Evaluate each expression if $a = -2$, $b = 3$, and $c = 4.2$.

1. $a - 2b + 3c$

**SOLUTION:**

$$a - 2b + 3c = (-2) - 2(3) + 3(4.2) = -2 - 6 + 12.6 = -8 + 12.6 = 4.6$$

**ANSWER:** 4.6

2. $2a + (b + 3)^2$

**SOLUTION:**

$$2a + (b + 3)^2 = 2(-2) + (3 + 3)^2 = 2(-2) + (6)^2 = -4 + 36 = 32$$

**ANSWER:** 32

3. $a + 3\left(b^2 - (a + c)\right)$

**SOLUTION:**

$$a + 3\left(b^2 - (a + c)\right) = -2 + 3\left[9 - (2.2)\right] = -2 + 3(6.8) = -2 + 20.4 = 18.4$$

**ANSWER:** 18.4

4. $5c - 2\left[(b - a) + c\right]$  

**SOLUTION:**

$$5c - 2\left[(b - a) + c\right] = 5(4.2) - 2\left[(3 - (-2)) + 4.2\right] = 21 - 2\left[5 + 4.2\right] = 21 - 2\left[9.2\right] = 21 - 18.4 = 2.6$$

**ANSWER:** 2.6

5. $4(2a + 3b) - 2c$

**SOLUTION:**

$$4(2a + 3b) - 2c = 4\left[2(-2) + 3(3)\right] - 2(4.2) = 4[-4 + 9] - 8.4 = 4[5] - 8.4 = 20 - 8.4 = 11.6$$

**ANSWER:** 11.6

6. $\frac{a^2 + 4c}{3b + 2a}$

**SOLUTION:**

$$\frac{a^2 + 4c}{3b + 2a} = \frac{(-2)^2 + 4(4.2)}{3(3) + 2(-2)} = \frac{4 + 16.8}{9 - 4} = \frac{20.8}{5} = 4.16$$

**ANSWER:** 4.16
7. \[ \frac{b^3 + ac}{ab + 2bc} \]

**SOLUTION:**

\[
\frac{b^3 + ac}{ab + 2bc} = \frac{3^3 + (-2)(4.2)}{(2)(3) + 2(3)(4.2)} = \frac{27 - 8.4}{-6 + 25.2} = \frac{18.6}{19.2} = 0.96875
\]

**ANSWER:**

0.96875

8. \[ \frac{3b + 2a}{5 - c} \]

**SOLUTION:**

\[
\frac{3b + 2a}{5 - c} = \frac{3(3) + 2(-2)}{5 - 4.2} = \frac{9 - 4}{0.8} = \frac{5}{0.8} = 6.25
\]

**ANSWER:**

6.25

9. \[ \frac{3a - 2c}{4ab} \]

**SOLUTION:**

\[
\frac{3a - 2c}{4ab} = \frac{3(-2) - 2(4.2)}{4(-2)(3)} = \frac{-6 - 8.4}{-24} = \frac{-14.4}{-24} = 0.6
\]

**ANSWER:**

0.6

10. **VOLLEYBALL** A player’s attack percentage \( A \) is calculated using the formula \( A = \frac{k-e}{t} \), where \( k \) represents the number of kills, \( e \) represents the number of attack errors including blocks, and \( t \) represents the total attacks attempted. Find the attack percentage given each set of values.

   a. \( k = 22, e = 11, t = 35 \)
   b. \( k = 33, e = 9, t = 50 \)

**SOLUTION:**

a. Substitute \( k = 22, e = 11, \) and \( t = 35 \) in the formula

\[
A = \frac{k-e}{t} = \frac{22-11}{35} = \frac{11}{35} \approx 0.314 \text{ or } 31.4\%
\]

The attack percentage is about 31.4%.

b. Substitute \( k = 33, e = 9, \) and \( t = 50 \) in the formula

\[
A = \frac{k-e}{t} = \frac{33-9}{50} = \frac{24}{50} = 0.48 \text{ or } 48\%
\]

The attack percentage is 48%.

**ANSWER:**

a. about 0.314 or 31.4%
b. 0.48 or 48%
Evaluate each expression if 
\( w = -3, x = 4, y = 2.6, \) and \( z = \frac{1}{3} \).

11. \( y + x - z \)

**SOLUTION:**

Substitute \( x = 4, y = 2.6 \) or \( \frac{13}{5} \), and \( z = \frac{1}{3} \) in the expression \( y + x - z \).

\[
y + x - z = \frac{13}{5} + 4 - \frac{1}{3}
\]

\[
= \frac{13(3) + 4(15) - 1(5)}{15}
\]

\[
= \frac{39 + 60 - 5}{15}
\]

\[
= \frac{94}{15}
\]

\[
= 6 \frac{4}{15}
\]

**ANSWER:**

\( 6 \frac{4}{15} \)

12. \( w - 2x + y + 2 \)

**SOLUTION:**

\[
w - 2x + y + 2 = w - 2x + \frac{y}{2}
\]

\[
= -3 - 2(4) + \frac{2.6}{2}
\]

\[
= -3 - 8 + 1.3
\]

\[
= -11 + 1.3
\]

\[
= -9.7
\]

**ANSWER:**

\(-9.7\)

13. \( 4(x - w) \)

**SOLUTION:**

\( 4(x - w) = 4(4 - (-3)) \)

\( = 4(7) \)

\( = 28 \)

**ANSWER:**

28

14. \( 6(y + x) \)

**SOLUTION:**

\( 6(y + x) = 6(2.6 + 4) \)

\( = 6(6.6) \)

\( = 39.6 \)

**ANSWER:**

39.6

15. \( 9z - 4y + 2w \)

**SOLUTION:**

\( 9z - 4y + 2w = 9\left(\frac{1}{3}\right) - 4(2.6) + 2(-3) \)

\( = 3 - 10.4 - 6 \)

\( = 3 - 16.4 \)

\( = -13.4 \)

**ANSWER:**

\(-13.4\)
1. Evaluate each expression if \( a = -2 \), \( b = 3 \), and \( c = 4.2 \).

**SOLUTION:**

\[
3y - 4z + x
\]

Substitute \( x = 4 \), \( y = 2.6 \), or \( z = \frac{13}{5} \), and \( \frac{1}{3} \) in the expression:

\[
3y - 4z + x = 3\left(\frac{13}{5}\right) - 4\left(\frac{1}{3}\right) + 4
\]

\[
= \frac{39}{5} - \frac{4}{3} + 4
\]

\[
= \frac{39(3) - 4(5) + 4(15)}{15}
\]

\[
= \frac{117 - 20 + 60}{15}
\]

\[
= \frac{117 + 40}{15}
\]

\[
= \frac{157}{15}
\]

\[
10\frac{7}{15}
\]

**ANSWER:**

10 \( \frac{7}{15} \)

62. The length of the hypotenuse is 10 cm.

**SOLUTION:**

Using the Pythagorean theorem:

\[ a^2 + b^2 = c^2 \]

\[ a^2 + b^2 = 10^2 \]

\[ a^2 + b^2 = 100 \]

Thus, the lengths of the legs are 6 cm and 8 cm.

63. Birmingham, Alabama and St. Petersburg Florida are average temperature range is the average the two:

**SOLUTION:**

\[ \text{Average Temperature} = \frac{A + B}{2} \]

**ANSWER:**

64. Determine room temperature range is the average of the

**SOLUTION:**

\[ \text{Room Temperature} = \frac{A + B}{2} \]

**ANSWER:**

65. The legs of a right triangle measure 6 centimeters

**SOLUTION:**

Using the Pythagorean theorem:

\[ a^2 + b^2 = c^2 \]

\[ 6^2 + b^2 = c^2 \]

\[ 36 + b^2 = c^2 \]

Thus, the length of the hypotenuse is.

66. The length of the hypotenuse is 10 cm.

**SOLUTION:**

Using the Pythagorean theorem:

\[ a^2 + b^2 = c^2 \]

\[ a^2 + b^2 = 10^2 \]

\[ a^2 + b^2 = 100 \]

Thus, the lengths of the legs are 6 cm and 8 cm.

67. The lengths of the three sides of a triangle are 10

**SOLUTION:**

Using the Pythagorean theorem:

\[ a^2 + b^2 = c^2 \]

\[ a^2 + b^2 = 10^2 \]

\[ a^2 + b^2 = 100 \]

Thus, the lengths of the legs are 6 cm and 8 cm.

7. **GAS MILEAGE**

The gasoline used by a car is measured in miles per gallon and is related to the distance traveled by the following formula.

\[ \text{miles per gallon} \times \text{number of gallons} = \text{distance traveled} \]

**a.** During a trip your car used a total of 46.2 gallons of gasoline. If your car gets 33 miles to the gallon, how far did you travel?

\[ \text{distance traveled} = \text{miles per gallon} \times \text{number of gallons} \]

\[ 33 \times 46.2 = \text{distance traveled} \]

\[ 1524.6 = \text{distance traveled} \]

**b.** Your friend has decided to buy a hybrid car that gets 60 miles per gallon. The gasoline tank holds 12 gallons. How far can the car go on one tank of gasoline?

**SOLUTION:**

\[ \text{distance traveled} = \text{miles per gallon} \times \text{number of gallons} \]

\[ 60 \times 12 = \text{distance traveled} \]

\[ 720 = \text{distance traveled} \]

**ANSWER:**

a. 1524.6 mi

b. 720 mi
Evaluate each expression if \( a = -4, b = -0.8, c = 5, \text{ and } d = \frac{1}{5} \).

18. \( \frac{a + b}{c - d} \)

**SOLUTION:**

\[
\frac{a + b}{c - d} = \frac{-4 + (-0.8)}{5 - \frac{1}{5}} = \frac{-4 - 0.8}{\frac{25 - 1}{5}} = \frac{-4.8}{\frac{24}{5}} = \frac{-4.8(5)}{24} = -1
\]

**ANSWER:**

-1

19. \( \frac{a - b}{bd} \)

**SOLUTION:**

\[
\frac{a - b}{bd} = \frac{-4 - (-0.8)}{(-0.8)(\frac{1}{5})} = \frac{-4 + 0.8}{-0.8(\frac{1}{5})} = \frac{-3.2}{-0.16} = 20
\]

**ANSWER:**

20

20. \( \frac{ac}{d + b} \)

**SOLUTION:**

\[
\frac{ac}{d + b} = \frac{(-4)(5)}{\frac{1}{5} + (-0.8)} = \frac{-20}{\frac{1}{5} - 0.8} = \frac{-20}{\frac{1 - 0.8(5)}{5}} = \frac{-20}{\frac{1 - 4}{5}} = \frac{-20}{\frac{-3}{5}} = \frac{-20(5)}{-3} = \frac{100}{3} = \frac{33\frac{1}{3}}{3}
\]

**ANSWER:**

\(33\frac{1}{3}\)
21. \( \frac{b^2c^2}{ac} \)

**SOLUTION:**

\[
\frac{b^2c^2}{ac} = \frac{(-0.8)^2(5)^2}{(-4)\left(\frac{1}{5}\right)}
\]

\[
= \frac{(0.64)(25)}{-4\left(\frac{1}{5}\right)}
\]

\[
= \frac{16}{4}\left(\frac{5}{4}\right)
\]

\[
= 16\left(\frac{5}{4}\right)
\]

\[
= (16)(5)
\]

\[
= -\frac{80}{4}
\]

\[
= -20
\]

**ANSWER:**

-20

---

22. \( \frac{b+6}{4(d+c)} \)

**SOLUTION:**

\[
\frac{b+6}{4(d+c)} = \frac{-0.8+6}{4\left(\frac{1+5}{5}\right)}
\]

\[
= \frac{-0.8+6}{4\left(\frac{6}{5}\right)}
\]

\[
= \frac{-0.8+6}{4\left(\frac{26}{5}\right)}
\]

\[
= \frac{-0.8+6}{4\left(\frac{52}{104}\right)}
\]

\[
= \frac{-0.8+6}{4\left(\frac{13}{26}\right)}
\]

\[
= \frac{-0.8+6}{4\left(\frac{1}{2}\right)}
\]

\[
= \frac{-0.8+6}{2}
\]

\[
= \frac{5.2}{2}
\]

\[
= \frac{26}{104}
\]

\[
= \frac{0.25}{0.25}
\]

**ANSWER:**

0.25
23. \( \frac{5(d + a)}{2ab^2} \)

**SOLUTION:**

\[
\frac{5(d + a)}{2ab^2} = \frac{5\left(\frac{1}{5} + (-4)\right)}{2(-4)(-0.8)^2} \\
= \frac{5\left(\frac{1}{5} - 4\right)}{(-8)(0.64)} \\
= \frac{5\left(\frac{1}{5} - 20\right)}{5} \\
= -5.12 \\
= \frac{-19}{5} \\
= -3.8 \\
\approx 3.71
\]

**ANSWER:**

\( \approx 3.71 \)

24. **SENSE-MAKING** The formula \( C = \frac{5(F - 32)}{9} \) can be used to convert temperatures in degrees Fahrenheit to degrees Celsius.

(a) Room temperature commonly ranges from 64°F to 73°F. Determine room temperature range in degrees Celsius.

(b) The normal average human body temperature is 98.6°F. A temperature above this indicates a fever. If your temperature is 42°C, do you have a fever? Explain your reasoning.

**SOLUTION:**

(a) Substitute 64 for \( F \) in the formula \( C = \frac{5(F - 32)}{9} \).

\( C = \frac{5(64 - 32)}{9} \)
\( = \frac{5(32)}{9} \)
\( = \frac{160}{9} \)
\( \approx 17.8 \)

Substitute 73 for \( F \) in the formula \( C = \frac{5(F - 32)}{9} \).

\( C = \frac{5(73 - 32)}{9} \)
\( = \frac{5(41)}{9} \)
\( = \frac{205}{9} \)
\( \approx 22.8 \)

The room temperature range is about 17.8°C to 22.8°C.

(b) Substitute 98.6 for \( F \) in the formula

\( C = \frac{5(98.6 - 32)}{9} \)
\( = \frac{5(66.6)}{9} \)
\( = \frac{333}{9} \)
\( = 37 \)

So, 98.6°F = 37°C. Since 42 > 37, this temperature indicates a fever.

**ANSWER:**

(a) 17.8°C to 22.8°C

(b) Yes; Sample answer: 98.6°F = 37°C, so a temperature above 37°C indicates a fever.
25. **GEOMETRY** The formula for the area \( A \) of a triangle with height \( h \) and base \( b \) is \( A = \frac{1}{2}bh \). Write an expression to represent the area of the triangle.

![Triangle Diagram](image)

**SOLUTION:**
\[
A = \frac{1}{2}bh = \frac{1}{2}(x + 7)(2x)
\]

**ANSWER:**
\[
\frac{1}{2}(x + 7)(2x)
\]

26. **FINANCIAL LITERACY** The profit that a business made during a year is $536,897,000. If the business divides the profit evenly for each share, estimate how much each share made if there are 10,995,000 shares.

**SOLUTION:**
Let \( x \) be the profit made by each share.
\[
x = \frac{536,897,000}{10,995,000}
\]
\[
\approx 48.83
\]
Each share made a profit of about $48.83.

**ANSWER:**
$48.83

27. **REASONING** The radius of Earth’s orbit is 93,000,000 miles.

a. Find the circumference of Earth’s orbit assuming that the orbit is a circle. The formula for the circumference of a circle is \( 2\pi r \).

**SOLUTION:**
\[
C = 2\pi r = 2\pi(93,000,000)
\]
\[
\approx 584,336,233.6
\]
The circumference of Earth’s orbit is about 584,336,233.6 miles.

b. Substitute \( C = 584,336,233.6 \) and \( V = 66,698 \) in the formula \( T = \frac{C}{V} \).

**SOLUTION:**
\[
T = \frac{584,336,233.6}{66,698}
\]
\[
\approx 8761
\]
Therefore, it takes about 8761 hours for Earth to revolve around the Sun.

c. Yes, because \( \frac{8761}{24} \approx 365 \) days or 1 year.

**ANSWER:**
a. 584,336,233.6 mi
b. 8761 h
c. Yes; \( \frac{8761}{24} = 365 \) days or 1 year
28. ANCIENT PYRAMID  The Great Pyramid in Cairo, Egypt, is approximately 146.7 meters high and each side of its base is approximately 230 meters.

**a.** Find the area of the base of the pyramid. Remember  \(A = lw\).

**b.** The volume of a pyramid is  \(\frac{1}{3}Bh\) where \(B\) is the area of the base and \(h\) is the height. What is the volume of the Great Pyramid?

**SOLUTION:**

**a.** Substitute \(l = 230\) and \(w = 230\) in the formula \(A = lw\).

\[
A = lw = (230)(230) = 52,900
\]

The base area of the pyramid is 52,900 m².

**b.** Substitute \(B = 52,900\) and \(h = 146.7\) in the formula \(V = \frac{1}{3}Bh\).

\[
V = \frac{1}{3}(52900)(146.7) = \frac{7760430}{3} = 2,586,810
\]

The volume of the pyramid is about 2,586,810 m³.

**ANSWER:**

a. 52,900 m²
b. 2,586,810 m³

Evaluate each expression if \(w = \frac{3}{4}, x = 8, y = -2,\) and \(z = 0.4\).

29. \(x^3 + 2y^4\)

**SOLUTION:**

\[
x^3 + 2y^4 = 8^3 + 2(-2)^4 = 512 + 2(16) = 512 + 32 = 544
\]

**ANSWER:**

544

30. \((x - 6z)^2\)

**SOLUTION:**

\[
(x - 6z)^2 = (8 - 6(0.4))^2 = (8 - 2.4)^2 = 5.6^2 = 31.36
\]

**ANSWER:**

31.36

31. \(2(6w - 2y) - 8z\)

**SOLUTION:**

\[
2(6w - 2y) - 8z = 2\left[6\left(\frac{3}{4}\right) - 2(-2)\right] - 8(0.4) = 2\left[\frac{18}{4} + 3\right] - 3.2 = 2\left[\frac{18 + 4(4)}{4}\right] - 3.2 = 2\left[\frac{34}{4}\right] - 3.2 = \frac{68}{4} - 3.2 = 17 - 3.2 = 13.8
\]

**ANSWER:**

13.8
1-1 Expressions and Formulas

32. \( \frac{(y + z)^2}{zw} \)

**SOLUTION:**

\[
\frac{(y + z)^2}{zw} = \frac{(-2 + 0.4)^2}{8\left(\frac{3}{4}\right)}
\]

\[
= \frac{(-1.6)^2}{(2)(3)}
\]

\[
= \frac{2.56}{6}
\]

\[
\approx 0.427
\]

**ANSWER:**

\( \approx 0.427 \)

33. \( \frac{12w - 6y}{z^2} \)

**SOLUTION:**

\[
\frac{12w - 6y}{z^2} = \frac{12\left(\frac{3}{4}\right) - 6(-2)}{(0.4)^2}
\]

\[
= \frac{(3)(3) + 12}{0.16}
\]

\[
= \frac{9 + 12}{0.16}
\]

\[
= \frac{21}{0.16}
\]

\[
= 131.25
\]

**ANSWER:**

131.25

34. \( \frac{wx + yz}{wx - yz} \)

**SOLUTION:**

\[
\frac{wx + yz}{wx - yz} = \frac{\left(\frac{3}{4}\right)(8) + (-2)(0.4)}{\left(\frac{3}{4}\right)(8) - (-2)(0.4)}
\]

\[
= \frac{(3)(2) - 0.8}{(3)(2) + 0.8}
\]

\[
= \frac{6 - 0.8}{6 + 0.8}
\]

\[
= \frac{5.2}{6.8}
\]

\[
\approx 0.765
\]

**ANSWER:**

\( \approx 0.765 \)

35. **GEOMETRY** The formula for the volume \( V \) of a cone with radius \( r \) and height \( h \) is \( V = \frac{1}{3}\pi r^2 h \). Write an expression for the volume of the cone shown.

![Cone Diagram](image)

**SOLUTION:**

\[
r = \frac{6x}{2} = 3x
\]

The expression for the radius of the cone is \( 3x \).

\[
V = \frac{1}{3}\pi (3x)^2 h
\]

\[
= \frac{1}{3}\pi (9x^2)(2x)
\]

\[
= \frac{18\pi x^3}{3}
\]

\[
= 6\pi x^3
\]

The expression for the volume of the cone is \( 6\pi x^3 \).

**ANSWER:**

\( 6\pi x^3 \)
36. **SEARCH ENGINES** Page rank is a numerical value that represents how important a page is on the Web. One formula used to calculate the page rank for a page is $PR = 0.15 + 0.85L$, where $L$ is the page rank of the linking page divided by the number of outbound links on the page. Determine the page rank of a page in which $L = 10$.

**SOLUTION:**
Substitute $L = 10$ in the formula $PR = 0.15 + 0.85L$.

$PR = 0.15 + 0.85(10)$

$= 0.15 + 8.5$

$= 8.65$

**ANSWER:**
8.65

37. **WEATHER** In 1898, A.E. Dolbear studied various species of crickets to determine their “chirp rate” based on temperatures. He determined that the formula $t = 50 + \frac{n - 40}{4}$ where $n$ is the number of chirps per minute, could be used to find the temperature $t$ in degrees Fahrenheit. What is the temperature if the number of chirps is 120?

**SOLUTION:**
Substitute $n = 120$ in the formula $t = 50 + \frac{n - 40}{4}$.

$t = 50 + \frac{120 - 40}{4}$

$= 50 + \frac{80}{4}$

$= 50 + 20$

$= 70$

The temperature is 70°F.

**ANSWER:**
70°F

38. **FOOTBALL** The following formula can be used to calculate a quarterback efficiency rating.

$$\left( \frac{C - 0.3}{A} + \frac{Y - 3}{4} + \frac{T}{A} + \frac{0.095 - I}{0.044} \right) \cdot \frac{100}{6}$$

- $C$ is the number of passes completed.
- $A$ is the number of passes attempted.
- $Y$ is passing yardage.
- $T$ is the number of touchdown passes.
- $I$ is the number of interceptions.

Find Peyton Manning’s efficiency rating to the nearest tenth for the season statistics shown.

**SOLUTION:**
Simplify the equation by multiplying each term by $\frac{A}{A}$.

Then find a common denominator in order to add the fractions.

$$\left( \frac{C - 0.3}{A} + \frac{Y - 3}{4} + \frac{T}{A} + \frac{0.095 - I}{0.044} \right) \cdot \frac{100}{6}$$

Substitute $C = 371$, $A = 555$, $Y = 4002$, $T = 27$, and $I = 12$.

$$\left( \frac{20C + 0.5A + 4Y + 100I}{A} \right) \frac{100}{6}$$

Substitute $C = 371$, $A = 555$, $Y = 4002$, $T = 27$, and $I = 12$.

$$\left( \frac{20(371) + 0.5(555) + 4(4002) + 100(12)}{555} \right) \frac{100}{6}$$

$$= \frac{7420 + 2275 + 16000 + 1200}{555} \cdot \frac{100}{6}$$

$$= \frac{49565}{555} \cdot \frac{100}{6}$$

$$= 27.87 \cdot 100$$

$$= 2787$$

$$\approx 95.0$$

Peyton Manning’s efficiency rating is about 95.0.

**ANSWER:**
95.0

39. **MOVIES** The average price for a movie ticket can be represented by $P = \frac{y^2}{400} + \frac{7y}{100} + 2.96$ where $y$ is the number of years since 1980.

**a.** Find the average price of a ticket in 1990, 2000, and 2010.

**b.** Another equation that can be used to represent
Evaluate each expression if $a = -2$, $b = 3$, and $c = 4.2$. 

1. SOLUTION: 
ANSWER: 
$rac{y^3 - y^2 + 6y}{2500} + 2.96$. 

2. SOLUTION: 
ANSWER: 
$rac{y^3 - y^2 + 6y}{2500} + 2.96$. 

66. SOLUTION: 
ANSWER: 
$rac{y^3 - y^2 + 6y}{2500} + 2.96$. 

Determine room temperature range is the average of the two: 

b. Substitute $y = 10$ in the equation $P = \frac{y^3 - y^2 + 6y}{2500} + 2.96$. 

So, the average price of a ticket in 1990 was $3.91. 
Substitute $y = 20$ in the equation $P = \frac{y^3 - y^2 + 6y}{2500} + 2.96$. 

So, the average price of a ticket in 2000 was $5.36. 
Substitute $y = 30$ in the equation $P = \frac{y^3 - y^2 + 6y}{2500} + 2.96$. 

So, the average price of a ticket in 2010 will be $7.31. 

Sample answer: The average prices found in part (a) become increasingly higher with time.

ANSWER: 

a. $3.91; 5.36; 7.31$

b. $4.42; 6.62; 11.62; 13.8$
1-1 Expressions and Formulas

40. GEOMETRY The area of a triangle can be found using Heron’s Formula,
\[ A = \sqrt{s(s-a)(s-b)(s-c)} \], where \( a, b, \) and \( c \) are the lengths of the three sides of the triangle, and \( s = \frac{a+b+c}{2} \). Find the area of the triangle at the right.

\[ 6 \text{ in.} \quad 11 \text{ in.} \quad 14 \text{ in.} \]

**SOLUTION:**
\[
s = \frac{11 + 14 + 6}{2} = \frac{31}{2} = 15.5
\]
\[
A = \sqrt{15.5(15.5-11)(15.5-14)(15.5-6)} = \sqrt{15.5(4.5)(1.5)(9.5)} = \sqrt{993.9375} \approx 31.5
\]
The area of the triangle is about 31.5 square inches.

**ANSWER:**
31.5 in\(^2\)

41. Evaluate \( y = \sqrt{\frac{b^2(1-x^2)}{a^2}} \) if \( a = 6, b = 8 \), and \( x = 3 \). Round to the nearest tenth.

**SOLUTION:**
\[
y = \sqrt{\frac{8^2(1-3^2)}{6^2}} = \sqrt{\frac{64(1-9)}{36}} = \sqrt{\frac{64(-8)}{36}} = \sqrt{\frac{27}{36}} = \sqrt{\frac{1728}{36}} = \sqrt{48} \approx 6.9
\]

**ANSWER:**
6.9

42. MULTIPLE REPRESENTATIONS You will write expressions using the formula for the volume of a cylinder. Recall that the volume of a cylinder can be found using the formula \( V = \pi r^2 h \), in which \( V \) = volume, \( r \) = radius, and \( h \) = height.

a. GEOMETRIC Draw two cylinders of different sizes.

b. TABULAR Use a ruler to measure the radius and height of each cylinder. Organize the measures for each cylinder into a table. Include a column in your table to calculate the volume of each cylinder.

c. VERBAL Write a verbal expression for the difference in volume of the two cylinders.

d. ALGEBRAIC Write and solve an algebraic expression for the difference in volume of the two cylinders.

**SOLUTION:**

a. Sample answer:
1-1 Expressions and Formulas

b. Sample answer: make a table to record the radius, height, and volume of each cylinder.

<table>
<thead>
<tr>
<th>cylinder</th>
<th>radius</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 in.</td>
<td>5 in.</td>
<td>$20\pi \approx 62.8$ in$^3$</td>
</tr>
<tr>
<td>2</td>
<td>4 in.</td>
<td>1 in.</td>
<td>$16\pi \approx 50.3$ in$^3$</td>
</tr>
</tbody>
</table>

c. Sample answer: $\pi$ times 2 squared times 5 minus $\pi$ times 4 squared times 1.

d. Sample answer:

$$\pi(2)^2(5) - \pi(4)^2(1) = 4\pi \approx 12.5 \text{ cm}^3$$

ANSWER:

a. Sample answer:

b. Sample answer:

<table>
<thead>
<tr>
<th>cylinder</th>
<th>radius</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 in.</td>
<td>5 in.</td>
<td>$20\pi \approx 62.8$ in$^3$</td>
</tr>
<tr>
<td>2</td>
<td>4 in.</td>
<td>1 in.</td>
<td>$16\pi \approx 50.3$ in$^3$</td>
</tr>
</tbody>
</table>

c. Sample answer: $\pi$ times 2 squared times 5 minus $\pi$ times 4 squared times 1.

d. Sample answer:

$$\pi(2)^2(5) - \pi(4)^2(1) = 4\pi \approx 12.5 \text{ cm}^3$$

ANSWER:

43. CRITIQUE Lauren and Rico are evaluating $\frac{-3d - 4c}{2ab}$ for $a = -2$, $b = -3$, $c = 5$, and $d = 4$. Is either of them correct? Explain your reasoning.

**SOLUTION:**

Lauren and Rico each wrote the equation correctly but Lauren is correct. She properly evaluated $-12 - 20 = -32$ while Rico evaluated this expression as 8.

**ANSWER:**

$-12 - 20 = -32$

44. CHALLENGE For any three distinct numbers $a$, $b$, and $c$, $a$ $b$ $c$ is defined as $a$ $b$ $c = \frac{-a - b - c}{c - b - a}$. Find $-2(-4)$ $5$.

**SOLUTION:**

$$-2(-4)$ $5 = \frac{-(-2) - (-4) - 5}{5 - (-4) - (-2)}$$

$$= \frac{2 + 4 - 5}{5 + 4 + 2}$$

$$= \frac{6 - 5}{11}$$

$$= \frac{1}{11}$$

**ANSWER:**

$\frac{1}{11}$
45. **REASONING** The following equivalent expressions represent the height in feet of a stone thrown downward off a bridge where \( t \) is the time in seconds after release. Which do you find most useful for finding the maximum height of the stone? Explain.

**a.** \(-4t^2 - 2t + 6\)

**b.** \(-2(2t + 1) + 6\)

**c.** \(-2(t - 1)(2t + 3)\)

**SOLUTION:**
Sample answer: \( b \); Since \( t \) is time, it must be nonnegative. So \(-2t(2t + 1)\) will be negative for all values of \( t \) other than 0. The maximum value of \(-2t(2t + 1)\) is 0, which occurs when \( t = 0 \). Thus, the maximum value of \(-2t(2t + 1) + 6\) is 6.

**ANSWER:**
Sample answer: \( b \); Since \( t \) is time, it must be nonnegative. So \(-2t(2t + 1)\) will be negative for all values of \( t \) other than 0. The maximum value of \(-2t(2t + 1)\) is 0, which occurs when \( t = 0 \). Thus, the maximum value of \(-2t(2t + 1) + 6\) is 6.

46. **CHALLENGE** Let \( m \), \( n \), \( p \), and \( q \) represent nonzero positive integers. Find a number in terms of \( m \), \( n \), \( p \), and \( q \) that is halfway between \( \frac{m}{n} \) and \( \frac{p}{q} \).

**SOLUTION:**
Halfway between \( \frac{m}{n} \) and \( \frac{p}{q} \) is the average of the two:

\[
\frac{1}{2} \left( \frac{m}{n} + \frac{p}{q} \right) = \frac{1}{2} \left( \frac{mq + pn}{nq} \right) = \frac{mq + pn}{2nq}
\]

**ANSWER:**
Halfway between \( \frac{m}{n} \) and \( \frac{p}{q} \) is the average the two:

\[
\frac{1}{2} \left( \frac{m}{n} + \frac{p}{q} \right) = \frac{mq + pn}{2nq} .
\]

47. **OPEN ENDED** Write an algebraic expression using \( x = -2 \), \( y = -3 \), and \( z = 4 \) and all four operations for which the value of the expression is 10.

**SOLUTION:**
Sample answer:

\[
y \left( \frac{-4z}{x^2} - x \right) + z
\]

\[
= -3 \left( \frac{-4(4)}{(-2)^2} - (-2) \right) + 4
\]

\[
= -3 \left( \frac{-16}{4} + 2 \right) + 4
\]

\[
= -3(-4 + 2) + 4
\]

\[
= 6 + 4 = 10
\]

**ANSWER:**
Sample answer: \( y \left( \frac{-4z}{x^2} - x \right) + z \)

48. **WRITING IN MATH** Provide an example of a formula used in everyday situations. Explain the usefulness of this formula and what happens if the formula is not used correctly.

**SOLUTION:**
Sample answer: A formula is used to calculate the price of filling a gasoline tank in which the price = number of gallons \( \times \) price per gallon. If calculated incorrectly, you may underestimate or overestimate how much you will need to pay.

**ANSWER:**
Sample answer: A formula is used to calculate the price of filling a gasoline tank in which the price = number of gallons \( \times \) price per gallon. If calculated incorrectly, you may underestimate or overestimate how much you will need to pay.
1-1 Expressions and Formulas

49. **WRITING IN MATH** Use the information for on-base percentage given at the beginning of the lesson to explain how formulas are used in baseball to calculate a player's stats. Explain why a formula for on-base percentage is more useful than a table of specific percentages.

**SOLUTION:**
A table of on-base percentages is limited to those situations listed, while a formula can be used to find any on-base percentage.

**ANSWER:**
A table of on-base percentages is limited to those situations listed, while a formula can be used to find any on-base percentage.

50. **SAT/ACT** If the area of a square with side \( x \) is 9, what is the area of a square of side \( 4x \)?
   
   - **A** 36
   - **B** 144
   - **C** 212
   - **D** 324
   - **E** 1296

   **SOLUTION:**
   \[
   A = (4x)^2 = 16x^2
   \]
   \[
   = 16(9)
   \]
   \[
   = 144
   \]
   The area of a square of side \( 4x \) is 144 square units. The correct choice is B.

   **ANSWER:**
   B

51. **SHORT RESPONSE** A coffee shop owner wants to open a second shop when his daily customer average reaches 800 people. He has calculated the daily customer average in the table below for each month since he has opened.

<table>
<thead>
<tr>
<th>Month</th>
<th>Daily Customer Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>225</td>
</tr>
<tr>
<td>2</td>
<td>298</td>
</tr>
<tr>
<td>3</td>
<td>371</td>
</tr>
<tr>
<td>4</td>
<td>444</td>
</tr>
<tr>
<td>5</td>
<td>444 + 73 = 517</td>
</tr>
<tr>
<td>6</td>
<td>517 + 73 = 590</td>
</tr>
<tr>
<td>7</td>
<td>590 + 73 = 663</td>
</tr>
<tr>
<td>8</td>
<td>663 + 73 = 736</td>
</tr>
<tr>
<td>9</td>
<td>736 + 73 = 809</td>
</tr>
</tbody>
</table>

   If the trend continues, during what month can he open a second shop?

   **SOLUTION:**
   Every month, the daily customer average increases by 73.

<table>
<thead>
<tr>
<th>Month</th>
<th>Daily Customer Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>444 + 73 = 517</td>
</tr>
<tr>
<td>6</td>
<td>517 + 73 = 590</td>
</tr>
<tr>
<td>7</td>
<td>590 + 73 = 663</td>
</tr>
<tr>
<td>8</td>
<td>663 + 73 = 736</td>
</tr>
<tr>
<td>9</td>
<td>736 + 73 = 809</td>
</tr>
</tbody>
</table>

   So, he can open a second shop during the 9th month.

   **ANSWER:**
   month 9
52. **GEOMETRY** In ΔDFG, FH and HG are angle bisectors and m∠D = 84. How many degrees are in ∠FHG?

**SOLUTION:**
Let m∠F = 2x, m∠G = 2y.

\[2x + 2y + 84 = 180\]
\[2(x + y + 42) = 180\]
\[x + y + 42 = 90\]
\[x + y + 42 - 42 = 90 - 42\]
\[x + y = 48\]

Use the Triangle Sum Theorem for the triangle FHG.

\[m∠FHG + (x + y) = 180\]
\[m∠FHG + 48 = 180\]
\[m∠FHG + 48 - 48 = 180 - 48\]
\[m∠FHG = 132\]

The correct choice is G.

**ANSWER:**
G

53. A skydiver in a computer game free-falls from a height of 3000 m at a rate of 55 meters per second. Which equation can be used to find h, the height of the skydiver after t seconds of free fall?

A h = −55t − 3000  
B h = −55t + 3000  
C h = 3000t − 55  
D h = 3000t + 55

**SOLUTION:**
Since the skydiver falls from a height of 3000 m at a rate of 55 meters per second, the height after t seconds of free fall represents the equation h = −55t + 3000. The correct choice is B.

**ANSWER:**
B

54. The lengths of the three sides of a triangle are 10, 14, and 18 inches. Determine whether this triangle is a right triangle.

**SOLUTION:**
Because the longest side is 18 inches, use 18 as c, the measure of the hypotenuse.

\[c^2 = a^2 + b^2\]

\[18^2 = 10^2 + 14^2\]

\[324 = 100 + 196\]

\[324 ≠ 296\]

Because \( c^2 ≠ a^2 + b^2 \), the triangle is not a right triangle.

**ANSWER:**
no

55. The legs of a right triangle measure 6 centimeters and 8 centimeters. Find the length of the hypotenuse.

**SOLUTION:**
\[c^2 = a^2 + b^2\]

\[c^2 = 6^2 + 8^2\]

\[c^2 = 36 + 64\]

\[c^2 = 100\]

\[c = \sqrt{100}\]

\[c = 10\]

The length of the hypotenuse is 10 cm.

**ANSWER:**
10 cm
56. **MAPS** On a map of the U.S., the cities Milwaukee, Wisconsin, and Charlotte, North Carolina are \( \frac{1}{2} \) inches apart. The actual distance between Milwaukee and Charlotte is 670 miles. If Birmingham, Alabama and St. Petersburg Florida are 465 miles apart, how far apart are they on the map?

**SOLUTION:**
Let \( x \) be the length of the reduced photo.

\[
\frac{6 \frac{1}{2}}{670} = \frac{x}{465} \\
\frac{6.5}{670} = \frac{x}{465} \\
x = \frac{6.5 \times 465}{670} = \frac{3022.5}{670} \\
\approx 4.5
\]

The length of the reduced photo will be \( 4 \frac{1}{2} \) inches.

**ANSWER:**
about \( 4 \frac{1}{2} \) in.

57. Factor \( 6x^2 + 12x \).

**SOLUTION:**
\[
6x^2 = 2 \cdot 3 \cdot x \cdot x \\
12x = 2 \cdot 2 \cdot 3 \cdot x
\]
The GCF of the terms \( 6x^2 \) and \( 12x \) is \( 6x \).
\[
6x^2 + 12x = 6x(x) + 6x(2) = 6x(x + 2)
\]

**ANSWER:**
\( 6x(x + 2) \)

58. Find the product of \( (a + 2)(a - 4) \).

**SOLUTION:**
Use the FOIL method to find the product.
\[
(a + 2)(a - 4) = (a)(a) + a(-4) + 2(a) + 2(-4) \\
= a^2 - 4a + 2a - 8 \\
= a^2 - 2a - 8
\]

**ANSWER:**
\( a^2 - 2a - 8 \)

59. **NUMBER** An integer is 2 less than a number, and another integer is 1 greater than double that same number. What are the two integers if their sum is 14?

**SOLUTION:**
Let \( x \) be a number. Therefore, the two integers in terms of \( x \) are \( (x - 2) \) and \( (2x + 1) \).
Let \( y = x - 2 \) and \( z = 2x + 1 \) and \( y + z = 14 \).
Substitute \( x = y + 2 \) in the equation \( z = 2x + 1 \).
\[
z = 2x + 1 \\
= 2(y + 2) + 1 \\
= 2y + 4 + 1 \\
= 2y + 5
\]
Since both \( z \) and \( y \) are integers, substitute any integer value for \( y \) and find the corresponding \( z \) value.
Sample answer: Substitute \( y = 3 \) in the equation \( z = 2y + 5 \).
\[
z = 2(3) + 5 \\
= 6 + 5 \\
= 11
\]
Therefore, the two integers are 3 and 11. Note that there are infinite number of such pairs of integers exist.

**ANSWER:**
3 and 11

**Evaluate each expression.**

60. \( \sqrt{4} \)

**SOLUTION:**
\[
\sqrt{4} = \sqrt{2^2} = 2
\]

**ANSWER:**
2
Evaluate each expression if \( a = -2 \), \( b = 3 \), and \( c = 4.2 \).

1. SOLUTION: \( 4.6 \)
   ANSWER: \( 4.6 \)

2. SOLUTION: \( -4 \)
   ANSWER: \( -4 \)

3. SOLUTION: 
   ANSWER: 

61. \( \sqrt{25} \)
   SOLUTION: 
   \[
   \sqrt{25} = \sqrt{5^2} \\
   = 5
   \]
   ANSWER: \( 5 \)

62. \( \sqrt{81} \)
   SOLUTION: 
   \[
   \sqrt{81} = \sqrt{9^2} \\
   = 9
   \]
   ANSWER: \( 9 \)

63. \( \sqrt{121} \)
   SOLUTION: 
   \[
   \sqrt{121} = \sqrt{11^2} \\
   = 11
   \]
   ANSWER: \( 11 \)

64. \( -\sqrt{9} \)
   SOLUTION: 
   \[
   -\sqrt{9} = -\sqrt{3^2} \\
   = -3
   \]
   ANSWER: \( -3 \)

65. \( -\sqrt{16} \)
   SOLUTION: 
   \[
   -\sqrt{16} = -\sqrt{4^2} \\
   = -4
   \]
   ANSWER: \( -4 \)

66. \( \sqrt{\frac{49}{100}} \)
   SOLUTION: 
   \[
   \sqrt{\frac{49}{100}} = \frac{\sqrt{49}}{\sqrt{100}} \\
   = \frac{7}{\sqrt{10^2}} \\
   = \frac{7}{10}
   \]
   ANSWER: \( \frac{7}{10} \)

67. \( \sqrt{\frac{25}{64}} \)
   SOLUTION: 
   \[
   \sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} \\
   = \frac{\sqrt{5^2}}{\sqrt{8^2}} \\
   = \frac{5}{8}
   \]
   ANSWER: \( \frac{5}{8} \)
1. 62

**SOLUTION:**
The number 62 is a real number. Since 62 can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is part of the set \{ –2, –1, 0, 1, 2, … \} so it is an integer. It is part of the set \{ 0, 1, 2, 3, … \} so it is a whole number and since it is not 0 it is also a natural number.

