3-1 Solving Systems of Equations

Solve each system of equations by using a table.

1. \( y = 3x - 4 \)
   \( y = -2x + 11 \)

**SOLUTION:**
The equations are in slope-intercept form. Make a table to find a solution that satisfies both equations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>((x, y_1))</th>
<th>((x, y_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>11</td>
<td>(0, -4)</td>
<td>(0, 11)</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>9</td>
<td>(1, -1)</td>
<td>(1, 9)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>7</td>
<td>(2, 2)</td>
<td>(2, 7)</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>5</td>
<td>(3, 5)</td>
<td>(3, 5)</td>
</tr>
</tbody>
</table>

Therefore, the solution of the system is (3, 5).

**ANSWER:**
(3, 5)

2. \( 4x - y = 1 \)
   \( 5x + 2y = 24 \)

**SOLUTION:**
Write each equation in slope-intercept form.

\[
\begin{align*}
4x - y &= 1 \\
-y &= 1 - 4x \\
y &= 4x - 1
\end{align*}
\]

Also:

\[
\begin{align*}
5x + 2y &= 24 \\
2y &= 24 - 5x \\
y &= \frac{-5}{2}x + 12
\end{align*}
\]

Make a table to find a solution that satisfies both equations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>((x, y_1))</th>
<th>((x, y_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>12</td>
<td>(0, -1)</td>
<td>(0, 12)</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>9.5</td>
<td>(1, 3)</td>
<td>(1, 9.5)</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
<td>(2, 7)</td>
<td>(2, 7)</td>
</tr>
</tbody>
</table>

Therefore, the solution of the system is (2, 7).

**ANSWER:**
(2, 7)
Solve each system of equations by using a table.

3. \( y = -3x + 6 \)
   \[ 2y = 10x - 36 \]

**SOLUTION:**
Graph the equations \( y = -3x + 6 \) and \( 2y = 10x - 36 \).

The lines intersect at the point \((3, -3)\). So, the solution of the system is \((3, -3)\).

**ANSWER:**
\((3, -3)\)

4. \( y = -x - 9 \)
   \[ 3y = 5x + 5 \]

**SOLUTION:**
Graph the equations \( y = -x - 9 \) and \( 3y = 5x + 5 \).

The lines intersect at the point \((-4, -5)\). So, the solution of the system is \((-4, -5)\).

**ANSWER:**
\((-4, -5)\)

5. \( y = 0.5x + 4 \)
   \[ 3y = 4x - 3 \]

**SOLUTION:**
Graph the equations \( y = 0.5x + 4 \) and \( 3y = 4x - 3 \).

The lines intersect at the point \((6, 7)\). So, the solution of the system is \((6, 7)\).

**ANSWER:**
\((6, 7)\)

6. \( -3y = 4x + 11 \)
   \[ 2x + 3y = -7 \]

**SOLUTION:**
Graph the equations \(-3y = 4x + 11\) and \(2x + 3y = -7\).

The lines intersect at the point \((-2, -1)\). So, the solution of the system is \((-2, -1)\).

**ANSWER:**
\((-2, -1)\)
7. $4x + 5y = -41$
   $3y - 5x = 5$

**SOLUTION:**
Graph the equations $4x + 5y = -41$ and $3y - 5x = 5$.

The lines intersect at the point $(-4, -5)$. So, the solution of the system is $(-4, -5)$.

**ANSWER:**
$(-4, -5)$

8. $8x - y = 50$
   $x + 4y = -2$

**SOLUTION:**
Graph the equations $8x - y = 50$ and $x + 4y = -2$.

The lines intersect at the point $(6, -2)$. So, the solution of the system is $(6, -2)$.

**ANSWER:**
$(6, -2)$

9. **CCSS MODELING** Refer to the table below.

<table>
<thead>
<tr>
<th>Digital Photos</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Online Store</strong></td>
<td></td>
</tr>
<tr>
<td>$0.15$ per photo + $2.70$ shipping</td>
<td></td>
</tr>
<tr>
<td><strong>Local Store</strong></td>
<td></td>
</tr>
<tr>
<td>$0.25$ per photo</td>
<td></td>
</tr>
</tbody>
</table>

**a.** Write equations that represent the cost of printing digital photos at each lab.
**b.** Under what conditions is the cost to print digital photos the same at both stores?
**c.** When is it best to use EZ Online Digital Photos and when is it best to use the local pharmacy?

**SOLUTION:**

**a.** Let $x$ be the charge for a digital photo and $y$ be the total cost.

At EZ Online Digital Photos:
$y = 0.15x + 2.70$

At Local Pharmacy:
$y = 0.25x$

**b.** Equate the total costs and find $x$.

$0.15x + 2.70 = 0.25x$

$0.10x = 2.70$

$x = 27$

When $x = 27$, $y = 6.75$.

Therefore, the cost will be the same at both stores when 27 photos are taken.

**c.**

$0.25x < 0.15x + 2.70$

$0.10x < 2.70$

$x < 27$

That is, you should use EZ Online Digital Photos if you are printing more than 27 digital photos and the local pharmacy if you are printing fewer than 27 photos.

**ANSWER:**

**a.** $y = 0.15x + 2.70, y = 0.25x$

**b.** $6.75$ for 27 photos

**c.** You should use EZ Online Digital Photos if you are printing more than 27 digital photos and the local pharmacy if you are printing fewer than 27 photos.
3-1 Solving Systems of Equations

Graph each system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

10. \[ y + 4x = 12 \]
\[ 3y = 8 - 12x \]

**SOLUTION:**

The lines are parallel. They do not intersect and there is no solution. So, the system is inconsistent.

**ANSWER:**

inconsistent

11. \[ -2x - 3y = 9 \]
\[ 4x + 6y = -18 \]

**SOLUTION:**

Because the equations are equivalent, their graphs are the same line. The system is consistent and dependent as it has an infinite number of solutions.

**ANSWER:**

consistent, dependent
Solve each system of equations by using substitution.

13. $x + 5y = 3$
   $3x - 2y = -8$

**SOLUTION:**

Solve the equation $x + 5y = 3$ for $x$.

$x = 3 - 5y$

Substitute $3 - 5y$ for $x$ in the equation $3x - 2y = -8$ and solve for $y$.

$3(3 - 5y) - 2y = -8$

$9 - 15y - 2y = -8$

$-17y = -17$

$y = 1$

Substitute 1 for $y$ into either original equation and solve for $x$.

$x + 5y = 3$

$x + 5(1) = 3$

$x = 3 - 5$

$x = -2$

The solution is $(-2, 1)$.

**ANSWER:**

$(-2, 1)$
14. \( y = 2x - 10 \)  
\( y = -4x + 8 \)

**SOLUTION:**
Substitute \( 2x - 10 \) for \( y \) in the equation  
\( y = -4x + 8 \) and solve for \( x \).

\[
\begin{align*}
2x - 10 &= -4x + 8 \\
2x + 4x &= 8 + 10 \\
6x &= 18 \\
x &= 3
\end{align*}
\]

Substitute 3 for \( x \) into either original equation and solve for \( y \).

\[
\begin{align*}
y &= 2x - 10 \\
y &= 2(3) - 10 \\
&= 6 - 10 \\
&= -4
\end{align*}
\]

The solution is (3, −4).

**ANSWER:**
(3, −4)

15. \( 2a + 8b = -8 \)  
\( 3a - 5b = 22 \)

**SOLUTION:**
Solve the equation \( 2a + 8b = -8 \) for \( a \).

\[
\begin{align*}
2a &= -8b - 8 \\
a &= -4b - 4
\end{align*}
\]

Substitute \( -4b - 4 \) for \( a \) in the equation \( 3a - 5b = 22 \) and solve for \( b \).

\[
\begin{align*}
3(-4b - 4) - 5b &= 22 \\
-12b - 12 - 5b &= 22 \\
-17b &= 34 \\
b &= -2
\end{align*}
\]

Substitute \( -2 \) for \( b \) into either original equation and solve for \( a \).

\[
\begin{align*}
2a + 8 &\times (-2) = -8 \\
2a - 16 &= -8 \\
2a &= 8 \\
a &= 4
\end{align*}
\]

The solution is (4, −2).

**ANSWER:**
(4, −2)
3-1 Solving Systems of Equations

16. \[ a - 3b = -22 \]
   \[ 4a + 2b = -4 \]

**SOLUTION:**

Solve the equation \( a - 3b = -22 \) for \( a \).

\[ a = 3b - 22 \]

Substitute \( 3b - 22 \) for \( a \) in the equation \( 4a + 2b = -4 \) and solve for \( b \).

\[
4(3b - 22) + 2b = -4 \\
12b - 88 + 2b = -4 \\
14b = 84 \\
b = 6
\]

Substitute 6 for \( b \) into either original equation and solve for \( a \).

\[
a - 3b = -22 \\
a - 3(6) = -22 \\
a - 18 = -22 \\
a = -4
\]

The solution is \((-4, 6)\).

**ANSWER:**

\((-4, 6)\)

17. \[ 6x - 7y = 23 \]
   \[ 8x + 4y = 44 \]

**SOLUTION:**

Solve the equation \( 8x + 4y = 44 \) for \( y \).

\[
4y = 44 - 8x \\
y = 11 - 2x
\]

Substitute \( 11 - 2x \) for \( y \) in the equation \( 6x - 7y = 23 \) and solve for \( x \).

\[
6x - 7(11 - 2x) = 23 \\
6x - 77 + 14x = 23 \\
20x = 100 \\
x = 5
\]

Substitute 5 for \( x \) into either original equation and solve for \( y \).

\[
8x + 4y = 44 \\
8(5) + 4y = 44 \\
40 + 4y = 44 \\
4y = 4 \\
y = 1
\]

The solution is \((5, 1)\).

**ANSWER:**

\((5, 1)\)
3-1 Solving Systems of Equations

18. \(9c - 3d = -33\)
   \(6c + 5d = -8\)

**SOLUTION:**
Solve the equation \(9c - 3d = -33\) for \(c\).

\[
9c = 3d - 33
\]
\[
c = \frac{1}{3}d - \frac{11}{3}
\]

Substitute \(\frac{1}{3}d - \frac{11}{3}\) for \(c\) in the equation \(6c + 5d = -8\) and solve for \(d\).

\[
6\left(\frac{1}{3}d - \frac{11}{3}\right) + 5d = -8
\]
\[
2d - 22 + 5d = -8
\]
\[
7d = 14
\]
\[
d = 2
\]

Substitute 2 for \(d\) into either original equation and solve for \(c\).

\[
9c - 3(2) = -33
\]
\[
9c - 6 = -33
\]
\[
9c = -27
\]
\[
c = -3
\]

The solution is \((-3, 2)\).

**ANSWER:**
\((-3, 2)\)

19. Solve each system of equations by using elimination.

\[-6w - 8z = -44\]
\[3w + 6z = 36\]

**SOLUTION:**
Multiply the equation \(3w + 6z = 36\) by 2.

\[2(3w + 6z) = 2(36)\]
\[6w + 12z = 72\]

Add the equations to eliminate one variable.

\[
\begin{align*}
-6w &- 8z = -44 \\
(+) \quad 6w + 12z & = 72 \\
\hline
4z & = 28 \\
\hline
z &= 7
\end{align*}
\]

Substitute 7 for \(z\) into either original equation and solve for \(w\).

\[
3w + 6z = 36
\]
\[
3w + 6(7) = 36
\]
\[
3w + 42 = 36
\]
\[
3w = -6
\]
\[
w = -2
\]

The solution is \((-2, 7)\).

**ANSWER:**
\((-2, 7)\)
20. \[ 4x - 3y = 29 \]
\[ 4x + 3y = 35 \]

**SOLUTION:**
Add the equations to eliminate \( y \).

\[
\begin{align*}
4x - 3y &= 29 \\
(+)
4x + 3y &= 35 \\
\hline
8x &= 64 \\
x &= 8
\end{align*}
\]

Substitute 8 for \( x \) into either original equation and solve for \( y \).

\[
\begin{align*}
4x + 3y &= 35 \\
4(8) + 3y &= 35 \\
32 + 3y &= 35 \\
3y &= 3 \\
y &= 1
\end{align*}
\]

The solution is (8, 1).

**ANSWER:**
(8, 1)

21. \[ 3a + 5b = -27 \]
\[ 4a + 10b = -46 \]

**SOLUTION:**
Multiply the equation \( 3a + 5b = -27 \) by \( -2 \).

\[
\begin{align*}
-2(3a + 5b) &= -2(-27) \\
-6a - 10b &= 54
\end{align*}
\]

Add the equations to eliminate one variable.

\[
\begin{align*}
-6a - 10b &= 54 \\
+ \\
4a + 10b &= -46 \\
\hline
-2a &= 8 \\
a &= -4
\end{align*}
\]

Substitute \(-4\) for \( a \) into either original equation and solve for \( b \).

\[
\begin{align*}
3a + 5b &= -27 \\
3(-4) + 5b &= -27 \\
-12 + 5b &= -27 \\
5b &= -15 \\
b &= -3
\end{align*}
\]

The solution is \((-4, -3)\).

**ANSWER:**
\((-4, -3)\)
22. \[8a - 3b = -11\]
\[5a + 2b = -3\]

**SOLUTION:**
The coefficients of \( b \)-variables are 3 and 2 and their least common multiple is 6, so multiply each equation by the value that will make the \( b \)-coefficient 6.

\[
\begin{align*}
8a - 3b &= -11 \quad \text{Multiply by 2} \rightarrow 16a - 6b = -22 \\
5a + 2b &= -3 \quad \text{Multiply by 3} \rightarrow (+) 15a + 6b = -9 \\
\end{align*}
\]

\[31a = -31\]
\[a = -1\]

Substitute \(-1\) for \( a \) into either original equation and solve for \( b \).

\[
\begin{align*}
5a + 2b &= -3 \\
5(-1) + 2b &= -3 \\
-5 + 2b &= -3 \\
2b &= 2 \\
b &= 1
\end{align*}
\]

The solution is \((-1, 1)\).

**ANSWER:**
\((-1, 1)\)

23. \[5a + 15b = -24\]
\[-2a - 6b = 28\]

**SOLUTION:**
The coefficients of \( a \)-variables are 5 and 2 (in absolute sense) and their least common multiple is 10, so multiply each equation by the value that will make the \( y \)-coefficient 10.

\[
\begin{align*}
5a + 15b &= -24 \quad \text{Multiply by 2} \rightarrow 10a + 30b = -48 \\
-2a - 6b &= 28 \quad \text{Multiply by 5} \rightarrow (-) 10a - 30b = 140 \\
\end{align*}
\]

\[0 = 92\]

Because \( 0 = 92 \) is not true, this system has no solution.

**ANSWER:**
No solution

24. \[6x - 4y = 30\]
\[12x + 5y = -18\]

**SOLUTION:**
The coefficients of \( y \)-variables are 4 and 5 and their least common multiple is 20, so multiply each equation by the value that will make the \( y \)-coefficient 20.

\[
\begin{align*}
6x - 4y &= 30 \quad \text{Multiply by 5} \rightarrow 30x - 20y = 150 \\
12x + 5y &= -18 \quad \text{Multiply by 4} \rightarrow (+) 48x + 20y = -72 \\
\end{align*}
\]

\[78x = 78\]
\[x = 1\]

Substitute 1 for \( x \) into either original equation and solve for \( y \).

\[
\begin{align*}
12x + 5y &= -18 \\
12(1) + 5y &= -18 \\
12 + 5y &= -18 \\
5y &= -30 \\
y &= -6
\end{align*}
\]

The solution is \((1, -6)\).

**ANSWER:**
\((1, -6)\)
3-1 Solving Systems of Equations

25. **MULTIPLE CHOICE** What is the solution of the linear system?
   
   \[ \begin{align*}
   4x + 3y &= 2 \\
   4x - 2y &= 12 \\
   \end{align*} \]

   A (8, –10)  
   B (2, –2)  
   C (–10, 14)  
   D no solution

   **SOLUTION:**
   
   Subtract the equations to eliminate one variable.
   
   \[
   \begin{array}{c}
   \frac{4x + 3y = 2}{4x - 2y = 12} \\
   \hline
   5y = -10 \\
   y = -2
   \end{array}
   \]

   Substitute \(-2\) for \(y\) into either original equation and solve for \(x\).

   \[
   \begin{align*}
   4x + 3(-2) &= 2 \\
   4x - 6 &= 2 \\
   4x &= 2 + 6 \\
   4x &= 8 \\
   x &= 2
   \end{align*}
   \]

   The solution is (2, –2). So the correct choice is B.

   **ANSWER:**
   
   B

26. **Solve each system of equations by using a table.**

   \[ \begin{align*}
   y &= 5x + 3 \\
   y &= x - 9 \\
   \end{align*} \]

   **SOLUTION:**

   \[
   \begin{array}{c|c|c|c|c}
   x & y & y & (x, y) & (x, y) \\
   \hline
   0 & 3 & -9 & (0, 3) & (0, -9) \\
   -1 & -2 & -10 & (-1, -2) & (-1, -10) \\
   -2 & -7 & -11 & (-2, -7) & (-2, -11) \\
   -3 & -12 & -12 & (-3, -12) & (-3, -12) \\
   \end{array}
   \]

   Therefore, the solution of the system is \((-3, -12)\).

   **ANSWER:**

   \((-3, -12)\)
27. \[ 3x - 4y = 16 \]
\[ -6x + 5y = -29 \]

**SOLUTION:**
Write each equation in slope-intercept form.

\[ 3x - 4y = 16 \]
\[ -4y = 16 - 3x \]
\[ y = \frac{3}{4}x - 4 \]

Also:
\[ -6x + 5y = -29 \]
\[ 5y = 6x - 29 \]
\[ y = \frac{6}{5}x - \frac{29}{5} \]

Make a table to find a solution that satisfies both equations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( (x, y_1) )</th>
<th>( (x, y_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>-5.8</td>
<td>(0, -4)</td>
<td>(0, -5.8)</td>
</tr>
<tr>
<td>1</td>
<td>-3.25</td>
<td>-4.6</td>
<td>(1, -3.25)</td>
<td>(1, -4.6)</td>
</tr>
<tr>
<td>2</td>
<td>-2.5</td>
<td>-3.4</td>
<td>(2, -2.5)</td>
<td>(2, -3.4)</td>
</tr>
<tr>
<td>3</td>
<td>-1.75</td>
<td>-2.2</td>
<td>(3, -1.75)</td>
<td>(3, -2.2)</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>-1</td>
<td>(4, -1)</td>
<td>(4, -1)</td>
</tr>
</tbody>
</table>

Therefore, the solution of the system is (4, -1).

**ANSWER:**
(4, -1)

28. \[ 2x - 5 = y \]
\[ -3x + 4y = 0 \]

**SOLUTION:**
Write each equation in slope-intercept form.

\[ y = 2x - 5 \]
\[ 4y = 3x \]
\[ y = \frac{3}{4}x \]

Make a table to find a solution that satisfies both equations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>( (x, y_1) )</th>
<th>( (x, y_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>(0, -5)</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
<td>0.75</td>
<td>(1, -3)</td>
<td>(1, 0.75)</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>1.5</td>
<td>(2, -1)</td>
<td>(2, 1.5)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2.25</td>
<td>(3, 1)</td>
<td>(3, 2.25)</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>(4, 3)</td>
<td>(4, 3)</td>
</tr>
</tbody>
</table>

Therefore, the solution of the system is (4, 3).

**ANSWER:**
(4, 3)
3-1 Solving Systems of Equations

29. **FUNDRAISER** To raise money for new uniforms, the band boosters sell T-shirts and hats. The cost and sale price of each item is shown. The boosters spend a total of $2000 on T-shirts and hats. They sell all of the merchandise, and make $3375. How many T-shirts did they sell?

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
<th>Sale Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Shirt</td>
<td>$6</td>
<td>$10</td>
</tr>
<tr>
<td>Hat</td>
<td>$4</td>
<td>$7</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \( x \) be the number of T-shirts.
Let \( y \) be the number of hats.
The system of equations is

Cost price: \( 6x + 4y = 2000 \) and
Sale price: \( 10x + 7y = 3375 \).

Solve the equation of cost price for \( y \).

\[
4y = -6x + 2000
\]
\[
y = -\frac{3}{2}x + 500
\]

Substitute \( -\frac{3}{2}x + 500 \) for \( y \) in the equation of sale price and solve for \( x \).

\[
10x + 7\left(-\frac{3}{2}x + 500\right) = 3375
\]
\[
10x - \frac{21}{2}x + 3500 = 3375
\]
\[
\frac{1}{2}x = -125
\]
\[
x = 250
\]

Thus, they sold 250 T-shirts.

**ANSWER:**
250 T-shirts

---

30. **Solve each system of equations by graphing.**

\[-3x + 2y = -6\]
\[-5x + 10y = 30\]

**SOLUTION:**
Graph the equations \(-3x + 2y = -6\) and \(-5x + 10y = 30\).

The lines intersect at the point (6, 6). So, the solution of the system is (6, 6).

**ANSWER:**
(6, 6)

31.

\[4x + 3y = -24\]
\[8x - 2y = -16\]

**SOLUTION:**
Graph the equations \(4x + 3y = -24\) and \(8x - 2y = -16\).

The lines intersect at the point (−3, −4). So, the solution of the system is (−3, −4).

**ANSWER:**
(−3, −4)
3.1 Solving Systems of Equations

32. \(6x - 5y = 17\)
   \(6x + 2y = 31\)

**SOLUTION:**
Graph the equations \(6x - 5y = 17\) and \(6x + 2y = 31\).

\[
\begin{array}{c|c|c}
\hline
x & y & \\
\hline
-1 & 19 & 31
\end{array}
\]

The lines intersect at the point (4, 5). So, the solution of the system is (4, 5).

**ANSWER:**
(4, 5)

33. \(-3x - 8y = 12\)
   \(12x + 32y = -48\)

**SOLUTION:**
Graph the equations \(-3x - 8y = 12\) and \(12x + 32y = -48\).

Because the equations are equivalent, their graphs are the same line. So, the system has an infinite number of solutions.

**ANSWER:**
Infinite solutions

34. \(y - 3x = -29\)
   \(9x - 6y = 102\)

**SOLUTION:**
Graph the equations \(y - 3x = -29\) and \(9x - 6y = 102\).

\[
\begin{array}{c|c|c}
\hline
x & y & \\
\hline
-3 & 1 & 12
\end{array}
\]

The lines intersect at the point (8, -5). So, the solution of the system is (8, -5).

**ANSWER:**
(8, -5)

35. \(-10x + 4y = 7\)
   \(2x - 5y = 7\)

**SOLUTION:**
Graph the equations \(-10x + 4y = 7\) and \(2x - 5y = 7\).

The lines intersect at the point (-1.5, -2). So, the solution of the system is (-1.5, -2).

**ANSWER:**
(-1.5, -2)
36. **CCSS MODELING** Jerilyn has a $10 coupon and a 15% discount coupon for her favorite store. The store has a policy that only one coupon may be used per purchase. When is it best for Jerilyn to use the $10 coupon, and when is it best for her to use the 15% discount coupon?

**SOLUTION:**
Let $x$ be the total cost (excluding discounts) of the items that Jerilyn purchased. If she uses the $10 coupon, the cost will be $(x - 10)$. If she uses the 15% coupon, the cost will be $0.85x$.

Assume: $x - 10 < 0.85x$

$x - 10 < 0.85x$

$0.15x < 10$

$x < 66.67$

So, when the total cost of the items purchased is less than $66.67, it is best to use the $10 coupon. The 15% discount is best when the purchase is more than $66.67.

**ANSWER:**
$10$ coupon for a purchase less than $66.67 and 15% discount coupon for a purchase over $66.67.

**37.** Graph each system of equations and describe it as consistent and independent, consistent and dependent, or inconsistent.

$y = 3x - 4$

$y = 6x - 8$

**SOLUTION:**
Graph the equations $y = 3x - 4$ and $y = 6x - 8$.

![Graph of equations](image)

The lines intersect at one point, so there is one solution. The system is consistent and independent.
3-1 Solving Systems of Equations

38. \[ y = 2x - 1 \\
    \] \[ y = 2x + 6 \]

**SOLUTION:**
Graph the equations \( y = 2x - 1 \) and \( y = 2x + 6 \).

[Diagram of two parallel lines, labeled with equations, showing no intersection.]

The lines are parallel. They do not intersect and there is no solution. So, the system is inconsistent.

**ANSWER:**

[Diagram of two parallel lines, labeled with equations, showing no intersection.]

39. \[ 2x + 5y = 10 \]
\[ -4x - 10y = 20 \]

**SOLUTION:**
Graph the equations \( 2x + 5y = 10 \) and \( -4x - 10y = 20 \).

[Diagram of two parallel lines, labeled with equations, showing no intersection.]

The lines are parallel. They do not intersect and there is no solution. So, the system is inconsistent.

**ANSWER:**

[Diagram of two parallel lines, labeled with equations, showing no intersection.]
3-1 Solving Systems of Equations

40. \(x - 6y = 12\)
   \(3x + 18y = 14\)

**SOLUTION:**
Graph the equations \(x - 6y = 12\) and \(3x + 18y = 14\).

\[
\begin{align*}
3x + 18y &= 14 \\
x - 6y &= 12
\end{align*}
\]

consistent and independent

The lines intersect at one point, so there is one solution. The system is consistent and independent.

**ANSWER:**

\[
\begin{align*}
3x + 18y &= 14 \\
x - 6y &= 12
\end{align*}
\]
consistent and independent

41. \(-5x - 6y = 13\)
   \(12y + 10x = -26\)

**SOLUTION:**
Graph the equations \(-5x - 6y = 13\) and \(12y + 10x = -26\).

\[
\begin{align*}
-5x - 6y &= 13 \\
12y + 10x &= -26
\end{align*}
\]
consistent and dependent

Because the equations are equivalent, their graphs are the same line. The system is consistent and dependent as it has an infinite number of solutions.

**ANSWER:**
Solve each system of equations by using substitution.

42. \( 8y - 3x = 15 \)
   \[-16y + 6x = -30 \]

**SOLUTION:**
Graph the equations \( 8y - 3x = 15 \) and \(-16y + 6x = -30 \).

Because the equations are equivalent, their graphs are the same line. The system is consistent and dependent as it has an infinite number of solutions.

**ANSWER:**
Infinite solutions

43. \( 9y + 3x = 18 \)
   \[-3y - x = -6 \]

**SOLUTION:**
Solve the equation \( 9y + 3x = 18 \) for \( x \).

\[
\begin{align*}
3x &= 18 - 9y \\
x &= 6 - 3y
\end{align*}
\]

Substitute \( 6 - 3y \) for \( x \) in the equation \(-3y - x = -6 \) and solve for \( y \).

\[
\begin{align*}
-3y - (6 - 3y) &= -6 \\
-3y - 6 + 3y &= -6 \\
0 &= 0
\end{align*}
\]

Because \( 0 = 0 \) is true, the system has infinite solutions.

**ANSWER:**
Infinite solutions
Solve each system of equations by using a table.

44. \(5x - 20y = 70\)
\(6x + 5y = -32\)

**SOLUTION:**
Solve the equation \(5x - 20y = 70\) for \(x\).

\[
5x = 70 + 20y \\
x = 14 + 4y
\]

Substitute \(14 + 4y\) for \(x\) in the equation \(6x + 5y = -32\) and solve for \(y\).

\[
6(14 + 4y) + 5y = -32 \\
84 + 24y + 5y = -32 \\
84 + 29y = -32 \\
29y = -116 \\
y = -4
\]

Substitute \(-4\) for \(y\) into either original equation and solve for \(x\).

\[
5x - 20y = 70 \\
5x - 20(-4) = 70 \\
5x + 80 = 70 \\
5x = -10 \\
x = -2
\]

The solution is \((-2, -4)\).

**ANSWER:**
\((-2, -4)\)

45. \(-4x - 16y = -96\)
\(7x + 3y = 68\)

**SOLUTION:**
Solve the equation \(7x + 3y = 68\) for \(x\).

\[
7x = 68 - 3y \\
x = \frac{68 - 3}{7}y
\]

Substitute \(\frac{68 - 3}{7}y\) for \(x\) in the equation \(-4x - 16y = -96\) and solve for \(y\).

\[
-4\left(\frac{68 - 3}{7}y\right) - 16y = -96 \\
-\frac{272}{7} + \frac{12}{7}y - 16y = -96 \\
-\frac{100}{7}y = -\frac{400}{7} \\
y = 4
\]

Substitute \(4\) for \(y\) into either original equation and solve for \(x\).

\[
7x + 3(4) = 68 \\
7x + 12 = 68 \\
7x = 56 \\
x = 8
\]

The solution is \((8, 4)\).

**ANSWER:**
\((8, 4)\)
46. \(-4a - 5b = 14\)
\[9a + 3b = -48\]

**SOLUTION:**
Solve the equation \(9a + 3b = -48\) for \(b\).

\[3b = -48 - 9a\]
\[b = -16 - 3a\]

Substitute \(-16 - 3a\) for \(b\) in the equation \(-4a - 5b = 14\) and solve for \(a\).

\[-4a - 5(-16 - 3a) = 14\]
\[-4a + 80 + 15a = 14\]
\[11a + 80 = 14\]
\[11a = -66\]
\[a = -6\]

Substitute \(-6\) for \(a\) into either original equation and solve for \(b\).

\[9(-6) + 3b = -48\]
\[-54 + 3b = -48\]
\[3b = 6\]
\[b = 2\]

The solution is \((-6, 2)\).

**ANSWER:**
\((-6, 2)\)

47. \(-9c - 4d = 31\)
\[6c + 6d = -24\]

**SOLUTION:**
Solve the equation \(6c + 6d = -24\) for \(c\).

\[6c = -24 - 6d\]
\[c = -4 - d\]

Substitute \((-4 - d)\) for \(c\) in the equation \(-9c - 4d = 31\) and solve for \(d\).

\[-9(-4 - d) - 4d = 31\]
\[36 + 9d - 4d = 31\]
\[5d = -5\]
\[d = -1\]

Substitute \(-1\) for \(d\) into either original equation and solve for \(c\).

\[6c + 6(-1) = -24\]
\[6c - 6 = -24\]
\[6c = -18\]
\[c = -3\]

The solution is \((-3, -1)\).

**ANSWER:**
\((-3, -1)\)
48. \[8f + 3g = 12\]
\[-32f - 12g = 48\]

**SOLUTION:**
Solve the equation \(8f + 3g = 12\) for \(g\).

\[3g = 12 - 8f\]
\[g = 4 - \frac{8}{3}f\]

Substitute \(4 - \frac{8}{3}f\) for \(g\) in the equation
\[-32f - 12g = 48\]
and solve for \(f\).

\[-32f - 12\left(4 - \frac{8}{3}f\right) = 48\]
\[-32f - 48 + 32f = 48\]
\[0 = 96\]

Because \(0 = 96\) is not true, this system has no solution.

**ANSWER:**
No solution

49. **TENNIS** At a park, there are 38 people playing tennis. Some are playing doubles, and some are playing singles. There are 13 matches in progress. A doubles match requires 4 players, and a singles match requires 2 players.

**a.** Write a system of two equations that represents the number of singles and doubles matches going on.
**b.** How many matches of each kind are in progress?

**SOLUTION:**

**a.** Let \(x\) be the number of doubles matches.
Let \(y\) be the number of singles matches.
The system of equations that represents the number of singles and doubles matches going on is:

\[x + y = 13\]
\[4x + 2y = 38\]

**b.** Solve the equation \(x + y = 13\) for \(x\).

\[x = 13 - y\]
Substitute \(13 - y\) for \(x\) in the equation
\[4x + 2y = 38\]
and solve for \(y\).

\[4(13 - y) + 2y = 38\]
\[52 - 4y + 2y = 38\]
\[52 - 2y = 38\]
\[-2y = -14\]
\[y = 7\]

Substitute 7 for \(y\) into either original equation and solve for \(x\).

\[x + 7 = 13\]
\[x = 6\]

There are 6 doubles matches and 7 singles matches are in progress.

**ANSWER:**

**a.** \(x + y = 13\) and \(4x + 2y = 38\)
**b.** 6 doubles matches and 7 singles matches
### 3-1 Solving Systems of Equations

#### Solve each system of equations by using elimination.

50. \(8x + y = 27\)
\(-3x + 4y = 3\)

**SOLUTION:**
Multiply the equation \(8x + y = 27\) by \(-4\).

\[-4(8x + y) = -4(27)\]
\[-32x - 4y = -108\]

Add the equations to eliminate one variable.

\[-32x - 4y = -108\]
\[\underline{(+)} -3x + 4y = 3\]
\[-35x = -105\]
\[x = 3\]

Substitute 3 for \(x\) into either original equation and solve for \(y\).

\[8x + y = 27\]
\[8(3) + y = 27\]
\[24 + y = 27\]
\[y = 3\]

The solution is \((3, 3)\).

**ANSWER:**
\((3, 3)\)

51. \(2a - 5b = -20\)
\(2a + 5b = 20\)

**SOLUTION:**
Add the equations to eliminate one variable.

\[2a - 5b = -20\]
\[\underline{(+)} 2a + 5b = 20\]
\[4a = 0\]
\[a = 0\]

Substitute 0 for \(a\) into either original equation and solve for \(b\).

\[2(0) + 5b = 20\]
\[5b = 20\]
\[b = 4\]

The solution is \((0, 4)\).

**ANSWER:**
\((0, 4)\)
3-1 Solving Systems of Equations

52. \(6j + 4k = -46\)
    \(2j + 4k = -26\)

**SOLUTION:**
Subtract the equations to eliminate one variable.

\[
\begin{align*}
6j + 4k &= -46 \\
(-) 2j + 4k &= -26 \\
4j &= -20 \\
j &= -5
\end{align*}
\]

Substitute \(-5\) for \(j\) into either original equation and solve for \(k\).

\[
\begin{align*}
2j + 4k &= -26 \\
2(-5) + 4k &= -26 \\
-10 + 4k &= -26 \\
4k &= -16 \\
k &= -4
\end{align*}
\]

The solution is \((-5, -4)\).

**ANSWER:**
\((-5, -4)\)

53. \(3x - 8y = 24\)
    
\(-12x + 32y = 96\)

**SOLUTION:**
Multiply the equation \(3x - 8y = 24\) by 4.

\[
\begin{align*}
4(3x - 8y) &= 4(24) \\
12x - 32y &= 96
\end{align*}
\]

Add the equations to eliminate one variable.

\[
\begin{align*}
12x - 32y &= 96 \\
(+)(-12x + 32y) &= (-96) \\
0 &= 192
\end{align*}
\]

Because 0 = 192 is not true, the system has no solution.

**ANSWER:**
No solution

54. \(5a - 2b = -19\)
    \(8a + 5b = -55\)

**SOLUTION:**
The coefficients of \(b\)-variables are 5 and 2 and their least common multiple is 10, so multiply each equation by the value that will make the \(b\)-coefficient 10.

\[
\begin{align*}
5a - 2b &= -19 \quad \text{Multiply by 5} \rightarrow 25a - 10b = -95 \\
8a + 5b &= -55 \quad \text{Multiply by 2} \rightarrow (+) 16a + 10b = -110 \\
41a &= -205 \\
a &= -5
\end{align*}
\]

Substitute \(-5\) for \(a\) into either original equation and solve for \(b\).

\[
\begin{align*}
8a + 5b &= -55 \\
8(-5) + 5b &= -55 \\
-40 + 5b &= -55 \\
5b &= -15 \\
b &= -3
\end{align*}
\]

The solution is \((-5, -3)\).

**ANSWER:**
\((-5, -3)\)
3-1 Solving Systems of Equations

55. \[r - 6t = 44\]
    \[9r + 12t = 0\]

   **SOLUTION:**
   Multiply the equation \(r - 6t = 44\) by 2.
   \[2(r - 6t) = 2(44)\]
   \[2r - 12t = 88\]

   Add the equations to eliminate one variable.
   \[\begin{align*}
   2r - 12t &= 88 \\
   (+) 9r + 12t &= 0 \\
   11r &= 88 \\
   r &= 8
   \end{align*}\]

   Substitute 8 for \(r\) into either original equation and solve for \(t\).
   \[9r + 12t = 0\]
   \[9(8) + 12t = 0\]
   \[72 + 12t = 0\]
   \[12t = -72\]
   \[t = -6\]

   The solution is \((8, -6)\).

   **ANSWER:**
   \((8, -6)\)

56. \[6d + 5f = -32\]
    \[5d - 9f = 26\]

   **SOLUTION:**
   The coefficients of \(f\)-variables are 5 and 9 and their least common multiple is 45, so multiply each equation by the value that will make the \(f\)-coefficient 45.
   \[6d + 5f = -32 \text{ Multiply by } 9 \Rightarrow 54d + 45f = -288\]
   \[5d - 9f = 26 \text{ Multiply by } 5 \Rightarrow (+) 25d - 45f = 130\]
   \[79d = -158\]
   \[d = -2\]

   Substitute \(-2\) for \(d\) into either original equation and solve for \(f\).
   \[6d + 5f = -32\]
   \[6(-2) + 5f = -32\]
   \[-12 + 5f = -32\]
   \[5f = -20\]
   \[f = -4\]

   The solution is \((-2, -4)\).

   **ANSWER:**
   \((-2, -4)\)
57. \[11u = 5v + 35\]
\[8v = -6u + 62\]

**SOLUTION:**
Rewrite the equations in the standard form \(Ax + By = C\).

\[11u - 5v = 35\]
\[6u + 8v = 62\]

The coefficients of \(v\)-variables are 5 and 8 and their least common multiple is 40, so multiply each equation by the value that will make the \(v\)-coefficient 40.

\[11u - 5v = 35\text{ Multiply by 8 }\rightarrow\]
\[88u - 40v = 280\]
\[6u + 8v = 62\text{ Multiply by 5 }\rightarrow\]
\[30u + 40v = 310\]
\[118u = 590\]
\[u = 5\]

Substitute 5 for \(u\) into either original equation and solve for \(v\).

\[8v = -6(5) + 62\]
\[8v = -30 + 62\]
\[8v = 32\]
\[v = 4\]

The solution is (5, 4).

**ANSWER:**
(5, 4)

58. \[-1.2c + 3.4d = 6\]
\[6c = -30 + 17d\]

**SOLUTION:**
Rewrite the equation \(6c = -30 + 17d\) in the standard form \(Ax + By = C\).

\[6c - 17d = -30\]

Multiply the equation \(-1.2c + 3.4d = 6\) by 5.

\[5(-1.2c + 3.4d) = 5(6)\]
\[-6c + 17d = 30\]

Add the equations to eliminate one variable.

\[-6c + 17d = 30\]
\[6c - 17d = -30\]
\[0 = 0\]

Because 0 = 0 is true, the system has infinite solutions.

**ANSWER:**
Infinite solutions
Solve each system of equations by using a table.

**SOLUTION:**
Write each equation in slope-intercept form.

\[ 12y = 5x - 15 \]
\[ 4.2y + 6.1x = 11 \]

**SOLUTION:**
Write each equation in the form \( y = mx + b \).

\[ 12y = 5x - 15 \quad 4.2y + 6.1x = 11 \]
\[ y = \frac{5x - 15}{12} \]
\[ 4.2y = -6.1x + 11 \]
\[ y = \frac{-6.1x + 11}{4.2} \]

Enter \( y = \frac{5x - 15}{12} \) as \( Y_1 \) and \( y = \frac{-6.1x + 11}{4.2} \) as \( Y_2 \).

Then graph the lines.

**KEYSTROKES:**
\[ Y = ( \begin{array}{c} 5 \cdot \text{X,T,0,n} \end{array} \) \]
\[ 1 \]
\[ 5 \]
\[ + \]
\[ 1 \]
\[ 2 \]
\[ \text{ENTER} \]
\[ (\begin{array}{c} -6 \cdot \text{X,T,0,n} \end{array} \) \]
\[ + \]
\[ 1 \]
\[ 1 \]
\[ \text{ENTER} \]
\[ 4 \]
\[ \cdot \]
\[ 2 \]
\[ \text{ENTER} \]
\[ \text{ZOOM} \]
\[ 6 \]

Find the intersection of the lines.

**KEYSTROKES:**
\[ \text{2nd} \quad [\text{CALC}] \quad 5 \quad \text{ENTER} \]
\[ \text{ENTER} \]
\[ \text{ENTER} \]

The coordinates of the intersection to the nearest hundredth are \((2.07, -0.39)\).

**ANSWER:**
\((2.07, -0.39)\)
3-1 Solving Systems of Equations

61. \[5.8x - 6.3y = 18\]
   \[-4.3x + 8.8y = 32\]

**SOLUTION:**
Write each equation in the form \(y = mx + b\).

\[
\begin{align*}
5.8x - 6.3y &= 18 \\
-6.3y &= -5.8x + 18 \\
y &= \frac{-5.8x + 18}{-6.3} \\
&= \frac{5.8x - 18}{6.3}
\end{align*}
\]
\[
\begin{align*}
-4.3x + 8.8y &= 32 \\
8.8y &= 4.3x + 32 \\
y &= \frac{4.3x + 32}{8.8}
\end{align*}
\]

Enter \(y = \frac{-5.8x + 18}{-6.3}\) as Y1 and \(y = \frac{4.3x + 32}{8.8}\) as Y2. Then graph the lines.

**KEYSTROKES:**

Y = ( (-) 5 ÷ 8

\[X,T,0,n\] + 18 ) ÷ (-) 6 ÷ 3 ENTER

( 4 ÷ 3 \[X,T,0,n\] + 32 ) ÷ 8 ÷ 8

ENTER ZOOM 3 ENTER ENTER

Find the intersection of the lines.

**KEYSTROKES:** 2nd [CALC] 5 ENTER

ENTER ENTER

The coordinates of the intersection to the nearest hundredth are \((15.03, 10.98)\).

**ANSWER:**

\((15.03, 10.98)\)

---

Solve each system of equations.

11p + 3q = 6

-0.75q - 2.75p = -1.5

**SOLUTION:**
Multiply the equation \(-2.75p - 0.75q = -1.5\) by 4.

\[4\left(-2.75p - 0.75q\right) = 4\left(-1.5\right)\]

\[-11p - 3q = -6\]

Add the equations to eliminate one variable.

\[\begin{align*}
11p + 3q &= 6 \\
\hspace{2cm} (+) \hspace{2cm} -11p - 3q &= -6 \\
\hspace{6cm} 0 &= 0
\end{align*}\]

Because \(0 = 0\) is true, the system has infinite solutions.

