Find the values of the six trigonometric functions for angle \( \theta \).

1. 

**SOLUTION:**

Opposite side = 8
Adjacent Side = 6
Let \( x \) be the hypotenuse.
By the Pythagorean theorem,

\[
x = \sqrt{8^2 + 6^2}
\]

= 10
Therefore, hypotenuse = 10.
The trigonometric ratios are:

\[
\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}
\]
\[
\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}
\]
\[
\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}
\]
\[
\csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}
\]
\[
\sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}
\]
\[
\cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}
\]
Substitute:

\[
\sin \theta = \frac{8}{10} \text{ or } \frac{4}{5}
\]
\[
\cos \theta = \frac{6}{10} \text{ or } \frac{3}{5}
\]
\[
\tan \theta = \frac{8}{6} \text{ or } \frac{4}{3}
\]
\[
\csc \theta = \frac{10}{8} \text{ or } \frac{5}{4}
\]

2. 

**SOLUTION:**

Adjacent side = 12
Hypotenuse = 16
opposite side = \( \sqrt{16^2 - 12^2} \)
= 4\sqrt{7}
The trigonometric ratios are:

\[
\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}
\]
\[
\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}
\]
\[
\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}
\]
\[
\csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}
\]
\[
\sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}
\]
\[
\cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}
\]
12-1 Trigonometric Functions in Right Triangles

The trigonometric ratios are:

\[ \sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} \]

\[ \cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} \]

\[ \tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} \]

\[ \csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} \]

\[ \sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} \]

\[ \cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} \]

In a right triangle, \( \angle A \) is acute. Find the values of the five remaining trigonometric functions.

3. \( \cos A = \frac{4}{7} \)

**SOLUTION:**

\[ \cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{4}{7} \]

Therefore:

\[ \text{Opposite side} = \sqrt{7^2 - 4^2} \]

\[ = \sqrt{33} \]

\[ \sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{\sqrt{33}}{7} \]

\[ \tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{\sqrt{33}}{4} \]

\[ \csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{\sqrt{33}}{\sqrt{33}} \]

\[ \sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{7}{4} \]

\[ \cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{4\sqrt{33}}{4} \]

**ANSWER:**

\[ \sin A = \frac{\sqrt{33}}{7}, \tan A = \frac{\sqrt{33}}{4}, \csc A = \frac{7\sqrt{33}}{33}, \]

\[ \sec A = \frac{7}{4}, \cot A = \frac{4\sqrt{33}}{33} \]
4. \( \tan A = \frac{20}{21} \)

**SOLUTION:**

\[
\tan A = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{20}{21}
\]

Therefore:

\[
\text{hypotenuse} = \sqrt{21^2 + 20^2} = 29
\]

\[
\sin A = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{20}{29}
\]

\[
\cos A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{21}{29}
\]

\[
\csc A = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{29}{20}
\]

\[
\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{29}{21}
\]

\[
\cot A = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{21}{20}
\]

**ANSWER:**

\[
\sin A = \frac{20}{29}, \cos A = \frac{21}{29}, \csc A = \frac{29}{20}, \\
\sec A = \frac{29}{21}, \cot A = \frac{21}{20}
\]

**Use a trigonometric function to find the value of \( x \). Round to the nearest tenth**

5. \[
\sin 60^\circ = \frac{22}{x} \\
x = \frac{22}{\sin 60^\circ} \\
x \approx 25.4
\]

**ANSWER:**

\[25.4\]

6. \[
\tan 52^\circ = \frac{x}{6} \\
x = 6 \tan 52^\circ \\
x \approx 7.7
\]

**ANSWER:**

\[7.7\]
12-1 Trigonometric Functions in Right Triangles

Find the values of the six trigonometric functions for angle \( \theta \).

1. SOLUTION:
   \[
   \cos 33^\circ = \frac{7}{x}
   \]
   \[
   x = \frac{7}{\cos 33^\circ}
   \]
   \[
   x \approx 8.3
   \]

   ANSWER: 8.3

Find the value of \( x \). Round to the nearest tenth.

8. SOLUTION:
   \[
   \tan x = \frac{15}{8}
   \]
   \[
   x = \tan^{-1}\left(\frac{15}{8}\right)
   \]
   \[
   \approx 61.9
   \]

   ANSWER: 61.9

Find the value of \( x \). Round to the nearest tenth.

9. SOLUTION:
   \[
   \sin x = \frac{6}{14}
   \]
   \[
   x = \sin^{-1}\left(\frac{6}{14}\right)
   \]
   \[
   x \approx 25.4
   \]

   ANSWER: 25.4

Find the value of \( x \). Round to the nearest tenth.

10. SOLUTION:
    \[
    \cos x = \frac{6}{16}
    \]
    \[
    x = \cos^{-1}\left(\frac{6}{16}\right)
    \]
    \[
    \approx 68.0
    \]

    ANSWER: 68.0
12-1 Trigonometric Functions in Right Triangles

11. CCSS SENSE-MAKING Christian found two trees directly across from each other in a canyon. When he moved 100 feet from the tree on his side (parallel to the edge of the canyon), the angle formed by the tree on his side, Christian, and the tree on the other side was 70°. Find the distance across the canyon.

**SOLUTION:**
Let \( x \) be the distance across the canyon.

\[
\tan 70^\circ = \frac{x}{100}
\]

\[
x \approx 274.7 \text{ ft}
\]

**ANSWER:**
about 274.7 ft

12. LADDERS The recommended angle of elevation for a ladder used in fire fighting is 75°. At what height on a building does a 21-foot ladder reach if the recommended angle of elevation is used? Round to the nearest tenth.

**SOLUTION:**
Let \( x \) be the height of the building.

\[
\sin 75^\circ = \frac{x}{21}
\]

\[
x \approx 20.3 \text{ ft}
\]

**ANSWER:**
20.3 ft

Find the values of the six trigonometric functions for angle \( \theta \).

**SOLUTION:**

\[
\text{adjacent side} = \sqrt{13^2 - 12^2} = 5
\]

The trigonometric ratios are:

\[
\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}
\]

\[
\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}
\]
Find the values of the six trigonometric functions for angle \( \theta \).

SOLUTION:

Substitute:

\[
\begin{align*}
\sin \theta &= \frac{12}{13} \\
\cos \theta &= \frac{5}{13} \\
\tan \theta &= \frac{12}{5} \\
csc \theta &= \frac{13}{12} \\
sec \theta &= \frac{13}{5} \\
cot \theta &= \frac{12}{5}
\end{align*}
\]

ANSWER:

\[
\begin{align*}
\sin \theta &= \frac{12}{13} \\
\cos \theta &= \frac{5}{13} \\
\tan \theta &= \frac{12}{5} \\
csc \theta &= \frac{13}{12} \\
sec \theta &= \frac{13}{5} \\
cot \theta &= \frac{12}{5}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= \frac{9}{41} \\
\cos \theta &= \frac{40}{41} \\
\tan \theta &= \frac{9}{40} \\
csc \theta &= \frac{41}{9} \\
sec \theta &= \frac{41}{40} \\
cot \theta &= \frac{40}{9}
\end{align*}
\]

**ANSWER:**

To find the values of the six trigonometric functions for angle \( \theta \), we use the definitions of sine, cosine, tangent, cosecant, secant, and cotangent:

\[
\begin{align*}
\sin \theta &= \frac{\text{Opposite}}{\text{Hypotenuse}} \\
\cos \theta &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\
\tan \theta &= \frac{\text{Opposite}}{\text{Adjacent}} \\
csc \theta &= \frac{\text{Hypotenuse}}{\text{Opposite}} \\
sec \theta &= \frac{\text{Hypotenuse}}{\text{Adjacent}} \\
cot \theta &= \frac{\text{Adjacent}}{\text{Opposite}}
\end{align*}
\]

The trigonometric ratios are:

\[
\begin{align*}
\sin \theta &= \frac{\text{Opposite}}{\text{Hypotenuse}} \\
\cos \theta &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\
\tan \theta &= \frac{\text{Opposite}}{\text{Adjacent}} \\
csc \theta &= \frac{\text{Hypotenuse}}{\text{Opposite}} \\
sec \theta &= \frac{\text{Hypotenuse}}{\text{Adjacent}} \\
cot \theta &= \frac{\text{Adjacent}}{\text{Opposite}}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= \frac{9}{41} \\
\cos \theta &= \frac{40}{41} \\
\tan \theta &= \frac{9}{40} \\
csc \theta &= \frac{41}{9} \\
sec \theta &= \frac{41}{40} \\
cot \theta &= \frac{40}{9}
\end{align*}
\]
12-1 Trigonometric Functions in Right Triangles

\[
\begin{align*}
\sin \theta &= \frac{9}{41}; \cos \theta = \frac{40}{41}; \\
\tan \theta &= \frac{9}{40}; \csc \theta = \frac{41}{9}; \\
\sec \theta &= \frac{41}{40}; \cot \theta = \frac{40}{9}
\end{align*}
\]

**SOLUTION:**

\[
\text{opposite side} = \sqrt{10^2 - 7^2} = \sqrt{51}
\]

The trigonometric ratios are:

\[
\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}
\]

\[
\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}
\]

\[
\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}
\]

\[
\csc \theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}
\]

\[
\sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}
\]

\[
\cot \theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}
\]

Substitute:

\[
\sin \theta = \frac{\sqrt{51}}{10}; \cos \theta = \frac{7}{10}; \\
\tan \theta = \frac{\sqrt{51}}{7}; \csc \theta = \frac{10\sqrt{51}}{51}; \\
\sec \theta = \frac{10}{7}; \cot \theta = \frac{7\sqrt{51}}{51}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= \frac{\sqrt{51}}{10}; \cos \theta = \frac{7}{10}; \\
\tan \theta &= \frac{\sqrt{51}}{7}; \csc \theta = \frac{10\sqrt{51}}{51}; \\
\sec \theta &= \frac{10}{7}; \cot \theta = \frac{7\sqrt{51}}{51}
\end{align*}
\]

**SOLUTION:**

\[
\text{hypotenuse} = \sqrt{9^2 + 6^2} = 3\sqrt{13}
\]

The trigonometric ratios are:

\[
\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}
\]

\[
\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}
\]

\[
\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}
\]
In a right triangle, \( \angle A \) and \( \angle B \) are acute. Find the values of the five remaining trigonometric functions.

17. \( \tan A = \frac{8}{15} \)

**SOLUTION:**

\[
\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{8}{15}
\]

Therefore:

\[
\text{hypotenuse} = \sqrt{15^2 + 8^2} = 17
\]

\[
\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{8}{17}
\]

\[
\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{15}{17}
\]

\[
\csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{17}{8}
\]

\[
\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{17}{15}
\]

\[
\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{15}{8}
\]

ANSWER:

\[
\sin A = \frac{8}{17}, \quad \cos A = \frac{15}{17}, \quad \csc A = \frac{17}{8}, \quad \sec A = \frac{17}{15}, \quad \cot A = \frac{15}{8}
\]
12-1 Trigonometric Functions in Right Triangles

18. \( \cos A = \frac{3}{10} \)

**SOLUTION:**

\[
\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{10}
\]

Therefore:

\[
\text{opposite side} = \sqrt{10^2 - 3^2} = \sqrt{91}
\]

\[
\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\sqrt{91}}{10}
\]

\[
\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\sqrt{91}}{3}
\]

\[
\csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{10\sqrt{91}}{91}
\]

\[
\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{10}{3}
\]

\[
\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{3\sqrt{91}}{91}
\]

**ANSWER:**

\[
\sin A = \frac{\sqrt{91}}{10}, \quad \tan A = \frac{\sqrt{91}}{3}, \quad \csc A = \frac{10\sqrt{91}}{91},
\]

\[
\sec A = \frac{10}{3}, \quad \cot A = \frac{3\sqrt{91}}{91}
\]

19. \( \tan B = 3 \)

**SOLUTION:**

\[
\tan B = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{3}{1}
\]

Therefore:

\[
\text{hypotenuse} = \sqrt{3^2 + 1^2} = \sqrt{10}
\]

\[
\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{3\sqrt{10}}{10}
\]

\[
\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\sqrt{10}}{10}
\]

\[
\csc B = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{\sqrt{10}}{3}
\]

\[
\sec B = \frac{\text{hypotenuse}}{\text{adjacent side}} = \sqrt{10}
\]

\[
\cot B = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{3}
\]

**ANSWER:**

\[
\sin B = \frac{3\sqrt{10}}{10}, \quad \cos B = \frac{\sqrt{10}}{10}, \quad \csc B = \frac{\sqrt{10}}{3},
\]

\[
\sec B = \sqrt{10}, \quad \cot B = \frac{1}{3}
\]
20. \( \sin B = \frac{4}{9} \)

**SOLUTION:**
\[
\sin B = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{4}{9}
\]

Therefore:
\[
\text{adjacent side} = \sqrt{9^2 - 4^2} = \sqrt{65}
\]
\[
\cos B = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{\sqrt{65}}{9}
\]
\[
\tan B = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{4\sqrt{65}}{65}
\]
\[
\csc B = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{9}{4}
\]
\[
\sec B = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{9\sqrt{65}}{65}
\]
\[
\cot B = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{\sqrt{65}}{4}
\]

**ANSWER:**
\[
\cos B = \frac{\sqrt{65}}{9}, \quad \tan B = \frac{4\sqrt{65}}{65}, \quad \csc B = \frac{9}{4},
\]
\[
\sec B = \frac{9\sqrt{65}}{65}, \quad \cot B = \frac{\sqrt{65}}{4}
\]

---

21.

Use a trigonometric function to find each value of \( x \). Round to the nearest tenth.

**SOLUTION:**
\[
\sin 45^\circ = \frac{9}{x}
\]
\[
\frac{1}{\sqrt{2}} = \frac{9}{x}
\]
\[
x \approx 12.7
\]

**ANSWER:**
12.7

22.

**SOLUTION:**
\[
\sin 64^\circ = \frac{x}{4}
\]
\[
x \approx 3.6
\]

**ANSWER:**
3.6
12-1 Trigonometric Functions in Right Triangles

23. 

\[ \tan 30^\circ = \frac{x}{18} \]

\[
\frac{1}{\sqrt{3}} = \frac{x}{18} \\
x \approx 10.4
\]

**ANSWER:** 10.4

24. 

\[ \cos 48^\circ = \frac{22}{x} \]

\[
x \approx 32.9
\]

**ANSWER:** 32.9

25. 

\[ \tan 32^\circ = \frac{x}{14} \]

\[
x \approx 8.7
\]

**ANSWER:** 8.7

26. 

\[ \cos 70^\circ = \frac{x}{15} \]

\[
x \approx 5.1
\]

**ANSWER:** 5.1
27. PARASAILING Refer to the beginning of the lesson and the figure shown. Find \( a \), the altitude of a person parasailing, if the tow rope is 250 feet long and the angle formed is 32\(^\circ\). Round to the nearest tenth.

\[
\text{SOLUTION:}
\sin 32^\circ = \frac{a}{250}
\]

\[
a \approx 132.5 \text{ ft}
\]

ANSWER:
132.5 ft

28. CCSS MODELING Devon wants to build a rope bridge between his treehouse and Cheng’s treehouse. Suppose Devon’s treehouse is directly behind Cheng’s treehouse. At a distance of 20 meters to the left of Devon’s treehouse, an angle of 52\(^\circ\) is measured between the two treehouses. Find the length of the rope.

\[
\text{SOLUTION:}
\tan 52^\circ = \frac{x}{20}
\]

\[
x \approx 25.6 \text{ m}
\]

ANSWER:
25.6 m

Find the value of \( x \). Round to the nearest tenth.

29.

\[
\text{SOLUTION:}
\sin x = \frac{5}{10} = \frac{1}{2}
\]

\[
x = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ
\]

ANSWER:
30
30. \[ \tan \theta = \frac{19}{8} \]
\[ \theta = \tan^{-1} \left( \frac{19}{8} \right) \approx 67.2^\circ \]

**ANSWER:**
67.2

32. \[ \cos \theta = \frac{4}{7} \]
\[ \theta = \cos^{-1} \left( \frac{4}{7} \right) \approx 55.2^\circ \]

**ANSWER:**
55.2

31. \[ \tan \theta = \frac{9}{12} = \frac{3}{4} \]
\[ \theta = \tan^{-1} \left( \frac{3}{4} \right) \approx 36.9^\circ \]

**ANSWER:**
36.9

33. \[ \cos \theta = \frac{27}{32} \]
\[ \theta = \cos^{-1} \left( \frac{27}{32} \right) \approx 32.5^\circ \]

**ANSWER:**
32.5
12-1 Trigonometric Functions in Right Triangles

34. 

\[
\text{SOLUTION:} \\
\sin x = \frac{10}{25} = \frac{2}{5} \\
x = \sin^{-1}\left(\frac{2}{5}\right) \\
\approx 23.6^\circ
\]

\text{ANSWER:} 23.6

35. SQUIRRELS Adult flying squirrels can make glides of up to 160 feet. If a flying squirrel glides a horizontal distance of 160 feet and the angle of descent is 9°, find its change in height.

\text{SOLUTION:} \\
Let h be the height.

\[
\tan 9^\circ = \frac{h}{160} \\
h \approx 25.3 \text{ ft}
\]

\text{ANSWER:} about 25.3 ft

36. HANG GLIDING A hang glider climbs at a 20° angle of elevation. Find the change in altitude of the hang glider when it has flown a horizontal distance of 60 feet.

\text{SOLUTION:} \\
Let x be the change in altitude.

\[
\tan 20^\circ = \frac{x}{60} \\
x \approx 21.8 \text{ ft}
\]

\text{ANSWER:} about 21.8 ft

Use trigonometric functions to find the values of x and y. Round to the nearest tenth.

37. 

\text{SOLUTION:} \\
\sin 46.5^\circ = \frac{x}{30.2} \\
x \approx 21.9

\cos 46.5^\circ = \frac{y}{30.2} \\
y \approx 20.8

\text{ANSWER:} \\
x = 21.9, y = 20.8
38. **SOLUTION:**
\[
\sin 50^\circ = \frac{71.8}{x} \\
x = \frac{71.8}{\sin 50^\circ} \\
x \approx 93.7
\]
\[
\tan 50^\circ = \frac{71.8}{y} \\
y \approx 60.2
\]
**ANSWER:** 
\(x = 93.7, y = 60.2\)

39. **SOLUTION:**
\[
\sin x = \frac{\frac{26}{4}}{81} \\
x = \sin^{-1} \left( \frac{\frac{26}{4}}{81} \right) \\
\approx 19.3
\]
\[
\cos y = \frac{\frac{26}{4}}{81} \\
y = \cos^{-1} \left( \frac{\frac{26}{4}}{81} \right) \\
\approx 70.7
\]
**ANSWER:** 
\(x = 19.3, y = 70.7\)

Solve each equation

40. \(\cos A = \frac{3}{19}\)

**SOLUTION:**
\[
\cos A = \frac{3}{19} \\
A = \cos^{-1} \left( \frac{3}{19} \right) \\
\approx 80.9
\]
**ANSWER:**
about 80.9
12-1 Trigonometric Functions in Right Triangles

41. \( \sin N = \frac{9}{11} \)

**SOLUTION:**
\[
\sin N = \frac{9}{11} \\
N = \sin^{-1} \left( \frac{9}{11} \right) \\
\approx 54.9
\]

**ANSWER:**
about 54.9

42. \( \tan X = 15 \)

**SOLUTION:**
\[
\tan X = 15 \\
X = \tan^{-1} (15) \\
\approx 86.2
\]

**ANSWER:**
about 86.2

43. \( \sin T = 0.35 \)

**SOLUTION:**
\[
\sin T = 0.35 \\
T = \sin^{-1} (0.35) \\
\approx 20.5
\]

**ANSWER:**
about 20.5

44. \( \tan G = 0.125 \)

**SOLUTION:**
\[
\tan G = 0.125 \\
G = \tan^{-1} (0.125) \\
\approx 7.1
\]

**ANSWER:**
about 7.1

45. \( \cos Z = 0.98 \)

**SOLUTION:**
\[
\cos Z = 0.98 \\
Z = \cos^{-1} (0.98) \\
\approx 11.5
\]

**ANSWER:**
about 11.5

46. **MONUMENTS** A monument casts a shadow 24 feet long. The angle of elevation from the end of the shadow to the top of the monument is 50°.

**a.** Draw and label a right triangle to represent this situation.

**b.** Write a trigonometric function that can be used to find the height of the monument.

**c.** Find the value of the function to determine the height of the monument to the nearest tenth.

**SOLUTION:**

\[ x \]
b. Let \( x \) be the height of the monument.

\[
\tan 50^\circ = \frac{x}{24}
\]

\[x \approx 28.6 \text{ ft}
\]

ANSWER:

\[\begin{array}{c}
\text{a.}
\end{array}\]

\[\begin{array}{c}
\text{b. } \tan 50^\circ = \frac{x}{24}
\end{array}\]

\[\begin{array}{c}
\text{c. } 28.6 \text{ ft}
\end{array}\]

47. **NESTS** Tabitha’s eyes are 5 feet above the ground as she looks up to a bird’s nest in a tree. If the angle of elevation is 74.5° and she is standing 12 feet from the tree’s base, what is the height of the bird’s nest? Round to the nearest foot.

**SOLUTION:**

\[\begin{array}{c}
\text{tan 74.5} = \frac{x}{12}
\end{array}\]

\[x \approx 43 \text{ ft}
\]

Therefore, the height of the nest = 43 + 5 = 48 ft.

**ANSWER:**

48 ft

48. **RAMPS** Two bicycle ramps each cover a horizontal distance of 8 feet. One ramp has a 20° angle of elevation, and the other ramp has a 35° angle of elevation, as shown at the right.

a. How much taller is the second ramp than the first? Round to the nearest tenth.

b. How much longer is the second ramp than the
12-1 Trigonometric Functions in Right Triangles

first? Round to the nearest tenth.

SOLUTION:
a. Let $x$ be the height of the first ramp and $y$ be the height of the second ramp.

$$\tan 20^\circ = \frac{x}{8}$$
$$x \approx 2.9$$
$$\tan 35^\circ = \frac{y}{8}$$
$$y \approx 5.6$$

$$y - x = 5.6 - 2.9$$
$$= 2.7$$

Therefore, the second ramp is 2.7 ft taller than the first ramp.

b. Let $l$ be the length of the first ramp and $m$ the length of the second ramp.

$$\cos 20^\circ = \frac{8}{l}$$
$$l \approx 8.5$$

$$\cos 35^\circ = \frac{8}{m}$$
$$m \approx 9.8$$
$$m - l = 9.8 - 8.5 = 1.3$$

Therefore, the second ramp is 1.3 ft longer than the first ramp.

ANSWER:
a. 2.7 ft
b. 1.3 ft

49. FALCONS A falcon at a height of 200 feet sees two mice $A$ and $B$, as shown in the diagram.

a. What is the approximate distance $z$ between the falcon and mouse $B$?

b. How far apart are the two mice?

SOLUTION:
a. $\cos 72^\circ = \frac{200}{z}$
$$z \approx 647.2 \text{ ft}$$

b. $\tan 62^\circ = \frac{x}{200}$

Also:

$$\tan 72^\circ = \frac{x + y}{200}$$

Therefore, the two mice are about 239.4 ft apart.

ANSWER:
a. about 647.2 ft
b. about 239.4 ft
12-1 Trigonometric Functions in Right Triangles

In \( \triangle ABC \), \( \angle C \) is a right angle. Use the given measurements to find the missing side lengths and missing angle measures of \( \triangle ABC \). Round to the nearest tenth if necessary.

50. \( m\angle A = 36^\circ, a = 12 \)

**SOLUTION:**

\[ A + B + C = 180^\circ \]

\[ 36^\circ + B + 90^\circ = 180^\circ \]

\[ B = 54^\circ \]

\[ \tan 36^\circ = \frac{12}{b} \]

\[ b \approx 16.5 \]

\[ \sin 36^\circ = \frac{12}{c} \]

\[ c \approx 20.4 \]

**ANSWER:**

\( m\angle B = 54^\circ, b = 16.5, c = 20.4 \)

51. \( m\angle B = 31^\circ, b = 19 \)

**SOLUTION:**

\[ A + B + C = 180^\circ \]

\[ A = 59^\circ \]

\[ \tan 31^\circ = \frac{19}{a} \]

\[ a \approx 31.6 \]

\[ \sin 31^\circ = \frac{19}{c} \]

\[ c \approx 36.9 \]

**ANSWER:**

\( m\angle A = 59^\circ, a = 31.6, c = 36.9 \)

52. \( a = 8, c = 17 \)

**SOLUTION:**

\[ \sin A = \frac{8}{17} \]

\[ A = \sin^{-1} \left( \frac{8}{17} \right) \]

\[ \approx 28.1 \]

\[ \cos B = \frac{8}{17} \]

\[ B = \cos^{-1} \left( \frac{8}{17} \right) \]

\[ \approx 61.9 \]

\[ b = \sqrt{17^2 - 8^2} \]

\[ = 15 \]

**ANSWER:**

\( m\angle A = 28.1^\circ, m\angle B = 61.9^\circ, b = 15 \)
53. \( \tan A = \frac{4}{5}, a = 6 \)

**SOLUTION:**

\[
\tan A = \frac{4}{5} \\
A = \tan^{-1} \left( \frac{4}{5} \right) \\
\approx 38.7 \\
A + B + C = 180^\circ \\
38.7^\circ + B + 90^\circ \approx 180^\circ \\
B \approx 51.3^\circ
\]

\[
\tan A = \frac{a}{b} \\
\tan 38.7 \approx \frac{6}{b} \\
b \approx 7.5 \\
\sin 38.7^\circ = \frac{6}{c} \\
c \approx 9.6
\]

**ANSWER:**

\( m\angle A = 38.7^\circ, m\angle B = 51.3^\circ, b = 7.5, c = 9.6 \)

54. **CHALLENGE** A line segment has endpoints \( A(2, 0) \) and \( B(6, 5) \), as shown in the figure. What is the measure of the acute angle \( \theta \) formed by the line segment and the \( x \)-axis? Explain how you found the measure.

**SOLUTION:**

About 51.3°; if a right triangle is drawn with \( \overline{AB} \) as the hypotenuse, then the side opposite angle \( \theta \) is 5 and the side adjacent angle \( \theta \) is 4. \( \tan A = \frac{5}{4} \), so \( A \approx 51.3^\circ \).

**ANSWER:**

About 51.3°; if a right triangle is drawn with \( \overline{AB} \) as the hypotenuse, then the side opposite angle \( \theta \) is 5 and the side adjacent angle \( \theta \) is 4. \( \tan A = \frac{5}{4} \), so \( A \approx 51.3^\circ \).
55. **CCSS ARGUMENTS** Determine whether the following statement is true or false. Explain your reasoning.

*For any acute angle, the sine function will never have a negative value.*

**SOLUTION:**

True. \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \) and the values of the opposite side and the hypotenuse of an acute triangle are positive, so the value of the sine function is positive.

**ANSWER:**

True; \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \) and the values of the opposite side and the hypotenuse of an acute triangle are positive, so the value of the sine function is positive.

56. **OPEN ENDED** In right triangle \( \triangle ABC \), \( \sin A = \sin C \). What can you conclude about \( \triangle ABC \)? Justify your reasoning.

**SOLUTION:**

\[ \sin A = \sin C \]

So:

\[ \frac{\text{sideopp} A}{\text{hyp}} = \frac{\text{sideopp} C}{\text{hyp}} \]

Therefore:

\[ \text{sideopp} A = \text{sideopp} C \]

Since the two sides have the same measure, the triangle is isosceles.

**ANSWER:**

\( \sin A = \sin C \), so \( \frac{\text{sideopp} A}{\text{hyp}} = \frac{\text{sideopp} C}{\text{hyp}} \). Since the hypotenuse is the same, the length of the side opposite angle \( A \) must equal the length of the side opposite angle \( C \). Since the two sides have the same measure, the triangle is isosceles.

57. **WRITING IN MATH** A roof has a slope of \( \frac{2}{3} \).

Describe the connection between the slope and the angle of elevation \( \theta \) that the roof makes with the horizontal. Then use an inverse trigonometric function to find \( \theta \).

**SOLUTION:**

Sample answer: The slope describes the ratio of the vertical rise to the horizontal run of the roof. The vertical rise is opposite the angle that the roof makes with the horizontal. The horizontal run is the adjacent side. So, the tangent of the angle of elevation equals the ratio of the rise to the run, or the slope of the roof; \( \theta \approx 33.7^\circ \).

**ANSWER:**

Sample answer: The slope describes the ratio of the vertical rise to the horizontal run of the roof. The vertical rise is opposite the angle that the roof makes with the horizontal. The horizontal run is the adjacent side. So, the tangent of the angle of elevation equals the ratio of the rise to the run, or the slope of the roof; \( \theta \approx 33.7^\circ \).
58. **EXTENDED RESPONSE** Your school needs 5 cases of yearbooks. Neighborhood Yearbooks lists a case of yearbooks at $153.85 with a 10% discount on an order of 5 cases. Yearbooks R Us lists a case of yearbooks at $157.36 with a 15% discount on 5 cases.

   **a.** Which company would you choose?

   **b.** What is the least amount that you would have to spend for the yearbooks?

   **SOLUTION:**
   
   a. Calculate the price of 5 cases of Neighborhood Yearbooks.
   
   \[5 \times 153.85 = 769.25\]
   
   After 10% discount:
   
   \[10\% \times 769.25 = 76.93\]
   
   So:
   
   \[769.25 - 76.93 = 692.32\]
   
   Calculate the price of 5 cases of Yearbooks R Us yearbooks.
   
   \[5 \times 157.36 = 786.8\]
   
   After 15% discount:
   
   \[15\% \times 786.8 = 118.02\]
   
   So:
   
   \[786.8 - 118.02 = 668.78\]
   
   Since $668.78 < $786.8, it will be profitable to buy books from Yearbooks R Us.

   **b.** The least amount is $668.78.

   **ANSWER:**
   
   a. Yearbooks R Us

   b. $668.78

59. **SHORT RESPONSE** As a fundraiser, the marching band sold T-shirts and hats. They sold a total of 105 items and raised $1170. If the cost of a hat was $10 and the cost of a T-shirt was $15, how many T-shirts were sold?

   **SOLUTION:**
   
   Let \(x\) be the number of T-shirts sold. Total number of items sold is 105.
   
   Therefore, number of hats sold is \(105 - x\).

   So:
   
   \[(105 - x) \times 10 + 15x = 1170\]
   
   Solve for \(x\):
   
   \[5x = 120\]
   
   \[x = 24\]
   
   Number of T-shirts sold is 24.

   **ANSWER:**
   
   24
12-1 Trigonometric Functions in Right Triangles

60. A hot dog stand charges price \( x \) for a hot dog and price \( y \) for a drink. Two hot dogs and one drink cost $4.50. Three hot dogs and two drinks cost $7.25.

Which matrix could be multiplied by \[
\begin{bmatrix}
4.50 \\
7.25
\end{bmatrix}
\]
to find \( x \) and \( y \)?

A \[
\begin{bmatrix}
-1 & 1 \\
2 & -1
\end{bmatrix}
\]
B \[
\begin{bmatrix}
2 & -1 \\
-3 & 2
\end{bmatrix}
\]
C \[
\begin{bmatrix}
1 & 2 \\
-1 & 3
\end{bmatrix}
\]
D \[
\begin{bmatrix}
1 & -1 \\
-1 & 2
\end{bmatrix}
\]

**SOLUTION:**
The system of equations represents the given situation:
\[
\begin{align*}
2x + y &= 4.50 \\
3x + 2y &= 7.25
\end{align*}
\]
Rewrite the system of equations.
\[
\begin{bmatrix}
2 & 1 \\
3 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
4.50 \\
7.25
\end{bmatrix}
\]
To find the values of \( x \) and \( y \), multiply the equation by inverse of matrix \[
\begin{bmatrix}
2 & 1 \\
3 & 2
\end{bmatrix}
\]
Inverse matrix of \[
\begin{bmatrix}
2 & 1 \\
3 & 2
\end{bmatrix}
\]
is \[
\begin{bmatrix}
2 & -1 \\
-3 & 2
\end{bmatrix}
\]
Option B is the correct answer.

**ANSWER:**
B

61. SAT/ACT  The length and width of a rectangle are in the ratio of 5:12. If the rectangle has an area of 240 square centimeters, what is the length, in centimeters, of its diagonal?

F 24
G 26
H 28
J 30
K 32

**SOLUTION:**
Let \( x \) be apart.
Therefore, the length and the width of the rectangle are 5\(x \) and 12\(x \).

So:
\[
5x(12x) = 240
\]
\[
60x^2 = 240
\]
\[
x = 2
\]
Therefore, the length and the width of the rectangle are 10 cm and 24 cm.
The diagonal of the rectangle is \[
\sqrt{24^2 + 10^2} \text{ or } 26\text{ cm}.
\]
Option G is the correct answer.

**ANSWER:**
G
12-1 Trigonometric Functions in Right Triangles

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

62. Jack thinks that it takes less than 10 minutes to ride his bike from his home to the store.

**SOLUTION:**
less than 10 minutes: \( \mu < 10 \),
not less than 10 minutes: \( \mu \geq 10 \)

The claim is \( \mu < 10 \), and it is the alternative hypothesis because it does not include equality. The null hypothesis is \( \mu \geq 10 \), which is the complement of \( \mu < 10 \).

\( H_0: \mu \geq 10 \)
\( H_a: \mu < 10 \) (claim)

**ANSWER:**
\( H_0: \mu \geq 10; H_a: \mu < 10 \) (claim)

63. A deli sign says that one 12-inch turkey sandwich contains three ounces of meat.

**SOLUTION:**
contains 3 ounces: \( \mu = 3 \)
does not contain 3 ounces: \( \mu \neq 3 \)

The claim is \( \mu = 3 \), and it is the null hypothesis because it does include equality. The alternative hypothesis is \( \mu \neq 3 \), which is the complement of \( \mu = 3 \).

\( H_0: \mu = 3 \) (claim)
\( H_a: \mu \neq 3 \)

**ANSWER:**
\( H_0: \mu = 3 \) (claim); \( H_a: \mu \neq 3 \)

64. Mrs. Thomas takes at least 15 minutes to prepare a cake.

**SOLUTION:**
at least 15 minutes: \( \mu \geq 15 \)
less than 15 minutes: \( \mu < 15 \)

The claim is \( \mu \geq 15 \), and it is the null hypothesis because it does include equality. The alternative hypothesis is \( \mu < 15 \), which is the complement of \( \mu \geq 15 \).

\( H_0: \mu \geq 15 \) (claim)
\( H_a: \mu < 15 \)

**ANSWER:**
\( H_0: \mu \geq 15 \) (claim); \( H_a: \mu < 15 \)

65. **SWIMMING POOL** The number of visits to a community swimming pool per year by a sample of 425 members is normally distributed with a mean of 90 and a standard deviation of 15.

a. About what percent of the members went to the pool at least 45 times?

b. What is the probability that a member selected at random went to the pool more than 120 times?

c. What percent of the members went to the pool between 75 and 105 times?

**SOLUTION:**

**ANSWER:**

a. 99.85%

b. 2.5%

c. 64%
12-1 Trigonometric Functions in Right Triangles

66. **POLLS** A polling company wants to estimate how many people are in favor of a new environmental law. The polling company polls 20 people. The probability that a person is in favor of the law is 0.5.

a. What is the probability that exactly 12 people are in favor of the new law?

b. What is the expected number of people in favor of the law?

**SOLUTION:**
a. Use the binomial distribution. The probability of \( x \) successes in \( n \) independent trials is given by \( P(x) = C(n, x) s^x f^{n-x} \)

Here, \( s = 0.5 \). So, \( f = 0.5 \).

\[
P(x = 12) = C(20, 12) (0.5)^{12} (0.5)^{8}
\]

\[
\approx 0.12
\]

b. The probability that a person is in favor of the law is 0.5.

So, out of 20 people, there will be 20 (0.5) or 10 people in favor of the law.

**ANSWER:**
a. 0.12

b. 10

---

Find each product. Include the appropriate units with your answer.

67. \[ 4.3 \text{ miles} \left( \frac{5280 \text{ feet}}{1 \text{ mile}} \right) \]

**SOLUTION:**

1 mile = 5280 feet

Therefore:

\[ 4.3 \times 5280 \left( \frac{5280}{1 \times 5280} \right) \text{ ft} = 22,704 \text{ ft} \]

**ANSWER:**

22,704 ft

68. \[ 8 \text{ gallons} \left( \frac{8 \text{ pints}}{1 \text{ gallon}} \right) \]

**SOLUTION:**

1 gallon = 8 pints

Therefore:

\[ 8 \times 8 \left( \frac{8}{8} \right) \text{ pt} = 64 \text{ pt} \]

**ANSWER:**

64 pt
69. \( \frac{5 \text{ dollars}}{3 \text{ meters}} \) 21 meters

**SOLUTION:**

\[
\frac{5 \text{ dollars}}{3 \text{ meters}} \times 21 \text{ meters} = \frac{5 \text{ dollars}}{3} \times 21 = 35 \text{ dollars}
\]

**ANSWER:**

35 dollars

70. \( \frac{18 \text{ cubic inches}}{5 \text{ seconds}} \) 24 seconds

**SOLUTION:**

\[
\frac{18 \text{ cubic inches}}{5 \text{ seconds}} \times 24 \text{ seconds} = \frac{18 \text{ cubic inches}}{5} \times 24 = 86.4 \text{ in}^3
\]

**ANSWER:**

86.4 in\(^3\)

71. 65 degrees \( \frac{10 \text{ centimeters}}{3 \text{ degrees}} \)

**SOLUTION:**

\[
65 \text{ degrees} \times \frac{10 \text{ centimeters}}{3 \text{ degrees}} = 216 \frac{2}{3} \text{ cm}
\]

**ANSWER:**

\( 216 \frac{2}{3} \text{ cm} \)
12-2 Angles and Angle Measure

1. $140^\circ$

**SOLUTION:**
$140^\circ$

**ANSWER:**

2. $-60^\circ$

**SOLUTION:**
$-60^\circ$

**ANSWER:**
12-2 Angles and Angle Measure

3. 390°

**SOLUTION:**
390°

**ANSWER:**

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

4. 25°

**SOLUTION:**
Positive angle: 25° + 360° = 385°
Negative angle: 25° – 360° = –335°

**ANSWER:**
385°, –335°

5. 175°

**SOLUTION:**
Positive angle: 175° + 360° = 535°
Negative angle: 175° – 360° = –185°

**ANSWER:**
535°, –185°

6. –100°

**SOLUTION:**
Positive angle: –100° + 360° = 260°
Negative angle: –100° – 360° = –460°

**ANSWER:**
260°, –460°

Rewrite each degree measure in radians and each radian measure in degrees.

7. \( \frac{\pi}{4} \)

**SOLUTION:**
\[
\frac{\pi}{4} = \frac{\pi}{4} \text{ radians} \times \frac{180^\circ}{\pi \text{ radians}} \\
= \frac{180^\circ}{4} \\
= 45^\circ
\]

**ANSWER:**
45°
12-2 Angles and Angle Measure

8. 225°

\[ 225° = 225 \cdot \frac{\pi \text{ radians}}{180} = \frac{225\pi}{180} \text{ or } \frac{5\pi}{4} \text{ radians} \]

\text{ANSWER:} \quad \frac{5\pi}{4}

9. −40°

\[ -40° = -40 \cdot \frac{\pi \text{ radians}}{180} = -\frac{40\pi}{180} \text{ or } -\frac{2\pi}{9} \text{ radians} \]

\text{ANSWER:} \quad -\frac{2\pi}{9}

10. CCSS REASONING A tennis player’s swing moves along the path of an arc. If the radius of the arc’s circle is 4 feet and the angle of rotation is 100°, what is the length of the arc? Round to the nearest tenth.

\text{SOLUTION:} \quad \text{Rewrite the central angle in radians.}

\[ 100° = 100 \cdot \frac{\pi \text{ radians}}{180} = \frac{100\pi}{180} = \frac{5\pi}{9} \text{ radians} \]

Substitute 4 for \( r \) and \( \frac{5\pi}{9} \) for \( \theta \) in the formula to find the arc length.

\[ s = r\theta \]

\[ = 4 \cdot \frac{5\pi}{9} = \frac{20\pi}{9} \approx 7.0 \text{ ft} \]

\text{ANSWER:} \quad 7.0 \text{ ft}
12-2 Angles and Angle Measure

Draw an angle with the given measure in standard position.

11. $75^\circ$

**SOLUTION:**
Draw the terminal side of the angle $75^\circ$ clockwise.

**ANSWER:**

12. $160^\circ$

**SOLUTION:**
Draw the terminal side of the angle $160^\circ$ counterclockwise.

**ANSWER:**
12-2 Angles and Angle Measure

13. \(-90^\circ\)

**SOLUTION:**
The angle is negative. Draw the terminal side of the angle \(90^\circ\) clockwise from the positive \(x\)-axis.

14. \(-120^\circ\)

**SOLUTION:**
The angle is negative. Draw the terminal side of the angle \(120^\circ\) clockwise from the positive \(x\)-axis.
15. 295°

**SOLUTION:**
Draw the terminal side of the angle 295° counterclockwise.

**ANSWER:**

16. 510°

**SOLUTION:**
510° = 360° + 150°

Draw the terminal side of the angle 150° counterclockwise past the positive x-axis.
12-2 Angles and Angle Measure

17. **GYMNASTICS**  A gymnast on the uneven bars swings to make a 240° angle of rotation.

**SOLUTION:**
Draw the terminal side of the angle 240° counterclockwise.

**ANSWER:**

18. **FOOD**  The lid on a jar of pasta sauce is turned 420° before it comes off.

**SOLUTION:**

\[ 420° = 360° + 60° \]

Draw the terminal side of the angle 60° counterclockwise past the positive x-axis.

**ANSWER:**

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

19. **50°**

**SOLUTION:**
Positive angle: \( 50° + 360° = 410° \)
Negative angle: \( 50° − 360° = −310° \)

**ANSWER:**

\( 410°, −310° \)
12-2 Angles and Angle Measure

20. \(95^\circ\)

**SOLUTION:**
Positive angle: \(95^\circ + 360^\circ = 455^\circ\)
Negative angle: \(95^\circ - 360^\circ = -265^\circ\)

**ANSWER:**
\(455^\circ, -265^\circ\)

21. \(205^\circ\)

**SOLUTION:**
Positive angle: \(205^\circ + 360^\circ = 565^\circ\)
Negative angle: \(205^\circ - 360^\circ = -155^\circ\)

**ANSWER:**
\(565^\circ, -155^\circ\)

22. \(350^\circ\)

**SOLUTION:**
Positive angle: \(350^\circ + 360^\circ = 710^\circ\)
Negative angle: \(350^\circ - 360^\circ = -10^\circ\)

**ANSWER:**
\(710^\circ, -10^\circ\)

23. \(-80^\circ\)

**SOLUTION:**
Positive angle: \(-80^\circ + 360^\circ = 280^\circ\)
Negative angle: \(350^\circ - 360^\circ = -440^\circ\)

**ANSWER:**
\(280^\circ, -440^\circ\)

24. \(-195^\circ\)

**SOLUTION:**
Positive angle: \(-195^\circ + 360^\circ = 165^\circ\)
Negative angle: \(-195^\circ - 360^\circ = -555^\circ\)

**ANSWER:**
\(165^\circ, -555^\circ\)

Rewrite each degree measure in radians and each radian measure in degrees.

25. \(330^\circ\)

**SOLUTION:**
\[
\begin{align*}
330^\circ &= 330 \cdot \frac{\pi}{180} \\
&= \frac{330\pi}{180} \\
&= \frac{11\pi}{6}
\end{align*}
\]

**ANSWER:**
\(\frac{11\pi}{6}\)

26. \(\frac{5\pi}{6}\)

**SOLUTION:**
\[
\begin{align*}
\frac{5\pi}{6} &= \frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} \\
&= \frac{900^\circ}{6} \\
&= 150^\circ
\end{align*}
\]

**ANSWER:**
\(150^\circ\)
12-2 Angles and Angle Measure

27. \(-\frac{\pi}{3}\)

\[\text{SOLUTION:}\]
\[-\frac{\pi}{3} = -\frac{\pi}{3} \cdot \frac{180}{\pi} \]
\[= -\frac{180}{3} \]
\[= -60^\circ\]

\[\text{ANSWER:}\]
\[-60^\circ\]

28. \(-50^\circ\)

\[\text{SOLUTION:}\]
\[-50 = -50 \cdot \frac{\pi}{180} \]
\[= -\frac{50\pi}{180} \text{ or } -\frac{5\pi}{18} \text{ radians}\]

\[\text{ANSWER:}\]
\[-\frac{5\pi}{18}\]

29. \(190^\circ\)

\[\text{SOLUTION:}\]
\[190 = 190 \cdot \frac{\pi}{180} \]
\[= \frac{190\pi}{180} \text{ or } \frac{19\pi}{18} \text{ radians}\]

\[\text{ANSWER:}\]
\[\frac{19\pi}{18}\]
12-2 Angles and Angle Measure

31. **SKATEBOARDING** The skateboard ramp at the 8 ft right is called a *quarter pipe*. The curved surface is determined by the radius of a circle. Find the length of the curved part of the ramp.

![Diagram of a quarter pipe skateboard ramp]

**SOLUTION:**
Rewrite the central angle in radians.

\[
90 = 90 \cdot \frac{\pi \text{ radians}}{180} = \frac{90\pi}{180} = \frac{\pi}{2} \text{ radians}
\]

Substitute 8 for \( r \) and \( \frac{\pi}{2} \) for \( \theta \) in the formula to find the arc length.

\[
s = r\theta = 8 \cdot \frac{\pi}{2} = 4\pi \\
\approx 12.6 \text{ ft}
\]

**ANSWER:**
about 12.6 ft

32. **RIVERBOATS** The paddlewheel of a riverboat has a diameter of 24 feet. Find the arc length of the circle made when the paddlewheel rotates 300°.

**SOLUTION:**
Rewrite the central angle in radians.

\[
300 = 300 \cdot \frac{\pi \text{ radians}}{180} = \frac{300\pi}{180} = \frac{5\pi}{3} \text{ radians}
\]

Radius = \( \frac{24}{2} = 12 \)

Substitute 12 for \( r \) and \( \frac{5\pi}{3} \) for \( \theta \) in the formula to find the arc length.

\[
s = r\theta = 12 \cdot \frac{5\pi}{3} = 20\pi \\
\approx 62.8 \text{ ft}
\]

**ANSWER:**
about 62.8 ft
**12-2 Angles and Angle Measure**

Find the length of each arc. Round to the nearest tenth.

33.  

**SOLUTION:**  
Substitute 5 for \( r \) and \( \frac{3\pi}{7} \) for \( \theta \) in the formula to find the arc length.  

\[
s = r\theta \\
= 5 \cdot \frac{3\pi}{7} \\
= \frac{15\pi}{7} \\
= 6.7 \text{ cm}
\]

**ANSWER:**  
6.7 cm

34.  

**SOLUTION:**  
Substitute 27 for \( r \) and \( \frac{10\pi}{9} \) for \( \theta \) in the formula to find the arc length.  

\[
s = r\theta \\
= 27 \cdot \frac{10\pi}{9} \\
= \frac{270\pi}{9} \\
= 30\pi \\
= 94.2 \text{ m}
\]

**ANSWER:**  
94.2 m

35. **CLOCKS** How long does it take for the minute hand on a clock to pass through \( 2.5\pi \) radians?  

**SOLUTION:**  
Rewrite the radian measure in degrees.  

\[2.5\pi = 2.5\pi \text{ radians} \cdot \frac{180}{\pi \text{ radians}} = 450^\circ\]

\[450^\circ = 360^\circ + 90^\circ\]

Rotation of \( 360^\circ \) makes an hour and rotation of \( 90^\circ \) makes 15 minutes.  
So, the minute hand takes 1 hour and 15 minutes to pass through \( 2.5\pi \) radians.  

**ANSWER:**  
1 h 15 min
36. CCSS PERSEVERANCE Refer to the beginning of the lesson. A shadow moves around a sundial 15° every hour.

a. After how many hours is the angle of rotation of the shadow \(rac{8\pi}{5}\) radians?

b. What is the angle of rotation in radians after 5 hours?

c. A sundial has a radius of 8 inches. What is the arc formed by a shadow after 14 hours? Round to the nearest tenth.

**SOLUTION:**

**a.** Rewrite the radian measure in degrees.

\[
\frac{8\pi}{5} = \frac{8\pi}{5} \text{ radians} \cdot \frac{180°}{\pi \text{ radians}}
\]

= 288°

For every hour the shadow moves 15° around a sundial.

So 288° rotation takes \(\frac{288}{15}\) hours or 19.2 hours.

**b.** Let \(x\) represents the angle of rotation (in radians) after 5 hours.

The equation that represents the situation is

\[
\frac{8\pi}{5} = \frac{19.2}{5}.
\]

\[
\frac{8\pi}{5x} = \frac{19.2}{5}
\]

40\(\pi\) = 96\(x\)

\[
x = \frac{40\pi}{96}
\]

\[
x = \frac{5\pi}{12}
\]

c. Let \(x\) represents the angle of rotation after 14 hours.

The equation that represents the situation is

\[
\frac{8\pi}{5} = \frac{19.2}{5}.
\]

\[
\frac{8\pi}{5x} = \frac{19.2}{5}
\]

112\(\pi\) = 96\(x\)

\[
x = \frac{112\pi}{96}
\]

\[
x = \frac{7\pi}{6}
\]

Substitute 8 for \(r\) and \(\frac{7\pi}{6}\) for \(\theta\) in the formula to find the arc length.

\[
s = r\theta
\]

\[
= 8 \cdot \frac{7\pi}{6}
\]

\[
\approx 29.3 \text{ in}
\]

**ANSWER:**

a. 19.2 h

b. \(\frac{5\pi}{12}\)

c. 29.3 in.

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

37. 620°

**SOLUTION:**

Positive angle: 620° – 360° = 260°

Negative angle: 620° – 2(360°) = –100°

**ANSWER:**

260°, –100°
12-2 Angles and Angle Measure

38. \(-400^\circ\)

**SOLUTION:**
Positive angle: \(-400^\circ + 2(360^\circ) = 320^\circ\)
Negative angle: \(-400^\circ + 360^\circ = -40^\circ\)

**ANSWER:**
\(320^\circ, -40^\circ\)

39. \(-\frac{3\pi}{4}\)

**SOLUTION:**
Positive angle: \(-\frac{3\pi}{4} + 2\pi = \frac{5\pi}{4}\)
Negative angle: \(-\frac{3\pi}{4} - 2\pi = -\frac{11\pi}{4}\)

**ANSWER:**
\(\frac{5\pi}{4}, -\frac{11\pi}{4}\)

40. \(\frac{19\pi}{6}\)

**SOLUTION:**
Positive angle: \(\frac{19\pi}{6} - 2\pi = \frac{7\pi}{6}\)
Negative angle: \(\frac{19\pi}{6} - 2(2\pi) = -\frac{5\pi}{6}\)

**ANSWER:**
\(\frac{7\pi}{6}, -\frac{5\pi}{6}\)

41. **SWINGS** A swing has a 165° angle of rotation.

a. Draw the angle in standard position.

b. Write the angle measure in radians.

c. If the chains of the swing are \(6\frac{1}{2}\) feet long, what is the length of the arc that the swing makes? Round to the nearest tenth.

d. Describe how the arc length would change if the lengths of the chains of the swing were doubled.