**ANSWER:** N, W, Z, Q, R

2. \( \frac{5}{4} \)

**SOLUTION:**
The number \( \frac{5}{4} \) is a real number. Since \( \frac{5}{4} \) can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is not a part of the set \{ –2, –1, 0, 1, 2, … \} so it is not an integer. Since it is not a part of the set \{ 0, 1, 2, 3, … \} it is not a whole number or a natural number.

**ANSWER:** Q, R

3. \( \sqrt{11} \)

**SOLUTION:**
\( \sqrt{11} \) is a real number. Since it is a nonterminating decimal it is irrational.

**ANSWER:** I, R

4. –12

**SOLUTION:**
The number -12 is a real number. Since -12 can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is part of the set \{ …–2, –1, 0, 1, 2, … \} so it is an integer. It is not part of the set \{ …0, 1, 2, 3, … \} so it is not a whole number and since it is not a whole number it is not a natural number either.

**ANSWER:** Z, Q, R

5. \( (6 \cdot 8) \cdot 5 = 6 \cdot (8 \cdot 5) \)

**SOLUTION:**
Associative Property of Multiplication; the way the factors are grouped does not affect the product.

**ANSWER:** Assoc. (\( \times \))

6. \( 7(9 - 5) = 7 \cdot 9 - 7 \cdot 5 \)

**SOLUTION:**
Distributive Property; the Distributive Property states that there is no difference between a term multiplied by each term in a group and the term multiplied by the group.

**ANSWER:** Dist.

7. \( 84 + 16 = 16 + 84 \)

**SOLUTION:**
Commutative Property of Addition; the Commutative Property of Addition states that the order in which terms are added does not affect the sum.

**ANSWER:** Comm. (+)
**Properties of Real Numbers**

8. \((12 + 5)6 = 12 \cdot 6 + 5 \cdot 6\)

**SOLUTION:**
Distributive Property; the Distributive Property states that there is no difference between a term multiplied by each term in a group and the term multiplied by the group.

**ANSWER:**
\(\text{Dist.}\)

**Find the additive inverse and multiplicative inverse for each number.**

9. \(-7\)

**SOLUTION:**
Since \(-7 + 7 = 0\), the additive inverse of \(-7\) is \(7\).
Since \((-7)\left(-\frac{1}{7}\right) = 1\), the multiplicative inverse of \(-7\) is \(-\frac{1}{7}\).

**ANSWER:**
\(7; -\frac{1}{7}\)

10. \(\frac{4}{9}\)

**SOLUTION:**
Since \(\frac{4}{9} + \left(-\frac{4}{9}\right) = 0\), the additive inverse of \(\frac{4}{9}\) is \(-\frac{4}{9}\).
Since \(\left(\frac{4}{9}\right)\left(\frac{9}{4}\right) = 1\), the multiplicative inverse of \(\frac{4}{9}\) is \(\frac{9}{4}\).

**ANSWER:**
\(-\frac{4}{9}; \frac{9}{4}\)

11. \(3.8\)

**SOLUTION:**
Since \(3.8 + (-3.8) = 0\), the additive inverse of 3.8 is \(-3.8\).
Since \(3.8\left(\frac{1}{3.8}\right) = 1\), the multiplicative inverse of 3.8 is \(\frac{1}{3.8}\).

**ANSWER:**
\(-3.8; \frac{1}{3.8}\)

12. \(\sqrt{5}\)

**SOLUTION:**
Since \(\sqrt{5} + (-\sqrt{5}) = 0\), the additive inverse of \(\sqrt{5}\) is \(-\sqrt{5}\).
Since \(\sqrt{5}\left(\frac{1}{\sqrt{5}}\right) = 1\), the multiplicative inverse of \(\sqrt{5}\) is \(\frac{1}{\sqrt{5}}\).

**ANSWER:**
\(-\sqrt{5}; \frac{1}{\sqrt{5}}\)

13. **REASONING** Melba is mowing lawns for $22 each to earn money for a video game console that costs $550.
   a. Write an expression to represent the total amount of money Melba earned during this week.
   b. Evaluate the expression from part a by using the Distributive Property.
   c. When do you think Melba will earn enough for the video game console? Is this reasonable? Explain.
1-2 Properties of Real Numbers

<table>
<thead>
<tr>
<th>Lawns Mowed in One Week</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Day</strong></td>
</tr>
<tr>
<td>Monday</td>
</tr>
<tr>
<td>Tuesday</td>
</tr>
<tr>
<td>Wednesday</td>
</tr>
<tr>
<td>Thursday</td>
</tr>
<tr>
<td>Friday</td>
</tr>
<tr>
<td>Saturday</td>
</tr>
<tr>
<td>Sunday</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**a.** To write the expression, add the number of lawns mowed throughout the week and multiply by $22.22(2 + 4 + 3 + 1 + 5 + 6 + 7)

**b.** Use the Distributive Property to evaluate the expression $22(2 + 4 + 3 + 1 + 5 + 6 + 7).

$22(2 + 4 + 3 + 1 + 5 + 6 + 7) = 22(2) + 22(4) + 22(3) + 22(1) + 22(5) + 22(6) + 22(7) = 44 + 88 + 66 + 22 + 110 + 132 + 154 = 616$

Melba earns $616.

**c.** If she continues to mow the same number of lawns, at the end of next week she will have the money. This may not be reasonable because not all the lawns she mowed this week may need to be mowed again next week.

**ANSWER:**

**a.** $22(2 + 4 + 3 + 1 + 5 + 6 + 7)$ or $22(2) + 22(4) + 22(3) + 22(1) + 22(5) + 22(6) + 22(7)$

**b.** $616$

**c.** If she continues to mow the same number of lawns, at the end of next week she will have the money. This may not be reasonable because not all the lawns she mowed this week may need to be mowed again next week.

**Simplify each expression.**

14. $5(3x + 6y) + 4(2x - 9y)$

**SOLUTION:**

$5(3x + 6y) + 4(2x - 9y) = 5(3x) + 5(6y) + 4(2x) + 4(-9y) = 15x + 30y + 8x - 36y = 15x + 8x + 30y - 36y = (15 + 8)x + (30 - 36)y = 23x - 6y$

**ANSWER:**

$23x - 6y$

15. $6(6a + 5b) - 3(4a + 7b)$

**SOLUTION:**

$6(6a + 5b) - 3(4a + 7b) = 6(6a) + 6(5b) - 3(4a) - 3(7b) = 36a + 30b - 12a - 21b = 36a - 12a + 30b - 21b = (36 - 12)a + (30 - 21)b = 24a + 9b$

**ANSWER:**

$24a + 9b$

16. $-4(6c - 3d) - 5(-2c - 4d)$

**SOLUTION:**

$-4(6c - 3d) - 5(-2c - 4d) = (-4)(6c) + (-4)(-3d) + (-5)(-2c) + (-5)(-4d) = -24c + 12d + 10c + 20d = -24c + 10c + 12d + 20d = (-24 + 10)c + (12 + 20)d = -14c + 32d$

**ANSWER:**

$-14c + 32d$
1-2 Properties of Real Numbers

17. \(-5(8x - 2y) - 4(-6x - 3y)\)

**SOLUTION:**
\[
\begin{align*}
-5(8x - 2y) - 4(-6x - 3y) &= -40x + 10y + 24x + 12y \\
&= (-40 + 24)x + (10 + 12)y \\
&= -16x + 22y
\end{align*}
\]

**ANSWER:**
\(-16x + 22y\)

Name the sets of numbers to which each number belongs.

18. \(-\frac{4}{3}\)

**SOLUTION:**
The number \(-\frac{4}{3}\) is a real number. Since \(-\frac{4}{3}\) can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is not a part of the set \{\(-2, -1, 0, 1, 2, \ldots\)\} so it is not an integer. Since it is not a part of the set \{0, 1, 2, 3, \ldots\} it is not a whole number or a natural number.

**ANSWER:**
Q, R

19. \(-8.13\)

**SOLUTION:**
The number -8.13 is a real number. Since -8.13 can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is not a part of the set \{\(-2, -1, 0, 1, 2, \ldots\)\} so it is not an integer. Since it is not a part of the set \{0, 1, 2, 3, \ldots\} it is not a whole number or a natural number.

**ANSWER:**
Q, R

20. \(\sqrt{25}\)

**SOLUTION:**
Since \(\sqrt{25} = 5\), this is a real number. Since 5 can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is part of the set \{\(-2, -1, 0, 1, 2, \ldots\)\} so it is an integer. It is part of the set \{0, 1, 2, 3, \ldots\} so it is a whole number and since it is not 0 it is also a natural number.

**ANSWER:**
N, W, Z, Q, R

21. \(0.61\)

**SOLUTION:**
The number 0.61 is a real number. Since 0.61 is a repeating decimal it can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is not a part of the set \{\(-2, -1, 0, 1, 2, \ldots\)\} so it is not an integer. Since it is not a part of the set \{0, 1, 2, 3, \ldots\} it is not a whole number or a natural number.

**ANSWER:**
Q, R

22. \(\frac{9}{3}\)

**SOLUTION:**
The number \(\frac{9}{3} = 3\) and is a real number. Since 3 can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is part of the set \{\(-2, -1, 0, 1, 2, \ldots\)\} so it is an integer. It is part of the set \{0, 1, 2, 3, \ldots\} so it is a whole number and since it is not 0 it is also a natural number.

**ANSWER:**
N, W, Z, Q, R
23. \(-\sqrt{144}\)

**SOLUTION:**
The number \(-\sqrt{144} = -12\) is a real number. Since \(-12\) can be expressed as a ratio \(\frac{a}{b}\) where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is part of the set \{\ldots-2, -1, 0, 1, 2, \ldots\} so it is an integer. It is not part of the set \{\ldots0, 1, 2, 3, \ldots\} so it is not a whole number and since it is not a whole number it is not a natural number either.

\(Z, Q, R\)

**ANSWER:**
\(Z, Q, R\)

24. \(\frac{21}{7}\)

**SOLUTION:**
The number \(\frac{21}{7} = 3\) and is a real number. Since 3 can be expressed as a ratio \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b\) is not 0 it is also a rational number. It is part of the set \{\ldots-2, -1, 0, 1, 2, \ldots\} so it is an integer. It is part of the set \{\ldots0, 1, 2, 3, \ldots\} so it is a whole number and since it is not 0 it is also a natural number.

\(N, W, Z, Q, R\)

**ANSWER:**
\(N, W, Z, Q, R\)

25. \(\sqrt{17}\)

**SOLUTION:**
\(\sqrt{17}\) is a real number. Since it is a nonterminating decimal it is irrational.

\(I, R\)

**ANSWER:**
\(I, R\)

26. \(-7y + 7y = 0\)

**SOLUTION:**
Additive Inverse Property; the Additive Inverse Property states that a number added to its opposite is zero.

**ANSWER:**
Inverse (+)

27. \(8\sqrt{11} + 5\sqrt{11} = (8 + 5)\sqrt{11}\)

**SOLUTION:**
Distributive Property; the Distributive Property states that there is no difference between a term multiplied by each term in a group and the term multiplied by the group.

**ANSWER:**
Dist.

28. \((16 + 7) + 23 = 16 + (7 + 23)\)

**SOLUTION:**
Associative Property of Addition; the Associative Property of Addition states that the way the factors are grouped does not affect the sum.

**ANSWER:**
Assoc. (+)

29. \(\left(\frac{22}{7}\right)\left(\frac{7}{22}\right) = 1\)

**SOLUTION:**
Multiplicative Inverse Property; the Multiplicative Inverse Property states that a number multiplied by its reciprocal is 1.

**ANSWER:**
Inverse \((\times)\)
1-2 Properties of Real Numbers

Find the additive inverse and multiplicative inverse for each number.

30. \(-8\)

**SOLUTION:**
Since \(-8 + 8 = 0\), the additive inverse of \(-8\) is 8.

Since \(-8 \left(\frac{1}{8}\right) = 1\), the multiplicative inverse of \(-8\) is \(\frac{1}{8}\).

**ANSWER:**
8; \(\frac{1}{8}\)

31. \(12.1\)

**SOLUTION:**
Since \(12.1 + (-12.1) = 0\), the additive inverse of \(12.1\) is \(-12.1\).

Since \(12.1 \left(\frac{1}{12.1}\right) = 1\), the multiplicative inverse of \(12.1\) is \(\frac{1}{12.1}\).

**ANSWER:**
\(-12.1; \frac{1}{12.1}\)

32. \(-0.25\)

**SOLUTION:**
Since \(-0.25 + 0.25 = 0\), the additive inverse of \(-0.25\) is 0.25.

Since \(-0.25 \left(-\frac{1}{0.25}\right) = -0.25(-4) = 1\), the multiplicative inverse of \(-0.25\) is \(-4\).

**ANSWER:**
0.25; \(-4\)

33. \(6\)

**SOLUTION:**
Since \(6 - 6 = 0\), the additive inverse of \(6\) is \(6\).

Since \(6 \left(\frac{1}{6}\right) = 1\), the multiplicative inverse of \(6\) is \(\frac{1}{6}\).

**ANSWER:**
6; \(\frac{1}{6}\)

34. \(-\frac{3}{8}\)

**SOLUTION:**
Since \(-\frac{3}{8} + \frac{3}{8} = 0\), the additive inverse of \(-\frac{3}{8}\) is \(\frac{3}{8}\).

Since \(-\frac{3}{8} \left(-\frac{8}{3}\right) = 1\), the multiplicative inverse of \(-\frac{3}{8}\) is \(\frac{8}{3}\).

**ANSWER:**
\(-\frac{3}{8}; \frac{8}{3}\)

35. \(\sqrt{15}\)

**SOLUTION:**
Since \(\sqrt{15} - \sqrt{15} = 0\), the additive inverse of \(\sqrt{15}\) is \(-\sqrt{15}\).

Since \(\sqrt{15} \left(\frac{1}{\sqrt{15}}\right) = 1\), the multiplicative inverse of \(\sqrt{15}\) is \(\frac{1}{\sqrt{15}}\).

**ANSWER:**
\(-\sqrt{15}; \frac{1}{\sqrt{15}}\)

36. **CONSTRUCTION** Jorge needs two different kinds
of concrete: quick drying and slow drying. The quick
drying concrete mix calls for \(2\frac{1}{2}\) pounds of dry
cement, and the slow-drying concrete mix calls for
\(1\frac{1}{4}\) pounds of dry cement. He needs 5 times more
quick-drying concrete and 3 times more slow-drying
cement than the mixes make.

a. How many pounds of dry cement mix will he
need?

b. Use the properties of real numbers to show how
Jorge could compute this amount mentally. Justify
each step.

**SOLUTION:**

a. Write an expression. Jorge needs 5 times the
amount of dry cement, \(2\frac{1}{2}\), for the quick-drying mix
plus 3 times the amount of dry cement, \(1\frac{1}{4}\), for the
slow-drying mix.

\[
5 \left(2\frac{1}{2}\right) + 3 \left(1\frac{1}{4}\right) = 5 \left(\frac{5}{2}\right) + 3 \left(\frac{5}{4}\right)
\]

\[
= \frac{25}{2} + \frac{15}{4}
\]

\[
= \frac{25 \cdot 4}{2 \cdot 4} + \frac{15}{4}
\]

\[
= \frac{50 + 15}{4}
\]

\[
= \frac{65}{4}
\]

\[
= 16\frac{1}{4}
\]

He will need \(16\frac{1}{4}\) pounds of dry cement.

b. Simplify each expression.

37. \(8b - 3c + 4b + 9c\)

**SOLUTION:**

\(8b - 3c + 4b + 9c = 8b + 4b - 3c + 9c\)

\(= (8 + 4)b + (-3 + 9)c\)

\(= 12b + 6c\)

**ANSWER:**

\(12b + 6c\)

38. \(-2a + 9d - 5a - 6d\)

**SOLUTION:**

\(-2a + 9d - 5a - 6d = -2a - 5a + 9d - 6d\)

\(= (-2 - 5)a + (9 - 6)d\)

\(= -7a + 3d\)

**ANSWER:**

\(-7a + 3d\)
39. \(4(4x - 9y) + 8(3x + 2y)\)

**SOLUTION:**
\[4(4x - 9y) + 8(3x + 2y) = 4(4x) + 4(-9y) + 8(3x) + 8(2y) = 16x - 36y + 24x + 16y = 16x + 24x - 36y + 16y = (16 + 24)x + (-36 + 16)y = 40x - 20y\]

**ANSWER:**
\[40x - 20y\]

40. \(6(9a - 3b) - 8(2a + 4b)\)

**SOLUTION:**
\[6(9a - 3b) - 8(2a + 4b) = 6(9a) - 6(-3b) + (-8)(2a) + (-8)(4b) = 54a - 18b - 16a - 32b = 54a - 16a - 18b - 32b = (54 - 16)a + (-18 - 32)b = 38a - 50b\]

**ANSWER:**
\[38a - 50b\]

41. \(-2(-5g + 6k) - 9(-2g + 4k)\)

**SOLUTION:**
\[-2(-5g + 6k) - 9(-2g + 4k) = (-2)(-5g) + (-2)(6k) + (-9)(-2g) + (-9)(4k) = 10g - 12k + 18g - 36k = 10g + 18g - 12k - 36k = (10 + 18)g + (-12 - 36)k = 28g - 48k\]

**ANSWER:**
\[28g - 48k\]

42. \(-5(10x + 8z) - 6(4x - 7z)\)

**SOLUTION:**
\[-5(10x + 8z) - 6(4x - 7z) = (-5)(10x) + (-5)(8z) + (-6)(4x) + (-6)(-7z) = -50x - 40z - 24x + 42z = -50x - 24x - 40z + 42z = (-50 - 24)x + (-40 + 42)z = -74x + 2z\]

**ANSWER:**
\[-74x + 2z\]

43. **FOOTBALL** Illustrate the Distributive Property by writing two expressions for the area of a college football field. Then find the area of the football field.

**SOLUTION:**
The width of the football field is 35 yards and the length is \((60 + 60)\) yards.
The expression for the area of the field is \(35(60 + 60)\) square yards. Use the Distributive Property to rewrite the expression.
\[53(60 + 60) = 53(120) = 6360\]
So, the area of the field is 6360 square yards.

**ANSWER:**
\[53(60 + 60); 53(60) + 53(60); 6360 \text{yd}^2\]
1-2 Properties of Real Numbers

44. **PETS** The chart shows the percent of dogs registered with the American Kennel Club that are of the eight most popular breeds.

<table>
<thead>
<tr>
<th>Breed</th>
<th>Percent of Registered Dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labrador Retrievers</td>
<td>14.2</td>
</tr>
<tr>
<td>Yorkshire Terriers</td>
<td>5.6</td>
</tr>
<tr>
<td>German Shepherds</td>
<td>5.0</td>
</tr>
<tr>
<td>Golden Retrievers</td>
<td>4.9</td>
</tr>
<tr>
<td>Beagles</td>
<td>4.5</td>
</tr>
<tr>
<td>Dachshunds</td>
<td>4.1</td>
</tr>
<tr>
<td>Boxers</td>
<td>4.1</td>
</tr>
<tr>
<td>Poodles</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>Total Registered Dogs</strong></td>
<td><strong>870,192</strong></td>
</tr>
</tbody>
</table>

Source: American Kennel Club

**SOLUTION:**

a. Expression representing the number of registered dogs of the top four breeds is 870,192(0.142 + 0.056 + 0.05 + 0.049).

b. Evaluate the expressions you wrote to find the number of registered dogs of the top four breeds.

**ANSWER:**

a. 870,192(0.142 + 0.056 + 0.05 + 0.049) = 870,192(0.142) + 870,192(0.056) + 870,192(0.05) + 870,192(0.049) = 123,567.264 + 48,730.152 + 43,008.6 + 42,639.406 = 258,447.024 So, the number of registered dogs of the top four breeds is about 258,447.

45. **FINANCIAL LITERACY** Billie is given $20 in lunch money by her parents once every two weeks. On some days, she packs her lunch, and on other days, she buys her lunch. A hot lunch from the cafeteria costs $4.50, and a cold sandwich from the lunch line costs $2.

a. Billie decides that she wants to buy a hot lunch on Thursday and Friday of the first week and on Wednesday of the second week. Use the Distributive Property to determine how much that will cost.

b. How many cold sandwiches can Billie buy with the amount left over?

c. Assuming that both weeks are Monday through Friday, how many times will Billie have to pack her lunch?

**SOLUTION:**

a. Billie buys a hot lunch on Thursday and Friday of the first week and Wednesday of the second week.

$4.50 + 4.50 + 4.50 = 4.50(1 + 1 + 1)$

$4.50 + 4.50 + 4.50 = 4.50(1+1+1)$

$= 4.50(3)$

$= 13.50$

Hot lunch will cost $13.50.

b. Subtract $13.50 from $20.

$20 - 13.50 = 6.50$

$\frac{6.50}{2} \approx 3$

So, Billie can buy 3 cold sandwiches with the amount left over.

c. Since Billie can buy 3 hot lunches and 3 cold sandwiches, she has to pack her lunch for 4 times if both the weeks are Monday through Friday.

**ANSWER:**

a. $13.50

b. 3

c. 4 times
1-2 Properties of Real Numbers

Simplify each expression.

46. \[ \frac{2}{5}(6c - 8d) + \frac{3}{4}(4c - 9d) \]

**SOLUTION:**
\[
\frac{2}{5}(6c - 8d) + \frac{3}{4}(4c - 9d)
\]
\[
= \frac{2}{5}(6c) + \frac{2}{5}(-8d) + \frac{3}{4}(4c) + \frac{3}{4}(-9d)
\]
\[
= \frac{12c}{5} - \frac{16d}{5} + \frac{12c}{4} - \frac{27d}{4}
\]
\[
= \frac{12c(4) - 16d(4) + 12c(5) - 27d(5)}{20}
\]
\[
= \frac{48c - 64d + 60c - 135d}{20}
\]
\[
= \frac{48c + 60c - 64d - 135d}{20}
\]
\[
= \frac{(48 + 60)c + (-64 - 135)d}{20}
\]
\[
= \frac{108c - 199d}{20}
\]
\[
= \frac{108c - 199d}{20}
\]
\[
= \frac{-199d}{20}
\]
\[
= \frac{27}{5}c - \frac{199}{20}d
\]

**ANSWER:**
\[
\frac{27}{5}c - \frac{199}{20}d
\]

47. \[ \frac{1}{3}(5x + 8y) + \frac{1}{4}(6x - 2y) \]

**SOLUTION:**
\[
\frac{1}{3}(5x + 8y) + \frac{1}{4}(6x - 2y)
\]
\[
= \frac{1}{3}(5x) + \frac{1}{3}(8y) + \frac{1}{4}(6x) + \frac{1}{4}(-2y)
\]
\[
= \frac{5x}{3} + \frac{8y}{3} + \frac{6x}{4} - \frac{2y}{4}
\]
\[
= \frac{5x}{3} + \frac{8y}{3} + \frac{3x}{2} - \frac{y}{2}
\]
\[
= \frac{5x(2) + 8y(2) + 3x(3) - y(3)}{6}
\]
\[
= \frac{10x + 16y + 9x - 3y}{6}
\]
\[
= \frac{10x + 9x + 16y - 3y}{6}
\]
\[
= \frac{(10 + 9)x + (16 - 3)y}{6}
\]
\[
= \frac{19x + 13y}{6}
\]
\[
= \frac{12}{6}x + \frac{13}{6}y
\]

**ANSWER:**
\[
\frac{19}{6}x + \frac{13}{6}y
\]

48. \[-6(3a + 5b) - 3(6a - 8c) \]

**SOLUTION:**
\[-6(3a + 5b) - 3(6a - 8c) \]
\[- = -6(3a) + (-5)(5b) - 3(6a) + (-8c) \]
\[- = -18a - 30b - 18a + 24c \]
\[- = -18a - 18a - 30b + 24c \]
\[- = (-18 - 18)a - 30b + 24c \]
\[- = -36a - 30b + 24c \]

**ANSWER:**
\[- 36a - 30b + 24c \]
1-2 Properties of Real Numbers

49. \(-9(3x + 8y) - 3(5x + 10z)\)

**SOLUTION:**

\[-9(3x + 8y) - 3(5x + 10z)\]

\[= (-9)(3x) + (-9)(8y) + (-3)(5x) + (-3)(10z)\]

\[= -27x - 72y - 15x - 20z\]

\[= -27x - 15x - 72y - 20z\]

\[= -42x - 72y - 20z\]

**ANSWER:**

\[-42x - 72y - 30z\]

50. **MODELING** Mary is making curtains out of the same fabric for 5 windows. The two larger windows are the same size, and the three smaller windows are the same size. One larger window requires

\[\frac{3}{4}\] yards of fabric, and one smaller window needs

\[2\frac{1}{3}\] yards of fabric.

a. How many yards of material will Mary need?

b. Use the properties of real numbers to show how Mary could compute this amount mentally.

**SOLUTION:**

a. Since one larger window requires \(\frac{3}{4}\) yards of fabric and a smaller window requires \(2\frac{1}{3}\) yards of fabric, the expression that represents the requirement of total yards of fabric is

\[2\left(\frac{3}{4}\right) + 3\left(2\frac{1}{3}\right)\]

\[= 2\left(\frac{3}{4}\right) + 3\left(\frac{7}{3}\right)\]

\[= \frac{15}{2} + \frac{7}{3}\]

\[= \frac{15 + 7(2)}{2}\]

\[= \frac{15 + 14}{2}\]

\[= \frac{29}{2}\]

\[= 14\frac{1}{2}\]

So, Mary requires \(14\frac{1}{2}\) yards of fabric.

b. 

\[\frac{3}{4} + \frac{1}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} + \frac{1}{2}\] (Definition of a mixed number)

\[= 2\left(\frac{3}{4}\right) + \frac{1}{2}\] (Distributive Property)

\[= 6 + \frac{3}{2} + \frac{1}{2}\] (Multiply)

\[= 6 + 1\frac{3}{2}\] (Commutative Property of Addition)

\[= 13\frac{3}{2}\] (Addition)

\[= 14\frac{1}{2}\]

**ANSWER:**

\[14\frac{1}{2}\] yd

51. **MULTIPLE REPRESENTATIONS** Consider the following real numbers.

\[-\sqrt{6}, 3, \frac{-15}{3}, 4, 1\pi, 0, \frac{3}{9}, \sqrt{36}\]

a. **TABULAR** Organize the numbers into a table according to the sets of numbers to which each belongs.

b. **ALGEBRAIC** Convert each number to decimal form. Then list the numbers from least to greatest.

c. **GRAPHICAL** Graph the numbers on a number line.

d. **VERBAL** Make a conjecture about using decimal form to list real numbers in order.

**SOLUTION:**

a. Sample answer:

\(-\sqrt{6}\) is an irrational number because the square root of 6 is not a perfect square.

3, or \(\frac{3}{1}\), is a rational number, integer, whole number, and natural number.

\(\frac{-15}{3}\) or \(-5\), is a rational number and integer.

\(\frac{41}{10}\), is a rational number.

\(\pi\) is an irrational number.
1-2 Properties of Real Numbers

0, or 1, 2, ..., is a rational number, integer, and whole number.

\(\frac{3}{8}\) is a rational number.

\(\sqrt{36}\), or 6, is a rational number, integer, whole number, and natural number.

<table>
<thead>
<tr>
<th>irrational</th>
<th>rational</th>
<th>integer</th>
<th>whole</th>
<th>natural</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\sqrt{6}, \pi)</td>
<td>(\frac{3}{8}, \frac{\sqrt{3}}{3}, 4.1, 0)</td>
<td>(\frac{3}{8}, \frac{\sqrt{3}}{3}, 0)</td>
<td>(\frac{3}{8}, \sqrt{36})</td>
<td>(\frac{3}{8}, \sqrt{36})</td>
</tr>
</tbody>
</table>

b. Use a calculator to find the decimal form of \(-\sqrt{6}\) and \(\pi\).

\[-\sqrt{6} \approx -2.449\]

\[3 = 3.0\]

\[-\frac{15}{3} = -5\]

\[4.1 = 4.1\]

\[\pi \approx 3.14\]

\[0 = 0,\]

\[\frac{3}{8} = 0.375\]

\[\sqrt{36} = 6\]

The numbers listed from least to greatest is

\[-\frac{15}{3}, -\sqrt{6}, 0, \frac{3}{8}, 3, \pi, 4.1, \sqrt{36}\]

c. Draw a number line with tick marks at integers from \(-6\) to 6. Then use the decimal forms in part b to graph each number.

d. Sample answer: By converting the real numbers into decimal form, they can be easily lined up and compared.

**ANSWER:**

a. Sample answer:

<table>
<thead>
<tr>
<th>irrational</th>
<th>rational</th>
<th>integer</th>
<th>whole</th>
<th>natural</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\sqrt{6}, \pi)</td>
<td>(\frac{3}{8}, \frac{\sqrt{3}}{3}, 4.1, 0)</td>
<td>(\frac{3}{8}, \frac{\sqrt{3}}{3}, 0)</td>
<td>(\frac{3}{8}, \sqrt{36})</td>
<td>(\frac{3}{8}, \sqrt{36})</td>
</tr>
</tbody>
</table>

b. \(-\sqrt{6} \approx -2.449,\)

\[3 = 3.0, \frac{-15}{3} = -5;\]

\[4.1 = 4.1, \pi \approx 3.14,\]

\[0 = 0, \frac{3}{8} = .375;\]

\[\sqrt{36} = 6, \frac{-15}{3}, -\sqrt{6};\]

\[0, \frac{3}{8}, 3, \pi, 4.1, \sqrt{36}\]

c. Sample answer: By converting the real numbers into decimal form, they can be easily lined up and compared.

**d.** Sample answer: By converting the real numbers into decimal form, they can be easily lined up and compared.

52. CLOTHING A department store sells shirts for $12.50 each. Dalila buys 2, Latisha buys 3, and Pilar buys 1.

a. Illustrate the Distributive Property by writing two expressions to represent the cost of these shirts.

b. Use the Distributive Property to find how much money the store received from selling these shirts.

**SOLUTION:**

a. 12.50\((2 + 3 + 1)\)

Use the Distributive Property to rewrite the expression.

12.50\((2 + 3 + 1)\) = 12.50 \cdot 2 + 12.50 \cdot 3 + 12.50 \cdot 1

b. 12.50\((2 + 3 + 1)\) = 12.50 \cdot 2 + 12.50 \cdot 3 + 12.50 \cdot 1

\[= 25 + 37.50 + 12.50\]

\[= 75\]

So, the store received $75.

**ANSWER:**

a. 12.50\((2 + 3 + 1)\); 12.50 \cdot 2 + 12.50 \cdot 3 + 12.50 \cdot 1

b. $75
1-2 Properties of Real Numbers

53. WHICH ONE DOESN’T BELONG? Identify the number that does not belong with the other three. Explain your reasoning.

\[ \sqrt{21} \approx 4.58258 \]
\[ \sqrt{35} \approx 5.91608 \]
\[ \sqrt{67} \approx 8.18535 \]
\[ \sqrt{81} = 9 \]

**SOLUTION:**
$\sqrt{21}$ is a real number. Since 62 is a real number, the number 62 is a real number. Since 62 is a real number, the number 62 is a real number. Since 62 is a real number, the number 62 is a real number.

**ANSWER:**
$\sqrt{81}$: It is a rational number, while the other three are irrational numbers.

54. CHALLENGE If \(12(5r + 6t) = w\), then in terms of \(w\), what is \(48(30r + 36t)\)?

**SOLUTION:**
\[
48(30r + 36t) = 48[6(5r + 6t)] \quad \text{Factor 6 from (30r + 36t).}
\]
\[= 24 \cdot 2 \cdot (5r + 6t) \quad \text{Factor 48 so one term is 12}
\]
\[= 24 \cdot 12(5r + 6t) \quad \text{Simplify}
\]
\[= 24 \cdot w \quad \text{Since } w = 12(5r + 6t)
\]
\[= 24w \quad \text{Simplify}
\]

**ANSWER:**
\(24w\)

55. ERROR ANALYSIS Luna and Sophia are simplifying \(4(14a - 10b) - 6(b + 4a)\). Is either of them correct? Explain your reasoning.

**SOLUTION:**
No; Luna did not distribute the negative sign to the second term and Sophia switched the \(a\) and \(b\) terms because usually \(a\) comes first.

\[
4(14a - 10b) - 6(b + 4a)
\]
\[= 4(14a) + 4(-10b) - 6b - 4a
\]
\[= 56a - 40b - 6b - 24a
\]
\[= 32a - 46b
\]

The correct answer is \(32a - 46b\).

**ANSWER:**
No; Luna did not distribute the negative sign to the second term and Sophia switched the \(a\) and \(b\) terms because usually \(a\) comes first. The correct answer is \(32a - 46b\).

56. REASONING Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

An irrational number is a real number underneath a radical sign.

**SOLUTION:**
Sometimes; \(\pi\) and \(e\) are two examples of irrational numbers that do not involve the radical symbol but \(\sqrt{117}\) is a real number while \(\sqrt{117}\) is irrational.

**ANSWER:**
Sometimes; \(\pi\) and \(e\) are two examples of irrational numbers that do not involve the radical symbol.
57. OPEN ENDED Determine whether the Closure Property of Multiplication applies to irrational numbers. If not, provide a counterexample.

**SOLUTION:**
Sample answer:
\[ \sqrt{5} \cdot \sqrt{5} = \sqrt{5^2} \]
\[ = \sqrt{25} \]
\[ = 5 \]
So, \( \sqrt{5} \cdot \sqrt{5} = 5 \), which is not irrational.

**ANSWER:**
Sample answer: \( \sqrt{5} \cdot \sqrt{5} = \sqrt{25} \) or 5 which is not irrational.

OPEN ENDED The set of all real numbers is dense, meaning between any two distinct members of the set there lies infinitely many other members of the set. Find an example of (a) a rational number, and (b) an irrational number between the given numbers.

58. 2.45 and 2.5

**SOLUTION:**
Sample answer:
(a) 2.46
(b) 2.484484484448\ldots

**ANSWER:**
Sample answer: (a) 2.46 and (b) 2.484484484448\ldots

59. \( \pi \) and \( \frac{10}{3} \)

**SOLUTION:**
Sample answer:
(a) 3.2; \( \pi \approx 3.14 \) and \( \frac{10}{3} \approx 3.33 \)
(b) \( \sqrt{10} \approx 3.16 \)

**ANSWER:**
Sample answer: (a) 3.2 and (b) \( \sqrt{10} \)

60. 1.9 and 2.01

**SOLUTION:**
Sample answer:
a. 2.001
b. 2.001000100001\ldots

**ANSWER:**
Sample answer: (a) 2.001 and (b) 2.001000100001\ldots

61. WRITING IN MATH Explain and provide examples to show why the Commutative Property does not hold true for subtraction or division.

**SOLUTION:**
Sample answer: The Commutative Property does not hold for subtraction or division because order matters with these two operations. In addition or multiplication, the order does not matter.

For example, \( 2 + 4 = 4 + 2 \) results in \( 6 = 6 \) and \( 2 \cdot 4 = 4 \cdot 2 \) results in \( 8 = 8 \). However, with subtraction:
\[ 2 - 4 \neq 4 - 2 \]
and division:
\[ \frac{2}{4} \neq \frac{4}{2} \]

**ANSWER:**
Sample answer: The Commutative Property does not hold for subtraction or division because order matters with these two operations. In addition or multiplication, the order does not matter.

For example, \( 2 + 4 = 4 + 2 \) and \( 2 \cdot 4 = 4 \cdot 2 \). However, with subtraction, \( 2 - 4 \neq 4 - 2 \), and with division, \( \frac{2}{4} \neq \frac{4}{2} \).
1-2 Properties of Real Numbers

62. **EXTENDED RESPONSE** Lenora bought several pounds of cashews and several pounds of almonds for a party. The cashews cost $8 per pound, and the almonds cost $6 per pound. Lenora bought a total of 7 pounds and paid a total of $48. Write and solve equations to determine the pounds of cashews and the pounds of almonds that Lenora purchased.

**SOLUTION:**
Let c be the number of pounds of cashews and a be the number of pounds of almonds.
Eight pounds of cashews and 6 pounds of almonds costs $48. So, \(8c + 6a = 48\).
Lenora bought 7 pounds of cashews and almonds.
So, \(c + a = 7\).

Substitute \(c = 7 - a\) in the equation \(8c + 6a = 48\).

\[
8c + 6a = 48 \\
8(7 - a) + 6a = 48 \\
56 - 8a + 6a = 48 \\
56 - 2a = 48 \\
56 - 2a - 56 = 48 - 56 \\
-2a = -8 \\
\frac{-2a}{-2} = \frac{-8}{-2} \\
a = 4
\]

Substitute \(a = 4\) in the equation \(c = 7 - a\).

\[
c = 7 - a \\
= 7 - 4 \\
= 3
\]

Therefore, Lenora bought 3 pounds of cashews and 4 pounds of almonds.

**ANSWER:**
8c + 6a = 48 and c + a = 7; c = 3 lb; a = 4 lb

63. **SAT/ACT** Find the 10th term in the series 2, 4, 7, 11, 16, …

A 41
B 46
C 56
D 67
E 72

**SOLUTION:**
The difference between the next term and the previous term is 1 more than the difference between the previous set of two terms.

\[
2 + 2 = 4 \\
4 + 3 = 7 \\
7 + 4 = 11 \\
11 + 5 = 16 \\
16 + 6 = 22 \\
22 + 7 = 29 \\
29 + 8 = 37 \\
37 + 9 = 46 \\
46 + 10 = 56
\]

So, the 10th term in the series is 56.
The correct choice is C.

**ANSWER:**
C
64. **GEOMETRY** What are the coordinates of point A in the parallelogram?

![Diagram of a parallelogram with coordinates labeled]

**SOLUTION:**
Since the points A and B are in the horizontal line segment, the y-coordinate of the point A is equal to the y-coordinate of B. From the figure, the x-coordinate of A is negative. Since OABC is a parallelogram, the distance between O and C is same as the distance between A and B. Since b > a, the x-coordinate of A is a – b. So, the coordinate of A is A(a – b, c). So, the correct choice is G.

**ANSWER:**
G

65. What is the domain of the function that contains the points (-3, 0), (0, 4), (-2, 5), and (6, 4)?

A {-3, 6}  
B {-3, -2, 0, 6}  
C {0, 4, 5, 6}  
D {-3, -2, 0, 4, 5, 6}

**SOLUTION:**
The domain is the set of x-coordinates.
D = {-3, -2, 0, 6}
So, the correct choice is B.

**ANSWER:**
B

66. Evaluate 8(4 – 2)^3.

**SOLUTION:**

\[ 8(4 – 2)^3 = 8(2)^3 = 8(2)(2)(2) = 8(8) = 64 \]

**ANSWER:**
64

67. Evaluate \( a + 3(b + c) - d \), if \( a = 5 \), \( b = 4 \), \( c = 3 \), and \( d = 2 \).

**SOLUTION:**
Substitute \( a = 5 \), \( b = 4 \), \( c = 3 \), and \( d = 2 \) in the expression \( a + 3(b + c) - d \).

\[
\begin{align*}
  a + 3(b + c) - d &= 5 + 3(4 + 3) - 2 \\
  &= 5 + 3(7) - 2 \\
  &= 5 + 21 - 2 \\
  &= 26 - 2 \\
  &= 24
\end{align*}
\]

**ANSWER:**
24

68. **GEOMETRY** The formula for the area \( A \) of a circle with diameter \( d \) is \( A = \pi \left( \frac{d}{2} \right)^2 \). Write an expression to represent the area of the circle.

**SOLUTION:**
Substitute \( d = (x + 3) \) in the formula \( A = \pi \left( \frac{d}{2} \right)^2 \).

\[
A = \pi \left( \frac{x + 3}{2} \right)^2
\]

The area of the circle is \( \pi \left( \frac{x + 3}{2} \right)^2 \).

**ANSWER:**
\( \pi \left( \frac{x + 3}{2} \right)^2 \)
69. **CONSTRUCTION** A 10-meter ladder leans against a building so that the top is 9.64 meters above the ground. How far from the base of the wall is the bottom of the ladder?

**SOLUTION:**
Use the Pythagorean Theorem.
Substitute \( a = 9.64 \) and \( c = 10 \) in the formula \( c^2 = a^2 + b^2 \).

\[
c^2 = a^2 + b^2 \\
10^2 = (9.64)^2 + b^2 \\
100 = 92.9296 + b^2 \\
b^2 = 100 - 92.9296 \\
b = \sqrt{7.0704} \\
b \approx 2.66
\]

The distance from the wall to the base of the ladder is about 2.66 meters.

