 + 8 + 2 or 20 students.
   a. Because 9 freshmen (Fr) are female (F), these events are not mutually exclusive.
   \[ P(\text{freshman or female}) = P(\text{Fr}) + P(\text{F}) - P(\text{Fr and F}) \]
   \[ P(\text{Fr or F}) = \frac{10}{20} + \frac{12}{20} - \frac{9}{20} = \frac{13}{20}. \]
   b. Because there are 6 sophomores (S) who are males (M), these events are not mutually exclusive.
   \[ P(\text{sophomore or male}) = P(\text{S}) + P(\text{M}) - P(\text{S and M}) \]
   \[ P(\text{S or M}) = \frac{8}{20} + \frac{8}{20} - \frac{6}{20} = \frac{1}{2}. \]
   c. The events are mutually exclusive, because a student cannot be both a freshman (F) and a sophomore (S).
   \[ P(\text{freshman or sophomore}) = P(\text{F}) + P(\text{S}) \]
   \[ P(\text{F or S}) = \frac{10}{20} + \frac{8}{20} = \frac{9}{10}. \]

   **ANSWER:**
   a. not mutually exclusive, \( \frac{13}{20} \)
   b. not mutually exclusive, \( \frac{1}{2} \)
   c. mutually exclusive, \( \frac{9}{10} \)
5. There are 40 vehicles on a rental car lot. All are either sedans or SUVs. There are 18 red vehicles, and 3 of them are sedans. There are 15 blue vehicles, and 9 of them are SUVs. Of the remaining vehicles, all are black and 2 are SUVs. A vehicle is selected at random.

a. $P(\text{blue or black})$

b. $P(\text{red or SUV})$

c. $P(\text{black or sedan})$

**SOLUTION:**
There are 18 red (R), 15 blue (Bl), and 7 black (Bk) vehicles of which 14 are sedans (S) and 26 are SUVs.

a. The events are mutually exclusive, because a vehicle is not both blue and black.

$$P(\text{blue or black}) = P(\text{Bl}) + P(\text{Bk})$$

$$P(\text{blue or black}) = \frac{15}{40} + \frac{7}{40} = \frac{11}{20}.$$  

b. Because there are red SUVs, these events are not mutually exclusive.

$$P(\text{red or SUV}) = P(\text{R}) + P(\text{SUV}) - P(\text{R and SUV})$$

$$P(\text{red or SUV}) = \frac{18}{40} + \frac{26}{40} - \frac{15}{40} = \frac{29}{40}.$$  

c. Because there are black sedans, these events are not mutually exclusive.

$$P(\text{black or sedan}) = P(\text{Bk}) + P(\text{S}) - P(\text{Bk and S})$$

$$P(\text{black or sedan}) = \frac{7}{40} + \frac{14}{40} - \frac{5}{40} = \frac{2}{5}.$$  

**ANSWER:**

a. mutually exclusive, $\frac{11}{20}$

b. not mutually exclusive, $\frac{29}{40}$

c. not mutually exclusive, $\frac{2}{5}$

6. **DRIVING** A survey of Longview High School students found that the probability of a student driving while texting was 0.16, the probability of a student getting into an accident while driving was 0.07, and the probability of a student getting into an accident while driving and texting was 0.05. What is the probability of a student driving while texting or getting into an accident while driving?

**SOLUTION:**
Since some students text while driving $T$ and have accidents $A$, these events are not mutually exclusive.

$$P(T \text{ or } A) = P(T) + P(A) - P(T \text{ and } A)$$

$$P(T \text{ or } A) = 0.16 + 0.07 - 0.05 \text{ or } 0.18$$

The probability is 0.18 or 18%.

**ANSWER:**
0.18 or 18%

7. **REASONING** Explain why the rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ can be used for both mutually exclusive and not mutually exclusive events.

**SOLUTION:**
When events are mutually exclusive, $P(A \text{ and } B)$ will always equal 0, so the probability will simplify to $P(A) + P(B)$.

**ANSWER:**
When events are mutually exclusive, $P(A \text{ and } B)$ will always equal 0, so the probability will simplify to $P(A) + P(B)$.  

ODDS Another measure of the chance that an event will occur is called odds. The odds of an event occurring is a ratio that compares the number of ways an event can occur \(s\) (successes) to the number of ways it cannot occur \(f\) (failure), or \(s\) to \(f\). The sum of the number of success and failures equals the number of possible outcomes.

8. A card is drawn from a standard deck of 52 cards. Find the odds in favor of drawing a heart. Then find the odds against drawing an ace.

**SOLUTION:**
For drawing a heart, \(s = 13\) and \(f = 52 - 13 = 39\). The odds of drawing a heart are 13 to 39 or 1 to 3. For not drawing an ace, \(s = 52 - 4 = 48\) and \(f = 4\). The odds against drawing an ace are 48 to 4 or 12 to 1.

**ANSWER:**
13 to 39 or 1 to 3; 48 to 4 or 12 to 1

9. Two fair coins are tossed. Find the odds in favor of both landing on heads. Then find the odds in favor of exactly one landing on tails.

**SOLUTION:**
For both heads, \(s = 1\) and \(f = 4 - 1 = 3\). The odds of both landing on heads are 1 to 3. For exactly one landing on tails, \(s = 2\) and \(f = 4 - 2 = 2\). The odds of exactly one landing on tails are 2 to 2 or 1 to 1.

**ANSWER:**
1 to 3; 1 to 1

10. The results of rolling a die 120 times are shown. Find the experimental odds against rolling a 1 or a 6. Then find the experimental odds in favor of rolling a number less than 3.

**SOLUTION:**
For not rolling a 1 or a 6, \(s = 120 - (16 + 8) = 96\) and \(f = 120 - 96 = 24\). The odds against rolling a 1 or 6 are 96 to 24 or 4 to 1. For rolling a number less than 3, \(s = 16 + 24 = 40\) and \(f = 120 - 40 = 80\). The odds of rolling a number less than 3 are 40 to 80 or 1 to 2.

**ANSWER:**
96 to 24 or 4 to 1; 40 to 80 or 1 to 2
Determine whether the events are independent or dependent. Then find the probability.