**ANSWER:**

Infinite solutions
3-1 Solving Systems of Equations

63. \(8r - 5t = -60\)
\(6r + 3t = -18\)

**SOLUTION:**
The coefficients of \(t\)-variables are 5 and 3 and their least common multiple is 15, so multiply each equation by the value that will make the \(t\)-coefficient 15.

\[
\begin{align*}
8r - 5t &= -60 \quad \text{Multiply by 3} & \quad 24r - 15t &= -180 \\
6r + 3t &= -18 \quad \text{Multiply by 5} & \quad (+) 30r + 15t &= -90 \\
54r &= -270 \\
r &= -5
\end{align*}
\]

Substitute \(-5\) for \(r\) into either original equation and solve for \(t\).

\[
\begin{align*}
6(-5) + 3t &= -18 \\
-30 + 3t &= -18 \\
3t &= 12 \\
t &= 4
\end{align*}
\]

The solution is \((-5, 4)\).

**ANSWER:**
\((-5, 4)\)

64. \(10r + 4v = 13\)
\(-4t - 7v = 11\)

**SOLUTION:**
The coefficients of \(t\)-variables are 10 and 4 and their least common multiple is 20, so multiply each equation by the value that will make the \(t\)-coefficient 20.

\[
\begin{align*}
10r + 4v &= 13 \quad \text{Multiply by 2} & \quad 20r + 8v &= 26 \\
-4t - 7v &= 11 \quad \text{Multiply by 5} & \quad (+) -20t - 35v &= 55 \\
-27v &= 81 \\
v &= -3
\end{align*}
\]

Substitute \(-3\) for \(v\) into either original equation and solve for \(t\).

\[
\begin{align*}
10r + 4(-3) &= 13 \\
10r - 12 &= 13 \\
10r &= 25 \\
t &= 2.5
\end{align*}
\]

The solution is \((2.5, -3)\).

**ANSWER:**
\((2.5, -3)\)
3-1 Solving Systems of Equations

65. \[ 6w = 12 - 4x \]
\[ 6x = -9w + 18 \]

**SOLUTION:**
Rewrite the equations in the standard form of the equation \( Ax + By = C \).

\[-4x + 6w = 12 \]
\[ 6x - 9w = 18 \]

The coefficients of \( x \)-variables are 4 and 6 and their least common multiple is 12, so multiply each equation by the value that will make the \( x \)-coefficient 12.

\[-4x + 6w = 12 \text{ Multiply by 3} \rightarrow -12x + 18w = 36 \]
\[ 6x - 9w = 18 \text{ Multiply by 2} \rightarrow (+) 12x - 18w = 36 \]

\[ 0 = 0 \]

Because \( 0 = 0 \) is true, the system has infinite solutions.

**ANSWER:**
infinite solutions

66. \[ \frac{3}{2}y + z = 3 \]

**SOLUTION:**
Solve the equation \[ \frac{3}{2}y + z = 3 \] for \( z \).

\[ z = 3 - \frac{3}{2}y \]

Substitute \[ 3 - \frac{3}{2}y \] for \( z \) in the equation

\[ -y - \frac{2}{3}z = -2 \]

\[ -y - \frac{2}{3} \left( 3 - \frac{3}{2}y \right) = -2 \]

\[ -y - 2 + y = -2 \]

\[ 0 = 0 \]

Because \( 0 = 0 \) is true, the system has infinite solutions.

**ANSWER:**
infinite solutions
3-1 Solving Systems of Equations

\[ \frac{5}{2}a - \frac{3}{4}b = 46 \]

\[ \frac{7}{8}a - 3b = 10 \]

**SOLUTION:**

Multiply the equation \( \frac{7}{8}a - 3b = 10 \) by \( -\frac{1}{4} \).

\[
-\frac{1}{4} \left( \frac{7}{8}a - 3b \right) = -\frac{1}{4} \left( 10 \right)
\]

\[
\frac{7}{32}a + \frac{3}{4}b = -\frac{5}{2}
\]

Add the equations to eliminate one variable.

\[
\begin{align*}
\frac{5}{2}a - \frac{3}{4}b &= 46 \\
\left( + \right) \frac{7}{32}a + \frac{3}{4}b &= -\frac{5}{2}
\end{align*}
\]

\[
\frac{87}{32}a = \frac{87}{2}
\]

\[ a = 16 \]

Substitute 16 for \( a \) into either original equation and solve for \( b \).

\[
\frac{7}{8}a - 3b = 10
\]

\[
-\frac{7}{8}(16) - 3b = 10
\]

\[
-14 - 3b = 10
\]

\[
-3b = 24
\]

\[ b = -8 \]

The solution is \((16, -8)\).

**ANSWER:**

\((16, -8)\)

68. **ROWING** Allison can row a boat 1 mile upstream (against the current) in 24 minutes. She can row the same distance downstream in 13 minutes. Assume that both the rowing speed and the speed of the current are constant.

a. Find the speed at which Allison is rowing and the speed of the current.

b. If Allison plans to meet her friends 3 miles upstream one hour from now, will she be on time? Explain.

**SOLUTION:**

a. Let \( x \) be the rowing speed.

Let \( y \) be the speed of the current.

Allison’s speed in upstream: \( \frac{60}{24} = 2.5 \) mph.

Allison’s speed in downstream: \( \frac{60}{13} = 4.62 \) mph.

The system of equations that represents this situation is

\[ x - y = 2.5 \quad \text{and} \]

\[ x + y = 4.62 \]

Add the equations to eliminate one variable.

\[
\begin{align*}
x - y &= 2.5 \\
\left( + \right) x + y &= 4.62
\end{align*}
\]

\[
2x = 7.12
\]

\[ x = 3.56 \]

Substitute 3.56 for \( x \) into either original equation and solve for \( y \).

\[
\begin{align*}
x + y &= 4.62 \\
3.56 + y &= 4.62
\end{align*}
\]

\[ y = 1.06 \]

The speed of the rowing boat is 3.56 mph and the speed of the current is 1.06 mph.

b. No, Allison will be late.

To row 3 miles upstream she needs 72 minutes.

So, \( 72 - 60 = 12 \) minutes.

She will be 12 minutes late.

**ANSWER:**

a. 3.56 mph; 1.06 mph

b. No; she will be 12 minutes late.

69. **CCSS MODELING** The table shows the winning times in seconds for the 100-meter dash at the Olympics between 1964 and 2008.
3-1 Solving Systems of Equations

<table>
<thead>
<tr>
<th>Years Since 1964, x</th>
<th>Men’s Gold Medal Time</th>
<th>Women’s Gold Medal Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.0</td>
<td>11.4</td>
</tr>
<tr>
<td>4</td>
<td>9.90</td>
<td>11.0</td>
</tr>
<tr>
<td>8</td>
<td>10.14</td>
<td>11.07</td>
</tr>
<tr>
<td>12</td>
<td>10.06</td>
<td>11.08</td>
</tr>
<tr>
<td>16</td>
<td>10.23</td>
<td>11.06</td>
</tr>
<tr>
<td>20</td>
<td>9.99</td>
<td>10.97</td>
</tr>
<tr>
<td>24</td>
<td>9.92</td>
<td>10.54</td>
</tr>
<tr>
<td>28</td>
<td>9.96</td>
<td>10.82</td>
</tr>
<tr>
<td>32</td>
<td>9.64</td>
<td>10.94</td>
</tr>
<tr>
<td>36</td>
<td>9.87</td>
<td>10.75</td>
</tr>
<tr>
<td>40</td>
<td>9.93</td>
<td>10.93</td>
</tr>
<tr>
<td>44</td>
<td>9.69</td>
<td>10.78</td>
</tr>
</tbody>
</table>

a. Write equations that represent the winning times for men and women since 1964. Assume that both times continue along the same trend.
b. Graph both equations. Estimate when the women’s performance will catch up to the men’s performance. Do you think that your prediction is reasonable? Explain.

**SOLUTION:**
a. Sample answer for men using (0, 10) and (44, 9.69):
\[ y_m = -0.00705x + 10 \]; sample answer for women using (0, 11.4) and (44, 10.78):
\[ y_w = -0.01409x + 11.4 \]
b.

Based on these data, the women’s performance will catch up to the men’s performance 198 years after 1964, or in the year 2162. The next Olympic year would be 2164; this prediction is not reasonable. It is unlikely that women’s times will ever catch up to men’s times because the times cannot continue to increase and decrease infinitely.

70. **JOBS** Levi has a job offer in which he will receive $800 per month plus a commission of 2% of the total price of cars he sells. At his current job, he receives $1200 per month plus a commission of 1.5% of his total sales. How much must he sell per month to make the new job a better deal?

**SOLUTION:**
Let \( x \) be the total price of cars Levi sells.

\[
800 + 0.02x > 1200 + 0.015x
\]

\[
0.02x - 0.015x > 1200 - 800
\]

\[
0.005x > 400
\]

\[
\frac{0.005x}{0.005} > \frac{400}{0.005}
\]

\[
x > 80000
\]

To make the new job a better deal, he has to sell at least $80,000 worth of cars.

**ANSWER:**
more than $80,000
71. **TRAVEL** A youth group went on a trip to an amusement park, travelling in two vans. The number of people in each van and the total cost of admission are shown in the table. Find the adult price and student price of admission.

<table>
<thead>
<tr>
<th>Adults</th>
<th>Students</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van A</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Van B</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \( x \) be the adult price of admission.
Let \( y \) be the student price of admission.
The system of equations that represents the situation is

\[
2x + 5y = 77 \quad \text{and} \quad 2x + 7y = 95.
\]

Multiply the equation \( 2x + 5y = 77 \) by \(-1\).

\[
-1(2x + 5y) = -1(77) \quad \Rightarrow \quad -2x - 5y = -77
\]

Add the equations to eliminate one variable.

\[
\begin{align*}
-2x - 5y &= -77 \\
2x + 7y &= 95
\end{align*}
\]

\[
2y = 18
\]

\[
y = 9
\]

Substitute 9 for \( y \) into either original equation and solve for \( x \).

\[
2x + 5(9) = 77 \quad \Rightarrow \quad 2x + 45 = 77 \quad \Rightarrow \quad 2x = 32 \quad \Rightarrow \quad x = 16
\]

The price of admission for adults is $16 and for students is $9.

**ANSWER:**
adult: $16; student: $9

**GEOMETRY** Find the point at which the diagonals of the quadrilaterals intersect.

**SOLUTION:**
The quadrilateral has the vertices \( A(2, 2) \), \( B(0, 8) \), \( C(10, 5) \), and \( D(8, 2) \).

The equation of the lines \( AC \) and \( BD \) are

\[
y = \frac{3}{8}x + \frac{5}{4} \quad \text{and} \quad y = -\frac{3}{4}x + 8.
\]

Substitute \( \frac{3}{8}x + \frac{5}{4} \) for \( y \) in the equation

\[
y = -\frac{3}{4}x + 8
\]

and solve for \( x \).

\[
\begin{align*}
\frac{3}{8}x + \frac{5}{4} &= -\frac{3}{4}x + 8 \\
\frac{9}{8}x &= 27 \\
x &= 6
\end{align*}
\]

Substitute 6 for \( x \) into either original equation and solve for \( y \).

\[
\begin{align*}
y &= -\frac{3}{4}x + 8 \\
y &= -\frac{3}{4}(6) + 8 \\
y &= -\frac{9}{2} + 8 \\
y &= \frac{7}{2} \\
y &= 3.5
\end{align*}
\]

The diagonals of the quadrilateral intersect at (6,
3-1 Solving Systems of Equations

73. **SOLUTION:**

The quadrilateral has the vertices A(3, 4), B(2, 9), C (11, 18), and D(6, 3).

The equation of the lines $AC$ and $BD$ are

\[ y = \frac{7}{4} x - \frac{5}{4} \quad \text{and} \quad y = -\frac{3}{2} x + 12. \]

Substitute $\frac{7}{4} x - \frac{5}{4}$ for $y$ in the equation

\[ y = \frac{3}{2} x + 12 \]

and solve for $x$.

\[
\frac{7}{4} x - \frac{5}{4} = \frac{3}{2} x + 12 \\
\frac{7}{4} x + \frac{3}{2} x = 12 + \frac{5}{4} \\
\frac{13}{4} x = \frac{53}{4} \\
x = \frac{53}{13}
\]

Substitute $\frac{53}{13}$ for $x$ into either original equation and solve for $y$.

\[ y = -\frac{3}{2} \left( \frac{53}{13} \right) + 12 \]

\[ y = -\frac{159}{26} + 12 \]

\[ y = \frac{153}{26} \]

The diagonals of the quadrilateral intersect at

\[ \left( \frac{53}{13}, \frac{153}{26} \right) \]

**ANSWER:**

\[ \left( \frac{53}{13}, \frac{153}{26} \right) \]

74. **ELECTIONS** In the election for student council, Candidate A received 55% of the total votes, while Candidate B received 1541 votes. If Candidate C received 40% of the votes that Candidate A received, how many total votes were cast?

**SOLUTION:**

Let $x$ be the total number of votes.

The equation that represents the situation is

\[ x - 1541 = 0.55x + 0.40(0.55x). \]

Solve for $x$.

\[ x - 1541 = 0.55x + 0.40(0.55x) \]

\[ x - 1541 = 0.55x + 0.22x \]

\[ x - 1541 = 0.77x \]

\[ 0.23x = 1541 \]

\[ x = 6700 \]

The total number of votes is 6700.

**ANSWER:**

6700 votes

75. **MULTIPLE REPRESENTATIONS** In this problem, you will explore systems of equations with three linear equations and two variables.
3-1 Solving Systems of Equations

3y + x = 16
y - 2x = -4
y + 5x = 10

a. **TABULAR** Make a table of $x$ and $y$-values for each equation.

b. **ANALYTICAL** Which values from the table indicate intersections? Is there a solution that satisfies all three equations?

c. **GRAPHICAL** Graph the three equations on a single coordinate plane.

d. **VERBAL** What conditions must be met for a system of three equations with two variables to have a solution? What conditions result in no solution?

**SOLUTION:**

a. The table of $x$ and $y$ values for each equation:

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{16}{3}$</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{14}{3}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{13}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

b. Equations 1 and 2 intersect at (4, 4), equations 2 and 3 intersect at (2, 0), and equations 1 and 3 intersect at (1, 5); there is no solution that satisfies all three equations.

c. The graph of three equations on a single coordinate plane is

d. If all three lines intersect at the same point, then the system has a solution. The system has no solution if the lines intersect at 3 different points, or if two or three lines are parallel.

**ANSWER:**

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$\frac{16}{3}$</td>
</tr>
<tr>
<td>$1$</td>
<td>$5$</td>
</tr>
<tr>
<td>$2$</td>
<td>$\frac{14}{3}$</td>
</tr>
<tr>
<td>$3$</td>
<td>$\frac{13}{3}$</td>
</tr>
<tr>
<td>$4$</td>
<td>$4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>$0$</td>
</tr>
<tr>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
</tr>
<tr>
<td>$3$</td>
</tr>
<tr>
<td>$4$</td>
</tr>
</tbody>
</table>
76. **CCSS CRITIQUE** Gloria and Syreeta are solving the system $6x - 4y = 26$ and $-3x + 4y = -17$. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Sample answer: Gloria is correct. Syreeta subtracted 26 from 17 instead of 17 from 26 and got $3x = -9$ instead of $3x = 9$. She proceeded to get a value of $-11$ for $y$. She would have found her error if she substituted the solution into the original equations.

**ANSWER:**
Sample answer: Gloria; Syreeta subtracted 26 from 17 instead of 17 from 26 and got $3x = -9$ instead of $3x = 9$. She proceeded to get a value of $-11$ for $y$. She would have found her error if she substituted the solution into the original equations.

77. **CHALLENGE** Find values of $a$ and $b$ for which the following system has a solution of $(b - 1, b - 2)$.

$$-8ax + 4ay = -12a$$
$$2bx - by = 9$$

**SOLUTION:**
Substitute $b - 1$ for $x$ and $b - 2$ for $y$ in the equation $-8ax + 4ay = -12a$ and solve for $b$.

$$-8a(b - 1) + 4a(b - 2) = -12a$$
$$-8ab + 8a + 4ab - 8a = -12a$$
$$-4ab = -12a$$
$$b = \frac{-12a}{-4a}$$
$$b = 3, a \neq 0$$

**ANSWER:**
$a \neq 0, b = 3$

78. **REASONING** If $a$ is consistent and dependent with $b$, $b$ is inconsistent with $c$, and $c$ is consistent and independent with $d$, then $a$ will sometimes, always, or never be consistent and independent with $d$. Explain your reasoning.

**SOLUTION:**
Sample answer: Always; $a$ and $b$ are the same line. $b$ is parallel to $c$, so $a$ is also parallel to $c$. Since $c$ and $d$ are consistent and independent, then $c$ is not parallel to $d$ and, thus, intersects $d$. Since $a$ and $c$ are parallel, then $a$ cannot be parallel to $d$, so, $a$ must intersect $d$ and must be consistent and independent with $d$.

**ANSWER:**
Sample answer: Always; $a$ and $b$ are the same line. $b$ is parallel to $c$, so $a$ is also parallel to $c$. Since $c$ and $d$ are consistent and independent, then $c$ is not parallel to $d$ and, thus, intersects $d$. Since $a$ and $c$ are parallel, then $a$ cannot be parallel to $d$, so, $a$ must intersect $d$ and must be consistent and independent with $d$. 

---
3-1 Solving Systems of Equations

79. **OPEN ENDED** Write a system of equations in which one equation needs to be multiplied by 3 and the other needs to be multiplied by 4 in order to solve the system with elimination. Then solve your system.

**SOLUTION:**
Sample answer:
The system of equations is

\[
\begin{align*}
4x + 5y &= 21 \\
3x - 2y &= 10.
\end{align*}
\]

\[
\begin{align*}
4x + 5y &= 21 \quad \text{and} \quad 3(4x + 5y = 21) \\
3x - 2y &= 10 \quad \text{and} \quad 4(3x - 2y = 10)
\end{align*}
\]

\[
\begin{align*}
12x + 15y &= 63 \\
(−)12x - 8y &= 40
\end{align*}
\]

\[
23y = 23 \\
y = 1
\]

\[
\begin{align*}
4x + 5(1) &= 21 \\
4x + 5 &= 21 \\
4x &= 16 \\
x &= 4
\end{align*}
\]

The solution of the system is (4, 1).

**ANSWER:**
Sample answer:
4x + 5y = 21 and 3x - 2y = 10;
(4, 1)

80. **WRITING IN MATH** Why is substitution sometimes more helpful than elimination, and vice versa?

**SOLUTION:**
Sample answer: It is more helpful to use substitution when one of the variables has a coefficient of 1 or if a coefficient can be reduced to 1 without turning other coefficients into fractions. Otherwise, elimination is more helpful because it will avoid the use of fractions in solving the system.

**ANSWER:**
Sample answer: It is more helpful to use substitution when one of the variables has a coefficient of 1 or if a coefficient can be reduced to 1 without turning other coefficients into fractions. Otherwise, elimination is more helpful because it will avoid the use of fractions in solving the system.

81. **SHORT RESPONSE** Simplify 3y(4x + 6y – 5)

**SOLUTION:**
3y(4x + 6y – 5) = 3y · 4x + 3y · 6y – 3y · 5

\[
= 12xy + 18y^2 - 15y
\]

**ANSWER:**
12xy + 18y^2 - 15y
82. **ACT/SAT** Which of the following best describes the graph of the equations?

\[ 4y = 3x + 8 \]
\[ -6x = -8y + 24 \]

A The lines are parallel.
B The lines are perpendicular.
C The lines have the same x-intercept.
D The lines have the same y-intercept.
E The lines are the same.

**SOLUTION:**
Write each equation in slope-intercept form.

\[ 4y = 3x + 8 \]
\[ y = \frac{3}{4}x + 2 \]

Also:

\[ -6x = -8y + 24 \]
\[ 8y = 6x + 24 \]
\[ y = \frac{3}{4}x + 3 \]

The slopes are equal. So the lines are parallel. The correct choice is **A**.

**ANSWER:**
A

---

83. **GEOMETRY** Which set of dimensions corresponds to a triangle similar to the one shown at the right?

- **F** 1 unit, 2 units, 3 units
- **G** 7 units, 11 units, 12 units
- **H** 10 units, 23 units, 24 units
- **J** 20 units, 48 units, 52 units

SOLUTION:
The dimensions of the given triangle and the dimensions given in choice **J** have a common ratio. So, the correct choice is **J**.

**ANSWER:**
J

---

84. Move-A-Lot Rentals will rent a moving truck for $100 plus $0.10 for every mile it is driven. Which equation can be used to find **C**, the cost of renting a moving truck and driving it for **m** miles?

- **A** \( C = 0.1(100 + m) \)
- **B** \( C = 100 + 0.1m \)
- **C** \( C = 100m + 0.1 \)
- **D** \( C = 100(m + 0.1) \)

**SOLUTION:**
Let **m** be the number of miles the truck is driven. So:
\[ C = 100 + 0.10m \]
The correct choice is **B**.

**ANSWER:**
B

---

85. **CRAFTS** Priscilla sells stuffed animals at a local craft show. She charges $10 for the small ones and $15 for the large ones. To cover her expenses, she needs to sell at least $350.

a. Write an inequality for this situation.

b. Graph the inequality.

c. If she sells 10 small and 15 large animals, will she cover her expenses?
3-1 Solving Systems of Equations

**SOLUTION:**

a. Let \( s \) represent the small animals and \( l \) represent the large ones. 
Priscilla needs to sell at least $350. 
So: 
\[ 10s + 15l \geq 350 \]

b. The boundary is the graph of \( 10s + 15l = 350 \). 
Since the inequality is \( \geq \), the boundary line will be solid. 
Test the point \((0, 0)\). 
\[ 10(0) + 15(0) \geq 350 \]
\[ 0 \geq 350 \quad \text{FALSE} \]
Shade the region that does not include \((0, 0)\).

c. Test the point \((10, 15)\). 
\[ 10(10) + 15(15) \geq 350 \]
\[ 100 + 225 \geq 350 \]
\[ 325 \geq 350 \quad \text{FALSE} \]
So, Priscilla will not cover her expenses if she sells 10 small and 15 large animals.

**ANSWER:**

a. \( 10s + 15l \geq 350 \)

b. ...
3-1 Solving Systems of Equations

SOLUTION:
The graph is a transformation of the graph of the parent function \( y = |x| \).
When a constant \( h \) is added to or subtracted from \( x \) before evaluating a parent function, the result \( f(x \pm h) \), is a translation left or right.
When a constant \( k \) is added to or subtracted from a parent function, the result \( f(x) \pm k \) is a translation of the graph up or down.
A reflection flips a figure over a line called the line of reflection. The reflection \(-f(x)\) reflects the graph of \( f(x) \) across the \( x\)-axis and the reflection \( f(-x) \) reflects the graph of \( f(x) \) across the \( y\)-axis.
The parent function is moved 3 units right and reflected across the \( x\)-axis.
So, the equation of the graph is \( y = -|x - 3| \).

ANSWER:
\[ y = -|x - 3| \]

Solve each equation. Check your solution.
89. \( 2p = 14 \)

SOLUTION:
\[ 2p = 14 \]
\[ p = 7 \]
Substitute \( p = 7 \) in the original equation and check.

\[ 2(7) = 14 \]
\[ 14 = 14 \]  \( \checkmark \)
So, the solution is \( p = 7 \).

ANSWER:
\[ 7 \]
3-1 Solving Systems of Equations

90. \(-14 + n = -6\)

**SOLUTION:**

\[-14 + n = -6\]
\[n = 8\]

Substitute \(n = 8\) in the original equation and check.

\[-14 + 8 = -6\]
\[-6 = -6\]  \(\checkmark\)

So, the solution is \(n = 8\).

**ANSWER:**

8

91. \(7a - 3a + 2a - a = 16\)

**SOLUTION:**

\[7a - 3a + 2a - a = 16\]
\[5a = 16\]
\[a = \frac{16}{5}\]
\[a = 3.2\]

Substitute \(a = 3.2\) in the original equation and check.

\[7(3.2) - 3(3.2) + 2(3.2) - 3.2 = 16\]
\[22.4 - 9.6 + 6.4 - 3.2 = 16\]
\[16 = 16\]  \(\checkmark\)

So, the solution is \(a = 3.2\).

**ANSWER:**

3.2

92. \(x + 9x - 6x + 4x = 20\)

**SOLUTION:**

\[x + 9x - 6x + 4x = 20\]
\[14x - 6x = 20\]
\[8x = 20\]
\[x = 2.5\]

Substitute \(x = 2.5\) in the original equation and check.

\[2.5 + 9(2.5) - 6(2.5) + 4(2.5) = 20\]
\[2.5 + 22.5 - 15 + 10 = 20\]
\[20 = 20\]  \(\checkmark\)

So, the solution is \(x = 2.5\).

**ANSWER:**

2.5

93. \(27 = -9(y + 5) + 6(y + 8)\)

**SOLUTION:**

\[27 = -9(y + 5) + 6(y + 8)\]
\[27 = -9y - 45 + 6y + 48\]
\[27 = -3y + 3\]
\[-9 = y - 1\]
\[-8 = y\]

Substitute \(y = -8\) in the original equation and check.

\[27 = -9(-8 + 5) + 6(-8 + 8)\]
\[27 = -9(-3) + 0\]
\[27 = 27\]  \(\checkmark\)

So, the solution is \(y = -8\).

**ANSWER:**

-8
3-1 Solving Systems of Equations

94. \(-7(p + 7) + 3(p - 4) = -17\)

\textbf{SOLUTION:}
\[
-7(p + 7) + 3(p - 4) = -17
\]
\[
-7p - 49 + 3p - 12 = -17
\]
\[
-4p - 61 = -17
\]
\[
-4p = 44
\]
\[
p = -11
\]

Substitute \(p = -11\) in the original equation and check.
\[
-7(-11 + 7) + 3(-11 - 4) = -17
\]
\[
-7(-4) + 3(-15) = -17
\]
\[
28 - 45 = -17
\]
\[
-17 = -17 \checkmark
\]

So, the solution is \(p = -11\).

\textbf{ANSWER:}
\[-11\]

\textbf{Determine whether the given point satisfies each inequality.}

95. \(4x + 5y \leq 15; (2, -2)\)

\textbf{SOLUTION:}
Substitute 2 for \(x\) and \(-2\) for \(y\) in the inequality.

\[
4x + 5y \leq 15
\]
\[
4(2) + 5(-2) \leq 15
\]
\[
8 - 10 \leq 15
\]
\[
-2 \leq 15 \checkmark
\]

The point \((2, -2)\) satisfies the inequality \(4x + 5y \leq 15\).

\textbf{ANSWER:}
\[\text{yes}\]

96. \(3x + 5y \geq 8; (1, 1)\)

\textbf{SOLUTION:}
Substitute 1 for \(x\) and 1 for \(y\) in the inequality.

\[
3x + 5y \geq 8
\]
\[
3(1) + 5(1) \geq 8
\]
\[
8 \geq 8 \checkmark
\]

The point \((1, 1)\) satisfies the inequality \(3x + 5y \geq 8\).

\textbf{ANSWER:}
\[\text{yes}\]

97. \(6x + 9y < -1; (0, 0)\)

\textbf{SOLUTION:}
Substitute 0 for \(x\) and 0 for \(y\) in the inequality.

\[
6x + 9y < -1
\]
\[
6(0) + 9(0) < -1
\]
\[
0 < -1 \times
\]

The point \((0, 0)\) does not satisfy the inequality \(6x + 9y < -1\).

\textbf{ANSWER:}
\[\text{no}\]
3-2 Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

1. \( y \leq 6 \)
   \( y > -3 + x \)

\textbf{SOLUTION:}
The graph of the system of inequalities is

\begin{align*}
\text{Graph: } y &= 6 \\
\text{Graph: } y &= -3 + x
\end{align*}

\textbf{ANSWER:}

\begin{align*}
\text{Graph: } y &= 6 \\
\text{Graph: } y &= -3 + x
\end{align*}

\begin{align*}
\text{Graph: } y &= 6 \\
\text{Graph: } y &= -3 + x
\end{align*}

2. \( y \leq -3x + 4 \)
   \( y \geq 2x - 1 \)

\textbf{SOLUTION:}
The graph of the system of inequalities is

\begin{align*}
\text{Graph: } y &= -3x + 4 \\
\text{Graph: } y &= 2x - 1
\end{align*}

\textbf{ANSWER:}

\begin{align*}
\text{Graph: } y &= -3x + 4 \\
\text{Graph: } y &= 2x - 1
\end{align*}

\begin{align*}
\text{Graph: } y &= -3x + 4 \\
\text{Graph: } y &= 2x - 1
\end{align*}

3. \( y > -2x + 4 \)
   \( y \leq -3x - 3 \)

\textbf{SOLUTION:}
The graph of the system of inequalities is

\begin{align*}
\text{Graph: } y &= -2x + 4 \\
\text{Graph: } y &= -3x - 3
\end{align*}

\textbf{ANSWER:}

\begin{align*}
\text{Graph: } y &= -2x + 4 \\
\text{Graph: } y &= -3x - 3
\end{align*}

\begin{align*}
\text{Graph: } y &= -2x + 4 \\
\text{Graph: } y &= -3x - 3
\end{align*}

4. CCSS REASONING The most Kala can spend on hot dogs and buns for her cookout is $35. A package of 10 hot dogs costs $3.50. A package of buns costs $2.50 and contains 8 buns. She needs to buy at least 40 hot dogs and 40 buns.
   \begin{enumerate}
   \item[\textbf{a.}] Graph the region that shows how many packages of each item she can purchase.
   \item[\textbf{b.}] Give an example of three different purchases she can make.
   \end{enumerate}

\textbf{SOLUTION:}
\begin{enumerate}
\item[\textbf{a.}] Let \( h \) be the number of package of hotdogs.
   Let \( b \) be the number of package of buns.
   The inequalities that represents the situation are
   \begin{align*}
   2.5b + 3.5h &\leq 35, \\
   h &\geq 4 \text{ and } \\
   b &\geq 5.
   \end{align*}

   The graph of the region that shows the number of packages of each item Kala can purchase is

\end{enumerate}
b. Sample answer: 4 packages of hotdogs, 5 packages of buns; 5 packages of hotdogs, 6 packages of buns; 6 packages of hotdogs, 5 packages of buns.

**ANSWER:**

**a.**

b. Sample answer: 4 packages of hotdogs, 5 packages of buns; 5 packages of hotdogs, 6 packages of buns; 6 packages of hotdogs, 5 packages of buns

**Find the coordinates of the vertices of the triangle formed by each system of inequalities.**

\[ y \geq 2x + 1 \]

5. \( y \leq 8 \)

\[ 4x + 3y \geq 8 \]

**SOLUTION:**

The graph of the system of inequalities is

The coordinate \((-4, 8)\) can be determined from the graph.

Solve the system of equations \(y = 8\) and \(y = 2x + 1\).

Substitute 8 for \(y\) in the equation \(y = 2x + 1\) and solve for \(x\).

\[
\begin{align*}
8 &= 2x + 1 \\
7 &= 2x \\
x &= 3.5
\end{align*}
\]

So, the coordinate is \((3.5, 8)\).

Solve the system of equations \(y = 2x + 1\) and \(4x + 3y = 8\).

Substitute \(2x + 1\) for \(y\) in the equation \(4x + 3y = 8\) and solve for \(x\).

\[
\begin{align*}
4x + 3(2x + 1) &= 8 \\
4x + 6x + 3 &= 8 \\
10x + 3 &= 8 \\
10x &= 5 \\
x &= 0.5
\end{align*}
\]

Substitute 0.5 for \(x\) in the equation \(y = 2x + 1\) and find \(y\).

\[
\begin{align*}
y &= 2(0.5) + 1 \\
y &= 1 + 1 \\
y &= 2
\end{align*}
\]

So, the coordinate is \((0.5, 2)\).

The vertices of the triangle are at \((3.5, 8)\), \((-4, 8)\), and \((0.5, 2)\).

**ANSWER:**
Solve each system of inequalities by graphing.

1.

SOLUTION:
The graph of the system of inequalities is

The region that does not contain (0, 0) is shaded.

6.

$6y \leq x + 28$
$y \geq 13x - 34$

SOLUTION:
The graph of the system of inequalities is

The coordinate (−4, 4) and (2, −8) can be determined from the graph.

Solve the system of equations

$6y = x + 28$ and $y = 13x - 34$.

Substitute $13x - 34$ for $y$ in the equation

$6y = x + 28$ and solve for $x$.

$6(13x - 34) = x + 28$

$78x - 204 = x + 28$

$77x = 232$

$x \approx 3$

Substitute 3 for $x$ in the equation $y = 13x - 34$ and find $y$.

$y = 13(3) - 34$

$= 39 - 34$

$= 5$

So, the coordinate is (3, 5).

The vertices of the triangle are at (−4, 4), (2, −8) and (3, 5).

ANSWER:
3-2 Solving Systems of Inequalities by Graphing

8. \( y > 3x - 5 \)
   \( y \leq 4 \)

**SOLUTION:**
Graph the system of inequalities in a coordinate plane.

![Graph of system of inequalities](image)

**ANSWER:**

9. \( y < -3x + 4 \)
   \( 3y + x > -6 \)

**SOLUTION:**
Graph the system of inequalities in a coordinate plane.

![Graph of system of inequalities](image)

**ANSWER:**

10. \( y \geq 0 \)
    \( y < x \)

**SOLUTION:**
Graph the system of inequalities in a coordinate plane.

![Graph of system of inequalities](image)

**ANSWER:**

11. \( 6x - 2y \geq 12 \)
    \( 3x + 4y > 12 \)

**SOLUTION:**
Graph the system of inequalities in a coordinate plane.

![Graph of system of inequalities](image)

**ANSWER:**
3-2 Solving Systems of Inequalities by Graphing

12. \(-8x > -2y - 1\)
   \(-4y \geq 2x - 5\)

   **SOLUTION:**
   Graph the system of inequalities in a coordinate plane.

   ![Graph of system of inequalities](image)

   **ANSWER:**

13. \(5y < 2x + 10\)
   \(y - 4x > 8\)

   **SOLUTION:**
   The graph of the system of inequalities is

   ![Graph of system of inequalities](image)

   **ANSWER:**

14. \(3y - 2x \leq -24\)
   \(y \geq \frac{2}{3}x - 1\)

   **SOLUTION:**
   Graph the system of inequalities in a coordinate plane.

   ![Graph of system of inequalities](image)

   **ANSWER:**

15. \(y > -\frac{2}{5}x + 2\)
   \(5y \leq -2x - 15\)

   **SOLUTION:**
   Graph the system of inequalities in a coordinate plane.

   ![Graph of system of inequalities](image)

   **ANSWER:**
3-2 Solving Systems of Inequalities by Graphing

16. RECORDING Jane’s band wants to spend no more than $575 recording their first CD. The studio charges at least $35 an hour to record. Graph a system of inequalities to represent this situation.

**SOLUTION:**
Let \( y \) be the cost to record the CD. Jane’s band wants to spend no more than $575. So:
\[
y \leq 575
\]
Let \( x \) represent the time taken to record CD in hours. The studio charges at least $35 an hour. So:
\[
y \geq 35x.
\]
Graph the system of inequalities.

![Graph of the system of inequalities](image)

**ANSWER:**

17. SUMMER TRIP Rondell has to save at least $925 to go to Rome with his Latin class in 8 weeks. He earns $9 an hour working at the Pizza Palace and $12 an hour working at a car wash. By law, he cannot work more than 25 hours per week. Graph two inequalities that Rondell can use to determine the number of hours he needs to work at each job if he wants to make the trip.

**SOLUTION:**
Let \( x \) be the number of hours that Rondell works at the Pizza Palace and \( y \) be the number of hours he works at the car wash. So:
\[
x + y \leq 8(25)
\]
\[
x + y \leq 200
\]
Also:
\[
9x + 12y \geq 925
\]
Graph the system of inequalities and find the solution.

![Graph of the system of inequalities](image)

**ANSWER:**
### 3-2 Solving Systems of Inequalities by Graphing

Find the coordinates of the vertices of the triangle formed by each system of inequalities.

18. \( y \geq 0 \)
   \( x + 2y < 4 \)

**SOLUTION:**
Graph the system of inequalities.

The coordinates of the vertices are (0, 2), (4, 0), and (0, 0).

**ANSWER:**

19. \( y \geq 3x - 7 \)
   \( y \leq 8 \)
   \(-4x + 4y < -4 \)

**SOLUTION:**
Graph the system of inequalities.

The coordinates of the vertices are (2, -1), (5, 8), and (-7, 8).

**ANSWER:**
3-2 Solving Systems of Inequalities by Graphing

Solve each system of inequalities by graphing.

20. \( y > -3x + 12 \)
   \( y \leq 9 \)

**SOLUTION:**
Graph the system of inequalities.

The coordinates of the vertices are (1, 9), (4, 0), and (4, 9).

**ANSWER:**

\[(1, 9), (4, 0), (4, 9)\]

21. \( -3x + 4y \leq 15 \)
   \( 2y + 5x > -12 \)
   \( 10y + 60 \geq 27x \)

**SOLUTION:**
Graph the system of inequalities.

The coordinates of the vertices are \((-3, 1.5), (0, -6), \) and \((5, 7.5)\).

**ANSWER:**

\[(-3, 1.5), (5, 7.5), (0, -6)\]
Solve each system of inequalities by graphing.

**SOLUTION:**
Graph the system of inequalities.

The coordinates of the vertices are (–6, –5), (–2, 4.5), and (7.5, –2).

**ANSWER:**

(–6, –5), (–2, 4.5), (7.5, –2)
24. **BAKING** Rebecca wants to bake cookies and cupcakes for a bake sale. She can bake 15 cookies at a time and 12 cupcakes at a time. She needs to make at least 120 baked goods, but no more than 360, and she wants to have at least three times as many cookies as cupcakes. What combination of batches of each could Rebecca make?

**SOLUTION:**
Let \( x \) be the number of cookies and \( y \) be the number of cupcakes.
So:
\[ 120 \leq x + y \leq 360 \]
And:
\[ x \geq 3y; x \geq 0; y \geq 0 \]
Graph the inequalities.

The dark shaded region is the region where all the inequalities are true.
Pick a point in this region. Since we need to have whole numbers of batches, choose an \( x \)-coordinate which is a multiple of 15 and a \( y \)-coordinate which is a multiple of 12.
Sample answer: \((225, 72)\).

To make 225 cookies takes \( \frac{225}{15} = 15 \) batches, and to make 72 cupcakes takes \( \frac{72}{12} = 6 \) batches.

**ANSWER:**
Sample answer: 15 batches of cookies and 6 batches of cupcakes.

25. **CELL PHONES** Dale has a maximum of 800 minutes on his cell phone plan that he can use each month. Daytime minutes cost $0.15, and nighttime minutes cost $0.10. Dale plans to use at least twice as many daytime minutes as nighttime minutes. If Dale uses at least 200 nighttime minutes and does not go over his limit, what is his maximum bill? His minimum bill?

**SOLUTION:**
Let \( x \) be the number of daytime minutes and \( y \) be the number of nighttime minutes.
\[ x + y \leq 800 \]
\[ 2y \leq x \]
\[ y \geq 200 \]
The optimize function is \( f(x, y) = 0.15x + 0.10y \).
Graph the inequalities in the same coordinate plane.

The vertices of the shaded region are \((600, 200)\), \((400, 200)\), and \(\left(\frac{1600}{3}, \frac{800}{3}\right)\).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((600, 200))</td>
<td>(0.15(600) + 0.10(200) = 110)</td>
</tr>
<tr>
<td>((400, 200))</td>
<td>(0.15(400) + 0.10(200) = 80)</td>
</tr>
<tr>
<td>(\left(\frac{1600}{3}, \frac{800}{3}\right))</td>
<td>(0.15\left(\frac{1600}{3}\right) + 0.10\left(\frac{800}{3}\right) \approx 107)</td>
</tr>
</tbody>
</table>

The maximum bill is $110 and the minimum bill is $80.

**ANSWER:**
maximum = $110, minimum = $80
26. **TREES** Trees are divided into four categories according to height and trunk circumference. In one forest, the trees are categorized by the heights and circumferences described in the table.

<table>
<thead>
<tr>
<th>Crown Class</th>
<th>dominant</th>
<th>co-dominant</th>
<th>intermediate</th>
<th>suppressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in feet)</td>
<td>over 72</td>
<td>56–72</td>
<td>40–55</td>
<td>under 39</td>
</tr>
<tr>
<td>Trunk Circumference (in inches)</td>
<td>over 60</td>
<td>48–60</td>
<td>34–48</td>
<td>under 35</td>
</tr>
</tbody>
</table>

a. Write and graph the system of inequalities that represents the range of heights \( h \) and circumferences \( c \) for a co-dominant tree.
b. Determine the crown class of a basswood that is 48 feet tall. Find the expected trunk circumference.

**SOLUTION:**
a. The height of co-dominant tree ranges from 56 to 72.
So:
\[ h \geq 56; h \leq 72 \]
The trunk circumference of the co-dominant tree ranges from 48 to 60.
Therefore:
\[ c \geq 48; c \leq 60. \]
b. 48 lies between 40 and 55. So, the class is intermediate and the expected trunk circumference is 34–48 in.

**ANSWER:**
a. \( h \geq 56, h \leq 72, c \geq 48, c \leq 60 \)
b. intermediate; 34–48 in.

27. **CCSS REASONING** On a camping trip, Jessica needs at least 3 pounds of food and 0.5 gallon of water per day. Marc needs at least 5 pounds of food and 0.5 gallon of water per day. Jessica’s equipment weighs 10 pounds, and Marc’s equipment weighs 20 pounds.
A gallon of water weighs approximately 8 pounds.
Each of them carries their own supplies, and Jessica is capable of carrying 35 pounds while Marc can carry 50 pounds.

a. Graph the inequalities that represent how much they can carry.
b. How many days can they camp, assuming that they bring all their supplies in at once?
c. Who will run out of supplies first?