**ANSWER:**
about 2.66 m

**Factor each polynomial.**
70. \( 14x^2 + 10x - 8 \)

**SOLUTION:**
\[
14x^2 = 2 \cdot 7 \cdot x \cdot x \\
10x = 2 \cdot 5 \cdot x \\
8 = 2 \cdot 2 \cdot 2
\]

The GCF of the terms \( 14x^2, 10x \) and \( 8 \) is 2.

\[
14x^2 + 10x - 8 = 2 \left( 7x^2 \right) + 2 \left( 5x \right) + 2(-4) \\
= 2 \left( 7x^2 + 5x - 4 \right)
\]

**ANSWER:**
\( 2 \left( 7x^2 + 5x - 4 \right) \)

71. \( 9x^2 - 3x + 18 \)

**SOLUTION:**
\[
9x^2 = 3 \cdot 3 \cdot x \cdot x \\
3x = 3 \cdot x \\
18 = 2 \cdot 3 \cdot 3
\]

The GCF of the terms \( 9x^2, 3x \) and \( 18 \) is 3.

\[
9x^2 - 3x + 18 = 3 \left( 3x^2 \right) + 3(-x) + 3(6) \\
= 3 \left( 3x^2 - x + 6 \right)
\]

**ANSWER:**
\( 3 \left( 3x^2 - x + 6 \right) \)
1-2 Properties of Real Numbers

73. \(10x^2 - 20x\)

**SOLUTION:**

\[
10x^2 = 2 \cdot 5 \cdot x \cdot x
\]

\[
20x = 2 \cdot 2 \cdot 5 \cdot x
\]

The GCF of the terms \(10x^2\) and \(20x\) is \(2 \cdot 5 \cdot x\) or \(10x\).

\[
10x^2 - 20x = 10x(x) + 10x(-2)
\]

\[
= 10x(x - 2)
\]

**ANSWER:**

\(10x(x - 2)\)

74. \(7x^2 - 14x - 21\)

**SOLUTION:**

\[
7x^2 = 7 \cdot x \cdot x
\]

\[
14x = 2 \cdot 7 \cdot x
\]

\[
21 = 3 \cdot 7
\]

The GCF of the terms \(7x^2\), \(14x\) and \(21\) is \(7\).

\[
7x^2 - 14x - 21 = 7(x^2) + 7(-2x) + 7(-3)
\]

\[
= 7(x^2 - 2x - 3)
\]

\[
= 7(x - 3)(x + 1)
\]

**ANSWER:**

\(7(x - 3)(x + 1)\)

75. \(12x^2 - 18x - 24\)

**SOLUTION:**

\[
12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x
\]

\[
18x = 2 \cdot 3 \cdot 3 \cdot x
\]

\[
24 = 2 \cdot 2 \cdot 2 \cdot 3
\]

The GCF of the terms \(12x^2\), \(18x\) and \(24\) is \(2 \cdot 3\) or 6.

\[
12x^2 - 18x - 24 = 6(2x^2) + 6(-3x) + 6(-4)
\]

\[
= 6(2x^2 - 3x - 4)
\]

**ANSWER:**

\(6(2x^2 - 3x - 4)\)

76. \((x + 2)(x - 3)\)

**SOLUTION:**

Use the FOIL method to find the product.

\[
(x + 2)(x - 3) = x(x) + x(-3) + 2(x) + 2(-3)
\]

\[
= x^2 - 3x + 2x - 6
\]

\[
= x^2 - x - 6
\]

**ANSWER:**

\(x^2 - x - 6\)

77. \((y + 2)(y - 1)\)

**SOLUTION:**

Use the FOIL method to find the product.

\[
(y + 2)(y - 1) = y(y) + y(-1) + 2(y) + 2(-1)
\]

\[
= y^2 - y + 2y - 2
\]

\[
= y^2 + y - 2
\]

**ANSWER:**

\(y^2 + y - 2\)
1-2 Properties of Real Numbers

78. \((a - 5)(a + 4)\)

**SOLUTION:**
Use the FOIL method to find the product.
\[
(a - 5)(a + 4) = a(a) + a(4) + (-5)(a) + (-5)(4)
\]
\[
= a^2 + 4a - 5a - 20
\]
\[
= a^2 - a - 20
\]

**ANSWER:**
\[a^2 - a - 20\]

79. \((b - 7)(b - 3)\)

**SOLUTION:**
Use the FOIL method to find the product.
\[
(b - 7)(b - 3) = b(b) + b(-3) + (-7)(b) + (-7)(-3)
\]
\[
= b^2 - 3b - 7b + 21
\]
\[
= b^2 - 10b + 21
\]

**ANSWER:**
\[b^2 - 10b + 21\]

80. \((n + 6)(n + 8)\)

**SOLUTION:**
Use the FOIL method to find the product.
\[
(n + 6)(n + 8) = n(n) + n(8) + 6(n) + 6(8)
\]
\[
= n^2 + 8n + 6n + 48
\]
\[
= n^2 + 14n + 48
\]

**ANSWER:**
\[n^2 + 14n + 48\]

81. \((p - 9)(p + 1)\)

**SOLUTION:**
Use the FOIL method to find the product.
\[
(p - 9)(p + 1) = p(p) + p(1) + (-9)(p) + (-9)(1)
\]
\[
= p^2 + p - 9p - 9
\]
\[
= p^2 - 8p - 9
\]

**ANSWER:**
\[p^2 - 8p - 9\]

Evaluate each expression if \(a = 3, \ b = \frac{2}{3}\), and \(c = -1.7\).

82. \(6b - 5\)

**SOLUTION:**
\[
6b - 5 = 6 \left( \frac{2}{3} \right) - 5
\]
\[
= \frac{12}{3} - 5
\]
\[
= 4 - 5
\]
\[
= -1
\]

**ANSWER:**
\[-1\]

83. \(\frac{1}{6} b + 1\)

**SOLUTION:**
\[
\frac{1}{6} b + 1 = \frac{1}{6} \left( \frac{2}{3} \right) + 1
\]
\[
= \frac{2}{18} + 1
\]
\[
= \frac{1}{9} + 1
\]
\[
= \frac{1+9}{9}
\]
\[
= \frac{10}{9}
\]

**ANSWER:**
\[\frac{10}{9}\]

84. \(2.3c - 7\)

**SOLUTION:**
\[
2.3c - 7 = 2.3(-1.7) - 7
\]
\[
= -3.91 - 7
\]
\[
= -10.91
\]

**ANSWER:**
\[-10.91\]
1-2 Properties of Real Numbers

85. \(-8(a - 4)\)

**SOLUTION:**
\[ -8(a - 4) = -8(3 - 4) \]
\[ = -8(-1) \]
\[ = 8 \]

**ANSWER:**
8

86. \(a + b + c\)

**SOLUTION:**
\[ a + b + c = \frac{3 + \frac{2}{3} + (-1.7)}{3} \]
\[ = \frac{3(3) + 2 - 1.7(3)}{3} \]
\[ = \frac{9 + 2 - 5.1}{3} \]
\[ = \frac{5.9}{3} \]
\[ \approx 1.967 \]

**ANSWER:**
\[ \approx 1.967 \]

87. \(\frac{a \cdot b}{c}\)

**SOLUTION:**
\[ \frac{a \cdot b}{c} = \frac{3 \cdot \frac{2}{3}}{-1.7} \]
\[ = \frac{2}{-1.7} \]
\[ \approx -1.176 \]

**ANSWER:**
\[ \approx -1.176 \]

88. \(a^2 - c\)

**SOLUTION:**
\[ a^2 - c = 3^2 - (-1.7) \]
\[ = 9 + 1.7 \]
\[ = 10.7 \]

**ANSWER:**
10.7

89. \(\frac{a \cdot c}{a}\)

**SOLUTION:**
\[ \frac{a \cdot c}{a} = \frac{3 \cdot (-1.7)}{3} \]
\[ = -1.7 \]

**ANSWER:**
\[ \approx -1.7 \]
Write an algebraic expression to represent each verbal expression.

1. the product of 12 and the sum of a number and negative 3

**SOLUTION:**
Let \( x \) be the number.
The sum of \( x \) and negative 3 is \( x + (-3) \).
The product of 12 and the sum of \( x \) and negative 3 is
\[
12 \left[ x + (-3) \right].
\]

**ANSWER:**
\[
12 \left[ x + (-3) \right]
\]

2. the difference between the product of 4 and a number and the square of the number

**SOLUTION:**
Let \( x \) be the number.
4 times of \( x \) is \( 4x \). The square of \( x \) is \( x^2 \).
The keyword ‘difference’ means subtraction.
So, the algebraic expression is \( 4x - x^2 \).

**ANSWER:**
\[ 4x - x^2 \]

Write a verbal sentence to represent each equation.

3. \( 5x + 7 = 18 \)

**SOLUTION:**
The sum of five times a number and 7 equals 18.

**ANSWER:**
The sum of five times a number and 7 equals 18.

4. \( x^2 - 9 = 27 \)

**SOLUTION:**
The difference between the square of a number and 9 is 27.

**ANSWER:**
The difference between the square of a number and 9 is 27.

5. \( 5y - y^3 = 12 \)

**SOLUTION:**
The difference between five times a number and the cube of that number is 12.

**ANSWER:**
The difference between five times a number and the cube of that number is 12.

6. \( \frac{x}{4} + 8 = -16 \)

**SOLUTION:**
Eight more than the quotient of a number and four is \(-16\).

**ANSWER:**
Eight more than the quotient of a number and four is \(-16\).

Name the property illustrated by each statement.

7. \( (8x - 3) + 12 = (8x - 3) + 12 \)

**SOLUTION:**
Reflexive Property; the Reflexive Property of Equality states that for any real number \( a \), \( a = a \).

**ANSWER:**
Reflexive Property

8. If \( a = -3 \) and \( -3 = d \) then \( a = d \).

**SOLUTION:**
Transitive Property; the Transitive Property of Equality states that for any real numbers \( a, b, \) and \( c \), if \( a = b \) and \( b = c \), then \( a = c \).

**ANSWER:**
Transitive Property
1-3 Solving Equations

Solve each equation. Check your solution.

9. \( z - 19 = 34 \)

**SOLUTION:**

\[
\begin{align*}
  & z - 19 = 34 \\
  & z - 19 + 19 = 34 + 19 \\
  & z = 53 \\
\end{align*}
\]

Substitute \( z = 53 \) in the equation.

\[
\begin{align*}
  & 53 - 19 = 34 \\
  & 34 = 34 \quad \checkmark
\end{align*}
\]

Therefore, the solution is \( z = 53 \).

**ANSWER:**

53

10. \( x + 13 = 7 \)

**SOLUTION:**

\[
\begin{align*}
  & x + 13 = 7 \\
  & x + 13 - 13 = 7 - 13 \\
  & x = -6 \\
\end{align*}
\]

Substitute \( x = -6 \) in the equation.

\[
\begin{align*}
  & -6 + 13 = 7 \\
  & 7 = 7 \quad \checkmark
\end{align*}
\]

The solution of the equation is \( x = -6 \).

**ANSWER:**

-6

11. \( -y = 8 \)

**SOLUTION:**

\[
\begin{align*}
  & -y = 8 \\
  & y = -8 \\
\end{align*}
\]

Substitute \( y = -8 \) in the equation.

\[
\begin{align*}
  & -(-8) = 8 \\
  & 8 = 8 \quad \checkmark
\end{align*}
\]

So, the solution is \( y = -8 \).

**ANSWER:**

-8

12. \( -6x = 42 \)

**SOLUTION:**

\[
\begin{align*}
  & -6x = 42 \\
  & -6x \div -6 = 42 \div -6 \\
  & x = -7 \\
\end{align*}
\]

Substitute \( x = -7 \) in the equation.

\[
\begin{align*}
  & -6(-7) = 42 \\
  & 42 = 42 \quad \checkmark
\end{align*}
\]

So, the solution is \( x = -7 \).

**ANSWER:**

-7

13. \( 5x - 3 = -33 \)

**SOLUTION:**

\[
\begin{align*}
  & 5x - 3 = -33 \\
  & 5x - 3 + 3 = -33 + 3 \\
  & 5x = -30 \\
  & 5x \div 5 = -30 \div 5 \\
  & x = -6 \\
\end{align*}
\]

Substitute \( x = -6 \) in the equation.

\[
\begin{align*}
  & 5(-6) - 3 = -33 \\
  & -30 - 3 = -33 \\
  & -33 = -33 \quad \checkmark
\end{align*}
\]

So, the solution is \( x = -6 \).

**ANSWER:**

-6
1-3 Solving Equations

14. \(-6y - 8 = 16\)

\[\text{SOLUTION:}\]
\[-6y - 8 = 16\]
\[-6y = 24\]
\[y = -4\]

Substitute \(y = -4\) in the equation.

\[-6(-4) - 8 = 16\]
\[24 - 8 = 16\]
\[16 = 16 \checkmark\]

So, the solution is \(y = -4\).

\[\text{ANSWER:}\]
\[-4\]

15. \(3(2a + 3) - 4(3a - 6) = 15\)

\[\text{SOLUTION:}\]
\[3(2a + 3) - 4(3a - 6) = 15\]
\[6a + 9 - 12a + 24 = 15\]
\[-6a + 33 = 15\]
\[-6a = -18\]
\[a = 3\]

Substitute \(a = 3\) in the equation.

\[3(2(3) + 3) - 4(3(3) - 6) = 15\]
\[3(6 + 3) - 4(9 - 6) = 15\]
\[15 = 15 \checkmark\]

So, the solution is \(a = 3\).

\[\text{ANSWER:}\]
\[3\]

16. \(5(c - 8) - 3(2c + 12) = -84\)

\[\text{SOLUTION:}\]
\[5(c - 8) - 3(2c + 12) = -84\]
\[5c - 40 - 6c - 36 = -84\]
\[-c - 76 = -84\]
\[-c = -8\]
\[c = 8\]

Substitute \(c = 8\) in the equation.

\[5(8 - 8) - 3(2(8) + 12) = -84\]
\[-3(28) = -84\]
\[-84 = -84 \checkmark\]

So, the solution is \(c = 8\).

\[\text{ANSWER:}\]
\[8\]

17. \(-3(-2x + 20) + 8(x + 12) = 92\)

\[\text{SOLUTION:}\]
\[-3(-2x + 20) + 8(x + 12) = 92\]
\[6x - 60 + 8x + 96 = 92\]
\[14x + 36 = 92\]
\[14x = 56\]
\[x = 4\]

Substitute \(x = 4\) in the equation.

\[-3(-2(-4) + 20) + 8(4 + 12) = 92\]
\[-3(12) + 8(16) = 92\]
\[92 = 92 \checkmark\]

So, the solution is \(x = 4\).

\[\text{ANSWER:}\]
\[4\]
1-3 Solving Equations

18. \(-4(3m - 10) - 6(-7m - 6) = -74\)

**SOLUTION:**
\[-4(3m - 10) - 6(-7m - 6) = -74\]
\[-12m + 40 + 42m + 36 = -74\]
\[30m + 76 = -74\]
\[30m = -150\]
\[m = -5\]
Substitute \(m = -5\) in the equation.
\[-4(3(-5) - 10) - 6(-7(-5) - 6) = -74\]
\[-4(-15 - 10) - 6(-35 - 6) = -74\]
\[100 - 174 = -74\]
\[-74 = -74\] \(\checkmark\)
So, the solution is \(m = -5\).

**ANSWER:**
\(-5\)

Solve each equation or formula for the specified variable.

19. \(8r - 5q = 3\) for \(q\)

**SOLUTION:**
\[8r - 5q = 3\]
\[-8r + 8r - 5q = 3 - 8r\]
\[-5q = 3 - 8r\]
\[-5q = 3 - 8r\]
\[q = \frac{3 - 8r}{-5}\]
\[q = \frac{8r - 3}{5}\]

**ANSWER:**
\[q = \frac{8r - 3}{5}\]

20. \(pv = nrt\) for \(n\)

**SOLUTION:**
\[pv = nrt\]
\[\frac{pv}{rt} = \frac{nrt}{rt}\]
\[\frac{pv}{rt} = n\]

**ANSWER:**
\[\frac{pv}{rt} = n\]

21. **MULTIPLE CHOICE** If \(\frac{y}{5} + 8 = 7\), what is the value of \(\frac{y}{5} - 2\)?

A \(-10\)
B \(-3\)
C \(1\)
D \(5\)

**SOLUTION:**
\[\frac{y}{5} + 8 = 7\]
\[\frac{y}{5} + 8 - 8 = 7 - 8\]
\[\frac{y}{5} = -1\]
\[\frac{y}{5} - 2 = -1 - 2\]
\[\frac{y}{5} - 2 = -3\]
The correct choice is B.

**ANSWER:**
B
1-3 Solving Equations

Write an algebraic expression to represent each verbal expression.
22. the difference between the product of four and a number and 6
   SOLUTION:
   Let the number be \( n \).
   The product of four and \( n \) is \( 4n \).
   The keyword ‘difference’ indicates subtraction.
   The algebraic expression is \( 4n - 6 \).

   ANSWER:
   \( 4n - 6 \)

23. the product of the square of a number and 8
   SOLUTION:
   Let the number be \( x \).
   The square of \( x \) is \( x^2 \).
   The algebraic expression is \( 8x^2 \).

   ANSWER:
   \( 8x^2 \)

24. fifteen less than the cube of a number
   SOLUTION:
   Let the number be \( x \).
   Cube of \( x \) is \( x^3 \).
   15 less than \( x^3 \) is \( x^3 - 15 \).

   ANSWER:
   \( x^3 - 15 \)

25. five more than the quotient of a number and 4
   SOLUTION:
   Let the number be \( x \). The quotient of \( x \) and 4 is \( \frac{x}{4} \).
   Five more than \( \frac{x}{4} \) is \( \frac{x}{4} + 5 \).

   ANSWER:
   \( \frac{x}{4} + 5 \)

Write a verbal sentence to represent each equation.
26. \( 8x - 4 = 16 \)
   SOLUTION:
   Four less than 8 times a number is 16.
   ANSWER:
   Four less than 8 times a number is 16.

27. \( \frac{x + 3}{4} = 5 \)
   SOLUTION:
   The quotient of the sum of 3 and a number and 4 is 5.
   ANSWER:
   The quotient of the sum of 3 and a number and 4 is 5.

28. \( 4y^2 - 3 = 13 \)
   SOLUTION:
   Three less than four times the square of a number is 13.
   ANSWER:
   Three less than four times the square of a number is 13.
29. **BASEBALL** During a recent season, Miguel Cabrera and Mike Jacobs of the Florida Marlins hit a combined total of 46 home runs. Cabrera hit 6 more home runs than Jacobs. How many home runs did each player hit? Define a variable, write an equation, and solve the problem.

**SOLUTION:**

\[
\begin{align*}
n = \text{number of home runs Jacobs hit.} \\
\text{Cabrera hit 6 more home runs than Jacobs. The keyword ‘more than’ mean addition.} \\
\text{So, number of home runs Cabrera hit} &= n + 6. \\
\text{Total number of home runs is 46.} \\
\text{Therefore:} \\
\end{align*}
\]

\[
\begin{align*}
n + n + 6 &= 46 \\
2n + 6 &= 46 \\
2n &= 40 \\
n &= 20
\end{align*}
\]

Jacobs hit 20 home runs and Cabrera hit 26 home runs.

**ANSWER:**

\[
\begin{align*}
n = \text{number of home runs Jacobs hit; } n + 6 = \text{number of home runs Cabrera hit; } 2n + 6 = 46; \text{ Jacobs: } 20 \\
\text{home runs, Cabrera: } 26 \text{ home runs.}
\end{align*}
\]

**Name the property illustrated by each statement.**

30. If \(x + 9 = 2\), then \(x + 9 - 9 = 2 - 9\)

**SOLUTION:**

Subtraction Property of Equality; the Subtraction Property of Equality states that for any real numbers \(a, b, \text{ and } c\), if \(a = b\), then \(a - c = b - c\).

**ANSWER:**

30. Subtr. (=)

31. If \(y = -3\), then \(7y = 7(-3)\)

**SOLUTION:**

Substitution Property of Equality; the Substitution Property of Equality states that if \(a = b\), then \(a\) may be replaced by \(b\) and \(b\) may be replaced by \(a\).

**ANSWER:**

Subst.

32. If \(g = 3h\) and \(3h = 16\), then \(g = 16\)

**SOLUTION:**

Transitive Property: the Transitive Property of Equality states that for any real numbers \(a, b, \text{ and } c\), if \(a = b\) and \(b = c\), then \(a = c\).

**ANSWER:**

Transitive Property

33. If \(-y = 13\), then \((-y) = -13\)

**SOLUTION:**

Multiplication Property of Equality; this states that for any real numbers \(a, b, \text{ and } c\), if \(a = b\), then \(a \cdot c = b \cdot c\).

**ANSWER:**

Mult. (=)

34. **MONEY** Aiko and Kendra arrive at the state fair with $32.50. What is the total number of rides they can go on if they each pay the entrance fee?

**SOLUTION:**

Let \(n\) be the total number of rides.

The entrance fee for two persons = 2($7.50) = $15.00

\[
\begin{align*}
n(2.50) + 15.00 &= 32.50 \\
2.50n + 15.00 &= 32.50 \\
2.50n &= 17.5 \\
n &= 7
\end{align*}
\]

So, Aiko and Kendra can go on a total of 7 rides.

**ANSWER:**

\[n = \text{number of rides}; 2(7.50) + n(2.50) = 32.50; 7\]
1-3 Solving Equations

Solve each equation. Check your solution.

35. \[3y + 4 = 19\]

\[\text{SOLUTION:}\]
\[3y + 4 = 19\]
\[3y + 4 - 4 = 19 - 4\]
\[3y = 15\]
\[\frac{3y}{3} = \frac{15}{3}\]
\[y = 5\]
Substitute \(y = 5\) in the original equation.
\[3(5) + 4 = 19\]
\[15 + 4 = 19\]
\[19 = 19\] ✓
The solution is \(y = 5\).

\[\text{ANSWER:}\]
\[5\]

36. \[-9x - 8 = 55\]

\[\text{SOLUTION:}\]
\[-9x - 8 = 55\]
\[-9x - 8 + 8 = 55 + 8\]
\[-9x = 63\]
\[\frac{-9x}{-9} = \frac{63}{-9}\]
\[x = -7\]
Substitute \(x = -7\) in the equation.
\[-9(-7) - 8 = 55\]
\[63 - 8 = 55\]
\[55 = 55\] ✓
The solution is \(x = -7\).

\[\text{ANSWER:}\]
\[-7\]

37. \[7y - 2y + 4 + 3y = -20\]

\[\text{SOLUTION:}\]
\[7y - 2y + 4 + 3y = -20\]
\[8y + 4 = -20\]
\[8y = -24\]
\[y = -3\]
Substitute \(y = -3\) in the equation.
\[7(-3) - 2(-3) + 4 + 3(-3) = -20\]
\[-21 + 6 + 4 - 9 = -20\]
\[-20 = -20\] ✓
The solution is \(y = -3\).

\[\text{ANSWER:}\]
\[-3\]

38. \[5g + 18 - 7g + 4g = 8\]

\[\text{SOLUTION:}\]
\[5g + 18 - 7g + 4g = 8\]
\[2g + 18 = 8\]
\[2g + 18 - 18 = 8 - 18\]
\[2g = -10\]
\[\frac{2g}{2} = \frac{-10}{2}\]
\[g = -5\]
Substitute \(g = -5\) in the original equation.
\[5(-5) + 18 - 7(-5) + 4(-5) = 8\]
\[-25 + 18 + 35 - 20 = 8\]
\[53 - 45 = 8\]
\[8 = 8\] ✓
The solution is \(g = -5\).

\[\text{ANSWER:}\]
\[-5\]
39. \(5(-2x - 4) - 3(4x + 5) = 97\)

**SOLUTION:**
\[
\begin{align*}
5(-2x - 4) - 3(4x + 5) & = 97 \\
-10x - 20 - 12x - 15 & = 97 \\
-22x - 35 & = 97 \\
-22x & = 132 \\
x & = -6 \\
\end{align*}
\]
Substitute \(x = -6\) in the equation.
\[
\begin{align*}
5(-2(-6) - 4) - 3(4(-6) + 5) & = 97 \\
5(12 - 4) - 3(-24 + 5) & = 97 \\
5(8) - 3(-19) & = 97 \\
40 + 57 & = 97 \\
97 & = 97 \checkmark
\end{align*}
\]
The solution is \(x = -6\).

**ANSWER:**
\(-6\)

40. \(-2(3y - 6) + 4(5y - 8) = 92\)

**SOLUTION:**
\[
\begin{align*}
-2(3y - 6) + 4(5y - 8) & = 92 \\
-6y + 12 + 20y - 32 & = 92 \\
14y - 20 & = 92 \\
14y & = 112 \\
y & = 8 \\
\end{align*}
\]
Substitute \(y = 8\) in the equation.
\[
\begin{align*}
-2(3(8) - 6) + 4(5(8) - 8) & = 92 \\
-2(18) + 4(32) & = 92 \\
-36 + 128 & = 92 \\
92 & = 92 \checkmark
\end{align*}
\]
The solution is \(y = 8\).

**ANSWER:**
\(8\)

41. \(\frac{2}{3}(6c - 18) + \frac{3}{4}(8c + 32) = -18\)

**SOLUTION:**
\[
\begin{align*}
\frac{2}{3}(6c - 18) + \frac{3}{4}(8c + 32) & = -18 \\
\frac{2}{3} \cdot 6c - \frac{2}{3} \cdot 18 + \frac{3}{4} \cdot 8c + \frac{3}{4} \cdot 32 & = -18 \\
4c - 12 + 6c + 24 & = -18 \\
10c + 12 & = -18 \\
10c & = -30 \\
c & = -3 \\
\end{align*}
\]
Substitute \(c = -3\) in the equation.
\[
\begin{align*}
\frac{2}{3}(6(-3) - 18) + \frac{3}{4}(8(-3) + 32) & = -18 \\
\frac{2}{3}(-36) + \frac{3}{4}(8) & = -18 \\
-24 + 6 & = -18 \\
-18 & = -18 \checkmark
\end{align*}
\]
The solution is \(c = -18\).

**ANSWER:**
\(-3\)

42. \(\frac{3}{5}(15d + 20) - \frac{1}{6}(18d - 12) = 38\)

**SOLUTION:**
\[
\begin{align*}
\frac{3}{5}(15d + 20) - \frac{1}{6}(18d - 12) & = 38 \\
9d + 12 - 3d + 2 & = 38 \\
6d + 14 & = 38 \\
6d & = 24 \\
d & = 4 \\
\end{align*}
\]
Substitute \(d = 4\) in the equation.
\[
\begin{align*}
\frac{3}{5}(15(4) + 20) - \frac{1}{6}(18(4) - 12) & = 38 \\
\frac{3}{5}(80) - \frac{1}{6}(60) & = 38 \\
48 - 10 & = 38 \\
38 & = 38 \checkmark
\end{align*}
\]
The solution is \(d = 4\).

**ANSWER:**
\(4\)
43. **GEOMETRY** The perimeter of a regular pentagon is 100 inches. Find the length of each side.

**SOLUTION:**
The perimeter of a regular pentagon is given by \( P = 5s \), where \( s \) is the side length.
Substitute \( P = 100 \).
\[
5s = 100
\]
\[
\frac{5s}{5} = \frac{100}{5}
\]
\[
s = 20
\]
The length of each side of the pentagon is 20 inches.

**ANSWER:**
\( s = \text{length of a side}; 5s = 100; 20 \text{ in.} \)

44. **MEDICINE** For Nina’s illness her doctor gives her a prescription for 28 pills. The doctor says that she should take 4 pills the first day and then 2 pills each day until her prescription runs out. For how many days does she take 2 pills?

**SOLUTION:**
Let \( x \) be the number of days Nina takes 2 pills.
Total number of pills = 28.
So:
\[
4 + 2x = 28
\]
\[
-4 + 4 + 2x = -4 + 28
\]
\[
2x = 24
\]
\[
\frac{2x}{2} = \frac{24}{2}
\]
\[
x = 12
\]
Nina takes 2 pills a day for 12 days.

**ANSWER:**
\( x = \text{the number of days she takes 2 pills}; 4 + 2x = 28; 12 \text{ days} \)

45. \( E = mc^2 \) for \( m \)

**SOLUTION:**
\[
E = mc^2
\]
\[
\frac{E}{c^2} = \frac{mc^2}{c^2}
\]
\[
E = m
\]

**ANSWER:**
\( m = \frac{E}{c^2} \)

46. \( c(a + b) - d = f \) for \( a \)

**SOLUTION:**
\[
c(a + b) - d = f
\]
\[
ca + cb - d = f
\]
\[
ca + cb = f + d
\]
\[
ca = f + d - cb
\]
\[
a = \frac{f + d - cb}{c}
\]
\[
= \frac{f + d}{c} - b
\]

**ANSWER:**
\( a = \frac{f + d}{c} - b \)

47. \( z = \pi q^3 h \) for \( h \)

**SOLUTION:**
\[
z = \pi q^3 h
\]
\[
\frac{z}{\pi q^3} = \frac{\pi q^3 h}{\pi q^3}
\]
\[
\frac{z}{\pi q^3} = h
\]

**ANSWER:**
\( h = \frac{z}{\pi q^3} \)
48. \( \frac{x+y}{z} - a = b \) for \( y 

SOLUTION:

\[
\frac{x+y}{z} = a + b \\
x+y = z(a+b) \\
y = z(a+b) - x
\]

ANSWER:

\( y = z(a+b) - x \)

49. \( y = ax^2 + bx + c \), for \( a 

SOLUTION:

\[
y = ax^2 + bx + c \\
y - bx - c = ax^2 \\
\frac{y - bx - c}{x^2} = a \\
\frac{y - bx - c}{x^2} = a
\]

ANSWER:

\( a = \frac{y - bx - c}{x^2} \)

50. \( wx + yz = bc \) for \( z 

SOLUTION:

\[
wz + yz = bc \\
-wx + wz + yz = bc - wx \\
yz = bc - wx \\
\frac{yz}{y} = \frac{bc - wx}{y} \\
z = \frac{bc - wx}{y}
\]

ANSWER:

\( z = \frac{bc - wx}{y} \)

51. **GEOMETRY** The formula for the volume of a cylinder with radius \( r \) and height \( h \) is \( \pi \) times the radius times the height.

a. Write this as an algebraic expression.

b. Solve the expression in part a for \( h \).

**SOLUTION:**

a. The keyword ‘times’ indicates multiplication. Let \( V \) be the volume of the cylinder.

\[ V = \pi \times r \times r \times h \]

b. Divide both sides by \( \pi r^2 \).

\[
\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \\
\frac{V}{\pi r^2} = h
\]

**ANSWER:**

a. \( V = \pi \times r \times r \times h \)

b. \( h = \frac{V}{\pi r^2} \)
1-3 Solving Equations

52. AWARDS BANQUET A banquet room can seat a maximum of 69 people. The coach, principal, and vice principal have invited the award-winning girls’ tennis team to the banquet. If the tennis team consists of 22 girls, how many guests can each student bring?

SOLUTION:
Let \( n \) be the number of guests each student can bring. The maximum number of people can be seated in the room is 69. The tennis team, coach, principal and vice principal gives 25 to attend the banquet.

\[
22n + 25 = 69 \quad \text{Solve for } n.
\]

\[
22n + 25 = 69 \\
22n = 44 \\
n = 2
\]

Therefore, each student can bring 2 guests.

ANSWER:
\( n = \text{number of guests that each student can bring; } 22n + 25 = 69; 2 \text{ guests} \)

Solve each equation. Check your solution.

53. \( 5x - 9 = 11x + 3 \)

SOLUTION:
\[
5x - 9 = 11x + 3 \\
-9 = 6x + 3 \\
6x = -12 \\
x = -2
\]

Check:
\[
5x - 9 = 11x + 3 \\
5(-2) - 9 = 11(-2) + 3 \\
-10 - 9 = -22 + 3 \\
-19 = -19
\]

The solution is \( x = -2 \).

ANSWER:
\(-2\)

54. \( \frac{1}{x} + \frac{1}{4} = \frac{7}{12} \)

SOLUTION:
\[
\frac{1}{x} + \frac{1}{4} = \frac{7}{12} \\
\frac{1}{x} + \frac{1}{4} = \frac{7}{12} \\
\frac{1}{x} = \frac{7}{12} - \frac{1}{4} \\
\frac{1}{x} = \frac{7}{12} - \frac{3}{12} \\
\frac{1}{x} = \frac{4}{12} \\
x = \frac{12}{4} \\
x = 3
\]

Check:
\[
\frac{1}{x} + \frac{1}{4} = \frac{7}{12} \\
\frac{1}{3} + \frac{1}{4} = \frac{7}{12} \\
\frac{4 + 3}{12} = \frac{7}{12} \\
\frac{7}{12} = \frac{7}{12}
\]

The solution is \( x = 3 \).

ANSWER:
3

55. \( 5.4(3k - 12) + 3.2(2k + 6) = -136 \)

SOLUTION:
\[
5.4(3k - 12) + 3.2(2k + 6) = -136 \\
16.2k - 64.8 + 6.4k + 19.2 = -136 \\
22.6k - 45.6 = -136 \\
22.6k = -90.4 \\
k = -4
\]

Check:
\[
5.4(3(-4) - 12) + 3.2(2(-4) + 6) = -136 \\
5.4(-12 - 12) + 3.2(-8 + 6) = -136 \\
5.4(-24) + 3.2(-2) = -136 \\
-129.6 - 6.4 = -136 \\
-136 = -136
\]

The solution is \( k = -4 \).

ANSWER:
-4
1-3 Solving Equations

56. \(8.2p - 33.4 = 1.7 - 3.5p\)

**SOLUTION:**

\[
\begin{align*}
8.2p - 33.4 &= 1.7 - 3.5p \\
8.2p + 3.5p - 33.4 &= 1.7 - 3.5p + 3.5p \\
11.7p - 33.4 &= 1.7 \\
11.7p + 33.4 &= 1.7 + 33.4 \\
p &= 3.51 \\
p &= 3
\end{align*}
\]

Check:

\[
\begin{align*}
8.2p - 33.4 &= 1.7 - 3.5p \\
8.2(3) - 33.4 &= 1.7 - 3.5(3) \\
24.6 - 33.4 &= 1.7 - 10.5 \\
-8.8 &= -8.8 \checkmark
\end{align*}
\]

The solution is \(p = 3\).

**ANSWER:** 3

57. \(\frac{4}{9}y + 5 = \frac{7}{9}y - 8\)

**SOLUTION:**

\[
\begin{align*}
\frac{4}{9}y + 5 &= \frac{7}{9}y - 8 \\
\frac{4}{9}y + \frac{7}{9}y + 5 &= \frac{7}{9}y + \frac{7}{9}y - 8 \\
\frac{11}{9}y + 5 &= -8 \\
\frac{11}{9}y &= -13 \\
y &= \frac{-117}{11} \checkmark
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{4}{9}y + 5 &= \frac{7}{9}y - 8 \\
\frac{4}{9}\left(\frac{-117}{11}\right) + 5 &= \frac{7}{9}\left(\frac{-117}{11}\right) - 8 \\
-\frac{468}{99} + 5 &= \frac{7819}{99} - 8 \\
-\frac{468}{99} + \frac{495}{99} &= \frac{7819}{99} - \frac{792}{99} \\
\frac{27}{99} &= \frac{27}{99} \checkmark
\end{align*}
\]

The solution is \(y = \frac{-117}{11}\).

**ANSWER:** \(\frac{-117}{11}\)
58. \[ \frac{3}{4} - \frac{1}{3} = \frac{2}{3} + \frac{1}{5} \]

**SOLUTION:**

\[
\begin{align*}
\frac{3}{4} - \frac{1}{3} &= \frac{2}{3} + \frac{1}{5} \\
\frac{3}{4} \cdot \frac{1}{3} &= \frac{2}{3} \cdot \frac{1}{5} \\
\frac{1}{12} &= \frac{1}{15} \\
12z &= 8 \\
\frac{1}{12} &= \frac{1}{15} \\
15 &= 15
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{3}{4} - \frac{1}{3} &= \frac{2}{3} + \frac{1}{5} \\
\frac{3}{4} \cdot \frac{1}{3} &= \frac{2}{3} \cdot \frac{1}{5} \\
\frac{1}{12} &= \frac{1}{15} \\
12z &= 8 \\
\frac{1}{12} &= \frac{1}{15} \\
15 &= 15
\end{align*}
\]

The solution is \( z = \frac{32}{5} \).

**ANSWER:**

\[ \frac{32}{5} \]

59. **FINANCIAL LITERACY** Benjamin spent $10,734 on his living expenses last year. Most of these expenses are listed at the right. Benjamin’s only other expense last year was rent. If he paid rent 12 times last year, how much is Benjamin’s rent each month?

<table>
<thead>
<tr>
<th>Expense</th>
<th>Annual Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric</td>
<td>$622</td>
</tr>
<tr>
<td>Gas</td>
<td>$428</td>
</tr>
<tr>
<td>Water</td>
<td>$240</td>
</tr>
<tr>
<td>Renter's Insurance</td>
<td>$144</td>
</tr>
</tbody>
</table>

**SOLUTION:**

Let \( x \) be the cost of rent each month.

Expense excluding rent = $622 + $428 + $240 + $144 = $1434

So:

\[ 12x + 1434 = 10,734 \]

\[ 12x = 9300 \]

\[ x = 775 \]

The rent is $775.

**ANSWER:**

\( x \) = the cost of rent each month; $622 + $428 + $240 + $144 + 12x = 10,734; $775 per month
60. BRIDGES The Sunshine Skyway Bridge spans Tampa Bay, Florida. Suppose one crew began building south from St. Petersburg, and another crew began building north from Bradenton. The two crews met 10,560 feet south of St. Petersburg approximately 5 years after construction began.

a. Suppose the St. Petersburg crew built an average of 176 feet per month. Together the two crews built 21,120 feet of bridge. Determine the average number of feet built per month by the Bradenton crew.

b. About how many miles of bridge did each crew build?

c. Is this answer reasonable? Explain.

**SOLUTION:**

a. Let \( x \) be represent the average number of feet built per month by the Bradenton crew.

The number of feet the St. Petersburg crew built in 5 years is \( 5 \times 12 \times 176 = 10,560 \).

Therefore:

\[
60x + 10,560 = 21,120
\]

\[
60x = 10,560
\]

\[
x = 176
\]

The Bradenton crew built an average of 176 feet per month.

b. Each crew built a distance of 10,560 feet.

5,280 feet = 1 mile

Therefore, each crew built a distance of 2 miles.

c. Yes; it seems reasonable that two crews working 4 miles apart would be able to complete the same amount of miles in the same amount of time.

**ANSWER:**

a. 176 ft

b. 2 mi

c. Yes; it seems reasonable that two crews working 4 miles apart would be able to complete the same amount of miles in the same amount of time.

61. MULTIPLE REPRESENTATIONS The absolute value of a number describes the distance of the number from zero.

a. GEOMETRIC Draw a number line. Label the integers from –5 to 5.

b. TABULAR Create a table of the integers on the number line and their distance from zero.

c. GRAPHICAL Make a graph of each integer \( x \) and its distance from zero \( y \) using the data points in the table.

d. For positive integers, the distance from zero is the same as the integer. For negative integers, the distance is the integer with the opposite sign because distance is always positive.

**ANSWER:**

a.

b.
1-3 Solving Equations

<table>
<thead>
<tr>
<th>Integer</th>
<th>Distance from Zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
</tr>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

d. For positive integers, the distance from zero is the same as the integer. For negative integers, the distance is the integer with the opposite sign because distance is always positive.

62. ERROR ANALYSIS Steven and Jade are solving

\[ A = \frac{1}{2} \cdot h \cdot (b_1 + b_2) \]

for \( b_2 \). Is either of them correct? Explain your reasoning.

**SOLUTION:**
Sample answer: Jade; in the last step, when Steven subtracted \( b_1 \) from each side, he mistakenly put the \( -b_1 \) in the numerator instead of after the entire fraction. To solve for \( b_2 \), \( b_1 \) must be subtracted from each side.

**ANSWER:**
Sample answer: Jade; in the last step, when Steven subtracted \( b_1 \) from each side, he mistakenly put the \( -b_1 \) in the numerator instead of after the entire fraction.

63. CHALLENGE Solve

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \] for \( y_1 \)

**SOLUTION:**
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \]

\[ d^2 - (x_2 - x_1)^2 = (y_2 - y_1)^2 \]

\[ \sqrt{d^2 - (x_2 - x_1)^2} = y_2 - y_1 \]

\[ y_1 = y_2 - \sqrt{d^2 - (x_2 - x_1)^2} \]

**ANSWER:**
\[ y_1 = y_2 - \sqrt{d^2 - (x_2 - x_1)^2} \]

64. REASONING Use what you have learned in this lesson to explain why the following number trick works.

- Take any number.
- Multiply it by ten.
- Subtract 30 from the result.
- Divide the new result by 5.
- Add 6 to the result.
- Your new number is twice your original.

**SOLUTION:**
This number trick is a series of steps that can be represented by an equation with \( x \) representing the number chosen.

\[ \frac{10x - 30}{5} + 6 = 2x \]

\[ \frac{10x - 30}{5} = 2x - 6 \]

\[ (2x - 6) + 6 = 2x \]

**ANSWER:**
Translating this number trick into an expression yields:

\[ \frac{10x - 30}{5} + 6 = 2x \]

\[ \frac{10x - 30}{5} = 2x - 6 \]

\[ (2x - 6) + 6 = 2x \]
1-3 Solving Equations

65. **OPEN ENDED** Provide one example of an equation involving the Distributive Property that has no solution and another example that has infinitely many solutions.

**SOLUTION:**
Sample answer: 
3(x – 4) = 3x + 5 This has no solution since it simplifies to an untrue equation.
2(3x – 1) = 6x – 2 This has infinite number of solutions since it simplifies to a true equation and x can be any real number.