1. A red die and a blue die are rolled. What is the probability of getting the result shown?

   ![Red and Blue Dice]

   SOLUTION:
   Since the outcome of tossing the red die does not affect the outcome of rolling the blue die, these events are independent.

   \[ P(3 \text{ and } 5) = P(3) \cdot P(5) \]
   \[ = \frac{1}{6} \cdot \frac{1}{6} \quad \text{Probability of independent events} \]
   \[ = \frac{1}{36} \quad P(3) = \frac{1}{6} \text{ and } P(5) = \frac{1}{6} \]

   The probability is \( \frac{1}{36} \).

   ANSWER:
   independent; \( \frac{1}{36} \)

2. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

   SOLUTION:
   Since the socks are being selected with out replacement, the events are dependent.

   \[ P(\text{blue}) = \frac{6}{18} = \frac{1}{3} \]
   \[ P(\text{black} | \text{blue}) = \frac{4}{17} \]

   \[ P(\text{blue} | (\text{blue and black})) = \frac{5}{16} \]

   \[ P(\text{blue and black}) = P(\text{black}) \cdot P(\text{blue} | \text{black}) \cdot P(\text{blue} | (\text{blue and black})) \]

   \[ = \frac{1}{3} \cdot \frac{4}{17} \cdot \frac{5}{16} = \frac{5}{204} \]

   The probability is \( \frac{5}{204} \) or about 0.025.

   ANSWER:
   dependent; \( \frac{5}{204} \) or about 0.025

A die is rolled twice. Find each probability.

3. \( P(2 \text{ and } 3) \)

   SOLUTION:
   \[ P(2 \text{ and } 3) = P(2) \cdot P(3) \quad \text{Probability of independent events} \]
   \[ = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \quad P(2) = \frac{1}{6} \text{ and } P(3) = \frac{1}{6} \]

   The probability is \( \frac{1}{36} \).

   ANSWER:
   \( \frac{1}{36} \)

4. \( P(\text{two 4s}) \)

   SOLUTION:
   \[ P(\text{four and four}) = P(4) \cdot P(4) \quad \text{Probability of independent events} \]
   \[ = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \quad P(4) = \frac{1}{6} \text{ and } P(4) = \frac{1}{6} \]

   The probability is \( \frac{1}{36} \).

   ANSWER:
   \( \frac{1}{36} \)

5. \( P(\text{no 6s}) \)

   SOLUTION:
   \[ P(\text{no 6 and no 6}) = P(6) \cdot P(6) \quad \text{Probability of independent events} \]
   \[ = \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36} \quad P(\text{no 6}) = \frac{5}{6} \]

   The probability is \( \frac{25}{36} \).

   ANSWER:
   \( \frac{25}{36} \)

6. \( P(\text{two of the same number}) \)

   SOLUTION:
   \[ P(\text{two of the same number}) = P(2 \text{ and } 2) + P(3 \text{ and } 3) + P(4 \text{ and } 4) + P(5 \text{ and } 5) + P(6 \text{ and } 6) \]
   \[ = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} \]
   \[ = \frac{5}{36} \]

   The probability is \( \frac{5}{36} \).

   ANSWER:
   \( \frac{5}{36} \)
A bag contains 8 blue marbles, 6 red marbles, and 5 green marbles. Three marbles are drawn one at a time. Find each probability.

7. The second marble is green, given that the first marble is blue and not replaced.

\[ P(\text{green|blue}) = \frac{P(\text{green and blue})}{P(\text{blue})} = \frac{\frac{40}{120} \cdot \frac{9}{19}}{\frac{40}{120}} = \frac{9}{19} \]

The probability is \(\frac{9}{19}\).

\[ \text{ANSWER: } \frac{9}{19} \]

8. The second marble is red, given that the first marble is green and is replaced.

\[ P(\text{red|green}) = \frac{P(\text{red and green})}{P(\text{green})} = \frac{\frac{31}{19} \cdot \frac{6}{19}}{\frac{31}{19}} = \frac{6}{19} \]

The probability is \(\frac{6}{19}\).

\[ \text{ANSWER: } \frac{6}{19} \]

9. The third marble is red, given that the first two are red and blue and not replaced.

\[ P(\text{red|red and blue}) = \frac{P(\text{red and red and blue})}{P(\text{red and blue})} = \frac{\frac{345}{514} \cdot \frac{5}{17}}{\frac{345}{514}} = \frac{5}{17} \]

The probability is \(\frac{5}{17}\).

\[ \text{ANSWER: } \frac{5}{17} \]
13. **CARDS** You draw a card from a standard deck of cards and show it to a friend. The friend tells you that the card is red. What is the probability that you correctly guess that the card is the ace of diamonds?

**SOLUTION:**
Given that the card is red, the probability it is an ace is \( \frac{1}{26} \).
The probability is \( \frac{1}{26} \).

**ANSWER:**
\( \frac{1}{26} \)

14. **HONOR ROLL** Suppose the probability that a student takes AP Calculus and is on the honor roll is 0.0035, and the probability that a student is on the honor roll is 0.23. Find the probability that a student takes AP Calculus given that he or she is on the honor roll.

**SOLUTION:**
\[
P(\text{AP Calc} \mid \text{honor roll}) = \frac{P(\text{AP Calc and honor roll})}{P(\text{honor roll})} = \frac{0.0035}{0.23} \approx 0.015
\]
The probability is about 0.015.

**ANSWER:**
about 0.015

15. **DRIVING TESTS** The table shows how students in Mr. Diaz’s class fared on their first driving test. Some took a class to prepare, while others did not. Find each probability.

<table>
<thead>
<tr>
<th></th>
<th>Class</th>
<th>No Class</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>passed</td>
<td>64</td>
<td>48</td>
<td>112</td>
</tr>
<tr>
<td>failed</td>
<td>18</td>
<td>32</td>
<td>50</td>
</tr>
<tr>
<td>Totals</td>
<td>82</td>
<td>80</td>
<td>162</td>
</tr>
</tbody>
</table>

**a.** Paige passed, given that she took the class.
**b.** Madison failed, given that she did not take the class.
**c.** Jamal did not take the class, given that he passed.