**SOLUTION:**

a. Draw the terminal side of the angle 165° counterclockwise.

b. 

\[
165^\circ = 165^\circ \cdot \frac{\pi}{180} \text{ radians} \\
= \frac{165\pi}{180} = \frac{11\pi}{12} \text{ radians}
\]

c. Substitute \(6\frac{1}{2}\) for \(r\) and \(\frac{11\pi}{12}\) for \(\theta\) in the formula to find the arc length.

\[
s = r\theta \\
= 6\frac{1}{2} \cdot \frac{11\pi}{12} \\
= \frac{13}{2} \cdot \frac{11\pi}{12} \\
\approx 18.7 \text{ ft}
\]

d. The arc length would double. Since \(s = r\theta\), if \(r\) is doubled and \(\theta\) remains unchanged, then the value of \(s\) is also doubled.

**ANSWER:**
12-2 Angles and Angle Measure

a.

b. \( \frac{11\pi}{12} \)

c. 18.7 ft

d. The arc length would double. Since \( s = r\theta \), if \( r \) is doubled and \( \theta \) remains unchanged, then the value of \( s \) is also doubled.

42. MULTIPLE REPRESENTATIONS Consider \( A(-4, 0), B(-4, 6), C(6, 0), \) and \( D(6, 8) \).

a. GEOMETRIC Draw \( \triangle EAB \) and \( \triangle ECD \) with \( E \) at the origin.

b. ALGEBRAIC Find the values of the tangent of \( \triangle BAE \) and the tangent of \( \triangle DEC \).

c. ALGEBRAIC Find the slope of \( \overline{BE} \) and \( \overline{ED} \).

d. VERBAL What conclusions can you make about the relationship between slope and tangent?

SOLUTION:

a. Plot the given coordinates and \( E \) at the origin and join them to form the triangles \( \triangle EAB \) and \( \triangle ECD \).

b. 
\[
\begin{align*}
\overline{BA} &= 6 - 0 = 6 \\
\overline{EA} &= 0 - 4 = -4 \\
\overline{DC} &= 8 - 0 = 8 \\
\overline{EC} &= 6 - 0 = 6 \\
\end{align*}
\]

\[
\tan \angle BAE = \frac{-6}{4} = -\frac{3}{2}
\]

\[
\tan \angle DEC = \frac{8}{6} = \frac{4}{3}
\]

c. Consider the endpoints \((-4, 6)\) and \((0, 0)\) of \( \overline{BE} \).

\[
\text{Slope of } \overline{BE} = \frac{0 - 6}{0 + 4} = \frac{-3}{2}
\]

Consider the endpoints \((6, 8)\) and \((0, 0)\) of \( \overline{ED} \).

\[
\text{Slope of } \overline{ED} = \frac{0 - 8}{0 - 6} = \frac{4}{3}
\]

d. Sample answer: In the coordinate plane, the tangent of the angle in standard position equals the
12-2 Angles and Angle Measure

slope of the terminal side of the angle.

ANSWER:

a. 

![Diagram](image)

b. \( \tan \angle BEA = \frac{3}{2}, \tan \angle DEC = \frac{4}{3} \)

c. slope of \( BE = -\frac{3}{2}, \) slope of \( ED = \frac{4}{3} \)

d. Sample answer: In the coordinate plane, the tangent of the angle in standard position equals the slope of the terminal side of the angle.

Rewrite each degree measure in radians and each radian measure in degrees.

43. \( \frac{21\pi}{8} \)

SOLUTION:

\[
\frac{21\pi}{8} = \frac{21\pi}{8} \text{ radians} \cdot \frac{180}{\pi \text{ radians}}
= \frac{3780^\circ}{8}
= 472.5^\circ
\]

ANSWER:

472.5°

44. 124°

SOLUTION:

\[
124 = 124 \cdot \frac{\pi \text{ radians}}{180}
= \frac{124\pi}{180} \text{ or } \frac{31\pi}{45} \text{ radians}
\]

ANSWER:

\[ \frac{31\pi}{45} \]

45. –200°

SOLUTION:

\[
-200 = -200 \cdot \frac{\pi \text{ radians}}{180}
= \frac{-200\pi}{180} \text{ or } \frac{-10\pi}{9} \text{ radians}
\]

ANSWER:

\[ \frac{-10\pi}{9} \]

46. 5

SOLUTION:

\[
5 = 5 \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}}
= \frac{900^\circ}{\pi}
\approx 286.5^\circ
\]

ANSWER:

\[ \frac{900}{\pi} \approx 286.5^\circ \]

47. CAROUSELS A carousel makes 5 revolutions per minute. The circle formed by riders sitting in the outside row has a radius of 17.2 feet. The circle
12-2 Angles and Angle Measure

formed by riders sitting in the inside row has a radius of 13.1 feet.

a. Find the angle \( \theta \) in radians through which the carousel rotates in one second.

b. In one second, what is the difference in arc lengths between the riders sitting in the outside row and the riders sitting in the inside row?

\[ s = r \theta \]

\[ = 13.1 \cdot \frac{\pi}{6} \]

\[ \approx 6.9 \text{ ft} \]

The difference in arc lengths between the riders sitting in the outside row and the riders sitting in the inside row is \( 2.1 \text{ ft} \cdot (9 - 6.9) \).

**ANSWER:**

- \( \frac{\pi}{6} \)
- 2.1 ft
12-2 Angles and Angle Measure

48. CCSS CRITIQUE Tarshia and Alan are writing an expression for the measure of an angle coterminal with the angle shown. Is either of them correct? Explain your reasoning.

SOLUTION:
Tarshia is correct; a coterminal angle can be found by adding a multiple of 360° or by subtracting a multiple of 360°. Alan incorrectly subtracted the original angle measure from 360°.

ANSWER:
Tarshia: a coterminal angle can be found by adding a multiple of 360° or by subtracting a multiple of 360°. Alan incorrectly subtracted the original angle measure from 360°.

49. CHALLENGE A line makes an angle of \( \frac{\pi}{2} \) radians with the positive x-axis at the point (2, 0). Find an equation for this line.

SOLUTION:
The equation of the line is \( x = 2 \).

ANSWER:
\( x = 2 \)

50. REASONING Express \( \frac{1}{8} \) of a revolution in degrees and in radians. Explain your reasoning.

SOLUTION:
\( \frac{1}{8} \) of a revolution is 45° or \( \frac{\pi}{4} \).

One revolution makes 360° or 2\( \pi \) radians.

\( \frac{1}{8} \) of a revolution in degrees = \( \frac{1}{8} \cdot 360° \) or 45°

\( \frac{1}{8} \) of a revolution in radians = \( \frac{1}{8} \cdot 2\pi \) or \( \frac{\pi}{4} \)

ANSWER:
45°, \( \frac{\pi}{4} \); \( \frac{1}{8} \) of 360° is 45° and \( \frac{1}{8} \) of 2\( \pi \) is \( \frac{\pi}{4} \)
12-2 Angles and Angle Measure

51. OPEN ENDED Draw and label an acute angle in standard position. Find two angles, one positive and one negative, that are coterminal with the angle.

SOLUTION:
Sample answer:

Positive angle: \(80^\circ + 360^\circ = 440^\circ\)
Negative angle: \(80^\circ - 360^\circ = -280^\circ\)

ANSWER:
Sample answer:

440° and −280°

52. REASONING Justify the formula for the length of an arc.

SOLUTION:
Use a proportion.

\[
\frac{\theta}{2\pi} = \frac{s}{2\pi r}
\]

\[
\theta = \frac{2\pi s}{2\pi r} = \frac{s}{r}
\]

ANSWER:
Use a proportion.

\[
\frac{\theta}{2\pi} = \frac{s}{2\pi r}
\]

\[
\theta = \frac{2\pi s}{2\pi r} = \frac{s}{r}
\]
53. **WRITING IN MATH** Use a circle with radius \( r \) to describe what one degree and one radian represent. Then explain how to convert between the measures.

**SOLUTION:**
One degree represents an angle measure that equals \( \frac{1}{360} \) rotation around a circle. One radian represents the measure of an angle in standard position that intercepts an arc of length \( r \). To change from degrees to radians, multiply the number of degrees by \( \frac{\pi \text{ radians}}{180^\circ} \). To change from radians to degrees, multiply the number of radians by \( \frac{180^\circ}{\pi \text{ radians}} \).

**ANSWER:**
One degree represents an angle measure that equals \( \frac{1}{360} \) rotation around a circle. One radian represents the measure of an angle in standard position that intercepts an arc of length \( r \). To change from degrees to radians, multiply the number of degrees by \( \frac{\pi \text{ radians}}{180^\circ} \). To change from radians to degrees, multiply the number of radians by \( \frac{180^\circ}{\pi \text{ radians}} \).

54. **SHORT RESPONSE** If \( (x + 6)(x + 8) - (x - 7)(x - 5) = 0 \), find \( x \).

**SOLUTION:**
\[
(x + 6)(x + 8) - (x - 7)(x - 5) = 0 \\
(x^2 + 14x + 48) - (x^2 - 12x + 35) = 0 \\
x^2 + 14x + 48 - x^2 + 12x - 35 = 0 \\
26x + 13 = 0 \\
26x = -13 \\
x = \frac{13}{26} \\
x = \frac{1}{2}
\]

**ANSWER:**
\[
\frac{1}{2}
\]
12-2 Angles and Angle Measure

55. Which of the following represents an inverse variation?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Y</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

SOLUTION:
Consider the option A.

\((x_1, y_1) = (2, 50)\)
\((x_2, y_2) = (5, 20)\)

Use a proportion that relates the values.

\[ \frac{y_1}{x_2} = \frac{y_2}{x_1} \]

Indirect variation

\[ \frac{50}{5} = \frac{20}{2} \]

\[ 10 = 10 \checkmark \]

Thus, option A represents an inverse variation.

A is the correct option.

ANSWER:
A

56. GEOMETRY If the area of the figure is 60 square units, what is the length of side \( \overline{XZ} \)?

SOLUTION:
Let the base of the triangle be \( x \).
Substitute 6 for height, \( x \) for base and 60 for area in the area formula.

\[ \text{Area of a triangle} = \frac{1}{2} \cdot \text{base} \cdot \text{height} \]

\[ 60 = \frac{1}{2} (x)(6) \]

\[ 60 = 3x \]

\[ x = \frac{60}{3} \]

\[ x = 20 \]

Use the Pythagorean equation to find the other side of the triangle.

\[ c^2 = a^2 + b^2 \]

\[ c^2 = 6^2 + 20^2 \]

\[ c^2 = 36 + 400 \]

\[ c^2 = 436 \]

\[ c = \sqrt{436} \]

\[ c = 2\sqrt{109} \]

G is the correct option.

ANSWER:
G
12-2 Angles and Angle Measure

57. SAT/ACT  The first term of a sequence is \(-6\), and every term after the first is 8 more than the term immediately preceding it. What is the value of the 101st term?

A 788  
B 794  
C 802  
D 808  
E 814

**SOLUTION:**
Substitute \(a_1 = -6\), \(d = 8\) and \(n = 101\) in the formula to find \(n^{th}\) term.

\[
a_n = a_1 + (n-1)d  
\]

\[
a_n = -6 + (101-1)8  
\]

\[
= -6 + 800  
\]

\[
= 794  
\]

B is the correct option.

**ANSWER:**
B

---

58. Find the values of the six trigonometric functions for angle \(\theta\).

**SOLUTION:**
Use the Pythagorean equation to find the unknown side of the triangle.

\[c^2 = a^2 + b^2\]

\[c^2 = 3^2 + 14^2\]

\[c^2 = 205\]

\[c = \sqrt{205}\]

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{205}} \quad \text{or} \quad \frac{3\sqrt{205}}{205}  
\]

\[
\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{14}{\sqrt{205}} \quad \text{or} \quad \frac{14\sqrt{205}}{205}  
\]

\[
\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{14}  
\]

\[
\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{205}}{3}  
\]

\[
\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{205}}{14}  
\]

\[
\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{14}{3}  
\]

**ANSWER:**

\[
\sin \theta = \frac{3}{\sqrt{205}} \quad \text{or} \quad \frac{3\sqrt{205}}{205}, \quad \cos \theta = \frac{14}{\sqrt{205}} \quad \text{or} \quad \frac{14\sqrt{205}}{205}, \quad \tan \theta = \frac{3}{14}, \quad \csc \theta = \frac{\sqrt{205}}{3}, \quad \sec \theta = \frac{\sqrt{205}}{14}, \quad \cot \theta = \frac{14}{3}  
\]
**12-2 Angles and Angle Measure**

**SOLUTION:**
Use the Pythagorean equation to find the unknown side of the triangle.

\[ c^2 = a^2 + b^2 \]
\[ 22^2 = a^2 + 15^2 \]
\[ 484 = a^2 + 225 \]
\[ a^2 = 259 \]
\[ a = \sqrt{259} \]

\[
\begin{align*}
\sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{259}}{22} \\
\cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{15}{22} \\
\tan \theta &= \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{259}}{15} \\
\csc \theta &= \frac{\text{hyp}}{\text{opp}} = \frac{22}{\sqrt{259}} \quad \text{or} \quad \frac{22\sqrt{259}}{259} \\
\sec \theta &= \frac{\text{hyp}}{\text{adj}} = \frac{22}{15} \\
\cot \theta &= \frac{\text{adj}}{\text{opp}} = \frac{15}{\sqrt{259}} \quad \text{or} \quad \frac{15\sqrt{259}}{259}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= \frac{\sqrt{259}}{22} \\
\cos \theta &= \frac{15}{22} \\
\tan \theta &= \frac{\sqrt{259}}{15} \\
\csc \theta &= \frac{22\sqrt{259}}{259} \\
\sec \theta &= \frac{22}{15} \\
\cot \theta &= \frac{15}{\sqrt{259}} \quad \text{or} \quad \frac{15\sqrt{259}}{259}
\end{align*}
\]
12-2 Angles and Angle Measure

Identify the null and alternative hypotheses for each statement. Then identify the statement that represents the claim.

61. Tom drinks at least eight glasses of water every day.

**SOLUTION:**

at least 8 glasses: \( \mu \geq 8 \)
less than 8 glasses: \( \mu < 8 \)

The claim is \( \mu \geq 8 \), and it is the null hypothesis because it does include equality. The alternative hypothesis is \( \mu < 8 \), which is the complement of \( \mu \geq 8 \).

\( H_0: \mu \geq 8 \) (claim)
\( H_a: \mu < 8 \)

**ANSWER:**

\( H_0: \mu \geq 8 \) (claim); \( H_a: \mu < 8 \)

62. Juanita says that she has two umbrellas in her car.

**SOLUTION:**

2 umbrellas: \( \mu = 2 \)
not 2 umbrellas: \( \mu \neq 2 \)

The claim is \( \mu = 2 \), and it is the null hypothesis because it does include equality. The alternative hypothesis is \( \mu \neq 2 \), which is the complement of \( \mu = 2 \).

\( H_0: \mu = 2 \) (claim)
\( H_a: \mu \neq 2 \)

**ANSWER:**

\( H_0: \mu = 2 \) (claim); \( H_a: \mu \neq 2 \)

63. MANUFACTURING The sizes of CDs made by a company are normally distributed with a standard deviation of 1 millimeter. The CDs are supposed to be 120 millimeters in diameter, and they are made for drives that are 122 millimeters wide.

a. What percent of the CDs would you expect to be greater than 120 millimeters?

b. If the company manufactures 1000 CDs per hour, how many of the CDs made in one hour would you expect to be between 119 and 122 millimeters?

c. About how many CDs per hour will be too large to fit in the drives?

**SOLUTION:**

a. Given \( \mu = 120 \) and \( \sigma = 1 \).

The percent of the CDs would you expect to be greater than 120 millimeters is

\[ P(x > 120) = 34\% + 13.5\% + 2\% + 0.5\% = 50\% . \]

b. Given \( \mu = 120 \) and \( \sigma = 1 \).

The probability of CDs that was made in one hour would be between 119 and 122 millimeters \( \mu - \sigma \) and \( \mu + 2\sigma \), that is, between 120 – 1 or 119 and 120 + 2(1) or 122.

\[ P(119 < x < 122) = 34\% + 34\% + 13.5\% = 81.5\% \]

81.5% of 1000 CDs = 815 CDs

c. Given \( \mu = 120 \) and \( \sigma = 1 \).

The probability CDs that will be too large to fit in the drives is greater than \( \mu + 2\sigma \), that is, 120 + 2(1) or 122.

\[ P(x > 122) = 2\% + 0.5\% = 2.5\% \]

2.5% of 1000 CDs = 25 CDs

**ANSWER:**

a. 50%

b. 815

c. 25
12-2 Angles and Angle Measure

64. **FINANCIAL LITERACY**  If the rate of inflation is 2%, the cost of an item in future years can be found by iterating the function \( c(x) = 1.02x \). Find the cost of a $70 digital audio player in four years if the rate of inflation remains constant.

**SOLUTION:**
Given \( a_1 = 70 \), \( r = 1.02 \) and \( n = 5 \)

\[
a_n = a_1 r^{n-1}
\]

\[
a_5 = 70(1.02)^{5-1}
\]

\[
= 70(1.02)^4
\]

\[
= 75.77
\]

**ANSWER:**
$75.77

66. \( a = 8, b = 17 \)

**SOLUTION:**
Substitute 8 for \( a \) and 17 for \( b \) in the Pythagorean equation and solve for \( c \).

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 8^2 + 17^2
\]

\[
c^2 = 353
\]

\[
c = \sqrt{353}
\]

**ANSWER:**
\( \sqrt{353} \)

65. \( a = 12, b = 15 \)

**SOLUTION:**
Substitute 12 for \( a \) and 15 for \( b \) in the Pythagorean equation and solve for \( c \).

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 12^2 + 15^2
\]

\[
c^2 = 369
\]

\[
c = \sqrt{369}
\]

\[
c = 3\sqrt{41}
\]

**ANSWER:**
\( 3\sqrt{41} \)

67. \( a = 14, b = 11 \)

**SOLUTION:**
Substitute 14 for \( a \) and 11 for \( b \) in the Pythagorean equation and solve for \( c \).

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 14^2 + 11^2
\]

\[
c^2 = 317
\]

\[
c = \sqrt{317}
\]

**ANSWER:**
\( \sqrt{317} \)
12-3 Trigonometric Functions of General Angles

The terminal side of $\theta$ in standard position contains each point. Find the exact values of the six trigonometric functions of $\theta$.

1. $(1, 2)$

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{1^2 + 2^2}$$
$$r = \sqrt{5}$$

Use $x = 1, y = 2,$ and $r = \sqrt{5}$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{2}{1}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{5}}{2}$$

$$\sec \theta = \frac{r}{x} = \sqrt{5}$$

$$\cot \theta = \frac{x}{y} = \frac{1}{2}$$

**ANSWER:**

$$\sin \theta = \frac{2\sqrt{5}}{5}, \cos \theta = \frac{\sqrt{5}}{5},$$

$$\tan \theta = 2, \csc \theta = \frac{\sqrt{5}}{2},$$

$$\sec \theta = \sqrt{5}, \cot \theta = \frac{1}{2}$$

2. $(-8, -15)$

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-8)^2 + (-15)^2}$$
$$r = \sqrt{289}$$
$$r = 17$$

Use $x = -8, y = -15,$ and $r = 17$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{-15}{17}$$

$$\cos \theta = \frac{x}{r} = \frac{-8}{17}$$

$$\tan \theta = \frac{y}{x} = \frac{15}{8}$$

$$\csc \theta = \frac{r}{y} = \frac{17}{15}$$

$$\sec \theta = \frac{r}{x} = \frac{17}{-8}$$

$$\cot \theta = \frac{x}{y} = \frac{8}{15}$$

**ANSWER:**

$$\sin \theta = \frac{-15}{17}, \cos \theta = \frac{-8}{17},$$

$$\tan \theta = \frac{15}{8}, \csc \theta = \frac{-17}{15},$$

$$\sec \theta = \frac{-17}{8}, \cot \theta = \frac{8}{15}$$
12-3 Trigonometric Functions of General Angles

3. $(0, -4)$

\textbf{SOLUTION:}
Find the value of $r$.

\begin{align*}
    r &= \sqrt{x^2 + y^2} \\
    &= \sqrt{0^2 + (-4)^2} \\
    &= \sqrt{16} \\
    &= 4
\end{align*}

Use $x = 0, y = -4,$ and $r = 4$ to write the six trigonometric ratios.

\begin{align*}
    \sin \theta &= \frac{y}{r} = -\frac{4}{4} \text{ or } -1 \\
    \cos \theta &= \frac{x}{r} = 0 \text{ or } 0 \\
    \tan \theta &= \frac{y}{x} = \frac{4}{0} \text{ undefined} \\
    \csc \theta &= \frac{r}{y} = -\frac{4}{4} \text{ or } -1 \\
    \sec \theta &= \frac{r}{x} = \frac{4}{0} \text{ undefined} \\
    \cot \theta &= \frac{x}{y} = 0 \text{ or } 0
\end{align*}

\textbf{ANSWER:}
\begin{align*}
    \sin \theta &= -1, \cos \theta = 0, \\
    \tan \theta &= \text{undefined}, \csc \theta = -1, \\
    \sec \theta &= \text{undefined}, \cot \theta = 0
\end{align*}

Sketch each angle. Then find its reference angle.

4. $300^\circ$

\textbf{SOLUTION:}

The terminal side of $300^\circ$ lies in Quadrant IV.

\begin{align*}
    \theta' &= 360^\circ - \theta \\
    &= 360^\circ - 300^\circ \\
    &= 60^\circ
\end{align*}

\textbf{ANSWER:}
$60^\circ$
5. $115^\circ$

**SOLUTION:**

The terminal side of $115^\circ$ lies in Quadrant II.

$$\theta' = 180^\circ - \theta$$

$$= 180^\circ - 115^\circ$$

$$= 65^\circ$$

**ANSWER:**

$65^\circ$

6. $-\frac{3\pi}{4}$

**SOLUTION:**

The coterminal angle of $\frac{5\pi}{4}$ lies in Quadrant III.

$$\theta' = \theta - \pi$$

$$= \frac{5\pi}{4} - \pi$$

$$= \frac{\pi}{4}$$

**ANSWER:**

$\frac{\pi}{4}$
12-3 Trigonometric Functions of General Angles

Find the exact value of each trigonometric function.

7. \( \sin \frac{3\pi}{4} \)

**SOLUTION:**
The terminal side of \( \frac{3\pi}{4} \) lies in Quadrant II.

\[ \theta' = \pi - \theta \]
\[ = \pi - \frac{3\pi}{4} \]
\[ = \frac{\pi}{4} \]

The sine function is positive in Quadrant II.

\[ \sin \frac{3\pi}{4} = \sin \frac{\pi}{4} \]
\[ = \sin 45^\circ \]
\[ = \frac{\sqrt{2}}{2} \]

**ANSWER:**
\[ \frac{\sqrt{2}}{2} \]

8. \( \tan \frac{5\pi}{3} \)

**SOLUTION:**
The terminal side of \( \frac{5\pi}{3} \) lies in Quadrant IV.

\[ \theta' = 2\pi - \theta \]
\[ = 2\pi - \frac{5\pi}{3} \]
\[ = \frac{\pi}{3} \]

The tangent function is negative in Quadrant IV.

\[ \tan \frac{5\pi}{3} = -\tan \frac{\pi}{3} \]
\[ = -\tan 60^\circ \]
\[ = -\sqrt{3} \]

**ANSWER:**
\[ -\sqrt{3} \]

9. \( \sec 120^\circ \)

**SOLUTION:**
The terminal side of \( 120^\circ \) lies in Quadrant II.

\[ \theta' = 180^\circ - \theta \]
\[ = 180^\circ - 120^\circ \]
\[ = 60^\circ \]

The secant function is negative in Quadrant II.

\[ \sec 120^\circ = -\sec 60^\circ \]
\[ = -2 \]

**ANSWER:**
\[ -2 \]
12-3 Trigonometric Functions of General Angles

10. \( \sin 300^\circ \)

**SOLUTION:**
The terminal side of \( 300^\circ \) lies in Quadrant IV.

\[
\theta' = 360^\circ - \theta
\]

\[
= 360^\circ - 300^\circ
\]

\[
= 60^\circ
\]

The sine function is negative in Quadrant IV.

\[
\sin 300^\circ = -\sin 60^\circ
\]

\[
= -\frac{\sqrt{3}}{2}
\]

**ANSWER:**
\[-\frac{\sqrt{3}}{2}\]

11. **ENTERTAINMENT** Alejandra opens her portable DVD player so that it forms a 125° angle. The screen is \( 5 \frac{1}{2} \) inches long.

- **a.** Redraw the diagram so that the angle is in standard position on the coordinate plane.

- **b.** Find the reference angle. Then write a trigonometric function that can be used to find the distance to the wall \( d \) that she can place the DVD player.

- **c.** Use the function to find the distance. Round to the nearest tenth.

**SOLUTION:**

**a.**

**b.** Reference angle: \( 180^\circ - 125^\circ = 55^\circ \)
The trigonometric function that can be used to find the distance to the wall is \( \cos 55^\circ = \frac{d}{5 \frac{1}{2}} \).

**c.**

\[
5 \frac{1}{2} \cos 55^\circ = d
\]

\[
d = 3.2 \text{ in}
\]

**ANSWER:**

- **a.**

- **b.** \( 55^\circ; \cos 55^\circ = \frac{d}{5 \frac{1}{2}} \)

- **c.** 3.2 in.
The terminal side of $\theta$ in standard position contains each point. Find the exact values of the six trigonometric functions of $\theta$.

12. (5, 12)

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{169}$$
$$= 13$$

Use $x = 5, y = 12$, and $r = 13$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{12}{13}$$
$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$
$$\tan \theta = \frac{y}{x} = \frac{12}{5}$$
$$\csc \theta = \frac{r}{y} = \frac{13}{12}$$
$$\sec \theta = \frac{r}{x} = \frac{13}{5}$$
$$\cot \theta = \frac{x}{y} = \frac{5}{12}$$

**ANSWER:**

$$\sin \theta = \frac{12}{13}, \cos \theta = \frac{5}{13}, \tan \theta = \frac{12}{5}, \csc \theta = \frac{13}{12}, \sec \theta = \frac{13}{5}, \cot \theta = \frac{5}{12}$$

13. (-6, 8)

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-6)^2 + 8^2}$$
$$= \sqrt{100}$$
$$= 10$$

Use $x = -6, y = 8$, and $r = 10$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$
$$\cos \theta = \frac{x}{r} = \frac{-3}{5}$$
$$\tan \theta = \frac{y}{x} = \frac{-4}{3}$$
$$\csc \theta = \frac{r}{y} = \frac{5}{4}$$
$$\sec \theta = \frac{r}{x} = \frac{-5}{3}$$
$$\cot \theta = \frac{x}{y} = \frac{-3}{4}$$

**ANSWER:**

$$\sin \theta = \frac{4}{5}, \cos \theta = \frac{-3}{5}, \tan \theta = \frac{-4}{3}, \csc \theta = \frac{5}{4}, \sec \theta = \frac{-5}{3}, \cot \theta = \frac{-3}{4}$$
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14. (3, 0)

**SOLUTION:**
Find the value of $r$.

\[
r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 0^2} = \sqrt{9} = 3
\]

Use $x = 3$, $y = 0$, and $r = 3$ to write the six trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{0}{3} = 0
\]
\[
\cos \theta = \frac{x}{r} = \frac{3}{3} = 1
\]
\[
\tan \theta = \frac{y}{x} = \frac{0}{3} = 0
\]
\[
\csc \theta = \frac{r}{y} = \frac{3}{0} \text{ undefined}
\]
\[
\sec \theta = \frac{r}{x} = \frac{3}{3} = 1
\]
\[
\cot \theta = \frac{x}{y} = \frac{3}{0} \text{ undefined}
\]

**ANSWER:**
\[\sin \theta = 0, \cos \theta = 1, \tan \theta = 0, \csc \theta = \text{undefined}, \sec \theta = 1, \cot \theta = \text{undefined}\]

15. (0, −7)

**SOLUTION:**
Find the value of $r$.

\[
r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-7)^2} = \sqrt{49} = 7
\]

Use $x = 0$, $y = -7$, and $r = 7$ to write the six trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{-7}{7} = -1
\]
\[
\cos \theta = \frac{x}{r} = \frac{0}{7} = 0
\]
\[
\tan \theta = \frac{y}{x} = \frac{-7}{0} \text{ undefined}
\]
\[
\csc \theta = \frac{r}{y} = \frac{7}{-7} = -1
\]
\[
\sec \theta = \frac{r}{x} = \frac{7}{0} \text{ undefined}
\]
\[
\cot \theta = \frac{x}{y} = \frac{0}{-7} = 0
\]

**ANSWER:**
\[\sin \theta = -1, \cos \theta = 0, \tan \theta = \text{undefined}, \csc \theta = -1, \sec \theta = \text{undefined}, \cot \theta = 0\]
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16. \((4, -2)\)

**SOLUTION:**
Find the value of \(r\).

\[
r = \sqrt{x^2 + y^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}
\]

Use \(x = 4, y = -2,\) and \(r = 2\sqrt{5}\) to write the six trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{-2}{2\sqrt{5}} = -\frac{\sqrt{5}}{5}
\]

\[
\cos \theta = \frac{x}{r} = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}
\]

\[
\tan \theta = \frac{y}{x} = -\frac{1}{2}
\]

\[
\csc \theta = \frac{r}{y} = -\sqrt{5}
\]

\[
\sec \theta = \frac{r}{x} = \frac{\sqrt{5}}{2}
\]

\[
\cot \theta = \frac{x}{y} = -2
\]

**ANSWER:**

\[
\sin \theta = -\frac{\sqrt{5}}{5}, \quad \cos \theta = \frac{2\sqrt{5}}{5},
\]

\[
\tan \theta = -\frac{1}{2}, \quad \csc \theta = -\sqrt{5},
\]

\[
\sec \theta = \frac{\sqrt{5}}{2}, \quad \cot \theta = -2
\]

17. \((-9, -3)\)

**SOLUTION:**
Find the value of \(r\).

\[
r = \sqrt{x^2 + y^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{90} = 3\sqrt{10}
\]

Use \(x = -9, y = -3,\) and \(r = 3\sqrt{10}\) to write the six trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{-3}{3\sqrt{10}} = -\frac{\sqrt{10}}{10}
\]

\[
\cos \theta = \frac{x}{r} = \frac{-9}{3\sqrt{10}} = -\frac{3\sqrt{10}}{10}
\]

\[
\tan \theta = \frac{y}{x} = \frac{1}{3}
\]

\[
\csc \theta = \frac{r}{y} = -\frac{\sqrt{10}}{3}
\]

\[
\sec \theta = \frac{r}{x} = -\frac{\sqrt{10}}{3}, \quad \cot \theta = 3
\]

**ANSWER:**

\[
\sin \theta = -\frac{\sqrt{10}}{10}, \quad \cos \theta = -\frac{3\sqrt{10}}{10},
\]

\[
\tan \theta = \frac{1}{3}, \quad \csc \theta = -\frac{\sqrt{10}}{3},
\]

\[
\sec \theta = -\frac{\sqrt{10}}{3}, \quad \cot \theta = 3
\]
Sketch each angle. Then find its reference angle.

18. 195°

**SOLUTION:**

\[ \theta = 195° \]

The terminal side of 195° lies in Quadrant III.

\[ \theta' = \theta - 180° \]
\[ = 195° - 180° \]
\[ = 15° \]

**ANSWER:**

15°

19. 285°

**SOLUTION:**

The terminal side of 285° lies in Quadrant IV.

\[ \theta' = 360° - \theta \]
\[ = 360° - 285° \]
\[ = 75° \]

**ANSWER:**

75°
20. \(-250^\circ\)

**SOLUTION:**

coterminal angle: \(-250^\circ + 360^\circ = 110^\circ\)

The terminal side of \(110^\circ\) lies in Quadrant II.

\[
\theta' = 180^\circ - \theta
\]

\[
= 180^\circ - 110^\circ
\]

\[
= 70^\circ
\]

**ANSWER:**

\(70^\circ\)

---

21. \(\frac{7\pi}{4}\)

**SOLUTION:**

The terminal side \(\frac{7\pi}{4}\) lies in Quadrant IV.

\[
\theta' = 2\pi - \theta
\]

\[
= 2\pi - \frac{7\pi}{4}
\]

\[
= \frac{\pi}{4}
\]

**ANSWER:**

\(\frac{\pi}{4}\)
22. \(-\frac{\pi}{4}\)

**SOLUTION:**

coterminal angle: \(-\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}\)

The terminal side \(\frac{7\pi}{4}\) lies in Quadrant IV.

\[
\theta' = 2\pi - \theta \\
\quad = 2\pi - \frac{7\pi}{4} \\
\quad = \frac{\pi}{4}
\]

**ANSWER:**
\(\frac{\pi}{4}\)

23. 400°

**SOLUTION:**

coterminal angle: \(400° - 360° = 40°\)

The terminal side of 40° lies in Quadrant I.

\[
\theta' = \theta \\
\quad = 40°
\]

**ANSWER:**
40°
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Find the exact value of each trigonometric function.

24. sin 210°

**SOLUTION:**
The terminal side of 210° lies in Quadrant III.

\[ \theta' = \theta - 180° \]
\[ = 210° - 180° \]
\[ = 30° \]

The sine function is negative in Quadrant III.

\[ \sin 210° = -\sin 30° \]
\[ = -\frac{1}{2} \]

**ANSWER:**
\[ -\frac{1}{2} \]

25. tan 315°

**SOLUTION:**
The terminal side of 315° lies in Quadrant IV.

\[ \theta' = 360° - \theta \]
\[ = 360° - 315° \]
\[ = 45° \]

The tangent function is negative in Quadrant IV.

\[ \tan 315° = -\tan 45° \]
\[ = -1 \]

**ANSWER:**
\[ -1 \]

26. cos 150°

**SOLUTION:**
The terminal side of 150° lies in Quadrant II.

\[ \theta' = 180° - \theta \]
\[ = 180° - 150° \]
\[ = 30° \]

The cosine function is negative in Quadrant II.

\[ \cos 150° = -\cos 30° \]
\[ = -\frac{\sqrt{3}}{2} \]

**ANSWER:**
\[ -\frac{\sqrt{3}}{2} \]

27. csc 225°

**SOLUTION:**
The terminal side of 225° lies in Quadrant III.

\[ \theta' = \theta - 180° \]
\[ = 225° - 180° \]
\[ = 45° \]

The cosecant function is negative in Quadrant III.

\[ \csc 225° = -\csc 45° \]
\[ = -\sqrt{2} \]

**ANSWER:**
\[ -\sqrt{2} \]
28. \( \sin \frac{4\pi}{3} \)

**SOLUTION:**

The terminal side \( \frac{4\pi}{3} \) lies in Quadrant III.

\[
\theta' = \theta - \pi \\
= \frac{4\pi}{3} - \pi \\
= \frac{\pi}{3}
\]

The sine function is negative in Quadrant III.

\[
\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} \\
= -\sin 60^\circ \\
= -\frac{\sqrt{3}}{2}
\]

**ANSWER:**

\[-\frac{\sqrt{3}}{2}\]

29. \( \cos \frac{5\pi}{3} \)

**SOLUTION:**

The terminal side \( \frac{5\pi}{3} \) lies in Quadrant IV.

\[
\theta' = 2\pi - \theta \\
= 2\pi - \frac{5\pi}{3} \\
= \frac{\pi}{3}
\]

The cosine function is positive in Quadrant IV.

\[
\cos \frac{5\pi}{3} = \cos \frac{\pi}{3} \\
= \cos 60^\circ \\
= \frac{1}{2}
\]

**ANSWER:**

\[\frac{1}{2}\]
30. \( \cot \frac{5\pi}{4} \)

**SOLUTION:**

The terminal side \( \frac{5\pi}{4} \) lies in Quadrant III.

\[
\theta' = \theta - \pi \\
= \frac{5\pi}{4} - \pi \\
= \frac{\pi}{4}
\]

The cotangent function is positive in Quadrant III.

\[
\cot \frac{5\pi}{4} = \cot \frac{\pi}{4} \\
= \cot 45' \\
= 1
\]

**ANSWER:**
1

31. \( \sec \frac{11\pi}{6} \)

**SOLUTION:**

The terminal side \( \frac{11\pi}{6} \) lies in Quadrant IV.

\[
\theta' = 2\pi - \theta \\
= 2\pi - \frac{11\pi}{6} \\
= \frac{\pi}{6}
\]

The secant function is positive in Quadrant IV.

\[
\sec \frac{11\pi}{6} = \sec \frac{\pi}{6} \\
= \sec 30' \\
= \frac{2\sqrt{3}}{3}
\]

**ANSWER:**
\[ \frac{2\sqrt{3}}{3} \]
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32. CCSS REASONING A soccer player x feet from the goalie kicks the ball toward the goal, as shown in the figure. The goalie jumps up and catches the ball 7 feet in the air.

a. Find the reference angle. Then write a trigonometric function that can be used to find how far from the goalie the soccer player was when he kicked the ball.

b. About how far away from the goalie was the soccer player?

SOLUTION:
a. Reference angle: 180°−154° = 26°

The trigonometric function \( \tan 26° = \frac{7}{x} \) can be used to find the distance from the goalie was the soccer player.

\[ x = \frac{7}{\tan 26°} \]
\[ x \approx 14.4 \text{ ft} \]

ANSWER:
a. 26°, \( \tan 26° = \frac{7}{x} \)
b. about 14.4 ft

33. SPRINKLER A sprinkler rotating back and forth shoots water out a distance of 10 feet. From the horizontal position, it rotates 145° before reversing its direction. At a 145° angle, about how far to the left of the sprinkler does the water reach?

\[ \cos 35° = \frac{d}{10} \]
\[ 10 \cdot \cos 35° = d \]
\[ 8.2 \approx d \]

The water reaches about 8.2 feet to the left of the sprinkler.

ANSWER:
about 8.2 ft

34. BASKETBALL The formula \( R = \frac{V_0^2 \sin 2\theta}{32} \) gives the distance of a basketball shot with an initial velocity of \( V_0 \) feet per second at an angle \( \theta \) with the ground.

a. If the basketball was shot with an initial velocity of 24 feet per second at an angle of 75°, how far will the basketball travel?

b. If the basketball was shot at an angle of 65° and traveled 10 feet, what was its initial velocity?
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c. If the basketball was shot with an initial velocity of 30 feet per second and traveled 12 feet, at what angle was it shot?

SOLUTION:

a. Substitute 24 for \( V_0 \) and \( 75^\circ \) for \( \theta \) in the given formula and simplify.

\[
R = \frac{V_0^2 \sin 2\theta}{32} = \frac{24^2 \sin 2(75^\circ)}{32} = \frac{576 \sin 150^\circ}{32} = 9 \text{ ft}
\]

b. Substitute \( 65^\circ \) for \( \theta \) and 10 for \( R \) in the given formula and solve for \( V_0 \).

\[
10 = \frac{V_0^2 \sin 2(65^\circ)}{32} \Rightarrow V_0^2 = \frac{320}{\sin 130^\circ} \Rightarrow V_0 = \sqrt{\frac{320}{\sin 130^\circ}} \approx 20.4 \text{ ft/s}
\]

c. Substitute 30 for \( V_0 \) and 12 for \( R \) in the given formula and solve for \( \theta \).

\[
12 = \frac{V_0^2 \sin 2\theta}{32} = \frac{30^2 \sin 2\theta}{32} = \frac{900}{384} \Rightarrow \sin 2\theta = \frac{900}{384} = 2.35 \Rightarrow 2\theta = \sin^{-1} \frac{384}{900} \\
2\theta \approx 25.3 \Rightarrow \theta = 12.6^\circ
\]

ANSWER:

a. 9 ft

b. about 20.4 feet per second

c. about 12.6°

35. PHYSICS A rock is shot off the edge of a ravine with a slingshot at an angle of \( 65^\circ \) and with an initial velocity of 6 meters per second. The equation that represents the horizontal distance of the rock \( x \) is \( x = v_0 (\cos \theta)t \), where \( v_0 \) is the initial velocity, \( \theta \) is the angle at which it is shot, and \( t \) is the time in seconds. About how far does the rock travel after 4 seconds?

SOLUTION:

Substitute \( V_0 = 6, \theta = 65^\circ \) and \( t = 4 \) in the given equation and solve for \( x \).

\[
x = V_0 (\cos \theta)t = 6(\cos 65^\circ)4 \approx 10.1 \text{ meters}
\]

The rock travels about 10.1 m after 4 seconds.

ANSWER:

about 10.1 m
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36. FERRIS WHEELS The Wonder Wheel Ferris wheel at Coney Island has a radius of about 68 feet and is 15 feet off the ground. After a person gets on the bottom car, the Ferris wheel rotates 202.5° counterclockwise before stopping. How high above the ground is this car when it has stopped?

**SOLUTION:**
Since the angle measured from the negative y-axis, the terminal angle is 202.5° – 90° or 112.5°.

Therefore, the reference angle is (θ') 180° – 112.5° or 67.5°.

Substitute r = 68 and θ = 67.5° in the sine ratio.

\[
\sin \theta = \frac{y}{r}
\]
\[
\sin 67.5° = \frac{y}{68}
\]
\[
y = 68 \sin 67.5°
\]
\[
y \approx 62.8
\]

The height above the ground to the car is 62.8 + 68 + 15 or 145.8 feet.

**ANSWER:**
145.8 ft

37. Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each terminal side, find the exact values of the remaining five trigonometric functions of θ.

**SOLUTION:**
\[\sin \theta = \frac{4}{5}, \text{ Quadrant II}\]

**Use the Pythagorean Theorem to find the adjacent angle.**
\[
\text{Adjacent side} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3
\]

Only sine function is positive in Quadrant II.

\[
\cos \theta = -\frac{3}{5}, \tan \theta = -\frac{4}{3}, \csc \theta = \frac{5}{4}, \sec \theta = -\frac{5}{3}, \cot \theta = -\frac{3}{4}
\]

**ANSWER:**
\[
\cos \theta = -\frac{3}{5}, \tan \theta = -\frac{4}{3}, \csc \theta = \frac{5}{4}, \sec \theta = -\frac{5}{3}, \cot \theta = -\frac{3}{4}
\]
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38. \( \tan \theta = -\frac{2}{3} \), Quadrant IV

\[
\text{SOLUTION:}
\]

Opposite side = 2
Adjacent side = 3

Use the Pythagorean Theorem to find the hypotenuse.

\[
\text{Hypotenuse} = \sqrt{2^2 + 3^2} = \sqrt{13}
\]

Only cosine function is positive in Quadrant IV.

\[
\begin{align*}
\sin \theta &= -\frac{2}{\sqrt{13}} \text{ or } -\frac{2\sqrt{13}}{13} \\
\cos \theta &= \frac{3}{\sqrt{13}} \text{ or } \frac{3\sqrt{13}}{13} \\
\csc \theta &= -\frac{\sqrt{13}}{2} \\
\sec \theta &= \frac{\sqrt{13}}{3} \\
\cot \theta &= -\frac{3}{2}
\end{align*}
\]

\[
\text{ANSWER:}
\]

\[
\begin{align*}
\sin \theta &= -\frac{2\sqrt{13}}{13}, \cos \theta &= \frac{3\sqrt{13}}{13}, \\
\csc \theta &= -\frac{\sqrt{13}}{2}, \sec \theta &= \frac{\sqrt{13}}{3}, \\
\cot \theta &= -\frac{3}{2}
\end{align*}
\]

39. \( \cos \theta = -\frac{8}{17} \), Quadrant III

\[
\text{SOLUTION:}
\]

Adjacent side = 8
Hypotenuse = 17

Use the Pythagorean Theorem to find the opposite side.

\[
\begin{align*}
\text{Opposite side} &= \sqrt{17^2 - 8^2} \\
&= \sqrt{225} \\
&= 15
\end{align*}
\]

Only tangent function is positive in Quadrant III.

\[
\begin{align*}
\sin \theta &= -\frac{15}{17} \\
\tan \theta &= \frac{15}{8} \\
\csc \theta &= -\frac{17}{15} \\
\sec \theta &= -\frac{17}{8} \\
\cot \theta &= \frac{8}{15}
\end{align*}
\]

\[
\text{ANSWER:}
\]

\[
\begin{align*}
\sin \theta &= -\frac{15}{17}, \tan \theta &= \frac{15}{8}, \\
\csc \theta &= -\frac{17}{15}, \sec \theta &= -\frac{17}{8}, \\
\cot \theta &= \frac{8}{15}
\end{align*}
\]
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40. \( \cot \theta = -\frac{12}{5} \), Quadrant IV

**SOLUTION:**

Opposite side = 5
Adjacent side = 12

Use the Pythagorean Theorem to find the hypotenuse.

\[
\text{Hypotenuse} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13
\]

Only cosine function is positive in Quadrant IV.

\[
\begin{align*}
\sin \theta &= -\frac{5}{13} \\
\cos \theta &= \frac{12}{13} \\
\tan \theta &= -\frac{5}{12} \\
\csc \theta &= -\frac{13}{5} \\
\sec \theta &= \frac{13}{12} \\
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= -\frac{5}{13}, \cos \theta = \frac{12}{13}, \\
\csc \theta &= -\frac{13}{5}, \sec \theta = \frac{13}{12}, \\
\tan \theta &= -\frac{5}{12} \\
\end{align*}
\]

Find the exact value of each trigonometric function.

41. \( \cot 270^\circ \)

**SOLUTION:**

Since the angle 270° is a quadrant angle, the coordinates of the point on its terminal side is \((0, -y)\).

\[
\begin{align*}
\cot 270^\circ &= \frac{x}{y} \\
&= \frac{0}{-y} \\
&= 0
\end{align*}
\]

**ANSWER:**

0

42. \( \csc 180^\circ \)

**SOLUTION:**

Since the angle 180° is a quadrant angle, the coordinates of the point on its terminal side is \((-x, 0)\).

Find the value of \( r \).

\[
r = \sqrt{a^2 + b^2} = \sqrt{(-x)^2 + 0^2} = x
\]

\[
\csc 180^\circ = \frac{r}{y} = \frac{0}{0}
\]

**ANSWER:**

undefined
43. \( \sin 570^\circ \)

**SOLUTION:**
The coterminial angle of 570° is 570° – 360° or 210°.
The terminal side of 210° lies in Quadrant III.

\[
\theta' = \theta - 180^\circ
\]
\[
= 210^\circ - 180^\circ
\]
\[
= 30^\circ
\]
The sine function is negative in Quadrant III.

\[
\sin 570^\circ = -\sin 30^\circ
\]
\[
= -\frac{1}{2}
\]

**ANSWER:**
\[-\frac{1}{2}\]

44. \( \tan \left( -\frac{7\pi}{6} \right) \)

**SOLUTION:**
The coterminial angle of \( \frac{5\pi}{6} \) is \( -\frac{7\pi}{6} + 2\pi = \frac{5\pi}{6} \).
The terminal side of \( \frac{5\pi}{6} \) lies in Quadrant II.

\[
\theta' = \pi - \theta
\]
\[
= \pi - \frac{5\pi}{6}
\]
\[
= \frac{\pi}{6}
\]
The tangent function is negative in Quadrant II.

\[
\tan \left( -\frac{7\pi}{6} \right) = -\tan \frac{\pi}{6}
\]
\[
= -\tan 30^\circ
\]
\[
= -\frac{\sqrt{3}}{3}
\]

**ANSWER:**
\[-\frac{\sqrt{3}}{3}\]
12-3 Trigonometric Functions of General Angles

45. $\cos\left(-\frac{11\pi}{6}\right)$

**SOLUTION:**

The coterminal angle of $-\frac{11\pi}{6}$ is $-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$.

The terminal side of $\frac{\pi}{6}$ lies in Quadrant I.

$\theta' = \theta$

$= \frac{\pi}{6}$

$\cos\left(-\frac{11\pi}{6}\right) = \cos\frac{\pi}{6}$

$= \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

**ANSWER:**

$\frac{\sqrt{3}}{2}$

46. $\cot\frac{9\pi}{4}$

**SOLUTION:**

The coterminal angle of $\frac{9\pi}{4}$ is $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$.

The terminal side of $\frac{\pi}{4}$ lies in Quadrant I.

$\theta' = \theta$

$= \frac{\pi}{4}$

$\cot\frac{9\pi}{4} = \cot\frac{\pi}{4}$

$= \cot 45^\circ$

$= 1$

**ANSWER:**

1

47. **CHALLENGE** For an angle $\theta$ in standard position, $\sin \theta = \frac{\sqrt{2}}{2}$ and $\tan \theta = -1$. Can the value of $\theta$ be $225^\circ$? Justify your reasoning.

**SOLUTION:**

No; for $\sin \theta = \frac{\sqrt{2}}{2}$ and $\tan \theta = -1$, the reference angle is $45^\circ$. However, for $\sin \theta$ to be positive and $\tan \theta$ to be negative, the reference angle must be in the second quadrant. So, the value of $\theta$ must be $135^\circ$ or an angle coterminal with $135^\circ$.

**ANSWER:**

No; for $\sin \theta = \frac{\sqrt{2}}{2}$ and $\tan \theta = -1$, the reference angle is $45^\circ$. However, for $\sin \theta$ to be positive and $\tan \theta$ to be negative, the reference angle must be in the second quadrant. So, the value of $\theta$ must be $135^\circ$ or an angle coterminal with $135^\circ$.
12-3 Trigonometric Functions of General Angles

48. CCSS ARGUMENTS  Determine whether \(3 \sin 60^\circ = \sin 180^\circ\) is true or false. Explain your reasoning.

**SOLUTION:**
False;
\[
3 \sin 60^\circ = 3 \cdot \frac{\sqrt{3}}{2} \quad \text{or} \quad 3 \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin 180^\circ = 0
\]

**ANSWER:**
False; \(3 \sin 60^\circ = 3 \cdot \frac{\sqrt{3}}{2} \quad \text{or} \quad 3 \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin 180^\circ = 0\)

49. REASONING  Use the sine and cosine functions to explain why \(\cot 180^\circ\) is undefined.

**SOLUTION:**
Sample answer: We know that \(\cot \theta = \frac{x}{y}, \sin \theta = \frac{y}{r}\) and \(\cos \theta = \frac{x}{r}\). Since \(\sin 180^\circ = 0\), it must be true that \(y = 0\). Thus \(\cot 180^\circ = \frac{x}{0}\), which is undefined.

**ANSWER:**
Sample answer:
We know that \(\cot \theta = \frac{x}{y}, \sin \theta = \frac{y}{r}\) and \(\cos \theta = \frac{x}{r}\). Since \(\sin 180^\circ = 0\), it must be true that \(y = 0\). Thus \(\cot 180^\circ = \frac{x}{0}\), which is undefined.

50. OPEN ENDED  Give an example of a negative angle \(\theta\) for which \(\sin \theta > 0\) and \(\cos \theta < 0\).

**SOLUTION:**
Sample answer: \(\theta = -200^\circ\)

**ANSWER:**
Sample answer: \(\theta = -200^\circ\)

51. WRITING IN MATH  Describe the steps for evaluating a trigonometric function for an angle \(\theta\) that is greater than \(90^\circ\). Include a description of a reference angle.

**SOLUTION:**
First, sketch the angle and determine in which quadrant it is located. Then use the appropriate rule for finding its reference angle \(\theta'\). A reference angle is the acute angle formed by the terminal side of \(\theta\) and the \(x\)-axis. Next, find the value of the trigonometric function for \(\theta'\). Finally, use the quadrant location to determine the sign of the trigonometric function value of \(\theta\).