**ANSWER:**
Sample answer: 3(x – 4) = 3x + 5; 2(3x – 1) = 6x – 2

66. **WRITING IN MATH** Compare and contrast the Substitution Property of Equality and the Transitive Property of Equality.

**SOLUTION:**
Sample answer: The Transitive Property utilizes the Substitution Property. While the Substitution Property is done with two values, that is, one being substituted for another, the Transitive Property deals with three values, determining that since two values are equal to a third value, then they must be equal.

**ANSWER:**
Sample answer: The Transitive Property utilizes the Substitution Property. While the Substitution Property is done with two values, that is, one being substituted for another, the Transitive Property deals with three values, determining that since two values are equal to a third value, then they must be equal.

67. The graph shows the solution of which inequality?

![Graph](image)

A. $y < \frac{2}{3}x + 4$
B. $y > \frac{2}{3}x + 4$
C. $y < \frac{3}{2}x + 4$
D. $y > \frac{3}{2}x + 4$

**SOLUTION:**
The slope of the line is $\frac{3}{2}$ and the y-intercept is 4. So, the equation corresponding to the inequality is $y = \frac{3}{2}x + 4$. Since the upper region of the line is shaded, the inequality is $y > \frac{3}{2}x + 4$. So, the correct choice is D.

**ANSWER:**
D
1-3 Solving Equations

68. SAT/ACT What is $\frac{1}{3}$ subtracted from its reciprocal?
   
   F $-2 \frac{2}{3}$
   G $-\frac{7}{12}$
   H $-\frac{1}{12}$
   J $\frac{1}{4}$
   K $\frac{3}{4}$

   SOLUTION:
   
   $1 - \frac{4}{3} = \frac{3}{3}$
   
   The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$.
   
   $\frac{3}{4} - \frac{4}{3} = \frac{3(3) - 4(4)}{12}$
   
   $= \frac{9 - 16}{12}$
   
   $= \frac{-7}{12}$
   
   So, the correct choice is G.

   ANSWER: G

69. GEOMETRY Which of the following describes the transformation of $\triangle ABC$ to $\triangle A'B'C'$?

   A. a reflection across the $y$-axis and a translation down 2 units
   B. a reflection across the $x$-axis and a translation down 2 units
   C. a rotation $90^\circ$ to the right and a translation down 2 units
   D. a rotation $90^\circ$ to the right and a translation right 2 units

   SOLUTION:

   Analyzing the graph shows that the image was reflected across the $y$-axis. The answer is A a reflection across the $y$-axis and a translation down 2 units.

   ANSWER: A

70. SHORT RESPONSE A local theater sold 1200 tickets during the opening weekend of a movie. On the following weekend, 840 tickets were sold. What was the percent decrease of tickets sold?

   SOLUTION:
   
   Difference = 1200 - 840 = 360
   
   Percentage of decrease = $\frac{360}{1200}$
   
   $= 0.3$
   
   $= 30\%$

   ANSWER: 30\%
1-3 Solving Equations

71. Simplify $3x + 8y + 5z - 2y - 6x + z$.

**SOLUTION:**

$$3x + 8y + 5z - 2y - 6x + z$$
$$= 3x - 6x + 8y - 2y + 5z + z$$
$$= -3x + 6y + 6z$$

**ANSWER:**

$-3x + 6y + 6z$

72. **BAKING** Tamera is making two types of bread.

The first type of bread needs $\frac{1}{2}$ cups of flour, and the second needs $\frac{3}{4}$ cups of flour. Tamera wants to make 2 loaves of the first recipe and 3 loaves of the second recipe. How many cups of flour does she need?

**SOLUTION:**

$$2 \left( \frac{1}{2} \right) + 3 \left( \frac{3}{4} \right) = 2 \left( \frac{5}{2} \right) + 3 \left( \frac{7}{4} \right)$$
$$= 5 + \frac{21}{4}$$
$$= \frac{41}{4}$$
$$= 10 \frac{1}{4} \text{ cups}$$

**ANSWER:**

$10 \frac{1}{4}$

73. **LANDMARKS** Suppose the Space Needle in Seattle, Washington, casts a 220-foot shadow at the same time a nearby tourist casts a 2-foot shadow. If the tourist is $5 \frac{1}{2}$ feet tall, how tall is the Space Needle?

**SOLUTION:**

Let $h$ be the height of the Space Needle.

$$\frac{h}{220} = \frac{5 \frac{1}{2}}{2}$$
$$h = \frac{11}{4} \cdot 220$$
$$h = 2420$$
$$h = 605$$

The height of the Space Needle is 605 feet.

**ANSWER:**

605 ft

74. Evaluate $a - \left[ c(b - a) \right]$ if $a = 5$, $b = 7$, and $c = 2$.

**SOLUTION:**

Substitute $a = 5$, $b = 7$, and $c = 2$.

$$5 - \left[ 2(7 - 5) \right] = 5 - \left[ 2(2) \right]$$
$$= 5 - 4$$
$$= 1$$

**ANSWER:**

1
Identify the additive inverse for each number or expression.

75. \(-4 \frac{1}{5}\)

**SOLUTION:**
\[-4 \frac{1}{5} = -\frac{4(5) + 1}{5} = -\frac{20 + 1}{5} = -\frac{21}{5}\]
Since \(-\frac{21}{5} + \frac{21}{5} = 0\), the additive inverse of \(-\frac{21}{5}\) is \(\frac{21}{5}\) or \(-\frac{21}{5}\).

**ANSWER:**
\(-\frac{21}{5}\)

76. 3.5

**SOLUTION:**
Since 3.5 \(- 3.5 = 0\), the additive inverse of 3.5 is \(-3.5\).

**ANSWER:**
\(-3.5\)

77. \(-2x\)

**SOLUTION:**
Since \((-2x) + 2x = 0\), the additive inverse of \(-2x\) is 2x.

**ANSWER:**
2x

78. 6 \(- 7y\)

**SOLUTION:**
The additive inverse of 6 is \(-6\) and the additive inverse of \(-7y\) is 7y.
The additive inverse of 6 \(-7y\) is \(-6 + 7y\).

**ANSWER:**
\(-6 + 7y\)

**79. \(\frac{2}{3}\)**

**SOLUTION:**
\(\frac{2}{3} = \frac{11}{3}\)
Since \(\frac{11}{3} \(- \frac{11}{3} = 0\), the additive inverse of \(\frac{2}{3}\) or \(\frac{11}{3}\) is \(-\frac{2}{3}\) or \(-\frac{11}{3}\).

**ANSWER:**
\(-\frac{2}{3}\)

80. \(-1.25\)

**SOLUTION:**
Since \((-1.25) + 1.25 = 0\), the additive inverse of \(-1.25\) is 1.25.

**ANSWER:**
1.25

81. 5x

**SOLUTION:**
Since 5x \(- 5x = 0\), the additive inverse of 5x is \(-5x\).

**ANSWER:**
\(-5x\)

82. 4 \(- 9x\)

**SOLUTION:**
The additive inverse of 4 is \(-4\) and the additive inverse of 9x is \(-9x\).
So, the additive inverse of 4 \(-9x\) is \(-4 + 9x\).

**ANSWER:**
\(-4 + 9x\)
Evaluate each expression if \( x = -4 \) and \( y = -9 \).

1. \( |x - 8| \)

\[ \text{SOLUTION:} \]
Substitute \(-4\) for \( x \) and solve.
\[ |x - 8| = |-4 - 8| \]
\[ = |-12| \]
\[ = 12 \]

\[ \text{ANSWER:} \]
12

2. \( |7y| \)

\[ \text{SOLUTION:} \]
Substitute \(-9\) for \( y \) and solve.
\[ |7y| = |7(-9)| \]
\[ = |-63| \]
\[ = 63 \]

\[ \text{ANSWER:} \]
63

3. \( -3|xy| \)

\[ \text{SOLUTION:} \]
Substitute \(-4\) for \( x \) and \(-9\) for \( y \) and solve.
\[ -3|xy| = -3|(-4)(-9)| \]
\[ = -3|36| \]
\[ = -3(36) \]
\[ = -108 \]

\[ \text{ANSWER:} \]
-108

4. \(-2|3x + 8| - 4\)

\[ \text{SOLUTION:} \]
Substitute \(-4\) for \( x \) and solve.
\[-2|3x + 8| - 4 = -2|3(-4) + 8| - 4 \]
\[ = -2|12| - 4 \]
\[ = -2(12) - 4 \]
\[ = -24 - 4 \]
\[ = -28 \]

\[ \text{ANSWER:} \]
-12
5. MODELING Most freshwater tropical fish thrive if the water is within 2°F of 78°F.
   a. Write an equation to determine the least and greatest optimal temperatures.
   b. Solve the equation you wrote in part a.
   c. If your aquarium’s thermometer is accurate to within plus or minus 1°F, what should the temperature of the water be to ensure that it reaches the minimum temperature? Explain.

**SOLUTION:**

a. Freshwater tropical fish thrive if the water is within 2°F of 78°F. So, substitute \( c = 78 \) and \( r = 2 \) in the equation \(|x - c| = r\).

\[ |x - 78| = 2 \]

So, the equation to determine the least and greatest optimal temperatures is \(|x - 78| = 2\).

b. Case 1: \[ x - 78 = 2 \]

\[ x = 80 \]

Case 2: \[ x - 78 = -2 \]

\[ x = 76 \]

So, the least temperature is 76°F and the greatest temperature is 80°F.

c. 77°F; This would ensure a minimum temperature of 76°F.

**ANSWER:**

a. \(|x - 78| = 2\)

b. least: 76°F, greatest: 80°F

c. 77°F; This would ensure a minimum temperature of 76°F.

---

6. **Solve each equation. Check your solutions.**

**SOLUTION:**

\[ |x + 8| = 12 \]

Case 1: \[ x + 8 = 12 \]

\[ x = 4 \]

Case 2: \[ x + 8 = -12 \]

\[ x = -20 \]

There appear to be two solutions, 4 and -20.

Check: Substitute each value in the original equation.

\[ |x + 8| = 12 \]

\[ |4 + 8| = 12 \]

\[ |12| = 12 \]

\[ 12 = 12 \checkmark \]

\[ |x + 8| = 12 \]

\[ |-20 + 8| = 12 \]

\[ |-12| = 12 \]

\[ 12 = 12 \checkmark \]

The solution set is \( \{4, -20\} \).

**ANSWER:**

\( \{4, -20\} \)

7. **Solve each equation. Check your solutions.**

**SOLUTION:**

\[ |y - 4| = 11 \]

Case 1: \[ y - 4 = 11 \]

\[ y = 15 \]

Case 2: \[ y - 4 = -11 \]

\[ y = -7 \]

There appear to be two solutions, 15 and -7.

Check: Substitute each value in the original equation.

\[ |y - 4| = 11 \]

\[ |15 - 4| = 11 \]

\[ 11 = 11 \checkmark \]

\[ |-7 - 4| = 11 \]

\[ 11 = 11 \checkmark \]

The solution set is \( \{15, -7\} \).

**ANSWER:**

\( \{15, -7\} \)
1-4 Solving Absolute Value Equations

8. \(|a - 5| + 4 = 9\)

**SOLUTION:**

\(|a - 5| + 4 = 9\)

\(|a - 5| + 4 - 4 = 9 - 4\)

\(|a - 5| = 5\)

Case 1: \(a - 5 = 5\)

\(a - 5 + 5 = 5 + 5\)

\(a = 10\)

Case 2: \(a - 5 = -5\)

\(a - 5 + 5 = -5 + 5\)

\(a = 0\)

There appear to be two solutions, 10 and 0.

Check: Substitute each value in the original equation.

\(|a - 5| + 4 = 9\)

\(|10 - 5| + 4 = 9\)

\(5 + 4 = 9\)

\(9 = 9\checkmark\)

\(|0 - 5| + 4 = 9\)

\(|-5| + 4 = 9\)

\(5 + 4 = 9\)

\(9 = 9\checkmark\)

The solution set is \(\{10, 0\}\).

**ANSWER:**

\(\{10, 0\}\)

9. \(|b - 3| + 8 = 3\)

**SOLUTION:**

\(|b - 3| + 8 = 3\)

\(|b - 3| + 8 - 8 = 3 - 8\)

\(|b - 3| = -5\)

Case 1: \(b - 3 = -5\)

\(b - 3 + 3 = -5 + 3\)

\(b = -2\)

Case 2: \(b - 3 = 5\)

\(b - 3 + 3 = 5 + 3\)

\(b = 8\)

There appear to be two solutions, \(-2\) and \(8\).

Check: Substitute each value in the original equation.

\(|b - 3| + 8 = 3\)

\(|-2 - 3| + 8 = 3\)

\(|-5| + 8 = 3\)

\(5 + 8 = 3\)

\(15 \neq 3\)

\(|8 - 3| + 8 = 3\)

\(|5 + 8 = 3\)

\(5 + 8 = 3\)

Because \(15 \neq 3\) and \(13 \neq 3\), the solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)
1-4 Solving Absolute Value Equations

10. \(3|2x - 3| - 5 = 4\)

**SOLUTION:**

\[
\begin{align*}
3|2x - 3| - 5 &= 4 \\
3|2x - 3| - 5 + 5 &= 4 + 5 \\
3|2x - 3| &= 9 \\
\frac{3|2x - 3|}{3} &= \frac{9}{3} \\
|2x - 3| &= 3
\end{align*}
\]

Case 1:  
\[
2x - 3 = 3 \\
2x = 6 \\
x = 3
\]

Case 2:  
\[
2x - 3 = -3 \\
2x = 0 \\
x = 0
\]

There appear to be two solutions, 3 and 0.

Check: Substitute each value in the original equation.

\[
\begin{align*}
3|2(3) - 3| - 5 &= 4 \\
3|2(0) - 3| - 5 &= 4 \\
3|6 - 3| - 5 &= 4 \\
3|0 - 3| - 5 &= 4 \\
9 - 5 &= 4 \\
4 &= 4 \checkmark \\
4 &= 4 \checkmark
\end{align*}
\]

The solution set is \(\{3, 0\}\).

**ANSWER:**  
\(\{3, 0\}\)

11. \(-2|5y - 1| = -10\)

**SOLUTION:**

\[
\begin{align*}
-2|5y - 1| &= -10 \\
\frac{-2|5y - 1|}{-2} &= \frac{-10}{-2} \\
|5y - 1| &= 5
\end{align*}
\]

Case 1:  
\[
5y - 1 = 5 \\
5y = 6 \\
y = \frac{6}{5}
\]

Case 2:  
\[
5y - 1 = -5 \\
5y = -4 \\
y = -\frac{4}{5}
\]

There appear to be two solutions, \(\frac{6}{5}\) and \(-\frac{4}{5}\).

Check: Substitute each value in the original equation.

\[
\begin{align*}
-2|\frac{6}{5} - 1| &= -10 \\
-2\left|\frac{6}{5} - 1\right| &= -10 \\
-2\left|\frac{6}{5} - \frac{5}{5}\right| &= -10 \\
-2\left|\frac{1}{5}\right| &= -10 \\
-2\left(\frac{1}{5}\right) &= -10 \\
-\frac{2}{5} &= -10 \checkmark \\
-10 &= -10 \checkmark
\end{align*}
\]

The solution set is \(\left\{\frac{6}{5}, -\frac{4}{5}\right\}\).

**ANSWER:**  
\(\left\{\frac{6}{5}, -\frac{4}{5}\right\}\)
12. $|a - 4| = 3a - 6$

**SOLUTION:**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a - 4 = 3a - 6$</td>
<td>$a - 4 = -(3a - 6)$</td>
</tr>
<tr>
<td>$a - 4 + 4 = 3a - 6 + 4$</td>
<td>$a - 4 = -3a + 6$</td>
</tr>
<tr>
<td>$a = 3a - 2$</td>
<td>$a - 4 + 4 = -3a + 6 + 4$</td>
</tr>
<tr>
<td>$a = 3a = 3a - 2 - 3a$</td>
<td>$a = -3a + 10$</td>
</tr>
<tr>
<td>$2a = -2$</td>
<td>$a + 3a = -3a + 10 + 3a$</td>
</tr>
<tr>
<td>$a = 1$</td>
<td>$4a = 10$</td>
</tr>
<tr>
<td>$a = \frac{10}{4}$</td>
<td>$a = \frac{10}{4}$</td>
</tr>
<tr>
<td>$a = \frac{5}{2}$</td>
<td>$a = \frac{5}{2}$</td>
</tr>
</tbody>
</table>

There appear to be two solutions, 1 and $\frac{5}{2}$.

Check: Substitute each value in the original equation.

$|a - 4| = 3a - 6$

$|a - 4| = 3a - 6$

$\frac{5}{2} - 4 = \frac{3}{2}$

$5 - 4(2) = \frac{15}{2} - 6$

$2 \neq -3$

$5 - 8 = \frac{15 - 6(2)}{2}$

$|3 - 3| = \frac{15 - 12}{2}$

$3 = \frac{3}{2}$

Since $3 \neq -3$, $a = 1$ is an extraneous solution. The solution is $\frac{5}{2}$ or 2.5.

**ANSWER:**

$\{2.5\}$

13. $|b + 5| = 2b + 3$

**SOLUTION:**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b + 5 = 2b + 3$</td>
<td>$b + 5 = -(2b + 3)$</td>
</tr>
<tr>
<td>$b + 5 - 5 = 2b + 3 - 5$</td>
<td>$b + 5 = -2b - 3$</td>
</tr>
<tr>
<td>$b = 2b - 2$</td>
<td>$b + 5 - 5 = -2b - 3 - 5$</td>
</tr>
<tr>
<td>$b - 2b = 2b - 2 - 2b$</td>
<td>$b = -2b - 8$</td>
</tr>
<tr>
<td>$-b = -2$</td>
<td>$b + 2b = -2b - 8 + 2b$</td>
</tr>
<tr>
<td>$b = 2$</td>
<td>$3b = -8$</td>
</tr>
<tr>
<td>$b = \frac{3}{2}$</td>
<td>$b = \frac{8}{3}$</td>
</tr>
</tbody>
</table>

There appear to be two solutions, 2 and $-\frac{8}{3}$.

Check: Substitute each value in the original equation.

$|b + 5| = 2b + 3$

$|b + 5| = 2b + 3$

$2 + 5 = 2(2) + 3$

$\frac{8}{3} + 5 = 2\left(-\frac{8}{3}\right) + 3$

$\left[\frac{7}{3}\right] + 4 + 3$

$-8 + 5(3) = \frac{-16}{3} + 3$

$7 = 7$

$\left[\frac{7}{3}\right] + 16 + 9$

$\frac{7}{3} = \frac{-7}{3}$

Since $\frac{7}{3} \neq -\frac{7}{3}$, $b = -\frac{8}{3}$ is an extraneous solution. So, the solution is 2.

**ANSWER:**

$\{2\}$
1-4 Solving Absolute Value Equations

Evaluate each expression if \( a = -3, \ b = -5, \) and \( c = 4.2. \)

14. \(|-3c|\)

**SOLUTION:**
Substitute 4.2 for \( c \) and solve. 
\[
|-3c| = |-3(4.2)| \\
= |-12.6| \\
= 12.6
\]

**ANSWER:**
12.6

15. \(|5b|\)

**SOLUTION:**
Substitute -5 for \( b \) and solve. 
\[
|5b| = |5(-5)| \\
= |-25| \\
= 25
\]

**ANSWER:**
25

16. \(|a-b|\)

**SOLUTION:**
Substitute -3 for \( a \) and -5 for \( b \) and solve. 
\[
|a-b| = |-3-(-5)| \\
= |-3+5| \\
= |2| \\
= 2
\]

**ANSWER:**
2

17. \(|b-c|\)

**SOLUTION:**
Substitute -5 for \( b \) and 4.2 for \( c \) and solve. 
\[
|b-c| = |-5-4.2| \\
= |-9.2| \\
= 9.2
\]

**ANSWER:**
9.2

18. \(|3b-4a|\)

**SOLUTION:**
Substitute -3 for \( a \) and -5 for \( b \) and solve. 
\[
|3b-4a| = |3(-5)-4(-3)| \\
= |-15+12| \\
= |-3| \\
= 3
\]

**ANSWER:**
3

19. \(2|4a-3c|\)

**SOLUTION:**
Substitute -3 for \( a \) and 4.2 for \( c \) and solve. 
\[
2|4a-3c| = 2|4(-3)-3(4.2)| \\
= 2|-12-12.6| \\
= 2|-24.6| \\
= 2(24.6) \\
= 49.2
\]

**ANSWER:**
49.2

20. \(|-3c-a|\)

**SOLUTION:**
Substitute -3 for \( a \) and 4.2 for \( c \) and solve. 
\[
|-3c-a| = |-3(4.2)-(-3)| \\
= |-12.6+3| \\
= |-9.6| \\
= -15.6
\]

**ANSWER:**
-15.6

21. \(|abc|\)

**SOLUTION:**
\[
|abc| = |(-3)(-5)(4.2)| \\
= |-63| \\
= -63
\]

**ANSWER:**
-63
22. **FOOD** To make cocoa powder, cocoa beans are roasted. The ideal temperature for roasting is 300°F, plus or minus 25°. Write and solve an equation describing the maximum and minimum roasting temperatures for cocoa beans.

**SOLUTION:**

\[ |x - 300| = 25 \]

Solve the equation \(|x - 300| = 25\).

Case 1:  
\[ x - 300 = 25 \]
\[ x = 325 \]

Case 2:  
\[ x - 300 = -25 \]
\[ x = 275 \]

So, the maximum temperature is 325°F and the minimum temperature is 275°F.

**ANSWER:**  
\(|x - 300| = 25\); maximum: 325°F; minimum: 275°F

**Solve each equation. Check your solutions.**

23. \(|z - 13| = 21\)

**SOLUTION:**

Case 1:  
\[ z - 13 = 21 \]
\[ z = 34 \]

Case 2:  
\[ z - 13 = -21 \]
\[ z = -8 \]

There appear to be two solutions, 34 and \(-8\).

Check: Substitute each value in the original equation.

\[ |z - 13| = 21 \]
\[ |34 - 13| = 21 \]
\[ |-8 - 13| = 21 \]
\[ |21| = 21 \]
\[ |-21| = 21 \]
\[ 21 = 21 \checkmark \]
\[ 21 = 21 \checkmark \]

The solution set is \(\{34, -8\}\).

**ANSWER:**  
\(\{34, -8\}\)

24. \(|w + 9| = 17\)

**SOLUTION:**

Case 1:  
\[ w + 9 = 17 \]
\[ w = 8 \]

Case 2:  
\[ w + 9 = -17 \]
\[ w = -26 \]

There appear to be two solutions, 8 and \(-26\).

\[ |w + 9| = 17 \]
\[ |8 + 9| = 17 \]
\[ |-26 + 9| = 17 \]
\[ 17 = 17 \checkmark \]
\[ 17 = 17 \checkmark \]

The solution set is \(\{8, -26\}\).

**ANSWER:**  
\(\{8, -26\}\)

25. \(9 = |d + 5|\)

**SOLUTION:**

Case 1:  
\[ d + 5 = 9 \]
\[ d = 4 \]

Case 2:  
\[ d + 5 = -9 \]
\[ d = -14 \]

There appear to be two solutions, 4 and \(-14\).

Check: Substitute each value in the original equation.

\[ 9 = |d + 5| \]
\[ 9 = |4 + 5| \]
\[ 9 = |-14 + 5| \]
\[ 9 = 9 \checkmark \]
\[ 9 = 9 \checkmark \]

The solution set is \(\{4, -14\}\).

**ANSWER:**  
\(\{4, -14\}\)
26. \(35 = |x - 6|\)

**SOLUTION:**

Case 1: 
\[35 = x - 6\]
\[35 + 6 = x - 6 + 6\]
\[41 = x\]

Case 2: 
\[35 = -(x - 6)\]
\[35 + 6 = x - 6 + 6\]
\[41 = x\]

There appear to be two solutions, 41 and -29.

Check: Substitute each value in the original equation.

\[35 = |41 - 6| = 35\]
\[35 = |35| = 35\]
\[35 = 35\]
\[35 = |x - 6|\]
\[35 = |x - 6|\]

The solution set is \([-29, 41]\).

**ANSWER:**
\([-29, 41]\)

27. \(5|q + 6| = 20\)

**SOLUTION:**

Case 1: 
\[\frac{5|q + 6|}{5} = \frac{20}{5}\]
\[|q + 6| = 4\]

Case 2: 
\[q + 6 = 4\]
\[q = -2\]

Case 2: 
\[q + 6 = -4\]
\[q = -10\]

There appear to be two solutions, -2 and -10.

Check: Substitute each value in the original equation.

\[5|q + 6| = 20\]
\[5|-2 + 6| = 20\]
\[5|4| = 20\]
\[5(-2 + 6) = 20\]
\[5(-4) = 20\]
\[5(4) = 20\]
\[20 = 20\]

The solution set is \([-2, -10]\).

**ANSWER:**
\([-2, -10]\)
Solving Absolute Value Equations

28. \(-3|r + 4| = -21\)

**SOLUTION:**
\(-3|r + 4| = -21\)
\(-\frac{3|r + 4|}{-3} = \frac{-21}{-3}\)
\(|r + 4| = 7\)

Case 1: \(r + 4 = 7\)
\(r + 4 - 4 = 7 - 4\)
\(r = 3\)

Case 2: \(r + 4 = -7\)
\(r + 4 - 4 = -7 - 4\)
\(r = -11\)

There appear to be two solutions, 3 and -11.
Check: Substitute each value in the original equation.

\(-3|3 + 4| = -21\)
\(-3|-11 + 4| = -21\)
\(-3|7| = -21\)
\(-3|-7| = -21\)
\(-3|7| = -21\)
\(-3|-7| = -21\)
\(-21 = -21\)

The solution set is \(\{3, -11\}\).

**ANSWER:**
\(\{3, -11\}\)

29. \(3|2a - 4| = 0\)

**SOLUTION:**
\(3|2a - 4| = 0\)
\(\frac{3|2a - 4|}{3} = \frac{0}{3}\)
\(|2a - 4| = 0\)
\(2a - 4 = 0\)
\(2a = 4\)
\(a = 2\)

Check: Substitute \(a = 2\) in the original equation.

\(3|2(2) - 4| = 0\)
\(3|4 - 4| = 0\)
\(3|0| = 0\)
\(0 = 0\)

The solution is \(a = 2\).

**ANSWER:**
\(\{2\}\)
30. \(8|5w - 1| = 0\)

**SOLUTION:**

\[
\begin{align*}
8|5w - 1| &= 0 \\
\frac{8}{8} &= \frac{0}{8} \\
|5w - 1| &= 0 \\
5w - 1 &= 0 \\
5w &= 1 \\
\frac{5w}{5} &= \frac{1}{5} \\
w &= \frac{1}{5}
\end{align*}
\]

Check:

\[
\begin{align*}
8|5w - 1| &= 0 \\
8\left|5\left(\frac{1}{5}\right) - 1\right| &= 0 \\
8\left|1 - 1\right| &= 0 \\
8|0| &= 0 \\
8(0) &= 0 \\
0 &= 0 \checkmark
\end{align*}
\]

The solution is \(\frac{1}{5}\).

**ANSWER:**

\[\left\{ \frac{1}{5} \right\}\]

31. \(2|3x - 4| + 8 = 6\)

**SOLUTION:**

\[
\begin{align*}
2|3x - 4| + 8 &= 6 \\
2|3x - 4| &= 6 - 8 \\
2|3x - 4| &= -2 \\
\frac{2}{2} &= \frac{-2}{2} \\
|3x - 4| &= -1
\end{align*}
\]

Case 1:

\[
\begin{align*}
3x - 4 &= -1 \\
x &= 1
\end{align*}
\]

Case 2:

\[
\begin{align*}
3x - 4 &= -(-1) \\
x &= \frac{5}{3}
\end{align*}
\]

There appear to be two solutions, 1 and \(\frac{5}{3}\).

Check: Substitute the values in the original equation.

\[
\begin{align*}
2|3(1) - 4| + 8 &= 6 \\
2\left|3\left(\frac{5}{3}\right) - 4\right| + 8 &= 6 \\
2|3 - 4| + 8 &= 6 \\
2|1| + 8 &= 6 \\
2(1) + 8 &= 6 \\
2 + 8 &= 6 \\
10 &= 6
\end{align*}
\]

Since 10 \(\neq\) 6, the solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)

32. \(4|7y + 2| - 8 = -7\)

**SOLUTION:**

\[
\begin{align*}
4|7y + 2| &= -7 + 8 \\
4|7y + 2| &= 1 \\
\frac{4}{4} &= \frac{1}{4} \\
|7y + 2| &= \frac{1}{4}
\end{align*}
\]

The resulting equations corresponding to these cases will be:

\[
\begin{align*}
7y + 2 &= \frac{1}{4} \\
7y &= \frac{1}{4} - 2 \\
7y &= -\frac{7}{4} \\
y &= -\frac{1}{4}
\end{align*}
\]

\[
\begin{align*}
7y + 2 &= -\frac{1}{4} \\
7y &= -\frac{1}{4} - 2 \\
7y &= -\frac{9}{4} \\
y &= -\frac{3}{4}
\end{align*}
\]

**ANSWER:**

\[\left\{ -\frac{1}{4}, -\frac{3}{4} \right\}\]
Evaluate each expression if \(x = -4\) and \(y = -9\).

\[4|\frac{7y+2}{4}|-8 = -7\]
\[4|\frac{7y+2}{4}|-8 +8 = -7 +8\]
\[4|\frac{7y+2}{4}| = 1\]
\[4|\frac{7y+2}{4}| = 1\]
\[\frac{7y+2}{4} = \pm\frac{1}{4}\]
\[|7y+2| = \frac{1}{4}\]

Case 1:
\[7y+2 = \frac{1}{4}\]
\[7y+2 = -\frac{1}{4}\]
\[7y+2 - 2 = \frac{1}{4} - 2\]
\[7y+2 - 2 = -\frac{1}{4} - 2\]
\[7y = \frac{1-2(4)}{4}\]
\[7y = -\frac{1-2(4)}{4}\]
\[7y = \frac{1-8}{4}\]
\[7y = -\frac{1-8}{4}\]
\[7y = -\frac{7}{4}\]
\[7y = -\frac{9}{4}\]
\[\frac{7y}{7} = -\frac{1}{4}\]
\[\frac{7y}{7} = -\frac{9}{28}\]
\[y = -\frac{1}{4}\]
\[y = -\frac{9}{28}\]

There appear to be two solutions, \(-\frac{1}{4}\) and \(-\frac{9}{28}\).

Check: Substitute each value in the original equation.

\[4|\frac{7y+2}{4}| = 1\]
\[\left|\frac{7(-\frac{1}{4})+2}{4}\right| = 1\]
\[\left|\frac{7\left(-\frac{9}{28}\right)+2}{4}\right| = 1\]
\[\left|\frac{4}{4} + 2 - \frac{8}{4}\right| = 1\]
\[\left|\frac{9}{4} + 2 - \frac{8}{4}\right| = 1\]
\[\left|\frac{4}{4} + 2 - \frac{8}{4}\right| = 1\]
\[\left|\frac{4}{4} + 2 - \frac{8}{4}\right| = 1\]
\[\left|\frac{4}{4} - \frac{8}{4}\right| = 1\]
\[\left|\frac{4}{4} - \frac{8}{4}\right| = 1\]
\[\left|\frac{1}{4} - \frac{8}{4}\right| = 1\]
\[\left|\frac{1}{4} - \frac{8}{4}\right| = 1\]
\[\left|\frac{1}{4} - \frac{8}{4}\right| = 1\]
\[\left|\frac{1}{4} - \frac{8}{4}\right| = 1\]

The solution set is \(\left\{-\frac{1}{4}, -\frac{9}{28}\right\}\).

ANSWER:
1-4 Solving Absolute Value Equations

\[ -3|3x - 2| - 12 = -6 \]
\[ -3|3(0) - 2| - 12 \neq -6 \]
\[ -3|0 - 2| - 12 \neq -6 \]
\[ -3|-2| - 12 \neq -6 \]
\[ -3|-2| - 12 + 12 \neq -6 + 12 \]
\[ -3|-2| \neq 6 \]
\[ -3(2) \neq 6 \]
\[ -6 \neq 6 \]
\[ -3|3x - 2| - 12 = -6 \]
\[ -3|3\left(\frac{4}{3}\right) - 2| - 12 \neq -6 \]
\[ -3|4 - 2| - 12 \neq -6 \]
\[ -3|2| - 12 \neq -6 \]
\[ -3|2| - 12 + 12 \neq -6 + 12 \]
\[ -3|2| \neq 6 \]
\[ -3(2) \neq 6 \]
\[ -6 \neq 6 \]

Since \(-6 \neq 6\), the solution set is \(\emptyset\).

**ANSWER:**
\(\emptyset\)

34. \(-5|3z + 8| - 5 = -20\)

**SOLUTION:**
\[-5|3z + 8| - 5 = -20 \]
\[-5|3z + 8| - 5 + 5 = -20 + 5 \]
\[-5|3z + 8| = -15 \]
\[-5|3z + 8| = -15 \]
\[-5 \neq -5 \]
\[|3z + 8| = 3 \]

**Case 1:**
\[
\begin{align*}
3z + 8 &= 3 \\
3z + 8 &= -3
\end{align*}
\]
\[
\begin{align*}
3z &= -5 \\
3z &= -11
\end{align*}
\]
\[
\begin{align*}
\frac{3z}{3} &= \frac{-5}{3} \\
\frac{3z}{3} &= \frac{-11}{3}
\end{align*}
\]
\[
\begin{align*}
z &= \frac{-5}{3} \\
z &= \frac{-11}{3}
\end{align*}
\]

There appear to be two solutions, \(\frac{-5}{3}\) and \(\frac{-11}{3}\).

Check: Substitute the values in the original equation.

\[
\begin{align*}
z &= \frac{-5}{3} \\
-5|3z + 8| - 5 &= -20 \\
-5\left|3\left(\frac{-5}{3}\right) + 8\right| - 5 &= -20 \\
-5\left|-\frac{5}{3} + 8\right| - 5 &= -20 \\
-5|-3| - 5 &= -20 \\
-5(-3) - 5 &= -20 \\
-15 &= -15 \checkmark
\end{align*}
\]

\[
\begin{align*}
z &= \frac{-11}{3} \\
-5|3z + 8| - 5 &= -20 \\
-5\left|3\left(\frac{-11}{3}\right) + 8\right| - 5 &= -20 \\
-5\left|-\frac{11}{3} + 8\right| - 5 &= -20 \\
-5|-3| - 5 &= -20 \\
-5(-3) - 5 &= -20 \\
-15 &= -15 \checkmark
\end{align*}
\]
1-4 Solving Absolute Value Equations

The solution set is \(\left\{ \frac{-5}{3}, \frac{-11}{3} \right\}\).  

**ANSWER:**

\(\left\{ \frac{-5}{3}, \frac{-11}{3} \right\}\)

35. MONEY The U.S. Mint produces quarters that weigh about 5.67 grams each. After the quarters are produced, a machine weighs them. If the quarter weighs 0.02 gram more or less than the desired weight, the quarter is rejected. Write and solve an equation to find the heaviest and lightest quarters the machine will approve.

**SOLUTION:**

Substitute \(c = 5.67\) and \(r = 0.02\) in the equation \(|x - c| = r\).

\(|x - 5.67| = 0.02\)

Solve the equation \(|x - 5.67| = 0.02\).

Case 1:

\[x - 5.67 = 0.02\]

\[x = 5.69\]

Case 2:

\[x - 5.67 = -0.02\]

\[x = 5.65\]

So, the heaviest quarters the machine will approve are those weighing 5.69 grams. The lightest quarters the machine will approve is 5.65 grams.

**ANSWER:**

\(|x - 5.67| = 0.02; \text{heaviest: } 5.69 \text{ g; lightest: } 5.65 \text{ g}\)

36. \(12 - t|3r + 2|\)

**SOLUTION:**

Substitute \(-6\) for \(r\) and \(3\) for \(t\) and solve.

\[12 - t|3r + 2| = 12 - 3|3(-6) + 2|\]

\[= 12 - 3|18 + 2|\]

\[= 12 - 3|16|\]

\[= 12 - 3(16)\]

\[= 12 - 48\]

\[= -36\]

**ANSWER:**

\(-36\)

37. \(2q + |2rt + q|\)

**SOLUTION:**

Substitute \(-8\) for \(q\), \(-6\) for \(r\), and \(3\) for \(t\) and solve.

\[2q + |2rt + q| = 2(-8) + |2(-6)(3) + (-8)|\]

\[= -16 + |36 - 8|\]

\[= -16 + 44\]

\[= 28\]

**ANSWER:**

\(28\)

38. \(-5t - q|8r - t|\)

**SOLUTION:**

Substitute \(-8\) for \(q\), \(-6\) for \(r\), and \(3\) for \(t\) and solve.

\[-5t - q|8r - t| = -5(3) - (-8)|8(-6) - 3|\]

\[= -15 + 8|48 - 3|\]

\[= -15 + 8|51|\]

\[= -15 + 8(51)\]

\[= -15 + 408\]

\[= 393\]

**ANSWER:**

\(393\)

Solve each equation. Check your solutions.
39. \(8x = 2|6x - 2|\)

**SOLUTION:**

\[
8x = 2|6x - 2|
\]

\[
\frac{8x}{2} = \frac{2|6x - 2|}{2}
\]

\[
4x = |6x - 2|
\]

Case 1:

\[
6x - 2 = 4x
\]

\[
6x - 2 + 2 = 4x + 2
\]

\[
6x = 4x + 2
\]

\[
6x - 4x = 4x + 2 - 4x
\]

\[
2x = 2
\]

\[
\frac{2x}{2} = \frac{2}{2}
\]

\[
x = 1
\]

Case 2:

\[
6x - 2 = -4x
\]

\[
6x - 2 + 2 = -4x + 2
\]

\[
6x = -4x + 2
\]

\[
6x + 4x = -4x + 2 + 4x
\]

\[
10x = 2
\]

\[
\frac{10x}{10} = \frac{2}{10}
\]

\[
x = \frac{2}{10}
\]

\[
x = \frac{1}{5}
\]

There appear to be two solutions, 1 and \(\frac{1}{5}\).

Check: Substitute the values in the original equation.

\[x = 1\]

\[x = \frac{1}{5}\]

The solution set is \(\left\{1, \frac{1}{5}\right\}\).

**ANSWER:**

\(\left\{1, \frac{1}{5}\right\}\)

40. \(-6y + 4 = |4y + 12|\)

**SOLUTION:**
1-4 Solving Absolute Value Equations

Case 1:

\[ 4y + 12 = -6y + 4 \]
\[ 4y + 12 + 6y = -6y + 4 + 6y \]
\[ 10y + 12 = 4 \]
\[ 10y + 12 - 12 = 4 - 12 \]
\[ 10y = -8 \]
\[ 10y \div 10 = \frac{-8}{10} \]
\[ y = \frac{-8}{10} \]
\[ y = -\frac{4}{5} \]

Case 2:

\[ 4y + 12 = -(-6y + 4) \]
\[ 4y + 12 = 6y - 4 \]
\[ 4y + 12 - 6y = 6y - 4 - 6y \]
\[ -2y + 12 = -4 \]
\[ -2y + 12 - 12 = -4 - 12 \]
\[ -2y = -16 \]
\[ -2y \div -2 = -\frac{16}{-2} \]
\[ y = 8 \]

There appear to be two solutions, \(-\frac{4}{5}\) and 8.

Check: Substitute the values in the original equation.

\[ y = -\frac{4}{5} \]

\[ -6y + 4 = |4y + 12| \]
\[ -6\left(\frac{-4}{5}\right) + 4 \geq |4\left(\frac{-4}{5}\right) + 12| \]
\[ \frac{24}{5} + 4 \geq \frac{-16}{5} + 12 \]
\[ \frac{24 + 4(5)}{5} \geq \frac{-16 + 12(5)}{5} \]
\[ \frac{24 + 20}{5} \geq \frac{-16 + 60}{5} \]
\[ \frac{44}{5} \geq \frac{44}{5} \]
\[ \frac{44}{5} = \frac{44}{5} \]
\[ y = 8 \]

\[ -6y + 4 = |4y + 12| \]
\[ -6(8) + 4 \geq |4(8) + 12| \]
\[ -48 + 4 \geq |48 + 12| \]
\[ -44 \geq |60| \]
\[ -44 \neq 60 \]

Since \(-44 \neq 60\), \(y = 8\) is an extraneous solution. The solution set is \(\left\{-\frac{4}{5}\right\}\).

**ANSWER:**

\(\left\{-\frac{4}{5}\right\}\)

41. \(8z + 20 = -|2z + 4|\)

**SOLUTION:**

\[-|2z + 4| = 8z + 20\]
\[|2z + 4| = -(8z + 20)\]
\[|2z + 4| = -8z - 20\]
1-4 Solving Absolute Value Equations

Case 1:
\[
2z + 4 = -8z - 20
\]
\[
2z + 4 + 8z = -8z - 20 + 8z
\]
\[
10z + 4 = -20
\]
\[
10z + 4 - 4 = -20 - 4
\]
\[
10z = -24
\]
\[
\frac{10z}{10} = \frac{-24}{10}
\]
\[
z = \frac{-12}{5}
\]

Case 2:
\[
2z + 4 = -( -8z - 20)
\]
\[
2z + 4 = 8z + 20
\]
\[
2z + 4 - 8z = 8z + 20 - 8z
\]
\[
-6z + 4 = 20
\]
\[
-6z + 4 - 4 = 20 - 4
\]
\[
-6z = 16
\]
\[
\frac{-6z}{-6} = \frac{16}{-6}
\]
\[
z = \frac{-8}{3}
\]

There appear to be two solutions, \(-\frac{12}{5}\) and \(-\frac{8}{3}\).