**SOLUTION:**
Calculate the column and row totals.

**a.** \( P(\text{pass} \mid \text{class}) = \frac{64}{112} \approx 0.5692 \)

**b.** \( P(\text{fail} \mid \text{no class}) = \frac{32}{80} = \frac{4}{10} = 0.4 \)

**c.** \( P(\text{no class} \mid \text{pass}) = \frac{40}{112} \approx 0.3571 \)

**ANSWER:**
\( \frac{32}{41} \)
\( \frac{4}{5} \)
\( \frac{3}{7} \)

16. **SCHOOL CLUBS** King High School tallied the number of students that were members of at least one after school club.
0-6 Multiplying Probabilities

<table>
<thead>
<tr>
<th>Gender</th>
<th>Clubs</th>
<th>No Clubs</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>male</td>
<td>156</td>
<td>242</td>
<td>398</td>
</tr>
<tr>
<td>female</td>
<td>312</td>
<td>108</td>
<td>420</td>
</tr>
<tr>
<td>Totals</td>
<td>468</td>
<td>350</td>
<td>818</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Calculate the column and row totals.

17. **FOOTBALL ATTENDANCE** The number of students who have attended a football game at North Coast High School is shown. Find each probability.

<table>
<thead>
<tr>
<th>Class</th>
<th>Freshman</th>
<th>Sophomore</th>
<th>Junior</th>
<th>Senior</th>
</tr>
</thead>
<tbody>
<tr>
<td>attended</td>
<td>48</td>
<td>90</td>
<td>224</td>
<td>254</td>
</tr>
<tr>
<td>not attended</td>
<td>182</td>
<td>141</td>
<td>36</td>
<td>8</td>
</tr>
<tr>
<td>Totals</td>
<td>230</td>
<td>231</td>
<td>260</td>
<td>262</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Calculate the column and row totals.

**ANSWER:**

- a. A student is a member of a club given that he is male.
- b. A student is not a member of a club given that she is female.
- c. A student is a male given that he is not a member.

**SOLUTION:**
Given that the student is a freshman, the probability of attending the game is about 79.1%.

**ANSWER:**

- a. 0.91 or about 91%.
- b. 0.77 or about 77.6%.
- c. 0.69 or about 69.1%.
Determine whether each pair of figures is similar, congruent, or neither.

1. **SOLUTION:**

   Because the corresponding angles of two triangles are congruent, and the measures of corresponding sides are proportional, the triangles are similar.

   **ANSWER:**

   similar

2. **SOLUTION:**

   Because the corresponding sides and angles are congruent, the triangles are congruent.

   **ANSWER:**

   congruent

3. **SOLUTION:**

   Because the sides of the polygons are neither congruent nor proportional, the polygons are neither congruent nor similar.

   **ANSWER:**

   neither

4. **SOLUTION:**

   Because the sides of the triangles are neither congruent nor proportional, the triangles are neither congruent nor similar.

   **ANSWER:**

   neither

5. **SOLUTION:**

   Because the corresponding sides of rectangles are proportional, the rectangles are similar.

   **ANSWER:**

   similar

6. **SOLUTION:**

   Because the corresponding sides and angles are congruent, the triangles are congruent.

   **ANSWER:**

   congruent
Each pair of polygons is similar. Find the values of $x$ and $y$.

7. 

SOLUTION:
Because the polygons are similar, the corresponding sides are proportional.

\[
x = \frac{6}{12} = \frac{9}{x} \\
x = 12 \left( \frac{6}{9} \right) \\
= \frac{72}{9} \\
= 8
\]

The value of $x$ is 8.

\[
y = \frac{9 \cdot 14}{6} = \frac{126}{6} = 21
\]

The value of $y$ is 21.

ANSWER:
8; 21

8. 

SOLUTION:
Because the polygons are similar, the corresponding sides are proportional.

\[
x + 12 = \frac{8}{5} \\
x + 12 = \frac{12 \cdot 8}{5} \\
x + 12 = \frac{96}{5} \\
x + 12 = 19.2 \\
x = 19.2 - 12 \\
x = 7.2
\]

The value of $x$ is 7.2.

\[
y = \frac{8}{5} \\
y = \frac{8 \cdot 13}{5} = \frac{104}{5} = 20.8
\]

The value of $y$ is 20.8.

ANSWER:
7.2; 20.8
Determine whether each pair of figures is similar, congruent, or neither.

1. SOLUTION: Because the polygons are similar, corresponding sides are proportional.

\[
x = \frac{10 + 7}{6} = \frac{17}{6} = \frac{17 \cdot 6}{10} = \frac{102}{10} = 10.2
\]

The value of \(x\) is 10.2.

\[
y = \frac{10 + 7}{8} = \frac{17}{8} = \frac{17 \cdot 8}{10} = \frac{136}{10} = 13.6
\]

The value of \(y\) is 13.6.

ANSWER: 10.2; 13.6

10. SHADOWS On a sunny day, Jason measures the length of his shadow and the length of a tree’s shadow. Use the figures at the right to find the height of the tree.

\[
x = \frac{7.5}{1.5} = \frac{7.5(1.5)}{2.5} = \frac{11.25}{2.5} = 4.5
\]

The height of the tree is 4.5 m.

ANSWER: 4.5 m

11. PHOTOGRAPHY A photo that is 4 inches wide by 6 inches long must be reduced to fit in a space 3 inches wide. How long will the reduced photo be?

\[
x = \frac{3}{6} = \frac{3 \cdot 6}{4} = \frac{18}{4} = \frac{4 \cdot 2}{4} = 4 \frac{1}{2}
\]

The length of the reduced photo will be of 4 \(\frac{1}{2}\) inches.

ANSWER: 4 \(\frac{1}{2}\) in.
12. **SURVEYING** Surveyors use instruments to measure objects that are too large or too far away to measure by hand. They can use the shadows that objects cast to find the height of the objects without measuring them. A surveyor finds that a telephone pole that is 25 feet tall is casting a shadow 20 feet long. A nearby building is casting a shadow 52 feet long. What is the height of the building?

**SOLUTION:**
Let \( x \) be the height of the building.

\[
\frac{\text{building height}}{\text{building shadow}} = \frac{\text{pole height}}{\text{pole shadow}}
\]

\[
\frac{x}{52} = \frac{25}{20}
\]

\[
x = \frac{25 \cdot 52}{20}
\]

\[
x = 65
\]

The height of the building is 65 feet.