**ANSWER:**
First, sketch the angle and determine in which quadrant it is located. Then use the appropriate rule for finding its reference angle \(\theta'\). A reference angle is the acute angle formed by the terminal side of \(\theta\) and the \(x\)-axis. Next, find the value of the trigonometric function for \(\theta'\). Finally, use the quadrant location to determine the sign of the trigonometric function value of \(\theta\).
12-3 Trigonometric Functions of General Angles

52. **GRIDDED RESPONSE** If the sum of two numbers is 21 and their difference is 3, what is their product?

**SOLUTION:**
Let the two unknown numbers be \(x\) and \(y\). The system of equations that represent the situation are \(x + y = 21\) and \(x - y = 3\).

\[
\begin{align*}
x + y &= 21 \\
x - y &= 3 \\
2x &= 24 \\
x &= 12
\end{align*}
\]

Substitute 12 for \(x\) in the first equation and solve for \(y\).

\[
\begin{align*}
x + y &= 21 \\
12 + y &= 21 \\
y &= 21 - 12 \\
y &= 9
\end{align*}
\]

The product of 9 and 12 is 108.

**ANSWER:** 108

53. **GEOMETRY** \(D\) is the midpoint of \(BC\), and \(A\) and \(E\) are the midpoints of \(BD\) and \(DC\), respectively. If the length of \(AE\) is 12, what is the length of \(BC\)?

\(A\) 6

\(B\) 12

\(C\) 24

\(D\) 48

**SOLUTION:**
\(D\) is the midpoint of \(BC\), so \(BD = DC\).

\(A\) and \(E\) are the midpoints of \(BD\) and \(DC\) respectively, so \(BA = AD = DE = EC\). They are at equal distance. Thus, the length of \(BC\) is \(4 \times 6\) or 24.

\(C\) is the correct option.

**ANSWER:**
\(C\)
54. The expression \((-6 + i)^2\) is equivalent to which of the following expressions?

\[ \begin{align*}
F & \quad -12i \\
G & \quad 36 - i \\
H & \quad 36 - 12i \\
J & \quad 35 - 12i
\end{align*} \]

**SOLUTION:**
\[
(-6 + i)^2 = (-6)^2 + (2 \cdot -6 \cdot i) + i^2 \\
= 36 - 12i - 1 \\
= 35 - 12i
\]

J is the correct option.

**ANSWER:**
J

55. SAT/ACT Of the following, which is least?

\[ \begin{align*}
A & \quad 1 + \frac{1}{4} \\
B & \quad 1 - \frac{1}{4} \\
C & \quad 1 + \frac{1}{4} \\
D & \quad 1 \times \frac{1}{4} \\
E & \quad \frac{1}{4} - 1
\end{align*} \]

**SOLUTION:**
\[
\begin{align*}
1 + \frac{1}{4} &= 1 \frac{1}{4} \\
1 - \frac{1}{4} &= \frac{3}{4} \\
\frac{1}{4} - 1 &= -\frac{3}{4} \\
1 \times \frac{1}{4} &= \frac{1}{4}
\end{align*}
\]

\(-\frac{3}{4}\) is the least number.

E is the correct option.

**ANSWER:**
E
12-3 Trigonometric Functions of General Angles

Rewrite each radian measure in degrees.

56. \( \frac{4}{3} \pi \)

**SOLUTION:**
\[
\frac{4\pi}{3} = \frac{4\pi}{3} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}}
= \frac{720^\circ}{3}
= 240^\circ
\]

**ANSWER:**
240°

57. \( \frac{11}{6} \pi \)

**SOLUTION:**
\[
\frac{11\pi}{6} = \frac{11\pi}{6} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}}
= \frac{1980^\circ}{6}
= 330^\circ
\]

**ANSWER:**
330°

58. \( -\frac{17}{4} \pi \)

**SOLUTION:**
\[
-\frac{17\pi}{4} = -\frac{17\pi}{4} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}}
= -\frac{3060^\circ}{4}
= -765^\circ
\]

**ANSWER:**
-765°

Solve each equation.

59. \( \cos a = \frac{13}{17} \)

**SOLUTION:**
\[
\cos a = \frac{13}{17}
\cos^{-1} \left( \frac{13}{17} \right) = a
a \approx 40.1^\circ
\]

**ANSWER:**
40.1°
60. \( \sin 30^\circ = \frac{b}{6} \)

**SOLUTION:**

\[
\sin 30^\circ = \frac{b}{6} \\
6 \sin 30^\circ = b \\
\begin{align*}
b &= 6 \times \frac{1}{2} \\
b &= 3
\end{align*}
\]

**ANSWER:** 3

61. \( \tan c = \frac{9}{4} \)

**SOLUTION:**

\[
\tan c = \frac{9}{4} \\
\tan^{-1} \left( \frac{9}{4} \right) = c \\
c \approx 66.0^\circ
\]

**ANSWER:** 66.0°

62. **ARCHITECTURE** A memorial being constructed in a city park will be a brick wall, with a top row of six gold-plated bricks engraved with the names of six local war veterans. Each row has two more bricks than the row above it. Prove that the number of bricks in the top \( n \) rows is \( n^2 + 5n \).

**SOLUTION:**

Sample answer:

Step 1: There are 6 bricks in the top row, and \( 1^2 + 5 (1) = 6 \), so the formula is true for \( n = 1 \).

Step 2: Assume that there are \( k^2 + 5k \) bricks in the top \( k \) rows for some positive integer \( k \).

Step 3: Since each row has 2 more bricks than the one above, the numbers of bricks in the rows form an arithmetic sequence.

The number of bricks in the \((k + 1)^{st}\) row is \( 6 + [(k + 1) - 1](2) \) or \( 2k + 6 \).

Then the number of bricks in the top \( k + 1 \) rows is \( k^2 + 5k + (2k + 6) \) or \( k^2 + 7k + 6 \).

\[
k^2 + 7k + 6 = (k + 1)^2 + 5(k + 1), \text{ which is the formula to be proved, where } n = k + 1.
\]

Therefore, the formula is true for \( n = k + 1 \).

Therefore, the number of bricks in the top \( n \) rows in \( n^2 + 5n \) for all positive integers \( n \).
63. **LEGENDS** There is a legend of a king who wanted to reward a boy for a good deed. The king gave the boy a choice. He could have $1,000,000 at once, or he could be rewarded daily for a 30-day month, with one penny on the first day, two pennies on the second day, and so on, receiving twice as many pennies each day as the previous day. How much would the second option be worth?

**SOLUTION:**
Substitute \( a_1 = 1 \) and \( r = 2 \) in the sum formula.

\[
S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{1 - 2^{30}}{1 - 2} = 1 - 1073741824 \approx 1073741824 \text{ pennies}
\]

The worth of the second option is $10,737,418.23.

**ANSWER:**
$10,737,418.23

64. \((2, -4), (10, 2)\)

**SOLUTION:**
Find the center.

\[
(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 10}{2}, \frac{-4 + 2}{2}\right) = (6, -1)
\]

Find the radius.

\[
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(10 - 6)^2 + (2 + 1)^2} = \sqrt{25} = 5
\]

The equation of the circle is \((x - 6)^2 + (y + 1)^2 = 25\).

**ANSWER:**
\((x - 6)^2 + (y + 1)^2 = 25\)
12-3 Trigonometric Functions of General Angles

65. \((-1, -10), (-7, 6)\)

**SOLUTION:**
Find the center.

\[
(h, k) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-1 - 7}{2}, \frac{-10 + 6}{2} \right) = (-4, -2)
\]

Find the radius.

\[
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-7 + 4)^2 + (6 + 2)^2} = \sqrt{73}
\]

The equation of the circle is \((x + 4)^2 + (y + 2)^2 = 73\).

**ANSWER:**
\((x + 4)^2 + (y + 2)^2 = 73\)

66. \((9, 0), (4, -7)\)

**SOLUTION:**
Find the center.

\[
(h, k) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{9 + 4}{2}, \frac{0 - 7}{2} \right) = (6.5, -3.5)
\]

Find the radius.

\[
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 6.5)^2 + (-7 + 3.5)^2} = \sqrt{18.5}
\]

The equation of the circle is \((x - 6.5)^2 + (y + 3.5)^2 = 18.5\).

**ANSWER:**
\((x - 6.5)^2 + (y + 3.5)^2 = 18.5\)
12-3 Trigonometric Functions of General Angles

Simplify each expression.

67. \[ \frac{5}{x^2 + 6x + 8} + \frac{x}{x^2 - 3x - 28} \]

**SOLUTION:**
\[
\frac{5}{x^2 + 6x + 8} + \frac{x}{x^2 - 3x - 28} = \frac{5}{(x + 2)(x + 4)} + \frac{x}{(x - 7)(x + 4)}
\]
\[
= \frac{5(x - 7) + x(x + 2)}{(x + 2)(x + 4)(x - 7)}
\]
\[
= \frac{5x - 35 + x^2 + 2x}{(x + 2)(x + 4)(x - 7)}
\]
\[
= \frac{x^2 + 7x - 35}{(x + 2)(x + 4)(x - 7)}
\]

**ANSWER:**
\[
\frac{x^2 + 7x - 35}{(x + 2)(x + 4)(x - 7)}
\]

68. \[ \frac{3x}{x^2 + 8x - 20} - \frac{6}{x^2 + 7x - 18} \]

**SOLUTION:**
\[
\frac{3x}{x^2 + 8x - 20} - \frac{6}{x^2 + 7x - 18} = \frac{3x}{(x + 10)(x - 2)} - \frac{6}{(x + 10)(x + 9)}
\]
\[
= \frac{3x(x + 9) - 6(x + 10)}{(x + 10)(x - 2)(x + 9)}
\]
\[
= \frac{3x^2 + 21x - 60}{(x + 10)(x - 2)(x + 9)}
\]
\[
= \frac{3(x^2 + 7x - 20)}{(x + 10)(x - 2)(x + 9)}
\]

**ANSWER:**
\[
\frac{3(x^2 + 7x - 20)}{(x + 10)(x + 9)(x - 2)}
\]

69. \[ \frac{4}{3x^2 + 12x} + \frac{2x}{x^2 - 2x - 24} \]

**SOLUTION:**
\[
\frac{4}{3x^2 + 12x} + \frac{2x}{x^2 - 2x - 24} = \frac{4}{3x(x + 4)} + \frac{2x}{(x - 6)(x + 4)}
\]
\[
= \frac{4(x - 6) + 2x(3x)}{3x(x + 4)(x - 6)}
\]
\[
= \frac{6x^2 + 4x - 24}{3x(x + 4)(x - 6)}
\]
\[
= \frac{2(3x^2 + 2x - 12)}{3(x + 4)(x - 6)}
\]

**ANSWER:**
\[
\frac{2(3x^2 + 2x - 12)}{3(x + 4)(x - 6)}
\]

Solve each equation or inequality. Round to the nearest ten-thousandth.

70. \[ 8^x = 30 \]

**SOLUTION:**
Use the property of equality for logarithmic functions.
\[ 8^x = 30 \]
\[ \log 8^x = \log 30 \]
\[ x \log 8 = \log 30 \]
\[ x = \frac{\log 30}{\log 8} \]
\[ x \approx 1.6356 \]

**ANSWER:**
1.6356
12-3 Trigonometric Functions of General Angles

71. \(5^3 = 64\)

**SOLUTION:**
Use the property of equality for logarithmic functions.

\[
\begin{align*}
5^3 &= 64 \\
\log 5^3 &= \log 64 \\
x \log 5 &= \log 64 \\
x &= \frac{\log 64}{\log 5} \\
x &\approx 2.5841
\end{align*}
\]

**ANSWER:**
2.5841

72. \(3^x + 2 = 41\)

**SOLUTION:**
Use the property of equality for logarithmic functions.

\[
\begin{align*}
3^{x+2} &= 41 \\
\log 3^{x+2} &= \log 41 \\
(x+2) \log 3 &= \log 41 \\
x + 2 &= \frac{\log 41}{\log 3} \\
x &= \frac{\log 41}{\log 3} - 2 \\
x &\approx 1.3802
\end{align*}
\]

**ANSWER:**
1.3802

73. \(\frac{1}{16}^4\)

**SOLUTION:**

\[
\begin{align*}
16^{-4} &= \frac{1}{16^4} \\
&= \frac{1}{\sqrt{16}} \\
&= \frac{1}{2^4} \\
&= \frac{1}{2}
\end{align*}
\]

**ANSWER:**
\(\frac{1}{2}\)

74. \(27^3\)

**SOLUTION:**

\[
\begin{align*}
27^3 &= (3^3)^3 \\
&= 3^{3 \cdot 3} \\
&= 3^4 \\
&= 81
\end{align*}
\]

**ANSWER:**
81
12-3 Trigonometric Functions of General Angles

75. \(25\)\(^{-\frac{5}{2}}\)

SOLUTION:
\[
25^{-\frac{5}{2}} = \frac{1}{25^{\frac{5}{2}}} = \frac{1}{(5^2)^{\frac{5}{2}}} = \frac{1}{5^{\frac{5}{2}}} = \frac{1}{5^5} = \frac{1}{3125}
\]

ANSWER:
\[
\frac{1}{3125}
\]

Solve for \(x\).

76. \(\frac{x + 2}{18} = \frac{x - 2}{9}\)

SOLUTION:
\[
\frac{x + 2}{18} = \frac{x - 2}{9} \\
9(x + 2) = 18(x - 2) \\
9x + 18 = 18x - 36 \\
9x = 54 \\
x = \frac{54}{9} \\
x = 6
\]

ANSWER:
6

77. \(\frac{x + 5}{x - 1} = \frac{7}{4}\)

SOLUTION:
\[
4(x + 5) = 7(x - 1) \\
4x + 20 = 7x - 7 \\
3x = 27 \\
x = \frac{27}{3} \\
x = 9
\]

ANSWER:
9

78. \(\frac{5}{x + 8} = \frac{15}{2x + 20}\)

SOLUTION:
\[
\frac{5}{x + 8} = \frac{15}{2x + 20} \\
5(2x + 20) = 15(x + 8) \\
10x + 100 = 15x + 120 \\
5x = -20 \\
x = -\frac{20}{5} \\
x = -4
\]

ANSWER:
-4
Find the area of \( \triangle ABC \) to the nearest tenth, if necessary.

1. 

\[
\text{SOLUTION:}
\text{Substitute } c = 7, b = 8 \text{ and } A = 86^\circ \text{ in the area formula.}
\]
\[
\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(8)(7)\sin 86^\circ \\
\approx 27.9 \text{ mm}^2
\]

\text{ANSWER:} 

27.9 \text{ mm}^2

2. 

\[
\text{SOLUTION:}
\text{Substitute } c = 4, \ a = 3 \text{ and } B = 30^\circ \text{ in the area formula.}
\]
\[
\text{Area} = \frac{1}{2}ac \sin B = \frac{1}{2}(3)(4)\sin 30^\circ \\
= 3 \text{ yd}^2
\]

\text{ANSWER:} 

3 \text{ yd}^2

3. \( A = 40^\circ, b = 11 \text{ cm, } c = 6 \text{ cm} \)

\[
\text{SOLUTION:}
\text{Substitute } c = 6, \ b = 11 \text{ and } A = 40^\circ \text{ in the area formula.}
\]
\[
\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(11)(6)\sin 40^\circ \\
\approx 21.2 \text{ cm}^2
\]

\text{ANSWER:} 

21.2 \text{ cm}^2
12-4 Law of Sines

4. \( B = 103^\circ, a = 20 \) in., \( c = 18 \) in.

**SOLUTION:**
Substitute \( c = 18, a = 20 \) and \( B = 103^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} ac \sin B \\
= \frac{1}{2} (20)(18) \sin 103 \\
= 175.4 \text{ in}^2
\]

**ANSWER:**
175.4 in\(^2\)

Solve each triangle. Round side lengths to the nearest tenth and angle measure to the nearest degree.

5. \( \triangle ABC \)

**SOLUTION:**
\[
m\angle E = 180 - (34 + 39) \\
= 180 - 73 \\
= 107
\]

Use the Law of Sines to find side lengths \( d \) and \( f \).

\[
\frac{\sin D}{d} = \frac{\sin E}{e} \\
\sin 39 = \frac{\sin 107}{12} \\
d = \frac{12 \sin 39}{\sin 107} \\
d \approx 7.9
\]

\[
\frac{\sin F}{f} = \frac{\sin E}{e} \\
\sin 34 = \frac{\sin 107}{12} \\
f = \frac{12 \sin 34}{\sin 107} \\
f \approx 7.0
\]

**ANSWER:**
\( E = 107^\circ, d \approx 7.9, f \approx 7.0 \)
7. Solve \( \triangle FGH \) if \( G = 80^\circ, H = 40^\circ, \) and \( g = 14. \)

**SOLUTION:**

\[
m\angle F = 180^\circ - (80^\circ + 40^\circ) = 60^\circ
\]

Use the Law of Sines to find side lengths \( f \) and \( h. \)

\[
\frac{\sin F}{f} = \frac{\sin G}{g}
\]

\[
\frac{\sin 60^\circ}{f} = \frac{\sin 80^\circ}{14}
\]

\[
f = 14 \times \frac{\sin 60^\circ}{\sin 80^\circ} \approx 12.3
\]

\[
\frac{\sin H}{h} = \frac{\sin G}{g}
\]

\[
\frac{\sin 40^\circ}{h} = \frac{\sin 80^\circ}{14}
\]

\[
h = 14 \times \frac{\sin 40^\circ}{\sin 80^\circ} \approx 9.1
\]

**ANSWER:**

\( F = 60^\circ, f \approx 12.3, \) \( h \approx 9.1 \)
**12-4 Law of Sines**

**CCSS PERSEVERANCE** Determine whether each \( \triangle ABC \) has no solution, one solution, or two solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

8. \( A = 95^\circ, a = 19, b = 12 \)

**SOLUTION:**
Because \( \angle A \) is obtuse and \( a > b \), one solution exists.

Use the Law of Sines to find \( m \angle B \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \\
\sin 95^\circ = \frac{\sin B}{12} \\
\sin B = \frac{12 \sin 95^\circ}{19} \\
B \approx 39^\circ
\]

\( m \angle C \approx 180^\circ - (95^\circ + 39^\circ) \) or 46°

Use the Law of Sines to find \( c \).

\[
\sin 95^\circ \approx \sin 46^\circ \\
\frac{19}{c} \approx \frac{19 \sin 46^\circ}{\sin 95^\circ} \\
c \approx 13.7
\]

**ANSWER:**
one; \( B \approx 39^\circ, C \approx 46^\circ, c \approx 13.7 \)

9. \( A = 60^\circ, a = 15, b = 24 \)

**SOLUTION:**
Since \( \angle A \) is acute and \( a < b \), find \( h \) and compare it to \( a \).

\[
h = b \sin A \\
= 24 \sin 60^\circ \\
\approx 20.8
\]

Since \( 15 < 20.8 \) or \( a < h \), there is no solution.

**ANSWER:**
o no solution
10. $A = 34^\circ$, $a = 8$, $b = 13$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[
h = b \sin A \\
= 13 \sin 34 \\
\approx 7.3
\]

Since $7.3 < 8 < 13$ or $h < a < b$, there are two solutions. So, there are two triangles to be solved.

<table>
<thead>
<tr>
<th>Case 1 $\angle B$ is acute</th>
<th>Case 2 $\angle B$ is obtuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin B = \sin A$ [ \frac{b}{a} ]</td>
<td>The sine function also has a positive value in Quadrant II.</td>
</tr>
<tr>
<td>$\sin B = \frac{13 \sin 34}{8}$</td>
<td>So, find an obtuse angle $B$ for which $\sin B = 0.9087$.</td>
</tr>
<tr>
<td>$\sin B = 0.9087$</td>
<td>$m\angle B = 180^\circ - 65^\circ$ or $115^\circ$</td>
</tr>
<tr>
<td>$B = 65^\circ$</td>
<td>$m\angle C = 180^\circ - (34^\circ + 115^\circ)$ or $31^\circ$</td>
</tr>
<tr>
<td>$\sin C = 180^\circ - (34^\circ + 65^\circ)$ or $81^\circ$</td>
<td>$\sin C = \frac{3 \sin 60}{c}$</td>
</tr>
<tr>
<td>$\sin C = \frac{3 \sin 60}{c}$</td>
<td>$c = \frac{3 \sin 60}{\sin 30}$</td>
</tr>
<tr>
<td>$c = 14.1$</td>
<td>$c \approx 5.2$</td>
</tr>
</tbody>
</table>

**ANSWER:**
two; $B \approx 65^\circ$, $C \approx 81^\circ$, $c \approx 14.1$; $B \approx 115^\circ$, $C \approx 31^\circ$, $c \approx 7.4$

11. $A = 30^\circ$, $a = 3$, $b = 6$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[
h = b \sin A \\
= 6 \sin 30 \\
= 3
\]

Since $a = h$, there is one solution and $m\angle B = 90^\circ$.

\[
m\angle C = 180^\circ - (30^\circ + 90^\circ) \text{ or } 60^\circ
\]

Use the Law of Sines to find $c$.

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \\
\frac{\sin 30}{3} = \frac{\sin 60}{c}
\]

\[
c = \frac{3 \sin 60}{\sin 30}
\]

\[
c \approx 5.2
\]

**ANSWER:**
one; $B \approx 90^\circ$, $C \approx 60^\circ$, $c \approx 5.2
12-4 Law of Sines

12. SPACE Refer to the beginning of the lesson. Find the distance between the Wahoo Crater and the Naukan Crater on Mars.

\[ \begin{align*}
\text{SOLUTION:} \\
\text{Use the Law of Sines to find } x. \\
\frac{\sin A}{a} &= \frac{\sin C}{c} \\
\frac{\sin 23}{1.2} &= \frac{\sin 102}{x} \\
x &= \frac{1.2 \sin 102}{\sin 23} \\
x &= 3 \\
\end{align*} \]

The distance between the Wahoo Crater and the Naukan Crater is 3 kilometers.

\text{ANSWER:} \\
3 \text{ kilometers}
12-4 Law of Sines

14. Find the area of \( \triangle ABC \) to the nearest tenth, if necessary.

**SOLUTION:** Substitute \( b = 16, \ c = 20 \) and \( A = 52^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} bc \sin A
\]
\[
= \frac{1}{2} (16)(20) \sin 52^\circ
\]
\[
\approx 126.1 \text{ ft}^2
\]

**ANSWER:**
126.1 ft\(^2\)

15. \( C \)

**SOLUTION:** Substitute \( a = 8, \ c = 10 \) and \( B = 113^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} ac \sin B
\]
\[
= \frac{1}{2} (8)(10) \sin 113^\circ
\]
\[
\approx 36.8 \text{ m}^2
\]

**ANSWER:**
36.8 m\(^2\)

16. \( A \)

**SOLUTION:** Substitute \( a = 14, \ b = 18 \) and \( C = 36^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} ab \sin C
\]
\[
= \frac{1}{2} (14)(18) \sin 36
\]
\[
\approx 74.1 \text{ cm}^2
\]

**ANSWER:**
74.1 cm\(^2\)

17. \( C = 25^\circ, \ a = 4 \) ft, \( b = 7 \) ft

**SOLUTION:** Substitute \( a = 4, \ b = 7 \) and \( C = 25^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} ab \sin C
\]
\[
= \frac{1}{2} (4)(7) \sin 25
\]
\[
\approx 5.9 \text{ ft}^2
\]

**ANSWER:**
5.9 ft\(^2\)
12-4 Law of Sines

18. \( A = 138^\circ, b = 10 \text{ in.}, \ c = 20 \text{ in.} \)

**SOLUTION:**
Substitute \( b = 10, \ c = 20 \) and \( A = 138^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2}bc \sin A \\
= \frac{1}{2}(10)(20)\sin 138 \\
\approx 66.9 \text{ in}^2
\]

**ANSWER:**
66.9 in\(^2\)

19. \( B = 92^\circ, \ a = 14.5 \text{ m}, \ c = 9 \text{ m} \)

**SOLUTION:**
Substitute \( a = 14.5, \ c = 9 \) and \( B = 92^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2}ac \sin B \\
= \frac{1}{2}(14.5)(9)\sin 92 \\
\approx 65.2 \text{ m}^2
\]

**ANSWER:**
65.2 m\(^2\)

20. \( C = 116^\circ, \ a = 2.7 \text{ cm}, \ b = 4.6 \text{ cm} \)

**SOLUTION:**
Substitute \( a = 2.7, \ b = 4.6 \) and \( C = 116^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2}ab \sin C \\
= \frac{1}{2}(2.7)(4.6)\sin 116 \\
\approx 5.6 \text{ cm}^2
\]

**ANSWER:**
5.6 cm\(^2\)
CCSS REASONING  Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

21.

**SOLUTION:**

\[ m \angle C = 180^\circ - \left(106^\circ + 44^\circ \right) \text{ or } 30^\circ \]

Use the Law of Sines to find side lengths \(c\) and \(b\).

\[
\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin 30^\circ}{8} \Rightarrow c = \frac{8 \sin 30^\circ}{\sin 44^\circ} \approx 5.8
\]

\[
\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin 106^\circ}{8} \Rightarrow b = \frac{8 \sin 106^\circ}{\sin 44^\circ} \approx 11.1
\]

**ANSWER:**

\( C = 30^\circ, b \approx 11.1, c \approx 5.8 \)

22.

**SOLUTION:**

\[ m \angle R = 180^\circ - \left(47^\circ + 53^\circ \right) \text{ or } 80^\circ \]

Use the Law of Sines to find side lengths \(r\) and \(t\).

\[
\frac{\sin R}{r} = \frac{\sin S}{s} = \frac{\sin 80^\circ}{13} \Rightarrow \]

\[ r = \frac{13 \sin 80^\circ}{\sin 47^\circ} \approx 17.5 \]

\[
\frac{\sin T}{t} = \frac{\sin S}{s} = \frac{\sin 53^\circ}{13} \Rightarrow \]

\[ t = \frac{13 \sin 53^\circ}{\sin 47^\circ} \approx 14.2 \]

**ANSWER:**

\( R = 80^\circ, r \approx 17.5, t \approx 14.2 \)
### 12-4 Law of Sines

**23.**

**SOLUTION:**

\[ m \angle L = 180^\circ - (70^\circ + 36^\circ) \text{ or } 74^\circ \]

Use the Law of Sines to find side lengths \( n \) and \( m \).

\[
\frac{\sin N}{n} = \frac{\sin L}{l} \]
\[
\frac{\sin 36}{5} = \frac{\sin 74}{n} \]
\[
5 \cdot \sin 36 = n \cdot \sin 74 \]
\[
n = \frac{5 \cdot \sin 36}{\sin 74} \approx 3.1 
\]

\[
\frac{\sin M}{m} = \frac{\sin L}{l} \]
\[
\frac{\sin 70}{5} = \frac{\sin 74}{m} \]
\[
m = \frac{5 \cdot \sin 70}{\sin 74} \approx 4.9 
\]

**ANSWER:**

\( L = 74^\circ, n \approx 3.1, m \approx 4.9 \)

**24.**

**SOLUTION:**

\[ m \angle B = 180^\circ - (112^\circ + 30^\circ) \text{ or } 38^\circ \]

Use the Law of Sines to find side lengths \( a \) and \( c \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \]
\[
\frac{\sin 30}{a} = \frac{\sin 38}{24} \]
\[
a = \frac{24 \cdot \sin 30}{\sin 38} \approx 19.5 
\]

\[
\frac{\sin C}{c} = \frac{\sin B}{b} \]
\[
\frac{\sin 112}{c} = \frac{\sin 38}{24} \]
\[
c = \frac{24 \cdot \sin 112}{\sin 38} \approx 36.1 
\]

**ANSWER:**

\( B = 38^\circ, a \approx 19.5, c \approx 36.1 \)
25. Solve \( \triangle HJK \) if \( H = 53^\circ, J = 20^\circ, \) and \( h = 31 \).

**SOLUTION:**

\[ m \angle K = 180^\circ - \left( 53^\circ + 20^\circ \right) \text{ or } 107^\circ \]

Use the Law of Sines to find side lengths \( j \) and \( k \).

\[
\frac{\sin J}{j} = \frac{\sin H}{h} \\
\frac{\sin 20^\circ}{j} = \frac{\sin 53^\circ}{31} \\
j = \frac{31 \sin 20^\circ}{\sin 53^\circ} \\
j \approx 13.3 \\
\]

\[
\frac{\sin K}{k} = \frac{\sin H}{h} \\
\frac{\sin 107^\circ}{k} = \frac{\sin 53^\circ}{31} \\
k = \frac{31 \sin 107^\circ}{\sin 53^\circ} \\
k \approx 37.1 \\
\]

**ANSWER:**

\( K = 107^\circ, j \approx 13.3, k \approx 37.1 \)

26. Solve \( \triangle NPQ \) if \( P = 109^\circ, Q = 57^\circ, \) and \( n = 22 \).

**SOLUTION:**

\[ m \angle N = 180^\circ - \left( 109^\circ + 57^\circ \right) \]
\[ = 180^\circ - 166^\circ \]
\[ = 14^\circ \]

Use the Law of Sines to find side lengths \( p \) and \( q \).

\[
\frac{\sin P}{p} = \frac{\sin N}{n} \\
\frac{\sin 109^\circ}{p} = \frac{\sin 14^\circ}{22} \\
p = \frac{22 \sin 109^\circ}{\sin 14^\circ} \\
p \approx 86.0 \\
\]

\[
\frac{\sin Q}{q} = \frac{\sin N}{n} \\
\frac{\sin 57^\circ}{q} = \frac{\sin 14^\circ}{22} \\
q = \frac{22 \sin 57^\circ}{\sin 14^\circ} \\
q \approx 76.3 \\
\]

**ANSWER:**

\( N = 14^\circ, p \approx 86.0, q \approx 76.3 \)
12-4 Law of Sines

27. Solve \( \triangle ABC \) if \( A = 50^\circ \), \( a = 2.5 \), and \( C = 67^\circ \).

**SOLUTION:**

\[ m\angle B = 180^\circ - (50^\circ + 67^\circ) \text{ or } 63^\circ \]

Use the Law of Sines to find side lengths \( b \) and \( c \).

\[
\begin{align*}
\frac{\sin B}{b} &= \frac{\sin A}{a} \\
\frac{\sin 63^\circ}{b} &= \frac{\sin 50^\circ}{2.5} \\
b &= \frac{2.5 \sin 63^\circ}{\sin 50^\circ} \\
&\approx 2.9
\end{align*}
\]

\[
\begin{align*}
\frac{\sin C}{c} &= \frac{\sin A}{a} \\
\frac{\sin 67^\circ}{c} &= \frac{\sin 50^\circ}{2.5} \\
c &= \frac{2.5 \sin 67^\circ}{\sin 50^\circ} \\
&\approx 3.0
\end{align*}
\]

**ANSWER:**

\( B = 63^\circ \), \( b \approx 2.9 \), \( c \approx 3.0 \)

28. Solve \( \triangle ABC \) if \( B = 18^\circ \), \( C = 142^\circ \), and \( b = 20 \).

**SOLUTION:**

\[ m\angle A = 180^\circ - (18^\circ + 142^\circ) \text{ or } 20^\circ \]

Use the Law of Sines to find side lengths \( a \) and \( c \).

\[
\begin{align*}
\frac{\sin B}{b} &= \frac{\sin A}{a} \\
\frac{\sin 18^\circ}{20} &= \frac{\sin 20^\circ}{a} \\
a &= \frac{20 \sin 20^\circ}{\sin 18^\circ} \\
&\approx 22.1
\end{align*}
\]

\[
\begin{align*}
\frac{\sin C}{c} &= \frac{\sin A}{a} \\
\frac{\sin 18^\circ}{20} &= \frac{\sin 142^\circ}{c} \\
c &= \frac{20 \sin 142^\circ}{\sin 18^\circ} \\
&\approx 39.8
\end{align*}
\]

**ANSWER:**

\( A = 20^\circ \), \( a \approx 22.1 \), \( c \approx 39.8 \)
Determine whether each triangle \( \triangle ABC \) has no solution, one solution, or two solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

29. \( A = 100^\circ, a = 7, b = 3 \)

**SOLUTION:**
Because \( \angle A \) is obtuse and \( a > b \), one solution exists.

Use the Law of Sines to find \( m \angle B \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 100^\circ}{7} = \frac{\sin B}{3} \Rightarrow \sin B = \frac{3 \sin 100^\circ}{7} \Rightarrow B = \sin^{-1} \left( \frac{3 \sin 100^\circ}{7} \right) \Rightarrow B \approx 25^\circ
\]

\( m \angle C \approx 180^\circ - (100^\circ + 25^\circ) \) or 55

Use the Law of Sines to find \( c \).

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \frac{\sin 100^\circ}{7} = \frac{\sin 55^\circ}{c} \Rightarrow c = \frac{7 \sin 55^\circ}{\sin 100^\circ} \Rightarrow c \approx 5.8
\]

**ANSWER:**
one; \( B \approx 25^\circ, C \approx 55^\circ, c \approx 5.8 \)

30. \( A = 75^\circ, a = 14, b = 11 \)

**SOLUTION:**
Because \( \angle A \) is acute and \( a > b \), one solution exists.

Use the Law of Sines to find \( m \angle B \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{\sin 75^\circ}{14} = \frac{\sin B}{11} \Rightarrow \sin B = \frac{11 \sin 75^\circ}{14} \Rightarrow B = \sin^{-1} \left( \frac{11 \sin 75^\circ}{14} \right) \Rightarrow B \approx 49^\circ
\]

\( m \angle C \approx 180^\circ - \left( 75^\circ + 49^\circ \right) \) or 56

Use the Law of Sines to find \( c \).

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \frac{\sin 75^\circ}{14} = \frac{\sin 56^\circ}{c} \Rightarrow c = \frac{14 \sin 56^\circ}{\sin 75^\circ} \Rightarrow c \approx 12.0
\]

**ANSWER:**
one; \( B \approx 49^\circ, C \approx 56^\circ, c \approx 12.0 \)
31. $A = 38^\circ$, $a = 21$, $b = 18$

**SOLUTION:**
Because $\angle A$ is acute and $a > b$, one solution exists.

Use the Law of Sines to find $m\angle B$.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \quad \Rightarrow \quad \frac{\sin 38^\circ}{21} = \frac{\sin B}{18} \\
\sin B = \frac{18 \sin 38^\circ}{21} \\
B = \sin^{-1}\left(\frac{18 \sin 38^\circ}{21}\right) \\
B \approx 32^\circ
\]

$m\angle C = 180^\circ - (38^\circ + 32^\circ)$ or 110

Use the Law of Sines to find $c$.

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \quad \Rightarrow \quad \frac{\sin 38^\circ}{21} = \frac{\sin 110^\circ}{c} \\
c = \frac{21 \sin 110^\circ}{\sin 38^\circ} \\
c \approx 32.1
\]

**ANSWER:**
one; $B \approx 32^\circ$, $C \approx 110^\circ$, $c \approx 32.1$

32. $A = 52^\circ$, $a = 9$, $b = 20$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[h = b \sin A\]
\[= 20 \sin 52^\circ\]
\[\approx 15.76\]

Since $9 < 15.76$ or $a < h$, there is no solution.

**ANSWER:**
no solution
12-4 Law of Sines

33. $A = 42^\circ$, $a = 5$, $b = 6$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[
h = b \sin A = 6 \sin 42^\circ \\
\approx 4.0
\]

Since $4 < 5 < 6$ or $h < a < b$, there are two solutions. So, there are two triangles to be solved.

**ANSWER:**
two; $B \approx 53^\circ$, $C \approx 85^\circ$, $c \approx 7.4$; $B \approx 127^\circ$, $C \approx 11^\circ$, $c \approx 1.4$

34. $A = 44^\circ$, $a = 14$, $b = 19$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[
h = b \sin A = 19 \sin 44^\circ \\
\approx 13.2
\]

Since $13.2 < 14 < 19$ or $h < a < b$, there are two solutions. So, there are two triangles to be solved.

**ANSWER:**
two; $B \approx 71^\circ$, $C \approx 65^\circ$, $c \approx 18.3$; $B \approx 109^\circ$, $C \approx 27^\circ$, $c \approx 9.1$

35. $A = 131^\circ$, $a = 15$, $b = 32$

**SOLUTION:**
Since $\angle A$ is obtuse and $a < b$, so there is no solution.

**ANSWER:**
no solution
12-4 Law of Sines

36. $A = 30^\circ$, $a = 17$, $b = 34$

**SOLUTION:**
Since $\angle A$ is acute and $a < b$, find $h$ and compare it to $a$.

\[
\begin{align*}
h &= b \sin A \\
&= 34 \sin 30 \\
&= 17
\end{align*}
\]

Since $a = h$, one solution exists.

Use the Law of Sines to find $m\angle B$.

\[
\begin{align*}
\frac{\sin A}{a} &= \frac{\sin B}{b} \\
\frac{\sin 30}{17} &= \frac{\sin B}{34} \\
\sin B &= \frac{34 \sin 30}{17} \\
\sin B &= 1 \\
B &= 90^\circ
\end{align*}
\]

\[
\begin{align*}
m\angle C &= 180^\circ - (30^\circ + 90^\circ) \\
&= 180^\circ - 120 \\
&= 60^\circ
\end{align*}
\]

Use the Law of Sines to find $c$.

\[
\begin{align*}
\frac{\sin A}{a} &= \frac{\sin C}{c} \\
\frac{\sin 30}{17} &= \frac{\sin 60}{c} \\
c &= \frac{17 \sin 60}{\sin 30} \\
c &\approx 29.4
\end{align*}
\]

**ANSWER:**
one; $B = 90^\circ$, $C = 60^\circ$, $c \approx 29.4$

---

GEOGRAPHY In Hawaii, the distance from Hilo to Kailua is 57 miles, and the distance from Hilo to Captain Cook is 55 miles.

![Diagram](image)

37. What is the measure of the angle formed at Hilo?

**SOLUTION:**
Use the Law of Sines to find the measure of the angle formed at Kailua.

\[
\begin{align*}
\frac{\sin 80}{57} &= \frac{\sin K}{55} \\
\sin K &= \frac{55 \sin 80}{57} \\
\sin K &\approx 0.9503 \\
K &\approx 72^\circ
\end{align*}
\]

The measure of the angle formed at Hilo is about $28^\circ$ ($180^\circ - 80^\circ - 72^\circ$).

**ANSWER:**
about $28^\circ$
38. What is the distance between Kailua and Captain Cook?

**SOLUTION:**
Substitute 80, 57 and 28 for \( C, c \) and \( H \) and then solve for \( h \).

\[
\frac{\sin 80}{57} = \frac{\sin 28}{h} \]

\[
h = \frac{57 \sin 28}{\sin 80} \]

\[
h \approx 27.2 \]

The distance between Kailua and Captain Cook is about 27.2 mi.

**ANSWER:**
about 27.2 mi

39. **Tornadoes**

Tornado sirens \( A, B, \) and \( C \) form a triangular region in one area of a city. Sirens \( A \) and \( B \) are 8 miles apart. The angle formed at siren \( A \) is 112°, and the angle formed at siren \( B \) is 40°. How far apart are sirens \( B \) and \( C \)?

**SOLUTION:**
Measure of the angle formed at siren \( C = 180° - (112° + 40°) = 28°. \)

\[
\frac{\sin C}{c} = \frac{\sin A}{a} \]

\[
\frac{\sin 28}{8} = \frac{\sin 112}{a} \]

\[
a = \frac{8 \sin 112}{\sin 28} \]

\[
a \approx 15.8 \]

Sirens \( B \) and \( C \) are about 15.8 miles apart.

**ANSWER:**
about 15.8 mi

40. **Mysteries**

The Bermuda Triangle is a region of the Atlantic Ocean between Bermuda, Miami, Florida, and San Juan, Puerto Rico. It is an area where ships and airplanes have been rumored to mysteriously disappear.

![Map of Bermuda Triangle](image)

**a.** What is the distance between Miami and Bermuda?

**SOLUTION:**
Use the Law of Sines to find the measure of angle at Bermuda..

\[
\frac{\sin 53}{965} = \frac{\sin B}{1038} \]

\[
\sin B = \frac{1038 \sin 53}{965} \]

\[
B = \sin^{-1} \left( \frac{1038 \sin 53}{965} \right) \]

\[
B \approx 59° \]

The measure of angle at San Juan \( \approx 180° - (53° + 59°) \) or 68°

Use the Law of Sines to find the distance between Miami and Bermuda.

\[
\frac{\sin 53}{965} = \frac{\sin 68}{b} \]

\[
b \approx \frac{965 \sin 68}{\sin 53} \]

\[
b \approx 1120.3 \text{ mi} \]
12-4 Law of Sines

Area \approx \frac{1}{2} ab \sin C \\
\approx \frac{1}{2} (1038)(965) \sin 68 \\
\approx 46366.1 \text{ mi}^2

ANSWER:

a. about 1120.3 mi

b. about 464,366.1 mi$^2$

41. BICYCLING One side of a triangular cycling path is 4 miles long. The angle opposite this side is 64°. Another angle formed by the triangular path measures 66°.

a. Sketch a drawing of the situation. Label the missing sides $a$ and $b$.

b. Write equations that could be used to find the lengths of the missing sides.

c. What is the perimeter of the path?

SOLUTION:

a.

b. 
\[
\sin 66^\circ = \frac{\sin 64^\circ}{a} \\
\sin 50^\circ = \frac{\sin 64^\circ}{b}
\]

c. about 11.5 mi

42. ROCK CLIMBING Savannah S and Leon L are standing 8 feet apart in front of a rock climbing wall, as shown at the right. What is the height of the wall? Round to the nearest tenth.
12-4 Law of Sines

**SOLUTION:**
Label the triangles.

Consider the \( \triangle ABL \), \( m \angle LAB \) is 30\(^\circ \), so
\( m \angle LAS \) is 15\(^\circ \).

Use the Law of Sines to find the length \( LA \).

\[
\frac{\sin A}{a} = \frac{\sin S}{s} \quad \sin15 = \frac{\sin45}{8} \quad s = \frac{8\sin45}{\sin15}
\]

In a 30\(^\circ\)-60\(^\circ\)-90\(^\circ\) triangle the sides are in the ratio 1: \( \sqrt{3} \): 2.

In the \( \triangle ABL \) side opposite to the \( m \angle B \) is about
\( \frac{8\sin45}{\sin15} \) ft, so \( h \) is about \( \frac{1}{2} \cdot \frac{8\sin45}{\sin15} \cdot \sqrt{3} \) ft or 18.9 ft.

**ANSWER:**
18.9 ft

---

43. CCSS CRITIQUE In \( \triangle RST \), \( R = 56^\circ \), \( r = 24 \), and \( t = 12 \). Cameron and Gabriela are using the Law of Sines to find \( T \). Is either of them correct? Explain your reasoning.

**Cameron**

\[
\frac{\sin T}{12} = \frac{\sin 56^\circ}{24} \quad \sin T \approx 0.4145 \quad T \approx 24.5^\circ
\]

**Gabriela**

Since \( r > t \), there is no solution.

**SOLUTION:**
Cameron is correct; \( R \) is acute and \( r > t \), so there is one solution.

**ANSWER:**
Cameron; \( R \) is acute and \( r > t \), so there is one solution.

44. OPEN ENDED Create an application problem involving right triangles and the Law of Sines. Then solve your problem, drawing diagrams if necessary.

**SOLUTION:**
See students’ work.

**ANSWER:**
See students’ work.
45. **CHALLENGE** Using the figure, derive the formula
\[ \text{Area} = \frac{1}{2}bc \sin A. \]

**SOLUTION:**
- \( \sin A = \frac{\text{opposite}}{\text{hypotenuse}} \)  
  Definition of sine
- \( \sin A = \frac{h}{c} \)  
  \( h = \text{opposite side}, c = \text{hypotenuse} \)
- \( c \sin A = h \)  
  Multiply both sides by \( c \).
- Area = \( \frac{1}{2} \times \text{base} \times \text{height} \)  
  Area of a triangle
- Area = \( \frac{1}{2}bh \)  
  \( b = \text{base}, h = \text{height} \)
- Area = \( \frac{1}{2}bc \sin A \)  
  Substitution

**ANSWER:**
- \( \sin A = \frac{\text{opposite}}{\text{hypotenuse}} \)  
  Definition of sine
- \( \sin A = \frac{h}{c} \)  
  \( h = \text{opposite side}, c = \text{hypotenuse} \)
- \( c \sin A = h \)  
  Multiply both sides by \( c \).
- Area = \( \frac{1}{2} \times \text{base} \times \text{height} \)  
  Area of a triangle
- Area = \( \frac{1}{2}bh \)  
  \( b = \text{base}, h = \text{height} \)
- Area = \( \frac{1}{2}bc \sin A \)  
  Substitution

46. **REASONING** Find the side lengths of two different triangles \( ABC \) that can be formed if \( A = 55^\circ \) and \( C = 20^\circ \).

**SOLUTION:**
Sample answer:
\[ m \angle B = 180^\circ - (55^\circ + 20^\circ) \]
\[ = 180^\circ - 75^\circ \]
\[ = 105^\circ \]
\[ \frac{\sin 55^\circ}{a} = \frac{\sin 105^\circ}{b} \]
\[ b = \frac{a \sin 105^\circ}{\sin 55^\circ} \]
\[ b \approx 1.18a \]
\[ \frac{\sin 55^\circ}{a} = \frac{\sin 20^\circ}{c} \]
\[ c = \frac{a \sin 20^\circ}{\sin 55^\circ} \]
\[ c \approx 0.42a \]

The side lengths \( a, b \) and \( c \) of the triangle will be in the ratio \( 1 : 1.18 : 0.42 \).

If \( a = 12 \), then \( b \approx 1.18 \times 12 \) or 14.2 and \( c \approx 0.42 \times 12 \) or 5.0.
If \( a = 6 \), then \( b \approx 1.18 \times 6 \) or 7.1 and \( c \approx 0.42 \times 62 \) or 2.5.

**ANSWER:**
Sample answer:
\[ a = 12, b \approx 14.2, c \approx 5.0; a = 6, b \approx 7.1, c \approx 2.5 \]
47. **WRITING IN MATH** Use the Law of Sines to explain why \( a \) and \( b \) do not have unique values in the figure shown.

![Triangle](triangle.png)

**SOLUTION:**
In the triangle, \( B = 115^\circ \). Using the Law of Sines, 
\[
\frac{\sin 50^\circ}{a} = \frac{\sin 115^\circ}{b}.
\]
This equation cannot be solved because there are two unknown sides. To solve a triangle using the Law of Sines, two sides and an angle must be given or two angles and a side opposite one of the angles must be given.

**ANSWER:**
In the triangle, \( B = 115^\circ \). Using the Law of Sines, 
\[
\frac{\sin 50^\circ}{a} = \frac{\sin 115^\circ}{b}.
\]
This equation cannot be solved because there are two unknown sides. To solve a triangle using the Law of Sines, two sides and an angle must be given or two angles and a side opposite one of the angles must be given.

48. **OPEN ENDED** Given that \( E = 62^\circ \) and \( d = 38 \), find a value for \( e \) such that no triangle \( DEF \) can exist. Explain your reasoning.

**SOLUTION:**
Sample answer:
\[
\frac{\sin 62^\circ}{e} = \frac{\sin D}{38}
\]
\[
e = \frac{38 \sin 62^\circ}{\sin D}
\]
\[
e \approx 33.6
\]
Since the value of \( \sin D \) is between 0 to 1, the value \( e \) should be greater than or equal to 33.6.

For \( e = 30 \), the triangle will not exist.
For no triangle to exist, the length of the side opposite angle \( E \) must be less than 33.6 to satisfy the Law of Sines.

**ANSWER:**
Sample answer: \( e = 30 \); for no triangle to exist, the length of the side opposite angle \( E \) must be less than 33.6 to satisfy the Law of Sines.
12-4 Law of Sines

49. SHORT RESPONSE  Given the graphs of \( f(x) \) and \( g(x) \), what is the value of \( f(g(4)) \)?

\[
\begin{align*}
\text{SOLUTION:} \\
\text{At } x = 4, \text{ the graph of } g(x) \text{ intersect the } x\text{-axis.} \\
\text{Therefore, } g(4) = 0 \\
\therefore \quad f(g(4)) = f(0) \\
\end{align*}
\]

At \( x = 0 \), the graph of \( f(x) \) intersect the \( y\)-axis at 2. So, \( f(g(4)) = 2 \).

\text{ANSWER:} 2

50. STATISTICS  If the average of seven consecutive odd integers is \( n \), what is the median of these seven integers?

A 0

B 7

C \( n \)

D \( n - 2 \)

\text{SOLUTION:} \\
Let \( x \) be the first odd integer.

So, the other integers are \( x + 2, x + 4, x + 6, x + 8, x + 10, x + 12 \).

The median of the data is \( x + 6 \).

The equation that represents the situation is

\[
\frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8) + (x + 10) + (x + 12)}{7} = n
\]

\[
\frac{7x + 42}{7} = n
\]

\[
7(x + 6) = 7n
\]

\[
x + 6 = n
\]

C is the correct option.

\text{ANSWER:} C
12-4 Law of Sines

51. One zero of \( f(x) = x^3 - 7x^2 - 6x + 72 \) is 4. What is the factored form of the expression \( x^3 - 7x^2 - 6x + 72 \)?

\[
\begin{align*}
F & \quad (x - 6)(x + 3)(x + 4) \\
G & \quad (x - 6)(x + 3)(x - 4) \\
H & \quad (x + 6)(x + 3)(x - 4) \\
J & \quad (x + 12)(x - 1)(x - 4) \\
\end{align*}
\]

**SOLUTION:**
Divide the polynomial by \( x - 4 \).
\[
x^3 - 7x^2 - 6x + 72 = (x^2 - 3x - 18)(x - 4)
\]
To factor the quadratic, find a pair of numbers with the product \(-18\) and the sum \(-3\).
\[
= (x - 6)(x + 3)(x - 4)
\]
G is the correct option.

**ANSWER:**
G

52. **SAT/ACT** Three people are splitting $48,000 using the ratio 5:4:3. What is the amount of the greatest share?

\[
\begin{align*}
A & \quad $12,000 \\
B & \quad $16,000 \\
C & \quad $20,000 \\
D & \quad $24,000 \\
E & \quad $30,000 \\
\end{align*}
\]

**SOLUTION:**
The total number of shares is 5 + 4 + 3 or 12.
\[
\begin{align*}
\frac{5}{12} & \quad \text{of } $48,000 \text{ is } $20,000. \\
\frac{4}{12} & \quad \text{of } $48,000 \text{ is } $16,000. \\
\frac{3}{12} & \quad \text{of } $48,000 \text{ is } $12,000. \\
\end{align*}
\]
$20,000 is the greatest share.

C is the correct option.

**ANSWER:**
C
12-4 Law of Sines

Find the exact value of each trigonometric function.