**SOLUTION:**
a. A gallon of water weighs approximately 8 pounds, so 0.5 gallon of water weighs about 4 pounds. Let \( x \) be the number of days and \( y \) be the number of pounds.
The system of inequalities that represent the situation is:

\[
\begin{align*}
y & \geq 9x + 20 \\
y & \geq 7x + 10 \\
y & \leq 35 \\
y & \leq 50
\end{align*}
\]

Graph the system of inequalities in the same coordinate plane.

\[
\begin{align*}
y & \geq 9x + 20 \\
y & \geq 7x + 10 \\
y & \leq 35 \\
y & \leq 50
\end{align*}
\]

b & c. Assume Jessica only uses the supplies she has carried, and Marc only uses the supplies he has carried. Then Marc will run out of supplies at the point where \( y = 9x + 20 \) intersects the line \( y = 50 \).
This point is \( \left( \frac{1}{3}, 50 \right) \), so Marc will run out of supplies after \( \frac{1}{3} \) days. Jessica will run out of supplies at the point where \( y = 7x + 10 \) intersects the line \( y = 35 \). This point is \( (3.57, 50) \), so Jessica will run out of supplies after 3.57 days. Therefore, Marc will run out of supplies first; Jessica can last about a day.
Solve each system of inequalities by graphing.

ANSWER:

29. \[ y \geq |6-x| \]
\[ |y| \leq 4 \]

SOLUTION:
Graph the system of inequalities.

ANSWER:

b. \( \frac{1}{3} \) days

c. Marc; Jessica could last about a quarter of a day longer than Marc.

Solve each system of inequalities by graphing.

28. \[ y \geq |2x + 4| - 2 \]
\[ 3y + x \leq 15 \]

SOLUTION:
Graph the system of inequalities.

ANSWER:

30. \[ |y| \geq x \]
\[ y < 2x \]

SOLUTION:
Graph the system of inequalities.

ANSWER:
3-2 Solving Systems of Inequalities by Graphing

\[ y > -3x + 1 \]
\[ 4y \leq x - 8 \]
\[ 3x - 5y < 20 \]

**SOLUTION:**
Graph the system of inequalities.

\[ |x| > y \]
\[ y \leq 6 \]
\[ y \geq -2 \]

**SOLUTION:**
Graph the system of inequalities.
3-2 Solving Systems of Inequalities by Graphing

34. \[2x + 3y \geq 6\]
\[y \leq |x - 6|\]

**SOLUTION:**
Graph the system of inequalities.

**ANSWER:**

35. \[8x + 4y < 10\]
\[y > |2x - 1|\]

**SOLUTION:**
Graph the system of inequalities.

**ANSWER:**
3-2 Solving Systems of Inequalities by Graphing

36. \[ y \geq |x - 2| + 4 \]
   \[ y \leq \lfloor x \rfloor - 3 \]

**SOLUTION:**
Graph the system of inequalities.

Since the inequalities have no shaded region in common, the system has no solution.

**ANSWER:**

37. **MUSIC** Steve is trying to decide what to put on his MP3 player. Audio books are 3 hours long and songs are 2.5 minutes long. Steve wants no more than 4 audio books on his MP3 player, but at least ten songs and one audio book. Each book costs $15.00 and each song costs $0.95. Steve has $63 to spend on books and music. Graph the inequalities to show possible combinations of books and songs that Steve can have.

**SOLUTION:**
Let \( x \) represent the number of audio books and \( y \) represent the number of songs that Steve puts on his MP3 player.
Steve wants at least one audio book but no more than 10, so \( 1 \leq x \leq 4 \).
He also wants at least 10 songs, so \( y \geq 10 \).
Since he only has $63 to spend, \( 15x + 0.95y \leq 63 \).

Graph the following system of inequalities and the shaded region will represent the possible combinations of audio books and songs Steve can purchase.
\( 1 \leq x \leq 4, \ y \geq 10, \ and \ 15x + 0.95y \leq 63 \)
3-2 Solving Systems of Inequalities by Graphing

38. JOBS Louie has two jobs and can work no more than 25 total hours per week. He wants to earn at least $150 per week. Graph the inequalities to show possible combinations of hours worked at each job that will help him reach his goal.

<table>
<thead>
<tr>
<th>Job</th>
<th>Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Busboy</td>
<td>$6.50</td>
</tr>
<tr>
<td>Clerk</td>
<td>$8.00</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \( x \) represent the number of hours that Louie works as a Busboy and \( y \) represent the number of hours he works as a clerk.
So:
\[ x + y \leq 25 \quad \text{and} \quad 6.5x + 8y \geq 150 \]
\[ x \geq 0; y \geq 0 \]

Graph the system of inequalities.

**ANSWER:**

39. TIME MANAGEMENT Ramir uses his spare time to write a novel and to exercise. He has budgeted 35 hours per week. He wants to exercise at least 7 hours a week but no more than 15. He also hopes to write between 20 and 25 hours per week. Write and graph a system of inequalities that represents this situation.

**SOLUTION:**
Let \( w \) = the number of hours writing, and let \( e \) = the number of hours exercising.
So:
\[ w + e \leq 35 \]
\[ 7 \leq e \leq 15 \]
\[ 20 \leq w \leq 25 \]
\[ w \geq 0; e \geq 0 \]

Graph the inequalities.

**ANSWER:**
3-2 Solving Systems of Inequalities by Graphing

Find the coordinates of the vertices of the figure formed by each system of inequalities.

\[ y \geq 2x - 12 \]
\[ y \leq -4x + 20 \]
\[ 4y - x \leq 8 \]
\[ y \geq -3x + 2 \]

**SOLUTION:**
Graph the inequalities.

The coordinates of the vertices are (0, 2), \( \left( \frac{5}{3}, -\frac{1}{3} \right) \), \( \left( 4 \frac{4}{17}, 3 \frac{1}{17} \right) \), and (2.8, -6.4)

**ANSWER:**
(0.2), \( \left( \frac{5}{3}, -\frac{1}{3} \right) \), \( \left( 4 \frac{4}{17}, 3 \frac{1}{17} \right) \) (2.8, -6.4)

\[ y \geq -x - 8 \]
\[ 2y \geq 3x - 20 \]
\[ 4y + x \leq 24 \]
\[ y \leq 4x + 22 \]

**SOLUTION:**
Graph the inequalities.

The coordinates of the vertices are \((-6, -2)\), \(\left( -3\frac{13}{17}, 6\frac{16}{17} \right)\), \(\left( 9\frac{1}{7}, 3\frac{5}{7} \right)\), and (0.8, -8.8)

**ANSWER:**
\((-6, -2)\), \(\left( -3\frac{13}{17}, 6\frac{16}{17} \right)\), \(\left( 9\frac{1}{7}, 3\frac{5}{7} \right)\), (0.8, -8.8)
2. \( y - x \geq -20 \)
3. \( y \geq -3x - 6 \)
4. \( y \leq -2x + 2 \)
5. \( y \leq 2x + 14 \)

**SOLUTION:**
Graph the inequalities.

The coordinates of the vertices are \((-4, 6), (-3, 8), (4, -7.6), \) and \( \left( \frac{1}{7}, -\frac{93}{7} \right) \).

**ANSWER:**
\((-4, 6), (-3, 8), (4, -7.6), \left( \frac{1}{7}, -\frac{93}{7} \right) \)

**43. FINANCIAL LITERACY** Mr. Hoffman is investing $10,000 in two funds. One fund will pay 6% interest, and a riskier second fund will pay 10% interest. What is the least amount Mr. Hoffman can invest in the risky fund and still earn at least $740 after one year?

**SOLUTION:**
Let \( x \) be the least amount Mr. Hoffman invests in the risky fund.
The remaining amount, \( 10,000 - x \) would be invested in the first fund.
So:
\[
6\%(10,000 - x) + 10\%(x) \geq 740
\]
\[
0.06x + 0.10x \geq 740
\]
\[
x \geq 3500
\]
So, a minimum of $3500 should be invested in the risky fund to earn at least $740.

**ANSWER:**
$3500

**44. DODGEBALL** A high school is selecting a dodgeball team to play in a fund-raising exhibition against their rival. There can be between 10 and 15 players on the team and there must be more girls than boys on the team.

a. Write and graph a system of inequalities to represent the situation.
b. List all of the possible combinations of boys and girls for the team.
c. Explain why there is not an infinite number of possibilities.

**SOLUTION:**
a. Let \( g \) be the number of girls and \( b \) be the number of boys in the team.
\[
10 \leq g + b \leq 15; g > b; g \geq 0; b \geq 0
\]
Graph the inequalities in the same coordinate plane.
3-2 Solving Systems of Inequalities by Graphing

b. (6, 4), (6, 5), (7, 3), (7, 4), (7, 5), (7, 6), (8, 2), (8, 3), (8, 4), (8, 5), (8, 6), (8, 7), (9, 1), (9, 2), (9, 3), (9, 4), (9, 5), (9, 6), (10, 0), (10, 1), (10, 2), (10, 3), (10, 4), (10, 5), (11, 0), (11, 1), (11, 2), (11, 3), (11, 4), (12, 0), (12, 1), (12, 2), (12, 3), (13, 0), (13, 1), (13, 2), (14, 0), (14, 1), (15, 0)
c. Sample answer: You cannot have a fraction of a person.

ANSWER:
a. 10 ≤ g + b ≤ 15; g > b; g ≥ 0; b ≥ 0;

The area defined by the inequalities is the region quadrilateral ABCD.
The coordinates A(2, 6), B(5, 0), C(-3, -4), and D(-6, 8) can be determined from the graph. To find the area, divide the region into ΔABE, ΔCDF and trapezoid AECF by drawing lines x = 2 and x = -3.

To find the area of each shape use the appropriate area formula.

ΔABE

\[ A = \frac{1}{2}bh \]

Find the height by using the length of KB where K is (2, 0). This height is 3 units long. The corresponding base is AE or 7.5.

\[ A = \frac{1}{2} \times 7.5 \times 3 \]

\[ A = 11.25 \]

ΔCDF

45. CHALLENGE Find the area of the region defined by the following inequalities.

\[ y \geq -4x - 16 \]
\[ 4y \leq 26 - x \]
\[ 3y + 6x \leq 30 \]
\[ 4y - 2x \geq -10 \]

SOLUTION:
Graph the inequalities in the same coordinate plane.

The coordinates of the vertices are (1.5, 3), (3.5, 7), (8, 3), (6, 2), (5, 1), (4, 0), (3, 1), (2, 2), (1, 3), (0, 4).

The solution is (0, 0).
3-2 Solving Systems of Inequalities by Graphing

\[ A = \frac{1}{2}bh \]

Find the height by using the length of \( DG \) where \( G \) is (-3, 8). This height is 3 units long. The corresponding base is \( CF \) or 11.25.

\[ A = \frac{1}{2} \]
\[ A = \frac{1}{2}(11.25)(3) \]
\[ A = 16.875 \]

Trapezoid AECF
\[ A = \frac{1}{2}h(b_1 + b_2) \]
Let \( b_1 = FC = 11.25, \ b_2 = AE = 7.5 \) and \( h = HJ = 5 \).
\[ A = \frac{1}{2}(5)(11.25 + 7.5) \]
\[ A = \frac{1}{2}(5)(18.75) \]
\[ A = 46.875 \]

Add the areas of the three figures together to find the area of the enclosed figure.
11.25 + 16.875 + 46.875 = 75
Therefore, the area of the enclosed figure is 75 square units.

**ANSWER:**
75 square units

46. **OPEN ENDED** Write a system of two inequalities in which the solution:

- a. lies only in the third quadrant.
- b. does not exist.
- c. lies only on a line.
- d. lies on exactly one point.

**SOLUTION:**
- a. Sample answer: \( y \leq -2, x < -1 \)
- b. Sample answer: \( y > 2, y < -2 \)
- c. Sample answer: \( y \geq |x|, y \leq |x| \)
- d. Sample answer: \( y \geq |x|, y < -|x| \)
The solution is (0, 0).

**ANSWER:**
- a. Sample answer: \( y \leq -2, x < -1 \)
- b. Sample answer: \( y > 2, y < -2 \)
- c. Sample answer: \( y \geq x, y \leq x \)
- d. Sample answer: \( y \geq |x|, y < -|x| \); solution at (0, 0)

47. **CHALLENGE** Write a system of inequalities to represent the solution shown. How many points with integer coordinates are solutions of the system?

**SOLUTION:**
Sample answer:
First determine the related equations. To do this, analyze the graph to find the slope and one point for each line.

The uppermost boundary has a slope of \( -\frac{2}{4} \) or \(-0.5\).
This line has a y-intercept of 4 so the equation is \( y = -0.5x + 4 \). Since the line is solid and the shading is below the line, the inequality is \( y \leq -0.5x + 4 \).
The boundary to the left of the origin has a slope of \(-\frac{6}{2} \) or -3. The y-intercept is -6 so the equation is \( y = -3x - 6 \). Since the line is solid and the shading is above the line, the inequality is \( y \geq -3x - 6 \).
Solve each system of inequalities by graphing.

1. SOLUTION: The graph of the region that shows the number of solutions is ...

To determine how many points with integer coordinates are solutions to the system, analyze the graph. Redraw the graph with each axis scaled by 1. Count every integer ordered pair on the boundary lines as well as all of the integer points within the area.

First count all of the ordered pairs that lie on the boundary lines. There are 12 points. Next count the ordered pairs on the axes to find that there are 12. There are 7 points in the first quadrant, 10 in Quadrant II, 2 in Quadrant III, and 4 in Quadrant IV. There are a total of 47 points with integer coordinates that are solutions to the system.

ANSWER: 

\[ y \geq 2x - 6; \]
 Sample answer: \[ y \leq -0.5x + 4; \text{ } 47 \]
 \[ y \geq -3x - 6 \]

48. CCSS ARGUMENTS Determine whether the following statement is true or false. If false, give a counterexample. A system of two linear inequalities has either no points or infinitely many points in its solution.

SOLUTION: True

ANSWER: true

49. WRITING IN MATH Write a how-to manual for determining where to shade when graphing a system of inequalities.

SOLUTION:
Sample answer: Shade each inequality in its standard way, by shading above the line if \( y > \) and shading below the line if \( y < \) (or you can use test points). Once you determine where to shade for each inequality, the area where every inequality needs to be shaded is the actual solution. This is only the shaded area.

ANSWER:
Sample answer: Shade each inequality in its standard way, by shading above the line if \( y > \) and shading below the line if \( y < \) (or you can use test points). Once you determine where to shade for each inequality, the area where every inequality needs to be shaded is the actual solution. This is only the shaded area.

50. WRITING IN MATH Explain how you would test to see whether \((-4, 6)\) is a solution of a system of inequalities.

SOLUTION:
Sample answer: Determine whether the point falls in the shaded area of the graphs and/or determine whether the values satisfy each inequality.

ANSWER:
Sample answer: Determine whether the point falls in the shaded area of the graphs and/or determine whether the values satisfy each inequality.
51. To be a member of the marching band, a student must have a grade-point average of at least 2.0 and must have attended at least five after-school practices. Choose the system of inequalities that best represents this situation.

\[ \begin{align*}
A & : \quad x \geq 2 \\
 & \quad y \geq 5 \\
 & \quad x \leq 2 \\
B & : \quad y \leq 5 \\
 & \quad x < 2 \\
C & : \quad y < 5 \\
 & \quad x > 2 \\
D & : \quad y > 5 \\
\end{align*} \]

**SOLUTION:**
The system of inequalities is:
\[ x \geq 2 \]
\[ y \geq 5 \]
The correct choice is A.

**ANSWER:**
A

52. **ACT/SAT** The table at the right shows a relationship between \( x \) and \( y \). Which equation represents this relationship?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

F \( y = 3x - 2 \)

G \( y = 3x + 2 \)

H \( y = 4x + 1 \)

J \( y = 4x + 2 \)

K \( y = 4x - 1 \)

**SOLUTION:**
Each output is two more than three times the input.
The correct choice is G.

**ANSWER:**
G

53. **SHORT RESPONSE** If \( 3x = 2y \) and \( 5y = 6z \), what is the value of \( x \) in terms of \( z \)?

**SOLUTION:**
\[ 3x = 2y \]
\[ y = \frac{3}{2}x \]

Substitute \( y = \frac{3}{2}x \) in the equation \( 5y = 6z \).
\[ 5 \left( \frac{3}{2}x \right) = 6z \]
\[ x = \frac{4}{5}z \]

**ANSWER:**
\[ \frac{4}{5}z \]

54. **GEOMETRY** Look at the graph below. Which of these statements describes the relationship between the two lines?

A They intersect at (6, 2).
B They intersect at (0, 2).
C They intersect at (3.5, 0).
D They intersect at (2, 6).

**SOLUTION:**
The lines intersect at the point (2, 6).
The correct choice is D.

**ANSWER:**
D
55. **GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines with equations \( y = 3, \ y = 7, \ y = 2x, \) and \( y = 2x - 13. \)

**SOLUTION:**
Graph the equations \( y = 3, \ y = 2x, \) and \( y = 2x - 13. \)

![Graph of the parallelogram](image)

The coordinates of the vertices are \((1.5, 3), (3.5, 7), (8, 3), \) and \((10, 7).\)

**ANSWER:**
\((1.5, 3), (3.5, 7), (8, 3), (10, 7)\)

---

56. \( x + y \geq 6 \)

**SOLUTION:**
The boundary of the graph is the graph of \( x + y = 6. \) Since the inequality symbol is \( \geq, \) the boundary is solid. Test the inequality with the point \((0, 0).\)

\[
\begin{align*}
x + y &\geq 6 \\
0 + 0 &\geq 6 \\
0 &\geq 6 \times
\end{align*}
\]

The region that does not contain \((0, 0)\) is shaded.
3-2 Solving Systems of Inequalities by Graphing

57. \(4x - 3y < 10\)

**SOLUTION:**
The boundary of the graph is the graph of \(4x - 3y = 10\). Since the inequality symbol is <, the boundary is dashed. Test the inequality with the point \((0, 0)\).

\[
\begin{align*}
4x - 3y &< 10 \\
4(0) - 3(0) &< 6 \\
0 &< 6 \checkmark
\end{align*}
\]

The region that contains \((0, 0)\) is shaded.

**ANSWER:**

\[
\begin{align*}
\text{The region that contains (0,0) is shaded.}
\end{align*}
\]

58. \(5x + 7y \geq -20\)

**SOLUTION:**
The boundary of the graph is the graph of \(5x + 7y = -20\). Since the inequality symbol is \(\geq\), the boundary is solid. Test the inequality with the point \((0, 0)\).

\[
\begin{align*}
5x + 7y &\geq -20 \\
5(0) + 7(0) &\geq -20 \\
0 &\geq -20 \checkmark
\end{align*}
\]

The region that contains \((0, 0)\) is shaded.

**ANSWER:**

\[
\begin{align*}
\text{The region that contains (0,0) is shaded.}
\end{align*}
\]
Graph each function. Identify the domain and range.

59. \( g(x) = \begin{cases} \ -1 & \text{if } x < 0 \\ & \\ -x + 2 & \text{if } x \geq 0 \end{cases} \)

\textit{SOLUTION:}

\[D = \{\text{all real numbers}\},
R = \{g(x) | g(x) \leq 2\}\]

\textit{ANSWER:}

\[D = \{\text{all real numbers}\},
R = \{g(x) | g(x) \leq 2\}\]

60. \( h(x) = \begin{cases} \ x + 3 & \text{if } x \leq -1 \\ & \text{if } x > -1 \end{cases} \)

\textit{SOLUTION:}

\[D = \{\text{all real numbers}\},
R = \{\text{all real numbers}\}\]

\textit{ANSWER:}

\[D = \{\text{all real numbers}\},
R = \{\text{all real numbers}\}\]
3-2 Solving Systems of Inequalities by Graphing

61. \( h(x) = \begin{cases} 
-1 & \text{if } x < -2 \\
1 & \text{if } x > 2 
\end{cases} \)

**SOLUTION:**

\[
\begin{align*}
\text{D} & = \{ x \mid x < -2 \text{ or } x > 2 \} \\
\text{R} & = \{-1, 1\}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\text{D} & = \{ x \mid x < -2 \text{ or } x > 2 \}, \text{ R} = \{-1, 1\}
\end{align*}
\]

62. **BOOK CLUB** For each meeting of the Putnam High School book club, $25 is taken from the activities account to buy snacks and materials. After their sixth meeting, there will be $350 left in the activities account.

a. If no money is put back into the account, what equation can be used to show how much money is left in the activities account after having \( x \) number of meetings?

b. How much money was originally in the account?

c. After how many meetings will there be no money left in the activities account?

**SOLUTION:**

a. The amount spent for 6 meetings is $150. The amount remaining in the account after 6 meetings is $350. Therefore, the initial amount in the account is $500.

Let \( y \) be the amount remaining in the account after \( x \) meetings.

So:

\[ y = 500 - 25x. \]

b. $500.

c. Replace \( y \) with 0 and find \( x \).

\[ 0 = 500 - 25x \]

\[ 20 = x \]

After 20 meetings, there will be no money left in the account.

**ANSWER:**

a. \( y = 500 - 25x \)

b. $500

c. 20

**Find each value if** \( f(x) = 2x + 5 \) and \( g(x) = 3x - 4 \).

63. \( f(-3) \)

**SOLUTION:**

\[ f(x) = 2x + 5 \]

Replace \( x \) with \(-3\).

\[ f(-3) = 2(-3) + 5 \]

\[ = -1 \]

**ANSWER:**

\(-1\)
3-2 Solving Systems of Inequalities by Graphing

64. \( g(-2) \)

**SOLUTION:**
\[ g(x) = 3x - 4 \]
Replace \( x \) with \(-2\).
\[ g(-2) = 3(-2) - 4 = -10 \]
**ANSWER:**
\(-10\)

65. \( f(-1) \)

**SOLUTION:**
\[ f(x) = 2x + 5 \]
Replace \( x \) with \(-1\).
\[ f(-1) = 2(-1) + 5 = 3 \]
**ANSWER:**
\(3\)

66. \( g(-0.5) \)

**SOLUTION:**
\[ g(x) = 3x - 4 \]
Replace \( x \) with \(-0.5\).
\[ g(-0.5) = 3(-0.5) - 4 = -1.5 - 4 = -5.5 \]
**ANSWER:**
\(-5.5\)

67. \( f(-0.25) \)

**SOLUTION:**
\[ f(x) = 2x + 5 \]
Replace \( x \) with \(-0.25\).
\[ f(-0.25) = 2(-0.25) + 5 = 4.5 \]
**ANSWER:**
\(4.5\)

68. \( g(-0.75) \)

**SOLUTION:**
\[ g(x) = 3x - 4 \]
Replace \( x \) with \(-0.75\).
\[ g(-0.75) = 3(-0.75) - 4 = -6.25 \]
**ANSWER:**
\(-6.25\)
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

\[ y \leq 5 \]
\[ x \leq 4 \]
\[ y \geq -x \]

\[ f(x, y) = 5x - 2y \]

**SOLUTION:**

The lines \( y = 5 \) and \( y = -x \) intersect at \((-5, 5)\).
The lines \( x = 4 \) and \( y = -x \) intersect at \((4, -4)\).
The lines \( y = 5 \) and \( x = 4 \) intersect at \((4, 5)\).
Therefore, the vertices of the feasible region are \((4, 5)\), \((4, -4)\) and \((-5, 5)\).
Substitute the points \((4, 5)\), \((4, -4)\) and \((-5, 5)\) in the function \( f(x, y) = 5x - 2y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(5x - 2y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((4, 5))</td>
<td>5(4) - 2(5)</td>
<td>10</td>
</tr>
<tr>
<td>((4, -4))</td>
<td>5(4) - 2(-4)</td>
<td>28</td>
</tr>
<tr>
<td>((-5, 5))</td>
<td>5(-5) - 2(5)</td>
<td>-35</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 28 and the minimum value is -35.

**ANSWER:**

\((4, 5), (4, -4), (-5, 5)\); max = 28, min = -35
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the objective function. Which system has a bounded region? Which system has an unbounded region? Explain your reasoning.

**SOLUTION:**

The lines, $y = -3x + 2$ and $y = -4$ intersect at $(2, -4)$. The lines, $9x + 3y = 24$ and $y = -4$ intersect at $(4, -4)$. Therefore, the vertices of the feasible region are $(2, -4)$ and $(4, -4)$.

Substitute the points $(2, -4)$ and $(4, -4)$ in the function $f(x, y) = 2x + 14y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$2x + 14y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2, -4)$</td>
<td>$2(2) + 14(-4)$</td>
<td>$-52$</td>
</tr>
<tr>
<td>$(4, -4)$</td>
<td>$2(4) + 14(-4)$</td>
<td>$-48$</td>
</tr>
</tbody>
</table>

So, the minimum value is $-52$. Consider another point on the feasible region, $(0, 3)$, yields a value of $14$, which is greater than $-48$. Therefore, there is no maximum value.

**ANSWER:**

$(2, -4)$, $(4, -4)$; max does not exist, min $= -52$
The lines $4y = 4x - 8$ and $y = -3$ intersect at $(-1, -3)$.
The lines $3x + 6y = 12$ and $4y = 4x - 8$ intersect at $\begin{pmatrix} 8 \\ 2 \\ 3' \\ 3 \end{pmatrix}$.
The lines $3x + 6y = 24$ and $y = 7$, are intersects at $(-10, 7)$.
Therefore, the vertices of the feasible region are $(-1, -3), \begin{pmatrix} 8 \\ 2 \\ 3' \\ 3 \end{pmatrix}$ and $(-10, 7)$.
Substitute the points $(-1, -3), (4, 2)$ and $(-6, 7)$ in the function $f(x, y) = -12x + 9y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$-12x + 9y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{pmatrix} 8 \ 2 \ 3' \ 3 \end{pmatrix}$</td>
<td>$-12(\frac{8}{3}) + 9(\frac{2}{3})$</td>
<td>$-26$</td>
</tr>
<tr>
<td>$(-1, -3)$</td>
<td>$-12(-1) + 9(-3)$</td>
<td>$-15$</td>
</tr>
<tr>
<td>$(-10, 7)$</td>
<td>$-12(-10) + 9(7)$</td>
<td>$183$</td>
</tr>
</tbody>
</table>

So, the minimum value is $-26$.
Consider another point on the feasible region, $(-12, 7)$, yields a value of 207, which is greater than 183. Therefore, there is no maximum value.

**Answer:**
3-3 Optimization with Linear Programming

\[ y \leq 2x + 6 \]
\[ y \geq 2x - 8 \]
\[ y \geq -2x - 18 \]
\[ f(x, y) = 5x - 4y \]

**SOLUTION:**

The lines \( y = 2x + 6 \) and \( y = -2x - 18 \) intersect at \((-6, -6)\).
The lines \( y = 2x + 18 \) and \( y = -2x - 18 \) intersect at \((-2.5, -13)\).
Therefore, the vertices of the feasible region are \((-6, -6)\) and \((-2.5, -13)\).
Substitute the points \((-6, -6)\) and \((-2.5, -13)\) in the function \( f(x, y) = 5x - 4y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(5x - 4y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-6, -6))</td>
<td>(5(-6) - 4(-6))</td>
<td>(-6)</td>
</tr>
<tr>
<td>((-2.5, -13))</td>
<td>(5(-2.5) - 4(-13))</td>
<td>(39.5)</td>
</tr>
</tbody>
</table>

So, the maximum value is 39.5.
Consider another point on the feasible region, \((8, 10)\), yields a value of \(-10\), which is less than \(-6\).
Therefore, there is no minimum value.

**ANSWER:**

\((-6, -6), (-2.5, -13), \text{ no min, } \text{ max} = 39.5\)

7. **CCSS PRECISION** The total number of workers’ hours per day available for production in a skateboard factory is 85 hours. There are 40 hours available for finishing decks and quality control each day. The table shows the number of hours needed in each department for two different types of skateboards.

<table>
<thead>
<tr>
<th>Skateboard</th>
<th>Manufacturing Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Drill Time</td>
</tr>
<tr>
<td>Pro Boards</td>
<td>1.5 hours</td>
</tr>
<tr>
<td>Specialty Boards</td>
<td>1 hour</td>
</tr>
</tbody>
</table>

a. Write a system of inequalities to represent the situation.
b. Draw the graph showing the feasible region.
c. List the coordinates of the vertices of the feasible region.
d. If the profit on a pro board is $50 and the profit on a specialty board is $65, write a function for the total profit on the skateboards.
e. Determine the number of each type of skateboard that needs to be made to have a maximum profit. What is the maximum profit?

**SOLUTION:**

a. Let \( g \) and \( c \) represent number of pro and specialty boards.
Constraints:

\[ 1.5g + c \leq 85 \]
\[ 2g + 0.5c \leq 40 \]
\[ g \geq 0 \]
\[ c \geq 0 \]

b. The vertices of the feasible region are \((0, 0)\), \((0.20)\) and \((80, 0)\).

d. Optimizing function:

\[ f(c, g) = 65c + 50g \]

e. Substitute the points \((0, 0)\), \((0.20)\) and \((80, 0)\) in the function \( f(x, y) = 65c + 50g \).
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>65c + 50g</th>
<th>f(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>65(0) + 50(0)</td>
<td>0</td>
</tr>
<tr>
<td>(0, 20)</td>
<td>65(0) + 50(20)</td>
<td>1000</td>
</tr>
<tr>
<td>(80, 0)</td>
<td>65(80) + 50(0)</td>
<td>5200</td>
</tr>
</tbody>
</table>

The maximum value is 5200 at (80, 0).
To maximize the profit 80 specialty boards and 0 pro boards to be made.
The maximum profit is $5200.

**ANSWER:**

\[ g \geq 0 \]
\[ c \geq 0 \]

**a.**
\[ 1.5g + c \leq 85 \]
\[ 2g + 0.5c \leq 40 \]

**b.**

\[ f(c, g) = 65c + 50g \]

**c.** (0, 0), (0, 20), (80, 0)

**d.** \[ f(c, g) = 65c + 50g \]

**e.** 80 specialty boards, 0 pro boards; $5200

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

\[ 1 \leq y \leq 4 \]
\[ 4y - 6x \geq -32 \]
\[ 2y \geq -x + 4 \]
\[ f(x, y) = -6x + 3y \]

**SOLUTION:**

The lines \( y = 4 \) and \( 2y = -x + 4 \) intersect at \((-4, 4)\).
The lines \( y = 4 \) and \( 4y - 6x = -32 \) intersect at \((8, 4)\).
The lines \( y = 1 \) and \( 2y = -x + 4 \) intersect at \((2, 1)\).
The lines \( y = 1 \) and \( 4y - 6x = -32 \) intersect at \((6, 1)\).
Therefore, the vertices of the feasible region are \((8, 4), (-4, 4), (6, 1)\) and \((2, 1)\).
Substitute the points \((8, 4), (-4, 4), (6, 1)\) and \((2, 1)\) in the function \(f(x, y) = -6x + 3y\).

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>(-6x + 3y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8, 4)</td>
<td>(-6(8) + 3(4))</td>
<td>(-36)</td>
</tr>
<tr>
<td>(6, 1)</td>
<td>(-6(6) + 3(1))</td>
<td>(-33)</td>
</tr>
<tr>
<td>(-4, 4)</td>
<td>(-6(-4) + 3(4))</td>
<td>(36)</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>(-6(2) + 3(1))</td>
<td>(-9)</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 36 and the minimum value is \(-36\).

**ANSWER:**

\((8, 4), (6, 1), (2, 1), (-4, 4)\); max = 36, min = \(-36\)
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function.

**SOLUTION:**

The lines $x = 2$ and $y = -2x - 6$ intersect at $(2, -10)$. The lines $x = 2$ and $4y = 2x + 32$ intersect at $(2, 9)$. The lines $x = -3$ and $y = -2x - 6$ intersect at $(-3, 0)$. The lines $x = -3$ and $4y = 2x + 32$ intersect at $(-3, 6.5)$. Therefore, the vertices of the feasible region are $(2, -10), (2, 9), (-3, 0)$, and $(-3, 6.5)$.

Substitute the points $(2, -10), (2, 9), (-3, 0)$, and $(-3, 6.5)$ in the function $f(x, y) = -4x - 9y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$-4x - 9y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2, -10)$</td>
<td>-4(-2) - 9(-10)</td>
<td>82</td>
</tr>
<tr>
<td>$(2, 9)$</td>
<td>-4(2) - 9(9)</td>
<td>-89</td>
</tr>
<tr>
<td>$(-3, 0)$</td>
<td>-4(-3) - 9(0)</td>
<td>-12</td>
</tr>
<tr>
<td>$(-3, 6.5)$</td>
<td>-4(3) - 9(6.5)</td>
<td>-70.5</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 82 and the minimum value is -89.

**ANSWER:**

Therefore, the maximum value is $82$, and the minimum value is $-89$. 

---

**SOLUTION:**

The lines $x = -2$, $y = 5$ and $y = 8$ intersect at $(-2, 5), (-2, 8)$. The lines $x = 4$ and $2x + 3y = 26$ intersect at $(4, 6)$. The lines $x = 4$ and $y = 5$ intersect at $(4, 5)$. The lines $y = 8$ and $2x + 3y = 26$ intersect at $(1, 8)$. Therefore, the vertices of the feasible region are $(1, 8), (4, 6), (4, 5), (-2, 5)$, and $(-2, 8)$.

Substitute the points $(1, 8), (4, 6), (4, 5), (-2, 5)$, and $(-2, 8)$ in the function $f(x, y) = 8x - 10y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$8x - 10y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 8)$</td>
<td>8(1) - 10(8)</td>
<td>-72</td>
</tr>
<tr>
<td>$(4, 6)$</td>
<td>8(4) - 10(6)</td>
<td>-28</td>
</tr>
<tr>
<td>$(4, 5)$</td>
<td>8(4) - 10(5)</td>
<td>-18</td>
</tr>
<tr>
<td>$(-2, 5)$</td>
<td>8(-2) - 10(5)</td>
<td>-66</td>
</tr>
<tr>
<td>$(-2, 8)$</td>
<td>8(-2) - 10(8)</td>
<td>-96</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is $-18$ and the minimum value is $-96$. 

**ANSWER:**

Therefore, the maximum value is $-18$, and the minimum value is $-96$. 

---

3-3 Optimization with Linear Programming
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values.

Therefore, the vertices of the feasible region are 

\((-8, -8), (6, -8), (4, -2), (-2, -2), (-8, -8);\) max = \(-8\) min = \(-152\)

\((1, 8), (4, 6), (4, 5), (-2, 5), (-2, 8);\) max = \(-18\) min = \(-96\)

\(-8 \leq y \leq -2\)

\(y \leq x\)

\(y \leq -3x + 10\)

\(f(x, y) = 5x + 14y\)

\(\begin{array}{c|c|c}
(x, y) & 5x + 14y & f(x, y) \\
\hline
(6, -8) & 5(6) + 14(-8) & -82 \\
(-8, -8) & 5(-8) + 14(-8) & -152 \\
(-2, -2) & 5(-2) + 14(-2) & -38 \\
(4, -2) & 5(4) + 14(-2) & -8 \\
\end{array}\)

Therefore, the maximum value is \(-8\) and the minimum value is \(-152\).
3-3 Optimization with Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the objective function.

12. \[ x + 4y \geq 2 \]
   \[ 2x + 4y \leq 24 \]
   \[ 2 \leq x \leq 6 \]
   \[ f(x,y) = 6x + 7y \]

**SOLUTION:**

The lines \( x = 2 \) and \( 2x + 4y = 24 \) intersect at \( (2, 5) \).
The lines \( x = 2 \) and \( x + 4y = 2 \) intersect at \( (2, 0) \).
The lines \( x = 6 \) and \( 2x + 4y = 24 \) intersect at \( (6, 3) \).
The lines \( x = 6 \) and \( x + 4y = 2 \) intersect at \( (6, -1) \).
Therefore, the vertices of the feasible region are \( (2, 5), (2, 0), (6, 3) \) and \( (6, -1) \).

Substitute the points \( (2, 5), (2, 0), (6, 3) \) and \( (6, -1) \) in the function \( f(x,y) = 6x + 7y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(6x + 7y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (2, 5) )</td>
<td>(6(2) + 7(5))</td>
<td>47</td>
</tr>
<tr>
<td>( (2, 0) )</td>
<td>(6(2) + 7(0))</td>
<td>12</td>
</tr>
<tr>
<td>( (6, 3) )</td>
<td>(6(6) + 7(3))</td>
<td>57</td>
</tr>
<tr>
<td>( (6, -1) )</td>
<td>(6(6) + 7(-1))</td>
<td>29</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 57 and the minimum value is 12.

**ANSWER:**

\( (2, 0), (6, -1), (6, 3), (2, 5) \); max = 57, min = 12

13. \[ 3 \leq y \leq 7 \]
   \[ 2y + x \leq 8 \]
   \[ y - 2x \leq 23 \]
   \[ f(x,y) = -3x + 5y \]

**SOLUTION:**

The lines \( y = 3 \) and \( 2y + x = 8 \) intersect at \( (2, 3) \).
The lines \( y = 3 \) and \( y - 2x = 23 \) intersect at \( (-10, 3) \).
The lines \( y = 7 \) and \( 2y + x = 8 \) intersect at \( (-6, 7) \).
The lines \( y = 7 \) and \( y - 2x = 23 \) intersect at \( (-8, 7) \).
Therefore, the vertices of the feasible region are \( (2, 3), (-10, 3), (-6, 7) \) and \( (-8, 7) \).

Substitute the points \( (2, 3), (-10, 3), (-6, 7) \) and \( (-8, 7) \) in the function \( f(x,y) = -3x + 5y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(-3x + 5y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (2, 3) )</td>
<td>(-3(2) + 5(3))</td>
<td>9</td>
</tr>
<tr>
<td>( (-10, 3) )</td>
<td>(-3(-10) + 5(3))</td>
<td>45</td>
</tr>
<tr>
<td>( (-6, 7) )</td>
<td>(-3(-6) + 5(7))</td>
<td>53</td>
</tr>
<tr>
<td>( (-8, 7) )</td>
<td>(-3(-8) + 5(7))</td>
<td>59</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 59 and the minimum value is 9.

**ANSWER:**

\( (-10, 3), (2, 3), (-6, 7), (-8, 7) \); max = 59, min = 9

Graph each system of inequalities. Name the
3-3 Optimization with Linear Programming

coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

\(-9 \leq x \leq -3\)
\(-9 \leq y \leq -5\)
\(3y + 12x \leq -75\)

\(f(x, y) = 20x + 8y\)

**SOLUTION:**

The lines \(y = -5\) and \(x = -9\) intersect at \((-9, -5)\).
The lines \(y = -5\) and \(3y + 12x = -75\) intersect at \((-5, -5)\).
The lines \(y = -9\) and \(x = -9\) intersect at \((-9, -9)\).
The lines \(y = -9\) and \(3y + 12x = -75\) intersect at \((-5, -9)\).

Therefore, the vertices of the feasible region are \((-9, -5), (-9, -9), (-5, -5)\) and \((-5, -9)\).

Substitute the points \((-9, -5), (-9, -9), (-5, -5)\) and \((-5, -9)\) in the function \(f(x, y) = 20x + 8y\).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(20x + 8y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-9, -5))</td>
<td>(20(-9) + 8(-5))</td>
<td>(-220)</td>
</tr>
<tr>
<td>((-9, -9))</td>
<td>(20(-9) + 8(-9))</td>
<td>(-252)</td>
</tr>
<tr>
<td>((-5, -9))</td>
<td>(20(-5) + 8(-9))</td>
<td>(-172)</td>
</tr>
<tr>
<td>((-5, -5))</td>
<td>(20(-5) + 8(-5))</td>
<td>(-140)</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is \(-140\) and the minimum value is \(-252\).

**ANSWER:**
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values.

The lines \(x = -8\) and \(3x + 6y = 36\) intersect at \((-8, 10)\).
The lines \(x = -8\) and \(2y + 12 = 3x\) intersect at \((-8, -18)\).
The lines \(3x + 6y = 36\) and \(2y + 12 = 3x\) intersect at \((6, 3)\).
Therefore, the vertices of the feasible region are \((-8, 10), (-8, -18)\) and \((6, 3)\).
Substitute the points \((-8, 10), (-8, -18)\) and \((6, 3)\) in the function \(f(x, y) = 10x - 6y\).

\[
\begin{array}{|c|c|c|}
\hline
(x, y) & 10x - 6y & f(x, y) \\
\hline
(-8, 10) & 10(-8) - 6(10) & -140 \\
(-8, -18) & 10(-8) - 6(-18) & 28 \\
(6, 3) & 10(6) - 6(3) & 42 \\
\hline
\end{array}
\]

Therefore, the maximum value is 42 and the minimum value is -140.

\(\text{ANSWER:}\)

\((6, 3), (-8, 10), (-8, -18); \text{max } 42, \text{min } -140\)
3-3 Optimization with Linear Programming

(2, 0), (5, 3), (–3, 8), (–6, 8), max = −10, min = −105

17. \[ x \geq -6 \]
\[ y + x \leq -1 \]
\[ 2x + 3y \geq -9 \]
\[ f(x, y) = -10x - 12y \]

**SOLUTION:**

The lines \( x = -6 \) and \( y + x = -1 \) intersect at \((-6, 5)\).
The lines \( x = -6 \) and \( 2x + 3y = -9 \) intersect at \((-6, 1)\).
The lines \( 2y + x = -8 \) and \( y + x = -1 \) intersect at \((6, -7)\).
Therefore, the vertices of the feasible region are \((-6, 5), \(-6, 1)\), and \((6, -7)\).

Substitute the points \((-6, 5), (-6, -1)\), and \((6, -7)\) in the function \( f(x, y) = -10x - 12y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(-10x - 12y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-6, 5))</td>
<td>(-10(-6) - 12(5))</td>
<td>0</td>
</tr>
<tr>
<td>((-6, 1))</td>
<td>(-10(-6) - 12(1))</td>
<td>48</td>
</tr>
<tr>
<td>((6, -7))</td>
<td>(-10(6) - 12(-7))</td>
<td>24</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 48 and the minimum value is 0.

**ANSWER:**

\((-6, 1), (6, -7), (-6, 5); \text{ max } = 48, \text{ min } = 0\)

\[ -5 \leq y \leq 17 \]
\[ y \leq 3x + 19 \]
\[ y \leq -4x + 15 \]
\[ f(x, y) = 8x - 3y \]

**SOLUTION:**

The lines \( y = -5 \) and \( y = -4x + 15 \) intersect at \((5, -5)\).
The lines \( y = -17 \) and \( y = -4x + 15 \) intersect at \((8, -17)\).
Therefore, the vertices of the feasible region are \((5, -5)\) and \((8, -17)\).
Substitute the points \((5, -5)\) and \((8, -17)\) in the function \( f(x, y) = 8x - 3y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(8x - 3y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5, -5))</td>
<td>(8(5) - 3(-5))</td>
<td>55</td>
</tr>
<tr>
<td>((8, -17))</td>
<td>(8(8) - 3(-17))</td>
<td>115</td>
</tr>
<tr>
<td>((-12, -17))</td>
<td>(8(-12) - 3(-17))</td>
<td>-45</td>
</tr>
<tr>
<td>((-8, -5))</td>
<td>(8(-8) - 3(-5))</td>
<td>-49</td>
</tr>
</tbody>
</table>

So, the minimum value is −49 and the maximum value is 115.