**ANSWER:**
65 ft
Find each missing measure. Round to the nearest tenth, if necessary.

1. 

SOLUTION:
\[ c^2 = a^2 + b^2 \]
\[ c^2 = 36^2 + 15^2 \]
\[ c^2 = 1296 + 225 \]
\[ c^2 = 1521 \]
\[ c = \sqrt{1521} \]
\[ c = 39 \]
The length of the hypotenuse is 39 ft.

ANSWER:
39 ft

2. 

SOLUTION:
\[ c^2 = a^2 + b^2 \]
\[ 40^2 = a^2 + 32^2 \]
\[ 1600 = a^2 + 1024 \]
\[ 1600 - 1024 = a^2 + 1024 - 1024 \]
\[ 576 = a^2 \]
\[ \sqrt{576} = a \]
\[ 24 = a \]
The length of the missing measure is 24 km.

ANSWER:
24 km

3. 

SOLUTION:
\[ c^2 = a^2 + b^2 \]
\[ 13^2 = 10^2 + b^2 \]
\[ 169 = 100 + b^2 \]
\[ 169 - 100 = 100 + b^2 - 100 \]
\[ 69 = b^2 \]
\[ b^2 = 69 \]
\[ b = \sqrt{69} \]
\[ b \approx 8.3 \]
The length of the missing measure is about 8.3 cm.

ANSWER:
8.3 cm

4. \( a = 3, \ b = 4, \ c = ? \)

SOLUTION:
\[ c^2 = a^2 + b^2 \]
\[ c^2 = 3^2 + 4^2 \]
\[ c^2 = 9 + 16 \]
\[ c^2 = 25 \]
\[ c = \sqrt{25} \]
\[ c = 5 \]

ANSWER:
5
### 0-8 The Pythagorean Theorem

5. $a = ?, b = 12, c = 13$

**SOLUTION:**

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
13^2 &= a^2 + 12^2 \\
169 &= a^2 + 144 \\
169 - 144 &= a^2 + 144 - 144 \\
25 &= a^2 \\
a^2 &= 25 \\
a &= \sqrt{25} \\
a &= 5
\end{align*}
\]

**ANSWER:**

5

6. $a = 14, b = ?, c = 50$

**SOLUTION:**

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
50^2 &= 14^2 + b^2 \\
2500 &= 196 + b^2 \\
2304 &= b^2 \\
b^2 &= 2304 \\
b &= \sqrt{2304} \\
b &= 48
\end{align*}
\]

**ANSWER:**

48

7. $a = 2, b = 9, c = ?$

**SOLUTION:**

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
c^2 &= 2^2 + 9^2 \\
c^2 &= 4 + 81 \\
c^2 &= 85 \\
c &= \sqrt{85} \\
c &\approx 9.2
\end{align*}
\]

**ANSWER:**

9.2

8. $a = 6, b = ?, c = 13$

**SOLUTION:**

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
13^2 &= 6^2 + b^2 \\
169 &= 36 + b^2 \\
169 - 36 &= 36 + b^2 - 36 \\
133 &= b^2 \\
b^2 &= 133 \\
b &= \sqrt{133} \\
b &\approx 11.5
\end{align*}
\]

**ANSWER:**

11.5

9. $a = ?, b = 7, c = 11$

**SOLUTION:**

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
11^2 &= a^2 + 7^2 \\
121 &= a^2 + 49 \\
121 - 49 &= a^2 + 49 - 49 \\
72 &= a^2 \\
a^2 &= 72 \\
a &= \sqrt{72} \\
a &\approx 8.5
\end{align*}
\]

**ANSWER:**

8.5
The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

10. 5 in., 7 in., 8 in.

**SOLUTION:**
Because the longest side is 8 inches, use 8 as \( c \), the measure of the hypotenuse.
\[
c^2 = a^2 + b^2
\]
\[
8^2 = 7^2 + 5^2
\]
\[
64 = 49 + 25
\]
\[
64 \neq 74
\]
Because \( c^2 \neq a^2 + b^2 \), the triangle is not a right triangle.

**ANSWER:**
no

11. 9 m, 12 m, 15 m

**SOLUTION:**
Because the longest side is 15 m, use 15 as \( c \), the measure of the hypotenuse.
\[
c^2 = a^2 + b^2
\]
\[
15^2 = 9^2 + 12^2
\]
\[
225 = 81 + 144
\]
\[
225 = 225 \checkmark
\]
Because \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

**ANSWER:**
yes

12. 6 cm, 7 cm, 12 cm

**SOLUTION:**
Because the longest side is 12 cm, use 12 as \( c \), the measure of the hypotenuse.
\[
c^2 = a^2 + b^2
\]
\[
12^2 = 6^2 + 7^2
\]
\[
144 = 36 + 49
\]
\[
144 \neq 85
\]
Because \( c^2 \neq a^2 + b^2 \), the triangle is not a right triangle.

**ANSWER:**
no

13. 11 ft, 12 ft, 16 ft

**SOLUTION:**
Because the longest side is 16 ft, use 16 as \( c \), the measure of the hypotenuse.
\[
c^2 = a^2 + b^2
\]
\[
16^2 = 11^2 + 12^2
\]
\[
256 = 121 + 144
\]
\[
256 \neq 265
\]
Because \( c^2 \neq a^2 + b^2 \), the triangle is not a right triangle.

**ANSWER:**
no

14. 10 yd, 24 yd, 26 yd

**SOLUTION:**
Because the longest side is 26 yards, use 26 as \( c \), the measure of the hypotenuse.
\[
c^2 = a^2 + b^2
\]
\[
26^2 = 24^2 + 10^2
\]
\[
676 = 576 + 100
\]
\[
676 = 676 \checkmark
\]
Because \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

**ANSWER:**
yes
15. 11 km, 60 km, 61 km

**SOLUTION:**
Because the longest side is 61 km, use 61 as \( c \), the measure of the hypotenuse.

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
61^2 &= 60^2 + 11^2 \\
3721 &= 3600 + 121 \\
3721 &= 3721 \checkmark
\end{align*}
\]
Because \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

**ANSWER:**
yes

16. **FLAGPOLES** If a flagpole is 30 feet tall and Mai-Lin is standing a distance of 15 feet from the flagpole, what is the distance from her feet to the top of the flagpole?