53. \( \sin 210^\circ \)

**SOLUTION:**
The terminal side of \( 210^\circ \) lies in Quadrant III.

Find the reference angle.

\[
\theta' = \theta - 180^\circ \\
= 210^\circ - 180^\circ \\
= 30^\circ
\]

The sine function is negative in Quadrant III.

\[
\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}
\]

**ANSWER:**

\[
-\frac{1}{2}
\]

54. \( \cos \frac{3\pi}{4} \)

**SOLUTION:**
The terminal side of \( \frac{3\pi}{4} \) lies in Quadrant II.

Find the reference angle.

\[
\theta' = \pi - \theta \\
= \pi - \frac{3\pi}{4} \\
= \frac{\pi}{4}
\]

The cosine function is negative in Quadrant II.

\[
\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} \\
= -\cos 45^\circ \\
= -\frac{\sqrt{2}}{2}
\]

**ANSWER:**

\[
-\frac{\sqrt{2}}{2}
\]

55. \( \cot 60^\circ \)

**SOLUTION:**
The terminal side of \( 60^\circ \) lies in Quadrant I.

The cotangent function is positive in Quadrant I.

\[
cot 60^\circ = \frac{\sqrt{3}}{3}
\]

**ANSWER:**

\[
\frac{\sqrt{3}}{3}
\]
12-4 Law of Sines

Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

56. $125^\circ$

**SOLUTION:**
Positive angle: $125^\circ + 360^\circ = 485^\circ$
Negative angle: $125^\circ - 360^\circ = -235^\circ$

**ANSWER:**
$485^\circ, -235^\circ$

57. $-32^\circ$

**SOLUTION:**
Positive angle: $-32^\circ + 360^\circ = 328^\circ$
Negative angle: $-32^\circ - 360^\circ = -392^\circ$

**ANSWER:**
$328^\circ, -392^\circ$

58. $\frac{2}{3}\pi$

**SOLUTION:**
Positive angle: $\frac{2}{3}\pi + 2\pi = \frac{8\pi}{3}$
Negative angle: $\frac{2}{3}\pi - 2\pi = -\frac{4\pi}{3}$

**ANSWER:**
$\frac{8}{3}\pi, -\frac{4}{3}\pi$

59. **CLOCKS** Jun’s grandfather clock is broken. When she sets the pendulum in motion by holding it against the side of the clock and letting it go, it swings 24 centimeters to the other side, then 18 centimeters back, then 13.5 centimeters, and so on. What is the total distance that the pendulum swings before it stops?

**SOLUTION:**
Find the common ratio.

$r = \frac{18}{24} = \frac{13.5}{18}$ or $0.75$

Substitute 24 for $a_1$ and 0.75 for $r$ in the sum formula.

$$S_n = \frac{a_1}{1 - r} = \frac{24}{1 - 0.75} = 96$$

The total distance traveled by the pendulum is 96 cm.

**ANSWER:**
96 cm
Find the sum of each infinite series, if it exists.

60. $64 + 48 + 36 + \ldots$

**SOLUTION:**
Find the common ratio.

$$r = \frac{48}{64} = \frac{3}{4}$$

Since $\left| \frac{3}{4} \right| < 1$, the series is convergent.

Use the formula to find the sum.

$$S = \frac{a_1}{1 - r} = \frac{64}{1 - \frac{3}{4}} = 64 \div \frac{1}{4} = 256$$

**ANSWER:**
256

61. $27 + 36 + 48 + \ldots$

**SOLUTION:**
Find the common ratio.

$$r = \frac{36}{27} = \frac{4}{3} = 1\frac{1}{3}$$

Since $\left| 1\frac{1}{3} \right| > 1$, the series is divergent. So, the sum does not exist.

**ANSWER:**
No sum exists.

62. $\sum_{n=1}^{\infty} 0.5(1.1)^n$

**SOLUTION:**
Here $r = 1.1$.

Since $\left| 1.1 \right| > 1$, the series is divergent. So, the sum does not exist.

**ANSWER:**
No sum exists.
63. **ASTRONOMY** At its closest point, Earth is 91.8 million miles from the center of the Sun. At its farthest point, Earth is 94.9 million miles from the center of the Sun. Write an equation for the orbit of Earth, assuming that the center of the orbit is the origin and the Sun lies on the x-axis.

**SOLUTION:**
The value of $a$ is one half the length of the major axis.

\[
a = \frac{1}{2}(91.8 + 94.9) = 93.35 \text{ million miles}
\]

\[
a^2 = 8714.22 \text{ million miles} = 8.714 \times 10^{15}
\]

The value of $c$ is the distance from the center of the ellipse to the focus. This distance is equal to $a$ minus the perihelion.

\[
c = 93.35 - 91.8 = 1.55 \text{ million miles}
\]

\[
c^2 = a^2 - b^2 = 8714.22 - b^2
\]

\[
2.40 = 8714.22 - b^2 = 8711.82 \text{ million miles} = 8.712 \times 10^{15}
\]

The equation of the ellipse is

\[
\frac{x^2}{8.714 \times 10^{15}} + \frac{y^2}{8.712 \times 10^{15}} = 1
\]

**ANSWER:**

\[
\frac{x^2}{8.714 \times 10^{15}} + \frac{y^2}{8.712 \times 10^{15}} = 1
\]

---

64. Simplify.

\[\sqrt{(x - 4)^2}\]

**SOLUTION:**

\[\sqrt{(x - 4)^2} = |x - 4|\]

Since the index 2 is even and the exponent 1 is odd, we must use absolute value.

**ANSWER:**

\[|x - 4|\]

65. Simplify.

\[\sqrt{(y + 2)^4}\]

**SOLUTION:**

\[\sqrt{(y + 2)^4} = \sqrt{(y + 2)^2} = (y + 2)^2\]

**ANSWER:**

\[(y + 2)^2\]

66. Simplify.

\[\sqrt[3]{(a - b)^6}\]

**SOLUTION:**

\[\sqrt[3]{(a - b)^6} = \sqrt[3]{(a - b)^2} = (a - b)^2\]

**ANSWER:**

\[(a - b)^2\]
12-4 Law of Sines

Evaluate each expression if \(w = 6, x = -4, y = 1.5, \text{ and } z = \frac{3}{4}\).

67. \(w^2 + y^2 - 6xz\)

\[
\begin{align*}
SOLUTION: \\
\text{Substitute } w = 6, x = -4, y = 1.5 \text{ and } z = \frac{3}{4} \text{ in the given expression and evaluate.} \\
w^2 + y^2 - 6xz &= 6^2 + 1.5^2 - 6(-4)\left(\frac{3}{4}\right) \\
&= 36 + 2.25 + 18 \\
&= 56.25 \\
\text{ANSWER:} \\
56.25
\end{align*}
\]

68. \(x^2 + z^2 + 5wy\)

\[
\begin{align*}
SOLUTION: \\
\text{Substitute } w = 6, x = -4, y = 1.5 \text{ and } z = \frac{3}{4} \text{ in the given expression and evaluate.} \\
x^2 + z^2 + 5wy &= (-4)^2 + \left(\frac{3}{4}\right)^2 + 5(6)(1.5) \\
&= 16 + \frac{9}{16} + 45 \\
&= \frac{64}{4} + \frac{9}{16} + \frac{720}{16} \\
&= \frac{61\frac{9}{16}}{16} \\
\text{ANSWER:} \\
61\frac{9}{16}
\end{align*}
\]

69. \(wy + xz + w^2 - x^2\)

\[
\begin{align*}
SOLUTION: \\
\text{Substitute } w = 6, x = -4, y = 1.5 \text{ and } z = \frac{3}{4} \text{ in the given expression and evaluate.} \\
w\cdot y + x\cdot z + w^2 - x^2 &= (6)(1.5) + (-4)\left(\frac{3}{4}\right) + 6^2 - (-4)^2 \\
&= 9 - 3 + 36 - 16 \\
&= 26 \\
\text{ANSWER:} \\
26
\end{align*}
\]
12-5 Law of Cosines

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. Solve for side lengths.
   \[ b^2 = a^2 + c^2 - 2ac \cos B \]
   \[ b^2 = 3^2 + 4^2 - 2(3)(4) \cos 92 \]
   \[ b^2 \approx 25.8 \]
   \[ b \approx 5.1 \]

   Use the Law of Sines to find a missing angle measure.
   \[ \frac{\sin A}{3} = \frac{\sin 92}{5.1} \]
   \[ \sin A \approx \frac{3 \sin 92}{5.1} \]
   \[ A \approx 36^\circ \]

   Find the measure of \( \angle C \).
   \[ m\angle C \approx 180 - (36 + 92) \]
   \[ \approx 52^\circ \]

   **ANSWER:**
   \( A \approx 36^\circ, \ C \approx 52^\circ, \ b \approx 5.1 \)

**SOLUTION:**
Use the Law of Cosines to find the measure of the largest angle, \( \angle A \).

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
20^2 &= 14^2 + 10^2 - 2(14)(10) \cos A \\
20^2 - 14^2 - 10^2 &= -2(14)(10) \cos A \\
-280 &= -2(14)(10) \cos A \\
112 &= \cos A \\
A &\approx 36^\circ
\end{align*}
\]

Use the Law of Sines to find the measure of angle, \( \angle B \).

\[
\begin{align*}
\sin B &= \frac{\sin 112}{14} \\
\sin B &\approx \frac{14 \sin 112}{20} \\
B &\approx 40^\circ
\end{align*}
\]

Find the measure of \( \angle C \).

\[
\begin{align*}
m\angle C &\approx 180 - (112 + 40) \\
&\approx 28^\circ
\end{align*}
\]

**ANSWER:**
\( A \approx 112^\circ, \ B \approx 40^\circ, \ C \approx 28^\circ \)
3. $a = 5, b = 8, c = 12$

**SOLUTION:**

Use the Law of Cosines to find the measure of the largest angle, $\angle C$.

\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

\[
12^2 - 5^2 + 8^2 - 2(5)(8)\cos C
\]

\[
\frac{12^2 - 5^2 - 8^2}{-2(5)(8)} = \cos C
\]

\[
133 \approx C
\]

Use the Law of Sines to find the measure of angle, $\angle B$.

\[
\frac{\sin B}{8} = \frac{\sin 133}{12}
\]

\[
\sin B \approx \frac{8 \sin 133}{12}
\]

$B \approx 29$

Find the measure of $\angle A$.

\[
m\angle A = 180 - (133 + 29)
\]

\[
\approx 18
\]

**ANSWER:**

$A \approx 18^\circ, B \approx 29^\circ, C \approx 133^\circ$

4. $B = 110^\circ, a = 6, c = 3$

**SOLUTION:**

Use the Law of Cosines to find the missing side length.

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
b^2 = 6^2 + 3^2 - 2(6)(3)\cos 110
\]

\[
b^3 \approx 57.3
\]

\[
b \approx 7.6
\]

Use the Law of Sines to find a missing angle measure.

\[
\frac{\sin A}{6} = \frac{\sin 110}{7.6}
\]

\[
\sin A \approx \frac{6 \sin 110}{7.6}
\]

\[
A \approx 48
\]

Find the measure of $\angle C$.

\[
m\angle C = 180 - (48 + 110)
\]

\[
\approx 22
\]

**ANSWER:**

$A \approx 48^\circ, C \approx 22^\circ, b \approx 7.6$
12-5 Law of Cosines

CCSS PRECISION  Determine whether each triangle should be solved by beginning with the Law of Sines or the Law of Cosines. Then solve the triangle.

5.

SOLUTION:
Since two sides and an angle opposite one of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

\[
\frac{\sin 107^\circ}{12} = \frac{\sin B}{8}
\]

\[
\sin B = \frac{8 \sin 107^\circ}{12}
\]

\[
\sin B \approx 0.6375
\]

\[
B \approx 40^\circ
\]

Find the measure of \( \angle C \).

\[
m\angle C \approx 180^\circ - (107^\circ + 40^\circ)
\]

\[
\approx 33^\circ
\]

Use Law of Sines to find \( c \).

\[
\frac{\sin 107^\circ}{12} = \frac{\sin 33^\circ}{c}
\]

\[
c \approx \frac{12 \sin 33^\circ}{\sin 107^\circ}
\]

\[
c \approx 6.8
\]

ANSWER:
Sines; \( B \approx 40^\circ, C \approx 33^\circ, c \approx 6.8 \)

6.

SOLUTION:
Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
b^2 = 5^2 + 4^2 - 2(5)(4) \cos 96^\circ
\]

\[
b \approx 6.7
\]

Use the Law of Sines to find a missing angle measure.

\[
\frac{\sin A}{5} = \frac{\sin 96^\circ}{6.7}
\]

\[
\sin A \approx \frac{5 \sin 96^\circ}{6.7}
\]

\[
A \approx 48^\circ
\]

Find the measure of \( \angle C \).

\[
m\angle C \approx 180^\circ - (48^\circ + 96^\circ)
\]

\[
\approx 36^\circ
\]

ANSWER:
Cosines; \( A \approx 48^\circ, C \approx 36^\circ, b \approx 6.7 \)
12-5 Law of Cosines

7. In $\triangle RST$, $R = 35^\circ$, $s = 16$, and $t = 9$.

**SOLUTION:**
Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

\[ r^2 = s^2 + t^2 - 2st \cos R \]
\[ r^2 = 16^2 + 9^2 - 2(16)(9) \cos 35 \]
\[ r \approx 10.1 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin T}{t} = \frac{\sin R}{r} \]
\[ \frac{\sin T}{9} = \frac{\sin 35}{10.1} \]
\[ \sin T \approx \frac{9 \sin 35}{10.1} \]
\[ T \approx 31^\circ \]

Find the measure of $\measuredangle S$.

\[ m\measuredangle S \approx 180^\circ - (35^\circ + 31^\circ) \text{ or } 114^\circ \]

**ANSWER:**

Cosines; $S \approx 114^\circ$, $T \approx 31^\circ$, $r \approx 10.1$

8. **FOOTBALL** In a football game, the quarterback is 20 yards from Receiver A. He turns 40° to see Receiver B, who is 16 yards away. How far apart are the two receivers?

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ c^2 = 16^2 + 20^2 - 2(16)(20) \cos 40 \]
\[ c^2 \approx 165.7 \]
\[ c \approx 12.9 \]

The two receivers are about 12.9 yards apart.

**ANSWER:**

about 12.9 yd
Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

SOLUTION:
Use the Law of Cosines to find the missing side length.

\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ c^2 = 3^2 + 2^2 - 2(3)(2) \cos 70 \]
\[ c^2 \approx 8.8958 \]
\[ c \approx 3.0 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin A}{3} \approx \frac{\sin 70}{3} \]
\[ \sin A \approx \frac{3 \sin 70}{3} \]
\[ A \approx 70 \]

Find the measure of \( \angle B \).

\[ m\angle B \approx 180 - (70 + 70) \]
\[ \approx 40 \]

ANSWER:
\( A \approx 70^\circ, B \approx 40^\circ, c \approx 3.0 \)

10.

SOLUTION:
Use the Law of Cosines to find the missing side length.

\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ b^2 = 14^2 + 12^2 - 2(14)(12) \cos 92 \]
\[ b^2 \approx 351.7262 \]
\[ b \approx 18.75 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin A}{14} \approx \frac{\sin 92}{18.8} \]
\[ \sin A \approx \frac{5 \sin 96}{6.7} \]
\[ A \approx 48 \]

Find the measure of \( \angle C \).

\[ m\angle C \approx 180 - (48 + 92) \]
\[ \approx 40 \]

ANSWER:
\( A \approx 48^\circ, C \approx 40^\circ, b \approx 18.8 \)
12-5 Law of Cosines

SOLUTION:
Use the Law of Cosines to find the measure of the largest angle, $\angle B$.

\[ b^2 = a^2 + c^2 - 2ac \cos B \]
\[ 13^2 = 7^2 + 9^2 - 2(7)(9) \cos B \]
\[ \frac{13^2 - 7^2 - 9^2}{-2(7)(9)} = \cos B \]
\[ -0.3095 = \cos B \]
\[ 108^\circ \approx B \]

Use the Law of Sines to find the measure of angle, $\angle A$.

\[ \frac{\sin 108^\circ}{13} = \frac{\sin A}{7} \]
\[ \sin A \approx \frac{7 \sin 108^\circ}{13} \]
\[ A \approx 31^\circ \]

Find the measure of $\angle C$.

\[ m \angle C \approx 180^\circ - (108^\circ + 31^\circ) \]
\[ \approx 41^\circ \]

ANSWER:
A $\approx 31^\circ$, $B \approx 108^\circ$, $C \approx 41^\circ$

SOLUTION:
Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ 14^2 = 10^2 + 8^2 - 2(10)(8) \cos A \]
\[ \frac{14^2 - 10^2 - 8^2}{-2(10)(8)} = \cos A \]
\[ 102 \approx A \]

Use the Law of Sines to find the measure of angle, $\angle B$.

\[ \frac{\sin 102^\circ}{14} \approx \frac{\sin B}{10} \]
\[ \sin B \approx \frac{10 \sin 102^\circ}{14} \]
\[ B \approx 44^\circ \]

Find the measure of $\angle C$.

\[ m \angle C \approx 180^\circ - (102^\circ + 44^\circ) \]
\[ \approx 34^\circ \]

ANSWER:
A $\approx 102^\circ$, $B \approx 44^\circ$, $C \approx 34^\circ$
12-5 Law of Cosines

13. $A = 116^\circ$, $b = 5$, $c = 3$

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ a^2 = 5^2 + 3^2 - 2(5)(3)\cos 116^\circ \]
\[ a^2 \approx 47.2 \]
\[ a \approx 6.9 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin B}{5} \approx \frac{\sin 116^\circ}{6.9} \]
\[ \sin B \approx \frac{5\sin 116^\circ}{6.9} \]
\[ B \approx 41^\circ \]

Find the measure of $\angle C$.

\[ m\angle C \approx 180^\circ - (116^\circ + 41^\circ) \]
\[ \approx 23^\circ \]

**ANSWER:**
$a \approx 6.9$, $B \approx 41^\circ$, $C \approx 23^\circ$

14. $C = 80^\circ$, $a = 9$, $b = 2$

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ c^2 = 9^2 + 2^2 - 2(9)(2)\cos 80^\circ \]
\[ c^2 \approx 78.7 \]
\[ c \approx 8.9 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin B}{2} \approx \frac{\sin 80^\circ}{8.9} \]
\[ \sin B \approx \frac{2\sin 80^\circ}{8.9} \]
\[ B \approx 13^\circ \]

Find the measure of $\angle A$.

\[ m\angle A \approx 180^\circ - (80^\circ + 13^\circ) \]
\[ \approx 87^\circ \]

**ANSWER:**
$c \approx 8.9$, $A \approx 87^\circ$, $B \approx 13^\circ$
Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. \( \Delta \) with sides \( a = 1350 \), \( b = 2700 \), \( c = 1350 \), \( \angle A = 30^\circ \), \( \angle B = 45^\circ \), \( \angle C \).

**SOLUTION:**
Use the Law of Sines to find the measure of the largest angle, \( \angle C \).

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

\[
\frac{1350}{\sin 30^\circ} = \frac{2700}{\sin 45^\circ} = \frac{1350}{\sin C}
\]

\[
C \approx 28^\circ
\]

Use the Law of Sines to find the measure of angle, \( \angle F \).

\[
\sin F = \frac{10 \sin 28^\circ}{11}
\]

\[
F \approx 65^\circ
\]

Find the measure of \( \angle H \).

\[
m\angle H \approx 180 - (94 + 65) \]

\[
\approx 21^\circ
\]

**ANSWER:**
\( F \approx 65^\circ, G \approx 94^\circ, H \approx 21^\circ \)

16. \( \Delta \) with sides \( a = 20 \), \( b = 13 \), \( c = 12 \).

**SOLUTION:**
Use the Law of Cosines to find the measure of the largest angle, \( \angle W \).

\[
w^2 = x^2 + y^2 - 2xy \cos W
\]

\[
20^2 = 13^2 + 12^2 - 2(13)(12) \cos W
\]

\[
106^\circ \approx W
\]

Use the Law of Sines to find the measure of angle, \( \angle X \).

\[
\sin X = \frac{13 \sin 106^\circ}{20}
\]

\[
X \approx 39^\circ
\]

Find the measure of \( \angle Y \).

\[
m\angle Y \approx 180 - (39 + 106) \]

\[
\approx 35^\circ
\]

**ANSWER:**
\( W \approx 106^\circ, X \approx 39^\circ, Y \approx 35^\circ \)
12-5 Law of Cosines

Determine whether each triangle should be solved by beginning with the Law of Sines or the Law of Cosines. Then solve the triangle.

SOLUTION:
Since two sides and an angle opposite on of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

\[
\frac{\sin 50}{14} = \frac{\sin C}{13} \Rightarrow \sin C = \frac{13 \sin 50}{14} \Rightarrow C \approx 45
\]

Find the measure of \(\angle A\).

\[
m\angle A \approx 180^\circ - \left( 50^\circ + 45^\circ \right) \approx 85
\]

Use Law of Sines to find \(a\).

\[
\frac{\sin 50}{14} \approx \frac{\sin 85}{a} \Rightarrow a \approx 14 \sin 85 \div \sin 50 \approx 18.2
\]

ANSWER:
Sines; \(C \approx 45^\circ, A \approx 85^\circ, a \approx 18.2\)

SOLUTION:
Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

\[
s^2 = r^2 + r^2 - 2rr \cos S
\]

\[
s^2 = 20^2 + 16^2 - 2(20)(16) \cos 106^\circ
\]

\[
s^2 \approx 832.4
\]

\[
s \approx 28.9
\]

Use the Law of Sines to find a missing angle measure.

\[
\frac{\sin R}{20} \approx \frac{\sin 106^\circ}{28.9} \Rightarrow R \approx 42
\]

Find the measure of \(\angle T\).

\[
m\angle T \approx 180^\circ - \left( 106^\circ + 42^\circ \right) \approx 32
\]

ANSWER:
Cosines; \(s \approx 28.9, R \approx 42^\circ, T \approx 32^\circ\)
12-5 Law of Cosines

SOLUTION:
Since three sides of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle, \( \angle A \).

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
22^2 = 11^2 + 15^2 - 2(11)(15) \cos B
\]

\[
\frac{22^2 - 11^2 - 15^2}{-2(11)(15)} = \cos B
\]

\[
115 \approx B
\]

Use the Law of Sines to find the measure of angle, \( \angle A \).

\[
\frac{\sin 115}{22} = \frac{\sin A}{11}
\]

\[
\sin A \approx \frac{11 \sin 115}{22}
\]

\[
\sin A \approx 0.4532
\]

\[
A \approx 27\degree
\]

Find the measure of \( \angle C \).

\[
m\angle C = 180\degree - (27\degree + 115\degree)
\]

\[
\approx 38\degree
\]

ANSWER:
Cosines; \( A \approx 27\degree, B \approx 115\degree, C \approx 38\degree \)

SOLUTION:
Since two angles and any side of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

Find the measure of \( \angle N \).

\[
m\angle N = 180\degree - (47\degree + 80\degree)
\]

\[
= 53\degree
\]

Find the length of \( m \) and \( p \).

\[
\frac{\sin 53\degree}{31} = \frac{\sin 47\degree}{m}
\]

\[
m = \frac{31 \sin 47\degree}{\sin 53\degree}
\]

\[
m \approx 28.4
\]

\[
\frac{\sin 53\degree}{31} = \frac{\sin 80\degree}{p}
\]

\[
p = \frac{31 \sin 80\degree}{\sin 53\degree}
\]

\[
p \approx 38.2
\]

ANSWER:
Sines; \( N \approx 53\degree, p \approx 38.2, m \approx 28.4 \)
21. In \( \triangle ABC \), \( C = 84^\circ \), \( c = 7 \), and \( a = 2 \).

**SOLUTION:**
Since two sides and an angle opposite on of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

Find the measure of angle, \( \angle A \).

\[
\frac{\sin 84}{7} = \frac{\sin A}{2}
\]
\[
\sin A = \frac{2 \sin 84}{7}
\]
\[
A \approx 17^\circ
\]

Find the measure of angle, \( \angle B \).

\[
m\angle B \approx 180^\circ - (17 + 84^\circ)
\]
\[
\approx 79^\circ
\]

Find the length of \( b \).

\[
\frac{\sin 84}{7} = \frac{\sin 79}{b}
\]
\[
b = \frac{7 \sin 79}{\sin 84}
\]
\[
b \approx 6.9
\]

**ANSWER:**
Sines; \( A \approx 17^\circ \), \( B \approx 79^\circ \), \( b \approx 6.9 \)

22. In \( \triangle HJK \), \( h = 18 \), \( j = 10 \), and \( k = 23 \).

**SOLUTION:**
Since three sides of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle, \( \angle K \).

\[
k^2 = h^2 + j^2 - 2hj \cos K
\]
\[
23^2 = 18^2 + 10^2 - 2(18)(10) \cos K
\]
\[
23^2 - 18^2 - 10^2 = \cos K
\]
\[
107 \approx K
\]

Use the Law of Sines to find the measure of angle \( \angle H \).

\[
\frac{\sin 107}{23} \approx \frac{\sin H}{18}
\]
\[
\sin H \approx \frac{18 \sin 107}{23}
\]
\[
H \approx 48^\circ
\]

Find the measure of \( \angle J \).

\[
m\angle J \approx 180^\circ - (107 + 48^\circ)
\]
\[
\approx 25^\circ
\]

**ANSWER:**
Cosines; \( H \approx 48^\circ \), \( J \approx 25^\circ \), \( K \approx 107^\circ \)
23. EXPLORATION  Refer to the beginning of the lesson. Find the distance between the ship and the shipwreck. Round to the nearest tenth.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

\[ a^2 = 520^2 + 338^2 - 2(520)(338)\cos 70^\circ \]

\[ a^2 \approx 264417.0792 \]

\[ a \approx 514.215 \]

The distance between the ship and the shipwreck is 514.2 meters.

ANSWER:
514.2 m

24. GEOMETRY  A parallelogram has side lengths 8 centimeters and 12 centimeters. One angle between them measures 42°. To the nearest tenth, what is the length of the shorter diagonal?

SOLUTION:
Use the Law of Cosines to find the shorter diagonal.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ a^2 = 8^2 + 12^2 - 2(8)(12)\cos 42^\circ \]
\[ a^2 \approx 65.3162 \]
\[ a \approx 8.1 \]

The length of the shorter diagonal is 8.1 cm.

ANSWER:
8.1 cm
25. **RACING** A triangular cross-country course has side lengths 1.8 kilometers, 2 kilometers, and 1.2 kilometers. What are the angles formed between each pair of sides?

**SOLUTION:**
Let \( a = 2, b = 1.8 \) and \( c = 1.2 \).

Use the Law of Cosines to find the measure of the largest angle.

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
2^2 &= 1.8^2 + 1.2^2 - 2(1.8)(1.2) \cos A \\
2^2 - 1.8^2 - 1.2^2 &= -2(1.8)(1.2) \cos A \\
81 &= A \end{align*}
\]

Use the Law of Sines to find the measure of \( \angle B \).

\[
\frac{\sin 81}{2} \approx \frac{\sin B}{1.8}
\]

\[
\sin B \approx \frac{1.8 \sin 81}{2}
\]

\[
B \approx 63°
\]

Find the measure of \( \angle C \).

\[
m\angle C \approx 180° - (81° + 63°)
\]

\[
\approx 36°
\]

**ANSWER:**

\( 81°, 36°, 63° \)

26. **CCSS MODELING** A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles.

a. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.

b. What is the area of the plot of land?

**SOLUTION:**

a. Let \( a = 1.25, b = 0.9 \) and \( c = 0.5 \).
12-5 Law of Cosines

27. LAND Some land is in the shape of a triangle. The distances between each vertex of the triangle are 140 yd, 210 yd and 300 yd, respectively. Use the Law of Cosines to find the area of the land to the nearest square yard.

**SOLUTION:**
Use the Law of Cosines to find the measure of the largest angle.

\[
\begin{align*}
\alpha^2 &= b^2 + c^2 - 2bc \cos \alpha \\
300^2 &= 210^2 + 140^2 - 2(210)(140)\cos \alpha \\
300^2 - 210^2 - 140^2 &= -2(210)(140)\cos \alpha \\
-2(140)(140) &= -2(210)(140)\cos \alpha \\
-0.4473 &= \cos \alpha \\
\alpha &\approx 117°
\end{align*}
\]

Substitute \(b = 210\), \(c = 140\) and \(\alpha = 117°\) in the area formula.

\[
\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)
\]

\[
\approx \sqrt{325(325-140)(325-210)(325-300)}
\]

\[
\approx 13,148 \text{ yd}^2
\]

The area of the land is 13,148 yd².

**ANSWER:**
13,148 yd²

28. RIDES Two bumper cars at an amusement park ride collide as shown.

![Diagram of bumper cars](image)

a. How far apart \(d\) were the two cars before they collided?

b. Before the collision, a third car was 10 feet from car 1 and 13 feet from car 2. Describe the angles formed by cars 1, 2, and 3 before the collision.

**SOLUTION:**
a. Let \(a = 5.5\), \(b = 7\), \(c = d\) and \(C = 118°\) use the Law of Cosines to find the missing side length.

\[
d^2 = a^2 + b^2 - 2ab \cos C
\]

\[
d^2 = 5.5^2 + 7^2 - 2(5.5)(7)\cos 118°
\]

\[
d^2 \approx 115.399
\]

\[
d \approx 10.7
\]

The distance between the two cars is about 10.7 ft.

b. Let \(a = 13\), \(b = 10\) and \(c = 10.7\) use the Law of Cosines to find the measure of the largest angle.

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
13^2 = 10^2 + 10.7^2 - 2(10)(10.7)\cos A
\]

\[
\cos A = \frac{13^2 - 10^2 - 10.7^2}{-2(10)(10.7)}
\]

\[
A \approx 78°
\]

Use the Law of Sines to find the measure of \(\angle B\).

\[
\frac{\sin 78°}{13} = \frac{\sin B}{10}
\]

\[
\sin B \approx \frac{10 \sin 78°}{13}
\]

\[
B \approx 49°
\]

Find the measure of \(\angle C\).

\[
m\angle C \approx 180° -(78° + 49°)
\]

\[
\approx 53°
\]

**ANSWER:**
a. about 10.7 ft

b. 78°, 49°, 53°

29. PICNICS A triangular picnic area is 11 yards by 14 yards by 10 yards.

a. Sketch and label a drawing to represent the picnic
12-5 Law of Cosines

area.

b. Describe how you could find the area of the picnic area.

c. What is the area? Round to the nearest tenth.

**SOLUTION:**

a.

b. Sample answer: Use the Law of Cosines to find the measure of $\angle A$. Then use the formula $\text{Area} = \frac{1}{2}bc\sin A$.

c. $54.6 \text{ yd}^2$

- Substitute $b = 10$, $c = 11$ and $A = 83^\circ$ in the area formula.

\[
\text{Area} = \frac{1}{2}bc\sin A
\]

\[
= \frac{1}{2} (10)(11) \sin 83
\]

\[
\approx 54.6 \text{ yd}^2
\]

**ANSWER:**

a.
12-5 Law of Cosines

30. **WATERSPORTS** A person on a personal watercraft makes a trip from point A to point B to point C traveling 28 miles per hour. She then returns from point C back to her starting point traveling 35 miles per hour. How many minutes did the entire trip take? Round to the nearest tenth.

![Triangle Diagram]

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[ b^2 = 0.15^2 + 0.25^2 - 2(0.15)(0.25)\cos 30 \]
\[ b^2 \approx 0.1332 \]
\[ b \approx 0.36 \text{ mi} \]

The time taken for the trip from point A to point B is
\[ \frac{0.25}{28} \approx 0.0089 \text{ per hour.} \]

The time taken for the trip from point B to point C is
\[ \frac{0.15}{28} \approx 0.0054 \text{ per hour.} \]

The time taken for the trip from point C to point A is
\[ \frac{0.36}{35} \approx 0.0103 \text{ per hour.} \]

Time taken for the entire trip is 0.0246 hrs or 1.5 minutes.

**ANSWER:**
1.5 min

---

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

![Triangular Diagram]

**SOLUTION:**
Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin 104^\circ}{12.4} = \frac{\sin B}{8.1} \]
\[ \sin B = \frac{8.1 \sin 104^\circ}{12.4} \]
\[ B \approx 39^\circ \]

Find the measure of \( \angle C \).

\[ m\angle C \approx 180^\circ - (104 + 39) \]
\[ \approx 37^\circ \]

Use the Law of Sines to find \( c \).

\[ \frac{\sin 104^\circ}{12.4} \approx \frac{\sin 37^\circ}{c} \]
\[ c \approx \frac{12.4 \sin 37^\circ}{\sin 104^\circ} \]
\[ c \approx 7.7 \text{ mi} \]

**ANSWER:**
\( B \approx 39^\circ, C \approx 37^\circ, c \approx 7.7 \text{ mi} \)
12-5 Law of Cosines

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[ q^2 = r^2 + s^2 - 2rs \cos Q \]

\[ q^2 = 36.2^2 + 28^2 - 2(36.2)(28) \cos 25 \]

\[ q \approx 16.0 \]

Use the Law of Sines to find the measure of angle \( \angle R \).  

\[ \frac{\sin R}{36.2} = \frac{\sin 25}{16} \]

\[ \sin R \approx \frac{36.2 \sin 25}{16} \]

\[ R \approx 73^\circ \]

Since \( R \) is obtuse \( R = 180 - 73 = 107^\circ \)

Find the measure of \( \angle S \).

\[ m\angle S \approx 180 - (107 + 25) \text{ or } 48 \]

**ANSWER:**  
\( R \approx 107^\circ, S \approx 48^\circ, q \approx 16.0 \)

**SOLUTION:**
Use the Law of Cosines to find the measure of the largest angle, \( \angle A \).

\[ g^2 = h^2 + f^2 - 2hf \cos G \]

\[ 21.6^2 = 20.8^2 + 15.2^2 - 2(20.8)(15.2) \cos G \]

\[ 21.6^2 - 20.8^2 - 15.2^2 = \cos G \]

\[ \frac{72}{10} = G \]

Use the Law of Sines to find the measure of angle \( \angle H \).

\[ \frac{\sin H}{20.8} = \frac{\sin 72}{21.6} \]

\[ \sin H \approx \frac{20.8 \sin 72}{21.6} \]

\[ H \approx 66^\circ \]

Find the measure of \( \angle F \).

\[ m\angle F \approx 180 - (72 + 66 ) \]

\[ \approx 42^\circ \]

**ANSWER:**  
\( F \approx 42^\circ, G \approx 72^\circ, H \approx 66^\circ \)

34. **CHALLENGE** Use the figure and the Pythagorean Theorem to derive the Law of Cosines. Use the hints below.
12-5 Law of Cosines

• First, use the Pythagorean Theorem for $\triangle DBC$.

• In $\triangle ADB$, $c^2 = x^2 + h^2$.

$\cos A = \frac{x}{c}$

**SOLUTION:**

$a^2 = (b - x)^2 + h^2$ Use the Pythagorean Theorem for $\triangle DBC$.

$= b^2 - 2bx + x^2 + h^2$ Expand $(b - x)^2$.

$= b^2 - 2bx + c^2$ In $\triangle ADB$, $c^2 = x^2 + h^2$.

$= b^2 - 2b(c \cos A) + c^2 \cos A = \frac{x}{c}$, so $x = c \cos A$.

$= b^2 + c^2 - 2bc \cos A$ Commutative Property

**ANSWER:**

$a^2 = (b - x)^2 + h^2$ Use the Pythagorean Theorem for $\triangle DBC$.

$= b^2 - 2bx + x^2 + h^2$ Expand $(b - x)^2$.

$= b^2 - 2bx + c^2$ In $\triangle ADB$, $c^2 = x^2 + h^2$.

$= b^2 - 2b(c \cos A) + c^2 \cos A = \frac{x}{c}$, so $x = c \cos A$.

$= b^2 + c^2 - 2bc \cos A$ Commutative Property

35. **CCSS ARGUMENTS** Three sides of a triangle measure 10.6 centimeters, 8 centimeters, and 14.5 centimeters. Explain how to find the measure of the largest angle. Then find the measure of the angle to the nearest degree.

**SOLUTION:**

The longest side is 14.5 centimeters. Use the Law of Cosines to find the measure of the angle opposite the longest side.

**ANSWER:**

The longest side is 14.5 centimeters. Use the Law of Cosines to find the measure of the angle opposite the longest side; 102°.

36. **OPEN ENDED** Create an application problem involving right triangles and the Law of Cosines. Then solve your problem, drawing diagrams if necessary.

**SOLUTION:**

See students’ work.

**ANSWER:**

See students’ work.
12-5 Law of Cosines

37. **WRITING IN MATH** How do you know which method to use when solving a triangle?

**SOLUTION:**
Sample answer: To solve a right triangle, you can use the Pythagorean Theorem to find side lengths and trigonometric ratios to find angle measures and side lengths. To solve a nonright triangle, you can use the Law of Sines or the Law of Cosines, depending on what information is given. When two angles and a side are given or when two sides and an angle opposite one of the sides are given, you can use the Law of Sines. When two sides and an included angle or three sides are given, you can use the Law of Cosines.

**ANSWER:**
Sample answer: To solve a right triangle, you can use the Pythagorean Theorem to find side lengths and trigonometric ratios to find angle measures and side lengths. To solve a nonright triangle, you can use the Law of Sines or the Law of Cosines, depending on what information is given. When two angles and a side are given or when two sides and an angle opposite one of the sides are given, you can use the Law of Sines. When two sides and an included angle or three sides are given, you can use the Law of Cosines.

38. **SAT/ACT** If \( c \) and \( d \) are different positive integers and \( 4c + d = 26 \), what is the sum of all possible values of \( c \)?

\[ A \ 6 \]
\[ B \ 10 \]
\[ C \ 15 \]
\[ D \ 21 \]
\[ E \ 28 \]

**SOLUTION:**
\[
4c + d = 26 \\
d = 26 - 4c
\]

Since the values of \( c \) and \( d \) are different positive integers, the values of \( c \) are 1, 2, 3, 4, 5 and 6. Therefore, the sum of all possible values of \( c \) is \( 1 + 2 + 3 + 4 + 5 + 6 \) or 21.

D is the correct option.

**ANSWER:**
D
12-5 Law of Cosines

39. If \( 6^y = 21 \), what is \( y \)?

\[
\begin{align*}
F & \quad \log 12 - \log 6 \\
G & \quad \frac{\log 21}{\log 6} \\
H & \quad \frac{\log 6}{\log 21} \\
J & \quad \log \left( \frac{6}{21} \right)
\end{align*}
\]

**SOLUTION:**

\[
\begin{align*}
6^y &= 21 \\
\log 6^y &= \log 21 \\
y \cdot \log 6 &= \log 21 \\
y &= \frac{\log 21}{\log 6}
\end{align*}
\]

\( y \) is the correct option.

**ANSWER:**

\( G \)

40. **GEOMETRY** Find the perimeter of the figure.

\[
\begin{align*}
\text{SOLUTION:} \\
\text{Two sides of the triangle are equal, so the angle opposite to the side length 12 units is 60°.} \\
\text{Measure of the third angles is also 60°, so it is an equilateral triangle.} \\
\text{The side lengths of an equilateral triangle are equal.} \\
\text{Thus, the perimeter of the figure is } 12 + 12 + 12 \text{ or } 36 \text{ units.}
\end{align*}
\]

**ANSWER:**

\( C \)
12-5 Law of Cosines

41. SHORT RESPONSE  Solve the equation for \( x \).

\[
\frac{1}{x-1} + \frac{5}{8} = \frac{23}{6x}
\]

**SOLUTION:**

\[
\begin{align*}
\frac{1}{x-1} + \frac{5}{8} &= \frac{23}{6x} \\
\frac{8 + 5(x-1)}{8(x-1)} &= \frac{23}{6x} \\
5x + 3 &= 23 \\
8x - 8 &= 6x \\
(5x + 3)6x &= 23(8x - 8) \\
30x^2 + 18x &= 184x - 184 \\
30x^2 - 166x + 184 &= 0
\end{align*}
\]

\[x = 4 \text{ or } \frac{23}{15}\]

**ANSWER:**

4, \( \frac{23}{15} \)

Find the area of \( \triangle ABC \) to the nearest tenth.

![Diagram of \( \triangle ABC \)]

**SOLUTION:**

Substitute \( c = 11, \ b = 12 \) and \( A = 81^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2} bc \sin A
\]

\[
= \frac{1}{2} (12)(11) \sin 81
\]

\[\approx 65.2 \text{ cm}^2\]

**ANSWER:**

65.2 cm²
12-5 Law of Cosines

SOLUTION:
Substitute \( c = 5 \), \( a = 6 \) and \( A = 30^\circ \) in the area formula.

\[
\text{Area} = \frac{1}{2}ac \sin B \\
= \frac{1}{2}(6)(5)\sin 30^\circ \\
\approx 7.5 \text{ yd}^2
\]

ANSWER:
7.5 \text{ yd}^2

The terminal side of \( \theta \) in standard position contains each point. Find the exact values of the six trigonometric functions of \( \theta \).

45. \((8, 5)\)

SOLUTION:
Find the value of \( r \).

\[
r = \sqrt{x^2 + y^2} \\
= \sqrt{8^2 + 5^2} \\
= \sqrt{89}
\]

Use \( x = 8 \), \( y = 5 \), and \( r = \sqrt{89} \) to write the six trigonometric ratios.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} = \frac{5}{\sqrt{89}} \\
\cos \theta &= \frac{x}{r} = \frac{8}{\sqrt{89}} \\
\tan \theta &= \frac{y}{x} = \frac{5}{8} \\
csc \theta &= \frac{r}{y} = \frac{\sqrt{89}}{5} \\
sec \theta &= \frac{r}{x} = \frac{\sqrt{89}}{8} \\
cot \theta &= \frac{x}{y} = \frac{8}{5}
\end{align*}
\]

ANSWER:
\[
\begin{align*}
\sin \theta &= \frac{5\sqrt{89}}{89}, \cos \theta = \frac{8\sqrt{89}}{89}, \\
\tan \theta &= \frac{5}{8}, \csc \theta = \frac{\sqrt{89}}{5}, \\
sec \theta &= \frac{\sqrt{89}}{8}, \cot \theta = \frac{8}{5}
\end{align*}
\]
12-5 Law of Cosines

46. \((-4, -2)\)

**SOLUTION:**

Find the value of \(r\).

\[
 r = \sqrt{x^2 + y^2}
 = \sqrt{(-4)^2 + (-2)^2}
 = 2\sqrt{5}
\]

Use \(x = -4, y = -2,\) and \(r = 2\sqrt{5}\) to write the six trigonometric ratios.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} = \frac{-2}{2\sqrt{5}} \text{ or } -\frac{\sqrt{5}}{5} \\
\cos \theta &= \frac{x}{r} = \frac{-4}{2\sqrt{5}} \text{ or } -\frac{2\sqrt{5}}{5} \\
\tan \theta &= \frac{y}{x} = \frac{-2}{-4} \text{ or } 0.5 \\
\csc \theta &= \frac{r}{y} = \frac{2\sqrt{5}}{-2} \text{ or } -\sqrt{5} \\
\sec \theta &= \frac{r}{x} = \frac{2\sqrt{5}}{-4} \text{ or } -\frac{\sqrt{5}}{2} \\
\cot \theta &= \frac{x}{y} = \frac{-4}{-2} \text{ or } 2
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= -\frac{\sqrt{5}}{5}, \cos \theta = -\frac{2\sqrt{5}}{5}, \\
\tan \theta &= 0.5, \csc \theta = -\sqrt{5}, \\
\sec \theta &= -\frac{\sqrt{5}}{2}, \cot \theta = 2
\end{align*}
\]

47. \((6, -9)\)

**SOLUTION:**

Find the value of \(r\).

\[
 r = \sqrt{x^2 + y^2}
 = \sqrt{6^2 + (-9)^2}
 = 3\sqrt{13}
\]

Use \(x = 6, y = -9,\) and \(r = 3\sqrt{13}\) to write the six trigonometric ratios.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} = \frac{-9}{3\sqrt{13}} \text{ or } -\frac{3\sqrt{13}}{13} \\
\cos \theta &= \frac{x}{r} = \frac{6}{3\sqrt{13}} \text{ or } \frac{2\sqrt{13}}{13} \\
\tan \theta &= \frac{y}{x} = \frac{-9}{6} \text{ or } -1.5 \\
\csc \theta &= \frac{r}{y} = \frac{3\sqrt{13}}{3} \\
\sec \theta &= \frac{r}{x} = \frac{3\sqrt{13}}{2} \\
\cot \theta &= \frac{x}{y} = \frac{6}{9} \text{ or } \frac{2}{3}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\sin \theta &= -\frac{3\sqrt{13}}{13}, \cos \theta = \frac{2\sqrt{13}}{13}, \\
\tan \theta &= -1.5, \csc \theta = -\frac{\sqrt{13}}{3}, \\
\sec \theta &= \frac{\sqrt{13}}{2}, \cot \theta = -\frac{2}{3}
\end{align*}
\]

48. **ATHLETIC SHOES** The prices for a random sample of athletic shoes are shown.

<table>
<thead>
<tr>
<th>Price (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>150</td>
</tr>
</tbody>
</table>
12-5 Law of Cosines

a. Use a graphing calculator to create a box-and-whisker plot. Then describe the shape of the distribution.

b. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice.

**SOLUTION:**

a. ![Box-and-Whisker Plot]

The distribution is positively skewed.

b. Sample answer: The distribution is positively skewed, so use the five-number summary. The range is $70 to $400. The median is $190, and half of the data are between $120 and $250.

**ANSWER:**

a. ![Box-and-Whisker Plot]

The distribution is positively skewed.

b. Sample answer: The distribution is positively skewed, so use the five-number summary. The range is $70 to $400. The median is $190, and half of the data are between $120 and $250.

49. BUSINESS During the month of June, MediaWorld had revenue of $2700 from sales of a certain DVD box set. During the July Blowout Sale, the set was on sale for $10 off. Revenue from the set was $3750 in July with 30 more sets sold than were sold in June. Find the price of the DVD set for June and the price for July.

**SOLUTION:**

Let $p$ be the price in June and $q$ the quantity sold in June. Represent the problem with a system on nonlinear equations.

\[
\begin{align*}
\text{June} & : \quad pq = 2700 \\
\text{July} & : \quad (p - 10)(q + 30) = 3750
\end{align*}
\]

Solve the system.

\[
\begin{align*}
q & = \frac{2700}{p} \\
(p - 10)(q + 30) & = 3750 \\
30p + pq - 10q & = 4050 \\
30p + 2700 - \frac{27000}{p} & = 4050 \\
30p - \frac{27000}{p} & = 1350 \\
30p^2 - 27000 & = 1350p \\
p^2 - 45p - 900 & = 0 \\
(p - 60)(p + 15) & = 0 \\
p & = 60 \text{ or } 15
\end{align*}
\]

The price in June was $60. Therefore, the price in July was $50.

**ANSWER:**

$60, $50
12-5 Law of Cosines

Without writing the equation in standard form, state whether the graph of each equation is a parabola, circle, ellipse, or hyperbola.

50. \(x^2 + y^2 - 8x - 6y + 5 = 0\)

**SOLUTION:**
\(A = 1, B = 0,\) and \(C = 1\)

The discriminant is \((0)^2 - 4(1)(1)\) or \(-4\).

Because the discriminant is less than 0 and \(B = 0\) and \(A = C\), the conic is a circle.

**ANSWER:**
circle

51. \(3x^2 - 2y^2 + 32y - 134 = 0\)

**SOLUTION:**
\(A = 3, B = 0,\) and \(C = -2\)

The discriminant is \((0)^2 - 4(3)(-2)\) or 24.

Because the discriminant is greater than 0, the conic is a hyperbola.

**ANSWER:**
hyperbola

52. \(y^2 + 18y - 2x = -84\)

**SOLUTION:**
\(A = 0, B = 0,\) and \(C = 1\)

The discriminant is \((0)^2 - 4(0)(1)\) or 0.

Because the discriminant is equal to 0, the conic is a parabola.

**ANSWER:**
parabola
12-5 Law of Cosines

Sketch each angle. Then find its reference angle.

53. 245°

**SOLUTION:**

\[
\theta' = \theta - 180° \\
= 245° - 180° \\
= 65°
\]

The terminal side of 245° lies in Quadrant III.

**ANSWER:**

54. –15°

**SOLUTION:**

coterminal angle: \(-15° + 360° = 345°\)

The coterminall side of 345° lies in Quadrant IV.

\[
\theta' = 360° - \theta \\
= 360° - 345° \\
= 15°
\]

**ANSWER:**
55. $\frac{5\pi}{4}$

**SOLUTION:**

The terminal side $\frac{5\pi}{4}$ lies in Quadrant III.

\[
\theta' = \theta - \pi \\
= \frac{5\pi}{4} - \pi \\
= \frac{\pi}{4}
\]

**ANSWER:**
12-6 Circular and Periodic Functions

**CCSS STRUCTURE** The terminal side of angle \( \theta \) in standard position intersects the unit circle at each point \( P \). Find \( \cos \theta \) and \( \sin \theta \).

1. \( P \left( \frac{15}{17}, \frac{8}{17} \right) \)

**SOLUTION:**

\[
P \left( \frac{15}{17}, \frac{8}{17} \right) = P(\cos \theta, \sin \theta)
\]

\[
\cos \theta = \frac{15}{17}, \quad \sin \theta = \frac{8}{17}
\]

**ANSWER:**

\[
\cos \theta = \frac{15}{17}, \quad \sin \theta = \frac{8}{17}
\]

2. \( P \left( \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right) \)

**SOLUTION:**

\[
P \left( \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right) = P(\cos \theta, \sin \theta)
\]

\[
\cos \theta = -\frac{\sqrt{2}}{2}, \quad \sin \theta = \frac{\sqrt{2}}{2}
\]

**ANSWER:**

\[
\cos \theta = -\frac{\sqrt{2}}{2}, \quad \sin \theta = \frac{\sqrt{2}}{2}
\]

**Determine the period of each function.**

3. 

**SOLUTION:**

The pattern repeats after 2, 4, 6, 8, \ldots units. So, the period of the function is 2.

**ANSWER:**

2

4. 

**SOLUTION:**

The pattern repeats after 4\( \pi \), 8\( \pi \), \ldots units. So, the period of the function is 4\( \pi \).

**ANSWER:**

4\( \pi \)

5. **SWINGS** The height of a swing varies periodically as the function of time. The swing goes forward and reaches its high point of 6 feet. It then goes backward and reaches 6 feet again. Its lowest point is 2 feet. The time it takes to swing from its high point to its low point is 1 second.

   a. How long does it take for the swing to go forward and back one time?

   b. Graph the height of the swing \( h \) as a function of time \( t \).
12-6 Circular and Periodic Functions

SOLUTION:
a. It takes 4 seconds for the swing to go forward and back one time.

b. Sample answer: Let the x-axis represents the time and the y-axis represents the height in feet.
The maximum point is 6 ft, and the minimum point is 2 ft.
It takes 1 second to reach from the maximum to the minimum. So, period is 2 seconds.

Find the exact value of each function.

6. $\sin \frac{13\pi}{6}$

SOLUTION:

$\sin \frac{13\pi}{6} = \sin \left( \frac{\pi + 12\pi}{6} \right)$

$= \sin \frac{\pi}{6}$

$= \frac{1}{2}$

ANSWER:

$\frac{1}{2}$

7. $\sin (-60^\circ)$

SOLUTION:

$\sin (-60^\circ) = -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

ANSWER:

$-\frac{\sqrt{3}}{2}$

8. $\cos 540^\circ$

SOLUTION:

$\cos 540^\circ = \cos (360^\circ + 180^\circ)$

$= \cos 180^\circ$

$= -1$

ANSWER:

$-1$
The terminal side of angle \( \theta \) in standard position intersects the unit circle at each point \( P \). Find \( \cos \theta \) and \( \sin \theta \).