**ANSWER:**

\((5, -5), (8, -17), (-12, -17), (-8, -5): \text{ max } = 115, \text{ min } = -49\)
Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function.

\[ \begin{align*}
-8 \leq x \leq 16 \\
y \geq 2x - 10 \\
2y + x \leq 80
\end{align*} \]

\[ f(x, y) = 12x + 15y \]

**SOLUTION:**

The lines \( x = -8 \) and \( 2y + x = 80 \) intersect at \((-8, 44)\).
The lines \( x = -8 \) and \( y = 2x - 10 \) intersect at \((-8, -26)\).
The lines \( x = 16 \) and \( 2y + x = 80 \) intersect at \((16, 32)\).
The lines \( x = 16 \) and \( y = 2x - 10 \) intersect at \((16, 22)\).
Therefore, the vertices of the feasible region are \((-8, 44), (-8, -26), (16, 32) \) and \((16, 22)\).
Substitute the points \((-8, 44), (-8, -26), (16, 32) \) and \((16, 22)\) in the function \( f(x, y) = 12x + 15y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(12x - 15y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-8, 44))</td>
<td>12(-8) + 15(44)</td>
<td>564</td>
</tr>
<tr>
<td>((-8, -26))</td>
<td>12(-8) + 15(-26)</td>
<td>-486</td>
</tr>
<tr>
<td>((16, 32))</td>
<td>12(16) + 15(32)</td>
<td>672</td>
</tr>
<tr>
<td>((16, 22))</td>
<td>12(16) + 15(22)</td>
<td>522</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 672 and the minimum value is \(-486\).

**ANSWER:**

\[ \begin{align*}
-8 \leq x \leq 16 \\
y \leq x + 4 \\
y \geq x - 4 \\
2y + x \leq 80
\end{align*} \]

\[ f(x, y) = -10x + 9y \]

**SOLUTION:**

The lines \( y = x + 4 \) and \( y = -x + 10 \) intersect at \((3, 7)\).
The lines \( y = x - 4 \) and \( y = -x + 10 \) intersect at \((7, 3)\).
The lines \( y = x + 4 \) and \( y = -x - 10 \) intersect at \((-7, -3)\).
The lines \( y = x - 4 \) and \( y = -x - 10 \) intersect at \((-3, -7)\).
Therefore, the vertices of the feasible region are \((3, 7), (7, 3), (-7, -3) \) and \((-3, -7)\).
Substitute the points \((3, 7), (7, 3), (-7, -3) \) and \((-3, -7)\) in the function \( f(x, y) = -10x + 9y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(-10x + 9y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((3, 7))</td>
<td>-10(3) + 9(7)</td>
<td>33</td>
</tr>
<tr>
<td>((7, 3))</td>
<td>-10(7) + 9(3)</td>
<td>-43</td>
</tr>
<tr>
<td>((-7, -3))</td>
<td>-10(-7) + 9(-3)</td>
<td>43</td>
</tr>
<tr>
<td>((-3, -7))</td>
<td>-10(-3) + 9(-7)</td>
<td>-33</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 43 and the minimum value is \(-43\).

**ANSWER:**
3-3 Optimization with Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum value.

Therefore, the maximum value is 60 and the minimum value is –112.

\[ f(x, y) = 5x + 4y \]

The lines \( x = -6 \) and \( y = x + 3y = 14 \) intersect at \((-6, 3)\). The lines \( x = 2 \) and \( x + 3y = 14 \) intersect at \((2, 4)\). The lines \( x = 2 \) and \( y = x + 1 - 2 \) intersect at \((2, 1)\). The lines \( y = 0 \) and \( y = x + 1 - 2 \) intersect at \((1, 0)\). The lines \( y = 0 \) and \( y = -x + 1 - 2 \) intercept at \((-3, 0)\). The lines \( x = -6 \) and \( y = -(x + 1) - 2 \) intersect at \((-6, 3)\). The absolute value function also be formed because of the unbounded region. 

Consider another point on the feasible region, \((2, 6)\) is

The lines \( x = -6 \) and \( y = 6 \) intersect at \((1, 6)\). Therefore, the vertices of the feasible region are \((-4, 8)\), \((-4, 6)\), \((2, -8)\), \((5, -1)\) and \((1, 6)\).

Therefore, the maximum value is 28 and the minimum value is 9.

Substitute the points \((1, 3)\), \((3, 2)\), \((2, -2)\), \((5, -1)\) and \((1, 6)\) in the function \( f(x, y) = 12x + 8y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(12x + 8y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-4, -8))</td>
<td>(12(-4) + 8(-8))</td>
<td>(-112)</td>
</tr>
<tr>
<td>((-4, 6))</td>
<td>(12(-4) + 8(-6))</td>
<td>(-96)</td>
</tr>
<tr>
<td>((2, -8))</td>
<td>(12(2) + 8(-8))</td>
<td>(-40)</td>
</tr>
<tr>
<td>((5, -1))</td>
<td>(12(5) + 8(-1))</td>
<td>52</td>
</tr>
<tr>
<td>((1, 6))</td>
<td>(12(1) + 8(6))</td>
<td>60</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 48 and the minimum value is 36.

\[ f(x, y) = 16x - 22y \]

Therefore, the maximum value is 5200 at \((80, 0)\). The lines \( x = 8 \) and \( y = 3 \) are intersects at \((8, 7)\) and \((6, 3)\) in the function \( f(x, y) = 16x - 22y \).

The lines \( x = -6 \) and \( y = 6 \) intersect at \((1, 6)\). Therefore, the vertices of the feasible region are \((-4, 8)\), \((-4, 6)\), \((2, -8)\), \((5, -1)\) and \((1, 6)\).

Substitute the points \((2, 2)\), \((1, 4)\), \((-3, 1)\), \((3, 3)\), \((-6, 3)\) and \((-6, 6)\) in the function \( f(x, y) = 12x + 8y \).

The lines \( x = -3 \) and \( y = -x + 1 - 2 \) intersect at \((-3, 0)\). The lines \( x = -6 \) and \( y = x + 1 - 2 \) intersect at \((-6, 3)\). Therefore, the maximum value is 36 and the minimum value is 4.

\[ f(x, y) = 6x + 2y \]

Therefore, the maximum value is 20 and the minimum value is 9.

Substitute the points \((1, 3)\), \((3, 2)\), \((2, -2)\), \((5, -1)\) and \((1, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = 8 \) and \( y = 3 \) are intersects at \((8, 7)\) and \((6, 3)\) in the function \( f(x, y) = 6x + 2y \).

Therefore, the maximum value is 26 and the minimum value is 5.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); \( f(x, y) = 12x + 8y \)

Therefore, the maximum value is 43 and the minimum value is –43.

\[ f(x, y) = 16x - 22y \]

Therefore, the maximum value is 59 and the minimum value is 4.

Substitute the points \((2, 2)\), \((1, 4)\), \((-3, 1)\), \((3, 3)\), \((-6, 3)\) and \((-6, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = -3 \) and \( y = -x + 1 - 2 \) intersect at \((-3, 0)\). The lines \( x = -6 \) and \( y = x + 1 - 2 \) intersect at \((-6, 3)\). Therefore, the maximum value is 20 and the minimum value is 9.

Substitute the points \((1, 3)\), \((3, 2)\), \((2, -2)\), \((5, -1)\) and \((1, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = 8 \) and \( y = 3 \) are intersects at \((8, 7)\) and \((6, 3)\) in the function \( f(x, y) = 6x + 2y \).

Therefore, the maximum value is 26 and the minimum value is 5.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); max = 43, min = –43

\[ f(x, y) = 12x + 8y \]

Therefore, the maximum value is 43 and the minimum value is –43.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); max = 43, min = –43

\[ f(x, y) = 16x - 22y \]

Therefore, the maximum value is 59 and the minimum value is 4.

Substitute the points \((2, 2)\), \((1, 4)\), \((-3, 1)\), \((3, 3)\), \((-6, 3)\) and \((-6, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = -3 \) and \( y = -x + 1 - 2 \) intersect at \((-3, 0)\). The lines \( x = -6 \) and \( y = x + 1 - 2 \) intersect at \((-6, 3)\). Therefore, the maximum value is 20 and the minimum value is 9.

Substitute the points \((1, 3)\), \((3, 2)\), \((2, -2)\), \((5, -1)\) and \((1, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = 8 \) and \( y = 3 \) are intersects at \((8, 7)\) and \((6, 3)\) in the function \( f(x, y) = 6x + 2y \).

Therefore, the maximum value is 26 and the minimum value is 5.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); max = 43, min = –43

\[ f(x, y) = 12x + 8y \]

Therefore, the maximum value is 43 and the minimum value is –43.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); max = 43, min = –43

\[ f(x, y) = 16x - 22y \]

Therefore, the maximum value is 59 and the minimum value is 4.

Substitute the points \((2, 2)\), \((1, 4)\), \((-3, 1)\), \((3, 3)\), \((-6, 3)\) and \((-6, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = -3 \) and \( y = -x + 1 - 2 \) intersect at \((-3, 0)\). The lines \( x = -6 \) and \( y = x + 1 - 2 \) intersect at \((-6, 3)\). Therefore, the maximum value is 20 and the minimum value is 9.

Substitute the points \((1, 3)\), \((3, 2)\), \((2, -2)\), \((5, -1)\) and \((1, 6)\) in the function \( f(x, y) = 6x + 2y \).

The lines \( x = 8 \) and \( y = 3 \) are intersects at \((8, 7)\) and \((6, 3)\) in the function \( f(x, y) = 6x + 2y \).

Therefore, the maximum value is 26 and the minimum value is 5.

Substitute the points \((3, 7)\), \((7, 3)\), \((-3, -7)\), \((-7, -3)\); max = 43, min = –43

\[ f(x, y) = 12x + 8y \]
3-3 Optimization with Linear Programming

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>5x + 4y</th>
<th>f(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4, 6)</td>
<td>5(-4) + 4(6)</td>
<td>4</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>5(2) + 4(4)</td>
<td>26</td>
</tr>
<tr>
<td>(2, 1)</td>
<td>5(2) + 4(1)</td>
<td>14</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>5(1) + 4(0)</td>
<td>5</td>
</tr>
<tr>
<td>(-3, 0)</td>
<td>5(-3) + 4(0)</td>
<td>-15</td>
</tr>
<tr>
<td>(-6, 3)</td>
<td>5(-6) + 4(3)</td>
<td>-18</td>
</tr>
<tr>
<td>(-6, 6)</td>
<td>5(-6) + 6(6)</td>
<td>6</td>
</tr>
</tbody>
</table>

Therefore, the maximum value is 26 and the minimum value is -18.

**ANSWER:**

[Graph of the feasible region]

(-4, 6), (2, 4), (2, 1), (1, 0), (-3, 0), (-6, 3), (-6, 6); max = 26, min = -18

23. **COOKING** Jenny’s Bakery makes two types of birthday cakes: yellow cake, which sells for $25, and strawberry cake, which sells for $35. Both cakes are the same size, but the decorating and assembly time required for the yellow cake is 2 hours, while the time is 3 hours for the strawberry cake. There are 450 hours of labor available for production. How many of each type of cake should be made to maximize revenue?

**SOLUTION:**

Let x and y be the number of strawberry and yellow cakes.

Optimizing function:

\[ f(x, y) = 35x + 25y \]

Constraints:

\[ x \geq 0 \]
\[ y \geq 0 \]
\[ 3x + 2y \leq 450 \]

The vertices of the feasible region are (0, 0), (0, 225) and (150, 0).

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>35x + 25y</th>
<th>f(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 225)</td>
<td>35(0) + 25(225)</td>
<td>5625</td>
</tr>
<tr>
<td>(0, 0)</td>
<td>35(0) + 25(0)</td>
<td>0</td>
</tr>
<tr>
<td>(150, 0)</td>
<td>35(150) + 25(0)</td>
<td>5250</td>
</tr>
</tbody>
</table>

The maximum value is 5625. Therefore, 225 yellow cakes and 0 strawberry cakes will maximize the revenue.

**ANSWER:**

225 yellow cakes, 0 strawberry cakes
24. BUSINESS The manager of a travel agency is printing brochures and fliers to advertise special discounts on vacation spots during the summer months. Each brochure costs $0.08 to print, and each flier costs $0.04 to print. A brochure requires 3 pages, and a flier requires 2 pages. The manager does not want to use more than 600 pages, and she needs at least 50 brochures and 150 fliers. How many of each should she print to minimize the cost?

**SOLUTION:**
Let \( x \) and \( y \) be the number of brochures and fliers. Optimizing function:

\[
 f(x, y) = 0.08x + 0.04y
\]

Constraints:
\[
\begin{align*}
x &\geq 50 \\
y &\geq 150 \\
3x + 2y &\leq 600
\end{align*}
\]

The vertices of the feasible region are \((50, 150)\), \((100, 150)\) and \((50, 225)\).
Substitute the points \((50, 150)\), \((100, 150)\) and \((50, 225)\) in the function \( f(x, y) = 0.08x + 0.04y \).

\[
\begin{array}{|c|c|c|}
\hline
(x, y) & 0.08x + 0.04y & f(x, y) \\
\hline
(50, 150) & 0.08(50) + 0.04(150) & 10 \\
(50, 225) & 0.08(50) + 0.04(225) & 13 \\
(100, 150) & 0.08(100) + 0.04(150) & 14 \\
\hline
\end{array}
\]

The minimum value 10 at \((50, 150)\).
To minimize the cost, she would print 50 brochures and 150 fliers.

**ANSWER:**
50 brochures, 150 fliers

25. CCSS PRECISION Sean has 20 days to paint play houses and sheds. The sheds can be painted at a rate of 2.5 per day, and the play houses can be painted at a rate of 2 per day. He has 45 structures that need to be painted.

a. Write a system of inequalities to represent the possible ways Sean can paint the structures.
b. Draw a graph showing the feasible region and list the coordinates of the vertices of the feasible region.
c. If the profit is $26 per shed and $30 per play house, how many of each should he paint?
d. What is the maximum profit?
26. MOVIES Employees at a local movie theater work 8-hour shifts from noon to 8 P.M. or from 4 P.M. to midnight. The table below shows the number of employees needed and their corresponding pay. Find the numbers of day-shift workers and night-shift workers that should be scheduled to minimize the cost. What is the minimal cost?

<table>
<thead>
<tr>
<th>Time</th>
<th>noon to 4 P.M.</th>
<th>4 P.M. to 8 P.M.</th>
<th>8 P.M. to midnight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employees</td>
<td>at least 5</td>
<td>at least 14</td>
<td>6</td>
</tr>
<tr>
<td>Needed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate per Hour</td>
<td>$5.50</td>
<td>$7.50</td>
<td>$7.50</td>
</tr>
</tbody>
</table>

**SOLUTION:**

Let \( x \) be the number of day-shift workers and let \( y \) be the number of night-shift workers.

Optimizing function:

\[
f(x, y) = 5.50(4)x + 7.50(4)(x + y) + 7.50(4)y \\
= 22x + 30(x + y) + 30y \\
= 52x + 60y
\]

Constraints:

\[
x \geq 5 \\
x + y \geq 14 \\
y \geq 6
\]

The vertices of the feasible region are (5,9) and (8,6).

Substitute the coordinates in the optimizing function.

\[
f(5,9) = 52(5) + 60(9) = 260 + 540 = 800
\]

\[
f(8,6) = 52(8) + 60(6) = 416 + 360 = 776
\]

So, the theater should schedule 8 day-shift and 6 night-shift workers. The minimal cost is $776.

**ANSWER:**

8 day-shift and 6 night-shift workers; $776

27. BUSINESS Each car on a freight train can hold 4200 pounds of cargo and has a capacity of 480 cubic feet. The freight service handles two types of packages: small, which weigh 25 pounds and are 3 cubic feet each, and large, which are 50 pounds and are 5 cubic feet each. The freight service charges $5 for each small package and $8 for each large package.

a. Find the number of each type of package that should be placed on a train car to maximize revenue.

b. What is the maximum revenue per train car?

c. In this situation, is maximizing the revenue necessarily the best thing for the company to do? Explain.

**SOLUTION:**

a. Let \( x \) be the number of small packages. Let \( y \) be the number of large packages.

Optimizing function:

\[
f(x, y) = 5x + 8y
\]

Constraints:

\[
25x + 50y \leq 4200 \\
3x + 5y \leq 480 \\
x \geq 0 \\
y \geq 0
\]

The vertices of the feasible region are (0,0), (0, 84), (160, 0) and (120, 24).

Substitute the points (0,0), (0, 84), (160, 0) and (120, 24) in the function \( f(x, y) = 5x + 8y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>( f(x, y) )</th>
<th>( f(x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(120, 24)</td>
<td>5(120) + 8(24)</td>
<td>792</td>
</tr>
<tr>
<td>(160, 0)</td>
<td>5(160) + 8(0)</td>
<td>800</td>
</tr>
<tr>
<td>(0, 84)</td>
<td>5(0) + 8(84)</td>
<td>336</td>
</tr>
<tr>
<td>(0, 0)</td>
<td>5(0) + 8(0)</td>
<td>0</td>
</tr>
</tbody>
</table>

160 small packages and 0 large packages.

b. The maximum value is 800. That is the maximum revenue per train car is $800.

c. No. If revenue is maximized, the company will not deliver any large packages, and customers with large packages to ship will probably choose another carrier.

**ANSWER:**
3-3 Optimization with Linear Programming

a. 160 small packages, 0 large packages
b. $800
c. No; if revenue is maximized, the company will not deliver any large packages, and customers with large packages to ship will probably choose another carrier.

28. RECYCLING A recycling plant processes used plastic into food or drink containers. The plant processes up to 1200 tons of plastic per week. At least 300 tons must be processed for food containers, while at least 450 tons must be processed for drink containers. The profit is $17.50 per ton for processing food containers and $20 per ton for processing drink containers. What is the profit if the plant maximizes processing?

**SOLUTION:**
Let \( x \) be the number of tons of food containers processed.
Let \( y \) be the number of tons of drink containers processed.

Optimizing function:

\[
f(x, y) = 17.5x + 20y
\]

Constraints:

\[
x + y \leq 1200
\]
\[
x \geq 300
\]
\[
y \geq 450
\]

The vertices of the feasible region are (300, 450), (300, 900) and (750, 450).
Substitute the points (300, 450), (300, 900) and (750, 450) in the function \( f(x, y) = 17.5x + 20y \).

\[
\begin{array}{ccc}
(x, y) & 17.5x + 20y & f(x, y) \\
(300, 450) & 17.5(300) + 20(450) & 14250 \\
(750, 450) & 17.5(750) + 20(450) & 22125 \\
(300, 900) & 17.5(300) + 20(900) & 23250 \\
\end{array}
\]

Therefore, the maximum value is 23250.
If the plant maximized processing, the profit is $23,250.

**ANSWER:**
$23,250

29. OPEN ENDED Create a set of inequalities that forms a bounded region with an area of 20 units\(^2\) and lies only in the fourth quadrant.

**SOLUTION:**
Sample answer: \(-2 \geq y \geq -6, 4 \leq x \leq 9\)

**ANSWER:**
Sample answer: \(-2 \geq y \geq -6, 4 \leq x \leq 9\)

30. CHALLENGE Find the area of the bounded region formed by the following constraints:

\[y \geq |x| - 3, y \leq -|x| + 3, \text{and } x \geq |y|\]

**SOLUTION:**
Vertices of the feasible region are (0, 0), (3, 0), (1.5, 1.5) and (1.5, -1.5).
This vertices form a square.
The side of the feasible region is 2.12132 unit.
Therefore, the area of the feasible region is \((2.12132)^2\) or 4.5 unit\(^2\).

**ANSWER:**
4.5 units\(^2\)
31. **CCSS ARGUMENTS** Identify the system of inequalities that is not the same as the other three. Explain your reasoning.

**a.**

**b.**

**c.**

**d.**

**SOLUTION:**

b: The feasible region of Graph b is unbounded while the other three are bounded.

**ANSWER:**

b: The feasible region of Graph b is unbounded while the other three are bounded.

32. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

An unbounded region will not have both a maximum and minimum value.

**SOLUTION:**

Sample answer: Always; if a point on the unbounded region forms a minimum, then a maximum cannot also be formed because of the unbounded region. There will always be a value in the solution that will produce a higher value than any projected maximum.

**ANSWER:**

Sample answer: Always; if a point on the unbounded region forms a minimum, then a maximum cannot also be formed because of the unbounded region. There will always be a value in the solution that will produce a higher value than any projected maximum.

33. **WRITING IN MATH** Upon determining a bounded feasible region, Ayumi noticed that vertices A(−3, 4) and B(5, 2) yielded the same maximum value for \( f(x, y) = 16y + 4x \). Kelvin confirmed that her constraints were graphed correctly and her vertices were correct. Then he said that those two points were not the only maximum values in the feasible region. Explain how this could have happened.

**SOLUTION:**

Sample answer: Even though the region is bounded, multiple maximums occur at A and B and all of the points on the boundary of the feasible region containing both A and B. This happened because that boundary of the region has the same slope as the function.

**ANSWER:**

Sample answer: Even though the region is bounded, multiple maximums occur at A and B and all of the points on the boundary of the feasible region containing both A and B. This happened because that boundary of the region has the same slope as the function.
3-3 Optimization with Linear Programming

34. Kelsey worked 350 hours during the summer and earned $2978.50. She earned $6.85 per hour when she worked at a video store and $11 per hour as an architectural intern. Let x represent the number of hours she worked at the video store and y represent the number of hours that she interned. Which system of equations represents this situation?

A 11x + 6.85y = 2978.50
B x + y = 350
C 6.85x + 11y = 2978.50
D 11x + 6.85y = 350

**SOLUTION:**
Total number of hours worked is 350.
That is, x + y = 350.
Total amount she earned is $2978.50.
That is 6.85x + 11y = 2978.50
Therefore, option B is the correct answer.

**ANSWER:** B

35. SHORT RESPONSE A family of four went out to dinner. Their bill, including tax, was $60. They left a 17% tip on the total cost of their bill. What is the total cost of the dinner including tip?

**SOLUTION:**
The tip is 17% of 60.
0.17 × 60 = 10.20
So, the tip amount is $10.20.
Therefore, the total cost of the dinner including tip is $60 + $10.20 = $70.20.

**ANSWER:** $70.20

36. ACT/SAT For a game she is playing, Liz must draw a card from a deck of 26 cards, one with each letter of the alphabet on it, and roll a die. What is the probability that Liz will draw a letter in her name and roll an odd number?

F $\frac{2}{3}$
G $\frac{1}{13}$
H $\frac{3}{52}$
J $\frac{1}{26}$
K $\frac{1}{52}$

**SOLUTION:**
Liz has 3 letters in her name.
The probability of getting one of those three letters is $\frac{3}{26}$.
On a die, there are 3 odd numbers.
The probability of getting an odd number is $\frac{1}{2}$.
Therefore, the probability that Liz will draw a letter in her name and roll an odd number is $\frac{3}{26} \times \frac{1}{2} = \frac{3}{52}$.
Option H is the correct answer.

**ANSWER:** H

37. GEOMETRY Which of the following best describes the graphs of y = 3x − 5 and 4y = 12x + 16?

A The lines have the same y-intercept.
B The lines have the same x-intercept.
C The lines are perpendicular.
D The lines are parallel.

**SOLUTION:**
The slope of the line y = 3x − 5 is 3.
The slope of the line 4y = 12x + 16 is 3.
Since the slopes are equal, the lines are parallel.
Option D is the correct answer.

**ANSWER:** D
3-3 Optimization with Linear Programming

Solve each system of inequalities by graphing.

38. \(3x + 2y \geq 6\)
   \(4x - y \geq 2\)

SOLUTION:

ANSWER:

39. \(4x - 3y < 7\)
   \(2y - x < -6\)

SOLUTION:

ANSWER:

40. \(3y \leq 2x - 8\)
   \(y \geq \frac{2}{3}x - 1\)

SOLUTION:

no solution

ANSWER:

no solution
41. BUSINESS Last year the chess team paid $7 per hat and $15 per shirt for a total purchase of $330. This year they spent $360 to buy the same number of shirts and hats because the hats now cost $8 and the shirts cost $16. Write and solve a system of two equations that represents the number of hats and shirts bought each year.

**SOLUTION:**
Let x represent number hats and y represent number of shirts.

The equations are
\[ 7x + 15y = 330 \text{ and } 8x + 16y = 360. \]

Solve.

\[
\begin{align*}
7x + 15y &= 330 \quad \rightarrow (1) \\
8x + 16y &= 360 \quad \rightarrow (2) \\
8 \times (1) & \quad 56x + 120y = 2640 \quad \rightarrow (3) \\
7 \times (2) & \quad 56x + 112y = 2520 \quad \rightarrow (4) \\
(3) & \quad 8y = 120 \\
& \quad y = 15
\end{align*}
\]

Substitute 15 for y in the first equation and solve for x.

\[
\begin{align*}
7x + 15(15) &= 330 \\
7x + 225 &= 330 \\
7x &= 105 \\
x &= 15
\end{align*}
\]

There are 15 hats and 15 shirts.

**ANSWER:**
\[ 7x + 15y = 330, 8x + 16y = 360; \text{hats: 15, shirts: 15} \]
**3-3 Optimization with Linear Programming**

43. passes through (−3, 5) and (3, 2)

**SOLUTION:**
Slope of the line passing through the points is (−3, 5) and (3, 2) is 
\[ m = \frac{2 - 5}{3 - (-3)} = \frac{-3}{6} = -\frac{1}{2}. \]

Substitute \(-\frac{1}{2}\) for \(m\) in the slope-intercept form.

\[ y = -\frac{1}{2}x + b \]

Substitute 3 for \(x\) and 2 for \(y\) and solve for \(b\).

\[ 2 = -\frac{1}{2}(3) + b \]
\[ b = 2 + \frac{3}{2} \]
\[ b = \frac{7}{2} \]

Therefore, the equation of the line passing through the points (−3, 5) and (3, 2) is 
\[ y = -\frac{1}{2}x + \frac{7}{2}. \]

**ANSWER:**
\[ y = -\frac{1}{2}x + \frac{7}{2} \]

44. Find the \(x\)-intercept and the \(y\)-intercept of the graph of each equation. Then graph the equation.

**SOLUTION:**
To find the \(x\)-intercept, substitute 0 for \(y\) and solve for \(x\).

\[ 5x + 3y = 15 \]

\[ 5x + 3(0) = 15 \]
\[ 5x = 15 \]
\[ x = 3 \]

To find the \(y\)-intercept, substitute 0 for \(x\) and solve for \(y\).

\[ 5(0) + 3y = 15 \]
\[ 3y = 15 \]
\[ y = 5 \]

Therefore, the \(x\)- and the \(y\)-intercepts are 3 and 5 respectively.
3-3 Optimization with Linear Programming

45. \(2x - 6y = 12\)

**SOLUTION:**
To find the x-intercept, substitute 0 for \(y\) and solve for \(x\).

\[
2x - 6(0) = 12
\]

\[
2x = 12
\]

\[
x = 6
\]

To find the y-intercept, substitute 0 for \(x\) and solve for \(y\).

\[
2(0) - 6y = 12
\]

\[
-6y = 12
\]

\[
y = -2
\]

Therefore, the x- and the y- intercepts are 6 and –2 respectively.

**ANSWER:**
6; –2

46. \(3x - 4y - 10 = 0\)

**SOLUTION:**
To find the x-intercept, substitute 0 for \(y\) and solve for \(x\).

\[
3x - 4(0) - 10 = 0
\]

\[
3x = 10
\]

\[
x = \frac{10}{3}
\]

To find the y-intercept, substitute 0 for \(x\) and solve for \(y\).

\[
3(0) - 4y - 10 = 0
\]

\[
-4y = 10
\]

\[
y = -\frac{5}{2}
\]

Therefore, the x- and the y- intercepts are \(\frac{10}{3}\) and \(-\frac{5}{2}\) respectively.

**ANSWER:**
\(\frac{10}{3}; -\frac{5}{2}\)
3-3 Optimization with Linear Programming

47. \(2x + 5y - 10 = 0\)

**SOLUTION:**
To find the \(x\)-intercept, substitute 0 for \(y\) and solve for \(x\).

\[
2x + 5(0) - 10 = 0
\]
\[
2x = 10
\]
\[
x = 5
\]

To find the \(y\)-intercept, substitute 0 for \(x\) and solve for \(y\).

\[
2(0) + 5y - 10 = 0
\]
\[
5y = 10
\]
\[
y = 2
\]

Therefore, the \(x\)- and the \(y\)-intercepts are 5 and 2 respectively.

**ANSWER:**
5; 2

48. \(y = x\)

**SOLUTION:**
To find the \(x\)-intercept, substitute 0 for \(y\) and solve for \(x\).

\(0 = x\)

To find the \(y\)-intercept, substitute 0 for \(x\) and solve for \(y\).

\(y = 0\)

Therefore, the \(x\)- and the \(y\)-intercepts are 0 and 0 respectively.
49. \( y = 4x - 2 \)

**SOLUTION:**
To find the \( x \)-intercept, substitute 0 for \( y \) and solve for \( x \).

\[
0 = 4x - 2 \\
4x = 2 \\
x = \frac{1}{2}
\]

To find the \( y \)-intercept, substitute 0 for \( x \) and solve for \( y \).

\[
y = 4(0) - 2 \\
y = -2
\]

Therefore, the \( x \)- and the \( y \)-intercepts are \( \frac{1}{2} \) and \(-2\) respectively.

**Answer:** \( \frac{1}{2}; -2 \)

---

50. \( x + y + z \)

**SOLUTION:**
Substitute \(-1\), \(3\) and \(7\) for \(x, y \) and \(z \) and simplify.

\[
x + y + z = -1 + 3 + 7 \\
= 9
\]

**Answer:** 9

---

51. \( 2x - y + 2z \)

**SOLUTION:**
Substitute \(-1\), \(3\) and \(7\) for \(x, y \) and \(z \) and simplify.

\[
2x - y + 2z = 2(-1) - (3) + 2(7) \\
= -2 - 3 + 14 \\
= 9
\]

**Answer:** 9

---

52. \(-x + 4y - 3z\)

**SOLUTION:**
Substitute \(-1\), \(3\) and \(7\) for \(x, y \) and \(z \) and simplify.

\[
-x + 4y - 3z = -(1) + 4(3) - 3(7) \\
= 1 + 12 - 21 \\
= -8
\]

**Answer:** \(-8\)

---

53. \(4x + 2y - z\)

**SOLUTION:**
Substitute \(-1\), \(3\) and \(7\) for \(x, y \) and \(z \) and simplify.

\[
4x + 2y - z = 4(-1) + 2(3) - (7) \\
= -4 + 6 - 7 \\
= -5
\]

**Answer:** \(-5\)
3-3 Optimization with Linear Programming

54. $5x - y + 4z$

**SOLUTION:**
Substitute $-1, 3$ and $7$ for $x, y$ and $z$ and simplify.

\[
5x - y + 4z = 5(-1) - (3) + 4(7)
\]
\[
= -5 - 3 + 28
\]
\[
= 20
\]

**ANSWER:**
20

55. $-3x - 3y + 3z$

**SOLUTION:**
Substitute $-1, 3$ and $7$ for $x, y$ and $z$ and simplify.

\[
-3x - 3y + 3z = -3(-1) - 3(3) + 3(7)
\]
\[
= 3 - 9 + 21
\]
\[
= 15
\]

**ANSWER:**
15
3-4 Systems of Equations in Three Variables

Solve each system of equations.

\[-3a - 4b + 2c = 28\]
1. \[a + 3b - 4c = -31\]
\[2a + 3c = 11\]

**SOLUTION:**
\[-3a - 4b + 2c = 28 \rightarrow (1)\]
\[a + 3b - 4c = -31 \rightarrow (2)\]
\[2a + 3c = 11 \rightarrow (3)\]

Eliminate one variable.
Multiply the first equation by 3 and the second equation by 4 then add.

\[(1) \times 3 \quad -9a - 12b + 6c = 84\]
\[(2) \times 4 \quad 4a + 12b - 16c = -124\]
\[-5a - 10c = -40 \rightarrow (4)\]

Solve the third and fourth equations.

\[(3) \times 5 \quad 10a + 15c = 55\]
\[(4) \times 2 \quad -10a - 20c = -80\]
\[5c = 25\]
\[c = 5\]

Substitute 5 for \(c\) in the third equation and solve for \(a\).

\[2a + 3(5) = 11\]
\[2a + 15 = 11\]
\[2a = -4\]
\[a = -2\]

Substitute –2 for \(a\) and 5 for \(c\) in the second equation, and solve for \(b\).

\[-2 + 3b - 4(5) = -31\]
\[3b - 22 = -31\]
\[3b = -9\]
\[b = -3\]

Therefore, the solution is \((-2, -3, 5)\).

**ANSWER:**
\((-2, -3, 5)\)
3-4 Systems of Equations in Three Variables

3. \(3y - 5z = -23\)
2. \(4x + 2y + 3z = 7\)
\[-2x - y - z = -3\]

**SOLUTION:**
\[3y - 5z = -23 \quad \rightarrow (1)\]
\[4x + 2y + 3z = 7 \quad \rightarrow (2)\]
\[-2x - y - z = -3 \quad \rightarrow (3)\]

Multiply the third equation by 2 and with the second equation.
\[(3) \times 2 \quad -4x - 2y - 2z = -6\]
\[(2) \quad 4x + 2y + 3z = 7\]

Substitute 1 for \(z\) in the first equation and solve for \(y\).
\[3y - 5(1) = -23\]
\[3y = -18\]
\[y = -6\]

Substitute \(-6\) for \(y\) and 1 for \(z\) in the third equation, and solve for \(x\).
\[-2x - (-6) - (1) = -3\]
\[-2x + 6 - 1 = -3\]
\[-2x = -8\]
\[x = 4\]

Therefore, the solution is (4, -6, 1).

**ANSWER:**
(4, -6, 1)

3. \(2x + y + 4z = 19\)
\[-5x - 2y + 8z = 62\]

**SOLUTION:**
\[3x + 6y - 2z = -6 \quad \rightarrow (1)\]
\[2x + y + 4z = 19 \quad \rightarrow (2)\]
\[-5x - 2y + 8z = 62 \quad \rightarrow (3)\]

Eliminate one variable.

Multiply the second equation by \(-2\) and add with the third equation.
\[2)(-2) \quad -4x - 2y - 8z = -38\]
\[-5x - 2y + 8z = 62\]
\[-9x - 4y = 24 \quad \rightarrow (4)\]

Multiply the first equation by 2 and add with the second equation.
\[(1) \times 2 \quad 6x + 12y - 4z = -12\]
\[2x + y + 4z = 19\]
\[8x + 13y = 7 \quad \rightarrow (5)\]

Solve the fifth and fourth equations.
\[(5) \times 9 \quad 72x + 117y = 63\]
\[(4) \times 8 \quad -72x - 32y = 192\]
\[85y = 255\]
\[y = 3\]

Substitute 3 for \(y\) in the fourth equation and solve for \(x\).
\[-9x - 4(3) = 24\]
\[-9x - 12 = 24\]
\[-9x = 36\]
\[x = -4\]

Substitute \(-4\) for \(x\) and 3 for \(y\) in the second equation, and solve for \(z\).
\[2(-4) + (3) + 4z = 19\]
\[-8 + 3 + 4z = 19\]
\[4z = 24\]
\[z = 6\]

Therefore, the solution is \((-4, 3, 6)\).

**ANSWER:**
\((-4, 3, 6)\)
3-4 Systems of Equations in Three Variables

**SOLUTION:**

\[ -4r - s + 3t = -9 \] \( \rightarrow (1) \)

\[ 3r + 2s - t = 3 \] \( \rightarrow (2) \)

\[ r + 3s - 5t = 29 \] \( \rightarrow (3) \)

Eliminate one variable.
Multiply the first equation by 2 and add with the second equation.

\[ (1) \times 2 \quad -8r - 2s + 6t = -18 \]

\[ (2) \quad 3r + 2s - t = 3 \]

\[ -5r + 5t = -15 \]

\[ r - t = 3 \rightarrow (4) \]

Multiply the second equation by \(-3\), multiply the third equation by 2 and add.

\[ (2) \times -3 \quad -9r - 6s + 3t = -9 \]

\[ (3) \times 2 \quad 2r + 6s - 10t = 58 \]

\[ -7r - 7t = 49 \]

\[ r + t = -7 \rightarrow (5) \]

To solve the fourth and fifth equations, add both equations.

\[ (4) + (5) \]

\[ 2r = -4 \]

\[ r = -2 \]

Substitute \(-2\) for \(r\) in the fifth equation and solve for \(t\).

\[ -2 + t = -7 \]

\[ t = -5 \]

Substitute \(-2\) for \(r\) and \(-5\) for \(t\) in the first equation, and solve for \(s\).

\[ -4(-2) - s + 3(-5) = -9 \]

\[ 8 - s - 15 = -9 \]

\[ s = 2 \]

Therefore, the solution is \((-2, 2, -5)\).

**ANSWER:**

\((-2, 2, -5)\)

\[ 3x + 5y - z = 12 \] \( \rightarrow (1) \)

\[ -2x - 3y + 5z = 14 \] \( \rightarrow (2) \)

\[ 4x + 7y + 3z = 38 \] \( \rightarrow (3) \)

Eliminate one variable.
Multiply the first equation by 2, multiply the second equation by 3 and add.

\[ (1) \times 2 \quad 6x + 10y - 2z = 24 \]

\[ (2) \times 3 \quad -6x - 9y + 15z = 42 \]

\[ y + 13z = 66 \rightarrow (4) \]

Multiply the second equation by 2 and add with the third equation.

\[ (2) \times 2 \quad -4x - 6y + 10z = 28 \]

\[ (3) \quad 4x + 7y + 3z = 38 \]

\[ y + 13z = 66 \rightarrow (5) \]

Multiply the first equation by \(-4\), multiply the third equation by 3, and add.

\[ (1) \times -4 \quad -12x - 20y + 4z = -48 \]

\[ (3) \times 3 \quad 12x + 21y + 9z = 114 \]

\[ y + 13z = 66 \rightarrow (6) \]

Since the equations 4, 5 and 6 are same, the system of equations has an infinite number of solutions.

**ANSWER:**

Infinite solutions

\[ 2a - 3b + 5c = 58 \]

\[ -5a + b - 4c = -51 \]

\[ -6a - 8b + c = 22 \]

**SOLUTION:**
Solve each system of equations.

\[ 2a - 3b + 5c = 58 \quad \rightarrow (1) \]
\[ -5a + b - 4c = -51 \quad \rightarrow (2) \]
\[ -6a - 8b + c = 22 \quad \rightarrow (3) \]

Eliminate one variable.

Multiply the second equation by 3 and add with the first equation.

\[
\begin{align*}
(2) \times 3 & \quad -15a + 3b - 12c = -153 \\
(1) & \quad 2a - 3b + 5c = 58 \\
& \quad -13a - 7c = -95 \\
& \quad 13a + 7c = 95 \quad \rightarrow (4)
\end{align*}
\]

Multiply the second equation by 8 and add with the third equation.

\[
\begin{align*}
(2) \times 8 & \quad -40a + 8b - 32c = -408 \\
(3) & \quad -6a - 8b + c = 22 \\
& \quad -46a - 31c = 386 \quad \rightarrow (5)
\end{align*}
\]

Solve the fourth and fifth equations.

\[
\begin{align*}
(5) \times 31 & \quad 403a + 217b = 2945 \\
(4) \times 7 & \quad -322a - 218b = 2702 \\
& \quad 81a = 243 \\
& \quad a = 3
\end{align*}
\]

Substitute 3 for \( a \) in the fourth equation and solve for \( c \).

\[
\begin{align*}
13(3) + 7c & = 95 \\
39 + 7c & = 95 \\
7c & = 56 \\
c & = 8
\end{align*}
\]

Substitute 3 for \( a \) and 8 for \( c \) in the second equation, and solve for \( b \).

\[
\begin{align*}
-5(3) + b - 4(8) & = -51 \\
-15 + b - 32 & = -51 \\
b - 47 & = -51 \\
b & = -4
\end{align*}
\]

Therefore, the solution is \((3, -4, 8)\).

**ANSWER:**

\((3, -4, 8)\)

7. **DOWNLOADING** Heather downloaded some television shows. A sitcom uses 0.3 gigabyte of memory; a drama, 0.6 gigabyte; and a talk show, 0.6 gigabyte. She downloaded 7 programs totaling 3.6 gigabytes. There were twice as many episodes of the drama as the sitcom.

a. Write a system of equations for the number of episodes of each type of show.

b. How many episodes of each show did she download?

**SOLUTION:**

a. Let \( s \), \( t \) and \( d \) be the number of sitcoms, talk shows and dramas respectively. The system of equations is:

\[
s + d + t = 7, \quad d = 2s \quad \text{and} \quad 0.3s + 0.6d + 0.6t = 3.6
\]

b. Solve \( s + d + t = 7 \), \( d = 2s \) and \( 0.3s + 0.6d + 0.6t = 3.6 \).

\[
s + d + t = 7 \quad \rightarrow (1) \\
d = 2s \quad \rightarrow (2) \\
0.3s + 0.6d + 0.6t = 3.6 \quad \rightarrow (3)
\]

Substitute \( 2s \) for \( d \) in the first and third equation.

\[
s + 2s + t = 7 \\
3s + t = 7 \quad \rightarrow (4) \\
0.3s + 0.6(2s) + 0.6t = 3.6 \\
1.5s + 0.6t = 3.6 \quad \rightarrow (5)
\]

Solve the fourth and fifth equations.

\[
(4) \times -0.6 & \quad -1.8s - 0.6t = -4.2 \\
(5) & \quad 1.5s + 0.6t = 3.6 \\
& \quad -0.3s = -0.6 \\
& \quad s = 2
\]

Substitute 2 for \( s \) in the second equation and solve for \( d \).

\[
d = 2(2) \\
= 4
\]

Therefore, she downloaded 2 episodes of sitcom, 4 television shows. A sitcom uses 0.3 gigabyte of television shows, 3.6 gigabytes. Therefore, she invested $55,000, $20,000 and $25,000 for this investment. Total interest amount is $6300. That is, he should spend $19.99 for an oil change and $20 for brake pad replacement. The system of equations is:

\[
\begin{align*}
x + y & = 30 \\
2x + 5y & = 285 \\
3x + 2y & = 250 \\
x + 6y & = 270
\end{align*}
\]

Multiply the first equation by 2 and add with the second equation.

\[
\begin{align*}
(1) \times 2 & \quad 2x + 2y = 60 \\
(2) & \quad 2x + 5y = 285 \\
& \quad 3y = 225 \\
y & = 75 \\
x + y & = 30 \\
x + 75 & = 30 \\
x & = -45
\end{align*}
\]

Substitute 3 for \( a \) in the fourth equation and solve for \( c \).

\[
\begin{align*}
13(3) + 7c & = 95 \\
39 + 7c & = 95 \\
7c & = 56 \\
c & = 8
\end{align*}
\]

Substitute 3 for \( a \) and 8 for \( c \) in the second equation, and solve for \( b \).

\[
\begin{align*}
-5(3) + b - 4(8) & = -51 \\
-15 + b - 32 & = -51 \\
b - 47 & = -51 \\
b & = -4
\end{align*}
\]

Therefore, the solution is \((3, -4, 8)\).
Substitute 2 for $s$ and 4 for $d$ in the first equation and solve for $t$.

$$2 + 4 + t = 7$$
$$t = 1$$

Therefore, she downloaded 2 episodes of sitcom, 4 episodes of drama and 1 episode of talk show.