**SOLUTION:**
Let \( x \) be the distance from Mai-Lin’s feet to the top of the flagpole.

Use the Pythagorean Theorem to find \( x \).

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
x^2 &= 30^2 + 15^2 \\
x^2 &= 900 + 225 \\
x^2 &= 1125 \\
x &= \sqrt{1125} \\
x &\approx 33.5
\end{align*}
\]
The distance from Mai-Lin’s feet to the top of the flagpole is about 33.5 ft.

**ANSWER:**
about 33.5 ft

17. **CONSTRUCTION** The walls of a recreation center are being covered with paneling. The doorway into one room is 0.9 meter wide and 2.5 meters high. What is the length of the longest rectangular panel that can be taken through this doorway?

**SOLUTION:**
Let \( x \) be the length of the longest rectangular panel that can be taken through the doorway.

Use the Pythagorean Theorem to find \( x \).

\[
\begin{align*}
&x^2 = (2.5)^2 + (0.9)^2 \\
&x^2 = 6.25 + 0.81 \\
&x^2 = 7.06 \\
x &= \sqrt{7.06} \\
x &\approx 2.66
\end{align*}
\]
The length of the longest rectangular panel that can be taken through the doorway is about 2.66 m.

**ANSWER:**
about 2.66 m
18. **OPEN ENDED** Create an application problem involving right triangles and the Pythagorean Theorem. Then solve your problem, drawing diagrams if necessary.

**SOLUTION:**
Sample answer: John places a ladder 5 feet away from his house. If the ladder is 10 feet long, how far up the side of the house will it reach?
Draw a diagram to represent the situation.

![Diagram](image)

Use the Pythagorean Theorem to solve for \(x\).

\[
\begin{align*}
\alpha^2 + \beta^2 &= \gamma^2 \\
\gamma^2 + 5^2 &= 10^2 \\
\gamma^2 + 25 &= 100 \\
\gamma^2 &= 75 \\
\gamma &\approx 8.7
\end{align*}
\]

The ladder reaches approximately 8.7 feet

**ANSWER:**
See students’ work.
Find the mean, median, and mode for each set of data.

1. number of pages in each novel assigned for summer reading:
   224, 272, 374, 478, 960, 394, 404, 308, 480, 624

   **SOLUTION:**
   To find the mean divide the sum of all the pages divided by the total of novels.

   \[
   \text{mean} = \frac{224 + 272 + 374 + 478 + 960 + 394 + 404 + 308 + 480 + 624}{10} = \frac{4518}{10} = 451.8 \text{ pages}
   \]

   To find the median. First arrange the page numbers in order.
   224, 272, 308, 374, 394, 404, 478, 480, 624, 960.
   Because there is an even number of pages, find the mean of the middle two.

   \[
   \text{median} = \frac{394 + 404}{2} = \frac{792}{2} = 399 \text{ pages}
   \]

   The mode is the value that occurs the most often in the data set. There is not a repetitive number of pages, so there is no mode.

   **ANSWER:**
   451.8 min, 399 min, no mode

2. height in centimeters of bean plants at the end of an experiment:
   14.5, 12, 16, 11, 14, 11, 10.5, 14, 11.5, 15, 13.5

   **SOLUTION:**
   To find the mean divide the sum of all the heights divided by the total of plants.

   \[
   \text{mean} = \frac{14.5 + 12 + 16 + 11 + 14 + 11 + 10.5 + 14 + 11.5 + 15 + 13.5}{11} = \frac{143}{11} = 13 \text{ cm}
   \]

   To find the median, arrange the heights in order.
   10.5, 11, 11, 11.5, 12, 13.5, 14, 14, 14.5, 15, 16.

   Because there is an odd number of plants, the median is the middle number. So, the median is 13.5 cm.

   The mode is the value that occurs the most often in the data set. In this case, it is 11 cm and 14 cm.

   **ANSWER:**
   13 cm, 13.5 cm, 11 cm and 14 cm
3. number of text messages sent each day during the last two weeks: 
18, 35, 53, 44, 26, 57, 23, 27, 47, 33, 4, 35, 39, 41

**SOLUTION:**
To find the mean, divide the sum of number of all text messages divided by the 14 days.

\[
\text{mean} = \frac{18 + 35 + 53 + 44 + 26 + 57 + 23 + 27 + 47 + 33 + 4 + 35 + 39 + 41}{14} = \frac{482}{14} = 34.4 \text{ text messages}
\]

To find the median, arrange the values in order. 4, 18, 23, 26, 27, 33, 35, 35, 38, 47, 41, 44, 53, 57.

Because there is an even number of values, find the mean of the middle two values.

\[
\text{median} = \frac{35 + 35}{2} = \frac{70}{2} = 35 \text{ text messages}
\]

The mode is the value that occurs the most often in the data set. In this case, it is 35 text messages.

**ANSWER:**
≈34.4 text messages, 35 text messages, 35 text messages

State whether the data in sets A and B represent sample or population data. Then find the range, variance, and standard deviation of each set. Use the standard deviations to compare the variability between the data sets.

4. **SOLUTION:**
12 wait times for Ride A and Ride B are given. For an amusement park ride, there must be more than 12 rides a day, so the data must be a sample.