9. \( P \left( \frac{6}{10}, -\frac{8}{10} \right) \)

**SOLUTION:**

\[ P \left( \frac{6}{10}, -\frac{8}{10} \right) = P(\cos \theta, \sin \theta) \]

\[ \cos \theta = \frac{6}{10} \text{ or } \frac{3}{5} \]

\[ \sin \theta = -\frac{8}{10} \text{ or } -\frac{4}{5} \]

**ANSWER:**

\[ \cos \theta = \frac{3}{5}, \sin \theta = -\frac{4}{5} \]

10. \( P \left( -\frac{10}{26}, \frac{24}{26} \right) \)

**SOLUTION:**

\[ P \left( -\frac{10}{26}, \frac{24}{26} \right) = P(\cos \theta, \sin \theta) \]

\[ \cos \theta = -\frac{10}{26} \text{ or } -\frac{5}{13} \]

\[ \sin \theta = \frac{24}{26} \text{ or } \frac{12}{13} \]

**ANSWER:**

\[ \cos \theta = -\frac{5}{13}, \sin \theta = \frac{12}{13} \]

11. \( P \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)

**SOLUTION:**

\[ P \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) = P(\cos \theta, \sin \theta) \]

\[ \cos \theta = \frac{\sqrt{3}}{2} \]

\[ \sin \theta = \frac{1}{2} \]

**ANSWER:**

\[ \cos \theta = \frac{\sqrt{3}}{2}, \sin \theta = \frac{1}{2} \]

12. \( P \left( \frac{\sqrt{6}}{5}, \frac{\sqrt{19}}{5} \right) \)

**SOLUTION:**

\[ P \left( \frac{\sqrt{6}}{5}, \frac{\sqrt{19}}{5} \right) = P(\cos \theta, \sin \theta) \]

\[ \cos \theta = \frac{\sqrt{6}}{5} \]

\[ \sin \theta = \frac{\sqrt{19}}{5} \]

**ANSWER:**

\[ \cos \theta = \frac{\sqrt{6}}{5}, \sin \theta = \frac{\sqrt{19}}{5} \]
Determine the period of each function.

13. 

**SOLUTION:**
The pattern repeats every 3 units. So, the period of the given function is 3.

**ANSWER:**
3

14. 

**SOLUTION:**
The pattern repeats every 8 units. So, the period of the given function is 8.

**ANSWER:**
8

15. 

**SOLUTION:**
The pattern repeats every 12 units. So, the period of the given function is 12.

**ANSWER:**
12

16. 

**SOLUTION:**
The pattern repeats every 6 units. So, the period of the given function is 6.

**ANSWER:**
6
12-6 Circular and Periodic Functions

SOLUTION:
The pattern repeats every 180°. So, the period of the given function is 180°.

ANSWER:
180°

SOLUTION:
The pattern repeats every 2π units. So the period of the given function is 2π.

ANSWER:
2π

19. WEATHER In a city, the average high temperature for each month is shown in the table.

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature (°F)</th>
<th>Month</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>36</td>
<td>July</td>
<td>85</td>
</tr>
<tr>
<td>Feb</td>
<td>41</td>
<td>Aug.</td>
<td>84</td>
</tr>
<tr>
<td>Mar</td>
<td>52</td>
<td>Sept.</td>
<td>78</td>
</tr>
<tr>
<td>Apr.</td>
<td>64</td>
<td>Oct.</td>
<td>66</td>
</tr>
<tr>
<td>May</td>
<td>74</td>
<td>Nov.</td>
<td>52</td>
</tr>
<tr>
<td>Jun.</td>
<td>82</td>
<td>Dec.</td>
<td>41</td>
</tr>
</tbody>
</table>

Source: The Weather Channel

a. Sketch a graph of the function representing this situation.

b. Describe the period of the function.

SOLUTION:
a.

b. The period of the function is 12 months or 1 year.

ANSWER:
12 mo or 1 yr
Find the exact value of each function.

20. \( \sin \left( \frac{7\pi}{3} \right) \)

**SOLUTION:**
\[
\sin \left( \frac{7\pi}{3} \right) = \sin \left( \frac{\pi + 6\pi}{3} \right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}
\]

**ANSWER:** \( \frac{\sqrt{3}}{2} \)

21. \( \cos(-60^\circ) \)

**SOLUTION:**
\[
\cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}
\]

**ANSWER:** \( \frac{1}{2} \)

22. \( \cos 450^\circ \)

**SOLUTION:**
\[
\cos 450^\circ = \cos (90^\circ + 360^\circ) = \cos 90^\circ = 0
\]

**ANSWER:** 0

23. \( \sin \left( \frac{11\pi}{4} \right) \)

**SOLUTION:**
\[
\sin \left( \frac{11\pi}{4} \right) = \sin \left( \frac{3\pi}{4} + \frac{8\pi}{4} \right) = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}
\]

**ANSWER:** \( \frac{\sqrt{2}}{2} \)

24. \( \sin (-45^\circ) \)

**SOLUTION:**
\[
\sin (-45^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}
\]

**ANSWER:** \( -\frac{\sqrt{2}}{2} \)

25. \( \cos 570^\circ \)

**SOLUTION:**
\[
\cos 570^\circ = \cos (210^\circ + 360^\circ) = \cos 210^\circ = -\frac{\sqrt{3}}{2}
\]

**ANSWER:** \( -\frac{\sqrt{3}}{2} \)

26. **CCSS SENSE-MAKING** In the engine at the
12-6 Circular and Periodic Functions

right, the distance $d$ from the piston to the center of the circle, called the crankshaft, is a function of the speed of the piston rod. Point $R$ on the piston rod rotates 150 times per second.

[Diagram of piston rod and crankshaft]

a. Identify the period of the function as a fraction of a second.

b. The shortest distance $d$ is 0.5 inch, and the longest distance is 3.5 inches. Sketch a graph of the function. Let the horizontal axis represent the time $t$. Let the vertical axis represent the distance $d$.

**SOLUTION:**

a. The period is the time it takes to complete one rotation. So, the period of the function as a fraction of a second is $\frac{1}{150}$.

b. Sample answer:
The maximum distance $d$ 3.5 inches, and the minimum distance $d$ is 0.5 inch.

[Graph of function]

**ANSWER:**

a. $\frac{1}{150}$

b. Sample answer:
27. TORNADOES A tornado siren makes 2.5 rotations per minute and the beam of sound has a radius of 1 mile. Ms. Miller’s house is 1 mile from the siren. The distance of the sound beam from her house varies periodically as a function of time.

a. Identify the period of the function in seconds.

b. Sketch a graph of the function. Let the horizontal axis represent the time \( t \) from 0 seconds to 60 seconds. Let the vertical axis represent the distance \( d \) the sound beam is from Ms. Miller’s house at time \( t \).

**SOLUTION:**
a. The period is the time it takes to complete one rotation.
So, the period of the function in seconds is \( \frac{60}{2.5} \) or 24 seconds.

b. Sample answer:

**ANSWER:**  
a. 24 seconds

28. FERRIS WHEEL A Ferris wheel in China has a diameter of approximately 520 feet. The height of a compartment \( h \) is a function of time \( t \). It takes about 30 seconds to make one complete revolution. Let the height at the center of the wheel represent the height at time 0. Sketch a graph of the function.

**SOLUTION:**
Sample answer:

Period of the function is 30 seconds.

**ANSWER:**  
Sample answer:

29. MULTIPLE REPRESENTATIONS The terminal side of an angle in standard position intersects the unit circle at \( P \), as shown in the figure.
12-6 Circular and Periodic Functions

a. GEOMETRIC Copy the figure. Draw lines representing 30°, 60°, 150°, 210°, and 315°.

b. TABULAR Use a table of values to show the slope of each line to the nearest tenth.

c. ANALYTICAL What conclusions can you make about the relationship between the terminal side of the angle and the slope? Explain your reasoning.

SOLUTION:

a.

b.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.6</td>
</tr>
<tr>
<td>60</td>
<td>1.7</td>
</tr>
<tr>
<td>120</td>
<td>-1.7</td>
</tr>
<tr>
<td>150</td>
<td>-0.6</td>
</tr>
<tr>
<td>210</td>
<td>0.6</td>
</tr>
<tr>
<td>315</td>
<td>-1</td>
</tr>
</tbody>
</table>

c. Sample answer: The slope corresponds to the tangent of the angle. For \( \theta = 120^\circ \), the \( x \)-coordinate of \( P \) is \(-\frac{1}{2}\) and the \( y \)-coordinate is \( \sqrt{3} \); slope = \frac{\text{change in } y}{\text{change in } x}. Since change in \( x = -\frac{1}{2} \) and change in \( y = \sqrt{3} \), slope = \frac{\sqrt{3}}{2} \approx 1.7.

ANSWER:
change in \( y = \frac{\sqrt{3}}{2} \), slope = \( \frac{\sqrt{3}}{2} \cdot \left( -\frac{1}{2} \right) = -\frac{\sqrt{3}}{4} \) or about \(-1.7\).

30. **POGO STICK**  A person is jumping up and down on a pogo stick at a constant rate. The difference between his highest and lowest points is 2 feet. He jumps 50 times per minute.

a. Describe the independent variable and dependent variable of the periodic function that represents this situation. Then state the period of the function in seconds.

b. Sketch a graph of the jumper’s change in height in relation to his starting point. Assume that his starting point is halfway between his highest and lowest points. Let the horizontal axis represent the time \( t \) in seconds. Let the vertical axis represent the height \( h \). 

**SOLUTION:**
a. Sample answer: Independent variable: time \( t \) in seconds.
Dependent variable: height \( h \) in feet;
The period of the function is \( \frac{60}{50} \) or 1.2 second.

b.

**ANSWER:**
a. Sample answer: independent variable: time \( t \) in seconds; dependent variable: height \( h \) in feet; period: 1.2 second

b.

**Find the exact value of each function.**

31. \( \cos 45° - \cos 30° \)

**SOLUTION:**

\[
\cos 45° - \cos 30° = \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} = \frac{\sqrt{2} - \sqrt{3}}{2}
\]

**ANSWER:**

\[ \frac{\sqrt{2} - \sqrt{3}}{2} \]

32. \( 6(\sin 30°)(\sin 60°) \)

**SOLUTION:**

\[
6(\sin 30°)(\sin 60°) = 6 \left( \frac{1}{2} \right) \left( \frac{\sqrt{3}}{2} \right) = \frac{3\sqrt{3}}{2}
\]

**ANSWER:**

\[ \frac{3\sqrt{3}}{2} \]
12-6 Circular and Periodic Functions

33. \[2 \sin \frac{4\pi}{3} - 3 \cos \frac{11\pi}{6}\]

**SOLUTION:**
\[2 \sin \frac{4\pi}{3} - 3 \cos \frac{11\pi}{6} = 2 \sin \left(\pi + \frac{\pi}{3}\right) - 3 \cos \left(2\pi - \frac{\pi}{6}\right)\]
\[= -2 \sin \frac{\pi}{3} - 3 \cos \frac{\pi}{6}\]
\[= -2 \left(\frac{\sqrt{3}}{2}\right) - 3 \left(\frac{\sqrt{3}}{2}\right)\]
\[= -\frac{5\sqrt{3}}{2}\]

**ANSWER:**
\[-\frac{5\sqrt{3}}{2}\]

34. \[\cos \left(-\frac{2\pi}{3}\right) + \frac{1}{3} \sin 3\pi\]

**SOLUTION:**
\[\cos \left(-\frac{2\pi}{3}\right) + \frac{1}{3} \sin 3\pi = \cos \frac{2\pi}{3} + \frac{1}{3} \sin (\pi + 2\pi)\]
\[= \frac{1}{2} + \frac{1}{3} \sin \pi\]
\[= \frac{1}{2} + \frac{1}{3}(0)\]
\[= \frac{1}{2}\]

**ANSWER:**
\[\frac{1}{2}\]

35. \[(\sin 45^\circ)^2 + (\cos 45^\circ)^2\]

**SOLUTION:**
\[(\sin 45^\circ)^2 (\cos 45^\circ)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2\]
\[= \frac{1}{2} + \frac{1}{2}\]
\[= 1\]

**ANSWER:**
\[1\]

36. \[\frac{(\cos 30^\circ)(\cos 150^\circ)}{\sin 315^\circ}\]

**SOLUTION:**
\[\frac{(\cos 30^\circ)(\cos 150^\circ)}{\sin 315^\circ} = \frac{(\cos 30^\circ)(\cos (180^\circ - 30^\circ))}{\sin (360^\circ - 45^\circ)}\]
\[= \frac{\cos 30^\circ(-\cos 30^\circ)}{-\sin 45^\circ}\]
\[= \frac{\sqrt{3}/2}{\frac{\sqrt{2}}{2}}\]
\[= \frac{3}{2\sqrt{2}}\]
\[= \frac{3\sqrt{2}}{4}\]

**ANSWER:**
\[\frac{3\sqrt{2}}{4}\]
37. **CCSS CRITIQUE**  Francis and Benita are finding the exact value of \( \cos \frac{-\pi}{3} \). Is either of them correct? Explain your reasoning.

**Francis**
\[
\cos \frac{-\pi}{3} = -\cos \frac{\pi}{3} = -0.5
\]

**Benita**
\[
\cos \frac{-\pi}{3} = \cos \left( \frac{-\pi}{3} + 2\pi \right)
= \cos \frac{5\pi}{3}
= 0.5
\]

**SOLUTION:**
Benita is correct.

Francis incorrectly wrote \( \cos \frac{-\pi}{3} = -\cos \frac{\pi}{3} \).

**ANSWER:**
Benita, Francis incorrectly wrote \( \cos \frac{-\pi}{3} = -\cos \frac{\pi}{3} \).

38. **CHALLENGE**  A ray has its endpoint at the origin of the coordinate plane, and point \( P\left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) \) lies on the ray. Find the angle \( \theta \) formed by the positive \( x \)-axis and the ray.

\[
P\left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) = P(\cos \theta, \sin \theta)
\]

**SOLUTION:**
\[
\cos \theta = \frac{1}{2}
\]
\[
\theta = 60^\circ
\]
\[
\sin \theta = -\frac{\sqrt{3}}{2}
\]
\[
\theta = -60^\circ
\]

In Quadrant IV, cosines will have positive values and sines will have negative values. So, the angle \( \theta \) is \(-60^\circ\).

**ANSWER:**
\[-60^\circ\]
39. **REASONING** Is the period of a sine curve sometimes, always, or never a multiple of $\pi$? Justify your reasoning.

**SOLUTION:**
The period of a sine curve is sometimes a multiple of $\pi$. The period of a sine curve could be $\frac{\pi}{2}$, which is not a multiple of $\pi$.

**ANSWER:**
Sometimes; the period of a sine curve could be $\frac{\pi}{2}$, which is not a multiple of $\pi$.

40. **OPEN ENDED** Draw the graph of a periodic function that has a maximum value of 10 and a minimum value of $-10$. Describe the period of the function.

**SOLUTION:**
Sample answer:

![Periodic Function Graph](image)

**ANSWER:**
Sample answer:

![Periodic Function Graph](image)
41. **WRITING IN MATH**  Explain how to determine the period of a periodic function from its graph. Include a description of a cycle.

**SOLUTION:**
The period of a periodic function is the horizontal distance of the part of the graph that is nonrepeating. Each nonrepeating part of the graph is one cycle.

**ANSWER:**
The period of a periodic function is the horizontal distance of the part of the graph that is nonrepeating. Each nonrepeating part of the graph is one cycle.

42. **SHORT RESPONSE**  Describe the translation of the graph of \( f(x) = x^2 \) to the graph of \( g(x) = (x + 4)^2 - 3 \).

**SOLUTION:**
Sample answer: Move the graph of \( f(x) \) 4 units to the left and 3 units down to obtain the graph of \( g(x) \).

**ANSWER:**
Sample answer: Move the graph of \( f(x) \) 4 units to the left and 3 units down to obtain the graph of \( g(x) \).

43. The rate of population decline of Hampton Cove is modeled by \( P(t) = 24,000e^{-0.0064t} \), where \( t \) is time in years from this year and 24,000 is the current population. In how many years will the population be 10,000?

A 14

B 104

C 137

D 375

**SOLUTION:**
Substitute 10,000 for \( P(t) \) in the equation and solve for \( t \).

\[
P(t) = 24000e^{-0.0064t} \\
10000 = 24000e^{-0.0064t} \\
\frac{10000}{24000} = e^{-0.0064t} \\
\ln\frac{10000}{24000} = -0.0064t \\
137 \approx t
\]

After about 137 years the population will be 10,000.

Option C is the correct answer.

**ANSWER:**
C
12-6 Circular and Periodic Functions

44. SAT/ACT If \( d^2 + 8 = 21 \), then \( d^2 - 8 = \)

F 0

G 5

H 13

J 31

K 161

**SOLUTION:**

\[ d^2 + 8 = 21 \]
\[ d^2 = 13 \]

Substitute 13 for \( d^2 \).

\[ d^2 - 8 = 13 - 8 \]
\[ = 5 \]

So, G is the correct option.

**ANSWER:**

G

45. **STATISTICS** If the average of three different positive integers is 65, what is the greatest possible value of one of the integers?

A 192

B 193

C 194

D 195

**SOLUTION:**

The sum of three different positive integers is 195. So, the greatest possible value of one of the integer is 192 (195 – (1 + 2)).

A is the correct option.

**ANSWER:**

A

46. **GRIDDED RESPONSE** If \( 8xy + 3 = 3 \), what is the value of \( xy \)?

**SOLUTION:**

\[ 8xy + 3 = 3 \]
\[ 8xy = 0 \]
\[ xy = 0 \]

**ANSWER:**

0
Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

47. 

**SOLUTION:**
Use the Law of Sines to find the measure of angle \( \angle A \).

\[
\sin A = \frac{\sin 82}{8} = \frac{14}{8} \\
\sin A = \frac{8 \sin 82}{14} \\
A \approx 34 
\]

Find the measure of \( \angle C \).

\[
\angle C \approx 180 - (34 + 82) \\
\approx 64 
\]

Use the Law of Sines to find \( c \).

\[
\frac{\sin 64}{c} \approx \frac{\sin 82}{14} \\
c \approx \frac{14 \sin 64}{\sin 82} \\
c \approx 12.7 
\]

**ANSWER:**
\( A \approx 34^\circ, C \approx 64^\circ, c \approx 12.7 \)

48. 

**SOLUTION:**
Use the Law of Cosines to find the missing side length.

\[
a^2 = b^2 + c^2 - 2bc \cos A \\
a^2 = 13^2 + 6^2 - 2(13)(6) \cos 110 \\
a \approx 16.1 
\]

Use the Law of Sines to find a missing angle measure.

\[
\frac{\sin B}{13} \approx \frac{\sin 110}{16.1} \\
\sin B \approx \frac{13 \sin 110}{16.1} \\
B \approx 49 
\]

Find the measure of \( \angle C \).

\[
\angle C \approx 180 - (49 + 110) \\
\approx 21 
\]

**ANSWER:**
\( a \approx 16.1, B \approx 49^\circ, C \approx 21^\circ \)
Determine whether each triangle has no solution, one solution, or two solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

50. \( A = 72^\circ, a = 6, b = 11 \)

**SOLUTION:**

Since \( \angle A \) is acute and \( a < b \), find \( h \) and compare it to \( a \).

\[ h = b \sin A = 11 \sin 72^\circ \approx 10.5 \]

Since \( 6 < 10.5 \) or \( a < h \), there is no solution.

**ANSWER:**

no solution
12-6 Circular and Periodic Functions

51. \(A = 46^\circ, a = 10, b = 8\)

**Solution:**
Since \(\angle A\) is acute and \(a > b\), there is one solution.

Use the Law of Sines to find \(m\angle B\).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{or} \quad \frac{\sin 46}{10} = \frac{\sin B}{8}
\]

\[
\sin B = \frac{8 \sin 46}{10}
\]

\[
B \approx 35^\circ
\]

Find the measure of \(\angle C\).

\[
\angle C \approx 180 - \left(46 + 35\right) = 99^\circ
\]

Use the Law of Sines to find \(c\).

\[
\frac{\sin 46}{10} \approx \frac{\sin 99}{c} \quad \text{or} \quad \frac{10 \sin 99}{\sin 46} \approx c
\]

\[
c \approx 13.7
\]

**Answer:**
one solution; \(B \approx 35^\circ, C \approx 99^\circ, c \approx 13.7\)

52. \(A = 110^\circ, a = 9, b = 5\)

**Solution:**
Because \(\angle A\) is obtuse and \(a > b\), one solution exists.

Use the Law of Sines to find \(m\angle B\).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{or} \quad \frac{\sin 110}{9} = \frac{\sin B}{5}
\]

\[
\sin B = \frac{5 \sin 110}{9}
\]

\[
B \approx 31^\circ
\]

Find the measure of \(\angle C\).

\[
\angle C \approx 180 - \left(110 + 31\right) = 39^\circ
\]

Use the Law of Sines to find \(c\).

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{or} \quad \frac{\sin 110}{9} = \frac{\sin 39}{c}
\]

\[
c \approx 6.0
\]

**Answer:**
one solution; \(B \approx 31^\circ, C \approx 39^\circ, c \approx 6.0\)
A binomial distribution has a 70% rate of success. There are 10 trials.

53. What is the probability that there will be 3 failures?

**SOLUTION:**
The probability of a success is 0.7.
The probability of a failure is $1 - 0.7$ or 0.3.
The probability of 3 failures is $\binom{10}{3} (0.7)^3 (0.3)^7$ or about 0.267.

**ANSWER:**
0.267

54. What is the probability that there will be at least 7 successes?

**SOLUTION:**
The probability that at least 7 successes
is $\binom{10}{7} (0.7)^7 (0.3)^3 + \ldots + \binom{10}{10} (0.7)^0 (0.3)^0$.

That is approximately 0.6496.

**ANSWER:**
0.6496

55. What is the expected number of successes?

**SOLUTION:**
Expected value of binomial distribution.

$$E(X) = np$$

$$= 10(0.7)$$

$$= 7$$

The expected number of success is 7.

**ANSWER:**
7
56. **GAMES** The diagram shows the board for a game in which spheres are dropped down a chute. A pattern of nails and dividers causes the disks to take various paths to the sections at the bottom. For each section, how many paths through the board lead to that section?

![Diagram of a board with spheres dropping through it]

**SOLUTION:**
Look at the fourth row of Pascal’s triangle.

```
   1
  1 1
 1 2 1
1 3 3 1
1 4 6 4 1
```

So, for each section they have 1, 4, 6, 4, 1 paths which lead to the section at the bottom.

**ANSWER:**
1, 4, 6, 4, 1

57. **SALARIES** Phillip’s current salary is $40,000 per year. His annual pay raise is always a percent of his salary at the time. What would his salary be if he got four consecutive 4% increases?

**SOLUTION:**
The recursive formula is \( a_n = a_{n-1} + (a_{n-1} \times 0.04) \).

\[
\begin{align*}
a_1 &= 40,000 \\
a_2 &= 40,000 + (40,000 \times 0.04) = 41,600 \\
a_3 &= 41,600 + (41,600 \times 0.04) = 43,264 \\
a_4 &= 43,264 + (43,264 \times 0.04) = 44,994.56 \\
a_5 &= 44,994.56 + (44,994.56 \times 0.04) = 46,794.34
\end{align*}
\]

His salary would be $46,794.34.

**ANSWER:**
$46,794.34
Find the exact solution(s) of each system of equations.

58. \( y = x + 2 \)
   \( y = x^2 \)

**SOLUTION:**
Substitute \( x^2 \) for \( y \) in the equation \( y = x + 2 \) and solve for \( x \).

\[
\begin{align*}
  x^2 &= x + 2 \\
  x^2 - x - 2 &= 0 \\
  (x - 2)(x + 1) &= 0 \\
  x &= 2 \text{ or } -1
\end{align*}
\]

Substitute 2 and -1 for \( x \) in anyone of the equation and find the respective \( y \) values.

\[
\begin{align*}
  y &= x + 2 \\
  y &= 2 + 2 \\
  y &= 4 \\
  y &= x + 2 \\
  y &= -1 + 2 \\
  y &= 1
\end{align*}
\]

The solutions of the system are (2, 4) and (-1, 1).

**ANSWER:**
(2, 4), (-1, 1)

59. \( 4x + y^2 = 20 \)
   \( 4x^2 + y^2 = 100 \)

**SOLUTION:**
Solve the system of equations. Subtract both the equations.

\[
\begin{align*}
  4x^2 + y^2 - (4x + y^2) &= 100 - 20 \\
  4x^2 - 4x &= 80 \\
  x^2 - x - 20 &= 0 \\
  (x - 5)(x + 4) &= 0 \\
  x &= 5 \text{ or } -4
\end{align*}
\]

Substitute 5 and -4 for \( x \) in anyone of the equation and find the respective \( y \) values.

\[
\begin{align*}
  4x + y^2 &= 20 \\
  4(5) + y^2 &= 20 \\
  20 + y^2 &= 20 \\
  y^2 &= 0 \\
  y &= 0 \\
  4x + y^2 &= 20 \\
  4(-4) + y^2 &= 20 \\
  -16 + y^2 &= 20 \\
  y^2 &= 36 \\
  y &= \pm 6
\end{align*}
\]

The solutions are (5, 0) and (-4, ±6).

**ANSWER:**
(5, 0), (-4, ±6)
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Simplify each expression.

60. \[ \frac{240}{1 - \frac{5}{4}} \]

**SOLUTION:**

\[ \frac{240}{1 - \frac{5}{4}} = \frac{240}{1 - 1.25} = \frac{240}{-0.25} = -960 \]

**ANSWER:**

960

61. \[ \frac{180}{2 - \frac{1}{3}} \]

**SOLUTION:**

\[ \frac{180}{2 - \frac{1}{3}} = \frac{180}{\frac{6}{3} - \frac{1}{3}} = \frac{180}{\frac{5}{3}} = \frac{180 \times 3}{5} = 108 \]

**ANSWER:**

108

62. \[ \frac{90}{2 - \frac{11}{4}} \]

**SOLUTION:**

\[ \frac{90}{2 - \frac{11}{4}} = \frac{90}{\frac{8}{4} - \frac{11}{4}} = \frac{90}{-\frac{3}{4}} = -90 \times \frac{4}{3} = -120 \]

**ANSWER:**

120
Find the amplitude and period of each function.
Then graph the function.

1. \( y = 4 \sin \theta \)

**SOLUTION:**
- amplitude: \(|a| = 4\) or 4
- period: \( \frac{360^\circ}{|b|} = \frac{360^\circ}{1} = 360^\circ \) or 360
- \( x \)-intercepts: (0,0)

\[
\left( \frac{1}{2}, \frac{360}{b}, 0 \right) = (180, 0) \\
\left( \frac{360}{b}, 0 \right) = (360, 0)
\]

**ANSWER:**
- amplitude: 4; period: 360°
12-7 Graphing Trigonometric Functions

3. \( y = \cos 2\theta \)

**SOLUTION:**

- **Amplitude:** \(|a| = |1| \) or 1
- **Period:** \( \frac{360}{|b|} = \frac{360}{2} \) or 180 \( \degree \)

**x-intercepts:**

\[
\left( \frac{1}{4} \cdot \frac{360}{b} , 0 \right) = (45 , 0)
\]

\[
\left( \frac{3}{4} \cdot \frac{360}{b} , 0 \right) = (135 , 0)
\]

**ANSWER:**

- Amplitude: 1; period: 180 \( \degree \)

4. \( y = \frac{1}{2} \cos 3\theta \)

**SOLUTION:**

- **Amplitude:** \(|a| = \frac{1}{2} \) or \( \frac{1}{2} \)
- **Period:** \( \frac{360}{|b|} = \frac{360}{3} \) or 120 \( \degree \)

**x-intercepts:**

\[
\left( \frac{1}{4} \cdot \frac{360}{b} , 0 \right) = (30 , 0)
\]

\[
\left( \frac{3}{4} \cdot \frac{360}{b} , 0 \right) = (90 , 0)
\]

**ANSWER:**

- Amplitude: \( \frac{1}{2} \); period: 120 \( \degree \)

5. **SPIDERS** When an insect gets caught in a spider web, the web vibrates with a frequency of 14 hertz.
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a. Find the period of the function.

b. Let the amplitude equal 1 unit. Write a sine equation to represent the vibration of the web $y$ as a function of time $t$. Then graph the equation.

**SOLUTION:**

a. The period of the function is $\frac{1}{14}$ or about 0.07.

b. Period $= \frac{2\pi}{|b|}$

$$\frac{1}{14} = \frac{2\pi}{|b|}$$

$$\frac{1}{14}|b| = 2\pi$$

$$b = 28\pi$$

Substitute 1 for $a$, $28\pi$ for $b$ and $t$ for $\theta$ in the general equation for the sine function.

$$y = a\sin\, b\theta$$

$$y = 1\sin\, 28\pi t$$

$$y = \sin\, 28\pi t$$

amplitude: $|a| = 1$ or 1

$x$-intercepts: (0, 0)

$$\left(\frac{1}{2}, \frac{2\pi}{b}, 0\right) = (0.036, 0)$$

$$\left(\frac{2\pi}{b}, 0\right) = (0.07, 0)$$

**ANSWER:**

b. $y = \sin\, 28\pi t$
12-7 Graphing Trigonometric Functions

Find the period of each function. Then graph the function.

6. \( y = 3 \tan \theta \)

**SOLUTION:**

period: \( \frac{180}{|b|} = \frac{180}{|1|} \) or 180

asymptotes: \( \frac{180}{2|b|} = \frac{180}{2|1|} \) or 90

Sketch asymptotes at 1·90 or 90, 3·90 or 270°, and so on.

Use \( y = \tan \theta \), draw one cycle every 180°.

**ANSWER:**

period: 180°

7. \( y = 2 \csc \theta \)

**SOLUTION:**

period: \( \frac{360}{|b|} = \frac{360}{|1|} \) or 360

Since \( 2 \csc \theta \) is a reciprocal of \( 2 \sin \theta \), the vertical asymptotes occur at the points where \( 2 \sin \theta = 0 \). So, the asymptotes are at \( \theta = 0° \) and \( \theta = 180° \).

Sketch \( y = 2 \csc \theta \).
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8. \( y = \cot 2\theta \)

\textbf{SOLUTION:}

\[
\text{period: } \frac{180°}{|b|} = \frac{180°}{|2|} \text{ or } 90°
\]

Since \( \cot 2\theta \) is a reciprocal of \( \tan 2\theta \), the vertical asymptotes occur at the points where \( \tan 2\theta = 0 \). So, the asymptotes are at \( \theta = 0°, \ 90°, \ 180°, \ 270° \) and so on.

Sketch \( y = \cot 2\theta \).

\[ y = \cot 2\theta \]

\textbf{ANSWER:}

period: 90°

\[ y = \cot 2\theta \]

9. \( y = 2 \cos \theta \)

\textbf{SOLUTION:}

\[
\text{amplitude: } |a| = |2| \text{ or } 2
\]

\[
\text{period: } \frac{360°}{|b|} = \frac{360°}{|1|} \text{ or } 360°
\]

\[ y = 2 \cos \theta \]

\textbf{x-intercepts:}

\[
\left( \frac{1}{4} \cdot \frac{360°}{b}, 0 \right) = (90°, 0)
\]

\[
\left( \frac{3}{4} \cdot \frac{360°}{b}, 0 \right) = (270°, 0)
\]

\[ y = 2 \cos \theta \]

\textbf{ANSWER:}

amplitude: 2; period: 360°
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10. \( y = 3 \sin \theta \)

**SOLUTION:**

amplitude: \(|a| = |3|\) or 3

period: \( \frac{360}{|b|} = \frac{360}{1} \) or 360

\( x \)-intercepts: (0,0)

\[ \left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = (180, 0) \]

\[ \left( \frac{360}{b}, 0 \right) = (360, 0) \]

**ANSWER:**
amplitude: 3; period: 360°

11. \( y = \sin 2\theta \)

**SOLUTION:**

amplitude: \(|a| = |1|\) or 1

period: \( \frac{360}{|b|} = \frac{360}{2} \) or 180

\( x \)-intercepts: (0,0)

\[ \left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = (90, 0) \]

\[ \left( \frac{360}{b}, 0 \right) = (180, 0) \]

**ANSWER:**
amplitude: 1; period: 180°
12. \( y = \cos 3\theta \)

**SOLUTION:**

amplitude: \(|a| = 1|\) or 1

period: \(\frac{360}{|b|} = \frac{360}{3}\) or 120°

x-intercepts:

\[
\begin{align*}
\left( \frac{1}{4} \cdot \frac{360}{b} \right, 0\right) &= (30°, 0) \\
\left( \frac{3}{4} \cdot \frac{360}{b} \right, 0\right) &= (90°, 0)
\end{align*}
\]

**ANSWER:**

amplitude: 1; period: 120°

13. \( y = \cos \frac{1}{2} \theta \)

**SOLUTION:**

amplitude: \(|a| = 1|\) or 1

period: \(\frac{360}{|b|} = \frac{360}{1/2}\) or 720°

x-intercepts:

\[
\begin{align*}
\left( \frac{1}{4} \cdot \frac{360}{b} \right, 0\right) &= (180°, 0) \\
\left( \frac{3}{4} \cdot \frac{360}{b} \right, 0\right) &= (540°, 0)
\end{align*}
\]

**ANSWER:**

amplitude: 1; period: 720°
12-7 Graphing Trigonometric Functions

14. \( y = \sin 4\theta \)

\[ \text{SOLUTION:} \]
- amplitude: \(|a| = 1| \text{ or } 1\]
- period: \(\frac{360^\circ}{|b|} = \frac{360^\circ}{4} \text{ or } 90^\circ\]
- \(x\)-intercepts: \((0,0)\)

\[ \left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = (45^\circ, 0) \]
\[ \left( \frac{360}{b}, 0 \right) = (90^\circ, 0) \]

\[ y = \sin 4\theta \]

\[ \theta \]

\begin{align*}
&\text{ANSWER:} \\
&\text{amplitude: } 1\text{; period: } 90^\circ
\end{align*}

15. \( y = \frac{3}{4} \cos \theta \)

\[ \text{SOLUTION:} \]
- amplitude: \(|a| = \frac{3}{4} | \text{ or } \frac{3}{4}\]
- period: \(\frac{360^\circ}{|b|} = \frac{360^\circ}{1} \text{ or } 360^\circ\]
- \(x\)-intercepts:

\[ \left( \frac{1}{4} \cdot \frac{360}{b}, 0 \right) = (90^\circ, 0) \]
\[ \left( \frac{3}{4} \cdot \frac{360}{b}, 0 \right) = (270^\circ, 0) \]

\[ y = \frac{3}{4} \cos \theta \]

\[ \theta \]

\[ \text{ANSWER:} \]
- amplitude: \(\frac{3}{4}\); period: \(360^\circ\)
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16. \( y = \frac{3}{2} \sin \theta \)

**SOLUTION:**
- Amplitude: \( |a| = \left| \frac{3}{2} \right| = \frac{3}{2} \)
- Period: \( \frac{360^\circ}{|b|} = \frac{360^\circ}{\frac{3}{2}} = 240^\circ \)
- \( x \)-intercepts: (0,0)

\[
\left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = (180, 0) \\
\left( \frac{360}{b}, 0 \right) = (360, 0) \\
\]

**ANSWER:**
- Amplitude: \( \frac{3}{2} \); period: \( 360^\circ \)

![Graph of \( y = \frac{3}{2} \sin \theta \)](image)

17. \( y = \frac{1}{2} \sin 2\theta \)

**SOLUTION:**
- Amplitude: \( |a| = \left| \frac{1}{2} \right| = \frac{1}{2} \)
- Period: \( \frac{360^\circ}{|b|} = \frac{360^\circ}{2} = 180^\circ \)
- \( x \)-intercepts: (0,0)

\[
\left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = (90, 0) \\
\left( \frac{360}{b}, 0 \right) = (180, 0) \\
\]

**ANSWER:**
- Amplitude: \( \frac{1}{2} \); period: \( 180^\circ \)

![Graph of \( y = \frac{1}{2} \sin 2\theta \)](image)
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18. \( y = 4 \cos 2\theta \)

**SOLUTION:**
- amplitude: \(|a| = |4| = 4\)
- period: \( \frac{360}{|b|} = \frac{360}{2} = 180\)°
- \(x\)-intercepts:
  - \( \left( \frac{1}{4} \cdot \frac{360}{b},0 \right) = (45,0) \)
  - \( \left( \frac{3}{4} \cdot \frac{360}{b},0 \right) = (135,0) \)

**ANSWER:**
- amplitude: 4; period: 180°

![Graph of y = 4 cos 2\theta](image)

19. \( y = 3 \cos 2\theta \)

**SOLUTION:**
- amplitude: \(|a| = |3| = 3\)
- period: \( \frac{360}{|b|} = \frac{360}{2} = 180\)°
- \(x\)-intercepts:
  - \( \left( \frac{1}{4} \cdot \frac{360}{b},0 \right) = (45,0) \)
  - \( \left( \frac{3}{4} \cdot \frac{360}{b},0 \right) = (135,0) \)

**ANSWER:**
- amplitude: 3; period: 180°

![Graph of y = 3 cos 2\theta](image)
20. \( y = 5 \sin \frac{2}{3} \theta \)

**SOLUTION:**
- Amplitude: \( |a| = |5| \) or 5
- Period: \( \frac{360}{|b|} = \frac{360}{2} \) or 540
- \( x \)-intercepts: (0,0)

\[
\left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = \left( 270, 0 \right)
\]
\[
\left( \frac{360}{b}, 0 \right) = \left( 540, 0 \right)
\]

**ANSWER:**
- Amplitude: 5; Period: 540°

![Graph of y = 5 sin (2/3) theta](image)

21. **CCSS REASONING** A boat on a lake bobs up and down with the waves. The difference between the lowest and highest points of the boat is 8 inches. The boat is at equilibrium when it is halfway between the lowest and highest points. Each cycle of the periodic motion lasts 3 seconds.

a. Write an equation for the motion of the boat. Let \( h \) represent the height in inches and let \( t \) represent the time in seconds. Assume that the boat is at equilibrium at \( t = 0 \) seconds.

b. Draw a graph showing the height of the boat as a function of time.

**SOLUTION:**
a. The period of the function is 3.

\[
Period = \frac{2\pi}{|b|}
\]
\[
3 = \frac{2\pi}{|b|}
\]
\[
3|b| = 2\pi
\]
\[
b = \frac{2\pi}{3}
\]

Substitute 4 for \( a \), \( \frac{2\pi}{3} \) for \( b \) and \( t \) for \( \theta \) in the general equation for the sine function.

\[
y = a \sin b \theta
\]
\[
y = 4 \sin \frac{2\pi}{3} t
\]

b. Amplitude: \( |a| = |4| \) or 4

\[
\left( \frac{1}{2} \cdot \frac{2\pi}{b}, 0 \right) = (1.5, 0)
\]
\[
\left( \frac{2\pi}{b}, 0 \right) = (3, 0)
\]
12-7 Graphing Trigonometric Functions

**ANSWER:**

a. \( h = 4 \sin \frac{2}{3} \pi t \)

b. \( h = 4 \sin \frac{2}{3} \pi t \)

**22. ELECTRICITY** The voltage supplied by an electrical outlet is a periodic function that oscillates, or goes up and down, between \(-165\) volts and \(165\) volts with a frequency of 50 cycles per second.

a. Write an equation for the voltage \( V \) as a function of time \( t \). Assume that at \( t = 0 \) seconds, the current is 165 volts.

b. Graph the function.

**SOLUTION:**

a. The period of the function per second is \( \frac{1}{50} \) or 0.02.

Period = \( \frac{2\pi}{|b|} \)

\[
0.02 = \frac{2\pi}{|b|} \\
0.02|b| = 2\pi \\
b = \frac{2\pi}{0.02} \\
b = 100\pi
\]

Substitute 165 for \( a \), \( 100\pi \) for \( b \) and \( t \) for \( \theta \) in the general equation for the cosine function.

\[ y = a \cos b\theta \]

\[ y = 165 \cos 100\pi t \]

b. amplitude: \( |a| = 165 \) or 165

**x-intercepts:**

\[
\left( \frac{1}{4} \cdot \frac{2\pi}{b}, 0 \right) = (0.005, 0) \\
\left( \frac{3}{4} \cdot \frac{2\pi}{b}, 0 \right) = (0.015, 0)
\]

**ANSWER:**

a. \( V = 165 \cos 100\pi t \)

b.
23. \( y = \tan \frac{1}{2} \theta \)

**SOLUTION:**
- Period: \( \frac{180}{|b|} = \frac{180}{1} = 180^\circ \) or 360°

Sketch asymptotes at
- \(-180^\circ\) or \(-180^\circ\), \(180^\circ\) or \(180^\circ\), \(360^\circ\) or \(360^\circ\), and so on.

Use \( y = \tan \theta \), draw one cycle every 360°.
24. \( y = 3 \sec \theta \)

**SOLUTION:**
Period of the function is 360°.

Since \( 3 \sec \theta \) is a reciprocal of \( 3 \cos \theta \), the vertical asymptotes occur at the points where \( 3 \cos \theta = 0 \). So, the asymptotes are at \( \theta = 90° \) and \( \theta = 270° \).

Sketch \( y = 3 \cos \theta \) and use it to graph \( y = 3 \sec \theta \).

**ANSWER:**
period: 360°

25. \( y = 2 \cot \theta \)

**SOLUTION:**

Period of the function is 180°.

\[
|\theta| = \frac{180°}{|n|} = 180° \quad \text{or} \quad 180°
\]

Since \( 2 \cot \theta \) is a reciprocal of \( 2 \tan \theta \), the vertical asymptotes occur at the points where \( 2 \tan \theta = 0 \). So, the asymptotes are at \( \theta = 0° \), \( \theta = 180° \), \( \theta = 360° \) and so on.

Sketch \( y = 2 \tan \theta \) and use it to graph \( y = 2 \cot \theta \).
26. \( y = \csc \frac{1}{2} \theta \)

**SOLUTION:**

period: \( \frac{360}{|b|} = \frac{360}{1} \) or 720°

Since \( \csc \frac{1}{2} \theta \) is a reciprocal of \( \sin \frac{1}{2} \theta \), the vertical asymptotes occur at the points where \( \sin \frac{1}{2} \theta = 0 \). So, the asymptotes are at \( \theta = 0^\circ \), \( \theta = 360^\circ \), \( \theta = 720^\circ \) and so on.

Sketch \( y = \sin \frac{1}{2} \theta \) and use it to graph \( y = \csc \frac{1}{2} \theta \).

**ANSWER:**

period: 720°

27. \( y = 2 \tan \theta \)

**SOLUTION:**

period: \( \frac{180}{|b|} = \frac{180}{1} \) or 180°

asymptotes: \( \frac{180}{2|b|} = \frac{180}{2} \) or 90°

Sketch asymptotes at \( -1 \cdot 90 \) or \(-90, 1 \cdot 90 \) or \( 90, 3 \cdot 90 \) or \( 270\), and so on.

Use \( y = \tan \theta \), draw one cycle every 180°.
28. \( y = \sec \frac{1}{3} \theta \)

**SOLUTION:**

period: \( \frac{360}{|b|} = \frac{360}{1} \) or 1080°

Since \( \sec \frac{1}{3} \theta \) is a reciprocal of \( \cos \frac{1}{3} \theta \), the vertical asymptotes occur at the points where \( \cos \frac{1}{3} \theta = 0 \).

So, the asymptotes are at \( \theta = -270°, 270°, 810° \) and so on.

Sketch \( y = \cos \frac{1}{3} \theta \) and use it to graph \( y = \sec \frac{1}{3} \theta \).

**ANSWER:**

period: 1080°

29. **EARTHQUAKES** A seismic station detects an earthquake wave that has a frequency of 0.5 hertz and an amplitude of 1 meter.

**ANSWER:**

a. Write an equation involving sine to represent the height of the wave \( h \) as a function of time \( t \). Assume that the equilibrium point of the wave, \( h = 0 \), is halfway between the lowest and highest points.

b. Graph the function. Then determine the height of the wave after 20.5 seconds.

**SOLUTION:**

a. The period of the function is \( \frac{1}{0.5} \) or 2.

\[ \text{Period} = \frac{2\pi}{|b|} \]

\[ 2 = \frac{2\pi}{|b|} \]

\[ 2|b| = 2\pi \]

\[ b = \pi \]

Substitute 1 for \( a \), \( \pi \) for \( b \) and \( t \) for \( \theta \) in the general equation for the sine function.

\[ y = a \sin b\theta \]

\[ y = \sin \pi t \]

\[ y = \sin \pi t \]

b. amplitude: \( |a| = 1 \) or 1

\[ x \text{-intercepts: } (0,0) \]

\[ \left( \frac{1}{2}, \frac{2\pi}{b}, 0 \right) = (1,0) \]

\[ \left( \frac{2\pi}{b}, 0 \right) = (2,0) \]
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a. \( h = \sin \pi t \)

b. 

\[ h = \sin \pi t \]

30. **CCSS PERSEVERANCE**  An object is attached to a spring as shown at the right. It oscillates according to the equation \( y = 20 \cos \pi t \), where \( y \) is the distance in centimeters from its equilibrium position at time \( t \).

a. Describe the motion of the object by finding the following: the amplitude in centimeters, the frequency in vibrations per second, and the period in seconds.

b. Find the distance of the object from its equilibrium position at \( t = \frac{1}{4} \) second.

c. The equation \( v = (-20 \text{ cm})(\pi \text{ rad/s}) \cdot \sin (\pi \text{ rad/s} \cdot t) \) represents the velocity \( v \) of the object at time \( t \).

Find the velocity at \( t = \frac{1}{4} \) second.

**SOLUTION:**

a. amplitude: \(|a| = 20\) or 20 cm

\[ \text{period: } \frac{2\pi}{|b|} = \frac{2\pi}{|\pi|} = 2 \text{ or } 2 \text{ seconds} \]

Frequency is the reciprocal of period.

So, the frequency is \( \frac{1}{2} \) or 0.5 vibrations per second.

b. Substitute \( t = \frac{1}{4} \) in the given equation and solve for \( y \).

\[
\begin{align*}
y &= 20 \cos \pi t \\
y &= 20 \cos \pi \left(\frac{1}{4}\right) \\
y &= 20 \cos 45 \\
y &\approx 14.1
\end{align*}
\]

The distance of the object from the equilibrium position is about 14.1 cm at \( t = \frac{1}{4} \) second.

c. Substitute \( t = \frac{1}{4} \) in the given equation and solve for \( v \).

\[
\begin{align*}
v &= -20\pi \sin \pi t \\
v &= -20\pi \sin \left(\frac{\pi}{4}\right) \\
&= -20\pi \left(\frac{1}{\sqrt{2}}\right) \\
&= -10\pi \sqrt{2} \\
&\approx -44.4
\end{align*}
\]

The velocity is about \(-44.4\) cm/s.

**ANSWER:**

a. amplitude: 20 cm; frequency: 0.5 vibrations per second; period: 2 seconds

b. about 14.1 cm

c. about \(-44.4\) cm/s

31. **PIANOS**  A piano string vibrates at a frequency of 130 hertz.

a. Write and graph an equation using cosine to model the vibration of the string \( y \) as a function of time \( t \).

Let the amplitude equal 1 unit.

b. Suppose the frequency of the vibration doubles. Do the amplitude and period increase, decrease, or
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remain the same? Explain.

SOLUTION:

a. The period of the function is \( \frac{1}{130} \).

\[
\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{130} = \frac{1}{65}
\]

\[
\frac{1}{130} |b| = 2\pi
\]

\[
b = 260\pi
\]

Substitute 1 for \( a \), \( 260\pi \) for \( b \) and \( t \) for \( \theta \) in the general equation for the cosine function.

\[
y = a \cos b\theta
\]

\[
y = \cos 260\pi t
\]

\[
y = \cos 260\pi t
\]

Graph the function.

b. The amplitude remains the same. The period decreases because it is the reciprocal of the frequency.

Find the amplitude, if it exists, and period of each function. Then graph the function.

ANSWER:

a. \( y = \cos 260\pi t \)
Find the amplitude and period of each function. Then graph the function.

1. \( y = 4 \sin \frac{2}{3} \theta \)

**SOLUTION:**
- Amplitude: \(|a| = 3\) or 3
- Period: \( \frac{360}{|b|} = \frac{360}{2} = 180^\circ \) or 540°
- \( x \)-intercepts: \((0,0)\)

\[ \left( \frac{1}{2} \cdot \frac{360}{b} , 0 \right) = (270, 0) \]
\[ \left( \frac{360}{b} , 0 \right) = (540, 0) \]

**ANSWER:**
- Amplitude: 3; period: 540°

2. \( y = \frac{1}{2} \cos \frac{3}{4} \theta \)

**SOLUTION:**
- Amplitude: \(|a| = \frac{1}{2}\) or \(\frac{1}{2}\)
- Period: \( \frac{360}{|b|} = \frac{360}{3} = 120^\circ \) or 480°
- \( x \)-intercepts:

\[ \left( \frac{1}{4} \cdot \frac{360}{b} , 0 \right) = (120, 0) \]
\[ \left( \frac{3}{4} \cdot \frac{360}{b} , 0 \right) = (360, 0) \]

**ANSWER:**
- Amplitude: \(\frac{1}{2}\); period: 480°
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34. \( y = 2 \tan \frac{1}{2} \theta \)

**SOLUTION:**

amplitude: does not exist

period: \[ \frac{180}{|b|} = \frac{180}{1} \text{ or } 360 \]

asymptotes: \[ \frac{180}{2|b|} = \frac{180}{2} \text{ or } 180 \]

Sketch asymptotes at

\(-180°, -180°, 180°, 360° \text{ or } 540°, \text{ and so on.} \]

Use \( y = \tan \theta \), draw one cycle every 360°.

\[ y = 2 \tan \frac{1}{2} \theta \]

**ANSWER:**

amplitude: does not exist; period: 360°

35. \( y = 2 \sec \frac{4}{5} \theta \)

**SOLUTION:**

amplitude: does not exist.

period: \[ \frac{360}{|b|} = \frac{360}{4} \text{ or } 450° \]

The vertical asymptotes occur at the points where \( 2 \cos \frac{4}{5} \theta = 0 \). So, the asymptotes are at \( \theta = 112.5° \)

and \( \theta = 337.5° \).

Sketch \( 2 \cos \frac{4}{5} \theta \) and use it to graph \( 2 \sec \frac{4}{5} \theta \).

\[ y = 2 \sec \frac{4}{5} \theta \]

**ANSWER:**

amplitude: does not exist; period: 450°
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36. \( y = 5 \csc 3\theta \)

**SOLUTION:**
amplitude: does not exist.
period: \( \frac{360^\circ}{|b|} = \frac{360^\circ}{3} \) or 120

The vertical asymptotes occur at the points where 5 sin 3\( \theta \) = 0. So, the asymptotes are at \( \theta = 60^\circ \) and \( \theta = 120^\circ \).

Sketch \( y = 5 \sin 3\theta \) and use it to graph \( y = 5 \csc 3\theta \).

**ANSWER:**
amplitude: does not exist; period: 120°

37. \( y = 2 \cot 6\theta \)

**SOLUTION:**
amplitude: does not exist
period: \( \frac{180^\circ}{|b|} = \frac{180^\circ}{6} \) or 30

The vertical asymptotes occur at the points where 2 tan 6\( \theta \) = 0. So, the asymptotes are at \( \theta = 0^\circ \), \( \theta = 30^\circ \), \( \theta = 90^\circ \) and so on.

Sketch \( y = 2 \tan 6\theta \) and use it to graph \( y = 2 \cot 6\theta \).

**ANSWER:**
amplitude: does not exist; period: 30°
Identify the period of the graph and write an equation for each function.

38. 

**SOLUTION:**
The period of the graph is 360°.
The amplitude of the graph is $\frac{3}{2}$.

Period = $\frac{360}{|b|}$

$360 = \frac{360}{|b|}$

$360 |b| = 360$

$b = 1$

Substitute $\frac{3}{2}$ for $a$, 1 for $b$ in the general equation for the cosine function.

$y = a \cos b \theta$

$y = \frac{3}{2} \cos (1) \theta$

$y = \frac{3}{2} \cos \theta$

**ANSWER:**

$360^\circ; y = \frac{3}{2} \cos \theta$

---

39. 