Check: Substitute the values in the original equation.
\[
z = \frac{-12}{5}
\]
\[
8\left(-\frac{12}{5}\right) + 20 = -\left|2\left(-\frac{12}{5}\right) + 4\right|
\]
\[
-\frac{96}{5} + 20 = -\left|\frac{-24}{5} + 4\right|
\]
\[
-\frac{-96 + 20(5)}{5} = -\left|\frac{-24 + 5(4)}{5}\right|
\]
\[
-\frac{-96 + 100}{5} = -\left|\frac{-24 + 20}{5}\right|
\]
\[
\frac{4}{5} \neq -\frac{4}{5}
\]
\[
z = \frac{-8}{3}
\]
\[
8\left(-\frac{8}{3}\right) + 20 = -\left|2\left(-\frac{8}{3}\right) + 4\right|
\]
\[
-\frac{64}{3} + 20 = -\left|\frac{-16}{3} + 4\right|
\]
\[
-\frac{-64 + 20(3)}{3} = -\left|\frac{-16 + 4(3)}{3}\right|
\]
\[
-\frac{-64 + 60}{3} = -\left|\frac{-16 + 12}{3}\right|
\]
\[
\frac{-4}{3} \neq -\frac{4}{3}
\]
\[
\frac{-4}{3} \neq -\frac{4}{3}
\]
\[
\frac{-4}{3} = \frac{-4}{3}
\]

Since \(\frac{4}{5} \neq -\frac{4}{5}\), \(y = \frac{-12}{5}\) is an extraneous solution.

The solution set is \(\left\{\frac{-8}{3}\right\}\).

\textbf{ANSWER:}
\[
\left\{\frac{-8}{3}\right\}
\]
42. \(-3y - 2 = |6y + 25|\)

**SOLUTION:**

Case 1:

\[
6y + 25 = -3y - 2
\]

\[
6y + 25 + 3y = -3y - 2 + 3y
\]

\[
9y + 25 = -2
\]

\[
9y + 25 - 25 = -2 - 25
\]

\[
9y = -27
\]

\[
\frac{9y}{9} = -27
\]

\[
y = -3
\]

Case 2:

\[
6y + 25 = -(3y - 2)
\]

\[
6y + 25 = 3y + 2
\]

\[
6y + 25 - 3y = 3y + 2 - 3y
\]

\[
3y + 25 = 2
\]

\[
3y + 25 - 25 = 2 - 25
\]

\[
3y = -23
\]

\[
\frac{3y}{3} = \frac{-23}{3}
\]

\[
y = -\frac{23}{3}
\]

There appear to be two solutions, \(-3\) and \(-\frac{23}{3}\).

Check: Substitute the values in the original equation.

\[
y = -3:
\]

\[
-3y - 2 = |6y + 25|
\]

\[
-3(-3) - 2 = |6(-3) + 25|
\]

\[
9 - 2 = |-18 + 25|
\]

\[
7 = 7\]

\[
y = \frac{23}{3}
\]

\[
-3y - 2 = |6y + 25|
\]

\[
-3\left(-\frac{23}{3}\right) - 2 = \frac{6\left(-\frac{23}{3}\right)}{1} + 25
\]

\[
23 - 2 = |46 + 25|
\]

\[
21 = |21|
\]

\[
21 = 21
\]

The solution set is \(\left\{-3, -\frac{23}{3}\right\}\).

**ANSWER:**

\(\left\{-3, -\frac{23}{3}\right\}\)

43. **SEA LEVEL** Florida is on average 100 feet above sea level. This level varies by as much as 245 feet depending on precipitation and your location. Write and solve an equation describing the maximum and minimum sea levels for Florida. Is this solution reasonable? Explain.

**SOLUTION:**

Substitute \(c = 100\) and \(r = 245\) in the equation

\[
|x - c| = r
\]

\[
|x - 100| = 245
\]

Solve the equation \(|x - 100| = 245\).

Case 1:

\[
x - 100 = 245
\]

\[
x - 100 + 100 = 245 + 100
\]

\[
x = 345
\]

Case 2:

\[
x - 100 = -245
\]

\[
x - 100 + 100 = -245 + 100
\]

\[
x = -145
\]

So, the maximum sea level for Florida is 345 ft above sea level and the minimum is -145 ft below sea level. No, the maximum is reasonable but the minimum is not. Florida’s lowest point should be at sea level where Florida meets the Atlantic Ocean and the Gulf of Mexico.

**ANSWER:**

\(|x - 100| = 245;\) maximum: 345 ft above sea level; minimum: -145 ft below sea level. No, the maximum is reasonable but the minimum is not. Florida’s lowest point should be at sea level where Florida meets the Atlantic Ocean and the Gulf of Mexico.

44. **MULTIPLE REPRESENTATIONS** Draw a
1-4 Solving Absolute Value Equations

number line.

**a. GEOMETRIC** Label any 5 integers on the number line points A, B, C, D, and F.

**b. TABULAR** Fill in each blank in the table with either > or < using the points from the number line.

<table>
<thead>
<tr>
<th>A &lt; B</th>
<th>A + C &lt; B + C</th>
<th>A + D &lt; B + D</th>
<th>A + F &lt; B + F</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &gt; A</td>
<td>B + C &gt; A + C</td>
<td>B + D &gt; A + D</td>
<td>B + F &gt; A + F</td>
</tr>
</tbody>
</table>

**c. VERBAL** Describe the patterns in the table.

**d. ALGEBRAIC** Describe the patterns algebraically, using the variable x to replace C, D, and F.

**SOLUTION:**

a. Draw a number line and label from -3 to 3. Place points A, B, C, D, and F on the number line. Sample answer:

```
-3 -2 -1 0 1 2 3
```

b. Fill in the table based on the number line drawn in part a. Since A is -2 and B is -1. A < B. Since C is 0, A + C = -2 + 0 or -2. B + C = -1 + 0 or -1. Therefore, A + C < B + C. Similar reasoning is used to complete the table.

<table>
<thead>
<tr>
<th>A &lt; B</th>
<th>A + C &lt; B + C</th>
<th>A + D &lt; B + D</th>
<th>A + F &lt; B + F</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &gt; A</td>
<td>B + C &gt; A + C</td>
<td>B + D &gt; A + D</td>
<td>B + F &gt; A + F</td>
</tr>
</tbody>
</table>

**c. Sample answer:** If A is less than B, then any number added to or subtracted from A will be less than the same number added to or subtracted from B. If B is greater than A, then any number added to or subtracted from B is greater than the same number added to or subtracted from A.

**d.** If A < B, then A + x < B + x. If A < B, then A – x < B – x.

If B > A, then B + x > A + x. If B > A, then B – x > A – x.

**ANSWER:**

**a. Sample answer:**

```
<table>
<thead>
<tr>
<th>A &lt; B</th>
<th>A + C &lt; B + C</th>
<th>A + D &lt; B + D</th>
<th>A + F &lt; B + F</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &gt; A</td>
<td>B + C &gt; A + C</td>
<td>B + D &gt; A + D</td>
<td>B + F &gt; A + F</td>
</tr>
</tbody>
</table>
```

45. **CRITIQUE** Ana and Ling are solving |3x + 14| = −6x. Is either of them correct? Explain your reasoning.

**A**

```
<table>
<thead>
<tr>
<th>3x + 14 = −6x</th>
</tr>
</thead>
</table>
```

**B**

```
<table>
<thead>
<tr>
<th>3x + 14 = −6x</th>
</tr>
</thead>
</table>
```

**SOLUTION:**
Both students solved the equation correctly. However Ana included an extraneous solution in her final answer. She would have caught this error if she had checked to see if her answers were correct by substituting the values into the original equation. Ling is correct.

**ANSWER:**
Ling: Ana included an extraneous solution. She would have caught this error if she had checked to see if her answers were correct by substituting the values into the original equation.
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46. **CHALLENGE** Solve $|2x - 1| + 3 = |5 - x|$. List all cases and resulting equations. (*Hint: There are four possible cases to examine as potential solutions.*)

**SOLUTION:**
The 4 potential solutions are:
1. $(2x - 1) \geq 0$ and $(5 - x) \geq 0$
2. $(2x - 1) \geq 0$ and $(5 - x) < 0$
3. $(2x - 1) < 0$ and $(5 - x) \geq 0$
4. $(2x - 1) < 0$ and $(5 - x) < 0$

The resulting equations corresponding to these cases are:
1. $2x - 1 + 3 = 5 - x : x = 1$
2. $2x - 1 + 3 = x - 5 : x = -7$
3. $1 - 2x + 3 = 5 - x : x = -1$
4. $1 - 2x + 3 = x - 5 : x = 3$

The solutions from case 1 and case 3 work. The others are extraneous. The solution set is $\{-1, 1\}$.

**ANSWER:**
The 4 potential solutions are:
1. $(2x - 1) \geq 0$ and $(5 - x) \geq 0$
2. $(2x - 1) \geq 0$ and $(5 - x) < 0$
3. $(2x - 1) < 0$ and $(5 - x) \geq 0$
4. $(2x - 1) < 0$ and $(5 - x) < 0$

The resulting equations corresponding to these cases are:
1. $2x - 1 + 3 = 5 - x : x = 1$
2. $2x - 1 + 3 = x - 5 : x = -7$
3. $1 - 2x + 3 = 5 - x : x = -1$
4. $1 - 2x + 3 = x - 5 : x = 3$

The solutions from case 1 and case 3 work. The others are extraneous. The solution set is $\{-1, 1\}$.

**REASONING** If $a$, $x$, and $y$ are real numbers, determine whether each statement is sometimes, always, or never true. Explain your reasoning.

47. If $|a| > 7$, then $|a + 3| > 10$.

**SOLUTION:**
Sometimes; this is only true for certain values of $a$.
For example, it is true for $a = 8$; if $8 > 7$, then $11 > 10$. However it is not true for $a = -8$; if $8 > 7$, then $5 \not> 10$.

**ANSWER:**
Sometimes; this is only true for certain values of $a$.
For example, it is true for $a = 8$; if $8 > 7$, then $11 > 10$. However it is not true for $a = -8$; if $8 > 7$, then $5 \not> 10$.

48. If $|x| < 3$, then $|x| + 3 > 0$.

**SOLUTION:**
Always; if $|x| < 3$, then $x$ is between $-3$ or $3$.
Adding 3 to the absolute value of any of the numbers in this set will produce a positive number.

**ANSWER:**
Always; if $|x| < 3$, then $x$ is between $-3$ or $3$.
Adding 3 to the absolute value of any of the numbers in this set will produce a positive number.

49. If $y$ is between 1 and 5, then $|y - 3| \leq 2$.

**SOLUTION:**
Always; starting with numbers between 1 and 5 and subtracting 3 will produce numbers between $-2$ and $2$. These all have an absolute value less than or equal to 2.

**ANSWER:**
Always; starting with numbers between 1 and 5 and subtracting 3 will produce numbers between $-2$ and $2$. These all have an absolute value less than or equal to 2.
1-4 Solving Absolute Value Equations

50. **OPEN ENDED** Write an absolute value equation of the form $|ax + b| = cx + d$ that has no solution. Assume that $a$, $b$, $c$, and $d \neq 0$.

**SOLUTION:**
Sample answer: An absolute value expression cannot be negative. Solve the equation and check the solutions.

$$|2x + 1| = x - 3, \text{ or } |3x + 10| = x - 5, \text{ or } |x - 1| = \frac{1}{2}x - 4$$

**ANSWER:**
Sample answer: $|2x + 1| = x - 3$, or 

$$|3x + 10| = x - 5, \text{ or } |x - 1| = \frac{1}{2}x - 4$$

51. **WRITING IN MATH** How are symbols used to represent mathematical ideas? Use an example to justify your reasoning.

**SOLUTION:**
Sample answer: Symbols can be used as a shorthand way to represent ideas such as operations, equality, absolute value, and the empty set. For example, instead of writing 5 minus the absolute value of $2x$ equals 10, you could write $5 - |2x| = 10$.

**ANSWER:**
Sample answer: Symbols can be used as a shorthand way to represent ideas such as operations, equality, absolute value, and the empty set. For example, instead of writing 5 minus the absolute value of $2x$ equals 10, you could write $5 - |2x| = 10$.

52. If $4x - y = 3$ and $2x + 3y = 19$, what is the value of $y$?

A 2
B 3
C 4
D 5

**SOLUTION:**

4x - y + y = 3 + y

$4x = y + 3$

$2(2x) = y + 3$

$2x = \frac{y + 3}{2}$

Substitute $2x = \frac{y + 3}{2}$ in the equation $2x + 3y = 19$.

$$2x + 3y = 19$$

$$\frac{y + 3}{2} + 3y = 19$$

$$\frac{y + 3 + 6y}{2} = 19$$

$$\frac{7y + 3}{2} = 19$$

$$2(\frac{7y + 3}{2}) = 2(19)$$

$$7y + 3 = 38$$

$$7y + 3 - 3 = 38 - 3$$

$$7y = 35$$

$$\frac{7y}{7} = 5$$

So, the correct choice is D.

**ANSWER:**

D
53. **GRIDDED RESPONSE** Two male and 2 female students from each of the 9th, 10th, 11th, and 12th grades comprise the Student Council. If a Student Council representative is chosen at random to attend a board meeting, what is the probability that the student will be either an 11th grader or male?

**SOLUTION:**
The total number of students in the Student Council is 4 + 4 + 4 + 4 or 16.

\[
P(\text{11th grader or male}) = \frac{4 + 8 - 2}{16} = \frac{10}{16} = \frac{5}{8}
\]

**ANSWER:**
\[
\frac{5}{8}
\]

54. Which equation is equivalent to \(4(9 – 3x) = 7 – 2(6 – 5x)\)?

F \(8x = 41\)

G \(22x = 41\)

H \(8x = 24\)

J \(22x = 24\)

**SOLUTION:**

\[
4(9 – 3x) = 7 – 2(6 – 5x)
\]

\[
36 – 12x = 7 – 2(6) + (–2)(–5x)
\]

\[
36 – 12x = 7 – 12 + 10x
\]

\[
36 – 12x = –5 + 10x
\]

\[
36 – 12x – 36 = –5 + 10x – 36
\]

\[
–12x = 10x – 41
\]

\[
–12x – 10x = 10x – 41 – 10x
\]

\[
–22x = –41
\]

\[
–(–22x) = –(–41)
\]

\[
22x = 41
\]

Therefore, \(22x = 41\) is equivalent to \(4(9 – 3x) = 7 – 2(6 – 5x)\).

So, the correct choice is **G**.

**ANSWER:**

G

55. **SAT/ACT** A square with side length 4 units has one vertex at the point \((1, 2)\). Which one of the following points cannot be diagonally opposite that vertex?

A \((-3, –2)\)

B \((-3, 6)\)

C \((5, –2)\)

D \((5, 6)\)

E \((1, 6)\)

**SOLUTION:**
Let \(d\) be the length of the diagonal of the square. Use the Pythagorean Theorem.

\[
d^2 = 4^2 + 4^2
\]

\[
d^2 = 16 + 16
\]

\[
d^2 = 32
\]

\[
d = \sqrt{32}
\]

Find the distance between the points \((1, 2)\) and \((-3, –2)\).

\[
d = \sqrt{(1+3)^2 + (2+2)^2}
\]

\[
= \sqrt{4^2 + 4^2}
\]

\[
= \sqrt{16 + 16}
\]

\[
= \sqrt{32}
\]

So, the point \((-3, –2)\) could be diagonally opposite to \((1, 2)\).

Consider the point \((-3, 6)\).

Find the distance between the points \((1, 2)\) and \((-3, 6)\).

\[
d = \sqrt{(1+3)^2 + (2-6)^2}
\]

\[
= \sqrt{4^2 + (-4)^2}
\]

\[
= \sqrt{16 + 16}
\]

\[
= \sqrt{32}
\]

So, the point \((-3, 6)\) could be diagonally opposite to \((1, 2)\).

Consider the point \((5, –3)\).

Find the distance between the points \((1, 2)\) and \((5, –2)\).
1-4 Solving Absolute Value Equations

Solve each equation. Check your solution.

56. \( 4x + 6 = 30 \)

**SOLUTION:**

\[
\begin{align*}
4x + 6 &= 30 \\
4x &+ 6 - 6 = 30 - 6 \\
4x &= 24 \\
\frac{4x}{4} &= \frac{24}{4} \\
x &= 6
\end{align*}
\]

Check: Substitute \( x = 6 \) in the original equation.

\[
\begin{align*}
4x + 6 &= 30 \\
4(6) + 6 &= 30 \\
24 + 6 &= 30 \\
30 &= 30\checkmark
\end{align*}
\]

The solution is \( x = 6 \).

**ANSWER:**

6

\[
d = \sqrt{(1-5)^2 + (2+2)^2} \\
= \sqrt{(-4)^2 + 4^2} \\
= \sqrt{16 + 16} \\
= \sqrt{32}
\]

So, the point (5, -2) could be diagonally opposite to (1, 2).

Consider the point (5, 6).

Find the distance between the points (1, 2) and (5, 6).

\[
d = \sqrt{(1-5)^2 + (2-6)^2} \\
= \sqrt{(-4)^2 + (-4)^2} \\
= \sqrt{16 + 16} \\
= \sqrt{32}
\]

So, the point (5, 6) could be diagonally opposite to (1, 2).

Consider the point (1, 6).

Find the distance between the points (1, 2) and (1, 6).

\[
d = \sqrt{(1-1)^2 + (2-6)^2} \\
= \sqrt{0^2 + (-4)^2} \\
= \sqrt{0 + 16} \\
= \sqrt{16}
\]

So, the point (1, 6) cannot be diagonally opposite to the vertex (1, 2). The correct choice is **E**.

**ANSWER:**

E
1-4 Solving Absolute Value Equations

57. \(5p - 10 = 4(7 + 6p)\)

\[
\text{SOLUTION:}
\begin{align*}
5p - 10 &= 4(7 + 6p) \\
5p - 10 &= 4(7) + 4(6p) \\
5p - 10 &= 28 + 24p \\
5p - 10 - 24p &= 28 + 24p - 24p \\
-19p - 10 &= 28 \\
-19p &= 38 \\
p &= -2
\end{align*}
\]

Check: Substitute \(p = -2\) in the original equation.

\[
5p - 10 = 4(7 + 6p)
\]

\[
5(-2) - 10 = 4(7 + 6(-2))
\]

\[
-10 = 4(7 - 12)
\]

\[
-20 = 4(-5)
\]

\[
-20 = -20 \checkmark
\]

The solution is \(p = -2\).

\text{ANSWER:} \\
-2

58. \(\frac{3}{5}y - 7 = \frac{2}{5}y + 3\)

\[
\text{SOLUTION:}
\begin{align*}
\frac{3}{5}y - 7 &= \frac{2}{5}y + 3 \\
\frac{3}{5}y - 7 + 7 &= \frac{2}{5}y + 3 + 7 \\
\frac{3}{5}y &= \frac{2}{5}y + 10 \\
\frac{3}{5}y - \frac{2}{5}y &= \frac{2}{5}y + 10 - \frac{2}{5}y \\
\left(\frac{3}{5} - \frac{2}{5}\right)y &= 10 \\
\left(\frac{1}{5}\right)y &= 10 \\
\frac{1}{5}y &= 10 \\
5\left(\frac{1}{5}y\right) &= 5(10) \\
y &= 50
\end{align*}
\]

Check: Substitute \(y = 50\) in the original equation.

\[
\frac{3}{5}y - 7 = \frac{2}{5}y + 3
\]

\[
\frac{3}{5}(50) - 7 = \frac{2}{5}(50) + 3
\]

\[
3(10) - 7 = 2(10) + 3
\]

\[
30 - 7 = 20 + 3
\]

\[
23 = 23 \checkmark
\]

The solution is \(y = 50\).

\text{ANSWER:}

50

59. **MONEY** Nhu is saving to buy a car. In the first 6 months, his savings were \(\frac{3}{4}\) the price of the car. In the second six months, Nhu saved \(\frac{1}{5}\) more than \(\frac{1}{5}\) the price of the car. He still needs \$370.

\text{a.} What is the price of the car?

\text{b.} What is the average amount of money Nhu saved
1-4 Solving Absolute Value Equations

The solution is \( y = 50 \).

**ANSWER:**

50

59. **MONEY** Nhu is saving to buy a car. In the first 6 months, his savings were $80 less than $6800, so he still needs $370.

**SOLUTION:**

b. $535.83
c. 1 mo

**Name the property illustrated by each equation.**

60. \((1 + 8) + 11 = 11 + (1 + 8)\)

**SOLUTION:**

Commutative Property of Addition; the Commutative Property of Addition states that the order in which terms are added does not affect the sum.

**ANSWER:**

Comm. (+)

61. \(z(9 - 4) = z \cdot 9 - z \cdot 4\)

**SOLUTION:**

Distributive Property; the Distributive Property states that there is no difference between a term multiplied by each term in a group and the term multiplied by the group.

**ANSWER:**

Distributive

**Simplify each expression.**

62. \(7a + 3b - 4a - 5b\)

**SOLUTION:**

\[7a + 3b - 4a - 5b = 7a - 4a + 3b - 5b = (7 - 4)a + (3 - 5)b = 3a - 2b\]

**ANSWER:**

3a - 2b

63. \(3x + 5y + 7x - 3y\)

**SOLUTION:**

\[3x + 5y + 7x - 3y = 3x + 7x + 5y - 3y = (3 + 7)x + (5 - 3)y = 10x + 2y\]

**ANSWER:**

10x + 2y
64. \(3(15x - 9y) + 5(4y - x)\)

**SOLUTION:**
\[3(15x - 9y) + 5(4y - x) = 3(15x) + 3(-9y) + 5(4y) + 5(-x) = 45x - 27y + 20y - 5x = 40x - 7y\]

**ANSWER:**
\[40x - 7y\]

65. \(2(10m - 7a) + 3(8a - 3m)\)

**SOLUTION:**
\[2(10m - 7a) + 3(8a - 3m) = 2(10m) + 2(-7a) + 3(8a) + 3(-3m) = 20m - 14a + 24a - 9m = 20m - 9m - 14a + 24a = (20 - 9)m + (-14 + 24)a = 11m + 10a\]

**ANSWER:**
\[11m + 10a\]

66. \(8(r + 7t) - 4(13t + 5r)\)

**SOLUTION:**
\[8(r + 7t) - 4(13t + 5r) = 8(r) + 8(7t) + (-4)(13t) + (-4)(5r) = 8r + 56t - 52t - 20r = 8r - 20r + 56t - 52t = (8 - 20)r + (56 - 52)t = -12r + 4t\]

**ANSWER:**
\[-12r + 4t\]

67. \(4(14c - 10d) - 6(d + 4c)\)

**SOLUTION:**
\[4(14c - 10d) - 6(d + 4c) = 4(14c) + 4(-10d) + (-6)(d) + (-6)(4c) = 56c - 40d - 6d - 24c = 56c - 46d - 24c = (56 - 24)c + (-40 - 6)d = 32c - 46d\]

**ANSWER:**
\[32c - 46d\]

68. **GEOMETRY** The formula for the surface area of a rectangular prism is \(S_A = 2lwh + 2lh + 2wh\), where \(l\) represents the length, \(w\) represents the width, and \(h\) represents the height. Find the surface area of the rectangular prism at the right.

**SOLUTION:**
Substitute \(l = 12, w = 5,\) and \(h = 7\) in the formula \(S_A = 2lwh + 2lh + 2wh\).

\[S_A = 2(12)(5) + 2(12)(7) + 2(5)(7) = 120 + 168 + 70 = 358\]

The surface area of the rectangular prism is 358 square inches.

**ANSWER:**
358 in\(^2\)
1-4 Solving Absolute Value Equations

Solve each equation.

69. \(15x + 5 = 35\)

\textbf{SOLUTION:}

\[
15x + 5 = 35 \\
15x = 30 \\
\frac{15x}{15} = \frac{30}{15} \\
x = 2
\]

\textbf{ANSWER:}

2

70. \(2.4y + 4.6 = 20\)

\textbf{SOLUTION:}

\[
2.4y + 4.6 = 20 \\
2.4y + 4.6 - 4.6 = 20 - 4.6 \\
2.4y = 15.4 \\
\frac{2.4y}{2.4} = \frac{15.4}{2.4} \\
y \approx 6.417
\]

\textbf{ANSWER:}

\( \approx 6.417 \)

71. \(8a + 9 = 6a - 7\)

\textbf{SOLUTION:}

\[
8a + 9 = 6a - 7 \\
8a + 9 - 9 = 6a - 7 - 9 \\
8a = 6a - 16 \\
8a - 6a = 6a - 16 - 6a \\
2a = -16 \\
\frac{2a}{2} = \frac{-16}{2} \\
a = -8
\]

\textbf{ANSWER:}

-8

72. \(3(w - 1) = 2w - 6\)

\textbf{SOLUTION:}

\[
3(w - 1) = 2w - 6 \\
3w - 3 = 2w - 6 \\
3w - 3 + 3 = 2w - 6 + 3 \\
3w = 2w - 3 \\
3w - 2w = 2w - 3 - 2w \\
w = -3
\]

\textbf{ANSWER:}

-3

73. \(\frac{1}{2}(2b - 4) = 2 + 8b\)

\textbf{SOLUTION:}

\[
\frac{1}{2}(2b - 4) = 2 + 8b \\
\frac{1}{2}(2b) + \frac{1}{2}(-4) = 2 + 8b \\
\frac{b - 2}{2} = 2 + 8b \\
b - 2 + 2 = 2 + 8b + 2 \\
b = 4 + 8b \\
b - 8b = 4 + 8b - 8b \\
-7b = 4 \\
\frac{-7b}{-7} = \frac{4}{-7} \\
b = -\frac{4}{7}
\]

\textbf{ANSWER:}

\(-\frac{4}{7}\)
74. \( \frac{1}{3} (6p - 24) = 18 + 3p \)

**SOLUTION:**

\[
\frac{1}{3} (6p - 24) = 18 + 3p \\
\frac{1}{3} (6p) + \frac{1}{3} (-24) = 18 + 3p \\
2p - 8 = 18 + 3p \\
2p - 8 + 8 = 18 + 3p + 8 \\
2p = 26 + 3p \\
2p - 3p = 26 + 3p - 3p \\
-p = 26 \\
\frac{-p}{-1} = \frac{26}{-1} \\
p = -26
\]

**ANSWER:**

-26
1-5 Solving Inequalities

Solve each inequality. Then graph the solution set on a number line.
1. \( b + 6 < 14 \)

\[ \text{SOLUTION:} \]
\[ b + 6 < 14 \]
\[ b + 6 - 6 < 14 - 6 \]
\[ b < 8 \]

To graph this inequality, draw an open circle at 8 and draw an arrow extending to the left.

\[ \text{ANSWER:} \]
\[ b < 8 \]

2. \( 12 - d > -8 \)

\[ \text{SOLUTION:} \]
\[ 12 - d > -8 \]
\[ 12 - d - 12 > -8 - 12 \]
\[ -d > -20 \]
\[ d < 20 \]

To graph this inequality, draw an open circle at 20 and draw an arrow extending to the left.

\[ \text{ANSWER:} \]
\[ d < 20 \]

3. \( 18 \leq -3x \)

\[ \text{SOLUTION:} \]
\[ 18 \leq -3x \]
\[ \frac{18}{-3} \geq \frac{-3x}{-3} \]
\[ -6 \geq x \]
\[ x \leq -6 \]

To graph this inequality, draw a solid circle at -6 and draw an arrow extending to the left.

\[ \text{ANSWER:} \]
\[ x \leq -6 \]

4. \( -5y \geq -35 \)

\[ \text{SOLUTION:} \]
\[ -5y \geq -35 \]
\[ \frac{-5y}{-5} \leq \frac{-35}{-5} \]
\[ y \leq 7 \]

To graph this inequality, draw a solid circle at 7 and draw an arrow extending to the left.

\[ \text{ANSWER:} \]
\[ y \leq 7 \]
Solve each inequality. Then graph the solution set on a number line.

1. 

**SOLUTION:**

\[-4w - 13 > -21\]

\[-4w - 13 + 13 > -21 + 13\]

\[-4w > -8\]

\[\frac{-4w}{-4} < \frac{-8}{-4}\]

\[w < 2\]

To graph this inequality, draw an open circle at 2 and draw an arrow extending to the left.

**ANSWER:**

\[w < 2\]

\[-5 -4 -3 -2 -1 0 1 2 3 4 5\]

6. \(8z - 9 \geq -15\)

**SOLUTION:**

\[8z - 9 \geq -15\]

\[8z - 9 + 9 \geq -15 + 9\]

\[8z \geq -6\]

\[\frac{8z}{8} \geq \frac{-6}{8}\]

\[z \geq -\frac{3}{4}\]

To graph this inequality, draw a solid circle at \(-\frac{3}{4}\) and draw an arrow extending to the right.

**ANSWER:**

\[z \geq -\frac{3}{4}\]

\[-5 -4 -3 -2 -1 0 1 2 3 4 5\]

7. \(s \geq \frac{s + 6}{5}\)

**SOLUTION:**

\[s \geq \frac{s + 6}{5}\]

\[5s \geq s + 6\]

\[5s - s \geq s - s + 6\]

\[4s \geq 6\]

\[\frac{4s}{4} \geq \frac{6}{4}\]

\[s \geq \frac{3}{2}\]

To graph this inequality, draw a solid circle at \(\frac{3}{2}\) and draw an arrow extending to the right.

**ANSWER:**

\[s \geq \frac{3}{2}\]

\[-5 -4 -3 -2 -1 0 1 2 3 4 5\]
1-5 Solving Inequalities

8. \( \frac{2x - 9}{4} \leq x + 2 \)

**SOLUTION:**

\[
\begin{align*}
\frac{2x - 9}{4} & \leq x + 2 \\
4 \left( \frac{2x - 9}{4} \right) & \leq 4(x + 2) \\
2x - 9 & \leq 4x + 8 \\
2x - 9 - 4x & \leq 4x + 8 - 4x \\
-2x & \leq 17 \\
-\frac{2x}{-2} & \geq \frac{17}{-2} \\
x & \geq -8.5
\end{align*}
\]

To graph this inequality, draw a solid circle at \(-8.5\) and draw an arrow extending to the right.

**ANSWER:**

\( x \geq -8.5 \)

---

9. **YARD WORK** Tara is delivering bags of mulch. Each bag weighs 48 pounds, and the push cart weighs 65 pounds. If her flat-bed truck is capable of hauling 2000 pounds, how many bags of mulch can Tara safely take on each trip?

**SOLUTION:**

Let \( x \) be the number of bags of mulch.

\[
\begin{align*}
48x + 65 & \leq 2000 \\
48x + 65 & \leq 2000 - 65 \\
48x & \leq 1935 \\
\frac{48x}{48} & \leq \frac{1935}{48} \\
x & \leq 40.3125
\end{align*}
\]

So, Tara can safely take 40 bags of mulch on each trip.

**ANSWER:**

40 bags

**Solve each inequality. Then graph the solution set on a number line.**

10. \( m - 8 > -12 \)

**SOLUTION:**

\[
\begin{align*}
m - 8 & > -12 \\
m - 8 + 8 & > -12 + 8 \\
m & > -4
\end{align*}
\]

To graph this inequality, draw an open circle at \(-4\) and draw an arrow extending to the right.

**ANSWER:**

\( m > -4 \)
1-5 Solving Inequalities

11. \( n + 6 \leq 3 \)

**SOLUTION:**

\[
\begin{align*}
n + 6 & \leq 3 \\
n + 6 - 6 & \leq 3 - 6 \\
n & \leq -3
\end{align*}
\]

To graph this inequality, draw a solid circle at -3 and draw an arrow extending to the left.

**ANSWER:**

\( n \leq -3 \)

12. \( 6r < -36 \)

**SOLUTION:**

\[
\begin{align*}
6r & < -36 \\
\frac{6r}{6} & < \frac{-36}{6} \\
r & < -6
\end{align*}
\]

To graph this inequality, draw an open circle at -6 and draw an arrow extending to the left.

**ANSWER:**

\( r < -6 \)

13. \( -12t \geq -6 \)

**SOLUTION:**

\[
\begin{align*}
-12t & \geq -6 \\
\frac{-12t}{-12} & \leq \frac{-6}{-12} \\
t & \leq \frac{1}{2}
\end{align*}
\]

To graph this inequality, draw a solid circle at \( \frac{1}{2} \) and draw an arrow extending to the left.

**ANSWER:**

\( t \leq \frac{1}{2} \)

14. \( \frac{-w}{4} \leq -7 \)

**SOLUTION:**

\[
\begin{align*}
\frac{-w}{4} & \leq -7 \\
4 \left( \frac{-w}{4} \right) & \leq -7(4) \\
-w & \leq -28 \\
-w & \geq -28 \\
-w & \geq -28 \\
w & \geq 28
\end{align*}
\]

To graph this inequality, draw a solid circle at 28 and draw an arrow extending to the right.

**ANSWER:**

\( w \geq 28 \)
1-5 Solving Inequalities

15. \( \frac{k}{3} - 14 < -5 \)

**SOLUTION:**

\[
\frac{k}{3} - 14 < -5
\]

\[
\frac{k}{3} - 14 + 14 < -5 + 14
\]

\[
\frac{k}{3} < 9
\]

\[
3 \left( \frac{k}{3} \right) < 3(9)
\]

\[
k < 27
\]

To graph this inequality, draw an open circle at 27 and draw an arrow extending to the left.

**ANSWER:**

\( k < 27 \)

16. \( 4x - 15 \leq 21 \)

**SOLUTION:**

\[
4x - 15 \leq 21
\]

\[
4x - 15 + 15 \leq 21 + 15
\]

\[
4x \leq 36
\]

\[
\frac{4x}{4} \leq \frac{36}{4}
\]

\[
x \leq 9
\]

To graph this inequality, draw a solid circle at 9 and draw an arrow extending to the left.

**ANSWER:**

\( x \leq 9 \)

17. \( -6z - 14 > -32 \)

**SOLUTION:**

\[
-6z - 14 > -32
\]

\[
-6z - 14 + 14 > -32 + 14
\]

\[
-6z > -18
\]

\[
\frac{-6z}{-6} < \frac{-18}{-6}
\]

\[
z < 3
\]

To graph this inequality, draw an open circle at 3 and draw an arrow extending to the left.

**ANSWER:**

\( z < 3 \)

18. \( -16 \geq 5(2z - 11) \)

**SOLUTION:**

\[
-16 \geq 5(2z - 11)
\]

\[
-16 \geq 5(2z) + 5(-11)
\]

\[
-16 \geq 10z - 55
\]

\[
-16 + 55 \geq 10z - 55 + 55
\]

\[
39 \geq 10z
\]

\[
\frac{10z}{10} \leq \frac{39}{10}
\]

\[
z \leq 3.9
\]

To graph this inequality, draw a solid circle at 4 and draw an arrow extending to the left.

**ANSWER:**

\( z \leq 3.9 \)
1-5 Solving Inequalities

19. $12 < -4(3c - 6)$
   
   **SOLUTION:**
   
   $12 < -4(3c) + (-4)(-6)$
   $12 < -12c + 24$
   $12 - 24 < -12c + 24 - 24$
   $-12 < -12c$
   $-12c > -12$
   $\frac{-12c}{-12} \leq \frac{-12}{-12}$
   $c < 1$

   To graph this inequality, draw an open circle at 1 and draw an arrow extending to the left.

   **ANSWER:**
   
   $c < 1$

20. $\frac{3y - 4}{0.2} - 8 > 12$
   
   **SOLUTION:**
   
   $\frac{3y - 4}{0.2} - 8 + 8 > 12 + 8$
   $\frac{3y - 4}{0.2} > 20$
   $(0.2)\left(\frac{3y - 4}{0.2}\right) > (0.2)(20)$
   $3y - 4 > 4$
   $3y > 8$
   $\frac{3y}{3} > \frac{8}{3}$
   $y > \frac{8}{3}$

   To graph this inequality, draw an open circle at $\frac{8}{3}$ and draw an arrow extending to the right.

   **ANSWER:**
   
   $y > \frac{8}{3}$
22. **GYMNASTICS** In a gymnastics competition, an athlete’s final score is calculated by taking 75% of the average technical score and adding 25% of the artistic score. All scores are out of 10, and one gymnast has a 7.6 average technical score. What artistic score does the gymnast need to have a final score of at least 8.0?

**SOLUTION:**
Let \( x \) be the artistic score of the gymnast.

\[
0.75(7.6) + 0.25(x) \geq 8
\]
\[
5.7 + 0.25x \geq 8
\]
\[
5.7 + 0.25x - 5.7 \geq 8 - 5.7
\]
\[
0.25x \geq 2.3
\]
\[
\frac{0.25}{0.25}x \geq \frac{2.3}{0.25}
\]
\[
x \geq 9.2
\]

So, the gymnast needs to have 9.2 of artistic score to have a final score of at least 8.0.

**ANSWER:** 9.2

**Define a variable and write an inequality for each problem. Then solve.**

23. Twelve less than the product of three and a number is less than 21.

**SOLUTION:**
Let \( x \) be the unknown number.

\[
3x - 12 < 21
\]
\[
3x - 12 + 12 < 21 + 12
\]
\[
3x < 33
\]
\[
\frac{3x}{3} < \frac{33}{3}
\]
\[
x < 11
\]

**ANSWER:** 
\( 3x - 12 < 21; x < 11 \)
1-5 Solving Inequalities

24. The quotient of three times a number and 4 is at least –16.

**SOLUTION:**

Let \( x \) be the unknown number.

\[
\frac{3x}{4} \geq -16
\]

\[
4 \left( \frac{3x}{4} \right) \geq 4(-16)
\]

\[
3x \geq -64
\]

\[
\frac{3x}{3} \geq \frac{-64}{3}
\]

\[
x \geq \frac{-64}{3}
\]

**ANSWER:**

\[
\frac{3x}{4} \geq -16; x \geq \frac{-64}{3}
\]

25. The difference of 5 times a number and 6 is greater than the number.

**SOLUTION:**

Let \( x \) be the unknown number.

\[
5x - 6 > x
\]

\[
5x - 6 + 6 > x + 6
\]

\[
5x > x + 6
\]

\[
5x - x > x + 6 - x
\]

\[
4x > 6
\]

\[
4 \cdot \frac{x}{4} > \frac{6}{4}
\]

\[
x > \frac{3}{2}
\]

\[
x > 1.5
\]

**ANSWER:**

\[
5x - 6 > x; x > 1.5
\]

26. The quotient of the sum of 3 and a number and 6 is less than –2.

**SOLUTION:**

Let \( x \) be the unknown number.

\[
\frac{x + 3}{6} < -2
\]

\[
6 \left( \frac{x + 3}{6} \right) < 6(-2)
\]

\[
x + 3 < -12
\]

\[
x + 3 - 3 < -12 - 3
\]

\[
x < -15
\]

**ANSWER:**

\[
\frac{x + 3}{6} < -2; x < -15
\]

27. **HIKING** Danielle can hike 3 miles in an hour, but she has to take a one-hour break for lunch and a one-hour break for dinner. If Danielle wants to hike at least 18 miles, solve \(3(x - 2) \geq 18\) to determine how many hours the hike should take.

**SOLUTION:**

\[
3(x - 2) \geq 18
\]

\[
\frac{3(x - 2)}{3} \geq \frac{18}{3}
\]

\[
x - 2 \geq 6
\]

\[
x - 2 + 2 \geq 6 + 2
\]

\[
x \geq 8
\]

Danielle has to hike for at least 8 hours.

**ANSWER:**

at least 8 hours
1-5 Solving Inequalities

Solve each inequality. Then graph the solution set on a number line.

28. \(18 - 3x < 12\)

**SOLUTION:**
\[
egin{align*}
18 - 3x &< 12 \\
-3x &< -6 \\
\frac{-3x}{-3} &> \frac{-6}{-3} \\
x &> 2
\end{align*}
\]

To graph this inequality, draw an open circle at 2 and draw an arrow extending to the right.

**ANSWER:**
\(x > 2\)

29. \(-8(4x + 6) < -24\)

**SOLUTION:**
\[
egin{align*}
-8(4x + 6) &< -24 \\
-32x - 48 &< -24 \\
-32x &< 24 \\
\frac{-32x}{-32} &> \frac{24}{-32} \\
x &> \frac{-3}{4}
\end{align*}
\]

To graph this inequality, draw an open circle at \(\frac{-3}{4}\) and draw an arrow extending to the right.

**ANSWER:**
\(x > \frac{-3}{4}\)

30. \(\frac{1}{4}n + 12 \geq \frac{3}{4}n - 4\)

**SOLUTION:**
\[
egin{align*}
\frac{1}{4}n + 12 &\geq \frac{3}{4}n - 4 \\
\frac{1}{4}n + 12 - \frac{3}{4}n &\geq \frac{3}{4}n - 4 - \frac{3}{4}n \\
\frac{1}{4}n - \frac{3}{4}n &\geq 3 - 4 \\
\frac{1}{4}n - \frac{3}{4}n &\geq -1 \\
\frac{1}{4}n &\geq -1 \\
\frac{1}{4}n - 4 &\geq -1 - 4 \\
\frac{1}{4}n &\geq -5 \\
4\left(\frac{1}{4}n\right) &\geq 4(-5) \\
\frac{1}{4}n &\geq -20 \\
\frac{1}{2}n &\geq -16 \\
\frac{1}{2}n - 32 &\geq -16 - 32 \\
\frac{1}{2}n &\geq -48 \\
2\left(\frac{1}{2}n\right) &\geq 2(-48) \\
n &\geq -96
\end{align*}
\]

To graph this inequality, draw a solid circle at -96 and draw an arrow extending to the right.