**ANSWER:**

a. $s + d + t = 7, d = 2s, 0.3s + 0.6d + 0.6t = 3.6$

b. 2 sitcoms, 4 dramas, 1 talk show

**Solve each system of equations.**

- $-5x + y - 4z = 60$
- $2x + 4y + 3z = -12$
- $6x - 3y - 2z = -52$

**SOLUTION:**

- $-5x + y - 4z = 60$ → (1)
- $2x + 4y + 3z = -12$ → (2)
- $6x - 3y - 2z = -52$ → (3)

Eliminate one variable.

Multiply the first equation by $-4$ and add with the second equation.

$$(1) \times -4 \rightarrow 20x - 4y + 16z = -240$$

Multiply the first equation by 3 and add with the third equation.

$$(1) \times 3 \rightarrow -15x + 3y - 12z = 180$$

Solve the fourth and fifth equations.

$$\begin{aligned}(4) \times 9 & \quad 198x + 171z = -2268 \\
(5) \times 22 & \quad -198x - 308z = 2816 \\
& \quad -137z = 548 \\
& \quad z = -4 \end{aligned}$$

Substitute $-4$ for $z$ in the fourth equation and solve for $x$.

$$22x + 19(-4) = -252$$
$$22x - 76 = -252$$
$$22x = -176$$
$$x = -8$$

Substitute $-8$ and $-4$ for $x$ and $z$ in the first equation and solve for $y$.

$$-5(-8) + y - 4(-4) = 60$$
$$40 + y + 16 = 60$$
$$y + 56 = 60$$
$$y = 4$$

Therefore, the solution is $(-8, 4, -4)$.

**ANSWER:**

$$(8, 4, -4)$$

9. $-3a - 2b + 7c = -15$

$-a + 4b + 2c = -13$

**SOLUTION:**

$$4a + 5b - 6c = 2$$ → (1)

$$-3a - 2b + 7c = -15$$ → (2)

$$-a + 4b + 2c = -13$$ → (3)

Eliminate one variable.

Multiply the third equation by $-3$ and add the second equation with that.

$$\begin{aligned}(3) \times -3 & \quad 3a - 12b - 6c = 39 \\
(1) & \quad -3a - 2b + 7c = -15 \\
& \quad -14b + c = 24 \rightarrow (5) \end{aligned}$$
3-4 Systems of Equations in Three Variables

Solve each system of equations.

1. Solve each system of equations.

SOLUTION: Eliminate one variable. Multiply the first equation by 3 and add with the second equation.

\[ \begin{align*}
(4) & \quad 21b + 2c = -50 \\
(5) & \quad -28b - 2c = -48
\end{align*} \]

Multiply the first and second equation by 3 and 5 respectively and subtract. 

\[ \begin{align*}
28c & = - \frac{100}{11} \\
& \Rightarrow c = -\frac{25}{11}
\end{align*} \]

Substitute \(-\frac{25}{11}\) for \(c\) in the first equation and solve for \(b\).

\[ \begin{align*}
21b - \frac{50}{11} & = -50 \\
b & = -\frac{500}{231}
\end{align*} \]

Therefore, the solution is \((-\frac{500}{231}, -\frac{25}{11}, 0)\).

ANSWER: \((-\frac{500}{231}, -\frac{25}{11}, 0)\)

10. Solve each system of equations.

SOLUTION: Eliminate one variable. Multiply the first equation by 2 and add with the second equation.

\[ \begin{align*}
-2x + 5y + 3z & = -25 \\
-4x - 3y - 8z & = -39 \\
6x + 8y - 5z & = 14
\end{align*} \]

Multiply the first equation by 3 and add with the third equation.

\[ \begin{align*}
(1) & \quad 3(-2x + 5y + 3z) = -75 \\
(2) & \quad 3(-4x - 3y - 8z) = -117 \\
-13y - 14z & = 11 \Rightarrow (4)
\end{align*} \]

Multiply the first equation by 3 and add with the third equation.

\[ \begin{align*}
(1) & \quad -6x + 15y + 9z = -75 \\
(2) & \quad 6x + 8y - 5z = 14 \\
23y + 4z & = -61 \Rightarrow (5)
\end{align*} \]

Solve the fourth and fifth equations.

\[ \begin{align*}
(4) & \quad 2 - 26y - 28z = 22 \\
(5) & \quad 161y + 28z = -427 \\
135y & = -405 \Rightarrow y = -3
\end{align*} \]

Substitute \(-3\) for \(y\) in the fifth equation and solve for \(z\).

\[ \begin{align*}
23(-3) + 4z & = -61 \\
-69 + 4z & = -61 \\
4z & = 8 \\
z & = 2
\end{align*} \]

Substitute \(-3\) and \(2\) for \(y\) and \(z\) in the first equation and solve for \(x\).

\[ \begin{align*}
-2x + 5(-3) + 3(2) & = -25 \\
-2x - 15 + 6 & = -25 \\
-2x & = -16 \\
x & = 8
\end{align*} \]

Therefore, the solution is \((8, -3, 2)\).

ANSWER: \((8, -3, 2)\)
second equation.

\[(1) \times (-4) \quad -16r - 24s + 4t = 72\]
\[(2) \quad 3r + 2s - 4t = -24\]
\[-13r - 22s = 48 \quad \rightarrow (4)\]

Multiply the third equation by 2, and add with the second equation.

\[(3) \times 2 \quad -10r + 6s + 4t = 30\]
\[(2) \quad 3r + 2s - 4t = -24\]
\[-7r + 8s = 6 \quad \rightarrow (5)\]

Solve the fourth and fifth equations

\[(4) \times 4 \quad -52r - 88s = 192\]
\[(5) \times 11 \quad -77r + 88s = 66\]
\[-129r = 258\]
\[r = -2\]

Substitute \(-2\) for \(r\) in the fifth equation and solve for \(s\).

\[-7(-2) + 8s = 6\]
\[14 + 8s = 6\]
\[8s = -8\]
\[s = -1\]

Substitute \(-2\) and \(-1\) for \(r\) and \(s\) in the first equation and solve for \(t\).

\[4(-2) + 6(-1) - t = -18\]
\[-8 - 6 - t = -18\]
\[-t = 4\]
\[t = 4\]

Therefore, the solution is \((-2, -1, 4)\).

**ANSWER:**

\((-2, -1, 4)\)

12. \(4x + 3y + 3z = 18\)
\[-3x + 6y - z = 8\]

**SOLUTION:**

\[-2x + 15y + z = 44\]
\[
\rightarrow (1)
\]
\[4x + 3y + 3z = 18 \quad \rightarrow (2)\]
\[-3x + 6y - z = 8 \quad \rightarrow (3)\]

Eliminate one variable. Multiply the first equation by 2 and add with the second equation.

\[(1) \times 2 \quad -4x + 30y + 2z = 88\]
\[(2) \quad 4x + 3y + 3z = 18\]
\[33y + 5z = 106 \quad \rightarrow (4)\]

Multiply the second equation by 2 and the third equation by 4 then add.

\[(2) \times 3 \quad 12x + 9y + 9z = 54\]
\[(3) \times 4 \quad -12x + 24y - 4z = 32\]
\[33y + 5z = 86 \quad \rightarrow (5)\]

Solve the fourth and fifth equation.

\[(4) - (5) \quad 0 = 20x\]

This is a false statement. Therefore, there is no solution.

**ANSWER:**

No solution
Solve each system of equations.

13. \(-12x + 3y - 5z = 8\)
   \(-4x + 7y + 7z = 34\)

**SOLUTION:**

\[
\begin{align*}
4x + 2y + 6z &= 13 \quad \rightarrow (1) \\
-12x + 3y - 5z &= 8 \quad \rightarrow (2) \\
-4x + 7y + 7z &= 34 \quad \rightarrow (3)
\end{align*}
\]

Eliminate one variable.

Add the first and the third equations.

\[
(1) + (3) \quad 9y + 13z = 47 \quad \rightarrow (4)
\]

Multiply the first equation by 3 and add with the second equation.

\[
\begin{align*}
(1) \times 3 \quad 12x + 6y + 18z &= 39 \\
(2) \quad -12x + 3y - 5z &= 8 \\
\hline
9y + 13z &= 47 \quad \rightarrow (5)
\end{align*}
\]

Multiply the third equation by \(-3\) and add with the second equation.

\[
\begin{align*}
(3) \times -3 \quad 12x - 21y - 21z &= -102 \\
(4) \quad -12x + 3y - 5z &= 8 \\
\hline
-18y - 26z &= -94 \\
9y + 13z &= 47 \quad \rightarrow (6)
\end{align*}
\]

Since the equations 4, 5 and 6 are same, the system has an infinite number of solutions.

**ANSWER:**

Infinite solutions

\[
8x + 3y + 6z = 43
\]

14. \(-3x + 5y + 2z = 32\)
   \[5x - 2y + 5z = 24\]

**SOLUTION:**

\[
\begin{align*}
8x + 3y + 6z &= 43 \quad \rightarrow (1) \\
-3x + 5y + 2z &= 32 \quad \rightarrow (2) \\
5x - 2y + 5z &= 24 \quad \rightarrow (3)
\end{align*}
\]

Eliminate one variable.

Multiply the second equation by \(-3\) and add with the first equation.

\[
\begin{align*}
(1) \quad 8x + 3y + 6z &= 43 \\
(2) \times -3 \quad 9x - 15y - 6z &= -96 \\
17x - 12y &= -53 \quad \rightarrow (4)
\end{align*}
\]

Multiply the second and third equation by 5 and \(-2\) respectively and add.

\[
\begin{align*}
(2) \times 5 \quad -15x + 25y + 10z &= 160 \\
(3) \times -2 \quad -10x + 4y - 10z &= -48 \\
\hline
-25x + 29y &= 112 \quad \rightarrow (5)
\end{align*}
\]

Solve the fourth and fifth equations.

\[
\begin{align*}
(4) \times 25 \quad 425x - 300y &= -1325 \\
(5) \times 17 \quad -425x + 493y &= 1904 \\
\hline
193y &= 579 \\
y &= 3
\end{align*}
\]

Substitute 3 for \(y\) in the fourth equation and solve for \(x\).

\[
\begin{align*}
17x - 12(3) &= -53 \\
17x &= -53 \\
17x &= -17 \\
x &= -1
\end{align*}
\]

Substitute \(-1\) and 3 for \(x\) and \(y\) in the first equation and solve for \(z\).

\[
\begin{align*}
8(-1) + 3(3) + 6z &= 43 \\
-8 + 9 + 6z &= 43 \\
6z &= 42 \\
z &= 7
\end{align*}
\]

Therefore, the solution is \((-1, 3, 7)\).

**ANSWER:**

\((-1, 3, 7)\)

15. \[5x + 3y + 2z = -11\]
   \[8x - 6y + 5z = 4\]
3-4 Systems of Equations in Three Variables

**SOLUTION:**

\[-6x - 5y + 4z = 53 \quad \rightarrow (1)\]
\[5x + 3y + 2z = -11 \quad \rightarrow (2)\]
\[8x - 6y + 5z = 4 \quad \rightarrow (3)\]

Eliminate one variable.

Multiply the first and second equation by 3 and 5 respectively then add.

\[(1) \times 3 \quad -18x - 15y + 12z = 159\]
\[(2) \times 5 \quad 25x + 15y + 10z = -55\]
\[\quad 7x + 22z = 104 \quad \rightarrow (4)\]

Multiply the second equation by 2 and add with the third equation.

\[(2) \times 2 \quad 10x + 6y + 4z = -22\]
\[(3) \quad 8x - 6y + 5z = 4\]
\[\quad 18x + 9z = -18\]
\[\quad 2x + z = -2 \quad \rightarrow (5)\]

Solve the fourth and fifth equations.

\[(4) \quad 7x + 22z = 104\]
\[(5) \times -22 \quad -44x - 22z = 44\]
\[\quad -37x = 148\]
\[\quad x = -4\]

Substitute -4 for x in the fifth equation and solve for z.

\[2(-4) + z = -2\]
\[\quad -8 + z = -2\]
\[\quad z = 6\]

Substitute -4 and 6 for x and z in the first equation and solve for y.

\[-6(-4) - 5y + 4(6) = 53\]
\[\quad 24 - 5y + 24 = 53\]
\[\quad -5y = 5\]
\[\quad y = -1\]

Therefore, the solution is (-4, -1, 6).
3-4 Systems of Equations in Three Variables

Solve each system of equations.

1. Eliminate one variable.

\begin{align*}
5(-8) - 5(-7) + 8c &= -45 \\
-40 + 35 + 8c &= -45 \\
8c &= -40 \\
c &= -5
\end{align*}

Therefore, the solution is \((-8, -7, -5)\).

**ANSWER:**
\((-8, -7, -5)\)

17. \(2x - y + z = 1\)

\(x + 2y - 4z = 3\)

\(4x + 3y - 7z = -8\)

**SOLUTION:**
\begin{align*}
2x - y + z &= 1 \quad \rightarrow (1) \\
x + 2y - 4z &= 3 \quad \rightarrow (2) \\
4x + 3y - 7z &= -8 \quad \rightarrow (3)
\end{align*}

Eliminate one variable.

Multiply the first equation by 4 and add with the second equation.

\begin{align*}
(1) \times 4 & \\
8x - 4y + 4z &= 4
\end{align*}

\begin{align*}
(2) & \\
x + 2y - 4z &= 3
\end{align*}

\begin{align*}
9x - 2y &= 7 \quad \rightarrow (4)
\end{align*}

Multiply the first equation by 7 and add with the third equation.

\begin{align*}
(1) \times 7 & \\
14x - 7y + 7z &= 7
\end{align*}

\begin{align*}
(3) & \\
4x + 3y - 7z &= -8
\end{align*}

\begin{align*}
18x - 4y &= -1 \\
9x - 2y &= -\frac{1}{2} \quad \rightarrow (5)
\end{align*}

Solve the fourth and the fifth equation.

\begin{align*}
(4) - (5) & \\
0 &= \frac{15}{2} \\
x &= \frac{-15}{2}
\end{align*}

This is a false statement. Therefore, there is no solution.

**ANSWER:**
No solution

---

**Note:** The original solution provided incorrect steps for solving the system of equations. The correct approach involves eliminating variables and solving the resulting system of equations to find the solution. The final answer indicates no solution due to a false statement resulting from an incorrect step. The revised solution corrects these errors and provides the correct method for solving such systems.
3-4 Systems of Equations in Three Variables

x + 2y = 12
18. 3y - 4z = 25
x + 6y + z = 20

**SOLUTION:**

\[ x + 2y = 12 \] \( \rightarrow (1) \)
\[ 3y - 4z = 25 \] \( \rightarrow (2) \)
\[ x + 6y + z = 20 \] \( \rightarrow (3) \)

Eliminate one variable.
Subtract the first equation from the third equation.

\[(3) \quad x + 6y + z = 20 \]
\[(1) \quad x + 2y = 12 \]
\[(3) - (1) \quad 4y + z = 8 \] \( \rightarrow (4) \)

Multiply the fourth equation by 4 and add with the second equation.

\[(4) \times 4 \quad 16y + 4z = 32 \]
\[(2) \quad 3y - 4z = 25 \]
\[ 19y = 57 \]
\[ y = 3 \]

Substitute 3 for y in the fourth equation and solve for z.

\[ 4(3) + z = 8 \]
\[ 12 + z = 8 \]
\[ z = -4 \]

Substitute 3 for y in the first equation and solve for x.

\[ x + 2(3) = 12 \]
\[ x + 6 = 12 \]
\[ x = 6 \]

Therefore, the solution is (6, 3, -4)

**ANSWER:**

(6, 3, -4)

\[ r - 3s + t = 4 \]
19. \[ 3r - 6s + 9t = 5 \]
\[ 4r - 9s + 10t = 9 \]

**SOLUTION:**

\[ r - 3s + t = 4 \] \( \rightarrow (1) \)
\[ 3r - 6s + 9t = 5 \] \( \rightarrow (2) \)
\[ 4r - 9s + 10t = 9 \] \( \rightarrow (3) \)

Eliminate one variable.
Multiply the first equation by -3 and add with the second equation.

\[(1) \times -3 \quad -3r + 9s - 3t = -12 \]
\[(2) \quad 3r - 6s + 9t = 5 \]
\[ 3s + 6t = -7 \] \( \rightarrow (4) \)

Multiply the second equation by -4 and add with the third equation.

\[(1) \times -4 \quad -4r + 12s - 4t = -16 \]
\[(3) \quad 4r - 9s + 10t = 9 \]
\[ 3s + 6t = -7 \] \( \rightarrow (5) \)

Multiply the second equation by -4 and the third equation by 3 then add.

\[(2) \times -4 \quad -12r + 24s - 36t = -20 \]
\[(3) \times 3 \quad 12r - 27s + 30t = 27 \]
\[ -3s - 6t = 7 \]
\[ 3s + 6t = 7 \] \( \rightarrow (6) \)

Since the equations 4, 5 and 6 are same, the system has an infinite number of solutions.

**ANSWER:**

Infinite solutions

20. **CCSS SENSE-MAKING** A friend e-mails you the results of a recent high school swim meet. The e-mail states that 24 individuals placed, earning a combined total of 53 points. First place earned 3 points, second place earned 2 points, and third place earned 1 point. There were as many first-place finishers as second- and third-place finishers combined.
a. Write a system of three equations that represents how many people finished in each place.
b. How many swimmers finished in first place, in second place, and in third place?
c. Suppose the e-mail had said that the athletes scored a combined total of 47 points. Explain why this statement is false and the solution is unreasonable.

**SOLUTION:**

a. Let \( x, y, \) and \( z \) be the number of swimmers finished in first place, in second place and in third place.

\[
x + y + z = 24 \\
3x + 2y + z = 53 \\
x = y + z
\]
n. Name the equations.

\[
x + y + z = 24 \quad \rightarrow (1) \\
3x + 2y + z = 53 \quad \rightarrow (2) \\
x = y + z \quad \rightarrow (3)
\]

Substitute \( x \) for \( y + z \) in the first equation and solve for \( x \).

\[
x + x = 24 \\
2x = 24 \\
x = 12
\]

Substitute 12 for \( x \) and 5 for \( y + z \) in the second equation and solve for \( y \).

\[
3(12) + y + 12 = 53 \\
36 + y + 12 = 53 \\
y = 5
\]

Substitute \( x \) and \( y \) values in the first equation and solve for \( z \).

\[
12 + 5 + z = 24 \\
z = 7
\]

7 simmers placed third, 5 simmers placed second, and 12 simmers placed first.

c. The statement is false because when you solve for second place, you get a negative as an answer and you cannot have a negative person.

**ANSWER:**

a. \( x + y + z = 24, \ 3x + 2y + z = 53, \ x = y + z \).
b. 7 swimmers placed third, 5 swimmers placed second, and 12 swimmers placed first.
c. The statement is false because when you solve for second place, you get a negative as an answer and you cannot have a negative person.

21. **AMUSEMENT PARKS** Nick goes to the amusement park to ride roller coasters, bumper cars, and water slides. The wait for the roller coasters is 1 hour, the wait for the bumper cars is 20 minutes long, and the wait for the water slides is only 15 minutes long. Nick rode 10 total rides during his visit. Because he enjoys roller coasters the most, the number of times he rode the roller coasters was the sum of the times he rode the other two rides. If Nick waited in line for a total of 6 hours and 20 minutes, how many of each ride did he go on?

**SOLUTION:**

Let \( x, y, \) and \( z \) be the number of raids in roller coaster, bumper car and water slide respectively. Nick rode 10 rides during his visit.

\[
x + y + z = 10 \quad \rightarrow (1)
\]

The number of times that Nick rode the roller coaster is the sum of the times he rode the other two rides. So:

\[
x = y + z \quad \rightarrow (2)
\]

He waited in line for a total of 6 hours 20 minutes.

\[
x + \frac{1}{3}y + \frac{1}{4}z = \frac{19}{3} \\
12x + 4y + 3z = 76 \quad \rightarrow (3)
\]

Substitute \( x \) for \( y + z \) in the first equation and solve for \( x \).

\[
x + x = 10 \\
2x = 10 \\
x = 5
\]

Substitute 5 for \( x \) in the second and the third equation and simplify.
3-4 Systems of Equations in Three Variables

12(5) + 4y + 3z = 76
60 + 4y + 3z = 76
4y + 3z = 16 \rightarrow (4)
y + z = 5 \rightarrow (5)

Multiply the fifth equation by -3 and add with the fourth equation.

(4) \quad 8y + 3z = 16
(5) \times -3 \quad -3y - 3z = -15
5y = 5
y = 1

Substitute 1 for y in the fifth equation and solve for z.

1 + z = 5
z = 4

Nick rode the roller coaster, bumper cars and water slides 5, 1 and 4 times respectively.

ANSWER: roller coasters: 5; bumper cars: 1; water slides: 4

22. BUSINESS Ramón usually gets one of the routine maintenance options at Annie’s Garage. Today however, he needs a different combination of work than what is listed.

a. Assume that the price of an option is the same price as purchasing each item separately. Find the prices for an oil change, a radiator flush, and a brake pad replacement.

b. If Ramón wants his brake pads replaced and his radiator flushed, how much should he plan to spend?

SOLUTION:
a. Let x, y and z be the price for oil change, brake pad replacement and radiator flush.

\[ x + z = 29.9 \]
\[ x + y = 39.99 \]
\[ x + y + z = 49.99 \]

Substitute 39.99 for \( x + y \) in the third equation and solve for \( z \).

\[ 39.99 + z = 49.99 \]
\[ z = 10 \]

Substitute 10 for \( z \) in the first equation and solve for \( x \).

\[ x + 10 = 29.99 \]
\[ x = 19.99 \]

Substitute 19.99 for \( x \) in the second equation and solve for \( y \).

\[ 19.99 + y = 39.99 \]
\[ y = 20 \]

Therefore, the cost for oil change is $19.99, the cost for brake pad replacement is $20 and the cost for radiator flush is $10.

b. The cost for brake pad replacement and radiator flush are $20 and $10. Therefore, he should spend $30.

ANSWER:
a. oil change: $19.99; brake pad replacement: $20; radiator flush: $10
b. $30

23. FINANCIAL LITERACY Kate invested $100,000 in three different accounts. If she invested $30,000 more in account A than account C and is expected to earn $6300 in interest, how much did she invest in each account?

<table>
<thead>
<tr>
<th>Account</th>
<th>Expected Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4%</td>
</tr>
<tr>
<td>B</td>
<td>8%</td>
</tr>
<tr>
<td>C</td>
<td>10%</td>
</tr>
</tbody>
</table>

SOLUTION:
Let \( a \), \( b \) and \( c \) be the amount invested in the Account
3-4 Systems of Equations in Three Variables

A, B and C respectively.

\[ a + b + c = 100000 \quad \rightarrow (1) \]

Kate invested $30,000 more in account A than account C.

Therefore, \( a = c + 30000 \quad \rightarrow (2) \)

Substitute \( c + 30000 \) for \( a \) in the first equation and simplify.

\[ c + 30000 + b + c = 100000 \]
\[ 2c + b = 70000 \quad \rightarrow (3) \]

Total interest amount is $6300. That is,

\[ 4\% \text{ of } a + 8\% \text{ of } b + 10\% \text{ of } c = 6300 \]
\[ 0.04a + 0.08b + 0.1c = 6300 \]

Substitute \( c + 30000 \) for \( a \) and simplify.

\[ 0.04(c + 30000) + 0.08b + 0.1c = 6300 \]
\[ 0.14c + 1200 + 0.08b = 6300 \]
\[ 0.14c + 0.08b = 5100 \quad \rightarrow (4) \]

Solve the third and fourth equations.

\[ (3) \times -0.8 \quad -0.16c - 0.08b = -5600 \]
\[ (4) \quad 0.14c + 0.08b = 5100 \]
\[ -0.2c = -500 \]
\[ c = 25000 \]

Substitute 25000 for \( c \) in the second equation and solve for \( a \).

\[ a = 25000 + 30000 \]
\[ = 55000 \]

Substitute 25000 for \( c \) in the third equation and solve for \( b \).

\[ 2(25000) + b = 70000 \]
\[ 50000 + b = 70000 \]
\[ b = 20000 \]

Therefore, she invested $55,000, $20,000 and $25,000 in the account A, B and C respectively.

ANSWER:
A: $55,000; B: $20,000; C: $25,000

24. CCSS REASONING Write a system of equations to represent the three rows of figures below. Use the system to find the number of red triangles that will balance one green circle.

\[ t + c = s, p + t = c, 2s = 3p \]

where \( t \) represents triangle, \( c \) represents circle, \( s \) represents square, and \( p \) represents pentagon; 5 red triangles

ANSWER:
\[ t + c = s, p + t = c, 2s = 3p \]

where \( t \) represents triangle, \( c \) represents circle, \( s \) represents square, and \( p \) represents pentagon; 5 red triangles

25. CHALLENGE The general form of an equation for a parabola is \( y = ax^2 + bx + c \), where \((x, y)\) is a point on the parabola. If three points on a parabola are \((2, -10), (-5, -101), \) and \((6, -90)\), determine the values of \( a, b, \) and \( c \) and write the general form of the equation.

SOLUTION:
Substitute the points \((2, -10), (-5, -101), \) and \((6, -90)\) in the equation \( y = ax^2 + bx + c \).

\[ -10 = a(2^2) + b(2) + c \]
\[ -10 = 4a + 2b + c \quad \rightarrow (1) \]
\[ -101 = a(-5)^2 + b(-5) + c \]
\[ -101 = 25a - 5b + c \quad \rightarrow (2) \]
\[ -90 = a(6^2) + b(6) + c \]
\[ -90 = 36a + 6b + c \quad \rightarrow (3) \]
Solve the equations 1, 2 and 3.

\[
\begin{align*}
(2) - (1) & : 21a - 7b = -91 \quad \rightarrow (4) \\
(3) - (1) & : 32a + 4b = -80 \quad \rightarrow (5)
\end{align*}
\]

Solve the fourth and fifth equations.

\[
\begin{align*}
(4) \times 4 & : 84a - 28b = -364 \\
(5) \times 7 & : 224a + 28b = -560
\end{align*}
\]

\[
308a = -924
\]

\[
a = -3
\]

Substitute \(-3\) for \(a\) in the fourth equation and solve for \(b\).

\[
21(-3) - 7b = -91
\]

\[
-63 - 7b = -91
\]

\[
-7b = -28
\]

\[
b = 4
\]

Substitute \(-3\) and \(4\) for \(a\) and \(b\) in the first equation.

\[
4(-3) + 2(4) + c = -10
\]

\[
-12 + 8 + c = -10
\]

\[
c = -6
\]

The value of \(a\), \(b\) and \(c\) are \(-3\), \(4\) and \(-6\) respectively.

Therefore, the equation of the parabola is \(y = -3x^2 + 4x - 6\).

**ANSWER:**

\[
y = -3x^2 + 4x - 6; a = -3, b = 4, c = -6
\]

26. **PROOF** Consider the following system and prove that if \(b = c = -a\), then \(ty = a\).

\[
rx + ty + vz = a
\]

\[
rx - ty + vz = b
\]

\[
rx + ty - vz = c
\]

**SOLUTION:**

First add \(a\) and \(b\) to eliminate \(ty\).

\[
a + b = (rx + ty + vz) + (rx - ty + vz)
\]

\[
a + b = 2rx + 2vz
\]

\[
0 = 2rx + 2vz
\]

\[
0 = rx + vz
\]

Use this equation to prove that \(a = ty\).

\[
rx + ty + vz = a \quad \text{Given}
\]

\[
by + (rx + vz) = a \quad \text{Commutative and Associative Properties } (+);
\]

\[
by + 0 = a \quad \text{Substitution}
\]

\[
by = a \quad \text{Simplify}
\]

**ANSWER:**

\[
a + b = (rx + ty + vz) + (rx - ty + vz)
\]

\[
a + b = 2rx + 2vz
\]

\[
a + (-c) = 2rx + 2vz
\]

\[
0 = 2rx + 2vz
\]

\[
0 = rx + vz
\]

\[
rx + ty + vz = a \quad \text{Given}
\]

\[
by + (rx + vz) = a \quad \text{Commutative and Associative Properties } (+);
\]

\[
by + 0 = a \quad \text{Substitution}
\]

\[
by = a \quad \text{Simplify}
\]

27. **OPEN ENDED** Write a system of three linear equations that has a solution of \((-5, 2, 6)\). Show that the ordered triple satisfies all three equations.

**SOLUTION:**

Sample answer:

\[
3x + 4y + z = -17
\]

\[
3(-5) + 4(-2) + 6 = -17
\]

\[
-15 + (-8) + 6 = -17
\]

\[
-23 + 6 = -17
\]

\[
-17 = -17
\]
3-4 Systems of Equations in Three Variables

Solve each system of equations.

1. Solve the system.

\[ 2x - 5y - 3z = -18 \]
\[ 2(-5) - 5(-2) - 3(6) = -18 \]
\[ -10 + 10 - 18 = -18 \]
\[ -18 = -18 \checkmark \]
\[ -x + 3y + 8z = 47 \]
\[ -(-5) + 3(-2) + 8(6) = 47 \]
\[ 5 - 6 + 48 = 47 \]
\[ -1 + 48 = 47 \]
\[ 47 = 47 \checkmark \]

**ANSWER:**
Sample answer:
\[ 3x + 4y + z = -17 \]
\[ 3(-5) + 4(-2) + 6 = -17 \]
\[ -15 + (-8) + 6 = -17 \]
\[ -23 + 6 = -17 \]
\[ -17 = -17 \checkmark \]
\[ 2x - 5y - 3z = -18 \]
\[ 2(-5) - 5(-2) - 3(6) = -18 \]
\[ -10 + 10 - 18 = -18 \]
\[ -18 = -18 \checkmark \]
\[ -x + 3y + 8z = 47 \]
\[ -(-5) + 3(-2) + 8(6) = 47 \]
\[ 5 - 6 + 48 = 47 \]
\[ -1 + 48 = 47 \]
\[ 47 = 47 \checkmark \]

28. **REASONING** Use the diagram below of the solution of systems of equations to consider a system on inequalities in three variables. Describe the solution of such a system.

**One Solution**
The three individual planes intersect at a specific point.

**Infinitely Many Solutions**
The planes intersect in a line. Every coordinate on the line represents a solution of the system.

**No Solution**
There are no points in common with all three planes.

**SOLUTION:**
The solution of an inequality in 3 variables would be the region of space on one side or the other of a plane, which the plane included if the inequality is \( \leq \) or \( \geq \). The solution of a system of inequalities in 3 variables would be the intersection of the regions of space that are solution to the individual inequalities in the system.

**ANSWER:**
The solution of an inequality in 3 variables would be the region of space on one side or the other of a plane, which the plane included if the inequality is \( \leq \) or \( \geq \). The solution of a system of inequalities in 3 variables would be the intersection of the regions of space that are solution to the individual inequalities in the system.
3-4 Systems of Equations in Three Variables

29. **WRITING IN MATH** Use your knowledge of solving a system of three linear equations with three variables to explain how to solve a system of four equations with four variables.

**SOLUTION:**
Sample answer: First, combine two of the original equations using elimination to form a new equation with three variables. Next, combine a different pair of the original equations using elimination to eliminate the same variable and form a second equation with three variables. Do the same thing with a third pair of the original equations. You now have a system of three equations with three variables. Follow the same procedure you learned in this section. Once you find the three variables, you need to use them to find the eliminated variable.

**ANSWER:**
Sample answer: First, combine two of the original equations using elimination to form a new equation with three variables. Next, combine a different pair of the original equations using elimination to eliminate the same variable and form a second equation with three variables. Do the same thing with a third pair of the original equations. You now have a system of three equations with three variables. Follow the same procedure you learned in this section. Once you find the three variables, you need to use them to find the eliminated variable.

30. What is the solution of the system of equations shown below?
\[
\begin{align*}
x - y + z &= 0 \\
-5x + 3y - 2z &= -1 \\
2x - y + 4z &= 11
\end{align*}
\]
A (0, 3, 3)
B (2, 5, 3)
C no solution
D infinitely many solutions

**SOLUTION:**
\[
\begin{align*}
x - y + z &= 0 & \rightarrow (1) \\
-5x + 3y - 2z &= -1 & \rightarrow (2) \\
2x - y + 4z &= 11 & \rightarrow (3)
\end{align*}
\]
Eliminate one variable.
Multiply the first equation by 2 and with the second equation.

\[
\begin{align*}
(1) \times 2 & \quad 2x - 2y + 2z = 0 \\
(2) & \quad -5x + 3y - 2z = -1 \\
& \quad -3x + y = -1 \quad \rightarrow (4)
\end{align*}
\]
Multiply the second equation by 2 and add with the third equation.
\[
\begin{align*}
(2) \times 2 & \quad -10x + 6y - 4z = -2 \\
(3) & \quad 2x - y + 4z = 11 \\
& \quad -8x + 5y = 9 \quad \rightarrow (5)
\end{align*}
\]
Solve the fourth and fifth equations.
\[
\begin{align*}
(4) \times -5 & \quad 15x - 5y = 5 \\
(5) & \quad -8x + 5y = 9 \\
& \quad 7x = 14 \\
& \quad x = 2
\end{align*}
\]
Substitute 2 for x in the fourth equation and solve for y.
\[
\begin{align*}
-3(2) + y &= -1 \\
-6 + y &= -1 \\
y &= 5
\end{align*}
\]
Substitute 2 and 5 for x and y in the first equation and solve for z.
\[
\begin{align*}
2 - 5 + z &= 0 \\
z &= 3
\end{align*}
\]
The solution is (2, 5, 3).
Option B is the correct answer.

**ANSWER:**
B

31. **ACT/SAT** The graph shows which system of equations?

![Graph showing system of equations](image)
3-4 Systems of Equations in Three Variables

A \begin{align*} y + 14 &= 4x \\
y &= 4 - 2x \\
-7 &= y - \frac{5}{3}x \\
y - 14 &= 4 \\
2x &= 4 + y \\
7 &= y - \frac{5}{3}x \\
y - 14 &= 4x \\
\end{align*}

B \begin{align*} y &= 4 + 2x \\
-7 &= y + \frac{5}{3}x \\
y - 4x &= 14 \\
y &= 2x + 4 \\
7 &= y + \frac{5}{3}x \\
y + 14x &= 4 \\
\end{align*}

C \begin{align*} -2y &= 4 + y \\
-7 &= y - \frac{5}{3}x \\
\end{align*}

\textbf{SOLUTION:}

The lines intersect at (3, –2). Substitute the point in each system of equations.

\begin{align*}
y + 14 &= 4x \\
-2 + 14 &= 4(3) \\
12 &= 12 \checkmark \\
y &= 4 - 2x \\
-2 &= 4 - 2(3) \\
-2 &= -2 \checkmark \\
-7 &= y - \frac{5}{3}x \\
-7 &= -2 - \frac{5}{3}(3) \\
-7 &= -7 \checkmark \\
\end{align*}

First system of equations satisfies the point (3, –2). Therefore, option A is the correct answer.

\textbf{ANSWER:} A

32. \textbf{EXTENDED RESPONSE} Use the graph to find the solution of the system of equations. Describe one way to check the solution.

\textbf{SOLUTION:}

The lines intersect at (2, 8). So the solution is (2, 8).

Sample answer: You can substitute (2, 8) into each of the equations and make sure the equations are true.

\textbf{ANSWER:} (2, 8); Sample answer: You can substitute (2, 8) into each of the equations and make sure the equations are true.

33. Which of the following represents a correct procedure for solving each equation?

\begin{align*}
-3(x - 7) &= -16 \\
-3x - 21 &= -16 \\
\text{F} \quad -3x &= 5 \\
\quad x &= \frac{-5}{3} \\
\quad 7 - 4x &= 3x + 27 \\
\quad 7 - 7x &= 27 \\
\text{G} \quad -7x &= \frac{-20}{7} \\
&= 20 \\
\quad 2(x - 4) &= 20 \\
\quad 2x - 8 &= 20 \\
\quad 2x &= 12 \\
\quad x &= 6 \\
\quad 6(2x + 1) &= 30 \\
\quad 12x + 6 &= 30 \\
\quad 12x &= 24 \\
\quad x &= 2 \\
\end{align*}

\textbf{SOLUTION:}
3-4 Systems of Equations in Three Variables

Solve each system of equations.

1. 

SOLUTION: 

Eliminate one variable. 
Multiply the first equation by 3 and add with the second equation. 

\(-3\left(\frac{5}{3} - 7\right) = -16\) 

\(5 + 21 = -16\) 

\(26 = -16 \neq 32\) 

Substitute 20 for \(x\) in the equation \(7 - 4x = 3x + 27\). 

\(7 - 4(20) = 3(20) + 27\) 

\(-73 = 87 \neq 0\) 

Substitute 6 for \(x\) in the equation \(2(x - 4) = 20\). 

\(2(6 - 4) = 20\) 

\(2(2) = 20\) 

\(4 = 20 \neq 3\) 

Substitute 2 for \(x\) in the equation \(6(2x + 1) = 30\). 

\(6(2(2) + 1) = 30\) 

\(6(5) = 20\) 

\(30 = 30\) 

Therefore, option J is the correct answer. 

ANSWER: 

J

A feasible region has vertices at \((-3, 2), (1, 3), (6, 1),\) and \((2, -2)\). Find the maximum and minimum values of each function.

34. \(f(x, y) = 2x - y\)

SOLUTION: 
Substitute the points \((-3, 2), (1, 3), (6, 1),\) and \((2, -2)\) in the function \(f(x, y) = 2x - y\). 

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(2x - y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3, 2))</td>
<td>(-2)</td>
<td>(-8)</td>
</tr>
<tr>
<td>((1, 3))</td>
<td>(-1)</td>
<td>(-1)</td>
</tr>
<tr>
<td>((6, 1))</td>
<td>(11)</td>
<td>(11)</td>
</tr>
<tr>
<td>((2, -2))</td>
<td>(2)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

The maximum value is 11 and the minimum value is \(-8\). 

ANSWER: 

11; \(-8\)

35. \(f(x, y) = x + 5y\)

SOLUTION: 
Substitute the points \((-3, 2), (1, 3), (6, 1),\) and \((2, -2)\) in the function \(f(x, y) = x + 5y\). 

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(x + 5y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3, 2))</td>
<td>((-3) + 5(2))</td>
<td>(7)</td>
</tr>
<tr>
<td>((1, 3))</td>
<td>((1) + 5(3))</td>
<td>(16)</td>
</tr>
<tr>
<td>((6, 1))</td>
<td>((6) + 5(1))</td>
<td>(11)</td>
</tr>
<tr>
<td>((2, -2))</td>
<td>((2) + 5(-2))</td>
<td>(-8)</td>
</tr>
</tbody>
</table>

The maximum value is 16 and the minimum value is \(-8\). 

ANSWER: 

16; \(-8\)
36. $f(x, y) = y - 4x$

**SOLUTION:**
Substitute the points $(-3, 2)$, $(1, 3)$, $(6, 1)$, and $(2, -2)$ in the function $f(x, y) = y - 4x$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$y - 4x$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-3, 2)$</td>
<td>$(2) - 4(-3)$</td>
<td>$14$</td>
</tr>
<tr>
<td>$(1, 3)$</td>
<td>$(3) - 4(1)$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$(6, 1)$</td>
<td>$(1) - 4(6)$</td>
<td>$-23$</td>
</tr>
<tr>
<td>$(2, -2)$</td>
<td>$(-2) - 4(2)$</td>
<td>$-10$</td>
</tr>
</tbody>
</table>

The maximum value is $14$ and the minimum value is $-23$.

**ANSWER:**
$14; -23$

37. $f(x, y) = -x + 3y$

**SOLUTION:**
Substitute the points $(-3, 2)$, $(1, 3)$, $(6, 1)$, and $(2, -2)$ in the function $f(x, y) = -x + 3y$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$-x + 3y$</th>
<th>$f(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-3, 2)$</td>
<td>$-(3) + 3(2)$</td>
<td>$9$</td>
</tr>
<tr>
<td>$(1, 3)$</td>
<td>$-(1) + 3(3)$</td>
<td>$8$</td>
</tr>
<tr>
<td>$(6, 1)$</td>
<td>$-(6) + 3(1)$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$(2, -2)$</td>
<td>$-(2) + 3(-2)$</td>
<td>$-8$</td>
</tr>
</tbody>
</table>

The maximum value is $9$ and the minimum value is $-8$.

**ANSWER:**
$9; -8$

38. **SKI CLUB** The ski club’s budget for the year is $4250. They are able to find skis for $75 per pair and boots for $40 per pair. They know they should buy more boots than skis because the skis are adjustable to several sizes of boots.

a. Give an example of three different purchases that the ski club can make.

b. Suppose the ski club wants to spend all of its budget. What combination of skis and boots should they buy? Explain.

**SOLUTION:**
a. Sample answer: $40$ boots, $35$ skis; $45$ boots, $32$ skis; $50$ boots, $30$ skis

b. $50$ boots and $30$ skis cost exactly $4250.