**SOLUTION:**
The period of the graph is 180°.
The amplitude of the graph is 5.

Period = $\frac{360}{|b|}$

$180 = \frac{360}{|b|}$

$180 |b| = 360$

$b = 2$

Substitute 5 for $a$, 2 for $b$ in the general equation for the sine function.

$y = a \sin b \theta$

$y = 5 \sin 2 \theta$

**ANSWER:**

$180^\circ; y = 5 \sin 2 \theta$
40. **SOLUTION:**
The period of the graph is 1800°.

The amplitude of the graph is 2.

\[
\text{Period} = \frac{360}{|b|} \\
1800 = \frac{360}{|b|} \\
|b| = \frac{360}{1800} \\
b = \frac{1}{5}
\]

Substitute 2 for \(a\), \(\frac{1}{5}\) for \(b\) in the general equation for the sine function.

\[
y = a \sin b\theta \\
y = 2 \sin \frac{1}{5} \theta
\]

**ANSWER:**
1800°; \(y = 2 \sin \frac{1}{5} \theta\)

41. **CHALLENGE** Describe the domain and range of \(y = a \cos \theta\) and \(y = a \sec \theta\), where \(a\) is any positive real number.

**SOLUTION:**
The domain of \(y = a \cos \theta\) is the set of all real numbers.
The domain of \(y = a \sec \theta\) is the set of all real numbers except the values for which \(\cos \theta = 0\).
The range of \(y = a \cos \theta\) is \(-a \leq y \leq a\).
The range of \(y = a \sec \theta\) is \(y \leq -a\) and \(y \geq a\).

**ANSWER:**
The domain of \(y = a \cos \theta\) is the set of all real numbers.
The domain of \(y = a \sec \theta\) is the set of all real numbers except the values for which \(\cos \theta = 0\).
The range of \(y = a \cos \theta\) is \(-a \leq y \leq a\).
The range of \(y = a \sec \theta\) is \(y \leq -a\) and \(y \geq a\).

42. **REASONING** Compare and contrast the graphs of \(y = \frac{1}{2} \sin \theta\) and \(y = \sin \frac{1}{2} \theta\).

**SOLUTION:**
The graph of \(y = \frac{1}{2} \sin \theta\) has an amplitude of \(\frac{1}{2}\) and a period of 360°.

The graph of \(y = \sin \frac{1}{2} \theta\) has an amplitude of 1 and a period of 720°.

**ANSWER:**
The graph of \(y = \frac{1}{2} \sin \theta\) has an amplitude of \(\frac{1}{2}\) and a period of 360°.

The graph of \(y = \sin \frac{1}{2} \theta\) has an amplitude of 1 and a period of 720°.
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43. **OPEN ENDED** Write a trigonometric function that has an amplitude of 3 and a period of 180°. Then graph the function.

**SOLUTION:**
Sample answer:

\[ y = 3 \sin 2\theta \]

**ANSWER:**
Sample answer: \( y = 3 \sin 2\theta \)

44. **WRITING IN MATH** How can you use the characteristics of a trigonometric function to sketch its graph?

**SOLUTION:**
Sample answer: Determine the amplitude and period of the function; find and graph any x-intercepts, extrema, and asymptotes; use the parent function to sketch the graph.

**ANSWER:**
Sample answer: Determine the amplitude and period of the function; find and graph any x-intercepts, extrema, and asymptotes; use the parent function to sketch the graph.

45. **SHORT RESPONSE** Find the 100,001st term of the sequence.

13, 20, 27, 34, 41, …

**SOLUTION:**
Substitute \( a_1 = 13 \) and \( d = 7 \) in the formula to find the \( n \)th term.

\[
 a_n = a_1 + (n - 1)d \\
 a_{100,001} = 13 + (100,001 - 1)7 \\
 = 13 + 100,000 \\
 = 700,013
\]

**ANSWER:**
700,013
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46. **STATISTICS** You bowled five games and had the following scores: 143, 171, 167, 133, and 156. What was your average?

A 147  
B 153  
C 154  
D 156

**SOLUTION:**
Average = \( \frac{143 + 171 + 167 + 133 + 156}{5} \) or 154

C is the correct option.

**ANSWER:**
C

47. Your city had a population of 312,430 ten years ago. If its current population is 418,270, by what percentage has it grown over the past 10 years?

F 25%  
G 34%  
H 66%  
J 75%

**SOLUTION:**
Percentage of growth over the past 10 years
\[
\text{Change in population} = \frac{418,270 - 312,430}{312,430} \times 100\%
\]
\[
= \frac{105,840}{312,430} \times 100\%
\]
\[
\approx 34\%
\]

G is the correct option.

**ANSWER:**
G
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48. **SAT/ACT** If \( h + 4 = b - 3 \), then \( (h - 2)^2 = \)

A \( h^2 + 4 \)

B \( b^2 - 6b + 3 \)

C \( b^2 - 18b + 81 \)

D \( b^2 - 14b + 49 \)

E \( b^2 - 10b + 25 \)

**SOLUTION:**
\[ \begin{align*}
h + 4 &= b - 3 \\
h &= b - 7 \\
\text{Substitute } b - 7 \text{ for } h.
\end{align*} \]

\[ \begin{align*}
(h - 2)^2 &= (b - 7 - 2)^2 \\
&= (b - 9)^2 \\
&= b^2 - 18b + 81
\end{align*} \]

C is the correct option.

**ANSWER:**
C

50. \( 3(\sin 45^\circ)(\sin 60^\circ) \)

**SOLUTION:**
\[ \begin{align*}
3(\sin 45^\circ)(\sin 60^\circ) &= 3 \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) \\
&= \frac{3\sqrt{6}}{4}
\end{align*} \]

**ANSWER:**
\( \frac{3\sqrt{6}}{4} \)

51. \( 4\sin \frac{4\pi}{3} - 2\cos \frac{\pi}{6} \)

**SOLUTION:**
\[ \begin{align*}
4\sin \frac{4\pi}{3} - 2\cos \frac{\pi}{6} &= 4 \left( -\frac{\sqrt{3}}{2} \right) - 2 \left( \frac{\sqrt{3}}{2} \right) \\
&= -2\sqrt{3} - \sqrt{3} \\
&= -3\sqrt{3}
\end{align*} \]

**ANSWER:**
\( -3\sqrt{3} \)

Find the exact value of each expression.

49. \( \cos 120^\circ - \sin 30^\circ \)

**SOLUTION:**
\[ \cos 120^\circ - \sin 30^\circ = -\frac{1}{2} - \frac{1}{2} = -1 \]

**ANSWER:**
-1
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Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

52.

SOLUTION:
Use the Law of Cosines to find the missing side length.

\[ c^2 = a^2 + b^2 - 2ab \cos C \]

\[ c^2 = 11.7^2 + 5.3^2 - 2(11.7)(5.3) \cos 24 \]

\[ c \approx 7.2 \]

Use the Law of Sines to find a missing angle measure.

\[ \sin B \approx \sin 24 \]
\[ 5.3 \approx 7.2 \]

\[ \sin B \approx \frac{5.3 \sin 24}{7.2} \]

\[ B \approx 17 \]

Find the measure of \( \angle A \).

\[ \angle A \approx 180^\circ - (17 + 24) \] or 139

ANSWER:
\( B \approx 17^\circ, A \approx 139^\circ, c \approx 7.2 \)

53.

SOLUTION:
Use the Law of Cosines to find the missing side length.

\[ q^2 = r^2 + s^2 - 2rs \cos Q \]

\[ q^2 = 61.2^2 + 15.5^2 - 2(61.2)(15.5) \cos 31^\circ \]

\[ q \approx 48.6 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin R}{61.2} = \frac{\sin 31^\circ}{48.6} \]

\[ R \approx 40.4^\circ \]

\[ R = 180^\circ - 40^\circ - 140^\circ \] (since R is an obtuse angle)

Find the measure of \( \angle S \)

\[ \angle S \approx 180^\circ - (140^\circ + 31^\circ) \]

\[ \approx 9^\circ \]

ANSWER:
\( R \approx 40^\circ, S \approx 9^\circ, q \approx 48.6 \)
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SOLUTION:
Use the Law of Cosines to find the measure of the largest angle \( \angle H \).

\[ h^2 = f^2 + g^2 - 2fg \cos H \]
\[ 17.1^2 = 13.6^2 + 16.2^2 - 2(13.6)(16.2) \cos H \]
\[ 17.1^2 - 13.6^2 - 16.2^2 = -2(13.6)(16.2) \cos H \]
\[ \frac{-2(13.6)(16.2)}{17.1^2 - 13.6^2 - 16.2^2} = \cos H \]
\[ \cos H \approx \frac{69}{H} \]

Use the Law of Sines to find the measure of angle, \( \angle G \).

\[ \frac{\sin G}{16.2} = \frac{\sin 69^\circ}{17.1} \]
\[ \sin G \approx \frac{16.2 \sin 69^\circ}{17.1} \]
\[ G \approx 62^\circ \]

Find the measure of \( \angle F \).

\[ \angle F \approx 180^\circ - (69^\circ + 62^\circ) \text{ or } 49^\circ \]

ANSWER:
\( F \approx 49^\circ, G \approx 62^\circ, H \approx 69^\circ \)

A binomial distribution has a 40% rate of success. There are 12 trials.

55. What is the probability that there will be exactly 5 failures?

SOLUTION:
The probability of a success is 0.4.
The probability of a failure is 1 – 0.4 or 0.6.
The probability of 3 failures is \( _{12}C_5(0.6)^5(0.4)^7 \) or about 0.101, or 10.1%.

ANSWER:
10.1%

56. What is the probability that there will be at least 8 successes?

SOLUTION:
The probability that at least 8 successes is
\[ _{12}C_8(0.4)^8(0.6)^4 + _{12}C_9(0.4)^9(0.6)^3 + ... + _{12}C_{12}(0.4)^{12}(0.6)^0 \]
That is approximately 0.057, or 5.7%.

ANSWER:
5.7%
57. What is the expected number of successes?

**SOLUTION:**
Expected value of binomial distribution.

\[ E(X) = np \]
\[ = 12(0.4) \]
\[ = 4.8 \]

The expected number of success is about 5.

**ANSWER:**
5

58. **BANKING** Rita has deposited $1000 in a bank account. At the end of each year, the bank posts interest to her account in the amount of 3% of the balance, but then takes out a $10 annual fee.

a. Let \( b_0 \) be the amount Rita deposited. Write a recursive equation for the balance \( b_n \) in her account at the end of \( n \) years.

**SOLUTION:**
a. The recursive equation for the balance \( b_n \) at the end of \( n \) years is

\[ b_n = \left( b_{n-1} + (b_{n-1} \cdot 0.03) \right) - 10 \]
\[ b_n = b_{n-1} + 0.03b_{n-1} - 10 \]
\[ b_n = 1.03b_{n-1} - 10 \]

b. 
\[ b_0 = $1000 \]
\[ b_1 = 1.03(1000) - 10 = $1020 \]
\[ b_2 = 1.03(1020) - 10 = $1040.6 \]
\[ b_3 = 1.03(1040.6) - 10 = $1061.818 \]
\[ b_4 = 1.03(1061.818) - 10 \approx $1083.67 \]

After four years Rita will have $1083.67 in her account.

**ANSWER:**
a. \( b_n = 1.03b_{n-1} - 10 \)

b. $1083.67
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Write an equation for an ellipse that satisfies each set of conditions.

59. center at (6, 3), focus at (2, 3), co-vertex at (6, 1)

**SOLUTION:**
Since the y-coordinate of the center and the focus are same, the orientation is horizontal.
From the given points: \( h = 6, k = 3, h - c = 2, k - b = 1 \).

Find the values of \( c \).
\[ 6 - c = 2 \]
\[ c = 4 \]

Find the value of \( b \).
\[ 3 - b = 1 \]
\[ b = 2 \]

Find the value of \( a^2 \).
\[ c^2 = a^2 - b^2 \]
\[ 16 = a^2 - 4 \]
\[ a^2 = 20 \]

Therefore, the equation of the ellipse is
\[ \frac{(x - 6)^2}{20} + \frac{(y - 3)^2}{4} = 1. \]

**ANSWER:**
\[ \frac{(x - 6)^2}{20} + \frac{(y - 3)^2}{4} = 1 \]

---

60. foci at (2, 1) and (2, 13), co-vertex at (5, 7)

**SOLUTION:**
Since the x-coordinates of the foci are same, the orientation is vertical.

The foci are equidistance from the center. So, the center is at \( (2, 7) \).

Therefore, \( h = 2, k = 7 \).

The co-vertex \( (h + b, k) \) is \( (5, 7) \).

Therefore, \( b = 3 \).

The value of \( c \) is distance between the center and foci.
\[ c = 6. \]

Find the value \( a^2 \).
\[ c^2 = a^2 - b^2 \]
\[ 6^2 = a^2 - 3^2 \]
\[ a^2 = 45 \]

The equation of the ellipse is
\[ \frac{(x - 2)^2}{9} + \frac{(y - 7)^2}{45} = 1. \]

**ANSWER:**
\[ \frac{(x - 2)^2}{9} + \frac{(y - 7)^2}{45} = 1 \]
Graph each function.

61. \( y = 2(x - 3)^2 - 4 \)

**SOLUTION:**
The vertex is at (3, –4). The axis of symmetry is at \( x = 3 \). Because \( a = 2 > 1 \), the graph is narrower than the graph of \( y = x^2 \).

**ANSWER:**

62. \( y = \frac{1}{3}(x + 5)^2 + 2 \)

**SOLUTION:**
The vertex is at (–5, 2). The axis of symmetry is at \( x = -5 \). Because \( 0 < a < 1 \), the graph is compressed vertically.

**ANSWER:**
63. \( y = -3(x + 6)^2 + 7 \)

**SOLUTION:**
The vertex is at \((-6, 7)\). The axis of symmetry is at \(x = -6\). Because \(a < 0\), the graph opens down and is stretched vertically.

**ANSWER:**

![Graph of \( y = -3(x + 6)^2 + 7 \)](image)
12-8 Translations of Trigonometric Graphs

State the amplitude, period, and phase shift for each function. Then graph the function.

1. \(y = \sin (\theta - 180^\circ)\)

\[\text{SOLUTION:}\]

Given \(a = 1, \; b = 1 \) and \(h = 180^\circ\).

Amplitude:

\[|a| = |1|\]
\[= 1\]

Period:

\[\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}\]
\[= 360^\circ\]

Phase shift:

\[h = 180^\circ\]

Graph \(y = \sin \theta\) shifted \(180^\circ\) to the right.

\[\text{ANSWER:}\]

\(1; \; 360^\circ; \; h = 180^\circ\)
12-8 Translations of Trigonometric Graphs

2. \( y = \tan \left( \theta - \frac{\pi}{4} \right) \)

SOLUTION:
Given \( b = 1 \) and \( h = \frac{\pi}{4} \).

Amplitude:
No amplitude

Period:
\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]

Phase shift:
\( h = \frac{\pi}{4} \)

Graph \( y = \tan \theta \) shifted \( \frac{\pi}{4} \) units to the right.

ANSWER:
no amplitude; \( 180^\circ; \) \( h = \frac{\pi}{4} \)

3. \( y = \sin \left( \theta - \frac{\pi}{2} \right) \)

SOLUTION:
Given \( a = 1 \), \( b = 1 \) and \( h = \frac{\pi}{2} \).

Amplitude:
|\( a | = |1| = 1

Period:
\[
\frac{2\pi}{|b|} = \frac{2\pi}{|1|} = 2\pi
\]

Phase shift:
\( h = \frac{\pi}{2} \)

Graph \( y = \sin \theta \) shifted \( \frac{\pi}{2} \) units to the right.

ANSWER:
1, \( 2\pi; \) \( h = \frac{\pi}{2} \)
4. \( y = \frac{1}{2} \cos(\theta + 90^\circ) \)

**SOLUTION:**

Given \( a = \frac{1}{2}, b = 1 \) and \( h = -90^\circ \).

Amplitude:

\[
|a| = \frac{1}{2}
\]

Period:

\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{1} = 360^\circ
\]

Phase shift:

\( h = -90^\circ \)

Graph \( y = \frac{1}{2} \cos \theta \) shifted \( 90^\circ \) to the left.

5. \( y = \cos \theta + 4 \)

**SOLUTION:**

Given \( a = 1, b = 1 \) and \( k = 4 \).

Amplitude:

\[
|a| = 1
\]

Period:

\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{1} = 360^\circ
\]

Vertical shift:

\( k = 4 \)

Midline:

\( y = 4 \)

To graph \( y = \cos \theta + 4 \), first draw the midline. Then use it to graph \( y = \cos \theta \) shifted 4 units up.
6. $y = \sin \theta - 2$

**SOLUTION:**
The amplitude, period, vertical shift, and midline of the function $y = \sin \theta - 2$ is given by

Amplitude:

$$|d| = 1$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{1}$$

$$= 360^\circ$$

Vertical shift: $k = -2$

Midline: $y = -2$

To graph $y = \sin \theta - 2$, first draw the midline. Then use it to graph $y = \sin \theta$ shifted 2 units down.

7. $y = \frac{1}{2} \tan \theta + 1$

**SOLUTION:**
Given $b = 1$ and $k = 1$.

Amplitude:
No amplitude

Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{1}$$

$$= 180^\circ$$

Vertical shift:

$k = 1$
12-8 Translations of Trigonometric Graphs

State the amplitude, period, and phase shift for each function. Then graph the function.

1. \( y = \sin (\theta - 180^\circ) \)

**SOLUTION:**
The maximum and the minimum height is 15ft and 3ft respectively. Therefore, the amplitude is. 

ANSWER:
no amplitude; 180°; k = 1; y = 1

8. \( y = \sec \theta - 5 \)

**SOLUTION:**
Given \( b = 1 \) and \( k = -5 \).

Amplitude:
No amplitude

Period:
\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} = 360^\circ
\]

Vertical shift:
\( k = -5 \)

Midline:
\( y = -5 \)

First, graph the midline. Then graph \( y = \sec \theta - 5 \) using the midline as reference.

ANSWER:
no amplitude; 360°; k = -5; y = -5
9. \( y = 2 \sin (\theta + 45^\circ) + 1 \)

**SOLUTION:**
Given \( a = 2, b = 1, h = -45^\circ \) and \( k = 1 \).

Amplitude:
\[ |a| = 2 \]
\[ = 2 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{1} \]
\[ = 360^\circ \]

Phase shift:
\( h = -45^\circ \)

Vertical shift:
\( k = 1 \)

Midline:
\( y = 1 \)

First, graph the midline. Since the amplitude is 2, draw dashed line 2 units above and 2 units below the midline. Then graph \( y = 2 \sin \theta + 1 \) using the midline as reference. Then shift the graph \(-45^\circ\) to the left.

**ANSWER:**
2; \( 360^\circ \); \( h = -45^\circ \); \( k = 1 \)

10. \( y = \cos 3(\theta - \pi) - 4 \)

**SOLUTION:**
Given \( a = 1, b = 3, h = \pi \) and \( k = -4 \).

Amplitude:
\[ |a| = |1| \]
\[ = 1 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{3} \]
\[ = 120^\circ \]

Phase shift: \( h = \pi \)

Vertical shift: \( k = -4 \)

Midline: \( y = -4 \)

First, graph the midline. Since the amplitude is 1, draw dashed line 1 unit above and 1 unit below the midline. Then graph \( y = \cos 3(\theta) - 4 \) using the midline as reference. Then shift the graph \( \pi \) units to the right.
State the amplitude, period, and phase shift for each function. Then graph the function.

1. \(y = \sin (\theta - 180°)\)

SOLUTION:
The maximum and the minimum height is 15ft and 3ft respectively. Therefore, the amplitude is 6 units.

ANSWER:
1; 120°; \(h = \pi\); \(k = -4\)

11. \(y = \frac{1}{4} \tan 2(\theta + 30°) + 3\)

SOLUTION:
Given \(a = \frac{1}{4}\), \(b = 2\), \(h = -30°\) and \(k = 3\).

Amplitude: No amplitude

Period:
\[
\frac{180°}{|b|} = \frac{180°}{2} = 90°
\]

Phase shift: \(h = -30°\)

Vertical shift: \(k = 3\)

Midline: \(y = 3\)

First, graph the midline. Then graph \(y = \frac{1}{4} \tan 2(\theta + 30°) + 3\) using the midline as reference. Then shift the graph 30 units to the left.

12. \(y = 4 \sin \left(\frac{1}{2} \left(\theta - \frac{\pi}{2}\right)\right) + 5\)

SOLUTION:
Given \(a = 4\), \(b = \frac{1}{2}\), \(h = \frac{\pi}{2}\) and \(k = 5\).

Amplitude:
\[
|a| = |4| = 4
\]

Period:
\[
\frac{2\pi}{|b|} = \frac{2\pi}{\frac{1}{2}} = 4\pi
\]
12-8 Translations of Trigonometric Graphs

Phase shift: \( h = \frac{\pi}{2} \)

Vertical shift: \( k = 5 \)

Midline: \( y = 5 \)

First, graph the midline. Then graph
\[ y = 4 \sin \left( \frac{1}{2} (\theta - \frac{\pi}{2}) \right) + 5 \] using the midline as reference.

Then shift the graph \( \frac{\pi}{2} \) units to the right.

**ANSWER:**

4; 4\( \pi \); \( h = \frac{\pi}{2} \); \( k = 5 \)

13. **EXERCISE** While doing some moderate physical activity, a person’s blood pressure oscillates between a maximum of 130 and a minimum of 90. The person’s heart rate is 90 beats per minute. Write a sine function that represents the person’s blood pressure \( P \) at time \( t \) seconds. Then graph the function.

**SOLUTION:**

Amplitude:
\[ |d| = |130 - 110| \]
\[ = 20 \]

Period:
Since the person’s heart rate is 90 beats per minute, the heart beats every \( \frac{60}{90} \) second. So, the period is \( \frac{60}{90} \) second.

\[ \frac{60}{90} = \frac{2\pi}{|b|} \]
\[ |b| = \frac{180\pi}{60} \]
\[ b = 3\pi \]

The midline lies halfway between the maximum and the minimum values.
\[ y = \frac{130 + 90}{2} \]
\[ = 110 \]

Therefore the vertical shift is \( k = 110 \).

Substitute 20 for \( a \), 3\( \pi \) for \( b \), 0 for \( h \), and 110 for \( k \) in
\[ h = a \sin b(t - h) + k \]
\[ h = 20 \sin 3\pi(t - 0) + 110 \]
\[ h = 20 \sin 3\pi t + 110 \]

Graph the function.
12-8 Translations of Trigonometric Graphs

1. \( y = \sin (\theta - 180°) \)

**SOLUTION:**
The maximum and the minimum height is 15 ft and 3 ft respectively. Therefore, the amplitude is 12 units.

**ANSWER:**

14. \( y = \cos (\theta + 180°) \)

**SOLUTION:**
Given \( a = 1, \ b = 1 \) and \( h = -180° \).

- **Amplitude:** 
  \[ |a| = |1| \]
  \[ = 1 \]

- **Period:** 
  \[ \frac{360°}{|b|} = \frac{360°}{|1|} \]
  \[ = 360° \]

- **Phase shift:**
  \( h = -180° \)

Graph \( y = \cos \theta \) shifted \( 180° \) to the left.

**ANSWER:**
1; 360°; \( h = -180° \)
12-8 Translations of Trigonometric Graphs

15. \( y = \tan (\theta - 90^\circ) \)

**SOLUTION:**
Given \( b = 1 \) and \( h = 90^\circ \).

**Amplitude:**
No amplitude

**Period:**
\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{1} = 180^\circ
\]

**Phase shift:**
\( h = 90^\circ \)

Graph \( y = \tan \theta \) shifted \( 90^\circ \) to the right.

**ANSWER:**
no amplitude; \( 180^\circ; h = 90^\circ \)

16. \( y = \sin (\theta + \pi) \)

**SOLUTION:**
Given \( a = 1, b = 1 \) and \( h = -\pi \).

**Amplitude:**
\[
|a| = 1
\]

**Period:**
\[
\frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi
\]

**Phase shift:**
\( h = -\pi \)

Graph \( y = \sin \theta \) shifted \( \pi \) units to the left.

**ANSWER:**
1; \( 2\pi; h = -\pi \)

17. \( y = 2\sin \left( \theta + \frac{\pi}{2} \right) \)

**SOLUTION:**
Given \( a = 2, \ b = 1 \) and \( h = -\frac{\pi}{2} \).

Amplitude:

\[
|a| = |2| = 2
\]

Period:

\[
\frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi
\]

Phase shift: \( h = -\frac{\pi}{2} \)

Graph \( y = 2\sin(\theta - 180^\circ) \) shifted \( \frac{\pi}{2} \) units to the left.

ANSWER:

\( 2; \ 2\pi; \ h = -\frac{\pi}{2} \)

18. \( y = \tan\left(\frac{1}{2}(\theta + 30^\circ)\right) \)

**SOLUTION:**

Given \( b = \frac{1}{2} \) and \( h = -30^\circ \).

Amplitude:

No amplitude

Period:

\[
180^\circ = 180^\circ = 360^\circ
\]

Phase shift:

\( h = -30^\circ \)

Graph \( y = \tan\theta \) shifted 30° to the left.

ANSWER:

no amplitude; 360°; \( h = -30^\circ \)
12-8 Translations of Trigonometric Graphs

19. \( y = 3 \cos \left( \theta - \frac{\pi}{3} \right) \)

**SOLUTION:**

Given \( a = 3, b = 1 \) and \( h = \frac{\pi}{3} \).

Amplitude:

\[
|a| = 3
\]

\[
= 3
\]

Period:

\[
\frac{2\pi}{|b|} = \frac{2\pi}{|1|}
\]

\[
= 2\pi
\]

Phase shift:

\( h = \frac{\pi}{3} \)

Graph \( y = 3 \cos \theta \) shifted \( \frac{\pi}{3} \) units to the right.

**ANSWER:**

3; 2\( \pi \); \( h = \frac{\pi}{3} \)
12-8 Translations of Trigonometric Graphs

20. \( y = \cos \theta + 3 \)

SOLUTION:
Given \( a = 1, b = 1 \) and \( k = 3 \).

Amplitude:
\[ |a| = 1 \]

= 1

Period:
\[ \frac{360°}{|b|} = \frac{360°}{1} \]

= 360°

Vertical shift: \( k = 3 \)

Midline: \( y = 3 \)

To graph \( y = \cos \theta + 3 \), first draw the midline. Then use it to graph \( y = \cos \theta \) shifted 3 units up.

ANSWER:
1; 360°; \( k = 3; y = 3 \)

21. \( y = \tan \theta - 1 \)

SOLUTION:
Given \( a = 1, b = 1 \) and \( k = -1 \).

Amplitude: No amplitude

Period:
\[ \frac{180°}{|b|} = \frac{180°}{1} \]

= 180°

Vertical shift: \( k = -1 \)

Midline: \( y = -1 \)

To graph \( y = \tan \theta - 1 \), first draw the midline. Then use it to graph \( y = \tan \theta \) shifted 1 unit down.

ANSWER:
no amplitude; 180°; \( k = -1; y = -1 \)
12-8 Translations of Trigonometric Graphs

22. \( y = \tan \theta + \frac{1}{2} \)

**SOLUTION:**

Given \( a = 1, b = 1 \) and \( k = \frac{1}{2} \).

Amplitude: No amplitude

Period:

\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]

Vertical shift: \( k = \frac{1}{2} \)

Midline: \( y = \frac{1}{2} \)

To graph \( y = \tan \theta + \frac{1}{2} \), first draw the midline. Then use it to graph \( y = \tan \theta \) shifted \( \frac{1}{2} \) units up.

**ANSWER:**

no amplitude; \( 180^\circ; k = \frac{1}{2}, y = \frac{1}{2} \)
12-8 Translations of Trigonometric Graphs

23. \( y = 2 \cos \theta - 5 \)

**SOLUTION:**
Given \( a = 2, b = 1 \) and \( k = -5 \).

Amplitude:
\[ |a| = 2 \]
\[ = 2 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} \]
\[ = 360^\circ \]

Vertical shift: \( k = -5 \)

Midline: \( y = -5 \)

To graph \( y = 2 \cos \theta - 5 \), first draw the midline. Then use it to graph \( y = 2 \cos \theta \) shifted 5 units down.

**ANSWER:**
2; 360°; \( k = -5; y = -5 \)

24. \( y = 2 \sin \theta - 4 \)

**SOLUTION:**
Given \( a = 2, b = 1 \) and \( k = -4 \).

Amplitude:
\[ |a| = 2 \]
\[ = 2 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} \]
\[ = 360^\circ \]

Vertical shift: \( k = -4 \)

Midline: \( y = -4 \)

To graph \( y = 2 \sin \theta - 4 \), first draw the midline. Then use it to graph \( y = 2 \sin \theta \) shifted 4 units down.

**ANSWER:**
2; 360°; \( k = -4; y = -4 \)
25. \( y = \frac{1}{3} \sin \theta + 7 \)

**SOLUTION:**

Given \( a = \frac{1}{3}, b = 1 \) and \( k = 7 \).

Amplitude:

\[ |a| = \left| \frac{1}{3} \right| = \frac{1}{3} \]

Period:

\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{1} = 360^\circ \]

Vertical shift: \( k = 7 \)

Midline: \( y = 7 \)

To graph \( y = \frac{1}{3} \sin \theta + 7 \), first draw the midline. Then use it to graph \( y = \frac{1}{3} \sin \theta \) shifted 7 units up.

**ANSWER:**

\( \frac{1}{3}; 360^\circ; k = 7; y = 7 \)

State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

26. \( y = 4 \sin(\theta - 60^\circ) - 1 \)

**SOLUTION:**

Given \( a = 4, b = 1, h = 60^\circ \) and \( k = -1 \).

Amplitude:

\[ |a| = |4| = 4 \]

Period:

\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{1} = 360^\circ \]

Phase shift: \( h = 60^\circ \)

Vertical shift: \( k = -1 \)

Midline: \( y = -1 \)

First, graph the midline. Then graph \( y = 4 \sin \theta - 1 \) using the midline as reference. Then shift the graph \( 60^\circ \) to the right.
27. \( y = \cos \left( \frac{\theta - 90^\circ}{2} \right) + 2 \)

**SOLUTION:**

Given \( a = 1, \ b = \frac{1}{2}, \ h = 90^\circ \) and \( k = 2 \).

Amplitude:

\[
|a| = 1
\]

Period:

\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{\frac{1}{2}} = 720^\circ
\]

Phase shift: \( h = 90^\circ \)

Vertical shift: \( k = 2 \)

28. \( y = \tan (\theta + 30^\circ) - 2 \)

**SOLUTION:**

Given \( a = 1, \ b = 1, \ h = -30^\circ \) and \( k = -2 \).

Amplitude: No amplitude

Period:

\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]

Phase shift: \( h = -30^\circ \)

Vertical shift: \( k = -2 \)
12-8 Translations of Trigonometric Graphs

Midline: \( y = -2 \)

First, graph the midline. Then graph
\[ y = \tan(\theta + 30\degree) - 2 \]
using the midline as reference. Then shift the graph 30° to the left.

ANSWER:
no amplitude; 180°; \( h = -30\degree \); \( k = -2 \)

ANSWER:

29. \( y = 2 \tan \left( \theta + \frac{\pi}{4} \right) - 5 \)

SOLUTION:
Given \( a = 2 \), \( b = 2 \), \( h = \frac{\pi}{4} \) and \( k = -5 \).

Amplitude: No amplitude

Period:

\[ \frac{\pi}{|b|} = \frac{\pi}{2} \]

Phase shift: \( h = -\frac{\pi}{4} \)

Vertical shift: \( k = -5 \)

Midline: \( y = -5 \)

First, graph the midline. Then graph
\[ y = 2 \tan 2(\theta) - 5 \]
using the midline as reference. Then shift the graph \( \frac{\pi}{4} \) units to the left.

\[ y = 2 \tan 2\left( \theta + \frac{\pi}{4} \right) - 5 \]

ANSWER:
no amplitude; \( \frac{\pi}{2} \); \( h = -\frac{\pi}{4} \); \( k = -5 \)

30. \( y = \frac{1}{2} \sin \left( \theta - \frac{\pi}{2} \right) + 4 \)
12-8 Translations of Trigonometric Graphs

**SOLUTION:**

Given \( a = \frac{1}{2}, b = 1, h = \frac{\pi}{2} \) and \( k = 4 \).

Amplitude:
\[
|a| = \frac{1}{2} \Rightarrow \frac{1}{2}
\]

Period:
\[
\frac{2\pi}{|b|} = \frac{2\pi}{1} \Rightarrow 2\pi
\]

Phase shift: \( h = \frac{\pi}{2} \)

Vertical shift: \( k = 4 \)

Midline: \( y = 4 \)

First, graph the midline. Then graph 
\[
y = \frac{1}{2} \sin \theta + 4
\]
using the midline as reference. Then shift the graph \( \frac{\pi}{2} \) units to the right.

**ANSWER:**
\[
\frac{1}{2}; 2\pi; h = \frac{\pi}{2}; k = 4
\]

31. \( y = \cos \left( \theta - 45^\circ \right) + \frac{1}{2} \)

**SOLUTION:**

Given \( a = 1, b = 3, h = 45^\circ \) and \( k = \frac{1}{2} \).

Amplitude:
\[
|a| = |1| \Rightarrow 1
\]

Period:
\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{3} \Rightarrow 120^\circ
\]

Phase shift: \( h = 45^\circ \)

Vertical shift: \( k = \frac{1}{2} \)

Midline: \( y = \frac{1}{2} \)

First, graph the midline. Then graph 
\[
y = \cos \left( \theta \right) + \frac{1}{2}
\]
using the midline as reference. Then shift the graph \( 45^\circ \) to the right.
12-8 Translations of Trigonometric Graphs

\[ y = \cos 3(\theta - 45^\circ) + \frac{1}{2} \]

\[ y = 3 + 5 \sin 2(\theta - \pi) \]

ANSWER:

1; 120°; \( h = 45^\circ \); \( k = \frac{1}{2} \)

ANSWER:

5; \( \pi \); \( h = \pi \); \( k = 3 \)

32. \( y = 3 + 5 \sin 2(\theta - \pi) \)

SOLUTION:

Given \( a = 5 \), \( b = 2 \), \( h = \pi \) and \( k = 3 \).

Amplitude:

\[ |a| = |5| \]

\[ = 5 \]

Period:

\[ \frac{2\pi}{|b|} = \frac{2\pi}{|2|} \]

\[ = \pi \]

Phase shift: \( h = \pi \)

Vertical shift: \( k = 3 \)

Midline: \( y = 3 \)

First, graph the midline. Then graph

\[ y = 3 + 5 \sin 2\theta \]

using the midline as reference. Then

shift the graph \( \pi \) units to the right.

33. \( y = -2 + 3 \sin \frac{1}{3}(\theta - \frac{\pi}{2}) \)

SOLUTION:

Given \( a = 3 \), \( b = \frac{1}{3} \), \( h = \frac{\pi}{2} \) and \( k = -2 \).

Amplitude:

\[ |a| = |3| \]

\[ = 3 \]

Period:

\[ \frac{2\pi}{|b|} = \frac{2\pi}{|\frac{1}{3}|} \]

\[ = 6\pi \]

Phase shift: \( h = \frac{\pi}{2} \)

Vertical shift: \( k = -2 \)
12-8 Translations of Trigonometric Graphs

Midline: \( y = -2 \)

First, graph the midline. Then graph
\[ y = 3 \sin \frac{1}{3} \left( \theta - \frac{\pi}{2} \right) \]

Then shift the graph \( \frac{\pi}{2} \) units to the right.

ANSWER:
\[ 3; 6\pi; h = \frac{\pi}{2}; k = -2 \]

34. TIDES The height of the water in a harbor rose to a maximum height of 15 feet at 6:00 p.m. and then dropped to a minimum level of 3 feet by 3:00 a.m. The water level can be modeled by the sine function. Write an equation that represents the height \( h \) of the water \( t \) hours after noon on the first day.

SOLUTION:
The maximum and the minimum height is 15 ft and 3 ft respectively.
Therefore, the amplitude is \( \left| \frac{15 - 3}{2} \right| \) or 6.

The time taken for half cycle is 9 hrs. Therefore, the period is 18 hrs.
Find the value of \( b \).
\[ 18 = \frac{2\pi}{|b|} \]
\[ b = \pm \frac{\pi}{9} \]

Since the period of the function is 18 hrs, one fourth of the period is 4.5 hrs.
Therefore, the horizontal shift is 6 – 4.5 or 1.5.
That is, \( h = 1.5 \).

The vertical shift is \( \frac{15 + 3}{2} \) or 9.

That is \( k = 9 \).
Substitute the values of \( a, b, h \) and \( k \) in the standard equation of the sine function.
\[ h = 6 \sin \left[ \frac{\pi}{9} (t - 1.5) \right] + 9 \]

ANSWER:
\[ h = 9 + 6 \sin \left[ \frac{\pi}{9} (t - 1.5) \right] \]

35. LAKES A buoy marking the swimming area in a lake oscillates each time a speed boat goes by. Its distance \( d \) in feet from the bottom of the lake is given by \( d = 1.8 \sin \frac{3\pi}{4} t + 12 \), where \( t \) is the time in
The time taken for half cycle is 9 hrs. Therefore, the period is 18 hrs.

Find the value of $b$.

Since the function has shifted to the right by 45 units, the minimum value of 2. Therefore, the amplitude is

\[ |a| = |1.8| = 1.8 \]

Period:

\[ \frac{2\pi}{|b|} = \frac{2\pi}{3\pi/4} = \frac{8}{3} \]

Phase shift: No phase shift

Vertical shift: $k = 12$

Midline: $y = 12$

First, graph the midline. Then graph

\[ d = 1.8\sin \left(\frac{3\pi}{4}t + 12\right) \]

using the midline as reference.

Since the maximum value is the value of the midline plus the amplitude, the maximum distance is

\[ d = 1.8 + 12 \text{ or } 13.8 \]

Since the minimum value is the value of the midline minus the amplitude, the minimum distance is

\[ d = 12 - 1.8 \text{ or } 10.2 \]

ANSWER:

\[ d = 1.8\sin \left(\frac{3\pi}{4}t + 12\right) \]

min: 10.2 ft; max: 13.8 ft

36. **FERRIS WHEEL** Suppose a Ferris wheel has a diameter of approximately 520 feet and makes one complete revolution in 30 minutes. Suppose the lowest car on the Ferris wheel is 5 feet from the ground. Let the height at the top of the wheel represent the height at time 0. Write an equation for the height of a car $h$ as a function of time $t$. Then graph the function.

**SOLUTION:**

The midline lies halfway between the maximum and the minimum values $y = \frac{525 + 5}{2}$ or 265

Therefore the vertical shift is $k = 265$.

Amplitude:

\[ |a| = \left| \frac{525 - 5}{2} \right| = 260 \]

Period:

Since the wheel makes one complete revolution in 30 minutes, the period is 30 minutes.

\[ \frac{2\pi}{|b|} = \frac{2\pi}{30} \]

\[ |b| = \frac{3\pi}{15} \]

\[ b = \frac{\pi}{15} \]
12-8 Translations of Trigonometric Graphs

Substitute 260 for \( a \), \( \frac{\pi}{15} \) for \( b \), 265 for \( t \) in
\( h = a \sin b(t - h) + k \).

\[
\begin{align*}
  h &= 260 \sin \left( \frac{\pi}{15} (t - 0) \right) + 265 \\
  h &= 260 \sin \left( \frac{\pi}{15} t \right) + 265
\end{align*}
\]

Graph the function.

Write an equation for each translation.

37. \( y = \sin x \), 4 units to the right and 3 units up

\[\text{SOLUTION:}\]
The sine function involving phase shifts and vertical shifts is \( y = a \sin b(x - h) + k \).

Given \( a = 1 \), \( b = 1 \), \( h = 4 \), \( k = 3 \).

Therefore, the equation is \( y = \sin(x - 4) + 3 \).

\[\text{\textbf{ANSWER:}}\]
\( y = \sin (x - 4) + 3 \)

38. \( y = \cos x \), 5 units to the left and 2 units down

\[\text{SOLUTION:}\]
The cosine function involving phase shifts and vertical shifts is \( y = a \cos b(x - h) + k \).

Given \( a = 1 \), \( b = 1 \), \( h = -5 \), \( k = -2 \).

Therefore, the equation is \( y = \cos(x + 5) - 2 \).

\[\text{\textbf{ANSWER:}}\]
\( y = \cos (x + 5) - 2 \)
12-8 Translations of Trigonometric Graphs

39. \( y = \tan x, \pi \) units to the right and 2.5 units up

**SOLUTION:**
The tangent function involving phase shifts and vertical shifts is
\[ y = a \tan b(x - h) + k. \]

Given \( a = 1, b = 1, h = \pi, k = 2.5. \)

Therefore, the equation is \( y = \tan(x - \pi) + 2.5 \).

**ANSWER:**
\[ y = \tan(x - \pi) + 2.5 \]

40. **JUMP ROPE** The graph approximates the height of a jump rope \( h \) in inches as a function of time \( t \) in seconds. A maximum point on the graph is (1.25, 68), and a minimum point is (2.75, 2).

\[
y = \frac{68 + 2}{2} = 35
\]

Therefore the vertical shift is \( k = 35 \).

Midline: \( y = 35 \)

Amplitude:
\[
|a| = \left| \frac{68 - 2}{2} \right| = 33
\]

The graph completes 1.5 cycles in 1.5 seconds (between 1.25 and 2.75).

Therefore, period is 1

c. Find the value of \( b \).

Period = \( \frac{2\pi}{|b|} \)
\[
1 = \frac{2\pi}{|b|} \]
\[
b = \pm 2\pi
\]

Substitute 33 for \( a, 2\pi \) for \( b \), 35 for \( k \) in

\[ h = a \sin b(t - h) + k. \]

\[
h = 33 \sin (t - 0) + 35
\]

\[
h = 33 \sin 2\pi t + 35
\]

**ANSWER:**
a. At 1.25 seconds, the height of the rope is 68 inches; at 2.75 seconds, the height of the rope is 2 inches.

\[
y = 35; 33, 1
\]

\[
h = 33 \sin 2\pi t + 35
\]

41. **CAROUSEL** A horse on a carousel goes up and down 3 times as the carousel makes one complete rotation. The maximum height of the horse is 55 inches, and the minimum height is 37 inches. The carousel rotates once every 21 seconds. Assume that the horse starts and stops at its median height.

\[
y = \tan x, \pi \] units to the right and 2.5 units up

**SOLUTION:**
The tangent function involving phase shifts and vertical shifts is
\[ y = a \tan b(x - h) + k. \]

Given \( a = 1, b = 1, h = \pi, k = 2.5. \)

Therefore, the equation is \( y = \tan(x - \pi) + 2.5 \).

**ANSWER:**
\[ y = \tan(x - \pi) + 2.5 \]
12-8 Translations of Trigonometric Graphs

a. Write an equation to represent the height of the horse \( h \) as a function of time \( t \) seconds.

\[ h = 9 \sin \left( \frac{2\pi}{7} (t-0) \right) + 46 \]

b. Graph the function.

c. Use your graph to estimate the height of the horse after 8 seconds. Then use a calculator to find the height to the nearest tenth.

**SOLUTION:**
a. Amplitude:
\[ |a| = |55 - 46| = 9 \]

Since the carousel rotates once every 21 seconds, and a horse on the carousel goes up and down three times in one rotation, the time taken for the horse to go up and down once is 7 seconds. So, the period is 7 seconds.

Find the value of \( b \).

\[ 7 = \frac{2\pi}{|b|} \]

\[ |b| = \frac{2\pi}{7} \]

\[ b = \frac{2\pi}{7} \]

The midline lies halfway between the maximum and the minimum values.

\[ y = \frac{55 + 37}{2} = 46 \]

Therefore the vertical shift is \( k = 46 \).

Midline: \( y = 46 \)

Substitute 9 for \( a \), \( \frac{2\pi}{7} \) for \( b \), 0 for \( h \), and 46 for \( k \) in

\[ h = a \sin b(t-h) + k \]

b. Graph the function.

c. Sample answer:
Substitute 8 for \( t \) to find the height.

\[ h = 9 \sin \left( \frac{2\pi}{7} (8) \right) + 46 \]

\[ = 9 \sin \left( \frac{16\pi}{7} \right) + 46 \]

\[ \approx 7.03 + 46 \]

\[ \approx 53.0 \]

Therefore the height of the horse after 8 seconds is about 53 inches.

**ANSWER:**
a. \( h = 9 \sin \left( \frac{2\pi}{7} t \right) + 46 \)

b. Graph the function.
42. **CCSS REASONING** During one month, the outside temperature fluctuates between 40°F and 50°F. A cosine curve approximates the change in temperature, with a high of 50°F being reached every four days.

**a.** Describe the amplitude, period, and midline of the function that approximates the temperature \( y \) on day \( d \).

**b.** Write a cosine function to estimate the temperature \( y \) on day \( d \).

**c.** Sketch a graph of the function.

**d.** Estimate the temperature on the 7th day of the month.

**SOLUTION:**

**a.** Amplitude:

\[
|a| = \frac{|50 - 40|}{2} = 5
\]

Since the change in temperature with a high of 50°F being reached every four days, the period is 4. The midline lies halfway between the maximum and the minimum values.

\[
y = \frac{50 + 40}{2} = 45
\]

Therefore the vertical shift is \( k = 45 \).

**Midline:** \( y = 45 \)

**b.** Find the value of \( b \).

\[
4 = \frac{2\pi}{|b|}
\]

\[
|b| = \frac{2\pi}{4}
\]

\[
b = \frac{\pi}{2}
\]

Write an equation for the function.

\[ h = a \cos b(d - h) + k \]

Substitute 5 for \( a \), \( \frac{\pi}{2} \) for \( b \), 0 for \( h \), and 45 for \( k \).

\[ h = 5 \cos \frac{\pi}{2}(d - 0) + 45 \]

\[ h = 5 \cos \frac{\pi}{2}d + 45 \]

**c.** Graph the function.

**d.** Substitute 7 for \( d \) to find the temperature.
Find a coordinate that represents a maximum for each graph.

43. \( y = -2 \cos \left( x - \frac{\pi}{2} \right) \)

**SOLUTION:**
Sample answer:
\[
\begin{align*}
y &= -2 \cos \left( x - \frac{\pi}{2} \right) \\
\frac{-y}{2} &= \cos \left( x - \frac{\pi}{2} \right)
\end{align*}
\]

The range of \( \cos \left( x - \frac{\pi}{2} \right) \) is \(-1 \leq \frac{-y}{2} \leq 1\).

\[
\begin{align*}
-1 &\leq \frac{-y}{2} \\
-2 &\leq -y \\
2 &\geq y \geq -2
\end{align*}
\]

Substitute 2 for \( y \) and solve for \( x \).

\[
\begin{align*}
2 &= -2 \cos \left( x - \frac{\pi}{2} \right) \\
-1 &= \cos \left( x - \frac{\pi}{2} \right) \\
x - \frac{\pi}{2} &= \cos^{-1} (-1) \\
x &= \pi + \frac{\pi}{2} \text{ or } 3\pi \div 2
\end{align*}
\]

The coordinate of the maximum point is \( \left( \frac{3\pi}{2}, 2 \right) \).

**ANSWER:**
Sample answer: \( \left( \frac{3\pi}{2}, 2 \right) \)
12-8 Translations of Trigonometric Graphs

44. \[ y = 4 \sin \left( x + \frac{\pi}{3} \right) \]

**SOLUTION:**
Sample answer:
\[ y = 4 \sin \left( x + \frac{\pi}{3} \right) \]
\[ \frac{y}{4} = \sin \left( x + \frac{\pi}{3} \right) \]

The range of \( \sin \left( x + \frac{\pi}{3} \right) \) is \(-1 \leq \frac{y}{4} \leq 1\).

\[-1 \leq \frac{y}{4} \leq 1\]
\[-4 \leq y \leq 4\]

Substitute 4 for \( y \) and solve for \( x \).
\[ 4 = 4 \sin \left( x + \frac{\pi}{3} \right) \]
\[ 1 = \sin \left( x + \frac{\pi}{3} \right) \]
\[ x + \frac{\pi}{3} = \sin^{-1} (1) \]
\[ x = \sin^{-1} (1) - \frac{\pi}{3} \]
\[ x = \frac{\pi}{2} - \frac{\pi}{3} \text{ or } \frac{\pi}{6} \]

The coordinate of the maximum point is \( \left( \frac{\pi}{6}, 4 \right) \).

**ANSWER:**
Sample answer: \( \left( \frac{\pi}{6}, 4 \right) \)

45. \[ y = 3 \tan \left( x + \frac{\pi}{2} \right) + 2 \]

**SOLUTION:**
Since the amplitude is undefined for the tangent functions, there is no maximum value for \( y = 3 \tan \left( x + \frac{\pi}{2} \right) + 2 \).

**ANSWER:**
no maximum values
12-8 Translations of Trigonometric Graphs

46. \( y = -3\sin\left(x - \frac{\pi}{4}\right) - 4 \)

**SOLUTION:**
Sample answer:
\[ y = -3\sin\left(x - \frac{\pi}{4}\right) - 4 \]
\[ -\frac{y + 4}{3} = \sin\left(x - \frac{\pi}{4}\right) \]

The range of \( \sin\left(x - \frac{\pi}{4}\right) \) is \(-1 \leq -\frac{y + 4}{3} \leq 1\).

\[-1 \leq -\frac{y + 4}{3} \leq 1 \]
\[-3 \leq -y + 4 \leq 3 \]
\[1 \leq -y \leq 7 \]
\[-7 \leq y \leq -1 \]

Substitute \(-1\) for \( y \) and solve for \( x \).

\[-1 = -3\sin\left(x - \frac{\pi}{4}\right) - 4 \]
\[3 = -3\sin\left(x - \frac{\pi}{4}\right) \]
\[x - \frac{\pi}{4} = \sin^{-1}(-1) \]
\[x = \sin^{-1}(-1) + \frac{\pi}{4} \]
\[= \frac{3\pi}{2} + \frac{\pi}{4} \text{ or } \frac{7\pi}{4} \]

The coordinate of the maximum point is \( \left(\frac{7\pi}{4}, -1\right) \).

**ANSWER:**
Sample answer: \( \left(\frac{7\pi}{4}, -1\right) \)

**Compare each pair of graphs.**

47. \( y = -\cos 3\theta \) and \( y = \sin 3(\theta - 90^\circ) \)

**SOLUTION:**
Given \( a = -1 \) and \( b = 3 \).

Amplitude:
\[ |a| = |1| = 1 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{3} = 120^\circ \]

Draw the graph of \( y = -\cos 3\theta \).

Given \( a = -1, \ b = 3 \) and \( h = 90^\circ \).

Amplitude:
\[ |a| = |1| = 1 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{3} = 120^\circ \]

Phase shift:
\( h = 90^\circ \)

Graph \( y = \sin 3\theta \) shifted \( 90^\circ \) units to the right.
The graphs are reflections of each other over the x-axis.

**ANSWER:**
The graphs are reflections of each other over the x-axis.

48. $y = 2 + 0.5 \tan \theta$ and $y = 2 + 0.5 \tan (\theta + \pi)$

**SOLUTION:**
Given $a = 0.5$, $b = 1$, $h = 0$ and $k = 2$.
Amplitude: No amplitude

Period:
\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]

Vertical shift: $k = 2$

Midline: $y = 2$

To graph $y = 2 + 0.5 \tan \theta$, first draw the midline. Then use it to graph $y = 0.5 \tan \theta$ shifted 2 units up.