**ANSWER:**
\(n \geq -96\)
1-5 Solving Inequalities

31. \(0.24y - 0.64 > 3.86\)

**SOLUTION:**

\[
\begin{align*}
0.24y - 0.64 & > 3.86 \\
0.24y & > 4.5 \\
\frac{0.24y}{0.24} & > \frac{4.5}{0.24} \\
y & > 18.75 \\
\end{align*}
\]

To graph this inequality, draw an open circle at 18.75 and draw an arrow extending to the right.

**ANSWER:**

\( y > 18.75 \)

32. \(10x - 6 \leq 4x + 42\)

**SOLUTION:**

\[
\begin{align*}
10x - 6 & \leq 4x + 42 \\
10x + 6 & \leq 4x + 42 + 6 \\
10x & \leq 4x + 48 \\
10x - 4x & \leq 4x + 48 - 4x \\
6x & \leq 48 \\
\frac{6x}{6} & \leq \frac{48}{6} \\
x & \leq 8 \\
\end{align*}
\]

To graph this inequality, draw a solid circle at 8 and draw an arrow extending to the left.

**ANSWER:**

\( x \leq 8 \)

33. \(-6v + 8 > -14v - 28\)

**SOLUTION:**

\[
\begin{align*}
-6v + 8 & > -14v - 28 \\
-6v + 8 - 8 & > -14v - 28 - 8 \\
-6v & > -14v - 36 \\
-6v + 14v & > -14v - 36 + 14v \\
8v & > -36 \\
\frac{8v}{8} & > \frac{-36}{8} \\
v & > -4.5 \\
\end{align*}
\]

To graph this inequality, draw an open circle at -4.5 and draw an arrow extending to the right.

**ANSWER:**

\( v > -4.5 \)
34. \( n > -\frac{3n-15}{8} \)

**SOLUTION:**

\[
8n > 8 \left( -\frac{3n-15}{8} \right)
\]

\[
8n > -3n - 15
\]

\[
8n + 3n > -3n - 15 + 3n
\]

\[
11n > -15
\]

\[
\frac{11n}{11} > \frac{-15}{11}
\]

\[
n > -\frac{15}{11}
\]

To graph this inequality, draw an open circle at \(-\frac{15}{11}\) and draw an arrow extending to the right.

**ANSWER:**

\( n > -\frac{15}{11} \)

35. \(-2r < -\frac{6-2r}{5}\)

**SOLUTION:**

\[
5(-2r) < 5 \left( -\frac{6-2r}{5} \right)
\]

\[
-10r < 6 - 2r + 2r
\]

\[
-10r < 6
\]

\[
-8r < 6
\]

\[
\frac{-8r}{-8} > \frac{6}{-8}
\]

\[
r > -\frac{3}{4}
\]

To graph this inequality, draw an open circle at \(-\frac{3}{4}\) and draw an arrow extending to the right.

**ANSWER:**

\( r > -\frac{3}{4}\)
36. \( \frac{9z - 4}{5} \leq \frac{7z + 2}{4} \)

**SOLUTION:**

\[
20 \left( \frac{9z - 4}{5} \right) \leq 20 \left( \frac{7z + 2}{4} \right) \\
4(9z - 4) \leq 5(7z + 2) \\
36z - 16 \leq 35z + 10 \\
36z - 35z \leq 16 + 10 \\
z \geq 6
\]

To graph this inequality, draw a solid circle at 6 and draw an arrow extending to the right.

**ANSWER:**

\( z \geq 6 \)

---

37. **MONEY** Jin is selling advertising space in *Central City Magazine* to local businesses. Jin earns 3% commission for every advertisement he sells plus a salary of $250 a week. If the average amount of money that a business spends on an advertisement is $500, how many advertisements must he sell each week to make a salary of at least $700 that week?

**a.** Write an inequality to describe this situation.

**b.** Solve the inequality and interpret the solution.

**SOLUTION:**

\[
250 + 0.03(500a) \geq 700 \\
-250 -250 \\
0.03(500a) \geq 450 \\
15a \geq 450 \\
\frac{15a}{15} \geq \frac{450}{15} \\
a \geq 30
\]

So, he must sell at least 30 advertisements to make a salary of at least $700 that week.

**ANSWER:**

\( a \geq 30 \); He must sell at least 30 advertisements.
Define a variable and write an inequality for each problem. Then solve.

38. One third of the sum of 5 times a number and 3 is less than one fourth the sum of six times that number and 5.

**SOLUTION:**
Let \( n \) be the unknown number. The words one third of the sum of 5 times a number and 3 represent the expression \( \frac{5n + 3}{3} \). The words one fourth the sum of six times that number and 5 represent the expression \( \frac{6n + 5}{4} \).

So, the inequality is \( \frac{5n + 3}{3} < \frac{6n + 5}{4} \).

\[
\begin{align*}
\frac{5n + 3}{3} &< \frac{6n + 5}{4} \\
12 \left( \frac{5n + 3}{3} \right) &< 12 \left( \frac{6n + 5}{4} \right) \\
4(5n + 3) &< 3(6n + 5) \\
20n + 12 &< 18n + 15 \\
20n + 12 - 18n &< 18n + 15 - 18n \\
2n + 12 &< 15 \\
2n + 12 - 12 &< 15 - 12 \\
2n &< 3 \\
\frac{2n}{2} &< \frac{3}{2} \\
n &< 1.5
\end{align*}
\]

**ANSWER:** \( \frac{5n + 3}{3} < \frac{6n + 5}{4} \); \( n < 1.5 \)

39. The sum of one third a number and 4 is at most the sum of twice that number and 12.

**SOLUTION:**
Let \( x \) be a variable. The words the sum of one third a number represent the expression \( \frac{x}{3} + 4 \). The words the sum of twice that number and 12 represent the expression \( 2x + 12 \).

So, the inequality is \( \frac{x}{3} + 4 \leq 2x + 12 \).

\[
\begin{align*}
\frac{x}{3} + 4 &< 2x + 12 \\
\frac{x}{3} + 4 - 4 &< 2x + 12 - 4 \\
\frac{x}{3} &< 2x + 8 \\
3 \left( \frac{x}{3} \right) &< 3(2x + 8) \\
x &< 6x + 24 \\
x - 6x &< 6x + 24 - 6x \\
-5x &< 24 \\
\frac{-5x}{-5} &> \frac{24}{-5} \\
x &> -4.8
\end{align*}
\]

**ANSWER:** \( \frac{x}{3} + 4 \leq 2x + 12; x \geq -4.8 \)
1-5 Solving Inequalities

40. SENSE-MAKING The sides of square $ABCD$ are extended to form rectangle $DEFG$. If the perimeter of the rectangle is at least twice the perimeter of the square, what is the maximum length of a side of square $ABCD$?

SOLUTION:
Let $x$ be the side length of the square $ABCD$.
So, the perimeter of the square is $4x$.
Therefore, the length of the rectangle is $(x + 10)$ and the width of the rectangle is $(x + 8)$. So, the perimeter of the rectangle is $2(x + 10) + 2(x + 8)$.
Since the perimeter of the rectangle is at least twice the perimeter of the square, the inequality is $2(x + 10) + 2(x + 8) \geq 2(4x)$.

Solve the inequality $2(x + 10) + 2(x + 8) \geq 2(4x)$.

\[
2(x + 10) + 2(x + 8) \geq 2(4x)
\]
\[
2x + 20 + 2x + 16 \geq 8x
\]
\[
4x + 36 \geq 8x
\]
\[
4x + 36 - 36 \geq 8x - 36
\]
\[
4x \geq 8x - 36
\]
\[
4x - 8x \geq 8x - 36 - 8x
\]
\[
-4x \geq -36
\]
\[
\frac{-4x}{-4} \leq \frac{-36}{-4}
\]
\[
x \leq 9
\]

Therefore, the maximum length of the side of square $ABCD$ is 9 inches.

ANSWER: 9 in.

41. MARATHONS Jamie wants to be able to run at least the standard marathon distance of 26.2 miles. A good rule for training is that runners generally have enough endurance to finish a race that is up to 3 times his or her average daily distance.

a. If the length of her current daily run is 5 miles, write an inequality to find the amount by which she needs to increase her daily run to have enough endurance to finish a marathon.
b. Solve the inequality and interpret the solution.

SOLUTION:
\[
a. \quad 3(5 + d) \geq 26.2
\]
\[
b. \quad 3(5 + 3d) \geq 26.2
\]
\[
15 + 3d \geq 26.2
\]
\[
15 + 3d - 15 \geq 26.2 - 15
\]
\[
3d \geq 11.2
\]
\[
\frac{3d}{3} \geq \frac{11.2}{3}
\]
\[
d \geq 3.73
\]

In order to have enough endurance to run a marathon, Jamie should increase the distance of her average daily run by at least 3.73 miles.

ANSWER:
\[
a. \quad 3(5 + d) \geq 26.2
\]
\[
b. \quad d \geq 3.73; \quad \text{In order to have enough endurance to run a marathon, Jamie should increase the distance of her average daily run by at least 3.73 miles.}
\]
1-5 Solving Inequalities

42. **MODELING** The costs for renting a car from Ace Car Rental and from Basic Car Rental are shown in the table. For what mileage does Basic have the better deal? Use the inequality 
\[38 + 0.1x > 42 + 0.05x.\] Explain why this inequality works.

<table>
<thead>
<tr>
<th>Rental Car Costs</th>
<th>Company</th>
<th>Cost per Day</th>
<th>Cost per Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ace</td>
<td>$38</td>
<td>$0.10</td>
<td></td>
</tr>
<tr>
<td>Basic</td>
<td>$42</td>
<td>$0.05</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**

\[38 + 0.1x > 42 + 0.05x\]

\[38 + 0.1x - 38 > 42 + 0.05x - 38\]

\[0.1x > 4 + 0.05x\]

\[0.1x - 0.05x > 4 + 0.05x - 0.05x\]

\[0.05x > 4\]

\[\frac{0.05x}{0.05} > \frac{4}{0.05}\]

\[x > 80\]

Basic has the better deal as long as you are traveling more than 80 miles. Yes, this is the correct inequality to use.

Sample explanation: It works because the inequality finds the mileage at which Ace’s charge is greater than Basic’s charge.

**ANSWER:**

Basic has the better deal as long as you are traveling more than 80 miles. Yes, this is the correct inequality to use. Sample explanation: It works because the inequality finds the mileage at which Ace’s charge is greater than Basic’s charge.

43. **MULTIPLE REPRESENTATIONS** In this exercise, you will explore graphing inequalities on a coordinate plane.

**a. TABULAR** Organize the following into a table.

Substitute 5 points into the inequality \[y \geq -\frac{1}{2}x + 3.\] State whether the resulting statement is **true** or **false**.

**b. GRAPHICAL** Graph \(y = -\frac{1}{2}x + 3\). Also graph the 5 points from the table. Label all points that resulted in a true statement with a T. Label all points that resulted in a false statement with an F.

**c. VERBAL** Describe the pattern produced by the points you have labeled. Make a conjecture about which points on the coordinate plane would result in true and false statements.

**SOLUTION:**

a. Sample answer: Choose the five points (0, 0), (1, 1), (2, 2), (3, 3), and (4, 4). Substitute each ordered pair into the inequality, \[y \geq -\frac{1}{2}x + 3,\] and determine whether each point results in a true or false statement.

<table>
<thead>
<tr>
<th>Point</th>
<th>Resulting Statement</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>0 ≥ 3</td>
<td>False</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>[1 \geq \frac{5}{2}]</td>
<td>False</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>2 ≥ 2</td>
<td>True</td>
</tr>
<tr>
<td>(3, 3)</td>
<td>[3 \geq \frac{3}{2}]</td>
<td>True</td>
</tr>
<tr>
<td>(4, 4)</td>
<td>4 ≥ 1</td>
<td>True</td>
</tr>
</tbody>
</table>

b. Sample answer:

![Graph of inequality](image)

c. Sample answer: The points on or above the line result in true statements, and the points below the line result in false statements. This is true for all points on the coordinate plane.

**ANSWER:**

a. Sample answer:
1-5 Solving Inequalities

<table>
<thead>
<tr>
<th>Point</th>
<th>Resulting Statement</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>0 ≥ 3</td>
<td>False</td>
</tr>
<tr>
<td>(1, 1)</td>
<td>1 ≥ (\frac{5}{2})</td>
<td>False</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>2 ≥ 2</td>
<td>True</td>
</tr>
<tr>
<td>(3, 3)</td>
<td>3 ≥ (\frac{3}{2})</td>
<td>True</td>
</tr>
<tr>
<td>(4, 4)</td>
<td>4 ≥ 1</td>
<td>True</td>
</tr>
</tbody>
</table>

b. Sample answer:

To graph this inequality, draw an open circle at \(x = 2\) and draw an arrow extending to the left.

To graph this inequality, draw a solid circle at \(x = 3\) and draw an arrow extending to the right.

To graph this inequality, draw an open circle at \(x = 3\) and draw an arrow extending to the left.

To graph this inequality, draw a solid circle at \(x = 20\) and draw an arrow extending to the left.

Since \(-4 < x < 5\), \(ya = -4\) and \(yb = 5\).

\[ya + yb = -4 + 5\]
\[y(a + b) = 1\]
\[(a + b)0.25 < (a + b)y < (a + b)4\]
\[(a + b)0.25 < 1 < (a + b)4\]
\[\frac{0.25}{0.25} < \frac{1}{0.25} < \frac{(a + b)4}{0.25}\]
\[0.25 < 1 < 16(a + b)\]

Therefore, \((a + b) < 4\).

ANSWER:
\[(a + b) < 4\]

45. ERROR ANALYSIS Madlynn and Emilie were comparing their homework. Is either of them correct? Explain your reasoning.

SOLUTION:
No; sample answer: Madlynn reversed the inequality sign when she added 1 to each side. Emilie did not reverse the inequality sign at all.

ANSWER:
No; sample answer: Madlynn reversed the inequality sign when she added 1 to each side. Emilie did not reverse the inequality sign at all.
1-5 Solving Inequalities

46. REASONING Determine whether the following statement is sometimes, always, or never true.
   Explain your reasoning.
   The opposite of the absolute value of a negative number is less than the opposite of that number.
   
   SOLUTION:
   Sample answer: Always; the opposite of the absolute value of a negative number will always be a negative value, while the opposite of a negative number will always be a positive value. A negative value will always be less than a positive value.
   
   ANSWER:
   Sample answer: Always; the opposite of the absolute value of a negative number will always be a negative value, while the opposite of a negative number will always be a positive value. A negative value will always be less than a positive value.

47. CHALLENGE Given \( \triangle ABC \) with sides 
   \( AB = 3x + 4 \), \( BC = 2x + 5 \), and \( AC = 4x \). determine the values of \( x \) such that \( \triangle ABC \) exists.

   SOLUTION:
   Using the Triangle Inequality Theorem, we know that the sum of the lengths of any 2 sides of a triangle must be greater than the length of the remaining side. This generates 3 inequalities to examine.

   \[
   \begin{align*}
   3x + 4 + 2x + 5 &> 4x \\
   5x + 9 &> 4x \\
   5x + 9 - 4x &> 4x - 4x \\
   x + 9 &> 0 \\
   x + 9 - 9 &> 0 - 9 \\
   x &> -9 \\
   2x + 5 + 4x &> 3x + 4 \\
   6x + 5 &> 3x + 4 \\
   6x + 5 - 3x &> 3x + 4 - 3x \\
   3x + 5 &> 4 \\
   3x + 5 - 5 &> 4 - 5 \\
   3x &> -1 \\
   3x - 1 &> 0 \\
   \frac{3x - 1}{3} &> \frac{0}{3} \\
   x &> -1 \\
   \\
   3x + 4 + 4x &> 2x + 5 \\
   7x + 4 &> 2x + 5 \\
   7x + 4 - 2x &> 2x + 5 - 2x \\
   5x + 4 &> 5 \\
   5x + 4 - 4 &> 5 - 4 \\
   5x &> 1 \\
   \frac{5x}{5} &> \frac{1}{5} \\
   x &> \frac{1}{5}
   \end{align*}
   \]

   In order for all 3 conditions to be true, \( x \) must be greater than 0.2.

   ANSWER:
   Using the Triangle Inequality Theorem, we know that the sum of the lengths of any 2 sides of a triangle must be greater than the length of the remaining side. This generates 3 inequalities to examine.

   \[
   \begin{align*}
   3x + 4 + 2x + 5 &> 4x \\
   3x + 4 + 4x &> 2x + 5 \\
   x &> -9 \\
   x &> 0.2 \\
   2x + 5 + 4x &> 3x + 4 \\
   x &> -\frac{1}{3}
   \end{align*}
   \]

   In order for all 3 conditions to be true, \( x \) must be greater than 0.2.
1-5 Solving Inequalities

48. OPEN ENDED Write an inequality for which the solution is all real numbers in the form $ax + b > c(x + d)$. Explain how you know this.

**SOLUTION:**
Sample answer: $4x + 5 > 4(x + 1)$; This has a solution set of all real numbers because it simplifies to $4x + 5 > 4x + 4$ or $5 > 4$. This indicates that for any real value of $x$ the inequality is equivalent to $1 > 0$, that is the left side will always be 1 greater than the right side.

**ANSWER:**
Sample answer: $4x + 5 > 4(x + 1)$; This has a solution set of all real numbers because it simplifies to $4x + 5 > 4x + 4$ or $5 > 4$. This indicates that for any real value of $x$ the inequality is equivalent to $1 > 0$, that is the left side will always be 1 greater than the right side.

49. WRITING IN MATH Why does the inequality symbol need to be reversed when multiplying or dividing by a negative number?

**SOLUTION:**
Sample answer: When one number is greater than another number, it is either more positive or less negative than that number. When these numbers are multiplied by a negative value, their roles are reversed. That is, the number that was more positive is now more negative than the other number. Thus, it is now less than that number and the inequality symbol needs to be reversed.

**ANSWER:**
Sample answer: When one number is greater than another number, it is either more positive or less negative than that number. When these numbers are multiplied by a negative value, their roles are reversed. That is, the number that was more positive is now more negative than the other number. Thus, it is now less than that number and the inequality symbol needs to be reversed.

50. SHORT RESPONSE Rogelio found a cookie recipe that requires $\frac{3}{4}$ cup of sugar and 2 cups of flour. How many cups of sugar would he need if he used 6 cups of flour?

**SOLUTION:**
Let $x =$ cups of sugar needed for 6 cups of flour.

\[
x = \frac{6}{\frac{3}{4}}
\]

\[
x = \frac{6 \cdot 4}{3}
\]

\[
x = \frac{24}{3}
\]

\[
x = 8
\]

\[
x = 2 \frac{1}{4}
\]

So, $2 \frac{1}{4}$ cups of sugar would be needed for 6 cups of flour.

**ANSWER:**

$2 \frac{1}{4}$
51. **STATISTICS** The mean score for Samantha’s first six algebra quizzes was 88. If she scored a 95 on her next quiz, what will her mean score be for all 7 quizzes?

A 89 C 91
B 90 D 92

**SOLUTION:**

Let \( x \) be the sum of the scores of first six algebra quizzes.

\[
\frac{x}{6} = 88
\]

\[
6 \left( \frac{x}{6} \right) = 6(88)
\]

\[
x = 528
\]

\[
x + 95 = \frac{528 + 95}{7}
\]

\[
= \frac{623}{7}
\]

\[
= 89
\]

So, the correct choice is A.

**ANSWER:** A

52. **SAT/ACT** The average of five numbers is 9. The average of 7 other numbers is 8. What is the average of all 12 numbers?

F \( \frac{5}{12} \)
G \( \frac{7}{2} \)
H \( \frac{7}{12} \)
J \( \frac{3}{4} \)
K \( \frac{11}{12} \)

**SOLUTION:**

Let \( x \) be the sum of the five numbers.

\[
\frac{x}{5} = 9
\]

\[
5 \left( \frac{x}{5} \right) = 5(9)
\]

\[
x = 45
\]

Let \( y \) be the sum of the 7 other numbers.

\[
\frac{y}{7} = 8
\]

\[
7 \left( \frac{y}{7} \right) = 7(8)
\]

\[
y = 56
\]

\[
x + y = \frac{45 + 56}{12}
\]

\[
= \frac{101}{12}
\]

\[
= \frac{5}{12}
\]

Therefore, the average of all 12 numbers is \( \frac{5}{12} \). So, the correct choice is F.

**ANSWER:** F
53. What is the complete solution of the equation \(|8 - 4x| = 40|?\)

A \(x = 8; x = 12\)
B \(x = 8; x = -12\)
C \(x = -8; x = -12\)
D \(x = -8; x = 12\)

\textbf{SOLUTION:}

\begin{align*}
\text{Case 1:} & \quad 8 - 4x = 40 & \quad 8 - 4x = -40 \\
8 - 4x - 8 = 40 - 8 & \quad 8 - 4x - 8 = -40 - 8 \\
-4x & = 32 & \quad -4x & = -48 \\
\frac{-4x}{-4} & = \frac{32}{-4} & \quad \frac{-4x}{-4} & = \frac{-48}{-4} \\
x & = -8 & \quad x & = 12
\end{align*}

\textbf{Check:}

\begin{align*}
|8 - 4x| & = 40 & \quad |8 - 4x| & = 40 \\
|8 - 4(-8)| & = 40 & \quad |8 - 4(12)| & = 40 \\
|8 + 32| & = 40 & \quad |8 - 48| & = 40 \\
|40| & = 40 & \quad |-40| & = 40 \\
40 & = 40\checkmark & \quad 40 & = 40\checkmark
\end{align*}

The solution set is \([-8, 12]\).

So, the correct choice is D.

\textbf{ANSWER:}

D

---

54. \(|x - 5| = 12|\)

\textbf{SOLUTION:}

\textbf{Case 1:}

\begin{align*}
x - 5 & = 12 \\
x & = 17
\end{align*}

\textbf{Case 2:}

\begin{align*}
x - 5 & = -12 \\
x & = -7
\end{align*}

There appear to be two solutions, 17 and -7.

\textbf{Check:} Substitute the values in the original equation.

\begin{align*}
|x - 5| & = 12 & \quad |x - 5| & = 12 \\
|17 - 5| & = 12 & \quad |-7 - 5| & = 12 \\
|12| & = 12 & \quad |-12| & = 12 \\
12 & = 12\checkmark & \quad 12 & = 12\checkmark
\end{align*}

The solution set is \(\{17, -7\}\).

\textbf{ANSWER:}

\(\{-7, 17\}\)
Solve each inequality. Then graph the solution set on a number line.

1.

**SOLUTION:**

To graph this inequality, draw a solid circle at 8 and draw an arrow extending to the left.

There appear to be two solutions, 17 and 13.

The solution set is \{17, 13\}.

**ANSWER:**

56. \[ |a + 6| = a \]

**SOLUTION:**

Since the absolute value of the sum of 6 and a number never equal to the number, the solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)

57. **ASTRONOMY** Pluto travels in a path that is not circular. Pluto’s farthest distance from the Sun is 4539 million miles, and its closest distance is 2756 million miles. Write an equation that can be solved to find the minimum and maximum distances from the Sun to Pluto.

**SOLUTION:**

Maximum distance – Minimum distance = 4539 – 2756

\[ r = \frac{1783}{2} = 891.5 \]

The value of \(c\) is 4539 – 891.5 or 3647.5.

Substitute \(r = 891.5\) and \(c = 3647.5\) in the equation \(|t - c| = r\).

\(|t - 3647.5| = 891.5\)

**ANSWER:**

\(|t - 3647.5| = 891.5\)

---

**1-5 Solving Inequalities**

55. \[ 7|3y - 4| = 35 \]

**SOLUTION:**

\[ 7|3y - 4| = 35 \]

\[ \frac{7|3y - 4|}{7} = \frac{35}{7} \]

\[ |3y - 4| = 5 \]

Case 1: \[ 3y - 4 = 5 \]

Case 2: \[ 3y - 4 = -5 \]

\[ 3y = 9 \]

\[ y = 3 \]

\[ y = -\frac{1}{3} \]

There appear to be two solutions, 3 and \(-\frac{1}{3}\).

**Check:** Substitute the values in the original equation.

\[ 7|3y - 4| = 35 \]

\[ 7|3(-\frac{1}{3}) - 4| = 35 \]

\[ 7|-1 - 4| = 35 \]

\[ 7|5| = 35 \]

\[ 75 = 35 \checkmark \]

**ANSWER:**

\[ \left\{ -\frac{1}{3}, 3 \right\} \]
58. **POPULATION** In 2005, the population of Bay City was 19,611. For each of the next five years, the population decreased by an average of 715 people per year.

a. What was the population in 2010?

b. If the population continues to decline at the same rate as from 2005 to 2010, what would you expect the population to be in 2025?

**SOLUTION:**

a. Since the population is decreased by an average of 715 people per year, the decrease in population for 5 years is 5(715) or 3575.

19,611 – 3,575 = 16,036

The population in 2010 was 16,036.

b. To find the decrease in population in 20 years, multiply 20 and 715.

20 · 715 = 14,300

So in 2025, the population would be 19,611 – 14,300 or 5,311.

**ANSWER:**

a. 16,036

b. 5311

59. **GEOMETRY** The formula for the surface area of a cylinder is \( S_A = 2\pi r^2 + 2\pi rh \).

a. Use the Distributive Property to rewrite the formula by factoring out the greatest common factor of the two terms.

b. Find the surface area for a cylinder with radius 3 centimeters and height 10 centimeters using both formulas. Leave the answer in terms of \( \pi \).

c. Which formula do you prefer? Explain your reasoning.

**SOLUTION:**

a. \( 2\pi r^2 = 2 \cdot \pi \cdot r \cdot r \)

\( 2\pi rh = 2 \cdot \pi \cdot r \cdot h \)

The GCF of the two terms is \( 2\pi r \).

\( S_A = 2\pi r(r) + 2\pi r(h) \)

\( = 2\pi r(r + h) \)

b. Substitute \( r = 3 \) and \( h = 10 \) in the formula

\( S_A = 2\pi r^2 + 2\pi rh \).

\( S_A = 2\pi r^2 + 2\pi rh \)

\( = 2\pi(3)^2 + 2\pi(3)(10) \)

\( = 18\pi + 60\pi \)

\( = 78\pi \)

Substitute \( r = 3 \) and \( h = 10 \) in the formula

\( S_A = 2\pi(3)(3 + 10) \)

\( = 2\pi(3)(13) \)

\( = 78\pi \)

Therefore, the surface area of the cylinder is \( 78\pi \) cm\(^2\).

c. **Sample answer:** The formula in part b is quicker.

**ANSWER:**

a. \( S_A = 2\pi r(r + h) \)

b. \( 78\pi \) cm\(^2\)

c. Sample answer: The formula in part b is quicker.
60. **CONSTRUCTION** The Sawyers are adding a family room to their house. The dimensions of the room are 26 feet by 28 feet. Show how to use the Distributive Property to mentally calculate the area of the room.

**SOLUTION:**

\[26 \times 28 = 26(20 + 8)\]

\[= 520 + 208 = 728\]

**ANSWER:**

\[26 \times 28 = 26(20 + 8) = 520 + 208 = 728\]

Solve each equation. Check your solutions.

61. \[|x| = 9\]

**SOLUTION:**

**Case 1:** \[x = 9\]

**Case 2:** \[x = -9\]

The solution set is \([-9, 9]\).

**ANSWER:**

\[-9, 9\]

62. \[|x + 3| = 10\]

**SOLUTION:**

**Case 1:** \[x + 3 = 10\]

\[x + 3 - 3 = 10 - 3\]

\[x = 7\]

**Case 2:** \[x + 3 = -10\]

\[x + 3 - 3 = -10 - 3\]

\[x = -13\]

There appear to be two solutions, 7 and -13.

**Check:** Substitute the values in the original equation.

\[|x + 3| = 10, \quad |x + 3| = 10\]

\[|7 + 3| = 10, \quad |-13 + 3| = 10\]

\[|10| = 10, \quad |-10| = 10\]

\[10 = 10\checkmark, \quad 10 = 10\checkmark\]

The solution set is \([-13, 7]\).

**ANSWER:**

\[-13, 7\]
63. \( |4y - 15| = 13 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>Case 1: ( 4y - 15 = 13 )</th>
<th>Case 2: ( 4y - 15 = -13 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4y = 28 )</td>
<td>( 4y = 2 )</td>
</tr>
<tr>
<td>( y = \frac{28}{4} )</td>
<td>( y = \frac{2}{4} )</td>
</tr>
<tr>
<td>( y = 7 )</td>
<td>( y = \frac{1}{2} )</td>
</tr>
</tbody>
</table>

There appear to be two solutions, \( \frac{1}{2} \) and 7.

**Check:** Substitute the values in the original equation.

\[
\begin{align*}
|4y - 15| &= 13 \\
|4\left(\frac{1}{2}\right) - 15| &= 13 \\
|2 - 15| &= 13 \\
|-13| &= 13 \\
13 &= 13 \checkmark
\end{align*}
\]

\[
\begin{align*}
|4y - 15| &= 13 \\
|4(7) - 15| &= 13 \\
|28 - 15| &= 13 \\
|13| &= 13 \\
13 &= 13 \checkmark
\end{align*}
\]

The solution set is \( \left\{ \frac{1}{2}, 7 \right\} \).

**ANSWER:**

\( \left\{ \frac{1}{2}, 7 \right\} \)

---

64. \( 18 = |3x - 9| \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>Case 1: ( 3x - 9 = 18 )</th>
<th>Case 2: ( 3x - 9 = -18 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x = 27 )</td>
<td>( 3x = -9 )</td>
</tr>
<tr>
<td>( x = \frac{27}{3} )</td>
<td>( x = \frac{-9}{3} )</td>
</tr>
<tr>
<td>( x = 9 )</td>
<td>( x = -3 )</td>
</tr>
</tbody>
</table>

There appear to be two solutions, –3 and 9.

**Check:** Substitute the values in the original equation.

\[
\begin{align*}
18 &= |3x - 9| \\
18 &= |3(-3) - 9| \\
18 &= |-9 - 9| \\
18 &= |-18| \\
18 &= 18 \checkmark
\end{align*}
\]

The solution set is \( \{-3, 9\} \).

**ANSWER:**

\( \{-3, 9\} \)
Solve each inequality. Then graph the solution set on a number line.

65. \[|16 - 4| w + 2|\]

**SOLUTION:**
\[16 = 4|w + 2|\]
\[16 = 4|w + 2|\]
\[\frac{16}{4} = \frac{4}{w + 2}\]
\[4 = |w + 2|\]

**Case 1:**
\[w + 2 = 4\]
\[w = 2\]
\[w = 2\]

**Case 2:**
\[w + 2 = 2\]
\[w = 2\]
\[w = 2\]

There appear to be two solutions, \(-6\) and \(2\).

**Check:** Substitute the values in the original equation.

\[\begin{array}{ll}
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
16 = 4|w + 2| & 16 = 4|w + 2| \\
\end{array}\]

The solution set is \([-6, 2]\).

**ANSWER:**
\([-6, 2]\)

66. \[|y + 3| + 4 = 20\]

**SOLUTION:**
\[|y + 3| + 4 = 20\]
\[|y + 3| + 4 - 4 = 20 - 4\]
\[|y + 3| = 16\]

**Case 1:**
\[y + 3 = 16\]
\[y + 3 = -16\]
\[y = 13\]
\[y = -19\]

**Case 2:**
\[y + 3 = 16\]
\[y + 3 = -16\]
\[y = 13\]
\[y = -19\]

There appear to be two solutions, \(-19\) and \(13\).

\[\begin{array}{ll}
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
|y + 3| + 4 = 20 & |y + 3| + 4 = 20 \\
\end{array}\]

The solution set is \([-19, 13]\).

**ANSWER:**
\([-19, 13]\)
1-6 Solving Compound and Absolute Value Inequalities

Solve each inequality. Graph the solution set on a number line.

1. \(-4 < g + 8 < 6\)

\textbf{SOLUTION:}
\begin{align*}
-4 &< g + 8 < 6 \\
-4 - 8 &< g + 8 - 8 < 6 - 8 \\
-12 &< g < -2 \\
\end{align*}

The solution of the inequality is \(\{g | -12 < g < -2\}\).

To graph, draw an open circle at \(-2\) and an open circle at \(-12\) and draw a line to connect the circles.

\textbf{ANSWER:}
\(\{g | -12 < g < -2\}\)

\[\begin{array}{c}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
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\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\end{array}
\]

\[\begin{array}{c}
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\hspace{1cm}
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\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\end{array}
\]

2. \(-9 \leq 4y - 3 \leq 13\)

\textbf{SOLUTION:}
\begin{align*}
-9 &\leq 4y - 3 \leq 13 \\
-9 + 3 &\leq 4y - 3 + 3 \leq 13 + 3 \\
-6 &\leq 4y \leq 16 \\
\frac{-6}{4} &\leq \frac{4y}{4} \leq \frac{16}{4} \\
-1.5 &\leq y \leq 4 \\
\end{align*}

The solution of the inequality is \(\{y | -1.5 \leq y \leq 4\}\).

To graph, draw a solid circle at \(-1.5\) and a solid circle at \(4\) and draw a line to connect the circles.

\textbf{ANSWER:}
\(\{y | -1.5 \leq y \leq 4\}\)

\[\begin{array}{c}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
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\hspace{1cm}
\end{array}
\]

\[\begin{array}{c}
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\hspace{1cm}
\hspace{1cm}
\end{array}
\]

3. \(z + 6 > 3\) or \(2z < -12\)

\textbf{SOLUTION:}
\begin{align*}
z + 6 &> 3 \quad \text{or} \quad 2z < -12 \\
z + 6 &- 6 > 3 - 6 \quad \text{or} \quad 2z < -12 \\
z &> -3 \quad \text{or} \quad z < -6 \\
\end{align*}

The solution of the inequality is \(\{z | z > -3 \quad \text{or} \quad z < -6\}\).

To graph, draw an open circle at \(-3\) and draw an arrow extending to the right. Then draw an open circle at \(-6\) and draw an arrow extending to the left.

\textbf{ANSWER:}
\(\{z | z > -3 \quad \text{or} \quad z < -6\}\)

\[\begin{array}{c}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
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\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\end{array}
\]

\[\begin{array}{c}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
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\hspace{1cm}
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\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\hspace{1cm}
\end{array}
\]
1-6 Solving Compound and Absolute Value Inequalities

4. \( m - 7 \geq -3 \text{ or } -2m + 1 \geq 11 \)

**SOLUTION:**
\[
\begin{align*}
  m - 7 & \geq -3 \quad \text{or} \quad -2m + 1 \geq 11 \\
  m - 7 + 7 & \geq -3 + 7 \quad \text{or} \quad -2m + 1 - 1 \geq 11 - 1 \\
  m & \geq 4 \quad \text{or} \quad -2m \leq 10 \\
  \frac{-2m}{-2} & \leq \frac{10}{-2} \\
  m & \geq 4 \quad \text{or} \quad m \leq -5
\end{align*}
\]

The solution of the inequality is
\[\{ m \mid m \geq 4 \text{ or } m \leq -5 \} .\]

To graph, draw a solid circle at 4 and an arrow extending to the right and a solid circle at -5 and an arrow extending to the left.

**ANSWER:**
\[
\{ m \mid m \geq 4 \text{ or } m \leq -5 \}
\]

**5. \( |c| \geq 8 \)**

**SOLUTION:**
\[
|c| \geq 8 \\
\begin{align*}
  c & \leq -8 \quad \text{or} \quad c \geq 8
\end{align*}
\]

The solution of the absolute value inequality is
\[\{ c \mid c \geq 8 \text{ or } c \leq -8 \} .\]

To graph, draw a solid circle at -8 and an arrow extending to the left and a solid circle at 8 and an arrow extending to the right.

**ANSWER:**
\[
\{ c \mid c \geq 8 \text{ or } c \leq -8 \}
\]

6. \( |q| \geq -1 \)

**SOLUTION:**
\[
|q| \geq -1
\]

The absolute value of a number is always non-negative. So, the inequality is true for any value of \( q \).

The solution of the absolute value inequality is
\[\{ q \mid \text{all real numbers} \} .\]

Since the solution set includes all real numbers, draw an arrow extending in each direction.

**ANSWER:**
\[
\{ q \mid \text{all real numbers} \}
\]

7. \( |z| < 6 \)

**SOLUTION:**
\[
-6 < z < 6
\]

The solution of the absolute value inequality is
\[\{ z \mid -6 < z < 6 \} .\]

To graph, draw an open circle at -6 and an open circle at 6 and draw a line to connect the circles.

**ANSWER:**
\[
\{ z \mid -6 < z < 6 \}
\]
Solve each inequality. Graph the solution set on a number line.

8. \(|x| \leq -4\)

**SOLUTION:**
The absolute value of a number is always non-negative.
So, no value of \(x\) satisfies the inequality.

The solution set is \(\emptyset\).

Since there is no solution, leave the graph blank.

ANSWER:
\[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]

9. \(|3v + 5| > 14\)

**SOLUTION:**

\[ |3v + 5| > 14 \]

\[ 3v + 5 > 14 \quad \text{or} \quad 3v + 5 < -14 \]

\[ 3v > 9 \quad \text{or} \quad 3v < -19 \]

\[ v > 3 \quad \text{or} \quad v < -\frac{19}{3} \]

The solution of the inequality is
\[ \{v \mid v > 3 \text{ or } v < -\frac{19}{3}\} \).

To graph, draw an open circle at \(-\frac{19}{3}\) and an arrow extending to the left and an open circle at 3 and an arrow extending to the right.

ANSWER:
\[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]

10. \(|4t - 3| \leq 7\)

**SOLUTION:**

\[ |4t - 3| \leq 7 \]

\[ -7 \leq 4t - 3 \leq 7 \]

\[ -7 + 3 \leq 4t - 3 + 3 \leq 7 + 3 \]

\[ -4 \leq 4t \leq 10 \]

\[ -\frac{4}{4} \leq \frac{4t}{4} \leq \frac{10}{4} \]

\[ -1 \leq t \leq 2.5 \]

The solution of the inequality is
\[ \{t \mid -1 \leq t \leq 2.5\} \)

To graph, draw a solid circle at -1 and a solid circle at 2.5 and draw a line to connect the circles.

ANSWER:
\[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]
11. **MONEY** Khalid is considering several types of paint for his bedroom. He estimates that he will need between 2 and 3 gallons. The table at the right shows the price per gallon for each type of paint Khalid is considering. Write a compound inequality and determine how much he could be spending.

<table>
<thead>
<tr>
<th>Paint Type</th>
<th>Price per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td>$21.98</td>
</tr>
<tr>
<td>Satin</td>
<td>$23.98</td>
</tr>
<tr>
<td>Semi-Gloss</td>
<td>$24.98</td>
</tr>
<tr>
<td>Gloss</td>
<td>$25.98</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \( c \) represent the amount that Khalid could be spending to paint his bedroom. He requires a minimum of 2 gallons and a maximum of 3 gallons of paint.
The cost of 2 gallons of the cheapest paint will be the minimum amount Khalid needs to spend.
And, the cost 3 gallons of the costliest paint will be the maximum amount Khalid needs to spend.
So:
\[
2 \times 21.98 \leq c \leq 3 \times 25.98
\]
\[
43.96 \leq c \leq 77.94
\]
Khalid needs a minimum of $43.96 and a maximum of $77.96 to paint his bedroom.

**ANSWER:**
\[
43.96 \leq c \leq 77.94
\]

---

12. **Solve each inequality. Graph the solution set on a number line.**

**SOLUTION:**
\[
8 < 2v - 4 < 16
\]
\[
8 + 4 < 2v - 4 + 4 < 16 + 4
\]
\[
12 < 2v < 20
\]
\[
6 < v < 10
\]
The solution of the inequality is
\[
\{ v \mid 6 < v < 10 \}
\]
To graph, draw an open circle at 6 and an open circle at 10 and draw a line to connect the circles.

**ANSWER:**
\[
\{ v \mid 6 < v < 10 \}
\]
13. \(-7 \leq 4d - 3 \leq -1\)

**SOLUTION:**

\[
\begin{align*}
-7 & \leq 4d - 3 & \leq -1 \\
-7 + 3 & \leq 4d -3 + 3 & \leq -1 + 3 \\
-4 & \leq 4d & \leq 2 \\
\frac{-4}{4} & \leq \frac{4d}{4} & \leq \frac{2}{4} \\
-1 & \leq d & \leq 0.5
\end{align*}
\]

The solution of the inequality is \(\{d \mid -1 \leq d \leq 0.5\}\).

To graph, draw a solid circle at 1 and a solid circle at 0.5 and draw a line to connect the circles.

**ANSWER:**

\(\{d \mid -1 \leq d \leq 0.5\}\)

\[
\begin{array}{cccccc}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

---

14. \(4r + 3 < -6\) or \(3r - 7 > 2\)

**SOLUTION:**

\[
\begin{align*}
4r + 3 & < -6 & \text{or} & & & \\
4r + 3 - 3 & < -6 - 3 & & & & \\
4r & < -9 & & & & \\
\frac{4r}{4} & < -\frac{9}{4} & & & & \\
r & < -\frac{9}{4} & & & & \\
\end{align*}
\]

\[
\begin{align*}
3r - 7 & > 2 & & & & \\
3r - 7 + 7 & > 2 + 7 & & & & \\
3r & > 9 & & & & \\
\frac{3r}{3} & > \frac{9}{3} & & & & \\
r & > 3 & & & & \\
\end{align*}
\]

The solution of the inequality is \(\{r \mid r < -\frac{9}{4} \text{ or } r > 3\}\).