The range is the difference between the greatest and least values in the set.
Ride A: 60 - 11 = 49 min
Ride B: 50 - 31 = 19 min

In order to find the variance and standard deviation, the mean is needed.

\[
\text{mean Ride A} = \frac{45 + 22 + 40 + 60 + 53 + 44 + 26 + 57 + 23 + 27 + 47 + 33 + 35 + 39 + 41}{12} = \frac{461}{12} = 38.4 \text{ min}
\]

<table>
<thead>
<tr>
<th>X</th>
<th>(X - \bar{X})</th>
<th>((X - \bar{X})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>45 - 38.4 = 6.6</td>
<td>6.6^2 = 43.36</td>
</tr>
<tr>
<td>22</td>
<td>22 - 38.4 = -16.4</td>
<td>(-16.4)^2 = 268.96</td>
</tr>
<tr>
<td>40</td>
<td>40 - 38.4 = 1.6</td>
<td>1.6^2 = 2.56</td>
</tr>
<tr>
<td>48</td>
<td>48 - 38.4 = 9.6</td>
<td>9.6^2 = 92.16</td>
</tr>
<tr>
<td>11</td>
<td>11 - 38.4 = -27.4</td>
<td>(-27.4)^2 = 750.76</td>
</tr>
<tr>
<td>51</td>
<td>51 - 38.4 = 12.6</td>
<td>12.6^2 = 158.76</td>
</tr>
<tr>
<td>36</td>
<td>36 - 38.4 = -2.4</td>
<td>(-2.4)^2 = 5.76</td>
</tr>
<tr>
<td>55</td>
<td>55 - 38.4 = 16.6</td>
<td>16.6^2 = 275.56</td>
</tr>
<tr>
<td>60</td>
<td>60 - 38.4 = 21.6</td>
<td>21.6^2 = 466.56</td>
</tr>
<tr>
<td>32</td>
<td>32 - 38.4 = -6.4</td>
<td>(-6.4)^2 = 40.96</td>
</tr>
<tr>
<td>24</td>
<td>24 - 38.4 = -14.4</td>
<td>(-14.4)^2 = 207.36</td>
</tr>
<tr>
<td>37</td>
<td>37 - 38.4 = -1.4</td>
<td>(-1.4)^2 = 1.96</td>
</tr>
</tbody>
</table>

Sum = 2314.92

The variance of Ride A is \(\frac{2314.92}{12} = 192.91\) min

The standard deviation of Ride A is \(\sqrt{210.45} = 14.5\) min

mean Ride B = \(\frac{35 + 50 + 32 + 31 + 35 + 45 + 45 + 49 + 40 + 43 + 37 + 45}{12} = \frac{487}{12} = 40.6\) min

<table>
<thead>
<tr>
<th>X</th>
<th>(X - \bar{X})</th>
<th>((X - \bar{X})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>35 - 40.6 = -5.6</td>
<td>(-5.6)^2 = 31.36</td>
</tr>
<tr>
<td>50</td>
<td>50 - 40.6 = 9.4</td>
<td>9.4^2 = 88.36</td>
</tr>
<tr>
<td>32</td>
<td>32 - 40.6 = -8.6</td>
<td>(-8.6)^2 = 73.96</td>
</tr>
<tr>
<td>31</td>
<td>31 - 40.6 = -9.6</td>
<td>(-9.6)^2 = 92.16</td>
</tr>
</tbody>
</table>
0-9 Measures of Center, Spread, and Position

Find the mean, median, and mode for each set of data.

1. number of pages in each novel assigned for summer reading
   - Mean: \( \frac{210.4 + 148 + 210.4 + 272 + 478 + 960 + 394 + 404 + 308 + 480 + 624}{11} = 287.14 \)
   - Median: 272
   - Mode: 210.4

2. number of sponsors in each charity walk

<table>
<thead>
<tr>
<th>Charity Walk A</th>
<th>Charity Walk B</th>
</tr>
</thead>
<tbody>
<tr>
<td>44 14 61</td>
<td>8 28 15</td>
</tr>
<tr>
<td>22 27 25</td>
<td>100 42 19</td>
</tr>
<tr>
<td>38 50 49</td>
<td>25 75 82</td>
</tr>
</tbody>
</table>

   - Mean (Walk A): \( \frac{8 + 28 + 150 + 42 + 19 + 25 + 75 + 82}{8} = 39.125 \approx 39.1 \) sponsors
   - Mean (Walk B): \( \frac{44 + 14 + 27 + 25 + 38 + 50 + 49}{9} = 330 \approx 33.3 \) sponsors
   - Median: (Walk A) 15, (Walk B) 25
   - Mode: (Walk A) 8, (Walk B) 15

3. number of miles run in each week

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>
   - Mean: \( \frac{10 + 15 + 20 + 25 + 30}{5} = 20 \)
   - Median: 20
   - Mode: 20

4. number of pages in each section of the book

<table>
<thead>
<tr>
<th>Section 1</th>
<th>Section 2</th>
<th>Section 3</th>
<th>Section 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>45</td>
<td>45</td>
<td>49</td>
</tr>
</tbody>
</table>
   - Mean: \( \frac{35 + 45 + 45 + 49}{4} = 43 \)
   - Median: 45
   - Mode: 45

5. In order to find the variance and standard deviation,

   - The variance of Ride B is \( \frac{464.92}{11} = 42.3 \) min
   - The standard deviation of Ride B is \( \sqrt{42.3} = 6.5 \) min
   - Since the sample standard deviation of Ride A is greater than that of Ride B, there is more variability in the sample wait times for Ride A than Ride B.

   **ANSWER:**

   Sample; Ride A: 49 min, \( \approx 210.4 \) min, \( \approx 14.5 \) min; Ride B: 19 min, \( \approx 42.3 \) min, \( \approx 6.5 \) min; since the sample standard deviation of Ride A is greater than that of Ride B, there is more variability in the sample wait times for Ride A than Ride B.

   **SOLUTION:**

   9 sponsors for Walk A and Walk B are given. A charity walk will have more than 9 walkers; the data given is for a sample.

   Walk A: 47, \( \approx 242.0 \), \( \approx 15.6 \);
   Walk B: 92, \( \approx 1115.4 \), \( \approx 33.4 \);

   The range is the difference between the greatest and least values in the set.
   - Walk A: 61 – 14 = 47 sponsors
   - Walk B: 100 – 8 = 92 sponsors

   In order to find the variance and standard deviation,
0-9 Measures of Center, Spread, and Position

sponsors
The standard deviation of Walk B is \( \sqrt{1115.4} = 33.4 \) sponsors

Since the sample standard deviation of Walk B is greater than that of Walk A, there is more variability in the number of sponsors obtained by participants in Walk B than in Walk A.

**ANSWER:**
Sample; Walk A: 47, \( \approx 242.0, \approx 15.6 \); Walk B: 92, \( \approx 1115.4, \approx 33.4 \); since the sample standard deviation of Walk B is greater than that of Walk A, there is more variability in the number of sponsors obtained by participants in Walk B than in Walk A.