Given $a = 0.5$, $b = 1$, $h = \pi$ and $k = 2$.
Amplitude: No amplitude

Period:
\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]

Phase shift: $h = -\pi$

Vertical shift: $k = 2$

Midline: $y = 2$

First, graph the midline. Then graph $y = 2 + 0.5 \tan (\theta + \pi)$ using the midline as reference. Then shift the graph $\pi$ units to the left.

Therefore, the graphs are identical.

**ANSWER:**
The graphs are identical.
The graphs are identical.

**ANSWER:**

The graphs are identical.

![Graph Comparison](image)

**Graph:** $y = 2 \sin \theta$ shifted $\frac{9}{\pi}$ units to the left.

**Phase shift:** $y = 2 \sin \left( \theta + \frac{9}{\pi} \right)$

**Given:** $h = 0$, $k = 9$, $\omega = \frac{9}{\pi}$, and $a = 0$.

**Period:**

$$P = \frac{2\pi}{|\omega|} = \frac{2\pi}{\frac{9}{\pi}} = \frac{2\pi \times \pi}{9}$$

**Amplitude:**

$$A = |h| = |0| = 0$$

**Solution:**

$$\left( \frac{9}{\pi} + \theta \right) \sin \left( \theta + \frac{9}{\pi} \right) = 2 \sin \left( \theta + \frac{9}{\pi} \right)$$
12-8 Translations of Trigonometric Graphs

Identify the period of each function. Then write an equation for the graph using the given trigonometric function.

50. sine

\[ \text{SOLUTION:} \]
Period: \(360^\circ\)

The function gets the maximum value of \(-4\) and the minimum value of \(-6\). Therefore, the amplitude is \( \left| \frac{-6 - (-4)}{2} \right| = 1 \) and the midline is at \( y = \frac{-6 - 4}{2} = -5 \).

Since the period is \(360^\circ\), \( b = \frac{360^\circ}{360^\circ} = 1 \).

Since the midline is \( y = -5 \), the graph is shifted vertically down by 5 units. That is, \( k = -5 \).

Therefore the equation is \( y = \sin \theta - 5 \).

\[ \text{ANSWER:} \]
\(360^\circ\); Sample answer: \( y = \sin \theta - 5 \)

51. cosine

\[ \text{SOLUTION:} \]
Period: \(360^\circ\)

The function gets the maximum value of 2 and the minimum value of \(-2\). Therefore, the amplitude is \( \left| \frac{-2 - 2}{2} \right| = 1 \) or 2 and the midline is at \( y = \frac{-2 + 2}{2} = 0 \).

Since the period is \(360^\circ\), \( b = \frac{360^\circ}{360^\circ} = 1 \).

Since the function has shifted to the left by 90 units, \( h = -90^\circ \).

Since the midline is \( y = 0 \), there is no vertical shift. That is \( k = 0 \).

Therefore the equation is \( y = 2 \cos(\theta + 90^\circ) \).

\[ \text{ANSWER:} \]
\(360^\circ\); Sample answer: \( y = 2 \cos(\theta + 90^\circ) \)
The time taken for half cycle is 9 hrs. Therefore, the period is 18 hrs. Find the value of b. Since the midline is 1, graph is shifted vertically up by 1 unit. That is k = 1. Therefore the equation is $y = 4 \cos (\theta) + 1$.

**ANSWER:**
360°; Sample answer: $y = 4 \cos (\theta) + 1$

---

53. sine

**SOLUTION:**
Period: 180°

The function gets the maximum value of 4 and the minimum value of 2. Therefore, the amplitude is $\frac{4 - 2}{2}$ or 1 and the midline is at $y = \frac{4 + 2}{2}$ or 3.

Since the period is 180°, $b = \frac{360°}{180°}$ or 2.

Since the function has shifted to the right by 45 units, $h = 45°$.

Since the midline is $y = 3$, graph is shifted vertically up by 1 unit. That is $k = 3$.

Therefore the equation is $y = \sin 2(\theta - 45°) + 3$.

**ANSWER:**
180°; Sample answer: $y = \sin 2(\theta - 45°) + 3$
12-8 Translations of Trigonometric Graphs

State the period, phase shift, and vertical shift. Then graph the function.

54. \( y = \csc (\theta + \pi) \)

**SOLUTION:**
Given \( a = 1, b = 1 \) and \( h = -\pi \).

Period:
\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} = 360^\circ
\]
Phase shift: \( h = -\pi \)
Vertical shift: No vertical shift

To graph \( y = \csc (\theta + \pi) \), shift the graph of \( y = \csc (\theta) \) to the left by \( \pi \) units.

**ANSWER:**
360°; \( h = -\pi \); no vertical shift

55. \( y = \cot \theta + 6 \)

**SOLUTION:**
Given \( a = 1, b = 1 \) and \( k = 6 \).

Period:
\[
\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|} = 180^\circ
\]
Phase shift: No phase shift.
Vertical shift: \( k = 6 \)
Midline: \( y = 6 \)

To graph \( y = \csc (\theta) + 6 \), first draw the midline. Then use it to graph \( y = \cot \theta \) shifted 6 units up.

**ANSWER:**
180°; no phase shift; \( k = 6 \)
12-8 Translations of Trigonometric Graphs

56. \( y = \cot \left( \theta - \frac{\pi}{6} \right) - 2 \)

**SOLUTION:**

Given \( a = 1, b = 1, h = \frac{\pi}{6} \) and \( k = -2 \).

Period:

\[
\frac{\pi}{|h|} = \frac{\pi}{|1|} = \pi
\]

Phase shift: \( h = \frac{\pi}{6} \)

Vertical shift: \( k = -2 \)

Midline: \( y = -2 \)

First, graph the midline. Then graph \( y = \cot(\theta) - 2 \) using the midline as reference. Then shift the graph \( \frac{\pi}{6} \) units to the right.

**ANSWER:**

\( \pi; \ h = \frac{\pi}{6}; \ k = -2 \)

57. \( y = \frac{1}{2} \csc 3(\theta - 45^\circ) + 1 \)

**SOLUTION:**

Given \( a = \frac{1}{2}, b = 3, h = 45^\circ \) and \( k = 1 \).

Period:

\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{|3|} = 120^\circ
\]

Phase shift: \( h = 45^\circ \)

Vertical shift: \( k = 1 \)

Midline: \( y = 1 \)

First, graph the midline. Then graph \( y = \frac{1}{2} \csc 3(\theta) + 1 \) using the midline as reference. Then shift the graph \( 45^\circ \) to the right.
12-8 Translations of Trigonometric Graphs

**ANSWER:**
120°; h = 45°; k = 1

**58.** \( y = 2 \csc \frac{1}{2}(\theta - 90°) \)

**SOLUTION:**
Given \( a = 2, b = \frac{1}{2} \) and \( h = 90° \).

Period:
\[
\frac{360°}{|b|} = \frac{360°}{\frac{1}{2}} = 720°
\]

Phase shift: \( h = 90° \)

Vertical shift: No vertical shift

Graph \( y = 2 \csc \frac{1}{2}(\theta) \). Then shift the graph 90° to the right.

**ANSWER:**
720°; \( h = 90° \); no vertical shift
59. \( y = 4 \sec 2\left(\theta + \frac{\pi}{2}\right) - 3 \)

**SOLUTION:**

Given \( a = 4, b = 2, h = \frac{-\pi}{2} \) and \( k = -3 \).

Period:
\[
\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi
\]

Phase shift: \( h = \frac{-\pi}{2} \)

Vertical shift: \( k = -3 \)

Midline: \( y = -3 \)

First, graph the midline. Then graph \( y = 4 \sec 2\theta - 3 \) using the midline as reference. Then shift the graph \( \frac{\pi}{2} \) to the left.

**ANSWER:**

\( \pi; \ h = \frac{-\pi}{2}; \ k = -3 \)

60. **CCSS ARGUMENTS** If you are given the amplitude and period of a cosine function, is it *sometimes, always, or never* possible to find the maximum and minimum values of the function? Explain your reasoning.

**SOLUTION:**

Sometimes, if the function is shifted vertically, then you also need to know the value of the midline. The maximum value is the value of the midline plus the amplitude. The minimum value is the midline value minus the amplitude.

**ANSWER:**

Sometimes, if the function is shifted vertically, then you also need to know the value of the midline. The maximum value is the value of the midline plus the amplitude. The minimum value is the midline value minus the amplitude.

61. **REASONING** Describe how the graph of \( y = 3 \sin 2\theta + 1 \) is different from \( y = \sin \theta \).

**SOLUTION:**

The graph of \( y = 3 \sin 2\theta + 1 \) has an amplitude of 3 rather than an amplitude of 1. It is shifted up 1 unit from the parent graph and is compressed so that it has a period of 180°.

**ANSWER:**

The graph of \( y = 3 \sin 2\theta + 1 \) has an amplitude of 3 rather than an amplitude of 1. It is shifted up 1 unit from the parent graph and is compressed so that it has a period of 180°.
12-8 Translations of Trigonometric Graphs

62. **WRITING IN MATH** Describe two different phase shifts that will translate the sine curve onto the cosine curve shown at the right. Then write an equation for the new sine curve using each phase shift.

![Graph of sine and cosine functions](image)

**SOLUTION:**
When \( y = \sin \theta \) is shifted 90° to the left, the sine curve will be translated onto a cosine curve.
Phase shift: \( h = -90° \)

Therefore, the equation of the curve is
\[
y = \sin \left( \theta - (-90°) \right) \quad \text{or} \quad \sin \left( \theta + 90° \right)
\]

Also, when \( y = \sin \theta \) is shifted 270° to the right, the sine curve will be translated onto a cosine curve.
Phase shift: \( h = 270° \)

Therefore, the equation of the curve is
\[
y = \sin \left( \theta - 270° \right)
\]

**ANSWER:**
Sample answer: a phase shift 90° left, \( y = \sin (\theta + 90°) \); a phase shift 270° right, \( y = \sin (\theta - 270°) \)

63. **OPEN ENDED** Write a periodic function that has an amplitude of 2 and midline at \( y = -3 \). Then graph the function.

**SOLUTION:**
Sample answer:
The sine function involving phase shifts and vertical shifts is \( y = a \sin b(\theta - h) + k \).
Here \( a = 2, b = 1, h = 0, k = -3 \).

Therefore the equation is \( y = 2 \sin \theta - 3 \).

**Amplitude:**
\[
|a| = |2| = 1
\]

**Period:**
\[
|b| = \frac{360°}{|1|} = 360°
\]

**Vertical shift:** \( k = -3 \)

**Midline:** \( y = -3 \)

To graph \( y = 2 \sin \theta - 3 \), first draw the midline. Then use it to graph \( y = 2 \sin \theta \) shifted 3 units down.
12-8 Translations of Trigonometric Graphs

64. **REASONING** How many different sine graphs pass through the origin \((n\pi, 0)\)? Explain your reasoning.

**SOLUTION:**
Infinitely many; any change in amplitude will create a different graph that has the same \(\theta\)-intercepts.

**ANSWER:**
Sample answer: Infinitely many; any change in amplitude will create a different graph that has the same \(\theta\)-intercepts.

65. **GRIDDED RESPONSE** The expression \(\frac{3x - 1}{4} + \frac{x + 6}{4}\) is how much greater than \(x\)?

**SOLUTION:**
Let \(y = \frac{3x - 1}{4} + \frac{x + 6}{4}\).

\[
y = \frac{3x - 1}{4} + \frac{x + 6}{4}
= \frac{4x + 5}{4}
= \frac{5}{4} + \frac{x}{4}
= x + 1.25
\]

Therefore, \(\frac{3x - 1}{4} + \frac{x + 6}{4}\) is 1.25 greater than \(x\).

**ANSWER:**
1.25

66. Expand \((a - b)^4\).

A \(a^4 - b^4\)

B \(a^4 - 4ab + b^4\)

C \(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\)

D \(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\)

**SOLUTION:**
\[
(a - b)^4 = \frac{4!}{0!(4 - 0)!} a^4b^0 - \frac{4!}{1!(4 - 1)!} a^3b^1 + \ldots - \frac{4!}{4!(4 - 4)!} a^0b^4
= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4
\]

Therefore, the correct option is D.

**ANSWER:**
D
12-8 Translations of Trigonometric Graphs

67. Solve \( \sqrt{x - 3} + \sqrt{x + 2} = 5 \).

   F 7
   G 0.7
   H 7, 13
   J no solution

   \textit{SOLUTION:}
   \[
   \sqrt{x - 3} + \sqrt{x + 2} = 5
   \]
   \[
   \sqrt{x - 3} = 5 - \sqrt{x + 2}
   \]
   Square on both the sides.
   \[
   x - 3 = 25 + x + 2 - 10\sqrt{x + 2}
   \]
   \[
   -3 = 27 - 10\sqrt{x + 2}
   \]
   \[
   -30 = -10\sqrt{x + 2}
   \]
   \[
   -3 = \sqrt{x + 2}
   \]
   Again, square on both the sides.
   \[
   (-3)^2 = (\sqrt{x + 2})^2
   \]
   \[
   9 = x + 2
   \]
   \[
   x = 7
   \]
   Therefore, the option is F.

   \textit{ANSWER:}
   F

68. \textbf{GEOMETRY} Using the figures below, what is the average of \( a, b, c, d, \) and \( f \)?

   \[
   \begin{align*}
   a^\circ & \quad b^\circ \\
   c^\circ & \quad f^\circ
   \end{align*}
   \]

   A 21
   B 45
   C 50
   D 54

   \textit{SOLUTION:}
   Since the sum of the measures of the triangle is \( 180^\circ \), we have \( a^\circ + b^\circ + c^\circ = 180^\circ \) and \( 90^\circ + f^\circ + d^\circ = 180^\circ \) or \( f^\circ + d^\circ = 90^\circ \)
   
   The average
   \[
   \frac{a^\circ + b^\circ + c^\circ + d^\circ + f^\circ}{5}
   \]
   \[
   = \frac{180^\circ + 90^\circ}{5}
   \]
   \[
   = \frac{270^\circ}{5}
   \]
   \[
   = 54^\circ
   \]
   Therefore, the correct option is D.

   \textit{ANSWER:}
   D
Find the amplitude and period of each function. Then graph the function.

69. \( y = 2 \cos \theta \)

**SOLUTION:**
Given \( a = 2 \) and \( b = 1 \)

Amplitude:
\[ |a| = 2 \]
\[ = 2 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{1} \]
\[ = 360^\circ \]

Graph the function \( y = 2 \cos \theta \).

**ANSWER:**
amplitude: 2; period: 360°

---

70. \( y = 3 \sin \theta \)

**SOLUTION:**
Given \( a = 3 \) and \( b = 1 \)

Amplitude:
\[ |a| = 3 \]
\[ = 3 \]

Period:
\[ \frac{360^\circ}{|b|} = \frac{360^\circ}{1} \]
\[ = 360^\circ \]

Graph the function \( y = 3 \sin \theta \).

**ANSWER:**
amplitude: 3; period: 360°
12-8 Translations of Trigonometric Graphs

71. $y = \sin 2\theta$

**SOLUTION:**
Given $a = 1$ and $b = 2$

Amplitude:
$|a| = 1$

Period:
$\frac{360^\circ}{|b|} = \frac{360^\circ}{2} = 180^\circ$

Graph the function $y = \sin 2\theta$.

**ANSWER:**
amplitude: 1; period: 180°

---

Find the exact value of each expression.

72. $\sin \frac{4\pi}{3}$

**SOLUTION:**
The terminal side of $\frac{4\pi}{3}$ lies in Quadrant III.

Find the measure of the reference angle.
$\theta = \theta - \pi$

$= \frac{4\pi}{3} - \pi$

$= \frac{\pi}{3}$

The sine function is negative in quadrant III.

$\sin \frac{4\pi}{3} = -\sin \left( \frac{\pi}{3} \right)$

$= -\frac{\sqrt{3}}{2}$

**ANSWER:**
$-\frac{\sqrt{3}}{2}$

---

73. $\sin(-30^\circ)$

**SOLUTION:**

$\sin(-30^\circ) = -\sin(30^\circ)$

$= -\frac{1}{2}$

**ANSWER:**
$-\frac{1}{2}$
12-8 Translations of Trigonometric Graphs

74. \( \cos 405^\circ \)

\[ \text{SOLUTION:} \]
\[
\cos(405^\circ) = \cos(45^\circ + 360^\circ) \\
= \cos(45^\circ) \\
= \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}
\]

\[ \text{ANSWER:} \]
\[ \frac{\sqrt{2}}{2} \]

Determine whether each situation describes a survey, an experiment, or an observational study. Then identify the sample, and suggest a population from which it may have been selected.

75. A group of 220 adults is randomly split into two groups. One group exercises for an hour a day and the other group does not. The body mass indexes are then compared.

\[ \text{SOLUTION:} \]
experiment; sample: people that exercise for an hour a day; population: all adults

\[ \text{ANSWER:} \]
experiment; sample: people that exercise for an hour a day; population: all adults

76. A soccer coach randomly selects some of his players and gives them a questionnaire asking about their daily sleeping habits.

\[ \text{SOLUTION:} \]
survey; sample: players that received the questionnaire; population: all soccer players

\[ \text{ANSWER:} \]
survey; sample: players that received the questionnaire; population: all soccer players

77. A teacher randomly selects 100 students who have part-time jobs and compares their grades.

\[ \text{SOLUTION:} \]
observational study; sample: 100 students selected; population: all students that have part-time jobs

\[ \text{ANSWER:} \]
observational study; sample: 100 students selected; population: all students that have part-time jobs

78. \textbf{GEOMETRY} Equilateral triangle \( ABC \) has a perimeter of 39 centimeters. If the midpoints of the sides are connected, a smaller equilateral triangle results. Suppose the process of connecting midpoints of sides and drawing new triangles is continued indefinitely.

\[ \text{a. Write an infinite geometric series to represent the sum of the perimeters of all of the triangles.} \]

\[ \text{b. Find the sum of the perimeters of all of the triangles.} \]

\[ \text{SOLUTION:} \]
\[ \text{a. The perimeter of the equilateral triangle } ABC \text{ is } 39 \text{ centimeters. Since an equilateral triangle is formed by joining the midpoints, the perimeter of that triangle will be half of the equilateral triangle } ABC \text{. If the process is repeated again and again, then the perimeter of each equilateral triangle will be half the perimeter of the previous one. Therefore the sum of the perimeters of all of the triangles is } \frac{39}{2} + \frac{39}{4} + \frac{39}{8} + \ldots \]

\[ \text{b. Find the value of } r. \]
12-8 Translations of Trigonometric Graphs

\[ r = \frac{19.5}{39} \]
\[ = 0.5 \]

Since \( |0.5| < 1 \), the sum exists.

Find the sum.

\[ s = \frac{a_1}{1 - r} \]

Substitute 39 for \( a_1 \), and 0.5 for \( r \).

\[ s = \frac{39}{1 - 0.5} \]
\[ = \frac{39}{0.5} \]
\[ = 78 \]

Therefore, the sum of the perimeters of all the triangles is 78 cm.

**ANSWER:**

a. \( 39 + 19.5 + 9.75 + \ldots \)

b. 78 cm

79. **CONSTRUCTION** A construction company will be fined for each day it is late completing a bridge. The daily fine will be $4000 for the first day and will increase by $1000 each day. Based on its budget, the company can only afford $60,000 in total fines. What is the maximum number of days it can be late?

**SOLUTION:**

The fine is $4000 for the first day, $5000 for the second day, $6000 for the third day and so on. Therefore the arithmetic sequence is

\[ 4000 + 5000 + 6000 + \ldots \]

\[ a_1 = 4000 \]
\[ d = 1000 \]

To find \( n \):

\[ S_n = \frac{n}{2} (2a_1 + (n-1)d) \]

Substitute 60,000 for \( S_n \), 4000 for \( a_1 \), and 1000 for \( d \).

\[ 60,000 = \frac{n}{2} (2(4000) + (n-1)1000) \]
\[ = \frac{n}{2} (7000 + 1000n) \]
\[ 60 = \frac{n}{2} (7 + n) \]

\[ n^2 + 7n - 120 = 0 \]
\[ (n - 8)(n + 15) = 0 \]

\[ n = 8 \text{ or } n = -15 \]

Since the number of days cannot be negative, the number of days it can delay is 8 days.

**ANSWER:**

8 days
12-8 Translations of Trigonometric Graphs

Find each value of $\theta$. Round to the nearest degree.

80. $\sin \theta = \frac{7}{8}$

**SOLUTION:**

\[
\sin \theta = \frac{7}{8} \\
\theta = \sin^{-1} \left( \frac{7}{8} \right)
\]

Use a calculator.

Keystrokes: 2nd [SIN^{-1}] 7 ÷ 8 ) ENTER

Output: 61.04497563

Therefore, $\theta \approx 61^\circ$.

**ANSWER:**

61°

81. $\tan \theta = \frac{9}{10}$

**SOLUTION:**

\[
\tan \theta = \frac{9}{10} \\
\theta = \tan^{-1} \left( \frac{9}{10} \right)
\]

Use a calculator.

Keystrokes: 2nd [TAN^{-1}] 9 ÷ 10 ) ENTER

Output: 41.9872125

Therefore, $\theta \approx 42^\circ$.

**ANSWER:**

42°

82. $\cos \theta = \frac{1}{4}$

**SOLUTION:**

\[
\cos \theta = \frac{1}{4} \\
\theta = \cos^{-1} \left( \frac{1}{4} \right)
\]

Use a calculator.

Keystrokes: 2nd [COS^{-1}] 1 ÷ 4 ) ENTER

Output: 75.52248781

Therefore, $\theta \approx 76^\circ$.

**ANSWER:**

76°
12-8 Translations of Trigonometric Graphs

83. \( \cos \theta = \frac{4}{5} \)

**SOLUTION:**

\[
\theta = \cos^{-1} \left( \frac{4}{5} \right)
\]

Use a calculator.

Keystrokes: 2nd \([\text{COS}^{-1}]\) 4 ÷ 5 Enter

Output: 36.86989765

Therefore, \( \theta \approx 37^\circ \).

**ANSWER:**

37°

84. \( \sin \theta = \frac{5}{6} \)

**SOLUTION:**

\[
\theta = \sin^{-1} \left( \frac{5}{6} \right)
\]

Use a calculator.

Keystrokes: 2nd \([\text{SIN}^{-1}]\) 5 ÷ 6 Enter

Output: 56.44269024

Therefore, \( \theta \approx 56^\circ \).

**ANSWER:**

56°

85. \( \tan \theta = \frac{2}{7} \)

**SOLUTION:**

\[
\theta = \tan^{-1} \left( \frac{2}{7} \right)
\]

Use a calculator.

Keystrokes: 2nd \([\text{TAN}^{-1}]\) 2 ÷ 7 Enter

Output: 15.9453959

Therefore, \( \theta \approx 16^\circ \).

**ANSWER:**

16°
12-9 Inverse Trigonometric Functions

Find each value. Write angle measures in degrees and radians.

1. \( \sin^{-1} \frac{1}{2} \)

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd \([\sin^{-1}]\) \(1 \div 2 \) ENTER

Therefore, \( \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ \) or \( \frac{\pi}{6} \).

**ANSWER:**
\( 30^\circ; \frac{\pi}{6} \)

2. \( \arctan \left( -\sqrt{3} \right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd \([\tan^{-1}]\) \((-\sqrt{3}) \) ENTER \(-60 \)

Therefore, \( \arctan \left( -\sqrt{3} \right) = -60^\circ \) or \( -\frac{\pi}{3} \).

**ANSWER:**
\(-60^\circ; -\frac{\pi}{3} \)

3. \( \arccos(-1) \)

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd \([\cos^{-1}]\) \((-1)\) ENTER \(180 \)

Therefore, \( \arccos(-1) = 180^\circ \) or \( \pi \).

**ANSWER:**
\( 180^\circ; \pi \)

Find each value. Round to the nearest hundredth if necessary.

4. \( \cos \left( \arcsin \frac{4}{5} \right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: COS 2nd \([\sin^{-1}]\) \( \frac{4}{5} \) ENTER \(0.6 \)

Therefore, \( \cos \left( \arcsin \frac{4}{5} \right) = 0.6 \).

**ANSWER:**
0.6
12-9 Inverse Trigonometric Functions

5. \( \tan(\cos^{-1} 1) \)

**SOLUTION:**
Use a calculator.

Keystrokes: TAN 2nd [COS^-1] 1 ) )
ENTER 0

Therefore, \( \tan(\cos^{-1} 1) = 0 \).

**ANSWER:**
0

6. \( \sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: SIN 2nd [SIN^-1] 2nd 
\[ \sqrt{3} \) ÷ 2 ) ) ENTER
0.8660254038

Therefore, \( \sin\left(\sin^{-1} \frac{\sqrt{3}}{2}\right) \approx 0.87 \).

**ANSWER:**
0.87

7. **MULTIPLE CHOICE** If \( \sin \theta = 0.422 \), find \( \theta \).

A 25°
B 42°
C 48°
D 65°

**SOLUTION:**
\[ \sin \theta = 0.422 \]
\[ \theta = \arcsin(0.422) \]

Use a calculator.

Keystrokes: 2nd [SIN^-1] 0 . 4 2 2 )
ENTER 24.96092039

Therefore, \( \theta \approx 25° \).

The option A is the correct option.

**ANSWER:**
A
12-9 Inverse Trigonometric Functions

Solve each equation. Round to the nearest tenth if necessary.

8. \( \cos \theta = 0.9 \)

**SOLUTION:**
\[
\cos \theta = 0.9 \\
\theta = \arccos(0.9)
\]
Use a calculator.

Keystrokes: 2nd \([\cos^{-1}]\) 0.9 ENTER 25.84193276

Therefore, \( \theta \approx 25.8^\circ \).

**ANSWER:**
25.8°

9. \( \sin \theta = -0.46 \)

**SOLUTION:**
\[
\sin \theta = -0.46 \\
\theta = \arcsin(-0.46)
\]
Use a calculator.

Keystrokes: 2nd \([\sin^{-1}]\) (-) 0.46 ENTER -27.3871075

Therefore, \( \theta \approx -27.4^\circ \).

**ANSWER:**
-27.4°

10. \( \tan \theta = 2.1 \)

**SOLUTION:**
\[
\tan \theta = 2.1 \\
\theta = \arctan(2.1)
\]
Use a calculator.

Keystrokes: 2nd \([\tan^{-1}]\) 2.1 ENTER 64.53665494

Therefore, \( \theta \approx 64.5^\circ \).

**ANSWER:**
64.5°
11. **SNOWBOARDING** A cross section of a superpipe for snowboarders is shown at the right. Write an inverse trigonometric function that can be used to find $\theta$, the angle that describes the steepness of the superpipe. Then find the angle to the nearest degree.

![Diagram of a superpipe](image.png)

**SOLUTION:**
Since the measures of the opposite side and the adjacent side are known, use the tangent function.

$$\tan \theta = \frac{6.2}{18}$$

$$\theta = \text{Arctan} \left( \frac{6.2}{18} \right)$$

Use a calculator.

Keystrokes: 2nd [TAN$^{-1}$] 6 . 2 ÷ 18 )
ENTER 19.0059842

So, the angle of the steepness of superpipe is $19^\circ$.

**ANSWER:**
$\text{Arctan} \left( \frac{6.2}{18} \right) ; 19^\circ$

---

Find each value. Write angle measures in degrees and radians.

12. $\text{Arcsin} \left( \frac{\sqrt{3}}{2} \right)$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [SIN$^{-1}$] 2nd $[\sqrt{\_}]$ 3 )
÷ 2 ) ENTER 60

Therefore, $\text{arcsin} \left( \frac{\sqrt{3}}{2} \right) = 60^\circ$ or $\frac{\pi}{3}$.

**ANSWER:**
$60^\circ$; $\frac{\pi}{3}$

13. $\text{Arccos} \left( \frac{\sqrt{3}}{2} \right)$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [COS$^{-1}$] 2nd $[\sqrt{\_}]$ 3 )
÷ 2 ) ENTER 30

Therefore, $\text{arccos} \left( \frac{\sqrt{3}}{2} \right) = 30^\circ$ or $\frac{\pi}{6}$.

**ANSWER:**
$30^\circ$; $\frac{\pi}{6}$
12-9 Inverse Trigonometric Functions

14. $\sin^{-1}(-1)$

**SOLUTION:**

Use a calculator.

Keystrokes: 2nd $[\sin^{-1}]$ $(-)$ 1 ) ENTER

Therefore, $\sin^{-1}(-1) = -90^\circ$ or $-\frac{\pi}{2}$.

**ANSWER:**

$-90^\circ$; $-\frac{\pi}{2}$

15. $\tan^{-1}\sqrt{3}$

**SOLUTION:**

Use a calculator.

Keystrokes: 2nd $[\tan^{-1}]$ 2nd $[\sqrt{}]$ 3 ) ENTER 60

Therefore, $\tan^{-1}\sqrt{3} = 60^\circ$ or $\frac{\pi}{3}$.

**ANSWER:**

$60^\circ$; $\frac{\pi}{3}$

16. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

**SOLUTION:**

Use a calculator.

Keystrokes: 2nd $[\cos^{-1}]$ $(-)$ 2nd $[\sqrt{}]$ 3 ) $\div$ 2 ) ENTER 150

Therefore, $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$ or $\frac{5\pi}{6}$.

**ANSWER:**

$150^\circ$; $\frac{5\pi}{6}$

17. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

**SOLUTION:**

Use a calculator.

Keystrokes: 2nd $[\tan^{-1}]$ $(-)$ 2nd $[\sqrt{}]$ 3 ) $\div$ 3 ) ENTER $-30$

Therefore, $\arctan\left(-\frac{\sqrt{3}}{3}\right) = -30^\circ$ or $-\frac{\pi}{6}$.

**ANSWER:**

$-30^\circ$; $-\frac{\pi}{6}$
12-9 Inverse Trigonometric Functions

Find each value. Round to the nearest hundredth if necessary.

18. tan (Cos⁻¹ 1)

**SOLUTION:**
Use a calculator.

Keystrokes: TAN 2nd [COS⁻¹] 1 ) ) ENTER 0

Therefore, tan(Cos⁻¹1) = 0.

**ANSWER:**
0

19. tan \left( \arcsin \left( \frac{1}{2} \right) \right)

**SOLUTION:**
Use a calculator.

Keystrokes: TAN 2nd [SIN⁻¹] \left( \frac{1}{2} \right) \div 2 ) ) ENTER -0.5773502692

Therefore, tan \left( \arcsin \left( \frac{1}{2} \right) \right) \approx -0.58.

**ANSWER:**
-0.58

20. \cos \left( \tan^{-1} \frac{3}{5} \right)

**SOLUTION:**
Use a calculator.

Keystrokes: COS 2nd [TAN⁻¹] 3 ÷ 5 ) ) ENTER 0.8574929257

Therefore, \cos \left( \tan^{-1} \frac{3}{5} \right) \approx 0.86.

**ANSWER:**
0.86

21. \sin \left( \arctan \sqrt{3} \right)

**SOLUTION:**
Use a calculator.

Keystrokes: SIN 2nd [TAN⁻¹] 2nd [\sqrt{3}] 3 ) ) ) ENTER 0.8660254038

Therefore, \sin \left( \arctan \sqrt{3} \right) \approx 0.87.

**ANSWER:**
0.87
22. $\cos\left(\sin^{-1}\frac{4}{9}\right)$

**SOLUTION:**
Use a calculator.

Keystrokes: COS 2nd $[\sin^{-1}]$ 4 ÷ 9 ) ) ENTER 0.8958064165

Therefore, $\cos\left(\sin^{-1}\frac{4}{9}\right) \approx 0.90$.

**ANSWER:**
0.90

23. $\sin\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$  

**SOLUTION:**
Use a calculator.

Keystrokes: SIN 2nd $[\cos^{-1}]$ (-) 2nd $[\sqrt{]}$ 2 ÷ 2 ) ) ENTER 0.7071067812

Therefore, $\sin\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] \approx 0.71$.

**ANSWER:**
0.71

24. $\tan \theta = 3.8$

**SOLUTION:**
$\tan \theta = 3.8$
$\theta = \arctan(3.8)$

Use a calculator.

Keystrokes: 2nd $[\tan^{-1}]$ 3.8 ) ENTER 75.25643716

Therefore, $\theta \approx 75.3^\circ$.

**ANSWER:**
75.3°

25. $\sin \theta = 0.9$

**SOLUTION:**
$\sin \theta = 0.9$
$\theta = \arcsin(0.9)$

Use a calculator.

Keystrokes: 2nd $[\sin^{-1}]$ 0.9 ) ENTER 64.15806724

Therefore, $\theta \approx 64.2^\circ$.

**ANSWER:**
64.2°
26. \( \sin \theta = -2.5 \)

**SOLUTION:**
Since the range of \( \sin \theta \) is \( \{ y \mid -1 \leq y \leq 1 \} \),
\( \sin \theta = -2.5 \) has no solution.

**ANSWER:**
no solution

27. \( \cos \theta = -0.25 \)

**SOLUTION:**
\( \cos \theta = -0.25 \)
\( \theta = \arccos(-0.25) \)

Use a calculator.

Keystrokes: 2nd [COS\(^{-1}\)] (-) 0.25 ENTER 104.4775122

Therefore, \( \theta \approx 104.5^\circ \).

**ANSWER:**
104.5°

28. \( \cos \theta = 0.56 \)

**SOLUTION:**
\( \cos \theta = 0.56 \)
\( \theta = \arccos(0.56) \)

Use a calculator.

Keystrokes: 2nd [COS\(^{-1}\)] 0.56 ENTER 55.94420226

Therefore, \( \theta \approx 55.9^\circ \).

**ANSWER:**
55.9°

29. \( \tan \theta = -0.2 \)

**SOLUTION:**
\( \tan \theta = -0.2 \)
\( \theta = \arctan(-0.2) \)

Use a calculator.

Keystrokes: 2nd [TAN\(^{-1}\)] (-) 0.2 ENTER -11.30993247

Therefore, \( \theta \approx -11.3^\circ \).

**ANSWER:**
-11.3°
12-9 Inverse Trigonometric Functions

30. CCSS SENSE-MAKING A boat is traveling west to cross a river that is 190 meters wide. Because of the current, the boat lands at point Q, which is 59 meters from its original destination point P. Write an inverse trigonometric function that can be used to find θ, the angle at which the boat veered south of the horizontal line. Then find the measure of the angle to the nearest tenth.

**SOLUTION:**
Since the measures of the opposite side and the adjacent side are known, use the tangent function.

\[
\tan \theta = \frac{59}{190}
\]

\[\theta = \arctan \left( \frac{59}{190} \right)\]

Use a calculator.

Keystrokes: 2nd [TAN⁻¹] 5 9 ÷ 1 9 0 ) ENTER 17.25094388

Therefore, \(\theta \approx 17.3^\circ\).

So, the angle at which the boat veered south of the horizontal line is 17.3°.

**ANSWER:**
\[\arctan \frac{59}{190} = \theta, 17.3^\circ\]

31. TREES A 24-foot tree is leaning 2.5 feet left of vertical, as shown in the figure. Write an inverse trigonometric function that can be used to find θ, the angle at which the tree is leaning. Then find the measure of the angle to the nearest degree.

**SOLUTION:**
Since the measures of the opposite side and the hypotenuse are known, use the sine function.

\[\sin \theta = \frac{2.5}{24}\]

\[\theta = \arcsin \left( \frac{2.5}{24} \right)\]

Use a calculator.

Keystrokes: 2nd [SIN⁻¹] 2 . 5 ÷ 2 4 ) ENTER 5.979156796

Therefore, \(\theta \approx 6^\circ\).

So, the angle at which the tree is leaning is about 6°.

**ANSWER:**
\[\arcsin \frac{2.5}{24} ; 6^\circ\]
32. **DRIVING** An expressway off-ramp curve has a radius of 52 meters and is designed for vehicles to safely travel at speeds up to 45 kilometers per hour (or 12.5 meters per second). The equation below represents the angle θ of the curve. What is the measure of the angle to the nearest degree?

\[
\tan \theta = \frac{(12.5 \text{ m/s})^2}{(52 \text{ m})(9.8 \text{ m/s}^2)}
\]

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [TAN\(^{-1}\)] 12 . 5 \(x^2\)  
\(\div (52 \times 9 . 8)\) ENTER 17.04622194

Therefore, \(\theta \approx 17^\circ\).

**ANSWER:** 17°

33. **TRACK AND FIELD** A shot-putter throws the shot with an initial speed of 15 meters per second. The expression \(\frac{15 \text{ m/s(\sin x)}}{9.8 \text{ m/s}^2}\) represents the time in seconds at which the shot reached its maximum height. In the expression, \(x\) is the angle at which the shot was thrown. If the maximum height of the shot was reached in 1.0 second, at what angle was it thrown? Round to the nearest tenth.

**SOLUTION:**
The expression to find time, \(t\) at which the shot reached its maximum height is \(t = \frac{15 \text{ m/s(\sin x)}}{9.8 \text{ m/s}^2}\).

Substitute 1.0 for \(t\) to find the angle, \(x\).

\[1.0 = \frac{15(\sin x)}{9.8}\]
\[x = \sin^{-1}\left(\frac{9.8}{15}\right)\]

Use a calculator.

Keystrokes: 2nd [SIN\(^{-1}\)] 9 . 8 \(\div 15\) ) ENTER 40.79339495

So, the angle at which the maximum height of the shot was thrown is 40.8°.

**ANSWER:** 40.8°
12-9 Inverse Trigonometric Functions

Solve each equation for $0 \leq \theta \leq 2\pi$.

34. $\csc \theta = 1$

**SOLUTION:**
$\csc \theta = 1$
$\sin \theta = 1$
$\theta = \sin^{-1}(1)$

Use a calculator.

Keystrokes: 2nd [SIN$^{-1}$] 1 ) ENTER 90

Therefore, $\theta = \frac{\pi}{2}$.

**ANSWER:**
$\frac{\pi}{2}$

35. $\sec \theta = -1$

**SOLUTION:**
$\sec \theta = -1$
$\cos \theta = -1$
$\theta = \cos^{-1}(-1)$

Use a calculator.

Keystrokes: 2nd [COS$^{-1}$] ( - ) 1 ) ENTER 180

Therefore, $\theta = \pi$.

**ANSWER:**
$\pi$

36. $\sec \theta = 1$

**SOLUTION:**
$\sec \theta = 1$
$\cos \theta = 1$
$\theta = \cos^{-1}(1)$

Use a calculator.

Keystrokes: 2nd [COS$^{-1}$] 1 ) ENTER 0

Therefore, $\theta = 0, 2\pi$.

**ANSWER:**
$0, 2\pi$

37. $\csc \theta = \frac{1}{2}$

**SOLUTION:**
$\csc \theta = \frac{1}{2}$
$\sin \theta = 2$

Since the range of $\sin \theta$ is $\{y \mid -1 \leq y \leq 1\}$,
$\csc \theta = \frac{1}{2}$ has no solution.

**ANSWER:**
no solution
12-9 Inverse Trigonometric Functions

38. $\cot \theta = 1$

**SOLUTION:**

\[
\cot \theta = 1 \\
\tan \theta = 1 \\
\theta = \tan^{-1}(1)
\]

Use a calculator.

Keystrokes: 2nd [TAN] 1 ) ENTER 45

Therefore, $\theta = \frac{\pi}{4}$.

**ANSWER:**

\[
\frac{\pi}{4}, \frac{5\pi}{4}
\]

39. $\sec \theta = 2$

**SOLUTION:**

\[
\sec \theta = 2 \\
\cos \theta = 1 \\
\theta = \cos^{-1}
\]

Use a calculator.

Keystrokes: 2nd [COS] 1 ) 2 ) ENTER 60

Therefore, $\theta = \frac{\pi}{3}$.

**ANSWER:**

\[
\frac{\pi}{3}, \frac{5\pi}{3}
\]

40. **MULTIPLE REPRESENTATIONS** Consider $y = \cos^{-1} x$.

\[
\text{a. GRAPHICAL} \quad \text{Sketch a graph of the function. Describe the domain and the range.}
\]

\[
\text{b. SYMBOLIC} \quad \text{Write the function using different notation.}
\]

\[
\text{c. NUMERICAL} \quad \text{Choose a value for } x \text{ between } -1 \text{ and } 0. \text{ Then evaluate the inverse cosine function. Round to the nearest tenth.}
\]

\[
\text{d. ANALYTICAL} \quad \text{Compare the graphs of } y = \cos x \text{ and } y = \cos^{-1} x.
\]

**SOLUTION:**

\[
\text{a.}
\]

\[
\text{b. } y = \arccos x
\]

\[
\text{c. Sample answer: Let } x = -0.2. \text{ To find the value of } y = \cos^{-1}(-0.2), \text{ use a calculator.}
\]

Keystrokes: 2nd [COS^{-1}] ( ) 2 ) ENTER 101.536959

Therefore, $y = 101.5^\circ$.

\[
\text{d. Sample answer: The graph of } y = \cos x \text{ has a domain of all real numbers and a range from } -1 \text{ to } 1. \text{ The graph of } y = \cos^{-1} x \text{ has a domain from } -1 \text{ to } 1 \text{ and a range of } 0 \text{ to } 180^\circ.
\]

**ANSWER:**

\[
\text{a.}
\]
12-9 Inverse Trigonometric Functions

domain: \(-1 \leq x \leq 1\); range: \(0 \leq y \leq \pi\)

b. \(y = \text{Arccos } x\)

c. Sample answer: \(x = -0.2; y = 101.5^\circ\)

d. Sample answer: The graph of \(y = \cos x\) has a domain of all real numbers and a range from \(-1\) to \(1\). The graph of \(y = \cos^{-1} x\) has a domain from \(-1\) to \(1\) and a range of \(0\) to \(180^\circ\).

41. CHALLENGE Determine whether \(\cos (\text{Arccos } x) = x\) for all values of \(x\) is true or false. If false, give a counterexample.

SOLUTION: 
false; \(x = 2\pi\)

ANSWER: 
false; \(x = 2\pi\)

42. CCSS CRITIQUE Desiree and Oscar are solving \(\cos \theta = 0.3\) where \(90 < \theta < 180\). Is either of them correct? Explain your reasoning.

Desiree
\[
\begin{align*}
\cos \theta &= 0.3 \\
\cos^{-1} 0.3 &= 142.5^\circ
\end{align*}
\]

Oscar
\[
\begin{align*}
\cos \theta &= 0.3 \\
\cos^{-1} 0.3 &= 72.5^\circ
\end{align*}
\]

SOLUTION: 
Sample answer: Neither; cosine is not positive in the second quadrant.

ANSWER: 
Sample answer: Neither; cosine is not positive in the second quadrant.

43. REASONING Explain how the domain of \(y = \sin^{-1} x\) is related to the range of \(y = \sin x\).

SOLUTION: 
The domain of \(y = \sin^{-1} x\) is \(-1 \leq x \leq 1\). This is the same as the range of \(y = \sin x\).

ANSWER: 
The domain of \(y = \sin^{-1} x\) is \(-1 \leq x \leq 1\). This is the same as the range of \(y = \sin x\).
44. OPEN ENDED Write an equation with an Arcsine function and an equation with a Sine function that both involve the same angle measure.

**SOLUTION:**
Sample answer:

\[
\text{Arcsin} \frac{1}{2} = 30°
\]

\[
\frac{1}{2} = \sin 30°
\]

**ANSWER:**

\[
\text{Arcsin} \frac{1}{2} = 30°
\]

Sample answer:

\[
\frac{1}{2} = \sin 30°
\]

45. WRITING IN MATH Compare and contrast the relations \( y = \tan^{-1} x \) and \( y = \tan^{-1} x \). Include information about the domains and ranges.

**SOLUTION:**
Sample answer: \( y = \tan^{-1} x \) is a relation that has a domain of all real numbers and a range of all real numbers except odd multiples of \( \frac{\pi}{2} \).

The relation is not a function. \( y = \tan^{-1} x \) is a function that has a domain of all real numbers and a range of \( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \).

**ANSWER:**
Sample answer: \( y = \tan^{-1} x \) is a relation that has a domain of all real numbers and a range of all real numbers except odd multiples of \( \frac{\pi}{2} \). The relation is not a function. \( y = \tan^{-1} x \) is a function that has a domain of all real numbers and a range of \( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \).

46. REASONING Explain how \( \sin^{-1} 8 \) and \( \cos^{-1} 8 \) are undefined while \( \tan^{-1} 8 \) is defined.

**SOLUTION:**

The range of \( y = \sin x \) and \( y = \cos x \) is \(-1 \leq x \leq 1\).

The range of \( y = \tan^{-1} x \) is all real numbers.

**ANSWER:**

Sample answer: The range of \( y = \sin x \) and \( y = \cos x \) is \(-1 \leq x \leq 1\). The range of \( y = \tan^{-1} x \) is all real numbers.
12-9 Inverse Trigonometric Functions

47. Simplify \( \frac{x}{2} + \frac{2}{x} \).  

\[
\begin{align*}
\text{A} & \quad \frac{1+x}{1-x} \\
\text{B} & \quad \frac{2}{x} \\
\text{C} & \quad \frac{1-x}{1+x} \\
\text{D} & \quad -x
\end{align*}
\]

**SOLUTION:**

\[
\begin{align*}
\frac{2}{x} + \frac{2}{x} & \quad = \frac{2+2x}{x-2} \\
& \quad = \frac{2(1+x)}{2-2x} \\
& \quad = \frac{2(1+x)}{2-2x} \\
& \quad = \frac{2(1+x)}{2-2x} \\
& \quad = \frac{1+x}{1-x}
\end{align*}
\]

The option A is the correct option.

**ANSWER:**

A

48. **SHORT RESPONSE** What is the equation of the graph?

![Graph of a circle](image)

**SOLUTION:**

The center of the circle is (3, -4) and the radius is 5 units.

The equation of the circle with center \((h, k)\) and radius, \(r\) is \((x - h)^2 + (y - k)^2 = r^2\).

Substitute 3 for \(h\), -4 for \(k\) and 5 for \(r\).

\[
\begin{align*}
(x - 3)^2 + (y - (-4))^2 & = 5^2 \\
(x - 3)^2 + (y + 4)^2 & = 25
\end{align*}
\]

**ANSWER:**

\((x - 3)^2 + (y + 4)^2 = 25\)
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49. If \( f(x) = 2x^2 - 3x \) and \( g(x) = 4 - 2x \), what is \( g[f(x)] \)?

**F** \( g[f(x)] = 4 + 6x - 8x^2 \)

**G** \( g[f(x)] = 4 + 6x - 4x^2 \)

**H** \( g[f(x)] = 20 - 26x + 8x^2 \)

**J** \( g[f(x)] = 44 - 38x + 8x^2 \)

**SOLUTION:**

\[
g[f(x)] = g(f(x)) \\
= g(2x^2 - 3x) \\
= 4 - 2(2x^2 - 3x) \\
= 4 - 4x^2 + 6x \\
= 4 + 6x - 4x^2
\]

The option G is the correct option.

**ANSWER:**

G

50. If \( g \) is a positive number, which of the following is equal to \( 12g \)?

**A** \( \sqrt{144g} \)

**B** \( \sqrt{12g^2} \)

**C** \( \sqrt{24g^2} \)

**D** \( 6\sqrt{4g^2} \)

**SOLUTION:**

\[
(12g)^2 = 144g^2 \\
12g = \sqrt{144g^2} \\
= \sqrt{36 \times 4 \times g^2} \\
= 6\sqrt{4g^2}
\]

The option D is the correct option.

**ANSWER:**

D

51. **RIDES** The Cosmoclock 21 is a huge Ferris wheel in Japan. The diameter is 328 feet. Suppose a rider enters the ride at 0 feet, and then rotates in 90° increments counterclockwise. The table shows the angle measures of rotation and the height above the ground of the rider.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
</tr>
<tr>
<td>90°</td>
<td>164</td>
</tr>
<tr>
<td>180°</td>
<td>164</td>
</tr>
<tr>
<td>270°</td>
<td>164</td>
</tr>
<tr>
<td>360°</td>
<td>0</td>
</tr>
</tbody>
</table>

**a.** A function that models the data is
12-9 Inverse Trigonometric Functions

\[ y = 164 \cdot \sin (x - 90^\circ) + 164. \] Identify the vertical shift, amplitude, period, and phase shift of the graph.

b. Write an equation using the sine that models the position of a rider on the Vienna Giant Ferris Wheel in Austria, with a diameter of 200 feet. Check your equation by plotting the points and the equation with a graphing calculator.

**SOLUTION:**
a. From the equation, we can find \( a = 164, b = 1, h = 90^\circ \) and \( k = 164 \).

Vertical shift:
Since \( k = 164 \), the vertical shift is 164.

Amplitude:
\[
|a| = |164| \\
= 164
\]

Period:
\[
\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|} \\
= 360^\circ
\]

Phase shift:
\( h = 90^\circ \)

b. Here, the diameter of the Giant Wheel is 200 feet. So the amplitude is 100 and the vertical shift is \( k = 100 \).

Substitute 100 for \( a \), 1 for \( b \), 90 for \( h \), and 100 for \( k \) in \( y = a \sin b(x - h) + k \).

\[ y = 100 \sin (x - 90) + 100 \]

Check the equation by plotting the points and graphing the equation using graphing calc.

**ANSWER:**
a. 164; 164; 360°, 90°

b. \( y = 100 \left[ \sin (x - 90^\circ) \right] + 100 \)

52. TIDES The world’s record for the highest tide is held by the Minas Basin in Nova Scotia, Canada, with a tidal range of 54.6 feet. A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. Write an equation to represent the height \( h \) of the tide. Assume that the tide is at equilibrium at \( t = 0 \), that the high tide is beginning, and that the tide completes one cycle in 12 hours.

**SOLUTION:**
A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. As the midline lies halfway between the maximum and the minimum values, the equation of the midline is \( y = 0 \).

Therefore, there is no vertical shift.

Amplitude:
\[
|a| = \left| \frac{54.6}{2} - 0 \right| \\
= 27.3
\]

Period:
Since the tide completes one cycle in 12 hours, the period is 12.

\[
|b| = \frac{2\pi}{12} \\
b = \pm \frac{\pi}{6}
\]

Substitute 27.3 for \( a \), \( \frac{\pi}{6} \) for \( b \), 0 for \( h \), and 0 for \( k \) in \( h = a \sin b(t - h) + k \).
12-9 Inverse Trigonometric Functions

Find each value. Write angle measures in degrees and radians.

1. SOLUTION: Use a calculator.

\[ h = 27.3 \sin \frac{\pi}{6} (t - 0) + 0 \]

\[ h = 27.3 \sin \frac{\pi}{6} t \]

**ANSWER:**

\[ h = 27.3 \sin \frac{\pi}{6} t \]

Solve each equation.

53. \( \log_3 5 + \log_3 x = \log_3 10 \)

**SOLUTION:**

\[
\log_3 5 + \log_3 x = \log_3 10
\]

\[
\log_3 (5x) = \log_3 10
\]

5x = 10 [Product property of logarithm]

\[
x = 2 [Property of equality]
\]

**ANSWER:**

2

54. \( \log_4 a + \log_4 9 = \log_4 27 \)

**SOLUTION:**

\[
\log_4 a + \log_4 9 = \log_4 27
\]

\[
\log_4 (9a) = \log_4 27
\]

9a = 27 [Product property of logarithm]

\[
a = 3 [Property of equality]
\]

**ANSWER:**

3

55. \( \log_{10} 16 - \log_{10} 2 = \log_{10} 2 \)

**SOLUTION:**

\[
\log_{10} 16 - \log_{10} 2 = \log_{10} 2
\]

\[
\log_{10} \left( \frac{16}{2} \right) = \log_{10} 2 [Quotient property of logarithm]
\]

\[
\frac{16}{2} = 2 [Property of equality]
\]

\[
4 = 16
\]

\[
t = 4
\]

**ANSWER:**

4

56. \( \log_7 24 - \log_7 (y + 5) = \log_7 8 \)

**SOLUTION:**

\[
\log_7 24 - \log_7 (y + 5) = \log_7 8
\]

\[
\log_7 \left( \frac{24}{y + 5} \right) = \log_7 8 [Quotient property of logarithm]
\]

\[
\frac{24}{y + 5} = 8 [Property of equality]
\]

\[
24 = 8(y + 5)
\]

\[
8y + 40 = 24
\]

\[
8y = -16
\]

\[
y = -2
\]

**ANSWER:**

-2
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Find the exact value of each trigonometric function.