**ANSWER:**
$40$ boots and $35$ skis; $45$ boots, $32$ skis; $50$ boots, $30$ skis

39. Solve each system of equations.

$x = y + 5$

$3x + y = 19$

**SOLUTION:**

\[ x = y + 5 \quad \rightarrow (1) \]

\[ 3x + y = 19 \quad \rightarrow (2) \]

Substitute $y + 5$ for $x$ in the second equation and solve for $y$.

\[ 3(y + 5) + y = 19 \]

\[ 3y + 15 + y = 19 \]

\[ 4y = 4 \]

\[ y = 1 \]

Substitute $1$ for $y$ in the first equation and solve for $x$.

\[ x = 1 + 5 \]

\[ = 6 \]

Therefore, the solution is $(6, 1)$.

**ANSWER:**
$(6, 1)$
3-4 Systems of Equations in Three Variables

40.  
\[ \begin{align*}
3x - 2y &= 1 \\
4x + 2y &= 20
\end{align*} \]

**SOLUTION:**
\[ \begin{align*}
3x - 2y &= 1 & \rightarrow (1) \\
4x + 2y &= 20 & \rightarrow (2)
\end{align*} \]

Add both the equations.

\[ (1) + (2) \quad 7x = 21 \]
\[ x = 3 \]

Substitute 3 for \( x \) in the second equation and solve for \( y \).

\[ 4(3) + 2y = 20 \]
\[ 12 + 2y = 20 \]
\[ 2y = 8 \]
\[ y = 4 \]

Therefore, the solution is (3, 4).

**ANSWER:**
(3, 4)

41.  
\[ \begin{align*}
5x + 3y &= 25 \\
4x + 7y &= -3
\end{align*} \]

**SOLUTION:**
\[ \begin{align*}
5x + 3y &= 25 & \rightarrow (1) \\
4x + 7y &= -3 & \rightarrow (2)
\end{align*} \]

Multiply the first and second equation by -4 and 5 respectively then add.

\[ (1) \times -4 \quad -20x - 12y = -100 \]
\[ (2) \times 5 \quad 20x + 35y = -15 \]
\[ 23y = -115 \]
\[ y = -5 \]

Substitute -5 for \( y \) in the first equation and solve for \( x \).

\[ 5x + 3(-5) = 25 \]
\[ 5x - 15 = 25 \]
\[ 5x = 40 \]
\[ x = 8 \]

Therefore, the solution is (8, -5).

**ANSWER:**
(8, -5)
41. **SOLUTION:**

Multiply the first and second equation by –4 and 5 respectively then add.

Substitute \(-5\) for \(y\) in the first equation and solve for \(x\).

\[
\begin{align*}
2x - 8y &= 2 \\
2x - 8 & \rightarrow (1) \\
2x - 8y &= 2 & \rightarrow (2)
\end{align*}
\]

Substitute \(x - 7\) for \(y\) in the second equation and solve for \(x\).

\[
\begin{align*}
2x - 8(x - 7) &= 2 \\
2x - 8x + 56 &= 2 \\
-6x &= -54 \\
x &= 9
\end{align*}
\]

Substitute 9 for \(x\) in the first equation and solve for \(y\).

\[
\begin{align*}
y &= 9 - 7 \\
&= 2
\end{align*}
\]

Therefore, the solution is \((9, 2)\).

**ANSWER:**

\((9, 2)\)

42. **SOLUTION:**

Substitute \(x - 7\) for \(y\) in the second equation and solve for \(x\).

Substitute 9 for \(x\) in the first equation and solve for \(y\).

Therefore, the solution is \((9, 2)\).

**ANSWER:**

\((9, 2)\)
1. CCSS MODELING Use the table that shows the city and highway gas mileage of five different types of vehicles.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>SUV</th>
<th>Minivan</th>
<th>Sedan</th>
<th>Compact</th>
<th>APV</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>23</td>
<td>21</td>
<td>21</td>
<td>42</td>
<td>61</td>
</tr>
<tr>
<td>Highway</td>
<td>25</td>
<td>24</td>
<td>32</td>
<td>49</td>
<td>70</td>
</tr>
</tbody>
</table>

**Source:** Auto Heights

a. Organize the gas miles in a matrix.
b. Which type of vehicle has the best gas mileage?
c. Add the elements of each row and interpret the results.
d. Add the elements of each column and interpret the results.

**SOLUTION:**

a. Write the city gas mileage in the first row and the highway gas mileage in the second row.

\[
\begin{bmatrix}
23 & 21 & 21 & 42 & 61 \\
25 & 24 & 32 & 49 & 70
\end{bmatrix}
\]
b. APV gives more gas mileage than others.
c. Sample answer: City: The sum is 168. However, this value is irrelevant since it is the sum of 5 different types of data. Highway: The sum is 200. However, this value is irrelevant since it is the sum of 5 different types of data.
d. Sample answer: The sums are 48, 45, 53, 91, and 131. These values are irrelevant since they are the sums of 2 different types of data.

**ANSWER:**

a. \[
\begin{bmatrix}
23 & 21 & 21 & 42 & 61 \\
25 & 24 & 32 & 49 & 70
\end{bmatrix}
\]
b. APV
c. Sample answer: City: The sum is 168. However, this value is irrelevant since it is the sum of 5 different types of data. Highway: The sum is 200. However, this value is irrelevant since it is the sum of 5 different types of data.
d. Sample answer: The sums are 48, 45, 53, 91, and 131. These values are irrelevant since they are the sums of 2 different types of data.

Perform the indicated operations. If the matrix does not exist, write impossible.

2. \[
\begin{bmatrix}
-8 & 2 & 6 \\
11 & -7 & 1
\end{bmatrix}
\]

**SOLUTION:**

Add corresponding elements.

\[
\begin{bmatrix}
-8 + 11 & 2 - 7 & 6 + 1
\end{bmatrix}
\]

Simplify.

\[
\begin{bmatrix}
3 & -5 & 7
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
3 & -5 & 7
\end{bmatrix}
\]

3. \[
\begin{bmatrix}
9 & -8 & 4 \\
12 & 2
\end{bmatrix}
\]

**SOLUTION:**

The dimensions of the matrixes are not equal. So, impossible to add the matrices.

**ANSWER:**

impossible

4. \[
\begin{bmatrix}
7 & -12 \\
15 & 4
\end{bmatrix}
\]

**SOLUTION:**

Subtract corresponding elements.

\[
\begin{bmatrix}
7 - 9 & -12 - 6 \\
15 - 4 & 4 - (-9)
\end{bmatrix}
\]

Simplify.

\[
\begin{bmatrix}
-2 & -18 \\
11 & 13
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
-2 & -18 \\
11 & 13
\end{bmatrix}
\]
5. \[
\begin{bmatrix}
5 & 13 & -6 \\
3 & -17 & 2
\end{bmatrix}
- 
\begin{bmatrix}
-2 & -18 & 8 \\
2 & -11 & 0
\end{bmatrix}
\]

**SOLUTION:**
Subtract corresponding elements.
\[
\begin{bmatrix}
5 - (-2) & 13 - (-18) & -6 - 8 \\
3 - 2 & -17 - (-11) & 2 - 0
\end{bmatrix}
\]
Simplify.
\[
\begin{bmatrix}
7 & 31 & -14 \\
1 & -6 & 2
\end{bmatrix}
\]
**ANSWER:**
\[
\begin{bmatrix}
7 & 31 & -14 \\
1 & -6 & 2
\end{bmatrix}
\]

Perform the indicated operations. If the matrix does not exist, write *impossible.*

6. \[
\begin{bmatrix}
6 & 4 & 0 \\
-4 & -6 & 7
\end{bmatrix}
\]

**SOLUTION:**
Distribute the scalar.
\[
\begin{bmatrix}
3(6) & 3(4) & 3(0) \\
3(-2) & 3(14) & 3(-8) \\
3(-4) & 3(-6) & 3(7)
\end{bmatrix}
\]
Multiply.
\[
\begin{bmatrix}
18 & 12 & 0 \\
-6 & 42 & -24 \\
-12 & -18 & 21
\end{bmatrix}
\]
**ANSWER:**
\[
\begin{bmatrix}
18 & 12 & 0 \\
-6 & 42 & -24 \\
-12 & -18 & 21
\end{bmatrix}
\]
3-5 Operations with Matrices

Use matrices $A$, $B$, $C$, and $D$ to find the following.

$$A = \begin{bmatrix} 6 & -4 \\ 3 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 8 & -1 \\ -2 & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} -4 & -6 \\ 12 & -7 \end{bmatrix} \quad D = \begin{bmatrix} 9 & 6 & 0 \\ -2 & 8 & 0 \end{bmatrix}$$

8. $4B - 2A$

**SOLUTION:**

Distribute the scalar in each matrix.

$$\begin{bmatrix} 4(8) & 4(-1) \\ 4(-2) & 4(7) \end{bmatrix} - \begin{bmatrix} 2(6) & 2(-4) \\ 2(3) & 2(-5) \end{bmatrix}$$

Multiply.

$$= \begin{bmatrix} 32 & -4 \\ -8 & 28 \end{bmatrix} - \begin{bmatrix} 12 & -8 \\ 6 & -10 \end{bmatrix}$$

Subtract the corresponding elements.

$$= \begin{bmatrix} 32 - 12 & -4 - (-8) \\ -8 - 6 & 28 - (-10) \end{bmatrix}$$

Simplify.

$$= \begin{bmatrix} 20 & 4 \\ -14 & 38 \end{bmatrix}$$

**ANSWER:**

$$\begin{bmatrix} 20 & 4 \\ -14 & 38 \end{bmatrix}$$

9. $-8C + 3A$

**SOLUTION:**

Distribute the scalar in each matrix.

$$\begin{bmatrix} -8(-4) & -8(-6) \\ -8(12) & -8(-7) \end{bmatrix} + \begin{bmatrix} 3(6) & 3(-4) \\ 3(3) & 3(-5) \end{bmatrix}$$

Multiply.

$$= \begin{bmatrix} 32 & 48 \\ -96 & 56 \end{bmatrix} + \begin{bmatrix} 18 & -12 \\ 9 & -15 \end{bmatrix}$$

Add the corresponding elements.

$$= \begin{bmatrix} 32 + 18 & 48 - 12 \\ -96 + 9 & 56 - 15 \end{bmatrix}$$

Simplify.

$$= \begin{bmatrix} 50 & 36 \\ -87 & 41 \end{bmatrix}$$

**ANSWER:**

$$\begin{bmatrix} 50 & 36 \\ -87 & 41 \end{bmatrix}$$

10. $-5B - 2D$

**SOLUTION:**

The dimensions of the matrix $B$ and $D$ are not equal.

So, it is impossible.

**ANSWER:**

Impossible
3-5 Operations with Matrices

11. \(-4C - 5B\)

**SOLUTION:**
Distribute the scalar in each matrix.
\[
\begin{array}{cc}
-4(-4) & -4(-6) \\
-4(12) & -4(-7) \\
\end{array} - \begin{array}{cc}
5(8) & 5(-1) \\
5(-2) & 5(7) \\
\end{array}
\]

Multiply.
\[
\begin{array}{cc}
16 & 24 \\
-48 & 28 \\
\end{array} - \begin{array}{cc}
40 & -5 \\
-10 & 35 \\
\end{array}
\]

Subtract corresponding elements.
\[
\begin{array}{cc}
16 - 40 & 24 - (-5) \\
-48 - (-10) & 28 - 35 \\
\end{array}
\]

Simplify.
\[
\begin{array}{cc}
-24 & 29 \\
-38 & -7 \\
\end{array}
\]

**ANSWER:**
\[
\begin{array}{cc}
-24 & 29 \\
-38 & -7 \\
\end{array}
\]

12. **GRADES** Geraldo, Olivia, and Nikki have had two tests in their math class. The table shows the test grades for each student.

<table>
<thead>
<tr>
<th>Student</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geraldo</td>
<td>85</td>
<td>72</td>
</tr>
<tr>
<td>Olivia</td>
<td>75</td>
<td>74</td>
</tr>
<tr>
<td>Nikki</td>
<td>96</td>
<td>83</td>
</tr>
</tbody>
</table>

**a.** Write a matrix for the information.

**b.** Find the sum of the scores from the two tests expressed as a matrix.

**c.** Express the difference in scores from test 1 to test 2 as a matrix.

**SOLUTION:**

\[
\begin{array}{cc}
85 \\
75 \\
96 \\
\end{array}
\]

**a.** Test 1: 75

**b.** Test 2: 74

**c.** 13
13. **SHOES** A consumer service company rated several pairs of shoes by cost, level of comfort, look, and longevity using a scale of 1–5, with 1 being low and 5 being high.

<table>
<thead>
<tr>
<th>Brand</th>
<th>Cost</th>
<th>Comfort</th>
<th>Look</th>
<th>Longevity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Write a 4 × 4 matrix to organize this information.
b. Which shoe would you buy based on this information, and why?
c. Would finding the sum of the rows or columns provide any useful information? Explain your reasoning.

**SOLUTION:**

\[
\begin{bmatrix}
3 & 2 & 2 & 1 \\
4 & 3 & 2 & 3 \\
5 & 5 & 4 & 4 \\
1 & 5 & 5 & 2
\end{bmatrix}
\]

a. Sample answer: Brand C; it was given the highest rating possible for cost and comfort, and a high rating for looks, and it will last a fairly long time.
b. Sample answer: Yes; finding the sum of the rows and then calculating the average will provide an easy way to compare the data.

**ANSWER:**

\[
\begin{bmatrix}
3 & 2 & 2 & 1 \\
4 & 3 & 2 & 3 \\
5 & 5 & 4 & 4 \\
1 & 5 & 5 & 2
\end{bmatrix}
\]

b. Sample answer: Brand C; it was given the highest rating possible for cost and comfort, and a high rating for looks, and it will last a fairly long time.
c. Sample answer: Yes; finding the sum of the rows and then calculating the average will provide an easy way to compare the data.

**Perform the indicated operations. If the matrix does not exist, write impossible.**

14. \[
\begin{bmatrix}
12 & -5 \\
-8 & -3
\end{bmatrix}
+ \begin{bmatrix}
-6 & 11 \\
3 + 2
\end{bmatrix}
\]

**SOLUTION:**

Add corresponding elements.

\[
\begin{bmatrix}
12 - 6 & -5 + 11 \\
-8 - 7 & -3 + 2
\end{bmatrix}
\]

Simplify.

\[
\begin{bmatrix}
6 & 6 \\
-15 & -1
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
6 & 6 \\
-15 & -1
\end{bmatrix}
\]

15. \[
\begin{bmatrix}
9 & 5 \\
-2 & 16
\end{bmatrix}
+ \begin{bmatrix}
-6 & -3 & 7 \\
12 & 2 & -4
\end{bmatrix}
\]

**SOLUTION:**

The dimensions of the matrixes are not equal. So, we cannot add the matrix. Impossible.

**ANSWER:**

impossible

16. **BUSINESS** The drink menu from a fast-food restaurant is shown at the right. The store owner has decided that all of the prices must be increased by 10%.

<table>
<thead>
<tr>
<th>Drink</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soda</td>
<td>$0.95</td>
<td>$1.00</td>
<td>$1.05</td>
</tr>
<tr>
<td>Iced tea</td>
<td>$0.75</td>
<td>$0.80</td>
<td>$0.85</td>
</tr>
<tr>
<td>Lemonade</td>
<td>$0.75</td>
<td>$0.80</td>
<td>$0.85</td>
</tr>
<tr>
<td>Coffee</td>
<td>$1.00</td>
<td>$1.10</td>
<td>$1.20</td>
</tr>
</tbody>
</table>

a. Write matrix C to represent the current prices.
b. What scalar can be used to determine a matrix N to represent the new prices?
c. Find N.
d. What is N − C? What does this represent in this situation?

**SOLUTION:**
3-5 Operations with Matrices

\[
\begin{bmatrix}
$0.95 & $1.00 & $1.05 \\
$0.75 & $0.80 & $0.85 \\
$0.75 & $0.80 & $0.85 \\
$1.00 & $1.10 & $1.20 \\
\end{bmatrix}
\]

a. 

b. The cost is increased by 10%. That is the new price is 110% of old price.

110% = 1.1

So, multiply 1.1 with the matrix to determine the new price.

c. Multiply the matrix by 1.1.

\[
\begin{bmatrix}
1.1(0.95) & 1.1(1.00) & 1.1(1.05) \\
1.1(0.75) & 1.1(0.80) & 1.1(0.85) \\
1.1(0.75) & 1.1(0.80) & 1.1(0.85) \\
1.1(1.00) & 1.1(1.10) & 1.1(1.20) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1.05 & 1.10 & 1.16 \\
0.83 & 0.88 & 0.94 \\
0.83 & 0.88 & 0.94 \\
1.10 & 1.21 & 1.32 \\
\end{bmatrix}
\]

d. Subtract the matrix C from matrix N.

\[
\begin{bmatrix}
$1.05 & $1.10 & $1.16 \\
$0.83 & $0.88 & $0.94 \\
$0.83 & $0.88 & $0.94 \\
$1.10 & $1.21 & $1.32 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
$0.95 & $1.00 & $1.05 \\
$0.75 & $0.80 & $0.85 \\
$0.75 & $0.80 & $0.85 \\
$1.00 & $1.10 & $1.20 \\
\end{bmatrix}
\]

Sample answer: this matrix represents the price increases for each item.

**ANSWER:**

\[
\begin{bmatrix}
$0.95 & $1.00 & $1.05 \\
$0.75 & $0.80 & $0.85 \\
$0.75 & $0.80 & $0.85 \\
$1.00 & $1.10 & $1.20 \\
\end{bmatrix}
\]

b. 1.1

\[
\begin{bmatrix}
$1.05 & $1.10 & $1.16 \\
$0.83 & $0.88 & $0.94 \\
$0.83 & $0.88 & $0.94 \\
$1.10 & $1.21 & $1.32 \\
\end{bmatrix}
\]

c. 

\[
\begin{bmatrix}
$0.83 & $0.88 & $0.94 \\
$0.83 & $0.88 & $0.94 \\
$1.10 & $1.21 & $1.32 \\
$0.10 & $0.10 & $0.11 \\
\end{bmatrix}
\]

d. 

\[
\begin{bmatrix}
$0.08 & $0.08 & $0.09 \\
$0.08 & $0.08 & $0.09 \\
$0.10 & $0.11 & $0.12 \\
\end{bmatrix}
\]

Sample answer: this matrix represents the price increases for each item.

Use matrices A, B, C, and D to find the following.

\[
\begin{bmatrix}
3 & 1 \\
-2 & 3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 \\
-3 & 4 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
3 & 2 \\
-1 & 3 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 & 1 \\
1 & 2 \\
\end{bmatrix}
\]

17. \(-3B + 2A\)

**SOLUTION:**

Distribute the scalar in each matrix.

\[
\begin{bmatrix}
-3(1) & -3(4) \\
-3(-3) & -3(-17) \\
\end{bmatrix}
\] + \[
\begin{bmatrix}
2(0) & 2(-7) \\
2(8) & 2(12) \\
\end{bmatrix}
\]

Multiply.

\[
\begin{bmatrix}
-33 & -12 \\
9 & 51 \\
\end{bmatrix}
\]

Add the corresponding elements.

\[
\begin{bmatrix}
-33 + 0 & -12 + (-14) \\
9 + 16 & 51 + 24 \\
\end{bmatrix}
\]

Simplify.

\[
\begin{bmatrix}
-33 & -26 \\
25 & 75 \\
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
-33 & -26 \\
25 & 75 \\
\end{bmatrix}
\]
3-5 Operations with Matrices

18. \(9C - 4D\)

**SOLUTION:**
Distribute the scalar in each matrix.
\[
\begin{bmatrix}
9(5) & 9(2) & 9(-2) \\
9(1) & 9(-2) & 9(15)
\end{bmatrix} - \begin{bmatrix}
4(-2) & 4(-8) & 4(0) \\
4(4) & 4(13) & 4(1)
\end{bmatrix}
\]
Multiply.
\[
\begin{bmatrix}
72 & 18 & -18 \\
9 & -81 & 120
\end{bmatrix} - \begin{bmatrix}
-8 & -32 & 0 \\
16 & 52 & 4
\end{bmatrix}
\]
Subtract the corresponding elements.
\[
\begin{bmatrix}
72 - (-18) & 18 - (-32) & -18 - 0 \\
9 - 16 & -81 - 52 & 135 - 4
\end{bmatrix}
\]
Simplify.
\[
\begin{bmatrix}
80 & 50 & -18 \\
-7 & -133 & 131
\end{bmatrix}
\]
**ANSWER:**
\[
\begin{bmatrix}
80 & 50 & -18 \\
-7 & -133 & 131
\end{bmatrix}
\]

19. \(2C + 11A\)

**SOLUTION:**
The dimensions of the matrix \(C\) and matrix \(A\) are not equal.
So, it is impossible.
**ANSWER:**
impossible

20. \(7A - 2B\)

**SOLUTION:**
Distribute the scalar in each matrix.
\[
\begin{bmatrix}
7(0) & 7(-7) \\
7(8) & 7(12)
\end{bmatrix} - \begin{bmatrix}
2(11) & 2(4) \\
2(-3) & 2(-17)
\end{bmatrix}
\]
Multiply.
\[
\begin{bmatrix}
0 & -49 \\
56 & 84
\end{bmatrix} - \begin{bmatrix}
22 & 8 \\
-6 & -34
\end{bmatrix}
\]
Subtract the corresponding elements.
\[
\begin{bmatrix}
0 - 22 & -49 - 8 \\
56 - (-6) & 84 - (-34)
\end{bmatrix}
\]
Simplify.
\[
\begin{bmatrix}
-22 & -57 \\
62 & 118
\end{bmatrix}
\]
**ANSWER:**
\[
\begin{bmatrix}
-22 & -57 \\
62 & 118
\end{bmatrix}
\]

21. **CCSS MODELING** Library A has 10,000 novels, 5000 biographies, and 5000 children’s books. Library B has 15,000 novels, 10,000 biographies, and 2500 children’s books. Library C has 4000 novels, 700 biographies, and 800 children’s books.

a. Express each library’s number of books as a matrix. Label the matrices \(A\), \(B\), and \(C\).

b. Find the total number of each type of book in all 3 libraries. Express as a matrix.

c. How many more books of each type does Library A have than Library C?

d. Find \(A + B\). Does the matrix have meaning in this situation? Explain.

**SOLUTION:**

\[
\text{Library A: } \begin{bmatrix} 10,000 \\ 5000 \end{bmatrix} ; \text{ Library B: } \begin{bmatrix} 15,000 \\ 10,000 \end{bmatrix} ; \text{ Library C: } \begin{bmatrix} 4,000 \\ 700 \end{bmatrix}
\]

b. Add all matrices.
3-5 Operations with Matrices

$$\begin{bmatrix} 10000 + 15000 + 4000 \\ 5000 + 10000 + 700 \\ 5000 + 2500 + 800 \end{bmatrix} = \begin{bmatrix} 29000 \\ 15700 \\ 8300 \end{bmatrix}$$

22. Perform the indicated operations. If the matrix does not exist, write impossible.

$$-2 \begin{bmatrix} -9.2 & -8.4 \\ 5.6 & -4.3 \end{bmatrix} - 4 \begin{bmatrix} 4.1 & -2.9 \\ 7.2 & -8.2 \end{bmatrix}$$

**SOLUTION:**

Distribute the scalar in each matrix.

$$= \begin{bmatrix} -2(-9.2) & -2(-8.4) \\ -2(5.6) & -2(-4.3) \end{bmatrix} - \begin{bmatrix} 4(4.1) & 4(-2.9) \\ 4(7.2) & 4(-8.2) \end{bmatrix}$$

Multiply.

$$= \begin{bmatrix} 18.4 & 16.8 \\ -11.2 & 8.6 \end{bmatrix} - \begin{bmatrix} 16.4 & -11.6 \\ 28.8 & -32.8 \end{bmatrix}$$

Subtract the corresponding elements.

$$= \begin{bmatrix} 18.4 - 16.4 & 16.8 - (-11.6) \\ -11.2 - 28.8 & 8.6 - (-32.8) \end{bmatrix}$$

Simplify.

$$= \begin{bmatrix} 2 & 28.4 \\ -40 & 41.4 \end{bmatrix}$$

**ANSWER:**

$$\begin{bmatrix} 2 & 28.4 \\ -40 & 41.4 \end{bmatrix}$$

---

c. Subtract the matrix C from the matrix A.

$$\begin{bmatrix} 10000 - 4000 \\ 5000 - 700 \\ 5000 - 800 \end{bmatrix} = \begin{bmatrix} 6000 \\ 4300 \\ 4200 \end{bmatrix}$$

d. 15,000

7500

Sample answer: The sum represents the combined size of the two libraries.

**ANSWER:**

a. Library A: 

\[ \begin{bmatrix} 10,000 \\ 5000 \\ 4,000 \end{bmatrix} \]

Library B: 

\[ \begin{bmatrix} 15,000 \\ 2500 \end{bmatrix} \]

Library C: 

\[ \begin{bmatrix} 29,000 \\ 700 \\ 800 \end{bmatrix} \]

b. 15,700

8300

6000

c. 4300

4200

25,000

d. 15,000

7500

Sample answer: The sum represents the combined size of the two libraries.
### 3-5 Operations with Matrices

23. \(-\frac{3}{4}\begin{bmatrix} 12 & -16 \\ 15 & 8 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 21 & 18 \\ -4 & -6 \end{bmatrix}\)

**SOLUTION:**
Distribute the scalar in each matrix.

\[
\begin{aligned}
\begin{bmatrix} -\frac{3}{4}(12) & -\frac{3}{4}(-16) \\ -\frac{3}{4}(15) & -\frac{3}{4}(8) \end{bmatrix} + & \begin{bmatrix} \frac{2}{3}(21) & \frac{2}{3}(18) \\ \frac{2}{3}(-4) & \frac{2}{3}(-6) \end{bmatrix} \\
= & \begin{bmatrix} -9 & 12 \\ -\frac{45}{4} & -6 \end{bmatrix} + \begin{bmatrix} 14 & 12 \\ -\frac{8}{3} & -4 \end{bmatrix} \\
= & \begin{bmatrix} -9 + 14 & 12 + 12 \\ -\frac{45}{4} + (-\frac{8}{3}) & -6 + (-4) \end{bmatrix} \\
= & \begin{bmatrix} 5 & 24 \\ -\frac{167}{12} & -10 \end{bmatrix} \\
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} 5 & 24 \\ -\frac{167}{12} & -10 \end{bmatrix}
\]

24. \(-3\begin{bmatrix} 18 & -6 & -8 \\ -5 & -3 & 12 \\ 0 & 3x & -y \end{bmatrix}\)

**SOLUTION:**

\[
\begin{aligned}
\begin{bmatrix} 18 & -6 & -8 \\ -3 & -5 & 12 \\ 0 & 3x & -y \end{bmatrix} = & \begin{bmatrix} -3(18) & -3(-6) & -3(-8) \\ -3(-5) & -3(-3) & -3(12) \\ -3(0) & -3(3x) & -3(-y) \end{bmatrix} \\
= & \begin{bmatrix} -54 & 18 & 24 \\ 15 & 9 & -36 \\ 0 & -9x & 3y \end{bmatrix}
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} -54 & 18 & 24 \\ 15 & 9 & -36 \\ 0 & -9x & 3y \end{bmatrix}
\]

25. \(8\begin{bmatrix} -a & 4b & c-b \\ -13 & 10 & -5c \end{bmatrix}\)

**SOLUTION:**

\[
\begin{aligned}
8\begin{bmatrix} -a & 4b & c-b \\ -13 & 10 & -5c \end{bmatrix} = & \begin{bmatrix} 8(-a) & 8(4b) & 8(c-b) \\ 8(-13) & 8(10) & 8(-5c) \end{bmatrix} \\
= & \begin{bmatrix} -8a & 32b & 8c-8b \\ -104 & 80 & -40c \end{bmatrix}
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} -8a & 32b & 8c-8b \\ -104 & 80 & -40c \end{bmatrix}
\]

26. \(-4\begin{bmatrix} -7 & -8 \\ -3 & -9 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 12 \end{bmatrix}\)

**SOLUTION:**

\[
\begin{aligned}
-4\begin{bmatrix} -7 & -8 \\ -3 & -9 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 12 \end{bmatrix} = & \begin{bmatrix} -4(-7) & -4(-8) \\ -4(-3) & -4(-9) \end{bmatrix} + \begin{bmatrix} 3(4) \\ 3(12) \end{bmatrix} \\
= & \begin{bmatrix} 28 & 24 \\ -16 & -36 \end{bmatrix} + \begin{bmatrix} 12 & 36 \\ -27 & -54 \end{bmatrix} \\
= & \begin{bmatrix} 16 & 60 \\ -42 & -90 \end{bmatrix}
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} 16 & 60 \\ -42 & -90 \end{bmatrix}
\]
27. \(-5 \left[ \begin{array}{cc} 4 & -8 \\ 8 & -9 \end{array} \right] + \left[ \begin{array}{cc} 4 & -2 \\ 8 & -9 \end{array} \right] \)

**SOLUTION:**

\(-5 \left[ \begin{array}{cc} 4 & -8 \\ 8 & -9 \end{array} \right] - \left[ \begin{array}{cc} 4 & -2 \\ 8 & -9 \end{array} \right] \)

\(-5 \begin{bmatrix} 4 & -8 \\ 8 & -9 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & -9 \end{bmatrix} \)

\(= \begin{bmatrix} -5(4) - 5(4) \\ -5(8) - 5(8) \end{bmatrix} \)

\(= \begin{bmatrix} -5(4) \\ -5(8) \end{bmatrix} \)

\(= \begin{bmatrix} -20 \\ -40 \end{bmatrix} \)

\(= \begin{bmatrix} 20 \\ -10 \end{bmatrix} \)

\(= \begin{bmatrix} 15 \\ -30 \end{bmatrix} \)

\(= \begin{bmatrix} -20 - 20 \\ -40 \end{bmatrix} \)

\(= \begin{bmatrix} -40 \\ -15 \end{bmatrix} \)

\(= \begin{bmatrix} 40 \\ -30 \end{bmatrix} \)

\(= \begin{bmatrix} 40 \\ -15 \\ 45 \end{bmatrix} \)

\(= \begin{bmatrix} 25 \\ 75 \end{bmatrix} \)

**ANSWER:**

\(\begin{bmatrix} -40 \\ 50 \\ -25 \\ 75 \end{bmatrix} \)

28. **WEATHER** The table shows snowfall in inches.

<table>
<thead>
<tr>
<th>City</th>
<th>Normal Snowfall</th>
<th>2007 Snowfall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jan</td>
<td>Feb</td>
</tr>
<tr>
<td>Grand Rapids, MI</td>
<td>21.1</td>
<td>12.2</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>13.3</td>
<td>11.3</td>
</tr>
<tr>
<td>Buffalo, NY</td>
<td>26.1</td>
<td>17.8</td>
</tr>
<tr>
<td>Pittsburgh, PA</td>
<td>12.3</td>
<td>8.5</td>
</tr>
</tbody>
</table>

**For 2007 Snowfall:**

<table>
<thead>
<tr>
<th>City</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Rapids</td>
<td>21.1</td>
<td>12.2</td>
<td>9.0</td>
</tr>
<tr>
<td>Boston</td>
<td>13.3</td>
<td>11.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Buffalo</td>
<td>26.1</td>
<td>17.8</td>
<td>12.4</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>12.3</td>
<td>8.5</td>
<td>7.9</td>
</tr>
</tbody>
</table>

**a.** Express the normal snowfall data and the 2007 data in two 4 \times 3 matrices.

**b.** Subtract the matrix of normal data from the matrix of 2007 data. What does the difference represent in the context of the situation?

**c.** Explain the meaning of positive and negative numbers in the difference matrix. What trends do you see in the data?

**SOLUTION:**

**a.** For Normal snowfall:

<table>
<thead>
<tr>
<th>City</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Rapids</td>
<td>21.1</td>
<td>12.2</td>
<td>9.0</td>
</tr>
<tr>
<td>Boston</td>
<td>13.3</td>
<td>11.3</td>
<td>8.3</td>
</tr>
<tr>
<td>Buffalo</td>
<td>26.1</td>
<td>17.8</td>
<td>12.4</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>12.3</td>
<td>8.5</td>
<td>7.9</td>
</tr>
</tbody>
</table>

**b.** Subtract the matrix of normal data from the matrix of 2007 data.

The matrix represents the difference from normal for each city and month.

**c.** A negative number means the city was below normal on snowfall for the month; a positive number means the city’s snowfall was above normal for the month. Sample answer: All the cities had below normal snowfall in January. Grand Rapids and
3-5 Operations with Matrices

Buffalo had snowfall well above normal in February.

29. CCSS MODELING The table shows some of the world, Olympic and American women’s freestyle swimming records.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>World</th>
<th>Olympic</th>
<th>American</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>24.13 s</td>
<td>24.13 s</td>
<td>24.63 s</td>
</tr>
<tr>
<td>100</td>
<td>53.52 s</td>
<td>53.52 s</td>
<td>53.99 s</td>
</tr>
<tr>
<td>200</td>
<td>1:56.34 min</td>
<td>1:57.65 min</td>
<td>1:57.41 min</td>
</tr>
<tr>
<td>800</td>
<td>8:16.22 min</td>
<td>8:16.67 min</td>
<td>8:16.22 min</td>
</tr>
</tbody>
</table>

Source: USA Swimming

a. Find the difference between the American and World records expressed as a column matrix.
b. What is the meaning of each row in the column?
c. In which events were the fastest times set at the Olympics?

**SOLUTION:**

\[
\begin{bmatrix}
24.63 - 24.13 \\
53.99 - 53.52 \\
1:56.34 - 1:57.65 \\
8:16.22 - 8:16.67
\end{bmatrix} = 
\begin{bmatrix}
0.5 s \\
0.47 s \\
0.87 s \\
0 s
\end{bmatrix}
\]

a. In the 50-m, the fastest American time is 0.5 second behind the world record. In the 100-m, the fastest American time is 0.47 second behind the world record. In the 200-m, the fastest American time is 0.87 second behind the world record. In the 800-m, it was an American who set the world record.
b. 50-m and 100-m

c. 50-m and 100-m

c. In which events were the fastest times set at the Olympics?

**ANSWER:**

\[
\begin{bmatrix}
0.5 s \\
0.47 s \\
0.87 s \\
0 s
\end{bmatrix}
\]

b. In the 50-m, the fastest American time is 0.5 second behind the world record. In the 100-m, the fastest American time is 0.47 second behind the world record. In the 200-m, the fastest American time is 0.87 second behind the world record. In the 800-m, it was an American who set the world record.
b. 50-m and 100-m

c. 50-m and 100-m

c. In which events were the fastest times set at the Olympics?

**30. MULTIPLE REPRESENTATIONS** In this problem, you will investigate using matrices to represent transformations.

a. **ALGEBRAIC** The matrix

\[
\begin{bmatrix}
-3 & -4 & 1 \\
8 & 6 & 0
\end{bmatrix}
\]

represents a triangle with vertices at

(-3, 8), (-4, 6), and (1, 0). Write a matrix to represent \( \triangle ABC \)

b. **GEOMETRIC** Multiply the vertex matrix you wrote by 2. Then graph the figure represented by the new matrix.

c. **ANALYTICAL** How do the figures compare? Make a conjecture about the result of multiplying the matrix by 0.5. Verify your conjecture.

**SOLUTION:**

a. The vertices of the triangle \( \triangle ABC \) is (4, 2), (-1, -1) and (-3, 1).

Therefore, the matrix represents the triangle \( \triangle ABC \) is

\[
\begin{bmatrix}
4 & -1 & -3 \\
2 & -1 & 1
\end{bmatrix}
\]

b. Multiply the matrix by 2.

\[
2 \begin{bmatrix}
4 & -1 & -3 \\
2 & -1 & 1
\end{bmatrix} = 
\begin{bmatrix}
2(4) & 2(-1) & 2(-3) \\
2(2) & 2(-1) & 2(1)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
8 & -2 & -6 \\
4 & -2 & 2
\end{bmatrix}
\]

Plot the points (8, 4), (-2, -2) and (-6, 2) on coordinate plane and connect them.
3.5 Operations with Matrices

1. **CCSS MODELING** Use the table that shows the city and highway gas mileage of five different types of vehicles.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>City Mileage</th>
<th>Highway Mileage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>Suv</td>
<td>22</td>
<td>28</td>
</tr>
<tr>
<td>Van</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>Truck</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Bus</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**ANSWER:**

\[-10a - b\]

52. **−7(x − y) + 5(y − x)**

**SOLUTION:**

**ANSWER:**

\[−12x + 12y\]

---

**PROOF**

To prove that matrix addition is commutative for \(2 \times 2\) matrices.

**SOLUTION:**

To show that the Commutative Property of Matrix Addition is true for \(2 \times 2\) matrices, let

\[
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}.
\]

Show that \(A + B = B + A\).

\[
A + B = \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix} = B + A
\]

**ANSWER:**

To show that the Commutative Property of Matrix Addition is true for \(2 \times 2\) matrices, let

\[
A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}.
\]

Show that \(A + B = B + A\).

\[
A + B = \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix} = B + A
\]
3-5 Operations with Matrices

32. PROOF Prove that matrix addition is associative for $2 \times 2$ matrices.

SOLUTION:

\[
(A + B) + C = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix}
\]

Substitution

\[
= \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix}
\]

Definition of matrix addition

\[
= \begin{bmatrix} a+e+i & b+f+j \\ c+g+k & d+h+l \end{bmatrix}
\]

Definition of matrix addition

\[
= (A + C) + B
\]

Substitution

\[
= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix}
\]

Answer:

\[
(A + B) + C = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix}
\]

Substitution

\[
= \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix}
\]

Definition of matrix addition

\[
= \begin{bmatrix} a+e+i & b+f+j \\ c+g+k & d+h+l \end{bmatrix}
\]

Definition of matrix addition

\[
= (A + C) + B
\]

Substitution

33. CHALLENGE Find the elements of $C$ if:

\[
A = \begin{bmatrix} -3 & 4 \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}, \text{ and } 3A - 4B + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}.
\]

SOLUTION:

\[
3A = \begin{bmatrix} -9 & -12 \\ 24 & 18 \end{bmatrix}
\]

\[
4B = \begin{bmatrix} 20 & -4 \\ 2 & -4 \end{bmatrix}
\]

\[
3A - 4B = \begin{bmatrix} -29 & 8 \\ 8 & 16 \end{bmatrix}
\]

\[
6C = \begin{bmatrix} -29 & -8 \\ 16 & 34 \end{bmatrix}
\]

\[
A = \begin{bmatrix} -3 & 4 \\ 8 & -6 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 5 & -1 \\ 2 & -4 \end{bmatrix}
\]

\[
3A - 4B + 6C = \begin{bmatrix} 13 & 22 \\ 10 & 4 \end{bmatrix}
\]

Answer:

\[
C = \begin{bmatrix} 7 & 5 \\ -1 & -5 \end{bmatrix}
\]
3.5 Operations with Matrices

34. REASONING Determine whether each statement is sometimes, always, or never true for matrices A and B. Explain your reasoning.
   a. If \( A + B \) exists, then \( A - B \) exists.
   b. If \( k \) is a real number, then \( kA \) and \( kB \) exist.
   c. If \( A - B \) does not exist, then \( B - A \) does not exist.
   d. If \( A \) and \( B \) have the same number of elements, then \( A + B \) exists.
   e. If \( kA \) exists and \( kB \) exists, then \( kA + kB \) exists.

**SOLUTION:**
   a. Always; if \( A + B \) exists, \( A \) and \( B \) have the same dimensions. If \( A \) and \( B \) have the same dimensions, then \( A - B \) exists.
   b. Always; if \( k \) is a real number, then by the definition of scalar multiplication,
      \[ kA = \begin{bmatrix} -3k & -4k \\ 8k & 6k \end{bmatrix} \quad \text{and} \quad kB = \begin{bmatrix} 5k & -k \\ 2k & -4k \end{bmatrix}. \]
   c. Always; if \( A - B \) does not exist, then \( A \) and \( B \) must have different dimensions. If \( A \) and \( B \) have different dimensions, then \( A + B \) does not exist.
   d. Sometimes; matrices must have the same dimensions for their sum to exist.
   e. Sometimes; matrices must have the same dimensions for their sum to exist.

**ANSWER:**
   a. Always; if \( A + B \) exists, \( A \) and \( B \) have the same dimensions. If \( A \) and \( B \) have the same dimensions, then \( A - B \) exists.
   b. Always; if \( k \) is a real number, then by the definition of scalar multiplication,
      \[ kA = \begin{bmatrix} -3k & -4k \\ 8k & 6k \end{bmatrix} \quad \text{and} \quad kB = \begin{bmatrix} 5k & -k \\ 2k & -4k \end{bmatrix}. \]
   c. Always; if \( A - B \) does not exist, then \( A \) and \( B \) must have different dimensions. If \( A \) and \( B \) have different dimensions, then \( A + B \) does not exist.
   d. Sometimes; matrices must have the same dimensions for their sum to exist.
   e. Sometimes; matrices must have the same dimensions for their sum to exist.

35. OPEN ENDED Give an example of matrices \( A \) and \( B \) if \( 4B - 3A = \begin{bmatrix} -6 & 5 \\ -2 & -1 \end{bmatrix} \)

**SOLUTION:**
Sample answer: Use the "guess and check" problem solving strategy to find values for \( A \) and \( B \) that satisfy the given equation. Check each element one at a time.
   \( 4B - 3A = -6 \). When \( B = 3 \) and \( A = 6 \), this equation is true.
   \( 4B - 3A = 5 \). When \( B = 2 \) and \( A = 1 \), this equation is true.
   \( 4B - 3A = -2 \). When \( B = 4 \) and \( A = 6 \), this equation is true.
   \( 4B - 3A = -1 \). When \( B = 2 \) and \( A = 3 \), this equation is true.

Organize this into matrices.
   \[ A = \begin{bmatrix} 6 & 1 \\ 6 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix} \]

**ANSWER:**
Sample Answer: \( A = \begin{bmatrix} 6 & 1 \\ 6 & 3 \end{bmatrix} \), \( B = \begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix} \)

36. WRITING IN MATH Explain how to find \( 4D - 3C \) for two given matrices, \( C \) and \( D \) with the same dimensions.

**SOLUTION:**
Sample answer: First, multiply every element in \( D \) by 4. Then, multiply every element in \( C \) by 3. Finally, subtract the elements in \( 3C \) from the corresponding elements in \( 4D \). The result is a matrix equivalent to \( 4D - 3C \).