<table>
<thead>
<tr>
<th>Number of Days Each Student Missed This Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
</tr>
<tr>
<td>10  8  5  9  7  6  5  10</td>
</tr>
<tr>
<td>5  13  0  15  9  7  9  10</td>
</tr>
<tr>
<td>14  11  8  4  7  2  9  10</td>
</tr>
<tr>
<td>9  11  14  8  12  10  1</td>
</tr>
<tr>
<td>Class B</td>
</tr>
<tr>
<td>5  8  13  7  9  4  10  2</td>
</tr>
<tr>
<td>12  6  7  8  12  14  12  6</td>
</tr>
<tr>
<td>12  9  6  11  3  8  5</td>
</tr>
<tr>
<td>3  10  5  13  9  1  8</td>
</tr>
</tbody>
</table>

6.

**SOLUTION:**
The data given is the number of days absent for each student in Class A and Class B. Since there is data for each student, we are given data for the population.

Class A: 15, \( \approx 13.7, 3.7 \);
Class B: 12, \( \approx 10.5, \approx 3.2 \);

The range is the difference between the greatest and least values in the set.
Class A: 15 – 0 = 15 days
Class B: 13 – 1 = 12 days

In order to find the variance and standard deviation, the mean is needed.

mean Class A
\( \frac{10 + 8 + 5 + 9 + 7 + 6 + 5 + 10}{8} = \frac{52}{8} = 6.5 \)

mean Class B
\( \frac{10 + 8 + 5 + 9 + 7 + 6 + 5 + 10}{8} = \frac{52}{8} = 6.5 \)

The variance of Class A is \( \frac{410.7}{29} = 14.2 \) days
The standard deviation of Class A is \( \sqrt{14.2} = 3.8 \) days

mean Class B
\( \frac{10 + 8 + 5 + 9 + 7 + 6 + 5 + 10}{8} = \frac{52}{8} = 6.5 \)
Find the mean, median, and mode for each set of data.

1. Number of pages in each novel assigned for summer reading:
   - Sample answer: Data recording errors, manufacturing errors

Since 17.4 > 17.35, it would not be an outlier.

c. Sample answers: Data recording errors, manufacturing errors

11. a. The IQR is the difference between the 1st and 3rd quartiles.

10. To find the value beyond the 1st and 3rd quartiles, use the formula for the IQR:

   \[ IQR = Q_3 - Q_1 \]

12. a. The range is the difference between the greatest and least data values.

Since the sample standard deviation of Walk B is larger than that of Walk A, the variability in the number of days absent for each student in Walk B is greater than in Walk A.

Since the sample standard deviation of Ride A is less than that of Ride B, there is more variability in the data set for Ride A.

Since the sample standard deviation of Class A is greater than that of Class B, there is more variability in the number of days that students missed during the school year for Class A than for Class B.

**ANSWER:**
Population; Class A: 15, \( \approx 13.7, 3.7 \); Class B: 12, \( \approx 10.5, 3.2 \); since the sample standard deviation of Class A is greater than that of Class B, there is more variability in the number of days that students missed during the school year for Class A than for Class B.

### Measures of Center, Spread, and Position

The variance of Class B is
\[
\frac{314.8}{29} = 10.9 \text{ days}
\]

The standard deviation of Class B is
\[
\sqrt{10.9} = 3.3 \text{ days}
\]
Find the minimum, lower quartile, median, upper quartile, and maximum of each data set. Then interpret this five-number summary.

<table>
<thead>
<tr>
<th>Number of Students in Each Math Class at Central High</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 27 26 26 26 19 27 24 23</td>
</tr>
<tr>
<td>24 26 18 28 29 29 26 24</td>
</tr>
<tr>
<td>19 28 25 24 20 22 22</td>
</tr>
<tr>
<td>24 23 23 25 25 29 28</td>
</tr>
</tbody>
</table>

7. **SOLUTION:**
Enter the data into L1.
Press `STAT` `ENTER` to display the 1-Var statistics.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{\uparrow}n=30)</td>
</tr>
<tr>
<td>minX=18</td>
</tr>
<tr>
<td>Q1=23</td>
</tr>
<tr>
<td>Med=25</td>
</tr>
<tr>
<td>Q3=27</td>
</tr>
<tr>
<td>maxX=29</td>
</tr>
</tbody>
</table>

The minimum is 18.  
The lower quartile is 23.  
The median is 25.  
The upper quartile is 27.  
The maximum is 29.

There are 18 students in the smallest math class at Central High and 29 students in the largest class. 25% of the classes have less than 23 students, 50% of the classes have less than 25 students, and 75% of the classes have less than 27 students.

**ANSWER:**  
18, 23, 25, 27, 29; Sample answer: There are 18 students in the smallest math class at Central High and 29 students in the largest class. 25% of the classes have less than 23 students, 50% of the classes have less than 25 students, and 75% of the classes have less than 27 students.

---

8. **SOLUTION:**
Enter the data into L1.
Press `STAT` `ENTER` to display the 1-Var statistics.

<table>
<thead>
<tr>
<th>1-Var Stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{\uparrow}n=50)</td>
</tr>
<tr>
<td>minX=17.8</td>
</tr>
<tr>
<td>Q1=20.4</td>
</tr>
<tr>
<td>Med=21.4</td>
</tr>
<tr>
<td>Q3=21.8</td>
</tr>
<tr>
<td>maxX=22.7</td>
</tr>
</tbody>
</table>

The minimum is 17.8.  
The lower quartile is 20.4.  
The median is 21.4.  
The upper quartile is 21.8.  
The maximum is 22.7.

The lowest mean score for a state is 17.8 and the highest mean score is 22.7. 25% of the states have a mean score that is less than 20.4, 50% of the states have a mean score that is less than 21.4, and 75% of the states have a mean score that is less than 22.7.

**ANSWER:**
17.8, 20.4, 21.4, 21.8, 22.7; Sample answer: The lowest mean score for a state is 17.8 and the highest mean score is 22.7. 25% of the states have a mean score that is less than 20.4, 50% of the states have a mean score that is less than 21.4, and 75% of the states have a mean score that is less than 22.7.