57. \( \cos 3\pi \)

**SOLUTION:**

\[
\cos 3\pi = \cos (2\pi + \pi) = \cos \pi
\]

Since the angle \( \pi \) is a quadrant angle, the coordinates of the point on its terminal side is \((-x, 0)\).

Find the value of \( r \).

\[
r = \sqrt{a^2 + b^2} = \sqrt{(-x)^2 + 0^2} = x
\]

\[
\cos 3\pi = \cos \pi = \frac{x}{r} = \frac{-x}{x} = -1
\]

**ANSWER:**

\(-1\)

58. \( \tan 120^\circ \)

**SOLUTION:**

The terminal side of \( 120^\circ \) lies in Quadrant II.

Find the measure of the reference angle.

\[
\theta' = 180^\circ - \theta = 180^\circ - 120^\circ = 60^\circ
\]

The tangent function is negative in quadrant II.

\[
\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}
\]

**ANSWER:**

\(-\sqrt{3}\)

59. \( \sin 300^\circ \)

**SOLUTION:**

The terminal side of \( 300^\circ \) lies in Quadrant III.

Find the measure of the reference angle.

\[
\theta' = 360^\circ - \theta = 360^\circ - 300^\circ = 60^\circ
\]

The sine function is negative in quadrant III.

\[
\sin 300^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}
\]

**ANSWER:**

\(-\frac{\sqrt{3}}{2}\)
60. \( \sec \frac{7\pi}{6} \)

**SOLUTION:**

The terminal side of \( \frac{7\pi}{6} \) lies in Quadrant III.

Find the measure of the reference angle.

\[
\theta^* = \frac{7\pi}{6} - \pi
\]

\[
= \frac{\pi}{6}
\]

The secant function is negative in quadrant III.

\[
\sec \frac{7\pi}{6} = -\sec \frac{\pi}{6}
\]

\[
= -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}
\]

\[
= -\frac{2\sqrt{3}}{3}
\]

**ANSWER:**

\[-\frac{2\sqrt{3}}{3}\]
Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 36^\circ$, $c = 9$

**SOLUTION:**
Find the measure of the third angle.

$m\angle B = 180^\circ - (36^\circ + 90^\circ)$

$= 54^\circ$

Use the Law of Sines.

Find the value of $a$.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad [\text{Law of Sines}]$$

$$\frac{\sin 36^\circ}{9} = \frac{\sin 90^\circ}{a}$$

$$a = \frac{9 \sin 36^\circ}{\sin 90^\circ}$$

$$\approx 5.3$$

Find the value of $b$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad [\text{Law of Sines}]$$

$$\frac{\sin 36^\circ}{5.3} = \frac{\sin 54^\circ}{b}$$

$$b = \frac{5.3 \sin 54^\circ}{\sin 36^\circ}$$

$$\approx 7.3$$

**ANSWER:**

$B = 54^\circ$, $a = 5.3$, $b = 7.3$

2. $a = 12$, $A = 58^\circ$

**SOLUTION:**
Find the measure of the third angle.

$m\angle B = 180^\circ - (58^\circ + 90^\circ)$

$= 32^\circ$

Use the Law of Sines to find side length $c$.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad [\text{Law of Sines}]$$

$$\frac{\sin 58^\circ}{12} = \frac{\sin 90^\circ}{c}$$

$$c = \frac{12 \sin 90^\circ}{\sin 58^\circ}$$

$$\approx 14.2$$

Find the value of $b$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad [\text{Law of Sines}]$$

$$\frac{\sin 58^\circ}{12} = \frac{\sin 32^\circ}{b}$$

$$b = \frac{12 \sin 32^\circ}{\sin 58^\circ}$$

$$= \frac{12(0.53)}{0.848}$$

$$\approx 7.5$$

**ANSWER:**

$B = 32^\circ$, $c = 14.2$, $b = 7.5$
3. \( B = 85^\circ, b = 8 \)

**SOLUTION:**

Find the measure of the third angle.

\[
m\angle A = 180^\circ - (85^\circ + 90^\circ) = 5^\circ
\]

Use the Law of Sines to find side length \( c \).

\[
\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{[Law of Sines]}
\]

\[
\frac{\sin 85^\circ}{8} = \frac{\sin 90^\circ}{c}
\]

\[
c = \frac{8 \sin 90^\circ}{\sin 85^\circ} \approx 8.0
\]

Find the value of \( b \).

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{[Law of Sines]}
\]

\[
\frac{\sin 5^\circ}{a} = \frac{\sin 85^\circ}{8}
\]

\[
a = \frac{8 \sin 5^\circ}{\sin 85^\circ} \approx 0.7
\]

**ANSWER:**

\( A = 5^\circ, c = 8.0, a = 0.7 \)

4. \( a = 9, c = 12 \)

**SOLUTION:**

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{[Law of Sines]}
\]

\[
\frac{\sin A}{9} = \frac{\sin 90^\circ}{12}
\]

\[
A = \sin^{-1} \left( \frac{9}{12} \right) \approx 49^\circ
\]

Use the Law of Sines to find the length \( b \).

Find the measure of the third angle.

\[
m\angle B = 180^\circ - (49^\circ + 90^\circ) = 41^\circ
\]

Find the value of \( b \).

\[
\frac{\sin C}{c} = \frac{\sin B}{b} \quad \text{[Law of Sines]}
\]

\[
\frac{\sin 90^\circ}{12} = \frac{\sin 41^\circ}{b}
\]

\[
b = \frac{12 \sin 41^\circ}{\sin 90^\circ} \approx 7.9
\]

**ANSWER:**

\( b = 7.9, B = 41^\circ, A = 49^\circ \)
Rewrite each degree measure in radians and each radian measure in degrees.

5. $325^\circ$

**SOLUTION:**

$$325^\circ = 325 \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{325\pi}{180} \text{ radians} = \frac{65\pi}{36} \text{ radians}$$

**ANSWER:**

$$\frac{65\pi}{36}$$

6. $-175^\circ$

**SOLUTION:**

$$-175^\circ = -175 \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{-175\pi}{180} \text{ radians} = \frac{-35\pi}{36} \text{ radians}$$

**ANSWER:**

$$\frac{-35\pi}{36}$$

7. $\frac{9\pi}{4}$

**SOLUTION:**

$$\frac{9\pi}{4} = \frac{9\pi}{4} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}} = 405^\circ$$

**ANSWER:**

$405^\circ$

8. $-\frac{5\pi}{6}$

**SOLUTION:**

$$-\frac{5\pi}{6} = -\frac{5\pi}{6} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ radians}} = -150^\circ$$

**ANSWER:**

$-150^\circ$

9. Determine whether $\triangle ABC$, with $A = 110^\circ$, $a = 16$, and $b = 21$, has no solution, one solution, or two solutions. Then solve the triangle, if possible. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

**SOLUTION:**

If $\angle A$ is obtuse and $a < b$, then there is no solution.

Given $\angle A$ is obtuse and $16 < 21$. So, there is no solution.

**ANSWER:**

no solution
Find the exact value of each function. Write angle measures in degrees.

10. \( \cos(-90^\circ) \)

**SOLUTION:**

\(-90^\circ = 360 - 90^\circ = 270^\circ\)

Since the angle 270° is a quadrant angle, the coordinates of the point on its terminal side is (0, -y).

Find the value of \( r \).

\[
r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-y)^2} = y
\]

\( \cos(-90^\circ) = \cos90^\circ = \frac{x}{r} = \frac{0}{y} = 0 \)

**ANSWER:**

0

11. \( \sin 585^\circ \)

**SOLUTION:**

\[
\sin 585^\circ = \sin(360^\circ + 225^\circ) = \sin 225^\circ
\]

The terminal side of 225° lies in Quadrant III.

Find the measure of the reference angle.

\[
\theta' = \theta - 180^\circ = 225^\circ - 180^\circ = 45^\circ
\]

The sine function is negative in quadrant III.

\[
\sin(585^\circ) = -\sin 45^\circ = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}
\]

**ANSWER:**

\[-\frac{\sqrt{2}}{2}\]
Solve \( \triangle ABC \) by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. \( \sec \left( -\frac{9\pi}{4} \right) \)

**SOLUTION:**

\[ \sec \left( -\frac{9\pi}{4} \right) = \sec \left( \frac{9\pi}{4} \right) \]
\[ = \sec \left( 2\pi + \frac{\pi}{4} \right) \]
\[ = \sec \left( \frac{\pi}{4} \right) = \sqrt{2} \]

**ANSWER:**

\( \sqrt{2} \)

14. \( \tan \left( \cos^{-1} \frac{4}{5} \right) \)

**SOLUTION:**

Use a calculator.

Keystrokes: TAN 2nd \( \left[ \cos^{-1} \right] \frac{4}{5} \) \) ENTER \( .7500000001 \)

Therefore, \( \tan \left( \cos^{-1} \frac{4}{5} \right) = \frac{3}{4} \).

**ANSWER:**

\( \frac{3}{4} \)
15. Arccos \( \frac{1}{2} \)

**SOLUTION:**

Use a calculator.

Keystrokes: 2nd [COS\(^{-1}\)] 1 ÷ 2 ) ENTER 60

Therefore,

\[ \text{Arccos} \left( \frac{1}{2} \right) = 60^\circ. \]

**ANSWER:**

60°

16. The terminal side of angle \( \theta \) in standard position intersects the unit circle at point \( P \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \). Find \( \cos \theta \) and \( \sin \theta \).

**SOLUTION:**

\( P \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) = P(\cos \theta, \sin \theta) \)

Therefore, \( \cos \theta = \frac{1}{2} \) and \( \sin \theta = \frac{\sqrt{3}}{2} \).

**ANSWER:**

\( \cos \theta = \frac{1}{2}, \sin \theta = \frac{\sqrt{3}}{2} \)

17. **MULTIPLE CHOICE** What angle has a tangent and sine that are both negative?

A 65°

B 120°

C 120°

D 310°

**SOLUTION:**

In the fourth quadrant both the sine and the tangent are negative. The angle 310° is lie in the third quadrant. Therefore, option D is the correct answer.

**ANSWER:**

D

18. **NAVIGATION** Airplanes and ships measure distance in nautical miles. The formula 1 nautical mile = 6077 – 31 \cos \theta\) feet, where \( \theta \) is the latitude in degrees, can be used to find the approximate length of a nautical mile at a certain latitude. Find the length of a nautical mile when the latitude is 120°.

**SOLUTION:**

Substitute 120° for \( \theta \) in the given equation and evaluate.

\[
\text{length} = 6077 - 31 \cos (120°)
\]

\[= 6092.5\]

Therefore, the length of a nautical mile when the latitude is 120° is 6092.5 feet.

**ANSWER:**

6092.5 ft

Find the amplitude and period of each function. Then graph the function.

19. \( y = 2 \sin 3\theta \)
**SOLUTION:**
Given $a = 2$ and $b = 3$.

Amplitude:

$$|a| = |2|$$

$$= 2$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{3}$$

$$= 120^\circ$$

Graph the function.

**ANSWER:**
amplitude = 2; period = 120°

---

20. $y = \frac{1}{2} \cos 2\theta$

**SOLUTION:**
Given $a = \frac{1}{2}$ and $b = 2$.

Amplitude:

$$|a| = \left|\frac{1}{2}\right|$$

$$= \frac{1}{2}$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{2}$$

$$= 180^\circ$$

Graph the function.

**ANSWER:**
amplitude = $\frac{1}{2}$; period = 180°
22. Determine whether ΔXYZ, with y = 15, z = 9, and X = 105°, should be solved by beginning with the Law of Sines or Law of Cosines. Then solve the triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

**SOLUTION:**
Since the measure of two sides and the included angle is given, Law of Cosines is used at the beginning to solve the triangle.

Substitute 15 for y, 9 for z, and 105° for X in 
\[ x^2 = y^2 + z^2 - 2yz \cos X. \]
\[ x^2 = (15)^2 + (9)^2 - 2(15)(9) \cos 105° \]
\[ = 225 + 81 - 270 \cos 105° \]
\[ x \approx 19.4 \]

Use the Law of Sines to find a missing angle measure.

\[ \frac{\sin X}{x} = \frac{\sin Y}{y} [\text{Law of Sines}] \]
\[ \frac{\sin 105°}{19.4} = \frac{\sin Y}{15} \]
\[ \sin Y = \frac{15 \sin 105°}{19.4} \]
\[ Y \approx 48° \]
Find the measure of the third angle.

\[ m∠Z \approx 180° - (105° + 48°) \text{ or } 27° \]

**ANSWER:**
Law of Cosines; \( Y \approx 48°, x \approx 19.4, Z \approx 27° \)

State the amplitude, period, and phase shift for each function. Then graph the function.

23. \( y = \cos (θ + 180°) \)

**SOLUTION:**
Given \( a = 1, b = 1 \) and \( h = -180° \).
Amplitude: 
\[ |a| = |1| \]
\[ = 1 \]

Period: 
\[ 360^\circ = \frac{360^\circ}{|b|} \]
\[ = 360^\circ \]

Phase shift: \( h = -180^\circ \)

Draw the graph of \( y = \cos \theta \). Then shift the graph \( 180^\circ \) to the left.

\[ y = \cos (\theta + 180) \]

**ANSWER:** 
1, 360°, -180°

24. \( y = \frac{1}{2} \tan \left( \theta - \frac{\pi}{2} \right) \)

**SOLUTION:**

Given \( a = \frac{1}{2}, b = 1 \) and \( h = \frac{\pi}{2} \).

Amplitude: 
Amplitude does not exist.

Period: 
\[ \frac{\pi}{|b|} = \frac{\pi}{1} \]
\[ = \pi \]

Phase shift: \( h = \frac{\pi}{2} \)

Draw the graph of \( y = \frac{1}{2} \tan \theta \). Then shift the graph \( \frac{\pi}{2} \) units to the right.

\[ y = \frac{1}{2} \tan (\theta - \frac{\pi}{2}) \]

**ANSWER:**

does not exist, \( \pi, \frac{\pi}{2} \)
25. **WHEELS** A water wheel has a diameter of 20 feet. It makes one complete revolution in 45 seconds. Let the height at the top of the wheel represent the height at time 0. Write an equation for the height of point $h$ in the diagram below as a function of time $t$.

**SOLUTION:**

The maximum and the minimum height is 20 ft and 0 ft.

As the midline lies halfway between the maximum and the minimum values, the equation of the midline is $y = \frac{20 - 0}{2} = 10$.

Therefore the vertical shift $k = 10$.

Amplitude:

$|a| = |20 - 10| = 10$

Period:

Since the wheel makes one complete revolution in 45 seconds, the period is 45 seconds.

$45 = \frac{2\pi}{|b|}$

$|b| = \frac{2\pi}{45}$

Substitute 10 for $a$, $\frac{2\pi}{45}$ for $b$, 0 for $h$, and 10 for $k$ in

$h = a \cos b(t - h) + k$

$= 10 \cos \frac{2\pi}{45} (t - 0) + 10$

$= 10 \cos \frac{2\pi}{45} t + 10$

**ANSWER:**

$h = 10 \cos \frac{2\pi}{45} t + 10$
State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. The Law of Cosines is used to solve a triangle when two angles and any sides are known.

*SOLUTION:* It is a false sentence.

*ANSWER:* false, Law of Sines

2. An angle on the coordinate plane is in standard position if the vertex is at the origin and one ray is on the positive x-axis.

*SOLUTION:* True

*ANSWER:* true

3. Coterminal angles are angles in standard position that have the same terminal side.

*SOLUTION:* True

*ANSWER:* true

4. A horizontal translation of a periodic function is called a phase shift.

*SOLUTION:* True

*ANSWER:* true

5. The inverse of the sine function is the cosecant function.

*SOLUTION:* It is a false sentence.

*ANSWER:* false, arcsine function

6. The cycle of the graph of a sine or cosine function equals half the difference between the maximum and minimum values of the function.

*SOLUTION:* It is a false sentence.

*ANSWER:* false, amplitude
Solve \( \triangle ABC \) by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

7. \( c = 12, \ b = 5 \)

**SOLUTION:**

Use the Law of Sines to find \( m\angle B \).

\[
\frac{\sin B}{b} = \frac{\sin C}{c}
\]

\[
\frac{\sin B}{5} = \frac{\sin 90}{12}
\]

\[
\sin B = \frac{5 \sin 90}{12}
\]

\[
B \approx 25^\circ
\]

\( m\angle A = 180^\circ - (90 + 25) \) or \( 65^\circ \)

Use the Law of Sines to find the side length \( a \).

\[
\frac{\sin A}{a} = \frac{\sin C}{c}
\]

\[
\frac{\sin 65}{a} = \frac{\sin 90}{12}
\]

\[
a \approx 10.9
\]

**ANSWER:**

\( a = 10.9; \ A = 65^\circ; \ B = 25^\circ \)

8. \( a = 10, \ B = 55^\circ \)

**SOLUTION:**

\( m\angle A = 180^\circ - (90 + 55) \) or \( 35^\circ \)

Use the Law of Sines to find the side lengths \( b \) and \( c \).

\[
\frac{\sin A}{a} = \frac{\sin C}{c}
\]

\[
\sin 35 = \frac{\sin 90}{10}
\]

\[
c = \frac{10 \sin 90}{\sin 35}
\]

\[
c \approx 17.4
\]

\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]

\[
\sin 35 = \frac{\sin 55}{10}
\]

\[
b = \frac{10 \sin 55}{\sin 35}
\]

\[
b \approx 14.3
\]

**ANSWER:**

\( A = 35^\circ; \ c = 17.4; \ b = 14.3 \)
9. $B = 75^\circ$, $b = 15$

**SOLUTION:**

$m\angle A = 180^\circ - (90^\circ + 75^\circ)$ or $15^\circ$

Use the Law of Sines to find the side lengths $a$ and $c$.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

\[
\frac{\sin 15^\circ}{15} = \frac{\sin 75^\circ}{a} \quad \frac{\sin 90^\circ}{15} = \frac{\sin 75^\circ}{c}
\]

\[
a = \frac{15 \sin 15^\circ}{\sin 75^\circ} \approx 4.0
\]

\[
c = \frac{15 \sin 90^\circ}{\sin 75^\circ} \approx 15.5
\]

**ANSWER:**

$A = 15^\circ$; $a = 4.0$; $c = 15.5$

10. $B = 45^\circ$, $c = 16$

**SOLUTION:**

$m\angle A = 180^\circ - (90^\circ + 45^\circ)$ or $45^\circ$

Use the Law of Sines to find the side lengths $a$ and $b$.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

\[
\frac{\sin 90^\circ}{16} = \frac{\sin 45^\circ}{b} \quad \frac{\sin 90^\circ}{16} = \frac{\sin 45^\circ}{a}
\]

\[
b = \frac{16 \sin 45^\circ}{\sin 90^\circ} \approx 11.3
\]

\[
a = \frac{16 \sin 45^\circ}{\sin 90^\circ} \approx 11.3
\]

**ANSWER:**

$a = 11.3$; $b = 11.3$; $A = 45^\circ$
11. $A = 35^\circ$, $c = 22$

**SOLUTION:**

\[ m \angle B = 180 - (90 + 35) \text{ or } 55^\circ \]

Use the Law of Sines to find the side lengths $a$ and $b$.

\[
\frac{\sin C}{C} = \frac{\sin B}{b} \\
\frac{\sin 90}{22} = \frac{\sin 55}{b} \\
b = \frac{22 \sin 55}{\sin 90} \\
b \approx 18.0
\]

\[
\frac{\sin C}{C} = \frac{\sin A}{a} \\
\frac{\sin 90}{22} = \frac{\sin 35}{a} \\
a = \frac{22 \sin 35}{\sin 90} \\
a \approx 12.6
\]

**ANSWER:**

$B = 55^\circ$; $a = 12.6$; $b = 18.0$

12. $\sin A = \frac{2}{3}$, $a = 6$

**SOLUTION:**

\[ \sin A = \frac{2}{3} \]

\[ A \approx 42^\circ \]

\[ m \angle B = 180 - (90 + 42) \text{ or } 48^\circ \]

Use the Law of Sines to find the side lengths $b$ and $c$.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} \\
\frac{\sin 42}{6} = \frac{\sin 48}{b} \\
b \approx 6.7
\]

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \\
\frac{\sin 42}{6} = \frac{\sin 90}{c} \\
c \approx 9.0
\]

**ANSWER:**

$A = 42^\circ$; $B = 48^\circ$; $b = 6.7$; $c = 9.0$
13. **TRUCK** The back of a moving truck is 3 feet off the ground. What length does a ramp off the back of the truck need to be in order for the angle of elevation of the ramp to be 20°?

**SOLUTION:**
Draw the diagram which represents the situation.

![Diagram](triangle.png)

Let \( x \) be the length of the ramp off.

Use the Law of Sines to find the length of \( x \).

\[
\frac{\sin 90}{x} = \frac{\sin 20}{3}
\]

\[
x = \frac{3 \sin 90}{\sin 20}
\]

\[
x \approx 8.8 \text{ ft}
\]

**ANSWER:** about 8.8 ft

**Rewrite each degree measure in radians and each radian measure in degrees.**

14. **215°**

**SOLUTION:**
\[
215° = 215 \cdot \frac{\pi \text{ radians}}{180} = \frac{215\pi}{180} \text{ or } \frac{43\pi}{36} \text{ radians}
\]

**ANSWER:**
\[
\frac{43\pi}{36}
\]

15. **\( \frac{5\pi}{2} \)**

**SOLUTION:**
\[
\frac{5\pi}{2} = \frac{5\pi}{2} \cdot \frac{180°}{\pi \text{ radians}} = \frac{5 \cdot 180°}{2} = 450°
\]

**ANSWER:**
450°

16. **−3\( \pi \)**

**SOLUTION:**
\[
−3\pi = −3\pi \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = −3 \cdot 180° = −540°
\]

**ANSWER:**
−540°

17. **−315°**

**SOLUTION:**
\[
−315° = −315 \cdot \frac{\pi \text{ radians}}{180} = \frac{−315\pi}{180} \text{ or } −\frac{7\pi}{4} \text{ radians}
\]

**ANSWER:**
\[
−\frac{7\pi}{4}
\]
Find one angle with positive measure and one angle with negative measure coterminal with each angle.

18. 265°

**SOLUTION:**
Positive angle: 265° + 360° = 625°

Negative angle: 265° − 360° = −95°

**ANSWER:** 625°, −95°

19. −65°

**SOLUTION:**
Positive angle: −65° + 360° = 295°

Negative angle: −65° − 360° = −425°

**ANSWER:** 295°, −425°

20. \( \frac{7\pi}{2} \)

**SOLUTION:**
Positive angle: \( \frac{7\pi}{2} + 2\pi = \frac{11\pi}{2} \)

Negative angle: \( \frac{7\pi}{2} − 4\pi = −\frac{\pi}{2} \)

**ANSWER:** \( \frac{11\pi}{2} \), −\( \frac{\pi}{2} \)

21. BICYCLE A bicycle tire makes 8 revolutions in one minute. The tire has a radius of 15 inches. Find the angle \( \theta \) in radians through which the tire rotates in one second.

**SOLUTION:**
One revolution makes 360°, so 8 revolutions make \( 2880° \times 8 \).
For the rotation of 2880° it takes 60 seconds.

Therefore, for one sec it makes \( \frac{2880}{60} \) or 48° angle of rotation.

Rewrite 48° in radians.

\[
48° = 48 \times \frac{\pi \text{ radians}}{180} = \frac{48\pi}{180} \text{ or } \frac{4\pi}{15} \text{ radians}
\]

**ANSWER:** \( \frac{4\pi}{15} \)
Study Guide and Review - Chapter 12

Find the exact value of each trigonometric function.

22. \( \cos 135^\circ \)

**SOLUTION:**
The terminal side of \( 135^\circ \) lies in Quadrant II.

\[
\theta' = 180^\circ - \theta \\
= 180^\circ - 135^\circ \\
= 45^\circ
\]

The cosine function is negative in Quadrant II.

\[
\cos 150^\circ = -\cos 45^\circ \\
= -\frac{\sqrt{2}}{2}
\]

**ANSWER:**
\[-\frac{\sqrt{2}}{2}\]

23. \( \tan 150^\circ \)

**SOLUTION:**
The terminal side of \( 150^\circ \) lies in Quadrant II.

\[
\theta' = 180^\circ - \theta \\
= 180^\circ - 150^\circ \\
= 30^\circ
\]

The tangent function is negative in Quadrant II.

\[
\tan 150^\circ = -\tan 30^\circ \\
= -\frac{\sqrt{3}}{3}
\]

**ANSWER:**
\[-\frac{\sqrt{3}}{3}\]

24. \( \sin 2\pi \)

**SOLUTION:**
Since the angle \( 2\pi \) is a quadrant angle, the coordinates of the point on its terminal side is \((x, 0)\).

Find the value of \( r \).

\[
r = \sqrt{a^2 + b^2} \\
= \sqrt{x^2 + 0^2} \\
r = x
\]

\[
\sin 2\pi = \frac{y}{r} \\
= \frac{0}{x} \\
= 0
\]

**ANSWER:**
0
25. \( \cos \frac{3\pi}{2} \)

**SOLUTION:**
Since the angle \( \frac{3\pi}{2} \) is a quadrant angle, the coordinates of the point on its terminal side is \((0, -y)\).

Find the value of \( r \).

\[
r = \sqrt{a^2 + b^2}
= \sqrt{0^2 + (-y)^2}
= y
\]

\[
\cos \frac{3\pi}{2} = \frac{x}{r} = \frac{0}{y} = 0
\]

**ANSWER:**
0

The terminal side of \( \theta \) in standard position contains each point. Find the exact values of the six trigonometric functions of \( \theta \).

26. \( P(-4, 3) \)

**SOLUTION:**
Find the value of \( r \).

\[
r = \sqrt{x^2 + y^2}
= \sqrt{(-4)^2 + 3^2}
= \sqrt{25}
= 5
\]

Use \( x = -4, y = 3, \) and \( r = 5 \) to write the six trigonometric ratios.

\[
\sin \theta = \frac{y}{r} = \frac{3}{5}
\cos \theta = \frac{x}{r} = -\frac{4}{5}
\tan \theta = \frac{y}{x} = -\frac{3}{4}
\csc \theta = \frac{r}{y} = \frac{5}{3}
\sec \theta = \frac{r}{x} = -\frac{5}{4}
\cot \theta = \frac{x}{y} = -\frac{4}{3}
\]

**ANSWER:**
\[
\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5},
\tan \theta = -\frac{3}{4}, \csc \theta = \frac{5}{3},
\sec \theta = -\frac{5}{4}, \cot \theta = -\frac{4}{3}
\]
27. $P(5, 12)$

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{169}$$

$$= 13$$

Use $x = 5, y = 12$, and $r = 13$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{12}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{5}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{12}{5}$$

$$\csc \theta = \frac{r}{y} = \frac{13}{12}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{5}{12}$$

**ANSWER:**

$$\sin \theta = \frac{12}{13}, \cos \theta = \frac{5}{13},$$

$$\tan \theta = \frac{12}{5}, \csc \theta = \frac{13}{12},$$

$$\sec \theta = \frac{13}{5}, \cot \theta = \frac{5}{12}$$

28. $P(16, -12)$

**SOLUTION:**
Find the value of $r$.

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{16^2 + (-12)^2}$$

$$= \sqrt{400}$$

$$= 20$$

Use $x = 16, y = -12$, and $r = 20$ to write the six trigonometric ratios.

$$\sin \theta = \frac{y}{r} = \frac{-12}{20} = \frac{-3}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{16}{20} = \frac{4}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-12}{16} = \frac{-3}{4}$$

$$\csc \theta = \frac{r}{y} = \frac{20}{-12} = \frac{5}{-3}$$

$$\sec \theta = \frac{r}{x} = \frac{20}{16} = \frac{5}{4}$$

$$\cot \theta = \frac{x}{y} = \frac{16}{-12} = \frac{-4}{3}$$

**ANSWER:**

$$\sin \theta = \frac{-3}{5}, \cos \theta = \frac{4}{5},$$

$$\tan \theta = \frac{-3}{4}, \csc \theta = \frac{-5}{3},$$

$$\sec \theta = \frac{5}{4}, \cot \theta = \frac{-4}{3}$$
29. **BALL.** A ball is thrown off the edge of a building at an angle of 70° and with an initial velocity of 5 meters per second. The equation that represents the horizontal distance of the ball \( x \) is \( x = v_0(\cos \theta)t \), where \( v_0 \) is the initial velocity, \( \theta \) is the angle at which it is thrown, and \( t \) is the time in seconds. About how far will the ball travel in 10 seconds?

**SOLUTION:**
Substitute 5 for \( v_0 \), 70° for \( \theta \), and 10 for \( t \) in the given equation and solve for \( x \).

\[
x = v_0(\cos \theta)t \\
x = 5(\cos 70°)10 \\
= 50(\cos 70°) \\
\approx 17.1 \text{ meters}
\]

**ANSWER:**
about 17.1 meters

---

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

30. \( C = 118°, c = 10, a = 4 \)

**SOLUTION:**
Because \( \angle C \) is obtuse and \( c > a \), one solution exists.

Use the Law of Sines to find \( m\angle A \).

\[
\frac{\sin A}{a} = \frac{\sin C}{c} \\
\frac{\sin A}{4} = \frac{\sin 118°}{10} \\
\sin A = \frac{4\sin 118°}{10} \\
A \approx 21°
\]

\( m\angle B \approx 180° - (21° + 118°) \) or 41°

Use the Law of Sines to find \( b \).

\[
\frac{\sin C}{c} = \frac{\sin B}{b} \\
\frac{\sin 118°}{10} \approx \frac{\sin 41°}{b} \\
b \approx \frac{10\sin 41°}{\sin 118°} \\
b \approx 7.4
\]

**ANSWER:**
one solution; \( A \approx 21°, B \approx 41°, b \approx 7.4 \)
31. \( A = 25^\circ, a = 15, c = 18 \)

**SOLUTION:**
Since \( \angle A \) is acute and \( a \leq c \), find \( h \) and compare it to \( a \).

\[
\begin{align*}
h &= c \sin A \\
&= 18 \sin 25 \\
&\approx 7.6
\end{align*}
\]

Since \( 7.6 < 15 < 18 \) or \( h < a < c \), there is two solutions. So, there are two triangles to be solved.

**ANSWER:**
two solutions; First solution: \( C = 30^\circ \), \( B = 125^\circ \), \( b = 29.1 \); second solution: \( C = 150^\circ \), \( B = 5^\circ \), \( b = 3.1 \)

32. \( A = 70^\circ, a = 5, c = 16 \)

**SOLUTION:**
Since \( \angle A \) is acute and \( a < c \), find \( h \) and compare it to \( a \).

\[
\begin{align*}
h &= c \sin A \\
&= 16 \sin 70^\circ \\
&\approx 15.0
\end{align*}
\]

Since \( 5 < 15 \) or \( a < h \), there is no solution.

**ANSWER:**
no solution
33. **BOAT** Kira and Mallory are standing on opposite sides of a river. How far is Kira from the boat? Round to the nearest tenth if necessary.

**SOLUTION:**
Measure of angle at boat is \(180^\circ - (85^\circ + 30^\circ)\) or 65°.

Let \(x\) be the distance between Kira and the boat.

Use the Law of Sines to find \(x\).

\[
\frac{\sin 85}{x} = \frac{\sin 65}{90}
\]

\[
x = \frac{90 \sin 85}{\sin 65}
\]

\(x \approx 98.9\) ft

**ANSWER:**
98.9 ft

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

![Diagram of triangle ABC with sides 16, 15, and 21]

**SOLUTION:**
The triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle \(\angle B\).

\[
h^2 = a^2 + c^2 - 2ac \cos B
\]

\[
21^2 = 15^2 + 16^2 - 2(15)(16) \cos B
\]

\[
\cos B = \frac{85}{16}
\]

Use the Law of Sines to find the measure of angle, \(\angle C\).

\[
\sin 85 \approx \frac{\sin C}{21}
\]

\[
\sin C \approx \frac{16 \sin 85}{21}
\]

\(C \approx 49^\circ\)

\(m\angle B \approx 180 - (85 + 49)\) or 46

**ANSWER:**
Law of Cosines; \(A \approx 46^\circ, B \approx 85^\circ, C \approx 49^\circ\)
36. \( C = 75^\circ, \ a = 5, \ b = 7 \)

**SOLUTION:**

The triangle should be solved by beginning with the Law of Cosines.

\[
c^2 = 5^2 + 7^2 - 2(5)(7)\cos 75
\]

\[
c \approx 7.5
\]

Use the Law of Sines to find a missing angle measure.

\[
\frac{\sin A}{\sin 75} = \frac{5}{7.5}
\]

\[
\sin A \approx \frac{5\sin 75}{7.5}
\]

\[
A \approx 40
\]

\[
m\angle B \approx 180 - (40 + 75) \approx 65^\circ
\]

**ANSWER:**

Law of Cosines; \( A = 40^\circ, \ B = 65^\circ, \ c = 7.5 \)
37. $A = 42^\circ$, $a = 9$, $b = 13$

**SOLUTION:**
The triangle should be solved by beginning with the Law of Sines.

$$\frac{\sin 42^\circ}{9} = \frac{\sin B}{13}$$

$$\sin B = \frac{13 \sin 42^\circ}{9}$$

$B \approx 75^\circ$ or $105^\circ$

$m^\circ C \approx 180 - (42 + 75)$ or $63$

or $m^\circ C \approx 180 - (42 + 105)$ or $33$

Use Law of Sines to find $c$.

$$\frac{\sin 42^\circ}{9} \approx \frac{\sin 63^\circ}{c}$$

$$c \approx \frac{9 \sin 63^\circ}{\sin 42^\circ}$$

$= 12.0$

$$\frac{\sin 42^\circ}{9} \approx \frac{\sin 33^\circ}{c}$$

$$c \approx \frac{9 \sin 33^\circ}{\sin 42^\circ}$$

$= 7.3$

**ANSWER:**
Law of Sines;

$B \approx 75^\circ$, $C \approx 63^\circ$, $c \approx 12.0$ or $B \approx 105^\circ$, $C \approx 33^\circ$, $c \approx 7.3$

38. $b = 8.2$, $c = 15.4$, $A = 35^\circ$

**SOLUTION:**
The triangle should be solved by beginning with the Law of Cosines.

$$a^2 = 8.2^2 + 15.4^2 - 2(8.2)(15.4) \cos 35^\circ$$

$a \approx 9.9$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin B}{8.2} \approx \frac{\sin 35^\circ}{9.9}$$

$$\sin B \approx \frac{8.2 \sin 35^\circ}{9.9}$$

$B \approx 28^\circ$

$m^\circ C \approx 180 - (35 + 28)$ or $117^\circ$

**ANSWER:**
Law of Cosines; $a \approx 9.9$, $B \approx 28^\circ$, $C \approx 117^\circ$
39. **FARMING** A farmer wants to fence a piece of his land. Two sides of the triangular field have lengths of 120 feet and 325 feet. The measure of the angle between those sides is 70°. How much fencing will the farmer need?

**SOLUTION:**
The triangle should be solved by the Law of Cosines.

Let \( x \) be the unknown side length.

\[
x^2 = 120^2 + 325^2 - 2(120)(325)\cos 70°
\]

\[
x^2 \approx 93347.429
\]

\[
x \approx 305.5 \text{ ft}
\]

The length of the fence is about 750.5 ft (120 ft + 325 ft + 305.5 ft).

**ANSWER:**
about 750.5 ft

**Find the exact value of each function.**

40. \( \cos (-210°) \)

**SOLUTION:**
\[
\cos(-210°) = \cos 210°
\]

\[
= -\frac{\sqrt{3}}{2}
\]

**ANSWER:**
\[-\frac{\sqrt{3}}{2}\]

41. \( (\cos 45°)(\cos 210°) \)

**SOLUTION:**
\[
(\cos 45°)(\cos 210°) = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)
\]

\[
= -\frac{\sqrt{6}}{4}
\]

**ANSWER:**
\[-\frac{\sqrt{6}}{4}\]

42. \( \sin \left(-\frac{7\pi}{4}\right) \)

**SOLUTION:**
\[
\sin \left(-\frac{7\pi}{4}\right) = -\sin \frac{7\pi}{4}
\]

\[
= -\left(\frac{\sqrt{2}}{2}\right)
\]

\[
= \frac{\sqrt{2}}{2}
\]

43. \( \left(\cos \frac{\pi}{2}\right)\left(\sin \frac{\pi}{2}\right) \)

**SOLUTION:**
\[
\left(\cos \frac{\pi}{2}\right)\left(\sin \frac{\pi}{2}\right) = (0)(1)
\]

\[
= 0
\]

**ANSWER:**
0
44. Determine the period of the function.

Find the amplitude, if it exists, and period of each function. Then graph the function.

46. \( y = 4 \sin 2\theta \)

**SOLUTION:**
Amplitude: \(|a| = |4|\) or 4

Period: \(\frac{360}{|b|} = \frac{360}{2} = 180\) or \(180\)

\(x\)-intercepts: (0,0)

\[
\left( \frac{1}{2} \cdot \frac{360}{b}, 0 \right) = \left( \frac{1}{2} \cdot 180, 0 \right) \text{ or } \left( 90, 0 \right)
\]

\[
\left( \frac{360}{b}, 0 \right) = \left( \frac{360}{2}, 0 \right) \text{ or } \left( 180, 0 \right)
\]

**ANSWER:**
amplitude: 4, period: 180°

45. A wheel with a diameter of 18 inches completes 4 revolutions in 1 minute. What is the period of the function that describes the height of one spot on the outside edge of the wheel as a function of time?

**SOLUTION:**
Period:
Since the wheel completes 4 revolutions in 1 minute, the time taken to make one revolution is \(\frac{60}{4} = 15\) seconds. So, the period is 15 seconds.

**ANSWER:**
15 seconds
47. \( y = \cos \frac{1}{2} \theta \)

**SOLUTION:**
Amplitude: \(|a| = |1| = 1\) or 1

Period: \( \frac{360}{|b|} = \frac{360}{\frac{1}{2}} = 720^\circ \)

**x-intercepts:**
\[
\left(1, \frac{360}{4}, 0\right) = \left(1, \frac{90}{1}, 0\right) = (180, 0) \quad \text{or} \quad (360, 0)
\]
\[
\left(3, \frac{360}{4}, 0\right) = \left(3, \frac{90}{1}, 0\right) = (540, 0) \quad \text{or} \quad (720, 0)
\]

ANSWER:
Amplitude: 1, period: 720°

48. \( y = 3 \csc \theta \)

**SOLUTION:**
Amplitude: not defined

Period: \( \frac{360}{|b|} = \frac{360}{1} = 360^\circ \) or 360

The vertical asymptotes occur at the points where \( 3\sin \theta = 0 \). So, the asymptotes are at \( \theta = 180^\circ \) and \( \theta = 360^\circ \). Sketch \( y = 3\sin \theta \) and use it to graph \( y = 3\csc \theta \).

ANSWER:
amplitude: not defined, period: 360°
49. \( y = 3 \sec \theta \)

**SOLUTION:**
Amplitude: not defined

Period: \( \frac{360}{|b|} = \frac{360}{1} \) or 360

The vertical asymptotes occur at the points where \( 3 \cos \theta = 0 \). So, the asymptotes are at \( \theta = 90 \) and \( \theta = 270 \). Sketch \( y = 3 \cos \theta \) and use it to graph \( y = 3 \sec \theta \).

**ANSWER:**
Amplitude: not defined, period: 360°

50. \( y = \tan 2\theta \)

**SOLUTION:**
Amplitude: not defined

Period: \( \frac{180}{|b|} = \frac{180}{2} \) or 90

The vertical asymptotes occur at the points where \( \tan 2\theta = 0 \). So, the asymptotes are at \( \theta = 45 \) and \( \theta = 135 \). Sketch \( y = 3 \cos \theta \) and use it to graph \( y = \tan 2\theta \).

**ANSWER:**
Amplitude: not defined, period: 90°
51. \( y = 2 \csc \frac{1}{2} \theta \)

**SOLUTION:**
Amplitude: not defined
Period: \( \frac{360}{|b|} = \frac{360}{1} \) or 720

The vertical asymptotes occur at the points
where \( 2 \sin \frac{1}{2} \theta = 0 \). So, the asymptotes are at \( \theta = \)
360° and \( \theta = 720° \). Sketch \( y = 2 \sin \frac{1}{2} \theta \) and use it to
graph \( y = 2 \csc \frac{1}{2} \theta \).

**ANSWER:**
amplitude: not defined, period: 720°

52. When Lauren jumps on a trampoline it vibrates with
a frequency of 10 hertz. Let the amplitude equal 5 feet. Write a sine equation to represent the vibration
of the trampoline \( y \) as a function of time \( t \).

**SOLUTION:**
The amplitude of the function is 5.
Since the period is the reciprocal of the frequency,
the period of the function is \( \frac{1}{10} \).

Period = \( \frac{2\pi}{|b|} \)
\[
\frac{1}{10} = \frac{2\pi}{|b|} \\
|b| = 20\pi \\
b = \pm 20\pi
\]
Substitute 5 for \( a \), 20\pi for \( b \) and \( t \) for \( \theta \) in the
general equation for the sine function.

\( y = a \sin b \theta \)
\( y = 5 \sin 20\pi t \)

**ANSWER:**
y = 5 sin 20 \pi t

State the vertical shift, amplitude, period, and
phase shift of each function. Then graph the
function.

53. \( y = 3 \sin [2 (\theta - 90°)] + 1 \)

**SOLUTION:**
Given \( a = 3 \), \( b = 2 \), \( h = 90° \), \( k = 1 \).

Amplitude:
\[ |a| = |3| \]
\[ = 3 \]

Period:
State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

Keystrokes: 2nd \[\sin^{-1}\] 2nd 3 ) 2 ) ENTER 60 

Therefore, 

ANSWER: 60°, 62.

54. \( y = \frac{1}{2} \tan\left[2(\theta - 30°)\right] - 3 \)

SOLUTION:

Given \( a = \frac{1}{2}, b = 2, h = 30°, k = -3. \)

Amplitude: undefined

Period:

\[
\frac{180°}{|b|} = \frac{180°}{2} = 90°
\]

Phase shift:

Since \( h = 30° \), phase shift is 30° right.

Vertical shift: \( k = -3 \), 3 units down

Midline: \( y = -3 \)

First, graph the midline. Then graph

\( y = \frac{1}{2} \tan\left[2(\theta - 30°)\right] - 3 \) using the midline as reference.

Then shift the graph 30° to the right.

ANSWER:

vertical shift: down 3, amplitude: undefined, period: 90°, phase shift: 30° right
55. \( y = 2 \sec \left[ 3\left( \theta - \frac{\pi}{2} \right) \right] + 2 \)

**SOLUTION:**

Given \( a = 2, b = 3, h = \frac{\pi}{2}, k = 2. \)

Amplitude: Not defined

Period: 
\[
\frac{2\pi}{|b|} = \frac{2\pi}{3}
\]

Phase shift:
Since \( h = \frac{\pi}{2}, \) phase shift is \( \frac{\pi}{2} \) units right.

Vertical shift: \( k = 2, \) 2 units up

Midline: \( y = 2 \)

First, graph the midline. Then graph 
\( y = 2 \sec \left[ 3\theta \right] + 2 \) using the midline as reference.

Then shift the graph \( \frac{\pi}{2} \) units to the right.

**ANSWER:**
vertical shift: up 2, amplitude: not defined, period: 
\[
\frac{2\pi}{3}, \quad \text{phase shift: } \frac{\pi}{2} \text{ right}
\]

56. \( y = \frac{1}{2} \cos \left[ \frac{1}{4}(\theta + \frac{\pi}{4}) \right] - 1 \)

**SOLUTION:**

Given \( a = \frac{1}{2}, b = \frac{1}{4}, h = -\frac{\pi}{4}, k = -1. \)

Amplitude:
\[
|a| = \frac{1}{2}
\]

Period: 
\[
\frac{2\pi}{|b|} = \frac{2\pi}{\frac{1}{4}} = 8\pi
\]

Phase shift:
Since \( h = -\frac{\pi}{4}, \) phase shift is \( \frac{\pi}{4} \) units left

Vertical shift: \( k = -1, \) one unit down

Midline: \( y = -1 \)

First, graph the midline. Then graph 
\( y = \frac{1}{2} \cos \left[ \frac{1}{4}(\theta) \right] - 1 \) using the midline as reference.

Then shift the graph \( \frac{\pi}{4} \) units to the left.
ANSWER:
vertical shift: down 1; amplitude: $\frac{1}{2}$; period: $4\pi$;
phase shift: $\frac{\pi}{4}$ left

57. $y = \frac{1}{3} \sin \left[ \frac{1}{3}(\theta - 90^\circ) \right] + 2$

SOLUTION:
Given $a = \frac{1}{3}$, $b = \frac{1}{3}$, $h = 90^\circ$, $k = 2$.

Amplitude:
$|a| = \frac{1}{3}$

Period:
$\frac{360^\circ}{|b|} = \frac{360^\circ}{\frac{1}{3}}$

= $1080^\circ$
58. The graph approximates the height $y$ of a rope that two people are twirling as a function of time $t$ in seconds. Write an equation for the function.

![Graph of a sine wave]

**SOLUTION:**
The graph goes from 0 to 8, so the amplitude is 4. The midline is at $y = 4$, so the vertical shift is 4. At $\theta = 0$, $y = 4$, so $y = 1$ at $\theta = 0$ when there is no amplitude. Therefore, this is a sine curve.

The period is from $t = 0$ to $t = 1$.

\[
\frac{360^\circ}{|b|} = 1
\]

\[b = 360^\circ\]

The equation is $y = 4 \sin 360t + 4$.

**ANSWER:**
$y = 4 \sin 360t + 4$

---

**Evaluate each inverse trigonometric function. Write angle measures in degrees and radians.**

59. $\sin^{-1}(1)$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd $[\sin^{-1}] 1$ ) ENTER 90

Therefore, $\sin^{-1}(1) = 90^\circ$ or $\frac{\pi}{2}$.

**ANSWER:**
$90^\circ$, $\frac{\pi}{2}$

60. $\arctan(0)$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd $[\tan^{-1}] 0$ ) ENTER 0

Therefore, $\arctan(0) = 0^\circ$ or 0.

**ANSWER:**
$0^\circ$, 0
61. Arc sin $\frac{\sqrt{3}}{2}$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [SIN$^{-1}$] 2nd $\left[\sqrt{\frac{3}{2}}\right]$ ENTER $60^\circ$

Therefore, $\text{Arc sin} \frac{\sqrt{3}}{2} = 60^\circ$ or $\frac{\pi}{3}$.

**ANSWER:**
$60^\circ$, $\frac{\pi}{3}$

62. $\text{Cos}^{-1} \frac{\sqrt{2}}{2}$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [COS$^{-1}$] 2nd $\left[\sqrt{\frac{1}{2}}\right]$ ENTER $45^\circ$

Therefore, $\text{Cos}^{-1} \frac{\sqrt{2}}{2} = 45^\circ$ or $\frac{\pi}{4}$.

**ANSWER:**
$45^\circ$, $\frac{\pi}{4}$

63. $\text{Tan}^{-1}$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [TAN$^{-1}$] 1 ENTER 45

Therefore, $\text{Tan}^{-1} = 45^\circ$ or $\frac{\pi}{4}$.

**ANSWER:**
$45^\circ$, $\frac{\pi}{4}$

64. $\text{Arccos} 0$

**SOLUTION:**
Use a calculator.

Keystrokes: 2nd [COS$^{-1}$] 0 ENTER 90

Therefore, $\text{Arccos} \theta = 90^\circ$ or $\frac{\pi}{2}$.

**ANSWER:**
$90^\circ$, $\frac{\pi}{2}$
65. **RAMPS** A bicycle ramp is 5 feet tall and 10 feet long, as shown below. Write an inverse trigonometric function that can be used to find \( \theta \), the angle the ramp makes with the ground. Then find the angle.

![Diagram of a ramp with labels 10 ft and 5 ft]

**SOLUTION:**
Since the measures of the opposite side and the hypotenuse are known, use the sine function.

\[
\sin \theta = \frac{5}{10}
\]

\[
\theta = \arcsin \left( \frac{5}{10} \right)
\]

Use a calculator.

Keystrokes: 2nd \( \text{[SIN}^{-1}] \) 5 ÷ 10 ) ENTER 30

Therefore, \( \theta \approx 30^\circ \).
So, the angle of the ramp that makes with the ground is 30°.

**ANSWER:**
\[
\sin^{-1} \frac{5}{10} = \theta ; 30^\circ
\]

---

Evaluate each inverse trigonometric function. Round to the nearest hundredth if necessary.

66. \( \tan \left( \cos^{-1} \frac{1}{3} \right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: TAN 2nd \([\cos^{-1}] \frac{1}{3} \) ÷ ENTER 2.828427125

Therefore, \( \tan \left( \cos^{-1} \frac{1}{3} \right) \approx 2.83 \).

**ANSWER:**
2.83

67. \( \sin \left( \arcsin -\frac{\sqrt{2}}{2} \right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: SIN 2nd \([\sin^{-1}] \) ( ) - 2nd \( \sqrt{2} \) ÷ 2 ) ENTER -0.7071067812

Therefore, \( \sin \left( \arcsin -\frac{\sqrt{2}}{2} \right) \approx -0.71 \).

**ANSWER:**
-0.71
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68. \( \sin\left(\tan^{-1} 0\right) \)

**SOLUTION:**
Use a calculator.

Keystrokes: SIN 2nd [TAN\(^{-1}\)] 0 ) )
ENTER 0

Therefore, \( \sin\left(\tan^{-1} 0\right) = 0 \).

**ANSWER:**
0

Solve each equation. Round to the nearest tenth if necessary.

69. \( \tan \theta = -1.43 \)

**SOLUTION:**
\( \tan \theta = -1.43 \)
\[ \theta = \arctan(-1.43) \]

Use a calculator.

Keystrokes: 2nd [TAN\(^{-1}\)] \( - \) 1 . 4 3 ) ENTER
-55.03487923

Therefore, \( \theta \approx -55.0^\circ \).

**ANSWER:**
-55.0\(^\circ\)

70. \( \sin \theta = 0.8 \)

**SOLUTION:**
\( \sin \theta = 0.8 \)
\[ \theta = \arcsin(0.8) \]

Use a calculator.

Keystrokes: 2nd [SIN\(^{-1}\)] . 8 ) ENTER
53.13010235

Therefore, \( \theta \approx 53.1^\circ \).

**ANSWER:**
53.1\(^\circ\)

71. \( \cos \theta = 0.41 \)

**SOLUTION:**
\( \cos \theta = 0.41 \)
\[ \theta = \arccos(0.41) \]

Use a calculator.

Keystrokes: 2nd [COS\(^{-1}\)] . 4 1 ) ENTER
65.7951652

Therefore, \( \theta \approx 65.8^\circ \).

**ANSWER:**
65.8\(^\circ\)