**ANSWER:**
Sample answer: First, multiply every element in \( D \) by 4. Then, multiply every element in \( C \) by 3. Finally, subtract the elements in \( 3C \) from the corresponding elements in \( 4D \). The result is a matrix equivalent to \( 4D - 3C \).
3-5 Operations with Matrices

37. What is the solution of the system of equations?

\[0.06p + 4q = 0.88\]
\[p - q = -2.25\]

A \((-0.912, -1.338)\)  
B \((0.912, -3.162)\)  
C \((-2, 0.25)\)  
D \((-2, -4.25)\)

**SOLUTION:**  
Multiply the second equation by 4 and add with the first equation.

\[0.06p + 4q = 0.88\]
\[4p - 4q = -9\]
\[4.06p = -8.12\]
\[p = -2\]

Substitute \(-2\) for \(p\) in the second equation and solve for \(q\).

\[-2 - q = -2.25\]
\[q = -2 + 2.25\]
\[= 0.25\]

Option C is the correct answer.

**ANSWER:**  
C

38. **SHORT RESPONSE** Find \(A + B\) if

\[A = \begin{bmatrix} -7 & 3 \\ 2 & 6 \end{bmatrix}\] and \(B = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}\)

**SOLUTION:**  
Add corresponding elements.

\[\begin{bmatrix} -7 + 4 & 3 + 2 \\ 2 + 0 & 6 + 1 \end{bmatrix}\]

Simplify.

\[\begin{bmatrix} -3 & 5 \\ 2 & 7 \end{bmatrix}\]

**ANSWER:**  
\[\begin{bmatrix} -3 & 5 \\ 2 & 7 \end{bmatrix}\]

39. **SAT/ACT** Solve for \(x\) and \(y\).

\[x + 3y = 16\]
\[7 - x = 12\]
\[F \ x = -5, y = 7\]
\[G \ x = 7, y = 3\]
\[H \ x = 7, y = 5\]
\[J \ x = 5, y = 7\]
\[K \ x = -5, y = 3\]

**SOLUTION:**  
Solve the second equation for \(x\).

\[7 - x = 12\]
\[x = 7 - 12\]
\[x = -5\]

Substitute \(-5\) for \(x\) in the first equation and solve for \(y\).

\[x + 3y = 16\]
\[-5 + 3y = 16\]
\[3y = 21\]
\[y = 7\]

The solution is \((-5, 7)\). Therefore, option F is the correct answer.

**ANSWER:**  
F
3-5 Operations with Matrices

40. **PROBABILITY** A local pizzeria offers 5 different meat toppings and 6 different vegetable toppings. You decide to get two vegetable toppings and one meat topping. How many different types of pizzas can you order?

A 60  
B 75  
C 120  
D 150

**SOLUTION:**
The number of ways to select two vegetable toppings out of 6 is \( \binom{6}{2} = 15 \).
The number of ways to select one meat topping out of 5 is \( \binom{5}{1} = 5 \).
The number of different types of pizzas can order is \( 15 \times 5 = 75 \).
Therefore, option B is the correct answer.

**ANSWER:**  
B

41. Solve each system of equations.

\[ 2x + 3y - z = -1 \]
\[ 5x + y + 4z = 30 \]
\[ -8x - 2y + 5z = -2 \]

**SOLUTION:**
\[ 2x + 3y - z = -1 \quad \rightarrow (1) \]
\[ 5x + y + 4z = 30 \quad \rightarrow (2) \]
\[ -8x - 2y + 5z = -2 \quad \rightarrow (3) \]

Multiply the first equation by 4 and add with the second equation.

\[ (1) \times 4 \quad 8x + 12y - 4z = -4 \]
\[ (2) \quad 5x + y + 4z = 30 \]
\[ \quad 13x + 13y = 26 \quad \rightarrow (4) \]

Multiply the first equation by 5 and add with the third equation.

\[ (1) \times 5 \quad 10x + 15y - 5z = -5 \]
\[ (3) \quad -8x - 2y + 5z = -2 \]
\[ \quad 2x + 13y = -7 \quad \rightarrow (5) \]

Subtract the fifth equation from the fourth equation and solve for \( x \).

\[ (4) - (5) \quad 11x = 33 \]
\[ \quad x = 3 \]

Substitute 3 for \( x \) in the fifth equation and solve for \( y \).

\[ 2(3) + 13y = -7 \]
\[ \quad 6 + 13y = -7 \]
\[ \quad 13y = -13 \]
\[ \quad y = -1 \]

Substitute 3 and -1 for \( x \) and \( y \) in the first equation and solve for \( z \).

\[ 2(3) + 3(-1) - z = -1 \]
\[ \quad 6 - 3 - z = -1 \]
\[ \quad z = 4 \]

Therefore, the solution is \((3, -1, 4)\).

**ANSWER:**
\((3, -1, 4)\)

42. Solve each system of equations.

\[ 3x - 4y + 6z = 26 \]
\[ 5x + 3y + 2z = 5 \]
\[ -2x + 5y - 3z = -9 \]

**SOLUTION:**
\[ 3x - 4y + 6z = 26 \quad \rightarrow (1) \]
\[ 5x + 3y + 2z = 5 \quad \rightarrow (2) \]
\[ -2x + 5y - 3z = -9 \quad \rightarrow (3) \]

Multiply the second equation by -3 and add with the first equation.

\[ (2) \times -3 \quad -15x - 9y - 6z = -15 \]
\[ (1) \quad 3x - 4y + 6z = 26 \]
\[ \quad -12x - 13y = 11 \quad \rightarrow (4) \]

Multiply the third equation by 2 and add with the first equation.
3-5 Operations with Matrices

\(3\times 2\) \quad \begin{align*} -4x + 10y - 6z &= -18 \\ 3x - 4y + 6z &= 26 \\ -x + 6y &= 8 \quad \rightarrow (5) \end{align*}\n
Multiply the fifth equation by \(-12\) and add with the fourth equation.

\(5\times -12\) \quad \begin{align*} 12x - 72y &= -96 \\ -12x - 13y &= 11 \\ -85y &= -85 \\ y &= 1 \end{align*}\n
Substitute \(1\) for \(y\) in the fifth equation and solve for \(x\).

\(-x + 6(1) &= 8 \\ -x + 6 &= 8 \\ x &= -2 \)

Substitute \(-2\) and \(1\) for \(x\) and \(y\) in the second equation and solve for \(z\).

\(5(-2) + 3(1) + 2z &= 5 \\ -10 + 3 + 2z &= 5 \\ 2z &= 12 \\ z &= 6 \)

The solution is \((-2, 1, 6)\).

\textbf{ANSWER:} \quad (-2, 1, 6)

5. \(x + 2y - 4z = 22\)

43. \(6x + 3y + 5z = 5\) 
\(-2x - 4y + z = 2\)

\textbf{SOLUTION:} \quad \begin{align*} 5x + 2y - 4z &= 22 \quad \rightarrow (1) \\ 6x + 3y + 5z &= 5 \quad \rightarrow (2) \\ -2x - 4y + z &= 2 \rightarrow (3) \end{align*}\n
Multiply the third equation by \(4\) and add with the first equation.

\(3\times 4\) \quad \begin{align*} -8x - 16y + 4z &= 8 \\ 5x + 2y - 4z &= 22 \\ -3x - 14y &= 30 \quad \rightarrow (4) \end{align*}\n
Multiply the third equation by \(-5\) and add with the second equation.

\(3\times -5\) \quad \begin{align*} 10x + 20y - 5z &= -10 \\ 6x + 3y + 5z &= 5 \\ 16x + 23y &= -5 \quad \rightarrow (5) \end{align*}\n
Solve the fourth and the fifth equation.

\(4\times 16\) \quad \begin{align*} -48x - 224y &= 480 \\ 5\times 3 \quad 48x + 69y &= -15 \\ -155y &= 465 \\ y &= -3 \end{align*}\n
Substitute \(-3\) for \(y\) in the fifth equation and solve for \(x\).

\(16x + 23(-3) &= -5 \\ 16x - 69 &= -5 \\ 16x &= 64 \\ x &= 4 \)

Substitute the values of \(x\) and \(y\) in the second equation and solve for \(z\).

\(6(4) + 3(-3) + 5z &= 5 \\ 24 - 9 + 5z &= 5 \\ 5z &= -10 \\ z &= -2 \)

Therefore, the solution is \((4, -3, -2)\).

\textbf{ANSWER:} \quad (4, -3, -2)
44. **PACKAGING** The Cookie Factory sells chocolate chip and peanut butter cookies in combination packages that contain between six and twelve cookies. At least three of each type of cookie should be in each package. How many of each type of cookie should be in each package to maximize the profit?

<table>
<thead>
<tr>
<th>Cookie</th>
<th>chocolate chip</th>
<th>peanut butter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$0.19</td>
<td>$0.13</td>
</tr>
<tr>
<td>Price</td>
<td>$0.44</td>
<td>$0.39</td>
</tr>
</tbody>
</table>

**SOLUTION:**

Let \( x \) and \( y \) be the number of chocolate chip and peanut butter cookies, respectively.

Optimizing function:

\[
f(x, y) = (0.44 - 0.19)x + (0.39 - 0.13)y = 0.25x + 0.26y
\]

Constraints:

\[
\begin{align*}
x & \geq 3 \\
y & \geq 3 \\
6 & \leq x + y \leq 12
\end{align*}
\]

The vertices of the solution region are (9, 3), (3, 9) and (3, 3).

Substitute the points (9, 3), (3, 9) and (3, 3) in the function \( f(x, y) = 0.25x + 0.26y \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(0.25x + 0.26y)</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9, 3)</td>
<td>3.09</td>
<td></td>
</tr>
<tr>
<td>(3, 3)</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>(9, 3)</td>
<td>3.03</td>
<td></td>
</tr>
</tbody>
</table>

The maximum 3.09 occurs at (3, 9).

Therefore, 3 chocolate chip cookies and 9 peanut butter cookies make the maximum profit.

**ANSWER:**

3 chocolate chip, 9 peanut butter
47. \[4x + 2y > 8\]
\[4y - 3x \leq 12\]

**SOLUTION:**

```
<table>
<thead>
<tr>
<th>4y - 3x = 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x + 2y = 8</td>
</tr>
</tbody>
</table>
```

**ANSWER:**

```
<table>
<thead>
<tr>
<th>4y - 3x = 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x + 2y = 8</td>
</tr>
</tbody>
</table>
```

48. **RAKING LEAVES** A student can earn $20 plus an extra $5 for each trash bag he or she completely fill with leaves. Write and solve an equation to determine how many bags the student will need to fill in order to earn $100.

**SOLUTION:**

Let \(x\) be the number of bags to fill.
The equation representing the situation is
\[20 + 5x = 100\].

Solve for \(x\).
\[5x = 100 - 20\]
\[5x = 80\]
\[x = 16\]

**ANSWER:**

\[20 + 5x = 100; \text{ 16 bags}\]

49. **SPORTS** There are 15,991 more student athletes in New York than in Illinois. Write and solve an equation to find the number of student athletes in Illinois.

**SOLUTION:**

Let \(x\) be the number of student athletes in Illinois. The equation representing the situation is
\[350349 - x = 15991\].

Solve for \(x\).
\[x = 350349 - 15991\]
\[= 334358\]

The number of student athletes in Illinois is 334,358.

**ANSWER:**

\[350,349 - x = 15,991; \text{ 334,358}\]

50. \[4(2x - 3y) + 2(5x - 6y)\]

**SOLUTION:**

\[4(2x - 3y) + 2(5x - 6y) = 8x - 12y + 10x - 12y\]
\[= 18x - 24y\]

**ANSWER:**

\[18x - 24y\]

51. \[-3(2a - 5b) - 4(4b + a)\]

**SOLUTION:**

\[-3(2a - 5b) - 4(4b + a) = -6a + 15b - 16b - 4a\]
\[= -10a - b\]

**ANSWER:**

\[-10a - b\]
52. \(-7(x - y) + 5(y - x)\)

**SOLUTION:**

\[-7(x - y) + 5(y - x) = -7x + 7y + 5y - 5x\]

\[= -12x + 12y\]

**ANSWER:**

\(-12x + 12y\)
Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. \(A_{2 \times 4} \cdot B_{4 \times 3}\)

**SOLUTION:**
The product is defined as the inner dimensions are equal. Its dimensions are \(2 \times 3\).

**ANSWER:**
\(2 \times 3\)

2. \(C_{5 \times 4} \cdot D_{5 \times 4}\)

**SOLUTION:**
The product is undefined as the inner dimensions are not equal.

**ANSWER:**
Undefined

3. \(E_{8 \times 6} \cdot F_{6 \times 10}\)

**SOLUTION:**
The product is defined as the inner dimensions are equal. Its dimensions are \(8 \times 10\).

**ANSWER:**
\(8 \times 10\)

Find each product, if possible.

4. \[
\begin{bmatrix}
2 & 1 \\
7 & -5
\end{bmatrix}
\begin{bmatrix}
-6 & 3 \\
-2 & -4
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
2 & 1 \\
7 & -5
\end{bmatrix}
\begin{bmatrix}
-6 & 3 \\
-2 & -4
\end{bmatrix} = \begin{bmatrix}
-14 & 2 \\
-32 & 41
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-14 & 2 \\
-32 & 41
\end{bmatrix}
\]

5. \[
\begin{bmatrix}
10 & -2 \\
-7 & 3
\end{bmatrix}
\begin{bmatrix}
1 & 4 \\
5 & -2
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
10 & -2 \\
-7 & 3
\end{bmatrix}
\begin{bmatrix}
1 & 4 \\
5 & -2
\end{bmatrix} = \begin{bmatrix}
0 & 44 \\
8 & -34
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
0 & 44 \\
8 & -34
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
9 & -2 \\
6 & -7
\end{bmatrix}
\begin{bmatrix}
-2 & 4 \\
6 & -7
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
9 & -2 \\
6 & -7
\end{bmatrix}
\begin{bmatrix}
-2 & 4 \\
6 & -7
\end{bmatrix} = \begin{bmatrix}
-30 & 50
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-30 & 50
\end{bmatrix}
\]

7. \[
\begin{bmatrix}
-9 \\
6
\end{bmatrix}
\begin{bmatrix}
-1 & 10 & 1
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
-9 \\
6
\end{bmatrix}
\begin{bmatrix}
-1 & 10 & 1
\end{bmatrix} = \begin{bmatrix}
9 & 90 & -9 \\
6 & -60 & 6
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
9 & 90 & -9 \\
6 & -60 & 6
\end{bmatrix}
\]

8. \[
\begin{bmatrix}
-8 & 7 & 4 \\
-5 & -3 & 8
\end{bmatrix}
\begin{bmatrix}
10 & 6 \\
8 & 4
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are not equal. So, the matrices cannot be multiplied.

**ANSWER:**
Undefined
9. \[
\begin{bmatrix}
2 & 8 \\
3 & -1
\end{bmatrix}
\times
\begin{bmatrix}
6 \\
-7
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
2 & 8 \\
3 & -1
\end{bmatrix}
\times
\begin{bmatrix}
6 \\
-7
\end{bmatrix} = \begin{bmatrix} -44 \\ 25 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-44 \\
25
\end{bmatrix}
\]

10. \[
\begin{bmatrix}
-4 & 3 & 2 \\
-1 & -5 & 4
\end{bmatrix}
\times
\begin{bmatrix}
2 & 1 & 6 \\
8 & 4 & -1 \\
5 & 3 & -2
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
-4 & 3 & 2 \\
-1 & -5 & 4
\end{bmatrix}
\times
\begin{bmatrix}
2 & 1 & 6 \\
8 & 4 & -1 \\
5 & 3 & -2
\end{bmatrix} = \begin{bmatrix} 26 & 14 & -31 \\ -22 & -9 & -9 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
26 & 14 & -31 \\
-22 & -9 & -9
\end{bmatrix}
\]

11. \[
\begin{bmatrix}
2 & 5 & 3 & -1 \\
-3 & 1 & 8 & -3
\end{bmatrix}
\times
\begin{bmatrix}
6 & -3 \\
-7 & 1 \\
2 & 0 \\
-1 & 0
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are equal. So:
\[
\begin{bmatrix}
2 & 5 & 3 & -1 \\
-3 & 1 & 8 & -3
\end{bmatrix}
\times
\begin{bmatrix}
6 & -3 \\
-7 & 1 \\
2 & 0 \\
-1 & 0
\end{bmatrix} = \begin{bmatrix} -16 & -1 \\ -6 & 10 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-16 & -1 \\
-6 & 10
\end{bmatrix}
\]

12. **CCSS SENSE-MAKING** The table shows the number of people registered for aerobics for the first quarter.
Quinn’s Gym charges the following registration fees: class-by-class, $165; 11-class pass, $110; unlimited pass, $239.

<table>
<thead>
<tr>
<th>Payment</th>
<th>Aerobics</th>
<th>Step Aerobics</th>
</tr>
</thead>
<tbody>
<tr>
<td>class-by-class</td>
<td>35</td>
<td>28</td>
</tr>
<tr>
<td>11-class pass</td>
<td>32</td>
<td>17</td>
</tr>
<tr>
<td>unlimited pass</td>
<td>18</td>
<td>12</td>
</tr>
</tbody>
</table>

a. Write a matrix for the registration fees and a matrix for the number of students.
b. Find the total amount of money the gym received from aerobics and step aerobic registrations.

**SOLUTION:**
a. \[
\begin{bmatrix}
165 & 110 & 239
\end{bmatrix}
\]

b. Money received from aerobics registrations:
\[
\begin{bmatrix}
165 & 110 & 239
\end{bmatrix} \times \begin{bmatrix} 35 \\ 110 \\ 239 \end{bmatrix} = 13,597 + 13,597 = 27,194
\]

Money received from step aerobic registrations:
\[
\begin{bmatrix}
165 & 110 & 239
\end{bmatrix} \times \begin{bmatrix} 35 \\ 110 \\ 239 \end{bmatrix} = 9,358 + 9,358 = 18,716
\]

Total = $13,597 + $9,358 = $22,955

**ANSWER:**
a. \[
\begin{bmatrix}
165 & 110 & 239
\end{bmatrix}
\]

b. $22,955
3-6 Multiplying Matrices

Use \( \mathbf{X} = \begin{bmatrix} -10 & -3 \\ 2 & -8 \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix}, \text{and} \mathbf{Z} = \begin{bmatrix} -5 & -1 \\ -8 & -4 \end{bmatrix} \) to determine whether the following equations are true for the given matrices.

13. \( \mathbf{XY} = \mathbf{YX} \)

\[
\mathbf{X} \mathbf{Y} = \begin{bmatrix} -10 & -3 \\ 2 & -8 \end{bmatrix} \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} 53 & -87 \\ -2 & -60 \end{bmatrix}
\]
\[
\mathbf{Y} \mathbf{X} = \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix} \begin{bmatrix} -10 & -3 \\ -8 & -2 \end{bmatrix} = \begin{bmatrix} 62 & -33 \\ 28 & -69 \end{bmatrix}
\]

Therefore:
\( \mathbf{XY} \neq \mathbf{YX} \)

**ANSWER:**
No: \( \begin{bmatrix} 53 & -87 \\ -2 & -60 \end{bmatrix} \neq \begin{bmatrix} 62 & -33 \\ 28 & -69 \end{bmatrix} \).

14. \( \mathbf{XYZ} = (\mathbf{XY})\mathbf{Z} \)

\[
\mathbf{Y} \mathbf{Z} = \begin{bmatrix} -5 & 6 \\ -1 & 9 \end{bmatrix} \begin{bmatrix} -8 & -4 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} -23 & -19 \\ -67 & -35 \end{bmatrix}
\]
\[
(\mathbf{XY}) \mathbf{Z} = \begin{bmatrix} 53 & -87 \\ -2 & -60 \end{bmatrix} \begin{bmatrix} -5 & -1 \\ -8 & -4 \end{bmatrix} = \begin{bmatrix} 431 & 295 \\ 490 & 242 \end{bmatrix}
\]

Therefore:
\( \mathbf{XYZ} = (\mathbf{XY})\mathbf{Z} \)

**ANSWER:**
Yes; \( \mathbf{Y} \mathbf{Z} = \begin{bmatrix} 431 & 295 \\ 490 & 242 \end{bmatrix} \) and 
\( (\mathbf{XY}) \mathbf{Z} = \begin{bmatrix} 431 & 295 \\ 490 & 242 \end{bmatrix} \).

Determine whether each matrix product is defined. If so, state the dimensions of the product.

15. \( \mathbf{P} \times 3 \times \mathbf{Q} \times 3 \times 4 \)

\[
\mathbf{S} \mathbf{O} \mathbf{L} \mathbf{U} \mathbf{T} \mathbf{I} \mathbf{O} \mathbf{N}:
\]
The product is defined as the inner dimensions are equal. Its dimensions are \( 2 \times 4 \).

**ANSWER:**
\( 2 \times 4 \)
3-6 Multiplying Matrices

16. $A_{5 \times 5} \cdot B_{5 \times 5}$

**SOLUTION:**
The product is defined as the inner dimensions are equal. Its dimensions are $5 \times 5$.

**ANSWER:**
$5 \times 5$

17. $M_{3 \times 1} \cdot N_{2 \times 3}$

**SOLUTION:**
The product is undefined as the inner dimensions are not equal.

**ANSWER:**
undefined

18. $X_{2 \times 6} \cdot Y_{6 \times 3}$

**SOLUTION:**
The product is defined as the inner dimensions are equal. Its dimensions are $2 \times 3$.

**ANSWER:**
$2 \times 3$

19. $J_{2 \times 1} \cdot K_{2 \times 1}$

**SOLUTION:**
The product is undefined as the inner dimensions are not equal.

**ANSWER:**
undefined

20. $S_{5 \times 2} \cdot T_{2 \times 4}$

**SOLUTION:**
The product is defined as the inner dimensions are equal. Its dimensions are $5 \times 4$.

**ANSWER:**
$5 \times 4$

Find each product, if possible.

21. $[1 \quad 6] \begin{bmatrix} -10 \\ 6 \end{bmatrix}$

**SOLUTION:**

\[
\begin{bmatrix} 1 & 6 \end{bmatrix} \begin{bmatrix} -10 \\ 6 \end{bmatrix} = [(1)(-10) + 6(6)]
= [-10 + 36]
= [26]
\]

**ANSWER:**
$[26]$

22. $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \begin{bmatrix} 2 & -7 \end{bmatrix}$

**SOLUTION:**

\[
\begin{bmatrix} 6 \\ -3 \end{bmatrix} \begin{bmatrix} 2 & -7 \end{bmatrix} = \begin{bmatrix} 6(2) & 6(-7) \\ -3(2) & -3(-7) \end{bmatrix}
= \begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}
\]

23. $\begin{bmatrix} -3 & -7 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 9 & -3 \end{bmatrix}$

**SOLUTION:**

\[
\begin{bmatrix} -3 & -7 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 4 \\ 9 & -3 \end{bmatrix} = \begin{bmatrix} (-3)(4) + (-7)(9) & (-3)(4) + (-7)(-3) \\ (-2)(4) + (-1)(9) & (-2)(4) + (-1)(-3) \end{bmatrix}
= \begin{bmatrix} -12 - 63 & -12 + 21 \\ -8 - 9 & -8 + 3 \end{bmatrix}
= \begin{bmatrix} -75 & 9 \\ -17 & -5 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix} -75 & 9 \\ -17 & -5 \end{bmatrix}
\]
3-6 Multiplying Matrices

24. \[
\begin{bmatrix}
-1 & 0 \\
5 & 2
\end{bmatrix}
\begin{bmatrix}
6 & -3 \\
7 & -2
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-1 & 0 \\
5 & 2
\end{bmatrix}
\begin{bmatrix}
6 & -3 \\
7 & -2
\end{bmatrix} =
\begin{bmatrix}
-1(6) & -1(-3) \\
5(6) + 2(7) & 5(-3) + 2(-2)
\end{bmatrix}
= \begin{bmatrix}
-6 & 3 \\
30 + 14 & -15 - 4
\end{bmatrix}
= \begin{bmatrix}
-6 & 3 \\
44 & -19
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-6 & 3 \\
44 & -19
\end{bmatrix}
\]

25. \[
\begin{bmatrix}
-1 & 0 & 6 \\
-4 & -10 & 4
\end{bmatrix}
\begin{bmatrix}
5 & -7 \\
-2 & -9
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are not equal. So, the matrices cannot be multiplied.

**ANSWER:**
Undefined

26. \[
\begin{bmatrix}
-6 & 4 & -9 \\
2 & 8 & 7
\end{bmatrix}
\begin{bmatrix}
7 \\
2 \\
4
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-6 & 4 & -9 \\
2 & 8 & 7
\end{bmatrix}
\begin{bmatrix}
7 \\
2 \\
4
\end{bmatrix} = \begin{bmatrix}
-6(7) + 4(2) + (-9)(4) \\
2(7) + 8(2) + 7(4)
\end{bmatrix}
= \begin{bmatrix}
-42 + 8 - 36 \\
14 + 16 + 28
\end{bmatrix}
= \begin{bmatrix}
-70 \\
58
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-70 \\
58
\end{bmatrix}
\]

27. \[
\begin{bmatrix}
2 & 9 & -3 \\
4 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
4 & 2 \\
-6 & 7 \\
-2 & 1
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
2 & 9 & -3 \\
4 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
4 & 2 \\
-6 & 7 \\
-2 & 1
\end{bmatrix} = \begin{bmatrix}
2(4) + 9(-6) + (-3)(2) \\
4(4) + (-1)(-6) \\
2(-2) + 9(7) + (-3)(1)
\end{bmatrix}
= \begin{bmatrix}
8 - 54 + 6 + 63 - 3 \\
16 + 6 \\
-4 + 64 - 7
\end{bmatrix}
= \begin{bmatrix}
40 & 64 \\
22 & 1
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-40 & 64 \\
22 & 1
\end{bmatrix}
\]

28. \[
\begin{bmatrix}
-4 \\
8
\end{bmatrix}
\begin{bmatrix}
-3 & -1
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-4 \\
8
\end{bmatrix}
\begin{bmatrix}
-3 & -1
\end{bmatrix} = \begin{bmatrix}
-4(-3) & -4(-1) \\
8(-3) & 8(-1)
\end{bmatrix}
= \begin{bmatrix}
12 & 4 \\
-24 & -8
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
12 & 4 \\
-24 & -8
\end{bmatrix}
\]

29. **TRAVEL** The Wolf family owns three bed and breakfasts in a vacation spot. A room with a single bed is $220 a night, a room with two beds is $250 a night, and a suite is $360.

<table>
<thead>
<tr>
<th>Available Rooms at a Wolf Bed and Breakfast</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &amp; B</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**a.** Write a matrix for the number of each type of room at each bed and breakfast. Then write a room-cost matrix.

**b.** Write a matrix for total daily income, assuming that all the rooms are rented.

**c.** What is the total daily income from all three bed and breakfasts, assuming that all the rooms are rented?
3-6 Multiplying Matrices

**SOLUTION:**

\[
\begin{align*}
\text{a.} & \quad I = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \\ 4 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 220 \\ 250 \\ 360 \end{bmatrix} \\
\text{b.} & \quad I \times C = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \\ 4 & 3 & 0 \end{bmatrix} \times \begin{bmatrix} 220 \\ 250 \\ 360 \end{bmatrix} = \begin{bmatrix} 1880 \\ 1550 \\ 1630 \end{bmatrix}
\end{align*}
\]

The total daily income is given by the matrix
\[
\begin{bmatrix} $1880 \\
$1550 \\
$1630 \end{bmatrix}
\]

**ANSWER:**

\[
\begin{align*}
\text{a.} & \quad I = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \\ 4 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 220 \\ 250 \\ 360 \end{bmatrix} \\
\text{b.} & \quad \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 1 \\ 4 & 3 & 0 \end{bmatrix} \begin{bmatrix} 220 \\ 250 \\ 360 \end{bmatrix} = \begin{bmatrix} 1880 \\ 1550 \\ 1630 \end{bmatrix} \\
\text{c.} & \quad $1880 \\
& \quad $1550 \\
& \quad $1630 \\
& \quad $5060
\end{align*}
\]

Use \( P = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}, Q = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix}, R = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} \), and \( k = 2 \) to determine whether the following equations are true for the given matrices.

30. \( k(PQ) = P(kQ) \)

**SOLUTION:**

\[
\begin{align*}
PQ &= \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 26 & 21 \\ 2 & -6 \end{bmatrix} \\
kPQ &= k \begin{bmatrix} 26 & 21 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 26k & 21k \\ 2k & -6k \end{bmatrix}
\end{align*}
\]

When \( k = 2 \):

\[
\begin{align*}
2PQ &= \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix} \\
kQ &= \begin{bmatrix} 6k & 4k \\ -2k & -5k \end{bmatrix} \\
P(kQ) &= \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 6k & 4k \\ -2k & -5k \end{bmatrix} = \begin{bmatrix} 26k & 21k \\ 2k & -6k \end{bmatrix}
\end{align*}
\]

When \( k = 2 \):

\[
\begin{align*}
P(kQ) &= \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix}
\end{align*}
\]

So:

\[
kPQ = P(kQ)
\]

**ANSWER:**

\[
\begin{align*}
\text{Yes:} & \quad k(PQ) = \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix} \quad \text{and} \quad P(kQ) = \begin{bmatrix} 52 & 42 \\ 4 & -12 \end{bmatrix}
\end{align*}
\]
3.6 Multiplying Matrices

31. $PQR = RQP$

**SOLUTION:**

$$QR = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 52 \\ 22 & -32 \end{bmatrix}$$

$$PQR = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 52 \\ 22 & -32 \end{bmatrix} = \begin{bmatrix} -22 & 240 \\ 44 & -12 \end{bmatrix}$$

$$QP = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 28 & 2 \\ -13 & -8 \end{bmatrix}$$

$$RQP = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 28 & 2 \\ -13 & -8 \end{bmatrix} = \begin{bmatrix} 34 & -40 \\ -220 & -44 \end{bmatrix}$$

So: $PQR \neq RQP$

**ANSWER:**

No; $PQR = \begin{bmatrix} -22 & 240 \\ 44 & -12 \end{bmatrix}$ and $RQP = \begin{bmatrix} 34 & -40 \\ -220 & -44 \end{bmatrix}$.

32. $PR + QR = (P + Q)R$

**SOLUTION:**

$$PR = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 22 & 20 \\ -8 & 14 \end{bmatrix}$$

$$QR = \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 52 \\ 22 & -32 \end{bmatrix}$$

$$PR + QR = \begin{bmatrix} 22 & 20 \\ -8 & 14 \end{bmatrix} + \begin{bmatrix} 0 & 52 \\ 22 & -32 \end{bmatrix} = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$$

$$P + Q = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 4 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ -1 & -3 \end{bmatrix}$$

$$(P + Q)R = \begin{bmatrix} 10 & 3 \\ -1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$$

So: $PR + QR = (P + Q)R$

**ANSWER:**

Yes; $PR + QR = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$ and $(P + Q)R = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}$.
3.6 Multiplying Matrices

33. \( R(P + Q) = PR + QR \)

**SOLUTION:**

\[
P + Q = \begin{bmatrix} 10 & 3 \\ -1 & -3 \end{bmatrix}
\]

\[
R(P + Q) = \begin{bmatrix} -6 & 4 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 10 & 3 \\ -1 & -3 \end{bmatrix}
= \begin{bmatrix} 34 & -6 \\ -64 & -30 \end{bmatrix}
\]

\[
PR + QR = \begin{bmatrix} 22 & 20 \\ -8 & 14 \end{bmatrix} \begin{bmatrix} 0 & 52 \\ 22 & -32 \end{bmatrix}
= \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix}
\]

So: \( R(P + Q) \neq PR + QR \)

**ANSWER:**

No; \( R(P + Q) = \begin{bmatrix} 34 & -6 \\ -64 & -30 \end{bmatrix} \) and \( PR + QR = \begin{bmatrix} 22 & 72 \\ 14 & -18 \end{bmatrix} \)

34. **CCSS SENSE-MAKING** Student Council is selling flowers for Mother’s Day. They bought 200 roses, 150 daffodils, and 100 orchids for the purchase prices shown. They sold all of the flowers for the sales prices shown.

<table>
<thead>
<tr>
<th>Flower</th>
<th>Purchase Price</th>
<th>Sales Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>rose</td>
<td>$1.67</td>
<td>$3.00</td>
</tr>
<tr>
<td>daffodil</td>
<td>$1.03</td>
<td>$2.25</td>
</tr>
<tr>
<td>orchid</td>
<td>$2.59</td>
<td>$4.50</td>
</tr>
</tbody>
</table>

**a.** Organize the data in two matrices, and use matrix multiplication to find the total amount that was spent on the flowers.

**b.** Write two matrices, and use matrix multiplication to find the total amount the student council received for the flower sale.

**c.** Use matrix operations to find how much money the student council made on their project.

**SOLUTION:**

\[
\begin{bmatrix} 200 & 150 & 100 \end{bmatrix} \begin{bmatrix} 1.67 \\ 1.03 \\ 2.59 \end{bmatrix} = \begin{bmatrix} 747.50 \\ 2,59 \end{bmatrix}
\]

The amount was $747.50.

\[
\begin{bmatrix} 200 & 150 & 100 \end{bmatrix} \begin{bmatrix} 3.00 \\ 2.25 \\ 4.50 \end{bmatrix} = \begin{bmatrix} 1387.50 \\ 4,50 \end{bmatrix}
\]

The amount the student council received was $1387.50.

\[
\text{Profit} = \begin{bmatrix} 1387.50 \\ 4,50 \end{bmatrix} - \begin{bmatrix} 747.50 \\ 2,59 \end{bmatrix}
= \begin{bmatrix} 640 \end{bmatrix}
\]

The profit was $640.

**ANSWER:**

a. $747.50
b. $1387.50
c. $640
3-6 Multiplying Matrices

35. AUTO SALES A car lot has four sales associates. At the end of the year, each sales associate gets a bonus of $1000 for every new car they have sold and $500 for every used car they have sold.

<table>
<thead>
<tr>
<th>Sales Associate</th>
<th>New Cars</th>
<th>Used Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mason</td>
<td>27</td>
<td>49</td>
</tr>
<tr>
<td>Westin</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Gallagher</td>
<td>9</td>
<td>56</td>
</tr>
<tr>
<td>Stadler</td>
<td>15</td>
<td>62</td>
</tr>
</tbody>
</table>

a. Use a matrix to determine which sales associate earned the most money.
b. What is the total amount of money the car lot spent on bonuses for the sales associates this year?

**SOLUTION:**

\[
\begin{bmatrix}
27 & 49 \\
35 & 36 \\
9 & 56 \\
15 & 62 \\
\end{bmatrix}
\begin{bmatrix}
1000 \\
500 \\
\end{bmatrix}
= 
\begin{bmatrix}
51,500 \\
53,000 \\
37,000 \\
46,000 \\
\end{bmatrix}
\]

Therefore, Westin earned the most money.

b. Find the sum of the bonus amounts earned by each associate.

\[51,500 + 53,000 + 37,000 + 46,000 = 187,500\]

So, the total amount is $187,500.

**ANSWER:**

\[
\begin{bmatrix}
27 & 49 \\
35 & 36 \\
9 & 56 \\
15 & 62 \\
\end{bmatrix}
\begin{bmatrix}
1000 \\
500 \\
\end{bmatrix}
= 
\begin{bmatrix}
51,500 \\
53,000 \\
37,000 \\
46,000 \\
\end{bmatrix}
\]

$187,500

36. XY

**SOLUTION:**

The inner dimensions are not equal. So, the product is undefined.

**ANSWER:**

undefined

37. YX

**SOLUTION:**

\[
\begin{bmatrix}
-10 \ -4.5y \\
2x \ +4 +3y^2 \\
3.6y + 26 \\
\end{bmatrix}
\begin{bmatrix}
36.75 \\
-6x \ -4.5y -12 \\
-83.4 \\
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
-10 \ -4.5y \\
2x \ +4 +3y^2 \\
3.6y + 26 \\
\end{bmatrix}
\begin{bmatrix}
36.75 \\
-6x \ -4.5y -12 \\
-83.4 \\
\end{bmatrix}
\]

38. ZY

**SOLUTION:**

The inner dimensions are not equal. So, the product is undefined.

**ANSWER:**

undefined
3-6 Multiplying Matrices

39. \(YZ\)

**SOLUTION:**

\[
YZ = \begin{bmatrix} -5 & -1.5 \\ x + 2 & y \\ 13 & 1.2 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ x + y \end{bmatrix}
\]

\[
= \begin{bmatrix} -1.5x - 1.5y + 15 \\ y^2 + xy - 3x - 6 \\ 1.2x + 1.2y - 39 \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix} -1.5x - 1.5y + 15 \\ y^2 + xy - 3x - 6 \\ 1.2x + 1.2y - 39 \end{bmatrix}
\]

40. \((YX)Z\)

**SOLUTION:**

\[
(YX)Z = \begin{bmatrix} -10 - 4.5y \\ 2x + 4 + 3y^2 \\ 3.6y + 26 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ x + y \end{bmatrix}
\]

\[
= \begin{bmatrix} -36.75x + 50.25y + 30 \\ -6x^2 - 18x - 10.5xy - 13.5y^2 - 12y - 12 \\ -83.4x + 69y - 12 \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix} -36.75x + 50.25y + 30 \\ -6x^2 - 18x - 10.5xy - 13.5y^2 - 12y - 12 \\ -83.4x + 94.2y - 78 \end{bmatrix}
\]

41. \((XZ)X\)

**SOLUTION:**

The inner dimensions of the matrices are not equal.
So, the product is undefined.

**ANSWER:**

undefined

42. \(X(ZZ)\)

**SOLUTION:**

The product of the matrices \(ZZ\) is not defined. So, the product \(X(ZZ)\) is undefined.

**ANSWER:**

undefined
3-6 Multiplying Matrices

44. CAMERAS Prices of digital cameras depend on features like optical zoom, digital zoom, and megapixels.

<table>
<thead>
<tr>
<th>Optical Zoom</th>
<th>6 MP</th>
<th>7 MP</th>
<th>10 MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 to 4</td>
<td>$189.99</td>
<td>$249.99</td>
<td>$349.99</td>
</tr>
<tr>
<td>5 to 6</td>
<td>$199.99</td>
<td>$289.99</td>
<td>$399.99</td>
</tr>
<tr>
<td>10 to 12</td>
<td>$299.99</td>
<td>$399.99</td>
<td>$499.99</td>
</tr>
</tbody>
</table>

a. The 10-mp cameras are on sale for 20% off, and the other models are 10% off. Write a new matrix for these changes.
b. Write a new matrix allowing for a 6.25% sales tax on the discounted prices.
c. Describe what the differences in these two matrices represent.

**SOLUTION:**

\[
\begin{bmatrix}
170.99 & 224.99 & 279.99 \\
179.99 & 260.99 & 319.99 \\
269.99 & 359.99 & 399.99
\end{bmatrix}
\]

\[
\begin{bmatrix}
181.68 & 239.05 & 297.49 \\
191.24 & 277.30 & 339.99 \\
286.86 & 382.49 & 424.99
\end{bmatrix}
\]

da. Use matrices to determine the total cost of each package.
b. The studio offers an early bird discount of 15% off any package. Find the early bird price for each package.

**SOLUTION:**
a. Package A:

\[
\begin{bmatrix}
7 \\
10 \\
14 \\
13 \\
8
\end{bmatrix}
\]

\[
\begin{bmatrix}
10 & 4 & 2 & 1 & 1 & 88
\end{bmatrix}
\]

\[
\begin{bmatrix}
421
\end{bmatrix}
\]

Total cost for package A is $421.

Package B:

\[
\begin{bmatrix}
7 \\
10 \\
14 \\
13 \\
8
\end{bmatrix}
\]

\[
\begin{bmatrix}
10 & 4 & 2 & 1 & 0 & 56
\end{bmatrix}
\]

\[
\begin{bmatrix}
274
\end{bmatrix}
\]

Total cost for package B is $274.

Package C:

45. BUSINESS The Kangy Studio has packages available for senior portraits.
### 3-6 Multiplying Matrices

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. Translate 2 units to the left and 6 units down.

**ANSWER:**

<table>
<thead>
<tr>
<th>8</th>
<th>4</th>
<th>2</th>
<th>0</th>
<th>0</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>95</td>
<td>13</td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total cost for package C is $150.

**Package D:**

<table>
<thead>
<tr>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>95</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>8</td>
</tr>
</tbody>
</table>

Total cost for package D is $68.

b. The early bird price for package A is $421 \times 0.85 = $357.85.
The early bird price for package B is $274 \times 0.85 = $232.90.
The early bird price for package C is $150 \times 0.85 = $127.50.
The early bird price for package D is $68 \times 0.85 = $57.80.

**ANSWER:**

a. A: $421; B: $274; C: $150; D: $68

b. A: $357.85; B: $232.90; C: $127.50; D: $57.80

46. **REASONING** If the product matrix \(AB\) has dimensions \(5 \times 8\), and \(A\) has dimensions \(5 \times 6\), what are the dimensions of matrix \(B\)?

**SOLUTION:**
The inner dimensions should be equal. So, the dimensions of the matrix \(B\) are \(6 \times 8\).

**ANSWER:**

\(6 \times 8\)

47. **CCSS ARGUMENTS** Show that each property of matrices.

a. Scalar Distributive Property
b. Matrix Distributive Property
c. Associative Property of Multiplication
d. Associative Property of Scalar Multiplication

**SOLUTION:**

\[c(A + B) = c\left[\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}\right] + \left[\begin{array}{c} w \\ x \\ y \\ z \end{array}\right] \]

Substitution

\[= \left[\begin{array}{c} c + w \\ b + x \\ c + y \\ d + e \\ e + z \end{array}\right] \]

Definition of matrix addition

\[= \left[\begin{array}{c} ca + cw \\ cb + cx \\ cd + cy \\ ce + cz \end{array}\right] \]

Definition of scalar multiplication

\[= \left[\begin{array}{c} ca \\ cb \\ cd \\ ce \end{array}\right] + \left[\begin{array}{c} aw \\ cx \\ ew \\ ez \end{array}\right] \]

Definition of matrix addition

\[= CA + CB \]

Substitution

b. Use the definitions of matrix multiplication, Distributive Property of Addition, and the definition of matrix addition.
3-6 Multiplying Matrices

**Determine whether each matrix product is defined. If so, state the dimensions of the product.**

1. ... of left 2 units and down 6 units.