Identify any outliers in each data set, and explain your reasoning. Then find the mean, median, mode, range, and standard deviation of the data set with and without the outlier.
Describe the effect on each measure.

9. fuel efficiency in miles per gallon of 15 randomly selected automobiles:
40, 36, 29, 45, 51, 36, 48, 34, 36, 22, 13, 42, 31, 44, 32, 34

**SOLUTION:**
Enter the data into L1.

**Keystrokes:**
Use the 1-Var statistics to identify Q₁ and Q₃.

![1-Var Stats](image)

\[ Q₁ = 31.5 \text{ and } Q₃ = 43. \]
\[ \text{IQR} = Q₃ - Q₁ \text{ or } 43 - 31.5 = 11.5. \]
Find and use the IQR to find the values beyond which any outlier would lie.
\[ Q₁ - 1.5(\text{IQR}) \text{ and } Q₃ + 1.5(\text{IQR}) \]
\[ 31.5 - 1.5(11.5) \quad 43 + 1.5(11.5) \]
\[ 14.25 \quad 60.25 \]

The only outlier on the low end is 13. There are no outliers on the upper end.

**With outlier:**

![1-Var Stats](image)

**Without outlier:**

![1-Var Stats](image)

Removing the outlier did not affect the median or mode. However, the removal did affect the mean, standard deviation, and range. The mean and standard deviation increased, and the range decreased.

**ANSWER:**
13; Sample answer: Any outliers would be less than 14.25 or greater than 60.25. Since 13 < 14.25, it is an outlier.

![Data Set](image)

Removing the outlier did not affect the median or mode. However, the removal did affect the mean, standard deviation, and range. The mean and standard deviation increased, and the range decreased.

10. number of posts to a certain blog each month during a particular year:
25, 23, 21, 27, 29, 19, 10, 21, 20, 18, 26, 23

**SOLUTION:**
Enter the data into L1.

**Keystrokes:**
Use the 1-Var statistics to identify Q₁ and Q₃.
Find the mean, median, and mode for each set of data.

1. number of pages in each novel assigned for summer reading:

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

   Since 17.4 > 17.35, it would not be an outlier.

   
   c. Sample answers: data recording errors, manufacturing errors

   
   2. Time for Ride A and Ride B:

<table>
<thead>
<tr>
<th>Ride</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 min</td>
</tr>
<tr>
<td>B</td>
<td>30 min</td>
</tr>
</tbody>
</table>

   Removing the outlier did not affect the mode. However, the removal did affect the mean, median, standard deviation, and range. The mean and median increased, and the standard deviation and range decreased.

   
   3. Values beyond Q1 and Q3:

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Mean</th>
<th>Median</th>
<th>Mode</th>
<th>Range</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>with outlier</td>
<td>≈21.8</td>
<td>22</td>
<td>21</td>
<td>19</td>
<td>≈4.8</td>
</tr>
<tr>
<td>without outlier</td>
<td>≈22.9</td>
<td>23</td>
<td>21</td>
<td>11</td>
<td>≈3.3</td>
</tr>
</tbody>
</table>

   Removing the outlier did not affect the mode. However, the removal did affect the mean, median, standard deviation, and range. The mean and median increased, and the standard deviation and range decreased.

   
   4. Time for Ride A and Ride B:

<table>
<thead>
<tr>
<th>Ride</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 min</td>
</tr>
<tr>
<td>B</td>
<td>30 min</td>
</tr>
</tbody>
</table>

   Removing the outlier did not affect the mode. However, the removal did affect the mean, median, standard deviation, and range. The mean and median increased, and the standard deviation and range decreased.

   
   10. Variability in the number of days that students missed:

<table>
<thead>
<tr>
<th>Day</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

   Removing the outlier did not affect the mode. However, the removal did affect the mean, median, standard deviation, and range. The mean and median increased, and the standard deviation and range decreased.

   
   11. CEREAL The weights, in ounces, of 20 randomly selected boxes of a certain brand of cereal are shown.

<table>
<thead>
<tr>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.4</td>
</tr>
</tbody>
</table>

   a. Identify any outliers in the data set, and explain your reasoning.

   b. If the outlier was removed and an additional cereal box that was 17.35 ounces was added, would this value be an outlier of the new data set? Explain.

   c. What are some possible causes of outliers in this situation?

   SOLUTION:

   a. Enter the data into L1.

   Keystrokes: STAT→ ENTER

   Use the 1-Var statistics to identify Q1 and Q3.
### 0-9 Measures of Center, Spread, and Position

#### 1-Var Stats

| MinX = 14.9 | Q1 = 16.1 | Med = 16.5 | Q3 = 16.6 | MaxX = 16.8 |

Q₁ = 16.1 and Q₃ = 16.6. IQR = Q₃ − Q₁ or 16.6 − 16.1 = 0.5.
Find and use the IQR to find the values beyond which any outlier would lie.
Q₁ − 1.5(IQR) and Q₃ + 1.5(IQR)
16.1 − 1.5(0.5) 16.6 + 1.5(0.5)
15.35 17.35

The interval beyond which any outliers would lie is 15.35 < x < 17.35. Since 14.9 < 15.35, it is an outlier.

#### 1-Var Stats

| MinX = 15.8 | Q1 = 16.15 | Med = 16.5 | Q3 = 16.65 | MaxX = 17.35 |

Q₁ = 16.15 and Q₃ = 16.65. IQR = Q₃ − Q₁ or 16.65 − 16.15 = 0.5.
Find and use the IQR to find the values beyond which any outlier would lie.
Q₁ − 1.5(IQR) and Q₃ + 1.5(IQR)
16.15 − 1.5(0.5) 16.65 + 1.5(0.5)
15.4 17.4

The new interval beyond which any outliers would lie would be 15.4 < x < 17.4. Since 17.4 > 17.35, it would not be an outlier.

**c. Outliers can be caused by data recording errors or manufacturing errors.**

**ANSWER:**

**a.** Yes; sample answer: Any outliers would be less than 15.35 or greater than 17.35. Since 14.9 < 15.35, it is an outlier.

**b.** No; sample answer: Any outliers would be less than 15.4 or greater than 17.4. Since 17.4 > 17.35, it would not be an outlier.

**c.** Sample answers: data recording errors, manufacturing errors.