To graph, draw an open circle at \(-\frac{9}{4}\) and an arrow extending to the left and an open circle at 3 and an arrow extending to the right.

**ANSWER:**

\(\{r \mid r < -\frac{9}{4} \text{ or } r > 3\}\)

\[
\begin{array}{cccccc}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
& & & & & & & & & & \\
\end{array}
\]
1-6 Solving Compound and Absolute Value Inequalities

15. \(6y - 3 < -27\) or \(-4y + 2 < -26\)

**SOLUTION:**

\[
\begin{align*}
6y - 3 &< -27 \\
6y &< -27 + 3 \\
6y &< -24 \\
\frac{6y}{6} &< \frac{-24}{6} \\
y &< -4 \\
\end{align*}
\]

\[
\begin{align*}
-4y + 2 &< -26 \\
-4y &< -26 - 2 \\
-4y &< -28 \\
\frac{-4y}{-4} &< \frac{-28}{-4} \\
y &> 7
\end{align*}
\]

The solution of the inequality is \(\{y| y < -4 \text{ or } y > 7\}\).

To graph, draw an open circle at -4 and an arrow extending to the left and an open circle at 7 and an arrow extending right.

**ANSWER:**

\(\{y| y < -4 \text{ or } y > 7\}\)

16. \(|6h| < 12\)

**SOLUTION:**

\[
\begin{align*}
|6h| &< 12 \\
-12 &< 6h < 12 \\
\frac{-12}{6} &< \frac{6h}{6} < \frac{12}{6} \\
-2 &< h < 2
\end{align*}
\]

The solution of the inequality is \(|h| < 2\).

To graph, draw an open circle at -2 and an open circle at 2 and draw a line to connect the circles.

**ANSWER:**

\(|h| < 2\)
17. \(|-4k| > 16\)  
**SOLUTION:**  
\[
\begin{align*}
|-4k| &> 16 \\
\frac{-4k}{4} &> \frac{-16}{4} \\
k &> -4 \\
\text{or} \\
\frac{-4k}{4} &< \frac{16}{4} \\
k &< 4
\end{align*}
\]
The solution of the inequality is \(\{k \mid k < -4 \text{ or } k > 4\}\).

To graph, draw an open circle at \(-4\) and an arrow extending to the left and an open circle at \(4\) and an arrow extending to the right.

**ANSWER:**  
\(\{k \mid k < -4 \text{ or } k > 4\}\)

18. \(|3x - 4| > 10\)  
**SOLUTION:**  
\[
\begin{align*}
|3x - 4| &> 10 \\
3x - 4 < -10 & \text{ or } 3x - 4 > 10 \\
3x &< -6 & 3x &> 14 \\
\frac{3x}{3} &< -2 & \frac{3x}{3} &> \frac{14}{3} \\
x &< -2 & x &> \frac{14}{3}
\end{align*}
\]
The solution of the inequality is \(\{x \mid x < -2 \text{ or } x > \frac{14}{3}\}\).

To graph, draw an open circle at \(-2\) and an arrow extending to the left and an open circle at \(\frac{14}{3}\) and an arrow extending to the right.

**ANSWER:**  
\(\{x \mid x < -2 \text{ or } x > \frac{14}{3}\}\)
1-6 Solving Compound and Absolute Value Inequalities

19. \(|8t + 3| \leq 4\)

**SOLUTION:**
\[
|8t + 3| \leq 4 \\
-4 \leq 8t + 3 \leq 4 \\
-4 - 3 \leq 8t + 3 - 3 \leq 4 - 3 \\
-7 \leq 8t \leq 1 \\
\frac{7}{8} \leq t \leq \frac{1}{8}
\]

The solution of the inequality is 
\[
\left\{ t \middle| \frac{7}{8} \leq t \leq \frac{1}{8} \right\}
\]
To graph, draw a solid circle at \(-\frac{7}{8}\) and a solid circle at \(\frac{1}{8}\) and draw a line to connect the circles.

**ANSWER:**
\[
\left\{ t \middle| \frac{7}{8} \leq t \leq \frac{1}{8} \right\}
\]

---

20. \(|-9n - 3| < 6\)

**SOLUTION:**
\[
|-9n - 3| < 6 \\
-6 < -9n - 3 < 6 \\
-6 + 3 < -9n - 3 + 3 < 6 + 3 \\
-3 < -9n < 9 \\
\frac{-3}{9} < \frac{-9n}{-9} < \frac{9}{9} \\
\frac{1}{3} > n > -1
\]

The solution of the inequality is 
\[
\left\{ n \middle| -1 < n < \frac{1}{3} \right\}
\]
To graph, draw an open circle at -1 and an open circle at \(\frac{1}{3}\) and draw a line to connect the circles.

**ANSWER:**
\[
\left\{ n \middle| -1 < n < \frac{1}{3} \right\}
\]
21. \(|-5j - 4| \geq 12\)

**SOLUTION:**

\[
-5j - 4 \leq -12 \quad \text{or} \quad -5j - 4 + 4 \leq -12 + 4
-5j \leq -8 \quad \text{or} \quad -5j \geq 8
\]

\[
-5j + \frac{8}{5} \leq 0 \quad \text{or} \quad -5j - \frac{8}{5} \geq 0
j \geq \frac{8}{5} \quad \text{or} \quad j \leq -\frac{16}{5}
\]

The solution of the inequality is

\[
\left\{ j \mid j \geq \frac{8}{5} \quad \text{or} \quad j \leq -\frac{16}{5} \right\}.
\]

To graph, draw a solid circle at \(-\frac{16}{5}\) and an arrow extending to the left and a solid circle at \(\frac{8}{5}\) and an arrow extending to the right.

![Graph of inequality]

**ANSWER:**

\[
\left\{ j \mid j \geq \frac{8}{5} \quad \text{or} \quad j \leq -\frac{16}{5} \right\}
\]

22. **ANATOMY** Forensic scientists use the equation \(h = 2.6f + 47.2\) to estimate the height \(h\) of a woman given the length in centimeters \(f\) of her femur bone.

a. Suppose the equation has a margin of error of \(\pm 3\) centimeters. Write an inequality to represent the height of a woman given the length of her femur bone.

b. If the length of a female skeleton’s femur is 50 centimeters, write and solve an absolute value inequality that describes the woman’s height in centimeters.

**SOLUTION:**

a. The margin of error is \(\pm 3\) cm.

So:

\[-3 \leq 2.6f + 47.2 \leq 3\]

\[\Rightarrow |2.6f + 47.2| \leq 3\]

b. Substitute \(f = 50\) in the equation \(h = 2.6f + 47.2\).

\[h = 2.6(50) + 47.2 = 177.2\]

The margin of error is \(\pm 3\) cm.

So:

\[|h - 177.2| < 3\]

\[-3 < h - 177.2 < 3\]

\[-3 + 177.2 < h < 3 + 177.2\]

\[174.2 < h < 180.2\]

The woman’s height ranges from 174.2 cm to 180.2 cm.

**ANSWER:**

a. \(|2.6f + 47.2| < 3\)

b. \(|h - 177.2| < 3\); 174.2 cm < \(h\) < 180.2 cm
23. \( |x - 1| \leq 5 \)

**SOLUTION:**
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \(-a < x + c < a\). This implies that \(|x + c| < a\).

\[
-4 \leq x \leq 6
\]
\[
-4 - 1 \leq x - 1 \leq 6 - 1
\]
\[
-5 \leq x - 1 \leq 5
\]
This implies:
\( |x - 1| \leq 5 \)

**ANSWER:**
\( |x - 1| \leq 5 \)

24. \( |3x - 2| < 7 \)

**SOLUTION:**
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \(x < -a\) or \(x > a\). This implies that \(|x + c| > a\).

\[
x \leq -4
\]
\[
x - 1 \leq -4 - 1 \text{ or } x - 1 \geq 6 - 1
\]
\[
x \leq -5
\]
This implies:
\( |x - 1| \geq 5 \)

**ANSWER:**
\( |x - 1| \geq 5 \)

25. \( -15 

**SOLUTION:**
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \(-a < x + c < a\). This implies that \(|x + c| < a\).

\[
-12 \leq x \leq -6
\]
\[
-12 + 9 \leq x + 9 \leq -6 + 9
\]
\[
-3 \leq x \leq 3
\]
This implies:
\( |x + 9| \leq 3 \)

**ANSWER:**
\( |x + 9| \leq 3 \)

26. \( -5 

**SOLUTION:**
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \(x < -a\) or \(x > a\). This implies that \(|x + c| > a\).

\[
x < -1 \text{ or } x > 3
\]
\[
x - 1 < -2 \text{ or } x - 1 > 2
\]
This implies:
\( |x - 1| > 2 \)

**ANSWER:**
\( |x - 1| > 2 \)
Solve each inequality. Graph the solution set on a number line.

1. SOLUTION:
The solution of the inequality is the set of points completely inside the box below.

27. \[ -20 -16 -12 -8 -4 0 4 8 12 16 20 \]

SOLUTION:
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \( x < -a \) or \( x > a \). This implies that \( |x + c| > a \).

\[
\begin{align*}
  x &\leq -8 \\
  x - 2 &\leq -10 \\
  x &\geq 12 \\
  x - 2 &\geq 10
\end{align*}
\]

This implies:
\[ |x - 2| \geq 10 \]

ANSWER:
\[ |x - 2| \geq 10 \]

28. \[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]

SOLUTION:
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \( -a < x + c < a \). This implies that \( |x + c| < a \).

\[
\begin{align*}
  -2 &< x < 10 \\
  -2 - 4 &< x - 4 < 10 - 4 \\
  -6 &< x - 4 < 6
\end{align*}
\]

This implies:
\[ |x - 4| < 6 \]

ANSWER:
\[ |x - 4| < 6 \]

29. \[ -5 -4 -3 -2 -1 0 1 2 3 4 5 \]

SOLUTION:
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \( x < -a \) or \( x > a \). This implies that \( |x + c| > a \).

\[
\begin{align*}
  x &< -4 \\
  x + 3 &< -1 \\
  x &> -2 \\
  x + 3 &> 1
\end{align*}
\]

This implies:
\[ |x + 3| > 1 \]

ANSWER:
\[ |x + 3| > 1 \]

30. \[ -10 -8 -6 -4 -2 0 2 4 6 8 10 \]

SOLUTION:
Write a compound inequality from the graph. Then determine what can be added or subtracted from each term so that it is in the form \( -a < x + c < a \). This implies that \( |x + c| < a \).

\[
\begin{align*}
  2 &\leq x \leq 8 \\
  2 - 5 &\leq x - 5 \leq 8 - 5 \\
  -3 &\leq x - 5 \leq 3
\end{align*}
\]

This implies:
\[ |x - 5| \leq 3 \]

ANSWER:
\[ |x - 5| \leq 3 \]
31. **DOGS** The Labrador retriever is one of the most recognized and popular dogs kept as a pet. Using the information given, write a compound inequality to describe the range of healthy weights for a fully grown female Labrador retriever.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Height (in.)</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>22.5–24.5</td>
<td>65–80</td>
</tr>
<tr>
<td>Female</td>
<td>21.5–23.5</td>
<td>55–70</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \( w \) represent the weight of a female Labrador. The weight of a fully grown female Labrador ranges from 55 units to 77 units. So:

\[ 55 \leq w \leq 70 \]

**ANSWER:**
\[ 55 \leq w \leq 70 \]

32. **GEOMETRY** The Exterior Angle Inequality Theorem states that an exterior angle measure is greater than the measure of either of its corresponding remote interior angles. Write two inequalities to express the relationships among the measures of the angles of \( \triangle ABC \).

**SOLUTION:**
Angle 4 is the exterior angle, and angles 1 and 2 are its corresponding remote interior angles. By the Exterior Angle Inequality Theorem:

\[ m\angle 4 > 1, \quad m\angle 4 > 2 \]

**ANSWER:**
\[ m\angle 4 > m\angle 1, \quad m\angle 4 > m\angle 2 \]

---

**1-6 Solving Compound and Absolute Value Inequalities**

Solve each inequality. Graph the solution set on a number line.

33. \[ 28 > 6k + 4 > 16 \]

**SOLUTION:**
\[
\begin{align*}
28 > 6k + 4 & > 16 \\
28 - 4 > 6k + 4 - 4 & > 16 - 4 \\
24 > 6k & > 12 \\
\frac{24}{6} > \frac{6k}{6} & > \frac{12}{6} \\
4 > k & > 2
\end{align*}
\]

The solution set is:

\[ \{ k \mid 2 < k < 4 \} \]

To graph, draw an open circle at 2 and an open circle at 4 and draw a line to connect the circles.
Solve each inequality. Graph the solution set on a number line.

1. \[ m - 7 > -12 \text{ or } -3m + 2 > 38 \]

**SOLUTION:**
\[
m - 7 > -12 \quad \text{or} \quad -3m + 2 > 38
\]
\[
m - 7 > -12 + 7 \\
-3m + 2 > 38 - 2
\]
\[
-3m > 36 \\
m < -12
\]

The solution set is \( \{m \mid m > -5 \text{ or } m < -12\} \).

To graph, draw an open circle at -12 and an arrow extending to the left and an open circle at -5 and an arrow extending to the right.

**ANSWER:**

35. \( | -6h | > 90 \)

**SOLUTION:**
\[
| -6h | > 90
\]
\[
-6h < -90 \quad \text{or} \quad -6h > 90
\]
\[
-\frac{6h}{-6} > \frac{90}{-6} \quad -\frac{6h}{-6} < \frac{90}{-6}
\]
\[
h > 15 \quad h < -15
\]

The solution set is: \( \{h \mid h < -15 \text{ or } h > 15\} \)

To graph, draw an open circle at -15 and an arrow extending to the left and an open circle at 15 and an arrow extending to the right.

**ANSWER:**

36. \( | -5k | > 15 \)

**SOLUTION:**
The absolute value of a number is always non-negative.
So, no value of \( k \) satisfies the inequality.
The solution set is \( \emptyset \).

Since there is no solution, leave the graph blank.

**ANSWER:**

37. \( 3|2z - 4| - 6 > 12 \)

**SOLUTION:**
\[
3|2z - 4| - 6 > 12
\]
\[
3|2z - 4| > 18
\]
\[
|2z - 4| > 6
\]
\[
2z - 4 < -6 \quad \text{or} \quad 2z - 4 > 6
\]
\[
2z < -2 \quad \text{or} \quad 2z > 10
\]
\[
z < -1 \quad \text{or} \quad z > 5
\]

The solution set is:
\( \{z \mid z < -1 \text{ or } z > 5\} \)

To graph, draw an open circle at -1 and an arrow extending to the left and an open circle at 5 and an arrow extending to the right.

**ANSWER:**
38. \( 6 \mid 4p + 2 \mid - 8 < 34 \)

**SOLUTION:**

\[
\begin{align*}
6 \mid 4p + 2 \mid &= 8 < 34 \\
6 \mid 4p + 2 \mid &= 42 \\
|4p + 2| &= 7 \\
-7 &< 4p + 2 < 7 \\
-9 &< 4p < 5 \\
\frac{9}{4} &< p < \frac{5}{4}
\end{align*}
\]

The solution set is:

\[ \left\{ p \mid -\frac{9}{4} < p < \frac{5}{4} \right\} \]

To graph, draw an open circle at \(-\frac{9}{4}\) and an open circle at \(\frac{5}{4}\) and draw a line to connect the circles.

**ANSWER:**

\[ \left\{ p \mid -\frac{9}{4} < p < \frac{5}{4} \right\} \]

---

39. \( \frac{5f - 2}{6} > 4 \)

**SOLUTION:**

\[
\begin{align*}
\frac{5f - 2}{6} &= 4 \\
5f - 2 &= 24 \\
5f &= 26 \\
f &= \frac{26}{5}
\end{align*}
\]

The solution set is:

\[ \left\{ f \mid f > \frac{26}{5}, \text{ or } f < -\frac{22}{5} \right\} \]

To graph, draw an open circle at \(-\frac{22}{5}\) and an arrow extending to the left and an open circle at \(\frac{26}{5}\) and an arrow extending to the right.

**ANSWER:**

\[ \left\{ f \mid f > \frac{26}{5}, \text{ or } f < -\frac{22}{5} \right\} \]
Solve each inequality. Graph the solution set on
a number line.

1. \[
\frac{2w + 8}{5} \geq 3
\]

\textbf{SOLUTION:}

\[
\frac{2w + 8}{5} \geq 3
\]

\[
|2w + 8| \geq 15
\]

\[
2w + 8 \leq -15 \quad \text{or} \quad 2w + 8 \geq 15
\]

\[
w \leq -\frac{23}{2} \quad \text{or} \quad w \geq \frac{7}{2}
\]

The solution set is:

\[
\left\{ w \mid w \leq -\frac{23}{2} \quad \text{or} \quad w \geq \frac{7}{2} \right\}
\]

To graph, draw a solid circle at \(-\frac{23}{2}\) and an arrow extending to the left and a solid circle at \(\frac{7}{2}\) and an arrow extending to the right.

\textbf{ANSWER:}

\[
\left\{ w \mid w \leq -\frac{23}{2} \quad \text{or} \quad w \geq \frac{7}{2} \right\}
\]

Write an algebraic expression to represent each verbal expression.

41. numbers that are at least 4 units from \(-5\)

\textbf{SOLUTION:}

Let \(x\) represent the numbers that are at least 4 units from \(-5\).

So:

\(-9 \leq x \quad \text{or} \quad x \geq -1\)

\(-4 \leq x + 5 \quad \text{or} \quad x + 5 \geq 4\)

This implies:

\[
|x + 5| \geq 4
\]

\textbf{ANSWER:}

\[
|x + 5| \geq 4
\]

42. numbers that are no more than \(\frac{3}{8}\) unit from 1

\textbf{SOLUTION:}

Let \(x\) represent the numbers that are no more than \(\frac{3}{8}\) unit from 1.

So:

\[
1 - \frac{3}{8} \leq x \leq 1 + \frac{3}{8}
\]

\[
\left[ -\frac{3}{8}, \frac{3}{8} \right]
\]

\textbf{ANSWER:}

\[
|x - 1| \leq \frac{3}{8}
\]

43. numbers that are at least 6 units but no more than 10 units from 2

\textbf{SOLUTION:}

Let \(x\) represent the numbers that are at least 6 units but no more than 10 units from 2.

So:

\[
2 - 10 \leq x \leq 2 + 10
\]

\[
|x - 2| \leq 10 \quad \text{or} \quad x \leq 2 - 6
\]

\textbf{ANSWER:}

\[
6 \leq |x - 2| \leq 10
\]

44. AUTO RACING NASCAR rules stipulate that a car must conform to a set of 32 templates, each shaped to fit a different contour of the car. When a template is placed on a car, the gap between it and the car cannot exceed the specified tolerance. Each template is marked on its edge with a colored line that indicates the tolerance for the template.
1-6 Solving Compound and Absolute Value Inequalities

<table>
<thead>
<tr>
<th>Line Color</th>
<th>Tolerance (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>0.07</td>
</tr>
<tr>
<td>Blue</td>
<td>0.25</td>
</tr>
<tr>
<td>Green</td>
<td>0.5</td>
</tr>
</tbody>
</table>

a. Suppose a certain template is 24.42 inches long. Use the information in the table at the right to write an absolute value inequality for templates with each line color.
b. Find the acceptable lengths for that part of a car if the template has each line color.
c. Graph the solution set for each line color on a number line.
d. The tolerance of which line color includes the tolerances of the other line colors? Explain your reasoning.

**SOLUTION:**
a. Let $x$ represent the length for the part of a car.
Red:

\[-0.07 \leq x - 24.42 \leq 0.07\]
\[|x - 24.42| \leq 0.07\]
Blue:

\[-0.25 \leq x - 24.42 \leq 0.25\]
\[|x - 24.42| \leq 0.25\]
Green:

\[-0.5 \leq x - 24.42 \leq 0.5\]
\[|x - 24.42| \leq 0.5\]
b. The acceptable length for the part of the car if the template has red line color:

\[24.42 - 0.07 \leq x \leq 24.42 + 0.07\]
\[24.35 \leq x \leq 24.49\]

The acceptable length for the part of the car if the template has blue line color:

\[24.42 - 0.25 \leq x \leq 24.42 + 0.25\]
\[24.17 \leq x \leq 24.67\]

The acceptable length for the part of the car if the template has blue line color:

\[24.42 - 0.5 \leq x \leq 24.42 + 0.5\]
\[23.92 \leq x \leq 24.92\]
c. Red: $24.35 \leq x \leq 24.49$
Blue: $24.17 \leq x \leq 24.67$
Green: $23.92 \leq x \leq 24.92$
d. Red. The red line color has the smallest tolerance, $0.07 < 0.25 < 0.5$. So, the other line colors would be well within their tolerances.

**ANSWER:**
a. red: $|x - 24.42| \leq 0.07$;
blue: $|x - 24.42| \leq 0.25$;
green: $|x - 24.42| \leq 0.5$
b. red: $24.35 \leq x \leq 24.49$;
blue: $24.17 \leq x \leq 24.67$;
green: $23.92 \leq x \leq 24.92$
c. Red; the red line color has the smallest tolerance, $0.07 < 0.25 < 0.5$ so the other line colors would be well within their tolerances.
1-6 Solving Compound and Absolute Value Inequalities

Solve each inequality. Graph the solution set on a number line.

45. \( n + 6 > 2n + 5 > n - 2 \)

**SOLUTION:**

\[
\begin{align*}
  n + 6 & > 2n + 5 > n - 2 \\
  n + 6 - n & > 2n + 5 - n > n - 2 - n \\
  6 & > n + 5 > -2 \\
  6 - 5 & > n + 5 - 5 > -2 - 5 \\
  1 & > n > -7
\end{align*}
\]

The solution set is:

\[ \{ n | -7 < n < 1 \} \]

To graph, draw an open circle at -7 and an open circle at 1 and draw a line to connect the circles.

**ANSWER:**

\[ \{ n | -7 < n < 1 \} \]

46. \( y + 7 < 2y + 2 < 0 \)

**SOLUTION:**

\[
\begin{align*}
y + 7 & < 2y + 2 \Rightarrow \\
y + 7 & < 2y + 2 \\
y + 7 - y & < 2y + 2 - y \\
7 & < y + 2 \\
5 & < y \\
2y + 2 & < 0 \\
2y + 2 - 2 & < 0 - 2 \\
2y & < -2 \\
y & < -1
\end{align*}
\]

This implies: \( y > 5 \) and \( y < -1 \)

No value of \( y \) satisfies the compound inequality. So the solution set is \( \emptyset \).

Since there is no solution leave the graph blank.

**ANSWER:**

\( \emptyset \)
47. $2x + 6 < 3(x - 1) \leq 2(x + 3)$

**SOLUTION:**

\[
2x + 6 < 3(x - 1) \leq 2(x + 3)
\]
\[
2x + 6 < 3x - 3 \leq 2x + 6
\]
\[
6 < x - 3 \leq 6
\]
\[
9 < x \leq 9
\]

No value of $x$ satisfies the compound inequality. So the solution set is $\emptyset$.
Since there is no solution leave the graph blank.

**ANSWER:**

\[\emptyset\]

48. $a - 16 \leq 2(a - 4) < a + 2$

**SOLUTION:**

\[
a - 16 \leq 2(a - 4) < a + 2
\]
\[
a - 16 \leq 2a - 8 < a + 2
\]
\[
-16 \leq a - 8 < 2
\]
\[
-8 \leq a < 10
\]

The solution set is $\{a | -8 \leq a < 10\}$.

To graph, draw a solid circle at $-8$ and an open circle at $10$ and draw a line to connect the circles.

**ANSWER:**

\[\{a | -8 \leq a < 10\}\]

49. $4g + 8 \geq g + 6$ or $7g - 14 \geq 2g - 4$

**SOLUTION:**

\[
4g + 8 \geq g + 6
\]
\[
7g - 14 \geq 2g - 4
\]
\[
3g + 8 \geq 6
\]
\[
7g - 14 \geq -4
\]
\[
3g \geq -2
\]
\[
5g \geq 10
\]
\[
g \geq \frac{-2}{3}
\]
\[
g \geq 2
\]

The union of the inequalities gives $g \geq \frac{-2}{3}$.
So the solution set of the inequality is:

\[
\{g | g \geq \frac{-2}{3}\}
\]

To graph, draw a solid circle at $\frac{-2}{3}$ and an arrow extending to the right.

**ANSWER:**

\[\{g | g \geq \frac{-2}{3}\}\]

50. $5t + 7 > 2t + 4$ and $3t + 3 < 24 - 4t$

**SOLUTION:**

\[
5t + 7 > 2t + 4
\]
\[
3t + 3 < 24 - 4t
\]
\[
3t + 7 > 4
\]
\[
7t < 21
\]
\[
t > -1
\]
\[
t < 3
\]

That is, $-1 < t < 3$.
The solution set is $\{t | -1 < t < 3\}$.

To graph, draw an open circle at $-1$ and an open circle at $3$ and draw a line to connect the circles.

**ANSWER:**

\[\{t | -1 < t < 3\}\]
51. **HEALTH** Hypoglycemia (low blood sugar) and hyperglycemia (high blood sugar) are potentially dangerous and occur when a person’s blood sugar fluctuates by more than 38 mg from the normal blood sugar level of 88 mg. Write and solve an absolute value inequality to describe blood sugar levels that are considered potentially dangerous.

**SOLUTION:**
Let \( s \) represent the blood sugar levels that are considered dangerous.

\[
\begin{align*}
    s &< 88 - 38 \text{ or } s > 88 + 38 \\
    s - 88 &< -38 \text{ or } s - 88 > 38
\end{align*}
\]

This implies:
\[ |s - 88| > 38 \]

The solution set is \( \{ s \mid s > 126 \text{ or } s < 50 \} \).

**ANSWER:**
\[ |s - 88| > 38; \{ s \mid s > 126 \text{ or } s < 50 \} \]

52. **AIR TRAVEL** The airline on which Drew is flying has weight restrictions for checked baggage. Drew is checking one bag.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 50 lb limit</td>
<td>free</td>
</tr>
<tr>
<td>20 lb over limit</td>
<td>$25</td>
</tr>
<tr>
<td>More than 20, but less than 50 lb over limit</td>
<td>$50</td>
</tr>
<tr>
<td>More than 50 lb over limit</td>
<td>not accepted</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Let \( x \) represent the weight of the bag. There is no charge for the checked baggage if \( x \leq 50 \).

To cost $25, \( x \) should lie between 50 and 70. That is, \( 50 < x < 70 \).

To cost 50, \( x \) should lie between 50 and 100. That is, \( 70 < x < 100 \).

If the baggage is not acceptable, then \( x \) should be greater than 100. That is, \( x > 100 \).

b. \( x = 68 \). So, the cost for the baggage would be $25.

**ANSWER:**

a. \( x \leq 50; 50 < x < 70; 70 < x < 100; x > 100 \)

b. $25
53. **ERROR ANALYSIS** David and Sarah are solving \( 4| -5x - 3| - 6 \geq 34 \). Is either of them correct? Explain your reasoning.

**SOLUTION:**
Sample answer: David is correct; when Sarah converted the absolute value into two inequalities, she mistakenly switched the inequality symbols.

**ANSWER:**
Sample answer: David; when Sarah converted the absolute value into two inequalities, she mistakenly switched the inequality symbols.

54. **CHALLENGE** Solve \(|x - 2| - |x + 2| > x\).

**SOLUTION:**
Use the Alternate Triangle Inequality \(|a - b| \geq |a| - |b|\).

\[
\begin{align*}
|x - 2 - (x + 2)| & \geq |x - 2| - |x + 2| > x \\
|x - 2 - x - 2| & \geq |x - 2| - |x + 2| > x \\
|\ - 4 \ & \geq |x - 2| - |x + 2| > x \\
4 & \geq |x - 2| - |x + 2| > x \\
\Rightarrow \quad x & < |x - 2| - |x + 2| \leq 4
\end{align*}
\]

Case 1: When \( x \geq 0 \) and \( x \leq 4 \):
Substitute the test value for \( x = 1 \).

\[
\begin{align*}
x & < |x - 2| - |x + 2| \\
1 & < |1 - 2| - |1 + 2| \\
& < |\ - 1 \| - 3| \\
& < (1) - (3) \\
& < - 2
\end{align*}
\]

Therefore, \( x \) cannot be positive.
Case 2: When \( x < 0 \):
Substitute the test value for \( x = -1 \).

\[
\begin{align*}
x & < |x - 2| - |x + 2| \\
-1 & < |-1 - 2| - |-1 + 2| \\
& < |-3| - |1| \\
& < (3) - (1) \\
& < 2
\end{align*}
\]

So, the solution is \( x < 0 \).

**ANSWER:**
\( x < 0 \)
1-6 Solving Compound and Absolute Value Inequalities

**REASONING** Determine whether each statement is true or false. If false, provide a counterexample.

55. The graph of a compound inequality involving an and statement is bounded on the left and right by two values of x.

**SOLUTION:**
False; sample answer: the graph of \( x > 2 \) and \( x > 5 \) is a ray bounded only on one end. Only one counterexample is needed to show that this statement is false. While it is true for many cases since it is false for this example, the statement is false.

**ANSWER:**
False; sample answer: the graph of \( x > 2 \) and \( x > 5 \) is a ray bounded only on one end.

56. The graph of a compound inequality involving an or statement contains a region of values that are not solutions.

**SOLUTION:**
False; sample answer: the graph of \( x > 2 \) or \( x < 3 \) includes the entire number line.

**ANSWER:**
False; sample answer: the graph of \( x > 2 \) or \( x < 3 \) includes the entire number line.

57. The graph of a compound inequality involving an and statement includes values that make all parts of the given statement true.

**SOLUTION:**
Since an “and” inequality is true if and only if both inequalities are true, this statement is true.

**ANSWER:**
True

58. **WRITING IN MATH** An alternate definition of absolute value is to define \(|a - b|\) as the distance between \( a \) and \( b \) on the number line. Explain how this definition can be used to solve inequalities of the form \(|x - c| < r\).

**SOLUTION:**
Sample answer: \(|x - c|\) represents the distance between some unknown value of the variable \( x \) and a point \( c \) on the number line. The solution set of the inequality is the set of all numbers such that the distance from the numbers to \( c \) is less than \( r \) units. Use a number line to find the numbers that are \( r \) units from \( c \) in either direction.

**ANSWER:**
Sample answer: \(|x - c|\) represents the distance between some unknown value of the variable \( x \) and a point \( c \) on the number line. The solution set of the inequality is the set of all numbers such that the distance from the numbers to \( c \) is less than \( r \) units. Use a number line to find the numbers that are \( r \) units from \( c \) in either direction.

59. **REASONING** The graphs of the solutions of two different absolute value inequalities are shown. Compare and contrast the absolute value inequalities.

![Graphs of solutions](image)

**SOLUTION:**
Sample answer: The graph on the left indicates a solution set from \(-3\) to \(5\). The graph on the right indicates a solution set of all numbers less than or equal to \(-3\) or greater than or equal to \(5\). The graph on the left is of an and inequality while the graph on the right is of an or inequality.

**ANSWER:**
Sample answer: The graph on the left indicates a solution set from \(-3\) to \(5\). The graph on the right indicates a solution set of all numbers less than or equal to \(-3\) or greater than or equal to \(5\).

60. **OPEN ENDED** Write an absolute value inequality with a solution of \( a \leq x \leq b \).

**SOLUTION:**
1-6 Solving Compound and Absolute Value Inequalities

Sample answer: first graph \( a \) and \( b \) on a number line. The solution of the inequality is the set of points between \( a \) and \( b \), inclusive. Mark the midpoint.

\[
\begin{array}{c|c}
\text{ } & \frac{a + b}{2} \\
\hline
a & b
\end{array}
\]

The distance between points \( a \) or \( b \) and the center is \( \frac{a + b}{2} \). So the absolute value inequality will be \( |x - \frac{a + b}{2}| \leq \frac{b - a + b}{2} \). Since the absolute value inequality had the solution \( a < x < b \), the right side of the inequality must be \( b - \frac{a + b}{2} \).

Check this inequality.

\[
|x - \frac{a + b}{2}| \leq b - \frac{a + b}{2}
\]

\[
x - \frac{a + b}{2} \leq b - \frac{a + b}{2}
\]

\[
x \leq b
\]

or

\[
x - \frac{a + b}{2} \leq -\left[b - \frac{a + b}{2}\right]
\]

\[
x - \frac{a + b}{2} \leq -b + \frac{a + b}{2}
\]

\[
x \leq -b + 2\left(\frac{a + b}{2}\right)
\]

\[
x \leq -b + a + b
\]

\[
x \leq a
\]

Therefore the inequality is \( |x - \frac{a + b}{2}| \leq b - \frac{a + b}{2} \).

**ANSWER:**
Sample answer:

\[
|x - \frac{a + b}{2}| \leq b - \frac{a + b}{2}
\]

61. WHICH ONE DOESN'T BELONG? Identify the compound inequality that is not the same as the other three. Explain your reasoning.

\[
\begin{array}{c|c|c}
-3 < x < 5 & x > 2 \text{ and } x < 3 & x > 5 \text{ and } x < 1
\end{array}
\]

\[
\begin{array}{c|c|c}
x > 1 & x > -4 \text{ and } x > -2 & x > -4 \text{ and } x > -2
\end{array}
\]

**SOLUTION:**
Each of these has a non-empty solution set except for \( x > 5 \) and \( x < 1 \). There are no values of \( x \) that are simultaneously greater than 5 and less than 1.

**ANSWER:**
Each of these has a non-empty solution set except for \( x > 5 \) and \( x < 1 \). There are no values of \( x \) that are simultaneously greater than 5 and less than 1.

62. WRITING IN MATH Summarize the difference between and compound inequalities and or compound inequalities.

**SOLUTION:**
Sample answer: A compound inequality that contains and is true if and only if both individual inequalities are true, for example, \( -3 < x < 1 \), while an inequality containing or only needs one of the individual inequalities to be true, for example, \( x < -3 \) or \( x > 1 \).

**ANSWER:**
Sample answer: A compound inequality that contains and is true if and only if both individual inequalities are true, while an inequality containing or only needs one of the individual inequalities to be true.

63. Which of the following best describes the graph of the equations below?

\[
24y = 8x + 11
\]

\[
36y = 12x + 11
\]

A The lines have the same \( x \)-intercept.
B The lines have the same \( y \)-intercept.
C The lines are parallel.
D The lines are perpendicular.

**SOLUTION:**
The slopes of the lines are equal, so the lines are parallel. The correct choice is C.

**ANSWER:**
C
1-6 Solving Compound and Absolute Value Inequalities

64. SAT/ACT Find an expression equivalent
to \( \left( \frac{3x^3}{y} \right)^3 \)

\[ F \quad \frac{9x^6}{3y} \]
\[ G \quad \frac{9x^9}{y^3} \]
\[ H \quad \frac{9x^6}{3y^3} \]
\[ J \quad \frac{27x^6}{3y} \]
\[ K \quad \frac{27x^9}{y^3} \]

**SOLUTION:**

\[
\left( \frac{3x^3}{y} \right)^3 = \left( \frac{3^3 \cdot (x^3)^3}{y^3} \right) = \frac{27x^9}{y^3}
\]

The correct choice is K.

**ANSWER:**
K

65. GRIDDED RESPONSE How many cubes that measure 4 centimeters on each side can be placed completely inside the box below?

![Box Diagram](image)

**SOLUTION:**

The volume of a rectangular prism is given by

\[ V = \ell w h \]

where \( \ell \) is the length, \( w \) is the width, and \( h \) is the height.

Box:

\[ \ell = 20 \text{ cm}, \quad w = 16 \text{ cm}, \quad h = 12 \text{ cm}. \]

Volume of the box:

\[ V_1 = 20 \times 16 \times 12 = 3840 \text{ cm}^3 \]

Volume of the cube:

\[ V_2 = 4 \times 4 \times 4 = 64 \text{ cm}^3 \]

The number of cubes that can be placed inside the box is give by \( \frac{V_1}{V_2} \).

Number of cubes \( = \frac{3840}{64} = 60 \).

**ANSWER:**
60
66. Which graph represents the solution set for \(|3x - 6| + 8 \geq 17? \)

\[|3x - 6| + 8 \geq 17\]
\[|3x - 6| \geq 9\]
\[3x - 6 \leq -9\] or \[3x - 6 \geq 9\]
\[x \leq -1\] or \[x \geq 5\]

The correct choice is A.

\textbf{ANSWER:} A

67. **HEALTH** The National Heart Association recommends that less than 30% of a person’s total daily caloric intake come from fat. One gram of fat yields nine Calories. Consider a healthy 21-year-old whose average caloric intake is between 2500 and 3300 Calories.

\textbf{a.} Write an inequality that represents the suggested fat intake for the person.

\textbf{b.} What is the greatest suggested fat intake for the person?

\textbf{SOLUTION:}
\textbf{a.} Let \(x\) represent the intake of fat in Calories.
\[30\% (2500) \leq x \leq 30\% (3300)\]
\[750 \leq x \leq 990\]

\textbf{b.} The maximum fat intake is 990 Calories.

One gram of fat yields nine Calories.

So, the greatest fat intake for the person is 110 grams.

\textbf{ANSWER:}
\textbf{a.} 750 \leq x \leq 990
\textbf{b.} 110g

68. **TRAVEL** Maggie is planning a 5-day trip to a family reunion. She wants to spend no more than $1000. Her plane ticket is $375, and the hotel is $85 per night.

\textbf{a.} Let \(f\) represent the cost of food for one day. Write an inequality to represent this situation.

\textbf{b.} Solve the inequality and interpret the solution.

\textbf{SOLUTION:}
\textbf{a.} The cost of the food for one day is $f$.

For 5 days, the cost of the food is $5f$.

The plane ticket costs $375.

For a 5-day stay in the hotel, it costs $425.

Total cost for a 5-day trip = $375 + $375 + $425 = 800 + 5f$.

Maggie wants to spend no more than $1000.

Therefore:

\[800 + 5f \leq 1000\]

\[b.\]

\[800 + 5f \leq 1000\]

\[5f \leq 200\]

\[f \leq 40\]

That is, Maggie can spend no more than $40 per day on food.

\textbf{ANSWER:}
\textbf{a.} 800 + 5f \leq 1000
\textbf{b.} She can spend no more than $40 per day on food.
1-6 Solving Compound and Absolute Value Inequalities

Solve each equation. Check your solutions.

69. \(4|\!x - 5| = 20\)

**SOLUTION:**

\[
4|\!x - 5| = 20
\]

\[
|\!x - 5| = 5
\]

\[
x - 5 = 5 \text{ or } -(x - 5) = 5
\]

\[
x = 10 \text{ or } x = 0
\]

Substitute the values in the equation:

- \(4|0 - 5| = 20\)
  - \(4(5) = 20\)
  - \(20 = 20 \checkmark\)

- \(4|10 - 5| = 20\)
  - \(4(5) = 20\)
  - \(20 = 20 \checkmark\)

The solution set is \(\{0, 10\}\).

**ANSWER:**

\(\{0, 10\}\)

70. \(|3y + 10| = 25\)

**SOLUTION:**

\[
|3y + 10| = 25
\]

\[
3y + 10 = 25 \text{ or } -(3y + 10) = 25
\]

\[
3y = 15 \text{ or } -3y = 35
\]

\[
y = 5 \text{ or } y = -\frac{35}{3}
\]

Substitute the values in the equation.

- \(|3\left(-\frac{35}{3}\right) + 10| = 25\)
  - \(|-35 + 10| = 25\)
  - \(25 = 25 \checkmark\)

- \(|3(5) + 10| = 25\)
  - \(|15 + 10| = 25\)
  - \(25 = 25 \checkmark\)

The solution set is \(\left\{-\frac{35}{3}, 5\right\}\).

**ANSWER:**

\(\left\{-\frac{35}{3}, 5\right\}\)

71. \(|7z + 8| = -9\)

**SOLUTION:**

The absolute value of a number cannot be negative. So there is no solution for the equation \(|7z + 8| = -9\).

The solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)
Name the property illustrated by each statement.

72. If \(5x = 7\), then \(5x + 3 = 7 + 3\).

\textbf{SOLUTION:}
Addition Property of Equality; the Addition Property of Equality states that for any real numbers \(a, b,\) and \(c,\) if \(a = b,\) then \(a + c = b + c.\)

\textbf{ANSWER:}
Addition (=)

73. If \(-3x + 9 = 11\) and \(6x + 2 = 11,\) then \(-3x + 9 = 6x + 2.\)

\textbf{SOLUTION:}
Transitive Property; the Transitive Property of Equality states that for any real numbers \(a, b,\) and \(c,\) if \(a = b\) and \(b = c,\) then \(a = c.\)

\textbf{ANSWER:}
Transitive (=)

74. If \(x + (-2) + (-4) = 5,\) then \(x + [-2 + (-4)] = 5.\)

\textbf{SOLUTION:}
Associative Property of Addition; the Associate Property of Addition states that the way in which the terms are grouped does not affect the sum.

\textbf{ANSWER:}
Assoc. (+)
1. Evaluate \( x + y^2 (2 + x) \) if \( x = 3 \) and \( y = -1 \).

\[
\text{SOLUTION:} \\
x + y^2 (2 + x) = 3 + (-1)^2 (2 + 3) \\
= 3 + (1)(5) \\
= 3 + 5 \\
= 8
\]

\*

\text{ANSWER:} 8

2. Simplify \(-4(3a + b) - 2(a - 5b)\).

\[
\text{SOLUTION:} \\
-4(3a + b) - 2(a - 5b) \\
= -4(3a) + (-4)(b) + (-2)(a) + (-2)(-5b) \\
= -12a - 4b - 2a + 10b \\
= -12a - 2a - 4b + 10b \\
= (-12 - 2)a + (-4 + 10)b \\
= -14a + 6b
\]

\*

\text{ANSWER:} -14a + 6b

3. \text{MULTIPLE CHOICE} If \( 3m + 5 = 23 \), what is the value of \( 2m - 3 \)?

\begin{align*}
\text{A} & \quad 105 \\
\text{B} & \quad 9 \\
\text{C} & \quad \frac{47}{3} \\
\text{D} & \quad 6
\end{align*}

\[
\text{SOLUTION:} \\
3m + 5 = 23 \\
3m + 5 - 5 = 23 - 5 \\
3m = 18 \\
\frac{3m}{3} = \frac{18}{3} \\
m = 6
\]

Substitute \( m = 6 \) in \( 2m - 3 \).

\[
2m - 3 = 2(6) - 3 \\
= 12 - 3 \\
= 9
\]

So, the correct choice is B.

\*

\text{ANSWER: B}

4. Solve \( r = \frac{1}{2} m^2 p \) for \( p \).

\[
\text{SOLUTION:} \\
r = \frac{1}{2} m^2 p \\
2r = 2 \left( \frac{1}{2} m^2 p \right) \\
2r = m^2 p \\
\frac{2r}{m^2} = \frac{m^2 p}{m^2} \\
\frac{2r}{m^2} = p
\]

\*

\text{ANSWER:} \quad p = \frac{2r}{m^2}
1. Evaluate if \( x = 3 \) and \( y = -1 \).

**SOLUTION:**
Let \( n \) be a number.
The words *twice the difference of a number and 11* represents the expressions \( 2(n - 11) \).

**ANSWER:**
\( 2(n - 11) \)

2. Simplify \( \frac{8a - 5b}{2} \).

**SOLUTION:**
\( \frac{8a - 5b}{2} = 4a - \frac{5}{2}b \)

**ANSWER:**
\( -14a + 6b \)

---

8. Solve \(-2b > \frac{18 - b}{5}\). Graph the solution set on a number line.

**SOLUTION:**
\[-2b > \frac{18 - b}{5} \]
\[5(-2b) > 5 \left( \frac{18 - b}{5} \right) \]
\[-10b > 18 - b \]
\[-10b + b > 18 - b + b \]
\[-9b > 18 \]
\[-9 < \frac{18}{-9} \]
\[ b < -2 \]

To graph this inequality, draw an open circle at \(-2\) and draw an arrow extending to the left.

---

9. **MONEY** Carson has \$35 to spend at the water park. The admission price is \$25 and each soda is \$2.50. Write an inequality to show how many sodas he can buy.

**SOLUTION:**
Let \( s \) represent the number of sodas he can buy.
\[ 35 \geq 25 + 2.50s \]

**ANSWER:**
\[ 35 \geq 25 + 2.50s \]
10. Solve \( r - 3 < -5 \) or \( 4r + 1 > 15 \). Graph the solution set.

\[ \begin{align*}
\text{SOLUTION:} & \quad 4r + 1 > 15 \\
& \quad 4r + 1 - 1 > 15 - 1 \\
& \quad r - 3 < -5 \quad \text{or} \quad 4r > 14 \\
& \quad r < -2 \\
& \quad r > \frac{7}{2}
\end{align*} \]

So, the solution of the inequalities is \( \left\{ r \mid r < -2 \text{ or } r > \frac{7}{2} \right\} \).

To graph this inequality, draw an open circle at \(-2\) with an arrow extending to the left and an open circle at \(\frac{7}{2}\) with an arrow extending to the right.

\[ \begin{array}{c}
\text{ANSWER:} \\
\left\{ r \mid r < -2 \text{ or } r > \frac{7}{2} \right\}
\end{array} \]

11. Solve \( |p - 4| \leq 11 \). Graph the solution set on a number line.

\[ \begin{align*}
\text{SOLUTION:} & \quad |p - 4| \leq 11 \\
& \quad -11 \leq p - 4 \leq 11 \\
& \quad -11 + 4 \leq p - 4 + 4 \leq 11 + 4 \\
& \quad -7 \leq p \leq 15
\end{align*} \]

The solution set is \( \left\{ p \mid -7 \leq p \leq 15 \right\} \).

To graph this inequality, draw solid circles at \(-7\) and \(15\) and connect them with a solid line segment.

\[ \begin{array}{c}
\text{ANSWER:} \\
\left\{ p \mid -7 \leq p \leq 15 \right\}
\end{array} \]
12. **MULTIPLE CHOICE** Which graph represents the solution set for $4 < 6t + 1 \leq 43$?

**SOLUTION:**

$4 < 6t + 1 \leq 43$

$4 - 1 < 6t + 1 - 1 \leq 43 - 1$

$3 < 6t \leq 42$

$\frac{3}{6} < t \leq \frac{42}{6}$

$\frac{1}{2} < t \leq 7$

The graph in the choice **F** represents the solution set $\{ t \mid \frac{1}{2} < t \leq 7 \}$. So, the correct choice is **F**.

**ANSWER:**

**F**

13. **MONEY** Sofia is buying new skis. She finds that the average price of skis is $500 but the actual price could differ from the average by as much as $250. Write and solve an absolute value inequality to describe this situation.

**SOLUTION:**

Let $p$ represent the actual price. The absolute value inequality representing the situation is $|p - 500| \leq 250$.

Solve the absolute value inequality.

$|p - 500| \leq 250$

$-250 \leq p - 500 \leq 250$

$-250 + 500 \leq p - 500 + 500 \leq 250 + 500$

$250 \leq p \leq 750$

**ANSWER:**

$|p - 500| \leq 250; 250 \leq p \leq 750$

14. **GARDENING** Andy is making 3 trapezoidal garden boxes for his backyard. Each trapezoid will be the size of the trapezoid below. He will place stone blocks around the borders of the boxes. How many feet of stones will Andy need?

**SOLUTION:**

The perimeter of each stone block = $(8 + 7 + 12 + 7)$ ft.

Therefore, perimeter of 3 stone blocks = $3(8 + 7 + 12 + 7)$ ft.

$3(8 + 7 + 12 + 7) = 3(8) + 3(7) + 3(12) + 3(7)$

$= 24 + 21 + 36 + 21$

$= 102$

So, Andy will need 102 feet of stones.

**ANSWER:**

102 ft

**Solve each equation.**

15. $| x + 4 | = 3$

**SOLUTION:**

Case 1: $x + 4 = 3$

$x + 4 = -3$

$x = -1$

Case 2: $x + 4 = -3$

$x + 4 = -3$

$x = -7$

There appear to be two solutions, $-1$ and $-7$.

Check: Substitute each value in the original equation.

$| x + 4 | = 3$

$| -1 + 4 | = 3$

$|3| = 3$

$3 = 3 \checkmark$

The solution set is $\{ -7, -1 \}$.

**ANSWER:**

$\{ -7, -1 \}$
16. $|3m + 2| = 1$

**SOLUTION:**

Case 1: 

$3m + 2 = 1$

$3m = -1$

$m = -\frac{1}{3}$

Case 2: 

$3m + 2 = -1$

$3m = -3$

$m = -1$

There appear to be two solutions, $-\frac{1}{3}$ and $-1$.

Check: Substitute each value in the original equation.

$|3m + 2| = 1$

$|\frac{-1}{3} + 2| = 1$

$|\frac{5}{3}| = 1$

$1 = 1 \checkmark$

$|3(-1) + 2| = 1$

$|-1 + 2| = 1$

$|1| = 1$

$1 = 1 \checkmark$

The solution set is $\left\{-1, -\frac{1}{3}\right\}$.

**ANSWER:**

$\left\{-1, -\frac{1}{3}\right\}$

17. $|3a + 2| = -4$

**SOLUTION:**

Since the absolute value of $|3a + 2|$ cannot be negative, the solution set is $\emptyset$.

**ANSWER:**

$\emptyset$

18. $|2t + 5| - 7 = 4$

**SOLUTION:**

$|2t + 5| - 7 = 4$

$|2t + 5| = 11$

Case 1: 

$2t + 5 = 11$

$2t = 6$

$t = 3$

Case 2: 

$2t + 5 = -11$

$2t = -16$

$t = -8$

There appear to be two solutions, 3 and $-8$.

Check: Substitute each value in the original equation.

$|2t + 5| - 7 = 4$

$|2(3) + 5| - 7 = 4$

$|11| = 4$

$4 = 4 \checkmark$

$|2(-8) + 5| - 7 = 4$

$|-11| = 4$

$4 = 4 \checkmark$

The solution set is $\{3, -8\}$.

**ANSWER:**

$\{3, -8\}$
19. \( |5n - 2| - 6 = -3 \)

**SOLUTION:**
\[
|5n - 2| - 6 = -3
\]
\[
|5n - 2| - 6 + 6 = -3 + 6
\]
\[
5n - 2 = 3
\]

Case 1:
\[
5n - 2 = 3
\]
\[
5n = 5
\]
\[
\frac{5n}{5} = \frac{5}{5}
\]
\[
n = 1
\]

Case 2:
\[
5n - 2 = -3
\]
\[
5n = -1
\]
\[
\frac{5n}{5} = \frac{-1}{5}
\]
\[
n = -\frac{1}{5}
\]

There appear to be two solutions, 1 and \(-\frac{1}{5}\). Check: Substitute each value in the original equation.
\[
|5(1) - 2| - 6 = -3 \quad |5(-\frac{1}{5}) - 2| - 6 = -3
\]
\[
|3| - 6 = -3 \quad |-2| - 6 = -3
\]
\[
3 - 6 = -3 \quad 3 - 6 = -3
\]
\[
-3 = -3 \checkmark \quad -3 = -3 \checkmark
\]

The solution set is \(\left\{-\frac{1}{5}, 1\right\}\).

**ANSWER:**
\[
\left\{-\frac{1}{5}, 1\right\}
\]

20. \( |p + 6| + 9 = 8 \)

**SOLUTION:**
\[
|p + 6| + 9 = 8
\]
\[
|p + 6| + 9 - 9 = 8 - 9
\]
\[
|p + 6| = -1
\]

Since the absolute value of any number cannot be negative, the solution set is \(\emptyset\).

**ANSWER:**
\[
\emptyset
\]

21. **GEOMETRY** The volume of a cylinder is given by the formula \(V = \pi r^2 h\). What is the volume of the cylinder below?

![Cylinder Diagram](image)

**SOLUTION:**
Radius \(r\) of the cylinder is 6 cm. Substitute \(r = 6\), and \(h = 9\) in the formula \(V = \pi r^2 h\).
\[
V = \pi r^2 h
\]
\[
= \pi (6)^2 (9)
\]
\[
= (36)(9)\pi
\]
\[
= 324\pi
\]
\[
\approx 1017.88
\]

The volume of the cylinder is about 1017.88 cubic centimeters.

**ANSWER:**
about 1017.88 cm³
22. Solve \(-3b - 5 \geq -6b - 13\). Graph the solution set on a number line.

**SOLUTION:**
\[
\begin{align*}
-3b - 5 & \geq -6b - 13 \\
-3b - 5 + 5 & \geq -6b - 13 + 5 \\
-3b & \geq -6b - 8 \\
-3b + 6b & \geq -6b - 8 + 6b \\
3b & \geq -8 \\
\frac{3b}{3} & \geq \frac{-8}{3} \\
b & \geq \frac{-8}{3}
\end{align*}
\]
To graph this inequality, draw a solid circle at \(-\frac{8}{3}\) and draw an arrow extending to the right.

**ANSWER:**
\[b \geq \frac{-8}{3}\]

23. Evaluate \(\frac{3(x + y)}{4xy^2}\) if \(x = \frac{2}{3}\) and \(y = -2\).

**SOLUTION:**
\[
\begin{align*}
3(x + y) & = 3 \left(\frac{2}{3} + (-2)\right) \\
4xy^2 & = 4 \left(\frac{2}{3}\right)(-2)^2 \\
3 \left(\frac{2}{3} + (-2)\right) & = \frac{3}{4} \left(\frac{2}{3}\right)(-2)^2 \\
& = \frac{3}{4} \left(\frac{2}{3}\right)(4) \\
& = \frac{3}{4} \left(\frac{2 - 2(3)}{3}\right) \\
& = \frac{3}{4} \left(\frac{2 - 6}{3}\right) \\
& = \frac{3}{4} \left(\frac{2 - 6}{3}\right) \\
& = \frac{3}{4} \left(\frac{2 - 6}{3}\right) \\
& = \frac{3}{4} \left(\frac{12}{3}\right) \\
& = \frac{3}{4} \left(\frac{12}{3}\right) \\
& = \frac{3}{4} \left(\frac{3}{32}\right) \\
& = \frac{3}{8}
\end{align*}
\]

**ANSWER:**
\[\frac{3}{8}\]
24. Name the set(s) of numbers to which $-\frac{1}{3}$ belongs.

**SOLUTION:**

The number $-\frac{1}{3}$ is a real number. Since $-\frac{1}{3}$ can be expressed as a ratio $\frac{a}{b}$ where $a$ and $b$ are integers and $b$ is not 0 it is also a rational number. It is not a part of the set {...-2, -1, 0, 1, 2, ...} so it is not an integer. Since it is not a part of the set {...0, 1, 2, 3, ...} it is not a whole number or a natural number.

Q, R

**ANSWER:**

Q, R

25. **MONEY** The costs for making necklaces at two craft stores are shown in the table. For what quantity of beads does The Accessory Store have a better deal? Use the inequality $15 + 3.25b < 20 + 2.50b$.

<table>
<thead>
<tr>
<th>Shop</th>
<th>Cost per Chain</th>
<th>Cost per Bead</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Accessory Store</td>
<td>$15</td>
<td>$1.25</td>
</tr>
<tr>
<td>Finishing Touch</td>
<td>$20</td>
<td>$2.50</td>
</tr>
</tbody>
</table>

**SOLUTION:**

\[
15 + 3.25b < 20 + 2.50b \\
15 + 3.25b - 15 < 20 + 2.50b - 15 \\
3.25b < 2.50b + 5 \\
3.25b - 2.50b < 2.50b + 5 - 2.50b \\
0.75b < 5 \\
\frac{0.75b}{0.75} < \frac{5}{0.75} \\
b < 6.66
\]

When you buy 6 or fewer beads, The Accessory Store is a better deal.

**ANSWER:**

When you buy 6 or fewer beads, The Accessory Store is a better deal.
State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. The absolute value of a number is always negative.
   
   **SOLUTION:**
   The absolute value of a number is always nonnegative. So, the sentence is false.
   
   **ANSWER:**
   false; nonnegative
   
2. \(\sqrt{12}\) belongs to the set of rational numbers.
   
   **SOLUTION:**
   The sentence is false because \(\sqrt{12}\) belongs to the set of irrational numbers.
   
   **ANSWER:**
   false; irrational
   
3. An equation is a statement that two expressions have the same value.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
   
4. A solution of an equation is a value that makes the equation false.
   
   **SOLUTION:**
   A solution of an equation is a value that makes the equation true. So, the sentence is false.
   
   **ANSWER:**
   false; true
   
5. The empty set contains no elements.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
   
6. A mathematical sentence containing one or more variables is called an open sentence.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
   
7. The graph of a compound inequality containing and is the union of the solution sets of the two inequalities.
   
   **SOLUTION:**
   The graph of a compound inequality containing or is the union of the solution sets of the two inequalities. So, the sentence is false.
   
   **ANSWER:**
   false; or
   
8. Variables are used to represent unknown quantities.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
   
9. The set of rational numbers includes terminating and repeating decimals.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
   
10. Expressions that contain at least one variable are called algebraic expressions.
   
   **SOLUTION:**
   The sentence is true.
   
   **ANSWER:**
   true
Evaluate each expression.

11. \([28 - (16 + 3)] ÷ 3\)

\[
\text{SOLUTION:} \\
\left[28 - (16 + 3)\right] ÷ 3 = [28 - 19] ÷ 3 \\
= 9 ÷ 3 \\
= 3
\]

\text{ANSWER: 3}

12. \(\frac{2}{3}(3^3 + 12)\)

\[
\text{SOLUTION:} \\
\frac{2}{3}(3^3 + 12) = \frac{2}{3}(27 + 12) \\
= \frac{2}{3}(39) \\
= 2(13) \\
= 26
\]

\text{ANSWER: 26}

13. \(\frac{15(9 - 7)}{3}\)

\[
\text{SOLUTION:} \\
\frac{15(9 - 7)}{3} = \frac{15(2)}{3} \\
= \frac{30}{3} \\
= 10
\]

\text{ANSWER: 10}

Evaluate each expression if \(w = 0.2, x = 10, y = \frac{1}{2}, \text{and } z = -4\)

14. \(4w - 8y\)

\[
\text{SOLUTION:} \\
4w - 8y = 4(0.2) - 8\left(\frac{1}{2}\right) \\
= 0.8 - 4 \\
= -3.2
\]

\text{ANSWER: -3.2}

15. \(z^2 + xy\)

\[
\text{SOLUTION:} \\
z^2 + xy = (-4)^2 + (10)\left(\frac{1}{2}\right) \\
= 16 + 5 \\
= 21
\]

\text{ANSWER: 21}

16. \(\frac{5w - xy}{z}\)

\[
\text{SOLUTION:} \\
\frac{5w - xy}{z} = \frac{5(0.2) - (10)\left(\frac{1}{2}\right)}{-4} \\
= \frac{1 - 5}{-4} \\
= \frac{-4}{-4} \\
= 1
\]

\text{ANSWER: 1}
17. GEOMETRY The formula for the volume of a cylinder is \( V = \pi r^2 h \), where \( V \) is volume, \( r \) is radius, and \( h \) is the height. What is the volume of a cylinder that is 6 inches high and has a radius of 3 inches?

SOLUTION:
\[
V = \pi r^2 h \\
V = \pi (3)^2 (6) \\
= \pi (9)(6) \\
= 54\pi \\
\approx 169.65
\]

The volume of the cylinder is about 169.65 cubic inches.

ANSWER:
\[ \approx 169.65 \text{ in}^3 \]

Name the sets of numbers to which each value belongs.

18. \( 1.3 \)

SOLUTION:
The number \( 1.3 \) is a real number. Since \( 1.3 \) can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is not a part of the set \{…\(-2, -1, 0, 1, 2, \ldots\)\} so it is not an integer. Since it is not a part of the set \{…0, 1, 2, 3, \ldots\} it is not a whole number or a natural number.

Q, R

ANSWER:
Q, R

19. \( \sqrt{4} \)

SOLUTION:
The number \( \sqrt{4} = 2 \) which is a real number. Since 2 can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is part of the set \{…\(-2, -1, 0, 1, 2, \ldots\)\} so it is an integer. It is part of the set \{…0, 1, 2, 3, \ldots\} so it is a whole number and since it is not 0 it is also a natural number.

N, W, Z, Q, R

ANSWER:
N, W, Z, Q, R

20. \( -\frac{3}{4} \)

SOLUTION:
The number \( -\frac{3}{4} \) is a real number. Since \( -\frac{3}{4} \) can be expressed as a ratio \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \) is not 0 it is also a rational number. It is not a part of the set \{…\(-2, -1, 0, 1, 2, \ldots\)\} so it is not an integer. Since it is not a part of the set \{…0, 1, 2, 3, \ldots\} it is not a whole number or a natural number.

Q, R

ANSWER:
Q, R

Simplify each expression.

21. \( 4x - 3y + 7x + 5y \)

SOLUTION:
\[
4x - 3y + 7x + 5y = 4x + 7x - 3y + 5y \\
= (4 + 7)x + (-3 + 5)y \\
= 11x + 2y
\]

ANSWER:
11x + 2y
22. \(2(a + 3) - 4a + 8b\)

**SOLUTION:**
\[
2(a + 3) - 4a + 8b = 2(a) + 2(3) - 4a + 8b \\
= 2a + 6 - 4a + 8b \\
= 2a - 4a + 8b + 6 \\
= (2 - 4)a + 8b + 6 \\
= -2a + 8b + 6
\]

**ANSWER:**
\(-2a + 8b + 6\)

23. \(4(2m + 5n) - 3(m - 7n)\)

**SOLUTION:**
\[
4(2m + 5n) - 3(m - 7n) \\
= 4(2m) + 4(5n) + (-3)(m) + (-3)(-7n) \\
= 8m + 20n - 3m + 21n \\
= 8m - 3m + 20n + 21n \\
= (8 - 3)m + (20 + 21)n \\
= 5m + 41n
\]

**ANSWER:**
\(5m + 41n\)

24. **MONEY** At Fun City Amusement Park, hot dogs sell for $3.50 and sodas sell for $2.50. Dion bought 3 hot dogs and 3 sodas during one day at the park.

a. Illustrate the Distributive Property by writing two expressions to represent the cost of the hot dogs and the sodas.

b. Use the Distributive Property to find how much money Dion spent on food and drinks.

**SOLUTION:**

a. Since Dion bought 3 of each, you can write the expression in two ways. Either add the costs of 1 hot dog and 1 soda together and multiply by 3 or multiply the cost of each item by 3 and then add. The expressions are: \(3(3.50 + 2.50)\) or \(3(3.50) + 3(2.50)\).

b. \[
3(3.50 + 2.50) = 3(3.50) + 3(2.50) \\
= 10.50 + 7.50 \\
= 18
\]

Dion spent $18 on food and drinks.

**ANSWER:**

a. \(3(3.50 + 2.50)\) or \(3(3.50) + 3(2.50)\)
b. $18

---

25. \(8 + 5r = -27\)

**SOLUTION:**
\[
8 + 5r = -27 \\
8 + 5r - 8 = -27 - 8 \\
5r = -35 \\
\frac{5r}{5} = \frac{-35}{5} \\
r = -7
\]

Check:
\[
8 + 5(-7) = -27 \\
8 - 35 = -27 \\
-27 = -27 \checkmark
\]

So, the solution of the equation is \(r = -7\).

**ANSWER:**
\(-7\)
26. \(4w + 10 = 6w - 13\)

**SOLUTION:**

\[
\begin{align*}
4w + 10 &= 6w - 13 \\
4w + 10 - 10 &= 6w - 13 - 10 \\
4w &= 6w - 23 \\
4w - 6w &= 6w - 23 - 6w \\
-2w &= -23 \\
\frac{-2w}{-2} &= \frac{-23}{-2} \\
w &= \frac{23}{2}
\end{align*}
\]

Check:

\[
\begin{align*}
4\left(\frac{23}{2}\right) + 10 &= 6\left(\frac{23}{2}\right) - 13 \\
2(23) + 10 &= 3(23) - 13 \\
46 + 10 &= 69 - 13 \\
56 &= 56 \checkmark
\end{align*}
\]

So, the solution of the equation is \(w = \frac{23}{2}\).

**ANSWER:**

\[
\frac{23}{2}
\]

27. \(\frac{x}{6} + \frac{x}{3} = \frac{3}{4}\)

**SOLUTION:**

\[
\begin{align*}
x + x(2) &= \frac{3}{4} \\
\frac{x}{6} &= \frac{3}{4} \\
\frac{3x}{6} &= \frac{3}{4} \\
\frac{x}{2} &= \frac{3}{4} \\
2\left(\frac{x}{2}\right) &= 2\left(\frac{3}{4}\right) \\
x &= \frac{3}{2}
\end{align*}
\]

Check:

\[
\begin{align*}
\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) &= \frac{3}{4} \\
3\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right) &= \frac{3}{4} \\
\frac{1}{4} + \frac{1}{2} &= \frac{3}{4} \\
\frac{1 + 1(2)}{4} &= \frac{3}{4} \\
\frac{1 + 2}{4} &= \frac{3}{4} \\
\frac{3}{4} &= \frac{3}{4} \checkmark
\end{align*}
\]

So, the solution of the equation is \(x = \frac{3}{2}\).

**ANSWER:**

\[
\frac{3}{2}
\]
28. \(6b - 5 = 3(b + 2)\)

**SOLUTION:**

\[
\begin{align*}
6b - 5 &= 3(b + 2) \\
6b - 5 &= 3b + 6 \\
6b - 5 + 5 &= 3b + 6 + 5 \\
6b &= 3b + 11 \\
6b - 3b &= 3b + 11 - 3b \\
3b &= 11 \\
\frac{3b}{3} &= \frac{11}{3} \\
b &= \frac{11}{3}
\end{align*}
\]

Check:

\[
\begin{align*}
6 \left( \frac{11}{3} \right) - 5 &= 3 \left( \frac{11}{3} + 2 \right) \\
2(11) - 5 &= 3 \left( \frac{11 + 2(3)}{3} \right) \\
22 - 5 &= 3 \left( \frac{11 + 6}{3} \right) \\
\frac{17}{3} &= \frac{17}{3} \\
17 &= 17 \checkmark
\end{align*}
\]

So, the solution of the equation is \(b = \frac{11}{3}\).

**ANSWER:**

\[
\frac{11}{3}
\]

29. **MONEY** It cost Lori $14 to go to the movies. She bought popcorn for $3.50 and a soda for $2.50. How much was her ticket?

**SOLUTION:**

\[
\begin{align*}
14 - 3.50 - 2.50 &= 14 - (3.50 + 2.50) \\
&= 14 - (6) \\
&= 8
\end{align*}
\]

The cost of the ticket was $8.

**ANSWER:**

$8

Solve each equation or formula for the specified variable.

30. \(2k - 3m = 16\) for \(k\)

**SOLUTION:**

\[
\begin{align*}
2k - 3m &= 16 \\
2k - 3m + 3m &= 16 + 3m \\
2k &= 16 + 3m \\
\frac{2k}{2} &= \frac{16 + 3m}{2} \\
k &= \frac{16 + 3m}{2}
\end{align*}
\]

**ANSWER:**

\[
\frac{16 + 3m}{2}
\]

31. \(\frac{r + 5}{mn} = p\) for \(m\)

**SOLUTION:**

\[
\begin{align*}
\frac{r + 5}{mn} &= p \\
m \left( \frac{r + 5}{mn} \right) &= mp \\
\frac{r + 5}{n} &= mp \\
\frac{r + 5}{np} &= \frac{mp}{p} \\
\frac{r + 5}{pn} &= m
\end{align*}
\]

**ANSWER:**

\[
\frac{r + 5}{pn}
\]
32. \[ A = \frac{1}{2} h(a + b) \text{ for } h \]

**SOLUTION:**

\[
A = \frac{1}{2} h(a + b) \\
2A = 2 \left( \frac{1}{2} h(a + b) \right) \\
2A = h(a + b) \\
\frac{2A}{a + b} = \frac{h(a + b)}{a + b} \\
\frac{2A}{a + b} = h
\]

**ANSWER:**

\[ h = \frac{2A}{a + b} \]

33. **GEOMETRY** Yu-Jun wants to fill the water container at the right. He knows that the radius is 2 inches and the volume is 100.48 cubic inches. What is the height of the water bottle? Use the formula for the volume of a cylinder, \( V = \pi r^2 h \), to find the height of the bottle.

**SOLUTION:**

Substitute \( r = 2 \), and \( V = 100.48 \) in the formula \( V = \pi r^2 h \).

\[
V = \pi r^2 h \\
100.48 = \pi (2)^2 h \\
100.48 = 4\pi h \\
\frac{100.48}{4\pi} = \frac{4\pi h}{4\pi} \\
h \approx 8
\]

The height of the bottle is about 8 inches.

**ANSWER:**

about 8 in.
35. \(4|a - 6| = 16\)

**SOLUTION:**

\[
4|a - 6| = 16
\]

\[
4|a - 6| = 16
\]

\[
\frac{\text{Case 1:}}{\text{Case 2:}}\]

\[
\begin{align*}
a - 6 &= 4 & a - 6 &= -4 \\
a - 6 + 6 &= 4 + 6 & a - 6 + 6 &= -4 + 6 \\
a &= 10 & a &= 2
\end{align*}
\]

There appear to be two solutions, 2 and 10.

Check: Substitute each value in the original equation.

\[
4|a - 6| = 16 \quad 4|a - 6| = 16
\]

\[
\frac{4|10 - 6|}{4|2 - 6|} = 16 \quad \frac{4|4|}{4|-4|} = 16
\]

\[
4(4) = 16 \quad 4(4) = 16
\]

\[
16 = 16\checkmark \quad 16 = 16\checkmark
\]

The solution set is \(\{2, 10\}\).

**ANSWER:**

\(\{2, 10\}\)

36. \(|3x + 7| = -15\)

**SOLUTION:**

Case 1:

\[
3x + 7 = -15
\]

\[
3x + 7 = -15
\]

\[
\text{Case 2:}\]

\[
3x + 7 = 15
\]

\[
3x + 7 = 15
\]

\[
\frac{3x}{3} = \frac{22}{3} \quad \frac{3x}{3} = \frac{8}{3}
\]

\[
\frac{x = \frac{22}{3}}{x = \frac{8}{3}}
\]

There appear to be two solutions, \(-\frac{22}{3}\) and \(\frac{8}{3}\).

Check: Substitute each value in the original equation.

\[
\frac{|3x + 7|}{|3x + 7|} = -15 \quad \frac{|3x + 7|}{|3x + 7|} = -15
\]

\[
\frac{3(-\frac{22}{3}) + 7}{3(\frac{8}{3}) + 7} = -15 \quad \frac{3(\frac{8}{3}) + 7}{\frac{8}{3} + 7} = -15
\]

\[
\frac{15}{15} = -15 \quad 15 \neq -15
\]

Because \(15 \neq -15\), the solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)
37. \( |b + 5| = 2b - 9 \)

**SOLUTION:**

Case 1:  
\[
b + 5 = 2b - 9
\]
\[
b + 5 = -(2b - 9)
\]
\[
b + 5 = 2b - 9 - 5
\]
\[
b = 2b - 14
\]
\[
b - 2b = 2b - 14 - 2b
\]
\[
b - 2b = -2b + 4
\]
\[
-b = -14
\]
\[
b = 14
\]
\[
\frac{3b}{3} = \frac{4}{3}
\]
\[
b = \frac{4}{3}
\]

Case 2:

\[
2b - 9
\]
\[
3b = 4
\]
\[
b = \frac{4}{3}
\]

There appear to be two solutions, 14 and \( \frac{4}{3} \).

Check: Substitute each value in the original equation.

\[
|b + 5| = 2b - 9
\]
\[
|14 + 5| = 2(14) - 9
\]
\[
|19| = 28 - 9
\]
\[
19 = 19
\]

Because \( \frac{4}{3} \neq 19 \), \( b = \frac{4}{3} \) is an extraneous solution.

So, the solution set is \( \{14\} \).

**ANSWER:**

\( \{14\} \)

38. **MEASUREMENT** Marcos is cutting ribbons for a craft project. Each ribbon needs to be \( \frac{3}{4} \) yard long. If each piece is always within plus or minus \( \frac{1}{16} \) yard, how long are the shortest and longest pieces of ribbon?

**SOLUTION:**

Let \( x \) be the length of the shortest and longest pieces of ribbon.

\[
|x - \frac{3}{4}| = \frac{1}{16}
\]

Solve the equation \( x - \frac{3}{4} = \frac{1}{16} \).

Case 1:

\[
x = \frac{1}{4} + \frac{3}{16}
\]
\[
x = \frac{1 + 4(3)}{16}
\]
\[
x = \frac{13}{16}
\]

Case 2:

\[
x = \frac{1}{4} - \frac{3}{16}
\]
\[
x = \frac{1 - 3(4)}{16}
\]
\[
x = \frac{11}{16}
\]

**ANSWER:**

\( \frac{11}{16} \) yd; \( \frac{13}{16} \) yd
Solve each inequality. Then graph the solution set on a number line.

39. \(-4a \leq 24\)

**SOLUTION:**

\[
\begin{align*}
-4a &\leq 24 \\
\frac{-4a}{-4} &\geq \frac{24}{-4} \\
a &\geq -6
\end{align*}
\]

To graph this inequality, draw a solid circle at \(-6\) and draw an arrow extending to the right.

**ANSWER:**

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-8 & -7 & -6 & -5 & -4 & -3 & -2 \\
0 & 1 & 2
\end{array}
\]

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-8 & -7 & -6 & -5 & -4 & -3 & -2 \\
0 & 1 & 2
\end{array}
\]

40. \(\frac{r}{5} - 8 > 3\)

**SOLUTION:**

\[
\begin{align*}
\frac{r}{5} - 8 &> 3 \\
\frac{r}{5} &> 3 + 8 \\
\frac{r}{5} &> 11 \\
5\left(\frac{r}{5}\right) &> 5(11) \\
r &> 55
\end{align*}
\]

To graph this inequality, draw an open circle at 55 and draw an arrow extending to the right.

**ANSWER:**

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
50 & 51 & 52 & 53 & 54 & 55 & 56 \\
57 & 58 & 59 & 60
\end{array}
\]

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
50 & 51 & 52 & 53 & 54 & 55 & 56 \\
57 & 58 & 59 & 60
\end{array}
\]

41. \(4 - 7x \geq 2(x + 3)\)

**SOLUTION:**

\[
\begin{align*}
4 - 7x &\geq 2(x + 3) \\
4 - 7x &\geq 2x + 6 \\
4 - 7x - 4 &\geq 2x + 6 - 4 \\
-7x &\geq 2x + 2 \\
-7x - 2x &\geq 2x + 2 - 2x \\
-9x &\geq 2 \\
\frac{-9x}{-9} &\leq \frac{2}{-9} \\
x &\leq \frac{2}{9}
\end{align*}
\]

To graph this inequality, draw a solid circle at \(-\frac{2}{9}\) and draw an arrow extending to the left.

**ANSWER:**

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-1 & 0 & 1
\end{array}
\]

\[
\begin{array}{ccccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-1 & 0 & 1
\end{array}
\]
42. \(-p - 13 < 3(5 + 4p) - 2\)

**SOLUTION:**

\[
\begin{align*}
-p - 13 & < 3(5 + 4p) - 2 \\
-p - 13 & < 15 + 12p - 2 \\
-p - 13 & < 13 + 12p \\
-p & < 12p + 26 \\
-13p & < 26 \\
p & > -13 \\
& > -2
\end{align*}
\]

To graph this inequality, draw an open circle at \(-2\) and draw an arrow extending to the right.

**ANSWER:**

\(p > -2\)

43. **MONEY** Ms. Hawkins is taking her science class on a field trip to a museum. She has $572 to spend on the trip. There are 52 students that will go to the museum. The museum charges $5 per student, and Ms. Hawkins gets in for free. If the students will have slices of pizza for lunch that cost $2 each, how many slices can each student have?

**SOLUTION:**

Let \(x\) be the number of slices can each student have.

\[
\begin{align*}
52(5 + 2x) & \leq 572 \\
52(5) + 52(2x) & \leq 572 \\
260 + 104x & \leq 572 \\
104x & \leq 312 \\
\frac{104x}{104} & \leq \frac{312}{104} \\
x & \leq 3
\end{align*}
\]

Therefore, each student can have 3 or fewer slices.

**ANSWER:**

3 or fewer slices each
Solve each inequality. Graph the solution set on a number line.

44. \(2m + 4 < 7\) or \(3m + 5 > 14\)

**SOLUTION:**

\[
\begin{align*}
2m + 4 &< 7 \\
2m - 4 &< 3 \\
\frac{2m}{2} &< \frac{3}{2} \\
m &< \frac{3}{2}
\end{align*}
\]

\[
\begin{align*}
3x + 5 &> 14 \\
x &> \frac{9}{3} \\
x &> 3
\end{align*}
\]

The solution set is \( \left\{ m \mid m < \frac{3}{2} \text{ or } m > 3 \right\} \).

To graph, draw an open circle at \( \frac{3}{2} \) and an arrow extending to the left and an open circle at 3 and an arrow extending to the right.

**ANSWER:**

\[
\left\{ m \mid m < \frac{3}{2} \text{ or } m > 3 \right\}
\]

45. \(-5 < 4x + 3 < 19\)

**SOLUTION:**

\[
\begin{align*}
-5 &< 4x + 3 < 19 \\
-8 &< 4x < 16 \\
-2 &< x < 4
\end{align*}
\]

The solution set is \( \left\{ x \mid -2 < x < 4 \right\} \).

To graph, draw an open circle at -2 and an open circle at 4 and draw a line to connect the circles.

**ANSWER:**

\[
\left\{ x \mid -2 < x < 4 \right\}
\]
46. $6y - 1 > 17$ or $8y - 6 \leq -10$

**SOLUTION:**

\[
\begin{align*}
6y - 1 &> 17 \\
6y - 1 &+ 1 > 17 + 1 \\
6y &> 18 \\
\frac{6y}{6} &> \frac{18}{6} \\
y &> 3
\end{align*}
\]

\[
\begin{align*}
8y - 6 &\leq -10 \\
8y - 6 + 6 &\leq -10 + 6 \\
8y &\leq -4 \\
\frac{8y}{8} &\leq \frac{-4}{8} \\
y &\leq -\frac{1}{2}
\end{align*}
\]

The solution set is \( \{ y \mid \frac{-1}{2} \leq y \leq 3 \} \).

To graph, draw a solid circle at \(-\frac{1}{2}\) and an arrow extending to the left and a solid circle at 3 and an arrow extending to the right.

**ANSWER:**

\( \{ y \mid y \leq -\frac{1}{2} \text{ or } y > 3 \} \)

47. \(-2 \leq 5(m - 3) < 9\)

**SOLUTION:**

\[
\begin{align*}
-2 &\leq 5(m - 3) < 9 \\
-2 &\leq 5m - 15 < 9 \\
-2 + 15 &\leq 5m - 15 + 15 < 9 + 15 \\
13 &\leq 5m < 24 \\
\frac{13}{5} &\leq m < \frac{24}{5}
\end{align*}
\]

The solution set is \( \{ m \mid \frac{13}{5} \leq m < \frac{24}{5} \} \).

To graph, draw a solid circle at \( \frac{13}{5} \) and an open circle at \( \frac{24}{5} \) and draw a line to connect the circles.

**ANSWER:**

\( \{ m \mid \frac{13}{5} \leq m < \frac{24}{5} \} \)
48. \(|a| + 2 < 15\)

**SOLUTION:**
\[
|a| + 2 < 15
\]
\[
|a| < 13
\]
\[
-13 < a < 13
\]

The solution set is \(\{a|-13 < a < 13\}\).

To graph, draw an open circle at \(-13\) and an open circle at \(13\) and draw a line to connect the circles.

**ANSWER:**
\(\{a|-13 < a < 13\}\)

49. \(|p - 14| \leq 19\)

**SOLUTION:**
\[
|p - 14| \leq 19
\]
\[
-19 \leq (p - 14) \leq 19
\]
\[
-19 + 14 \leq p - 14 + 14 \leq 19 + 14
\]
\[
-5 \leq p \leq 33
\]

The solution set is \(\{p|-5 \leq p \leq 33\}\).

To graph, draw a solid circle at \(-5\) and a solid circle at \(33\) and draw a line to connect the circles.

**ANSWER:**
\(\{p|-5 \leq p \leq 33\}\)

50. \(|6k - 1| < 15\)

**SOLUTION:**
\[
|6k - 1| < 15
\]
\[
-15 < (6k - 1) < 15
\]
\[
-15 + 1 < 6k - 1 + 1 < 15 + 1
\]
\[
-14 < 6k < 16
\]
\[
-\frac{14}{6} < \frac{6k}{6} < \frac{16}{6}
\]
\[
-\frac{7}{3} < k < \frac{8}{3}
\]

The solution set is \(\{k|-\frac{7}{3} < k < \frac{8}{3}\}\).

To graph, draw an open circle at \(-\frac{7}{3}\) and an open circle at \(\frac{8}{3}\) and draw a line to connect the circles.

**ANSWER:**
\(\{k|-\frac{7}{3} < k < \frac{8}{3}\}\)

51. \(|2r + 7| < -1\)

**SOLUTION:**
Since the absolute value of a number is always positive, the solution set of the inequality is \(\emptyset\).
Since there are no solutions, leave the graph blank.

**ANSWER:**
\(\emptyset\)
53. **MONEY** Cara is making a beaded necklace for a gift. She wants to spend between $20 and $30 on the necklace. The bead store charges $2.50 for large beads and $1.25 for small beads. If she buys 3 large beads, how many small beads can she buy to stay within her budget? Write and solve a compound inequality to describe the range of possible beads.

**SOLUTION:**

Let \( b \) represent the number of small beads she can buy to stay within the budget.

\[
20 \leq 3(2.50) + b(1.25) \leq 30
\]

Solve the compound inequality.

\[
20 \leq 3(2.50) + b(1.25) \leq 30
\]

\[
20 \leq 7.50 + 1.25b \leq 30
\]

\[
20 - 7.50 \leq 1.25b \leq 30 - 7.50
\]

\[
12.50 \leq 1.25b \leq 22.50
\]

\[
\frac{12.50}{1.25} \leq b \leq \frac{22.50}{1.25}
\]

\[
10 \leq b \leq 18
\]

The range of number of possible beads is \( 10 \leq b \leq 18 \).

**ANSWER:**

\[
20 \leq 2.50(3) + 1.25b \leq 30; \ 10 \leq b \leq 18
\]