**ANSWER:** Translated 2 units to the left and 6 units down.

---

- **3-6 Multiplying Matrices**

\[
\begin{align*}
C(A + B) & = \left[ \begin{array}{c}
\alpha_{11} \\
\alpha_{21} \\
\end{array} \right] + \left[ \begin{array}{c}
\beta_{11} \\
\beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} + \beta_{11} \\
\alpha_{21} + \beta_{21} \\
\end{array} \right] \\
& = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} \\
\alpha_{12} \beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} \beta_{12} + \alpha_{12} \beta_{22} \\
\alpha_{21} \beta_{12} + \alpha_{22} \beta_{22} \\
\end{array} \right]
\end{align*}
\]

Substitution

**Definition of matrix addition.**

- **c. Use the definition of matrix multiplication, Distributivity Property of Addition.**

\[
(AB)C = \left[ \begin{array}{c}
\alpha_{11} \\
\alpha_{21} \\
\end{array} \right] C = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} \\
\alpha_{12} \beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21} \\
\end{array} \right]
\]

Substitution

**Definition of matrix multiplication.**

- **d.**

\[
C(AB) = \left[ \begin{array}{c}
\alpha_{11} \\
\alpha_{21} \\
\end{array} \right] \left[ \begin{array}{c}
\beta_{11} \\
\beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21} \\
\alpha_{21} \beta_{11} + \alpha_{22} \beta_{21} \\
\end{array} \right]
\]

**Definition of matrix multiplication.**

**Substitution:**

**Definition of scalar multiplication.**

**ANSWER:**

- **a.**

\[
C(A + B) = C \left[ \begin{array}{c}
\alpha_{11} \\
\alpha_{21} \\
\end{array} \right] + C \left[ \begin{array}{c}
\beta_{11} \\
\beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21} \\
\alpha_{21} \beta_{11} + \alpha_{22} \beta_{21} \\
\end{array} \right]
\]

**Substitution**

**Definition of matrix addition.**

**Definition of scalar multiplication.**

- **b.**

\[
C(AB) = C \left[ \begin{array}{c}
\alpha_{11} \\
\alpha_{21} \\
\end{array} \right] \left[ \begin{array}{c}
\beta_{11} \\
\beta_{21} \\
\end{array} \right] = \left[ \begin{array}{c}
\alpha_{11} \beta_{11} + \alpha_{12} \beta_{21} \\
\alpha_{21} \beta_{11} + \alpha_{22} \beta_{21} \\
\end{array} \right]
\]

**Definition of matrix multiplication.**

**Substitution**

---

**eSolutions Manual - Powered by Cognero**
Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. \( \text{left 2 units and down 6 units.} \)

ANSWER: Translated 2 units to the left and 6 units down.

---

3-6 Multiplying Matrices

\[ C(A + B) = \begin{bmatrix} \frac{a}{c} & \frac{b}{c} \\ \frac{a}{d} & \frac{b}{d} \end{bmatrix} \]

Substitution

\[ \frac{a}{c} \cdot \frac{b}{c} + \frac{a}{d} \cdot \frac{b}{d} \]

Definition of Matrix Addition

\[ c(AB) = c \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \]

Defini

Substitution

\[ c(A + B) = c \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \]

Defini

Substitution

\[ c = c \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \]

Defini

Substitution

\[ (AB)C = \begin{bmatrix} a_{11}a_{12} & a_{11}a_{22} \\ a_{21}a_{12} & a_{21}a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \]

Defini

Substitution

\[ (AB)C = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} \]

Defini

Substitution

48. OPEN ENDED Write two matrices \( A \) and \( B \) such that \( AB = BA \).

SOLUTION:

Sample answer:

\[ A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \]

Substi

\[ B = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \]

ANSWER:

Sample Answer: \( A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \)
3-6 Multiplying Matrices

49. **CHALLENGE** Find the missing values in
\[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\begin{bmatrix}
  4 & 3 \\
  2 & 5
\end{bmatrix}
= 
\begin{bmatrix}
  10 & 11 \\
  20 & 29
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\begin{bmatrix}
  4 & 3 \\
  2 & 5
\end{bmatrix}
= 
\begin{bmatrix}
  10 & 11 \\
  20 & 29
\end{bmatrix}
\]
So:
\[
4a + 2b = 10
\]
\[
3a + 5b = 11
\]
And:
\[
4c + 2d = 20
\]
\[
3c + 5d = 29
\]
Solve the equations:
\[a = 2, \ b = 1, \ c = 3, \text{ and } d = 4\]

**ANSWER:**
\[a = 2, \ b = 1, \ c = 3, \ d = 4\]

50. **WRITING IN MATH** Use the data on Lisa Leslie found at the beginning of the lesson to explain how matrices can be used in sports statistics. Describe a matrix that represents the total number of points she has scored during her career and an example of a sport in which different point values are used in scoring.

**SOLUTION:**
Sports statistics are often listed in columns and matrices. In this case, you can find the total number of points scored by multiplying the point value matrix, which does not change, by the scoring matrix, which changes after each season. The total number of points for her career can be found by multiplying the scoring matrix S by the point matrix P. Basketball and wrestling use different point values in scoring.

**ANSWER:**
Sports statistics are often listed in columns and matrices. In this case, you can find the total number of points scored by multiplying the point value matrix, which does not change, by the scoring matrix, which changes after each season. The total number of points for her career can be found by multiplying the scoring matrix S by the point matrix P. Basketball and wrestling use different point values in scoring.

51. **GRIDDED RESPONSE** The average (arithmetic mean) of \(r, w, x, \text{ and } y\) is 8, and the average of \(x\) and \(y\) is 4. What is the average of \(r\) and \(w\)?

**SOLUTION:**
\[
\frac{r + w + x + y}{4} = 8
\]
\[
\frac{x + y}{2} = 4
\]
So:
\[
\frac{r + w + 8}{4} = 8
\]
\[
\frac{r + w + 8}{4} = 8
\]
\[
\frac{r + w + 8}{4} = 8
\]
\[
\frac{r + w + 8}{4} = 8
\]
\[
\frac{r + w}{2} = 12
\]
That is, the average of \(r\) and \(w\) is 12.

**ANSWER:**
12
52. Carla, Meiko, and Kayla went shopping to get ready for college. Their purchases and total amounts spent are shown in the table below.

<table>
<thead>
<tr>
<th>Person</th>
<th>Shirts</th>
<th>Pants</th>
<th>Shoes</th>
<th>Total Spent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carla</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>$149.79</td>
</tr>
<tr>
<td>Meiko</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>$183.19</td>
</tr>
<tr>
<td>Kayla</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>$181.14</td>
</tr>
</tbody>
</table>

Assume that all of the shirts were the same price, all of the pants were the same price, and all of the shoes were the same price. What was the price of each item?

A shirt, $12.95; pants, $15.99; shoes, $23.49
B shirt, $15.99; pants, $12.95; shoes, $23.49
C shirt, $15.99; pants, $23.49; shoes, $12.95
D shirt, $23.49; pants, $15.99; shoes, $12.95

**SOLUTION:**

Let $a$ be the price of a shirt, $b$ be the price of pants, and $c$ be the price of a pair of shoes.

\[
\begin{bmatrix}
3 & 4 & 2 \\
5 & 3 & 3 \\
6 & 5 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
= 
\begin{bmatrix}
149.79 \\
183.19 \\
181.14
\end{bmatrix}
\]

So:

\[
3a + 4b + 2c = 149.79 \\
5a + 3b + 3c = 183.19 \\
6a + 5b + c = 181.14
\]

Solve:

\[a = 12.95, \ b = 15.99, \ c = 23.49\]

The correct choice is **A**.

**ANSWER:**

A

53. **GEOMETRY** Rectangle $LMNQ$ has diagonals that intersect at point $P$. Which of the following represents point $P$?

- **F** $(2, 2)$
- **G** $(1, 1)$
- **H** $(0, 0)$
- **J** $(-1, -1)$

**SOLUTION:**

The diagonals of a rectangle bisect each other. So, $P$ is the midpoint of $LN$.

So:

\[
P \left( \frac{3 + 3}{2}, \frac{5 - 5}{2} \right) = P(0, 0)
\]

The correct choice is **H**.

**ANSWER:**

H

54. **SAT/ACT** What are the dimensions of the matrix that results from the multiplication shown?

\[
\begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i \\
j & k & l
\end{bmatrix}
\cdot
\begin{bmatrix}
7 \\
4 \\
6
\end{bmatrix}
\]

- **A** $1 \times 4$
- **B** $3 \times 3$
- **C** $3 \times 1$
- **D** $4 \times 1$
- **E** $4 \times 3$

**SOLUTION:**

The dimensions of the matrices are $4 \times 3$ and $3 \times 1$. So, the dimensions of the resultant matrix are $4 \times 1$. The correct choice is **D**.

**ANSWER:**

D
3-6 Multiplying Matrices

Perform the indicated operations. If the matrix does not exist, write **impossible**.

55. \[
\begin{bmatrix}
8 & -1 \\
-3 & 4
\end{bmatrix} \cdot \begin{bmatrix}
-5 & 2 \\
6 & 3
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
8 & -1 \\
-3 & 4
\end{bmatrix} \cdot \begin{bmatrix}
-5 & 2 \\
6 & 3
\end{bmatrix} = \begin{bmatrix}
32 & -4 \\
12 & 16
\end{bmatrix} \cdot \begin{bmatrix}
-10 & 20 \\
30 & 15
\end{bmatrix} = \begin{bmatrix}
-42 & -24 \\
-42 & -31
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-42 & -24 \\
-42 & -31
\end{bmatrix}
\]

56. \[
\begin{bmatrix}
2 & -5 \\
-1 & 3
\end{bmatrix} \cdot \begin{bmatrix}
-1 & -2 \\
6 & 4
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
2 & -5 \\
-1 & 3
\end{bmatrix} \cdot \begin{bmatrix}
-1 & -2 \\
6 & 4
\end{bmatrix} = \begin{bmatrix}
-4 & -10 \\
-2 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-1 & -6 \\
18 & 12
\end{bmatrix} = \begin{bmatrix}
5 & -4 \\
-20 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-1 & 4 \\
-20 & 6
\end{bmatrix} = \begin{bmatrix}
-5 & -20 \\
-100 & -30
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-5 & -20 \\
-100 & -30
\end{bmatrix}
\]

57. \[
\begin{bmatrix}
8 & 9 \\
5 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-2 & -6 \\
-3 & 1
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
8 & 9 \\
5 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-2 & -6 \\
-3 & 1
\end{bmatrix} = \begin{bmatrix}
-24 & -66 \\
-10 & -36
\end{bmatrix}
\]

58. \[
2x - 4y + 3z = -3
\]

- 7x + 5y - 4z = 11

- x - y - 2z = -21

**SOLUTION:**
\[
\begin{align*}
2x - 4y + 3z &= -3 \quad (1) \\
-7x + 5y - 4z &= 11 \quad (2) \\
x - y - 2z &= -21 \quad (3)
\end{align*}
\]

Eliminate one variable.

Multiply the third equation by -2 and add with the first equation.

\[
(3) \times -2 \quad -2x + 2y + 4z = 42
\]

+ (1) \quad 2x - 4y + 3z = -3

\[\text{(4) } -2y + 7z = 39\]

Multiply the first equation by 7 and the second equation by 2. Then add.

\[
(1) \times 7 \quad 14x - 28y + 21z = -21
\]

+ (2) \times 2 \quad -14x + 10y - 8z = 22

\[\text{(5) } -18y + 13z = 1\]

Solve for one variable using the the fourth and fifth equations.
Determine whether each matrix product is defined. If so, state the dimensions of the product.

(4) \times -9 \\
+ (5) \\
\therefore 18y - 6z = -35 \\
+ (-18y + 13z = 1) \\
z \quad -50z = -350 \\
\therefore z = 7 \\

Substitute 7 for z in the fourth equation and solve for y.

-2y + 7(7) = 39 \\
-2y + 49 = 39 \\
-2y = -10 \\
y = 5 \\

Substitute 5 for y and 7 for z in the third equation, and solve for x.

x - 5 - 2(7) = -21 \\
x - 5 - 14 = -21 \\
x - 19 = -21 \\
x = -2 \\

Therefore, the solution is (-2, 5, 7).

ANSWER: (-2, 5, 7)

-4x - 2y + 9z = -29 \\
10x - 12y + 7z = 51 \\
59, \ 3x + 5y - 14z = 25 \\

SOLUTION:

-4x - 2y + 9z = -29 \quad \rightarrow (1) \\
10x - 12y + 7z = 51 \quad \rightarrow (2) \\
3x + 5y - 14z = 25 \quad \rightarrow (3) \\

Eliminate one variable.
Multiply the first equation by 3 and the third equation by 4. Then add.

\begin{align*}
& (1) \times 3 \quad -12x - 6y + 27z = -87 \\
& + (3) \times 4 \quad 12x + 20y - 56z = 100 \\
& (4) \quad 14y - 29z = 13 \\
\end{align*}

Multiply the second equation by 3 and add with the equation by -10. Then add.

\begin{align*}
& (2) \times 3 \quad 30x - 36y + 21z = 153 \\
& + (5) \times -10 \quad -30x - 50y + 140z = -250 \\
& (5) \quad -86y + 161z = -97 \\
\end{align*}

Solve for one variable using the fourth and fifth equations.

\begin{align*}
& (4) \times 86 \quad 1204y - 2494z = 1118 \\
& + (5) \times 14 \quad -1204y + 225z = -1358 \\
& z \quad -240z = -240 \\
\end{align*}

Substitute 1 for z in the fourth equation and solve for y.

\begin{align*}
14y - 29(1) & = 13 \\
14y - 29 & = 13 \\
14y & = 42 \\
y & = 3 \\
\end{align*}

Substitute 3 for y and 1 for z in the third equation, and solve for x.

\begin{align*}
3x + 5(3) - 14(1) & = 25 \\
3x + 15 - 14 & = 25 \\
3x & = 24 \\
x & = 8 \\
\end{align*}

Therefore, the solution is (8, 3, 1).

ANSWER: (8, 3, 1)

-7x + 8y - z = 43 \\
3x - 2y + 5z = -43 \\
60. \ 2x - 4y + 6z = -50 \\

SOLUTION:

\begin{align*}
-7x + 8y - z = 43 \quad \rightarrow (1) \\
3x - 2y + 5z = -43 \quad \rightarrow (2) \\
2x - 4y + 6z = -50 \quad \rightarrow (3) \\
\end{align*}

Eliminate one variable.
Multiply the second equation by 4 and add with the
first equation.

\[(2) \times 4 \quad 12x - 8y + 20z = -172 \]
\[(+1) \quad -7x + 8y - z = 43 \]
\[(4) \quad 5x + 19z = -129 \]

Multiply the third equation by \(2\) and add with the first equation.

\[(3) \times 2 \quad 4x - 8y + 12z = -100 \]
\[(+2) \times 2 \quad -7x + 8y - z = 43 \]
\[(5) \quad -3x + 11z = -57 \]

Solve for one variable using the fourth and fifth equations.

\[(4) \times 3 \quad 15x + 57z = -387 \]
\[(+5) \times 5 \quad -15x + 55z = -285 \]
\[z \]
\[112z = -672 \]
\[z = -6 \]

Substitute -6 for \(z\) in the fourth equation and solve for \(y\).

\[5x + 19(-6) = -129 \]
\[5x = -15 \]
\[x = -3 \]

Substitute -3 for \(x\) and -6 for \(z\) in the third equation, and solve for \(y\).

\[2(-3) - 4y + 6(-6) = -50 \]
\[-6 - 4y - 36 = -21 \]
\[-4y = -8 \]
\[y = 2 \]

Therefore, the solution is \((-3, 2, -6)\).

**ANSWER:**
\((-3, 2, -6)\)

61. **MEDICINE** The graph shows how much Americans spent on doctors’ visits in some recent years and a prediction for 2014.

**SOLUTION:**

a. Find a regression equation for the data without the predicted value.

b. Use your equation to predict the expenditures for 2014.

c. Compare your prediction to the one given in the graph.

![Graph of national health expenditures](image_url)

**SOLUTION:**

a. A sample answer is given. Use a graphing calculator to make a scatter plot of the data. Enter 1999, 2000, 2001, 2002, 2003 into L1 and 1222.2, 1309.9, 1426.4, 1559.0, and 1678.9 in L2. Then press `LIST`, select the `CALC` menu, and select `LinReg(ax+b)`. The calculator returns an \(a\)-value of 116.25 and a \(b\)-value of 231176.97, so a regression equation for the data where \(x\) is the year and \(y\) is the national health expenditure in billions is \(y = 116.25x - 231,176.97\).

d. Replace \(x\) with 2014 in the equation \(y = 116.25x - 231,176.97\) to calculate the national health expenditure in billions for the year 2014.

\[y = 116.25(2014) - 231,176.97 = 2950.53 \]

Therefore, using this equation, we predict that in 2014 the national health expenditure will be \$2950.53 billion.

c. The value predicted by the equation, \$2950.53 billion, is significantly lower than the one given in the graph, \$3585.7 billion.

**ANSWER:**

a. Sample answer: \(y = 116.25x - 231,176.97\)

b. Sample answer: \$2950.53 billion

c. The value predicted by the equation is significantly lower than the one given in the graph.
3-6 Multiplying Matrices

62. How many different ways can the letters of the word
MATHEMATICS be arranged?

**SOLUTION:**

Number of ways = \( \frac{11!}{2!2!} \)

= 4,989,600

**ANSWER:** 4,989,600

Describe the transformation in each function. Then graph the function.

63. \( f(x) = |x - 4| + 3 \)

**SOLUTION:**

The graph of \( y = |x - 4| + 3 \) is a translation of the graph of \( y = |x| \) right 4 units and up 3 units.

**ANSWER:**

Translated 4 units to the right and 3 units up.

64. \( f(x) = 2|x + 3| - 5 \)

**SOLUTION:**

The graph of \( y = 2|x + 3| - 5 \) is a translation of the graph of \( y = |x| \) down 5 units and left 3 units. Also, a vertical stretch of the graph of \( y = |x| \) by a degree 2.

**ANSWER:**

Translated 3 units to the left and 5 units down and stretched vertically...
3-6 Multiplying Matrices

65. \( f(x) = (x + 2)^2 - 6 \)

**SOLUTION:**
The graph of \( y = (x + 2)^2 - 6 \) is a translation of the graph of \( y = x^2 \) left 2 units and down 6 units.

**ANSWER:**
Translated 2 units to the left and 6 units down.
Evaluate each determinant.

1. \[
\begin{vmatrix}
8 & 6 \\
5 & 7
\end{vmatrix}
\]

**SOLUTION:**
\[
\begin{vmatrix}
8 & 6 \\
5 & 7
\end{vmatrix} = 8(7) - 6(5) = 56 - 30 = 26
\]

**ANSWER:**
26

2. \[
\begin{vmatrix}
-6 & -6 \\
8 & 10
\end{vmatrix}
\]

**SOLUTION:**
\[
\begin{vmatrix}
-6 & -6 \\
8 & 10
\end{vmatrix} = (-6)10 - (-6)8 = -60 + 48 = -12
\]

**ANSWER:**
-12

3. \[
\begin{vmatrix}
-4 & 12 \\
9 & 5
\end{vmatrix}
\]

**SOLUTION:**
\[
\begin{vmatrix}
-4 & 12 \\
9 & 5
\end{vmatrix} = (-4)5 - 12(9) = -20 - 108 = -128
\]

**ANSWER:**
-128

4. \[
\begin{vmatrix}
16 & -10 \\
-8 & 5
\end{vmatrix}
\]

**SOLUTION:**
\[
\begin{vmatrix}
16 & -10 \\
-8 & 5
\end{vmatrix} = 16(5) - (-10)(-8) = 80 - 80 = 0
\]

**ANSWER:**
0
Evaluate each determinant using diagonals.

5. \[
\begin{vmatrix} 3 & -2 & 2 \\ -4 & 2 & -5 \\ -3 & 1 & 4 \end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix} 3 & -2 & | & 3 & -2 \\ -4 & 2 & | & -4 & 2 \\ -3 & 1 & | & -3 & 1 \end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
3(2)(4) &= 24 \\
(-2)(-5)(-3) &= -30 \\
2(-1)(1) &= -2 \\
3 &-2 & 2 & \cdot & 3 & -2 \\
-4 & 2 & -5 & \cdot & -4 & 2 \\
-3 & 1 & 4 & \cdot & -3 & 1
\end{align*}
\]

\[
\begin{align*}
3(2)(4) &= 24 \\
(-3)(2)(2) &= -12 \\
(-2)(-5)(-3) &= -30 \\
1(-5)(3) &= -15 \\
2(-1)(1) &= -2 \\
4(-4)(-2) &= 32
\end{align*}
\]

Find the sum of each group.

\[
24 - 30 - 8 = -14 \\
\quad -12 - 15 + 32 = 5
\]

Subtract the value of the second group from the first group.

\[-14 - 5 = -19
\]

Therefore, the value of the determinant is \(-19\).

**ANSWER:**
\(-19\)

6. \[
\begin{vmatrix} 2 & -3 & 5 \\ -4 & 6 & -2 \\ 4 & -1 & -6 \end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix} 2 & -3 & | & 2 & -3 \\ -4 & 6 & | & -4 & 6 \\ 4 & -1 & | & 4 & -1 \end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
2(6)(-6) &= -72 \\
(-3)(-2)(4) &= 24 \\
(5)(-4)(-1) &= 20
\end{align*}
\]

Find the sum of each group.

\[
-72 + 24 + 20 = -28 \\
120 + 4 - 72 = 52
\]

Subtract the value of the second group from the first group.

\[-28 - 52 = -80
\]

Therefore, the value of the determinant is \(-80\).

**ANSWER:**
\(-80\)
Evaluate each determinant.

1. 
   SOLUTION: 
   ANSWER: 26

2. 
   SOLUTION: 
   ANSWER: −12

3. Mi-Ling ordered 6 jelly-filled doughnuts. 
   ANSWER: 6

4. CCSS PERSEVERANCE The salary for each of
3-7 Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant.

9. \[
\begin{vmatrix}
  8 & 3 & 4 \\
  2 & 4 & 2 \\
  1 & 6 & 5 \\
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.
\[
\begin{vmatrix}
  8 & 4 & 8 & 3 \\
  2 & 4 & 2 & 4 \\
  1 & 6 & 5 & 1 & 6 \\
\end{vmatrix}
\]
Find the product of the element of the diagonal.
\[
(8)(4)(5) = 160 \quad (1)(4)(4) = 16
\]
\[
(3)(2)(1) = 6 \quad (6)(2)(8) = 96
\]
\[
(4)(2)(6) = 48 \quad (5)(2)(3) = 30
\]
Find the sum of each group.
\[
160 + 6 + 48 = 214 \quad 16 + 96 + 30 = 142
\]
Subtract the value of the second group from the first group.
\[
214 - 142 = 72
\]
Therefore, the value of the determinant is 72.

**ANSWER:**

72

10. \[
\begin{vmatrix}
  -4 & 3 & 0 \\
  1 & 5 & -2 \\
  -1 & -8 & -3 \\
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.
\[
\begin{vmatrix}
  -4 & 3 & 0 \\
  1 & 5 & 2 \\
  -1 & -8 & -3 \\
\end{vmatrix}
\]
Find the product of the element of the diagonal.
\[
(4)(3)(0) = 0 \quad (1)(5)(0) = 0
\]
\[
(3)(-2)(-1) = 6 \quad (-8)(-2)(-4) = -64
\]
\[
(0)(1)(-8) = 0 \quad (-3)(3)(3) = -9
\]
Find the sum of each group.
\[
60 + 6 + 0 = 66 \quad 0 - 64 - 9 = -73
\]
Subtract the value of the second group from the first group.
\[
66 - (-73) = 139
\]
Therefore, the value of the determinant is 139.

**ANSWER:**

139
### 3-7 Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant:

1. \[
\begin{vmatrix}
2 & -6 & -3 \\
7 & 9 & -4 \\
-6 & 4 & 9
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
2 & -6 & -3 \\
7 & 9 & -4 \\
-6 & 4 & 9
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
(2)(9)(9) &= 162 \\
(-6)(9)(-3) &= 162 \\
(-3)(7)(4) &= -84 \\
(9)(7)(-6) &= -378
\end{align*}
\]

Find the sum of each group.

\[
\begin{align*}
162 - 144 - 84 &= -66 \\
162 - 32 - 378 &= -248
\end{align*}
\]

Subtract the value of the second group from the first group.

\[-66 - (-248) = 182\]

Therefore, the value of the determinant is 182.

**ANSWER:**
182

2. \[
\begin{vmatrix}
-5 & -6 & -7 \\
4 & 0 & 5 \\
-3 & 8 & 2
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
-5 & -6 & -7 \\
4 & 0 & 5 \\
-3 & 8 & 2
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
(-5)(0)(2) &= 0 \\
(-6)(5)(-3) &= -90 \\
(-7)(4)(8) &= 224 \\
(2)(4)(-6) &= -48
\end{align*}
\]

Find the sum of each group.

\[
\begin{align*}
0 + 90 + 224 &= 314 \\
0 - 200 - 48 &= -248
\end{align*}
\]

Subtract the value of the second group from the first group.

\[314 - (-248) = 562\]

Therefore, the value of the determinant is 562.

**ANSWER:**
562
3-7 Solving Systems of Equations Using Cramer's Rule

Use Cramer’s Rule to solve each system of equations.

13. \(4x - 5y = 39\)
\(3x + 8y = -6\)

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{pmatrix}
ax + by = m \\
fx + gy = n
\end{pmatrix} \rightarrow
\begin{vmatrix}
a & b \\
f & g
\end{vmatrix}
\]

The solution of the system is

\[
x = \frac{m}{C} \quad \text{and} \quad y = \frac{n}{C}
\]
if \(C \neq 0\).

\[
C = \begin{vmatrix}
4 & -5 \\
3 & 8
\end{vmatrix}
\]
\[
|C| = (4)(8) - (-5)(3)
\]
\[
= 32 + 15
\]
\[
= 47
\]
\[
x = \frac{39}{47} \quad \text{and} \quad y = \frac{-5}{47}
\]
\[
x = \frac{39(8) - (-5)(-6)}{47} \quad \text{and} \quad y = \frac{(4)(-6) - (39)(3)}{47}
\]
\[
x = \frac{282}{47} \quad \text{and} \quad y = \frac{-141}{47}
\]
\[
x = \frac{282}{47} \quad \text{and} \quad y = -3
\]

Therefore, the solution of the system is (6, -3).

**ANSWER:**
(6, -3)

14. \(5x + 6y = 20\)
\(-3x - 7y = -29\)

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
ax + by = m \rightarrow \begin{vmatrix}
a & b \\
f & g
\end{vmatrix}
\]

The solution of the system is

\[
x = \frac{m}{C} \quad \text{and} \quad y = \frac{n}{C}
\]
if \(C \neq 0\).

\[
C = \begin{vmatrix}
5 & 6 \\
-3 & -7
\end{vmatrix}
\]
\[
|C| = (5)(-7) - (-3)(6)
\]
\[
= -35 + 18
\]
\[
= -17
\]
\[
x = \frac{20}{17} \quad \text{and} \quad y = \frac{-3}{17}
\]
\[
x = \frac{20(-7) - (-3)(6)}{17} \quad \text{and} \quad y = \frac{5(-29) - (-3)(20)}{17}
\]
\[
x = \frac{34}{17} \quad \text{and} \quad y = \frac{-85}{17}
\]
\[
x = -2 \quad \text{and} \quad y = -5
\]

Therefore, the solution of the system is (-2, 5).

**ANSWER:**
(-2, 5)
15. \(-8a - 5b = -27\), \(7a + 6b = 22\)

**SOLUTION:**

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{pmatrix}
 a & b \\
 f & g
\end{pmatrix}
\]

The solution of the system is

\[
x = \frac{m}{C} \quad \text{and} \quad y = \frac{n}{C} \quad \text{if} \quad C \neq 0.
\]

\[
C = \begin{vmatrix}
 -8 & -5 \\
 7 & 6
\end{vmatrix}
\]

\[
|C| = (-8)(6) - (-5)(7)
\]

\[
= -48 + 35
\]

\[
= -13
\]

\[
x = \frac{-27}{-13} \quad \text{and} \quad y = \frac{22}{-13}
\]

\[
x = 2 \quad \text{and} \quad y = -1
\]

Therefore, the solution of the system is \((4, -1)\).

**ANSWER:**

\((4, -1)\)

16. \(10c - 7d = -59\), \(6c + 5d = -63\)

**SOLUTION:**

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{pmatrix}
 a & b \\
 f & g
\end{pmatrix}
\]

The solution of the system is

\[
x = \frac{m}{C} \quad \text{and} \quad y = \frac{n}{C} \quad \text{if} \quad C \neq 0.
\]

\[
C = \begin{vmatrix}
 10 & -7 \\
 6 & 5
\end{vmatrix}
\]

\[
|C| = (10)(5) - (-7)(6)
\]

\[
= 50 + 42
\]

\[
= 92
\]

\[
x = \frac{-59}{92} \quad \text{and} \quad y = \frac{10}{92}
\]

\[
x = -0.64 \quad \text{and} \quad y = 0.11
\]

Therefore, the solution of the system is \((-8, -3)\).

**ANSWER:**

\((-8, -3)\)

17. **CCSS PERSEVERANCE** The “Bermuda Triangle” is an area located off the southeastern Atlantic coast of the United States, and noted for reports of unexplained losses of ships, small boats, and aircraft.
Evaluate each determinant.

1. SOLUTION: ANSWER: 26
2. SOLUTION: ANSWER: -12

a. Find the area of the triangle on the map.
b. Suppose each grid represents 175 miles. What is the area of the Bermuda Triangle?

SOLUTION:
a. The area of a triangle with vertices \((a, b), (c, d)\)
and \((f, g)\) is \(A = \frac{1}{2} \left| \begin{array}{ccc} a & b & 1 \\ c & d & 1 \\ f & g & 1 \end{array} \right| \).

The vertices of the Bermuda triangle are \((4, 4), (9, 1)\)
and \((9.5, 7)\).

\[
A = \frac{1}{2} \begin{vmatrix} 4 & 4 & 1 \\ 9 & 1 & 1 \\ 9.5 & 7 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} \begin{vmatrix} 4 & 4 & 1 \\ 9 & 1 & 1 \\ 9.5 & 7 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} \begin{vmatrix} 4 & 4 & 1 \\ 9 & 1 & 1 \\ 9.5 & 7 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2} [105 - 73.5]
\]

\[
= 15.75
\]

\[
A = 15.75 \text{ unit}^2.
\]

Mi-Ling ordered 6 jelly-filled doughnuts. ANSWER: 6

48. CCSS PERSEVERANCE The salary for each of

b. 1 unit = 175 miles
Therefore, 1 unit\(^2\) = 30625 mi\(^2\)
Therefore, 15.75 unit\(^2\) = 482343.75 mi\(^2\).

Area of the Bermuda triangle is 482,343.75 mi\(^2\).

ANSWER:

a. 15.75 units\(^2\)
b. 482,343.75 mi\(^2\)

Use Cramer’s Rule to solve each system of equations.

18. \(4x - 2y + 7z = 26\)
\(5x + 3y - 5z = -50\)
\(-7x - 8y - 3z = 49\)

SOLUTION:

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{align*}
ax + by + cz &= m \\
fx + gy + hz &= n \\
jx + ky + lz &= p
\end{align*}
\]

\[
\begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}
\]

The solution of the system is

\[
x = \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{|C|},
y = \frac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{|C|},
z = \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{|C|}
\]

if \(C \neq 0\).

\[
C = \begin{vmatrix} 5 & 3 & -5 \\ -7 & -8 & -3 \end{vmatrix}
\]

\[
|C| = -429
\]
Evaluate each determinant.

1.  
   SOLUTION: 
   ANSWER: 26

2.  
   SOLUTION: 
   ANSWER: -12

3.  
   Mi-Ling ordered 6 jelly-filled doughnuts. 
   ANSWER: 6

48. CCSS PERSEVERANCE The salary for each of
   3, 7).
Evaluate each determinant.

26

Solution:

\begin{align*}
\text{ANSWER:} & \quad 26 \\
\end{align*}

2. 

Solution:

\begin{align*}
\text{ANSWER:} & \quad -12 \\
\end{align*}

3. 

\begin{align*}
\text{Mi-Ling ordered 6 jelly-filled doughnuts.} \\
\text{ANSWER:} & \quad 6 \\
\end{align*}

48. CCSS PERSEVERANCE

The salary for each of Mr. Smith's art class took a bus trip to an 

Use Cramer’s Rule.

The solution of the system is

Therefore, the solution of the system is \((-3, 7, 2)\).

ANSWER:

\((-3, 7, 2)\)

21. \(-9x + 5y + 3z = 50\)

\(-2x + 7y + 5z = 46\)

Solution:

Use Cramer’s Rule.

The solution of the system is
Evaluate each determinant.

1. 
SOLUTION:
ANSWER: 26

2. 
SOLUTION: 
ANSWER: -12

3. 

Mi-Ling ordered 6 jelly-filled doughnuts. 
ANSWER: 6

48. CCSS PERSEVERANCE The salary for each of the group.

Let $C$ be the coefficient matrix of the system

\[
\begin{align*}
ax + by + cz &= m \\
fx + gy + hz &= n \\
jx + ky + lz &= p
\end{align*}
\]

The solution of the system is 

Therefore, the solution of the system is (4, 0, 8).

SOLUTION: 
Use Cramer’s Rule.

Therefore, the solution of the system is (6, 3, -4).

ANSWER:

22. $x + 2y = 12$
$3y - 4z = 25$
$x + 6y + z = 20$

SOLUTION: 
Use Cramer’s Rule.
3-7 Solving Systems of Equations Using Cramer's Rule

(6, 3, −4)

23. \(9a + 7b = -30\)
\(8b + 5c = 11\)
\(-3a + 10c = 73\)

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{align*}
ax + by + cz &= m \\
f(x + gy + hz) &= n \\
x + ky + lz &= p
\end{align*}
\]

The solution of the system is

\[
x = \frac{m b c}{|C|}, \quad y = \frac{a m c}{|C|}, \quad z = \frac{a b m}{|C|}
\]

if \(C \neq 0\).

\[
\begin{vmatrix}
9 & 7 & 0 \\
0 & 8 & 5 \\
-3 & 0 & 10
\end{vmatrix} = 615
\]

Therefore, the solution of the system is \((-1, -3, 7)\).

**ANSWER:**
\((-1, -3, 7)\)

24. \(2n + 3p - 4w = 20\)
\(4n - 6p + 5w = 13\)
\(3n + 2p + 4w = 15\)

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{align*}
ax + by + cz &= m \\
f(x + gy + hz) &= n \\
x + ky + lz &= p
\end{align*}
\]

The solution of the system is
3-7 Solving Systems of Equations Using Cramer's Rule

\[
\begin{vmatrix}
 m & b & c \\
 n & g & h \\
 p & k & l \\
\end{vmatrix},
\begin{vmatrix}
 a & m & c \\
 f & n & h \\
 j & p & l \\
\end{vmatrix},
\begin{vmatrix}
 a & b & m \\
 f & g & n \\
 j & k & p \\
\end{vmatrix}
\]

if \( C \neq 0 \).

\[
\begin{vmatrix}
 2 & 3 & -4 \\
 4 & -1 & 5 \\
 3 & 2 & 4 \\
\end{vmatrix}
= -75
\]

\[
\begin{vmatrix}
 20 & 3 & -4 \\
 13 & -1 & 5 \\
 15 & 2 & 4 \\
\end{vmatrix}
= -375
\]

\[
\begin{vmatrix}
 2 & 20 & -4 \\
 4 & 13 & 5 \\
 3 & 15 & 4 \\
\end{vmatrix}
= 5
\]

\[
\begin{vmatrix}
 2 & 3 & 20 \\
 4 & -1 & 13 \\
 3 & 2 & 15 \\
\end{vmatrix}
= 1
\]

Therefore, the solution of the system is \((5, 2, -1)\).

**ANSWER:**

\((5, 2, -1)\)

25. \(x + y + z = 12\)
\(6x - 2y - z = 16\)
\(3x + 4y + 2z = 28\)

**SOLUTION:**

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system.

\[
\begin{vmatrix}
 1 & 1 & 1 \\
 6 & -2 & -1 \\
 3 & 4 & 2 \\
\end{vmatrix}
= 15
\]

\[
\begin{vmatrix}
 12 & 1 & 1 \\
 16 & -2 & -1 \\
 28 & 4 & 2 \\
\end{vmatrix}
= 60
\]

\[
\begin{vmatrix}
 1 & 12 & 1 \\
 6 & 16 & -1 \\
 3 & 28 & 2 \\
\end{vmatrix}
= 0
\]

\[
\begin{vmatrix}
 1 & 1 & 12 \\
 6 & -2 & 16 \\
 3 & 4 & 28 \\
\end{vmatrix}
= 120
\]

Therefore, the solution of the system is \((4, 0, 8)\).

**ANSWER:**

\((4, 0, 8)\)
Evaluate each determinant.

26. \[
\begin{vmatrix}
-7 & 12 \\
5 & 6
\end{vmatrix}
\]

**SOLUTION:**

\[
\begin{align*}
\begin{vmatrix}
-7 & 12 \\
5 & 6
\end{vmatrix} &= (-7)(6) - (12)(5) \\
&= -42 - 60 \\
&= -102
\end{align*}
\]

**ANSWER:**

-102

27. \[
\begin{vmatrix}
-8 & -9 \\
11 & 12
\end{vmatrix}
\]

**SOLUTION:**

\[
\begin{align*}
\begin{vmatrix}
-8 & -9 \\
11 & 12
\end{vmatrix} &= (-8)(12) - (-9)(11) \\
&= -96 + 99 \\
&= 3
\end{align*}
\]

**ANSWER:**

3

28. \[
\begin{vmatrix}
-5 & 8 \\
-6 & -7
\end{vmatrix}
\]

**SOLUTION:**

\[
\begin{align*}
\begin{vmatrix}
-5 & 8 \\
-6 & -7
\end{vmatrix} &= (-5)(-7) - (8)(-6) \\
&= 35 + 48 \\
&= 83
\end{align*}
\]

**ANSWER:**

83

---

3-7 Solving Systems of Equations Using Cramer's Rule

29. \[
\begin{vmatrix}
3 & 5 & -2 \\
-1 & -4 & 6 \\
-6 & -2 & 5
\end{vmatrix}
\]

**SOLUTION:**

Rewrite the first two columns in the right of the determinant.

\[
\begin{align*}
\begin{vmatrix}
3 & 5 & -2 \\
-1 & -4 & 6 \\
-6 & -2 & 5
\end{vmatrix}
&= \begin{vmatrix}
3 & 5 \\
-1 & -4 \\
-6 & -2
\end{vmatrix} & \begin{vmatrix}
-2 \\
6 \\
-2
\end{vmatrix}
\end{align*}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
3(3) - 5(-4) + (-2)(6) &= 9 + 20 - 12 \\
&= 17
\end{align*}
\]

Find the sum of each group.

\[
-60 - 180 - 4 = -244
\]

Subtract the value of the second group from the first group.

\[
-244 - (-109) = -135
\]

Therefore, the value of the determinant is -135.

**ANSWER:**

-135
### 3-7 Solving Systems of Equations Using Cramer's Rule

<table>
<thead>
<tr>
<th>2 0 −6</th>
<th>2 0</th>
<th>30.</th>
<th>2</th>
<th>0</th>
<th>−6</th>
<th>31.</th>
<th>−5</th>
<th>−1</th>
<th>−2</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3 −4 −5</td>
<td>−3 −4 −5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−2 5 8</td>
<td>−2 5 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
2 & 0 & −6 \\
−3 & −4 & −5 \\
−2 & 5 & 8
\end{vmatrix}
\]

\[
\begin{vmatrix}
2 & 0 \\
−3 & −4 \\
−2 & 5
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
(2)(−4)(8) = −64
\]

\[
(0)(−5)(−2) = 0
\]

\[
(−6)(3)(−3) = 90
\]

\[
(2)(0)(0) = 0
\]

Find the sum of each group.

\[-64 + 0 + 90 = 26
\]

\[-48 + 0 + 0 = −98
\]

Subtract the value of the second group from the first group.

\[26 − (−98) = 124
\]

Therefore, the value of the determinant is 124.

**ANSWER:**

\[124\]

---

<table>
<thead>
<tr>
<th>−5 −1 −2</th>
<th>−5 −1 −2</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1 8 4</td>
<td>−1 8 4</td>
</tr>
<tr>
<td>0 −6 9</td>
<td>0 −6 9</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
−5 & −1 & −2 \\
−1 & 8 & 4 \\
0 & −6 & 9
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
(−5)(8)(9) = −360
\]

\[
(−1)(4)(0) = 0
\]

\[
(−2)(1)(−6) = 12
\]

\[
(0)(8)(−2) = 0
\]

\[
(−6)(4)(−5) = −120
\]

\[
(9)(1)(−1) = −9
\]

Find the sum of each group.

\[−360 + 0 + 12 = −348
\]

\[0 + 120 − 9 = 111
\]

Subtract the value of the second group from the first group.

\[−348 − 11 = −459
\]

Therefore, the value of the determinant is −459.

**ANSWER:**

\[−459\]
Evaluate each determinant.

32. \[ \begin{vmatrix} 6 & -3 & -5 \\ 0 & -7 & 0 \\ 3 & -6 & -4 \end{vmatrix} \]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[ \begin{vmatrix} 6 & -3 & 6 & -3 \\ 0 & -7 & 0 & -7 \\ 3 & -6 & 3 & -6 \end{vmatrix} \]

Find the product of the element of the diagonal.

\[
\begin{align*}
(6)(-7)(-4) &= 168 \\
(-3)(0)(3) &= 0 \\
(-5)(0)(-6) &= 0 \\
(3)(-7)(-5) &= 105
\end{align*}
\]

Find the sum of each group.

\[
\begin{align*}
168 + 0 + 0 &= 168 \\
105 + 0 + 0 &= 105
\end{align*}
\]

Subtract the value of the second group from the first group.

\[ 168 - 105 = 63 \]

Therefore, the value of the determinant is 63.

**ANSWER:**
63

33. \[ \begin{vmatrix} -8 & -3 & -9 \\ 0 & 0 & 0 \\ 8 & -2 & -4 \end{vmatrix} \]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[ \begin{vmatrix} -8 & -3 & -8 & -3 \\ 0 & 0 & 0 & 0 \\ 8 & -2 & 8 & -2 \end{vmatrix} \]

Find the product of the element of the diagonal.

\[
\begin{align*}
(-8)(0)(-4) &= 0 \\
(-3)(0)(3) &= 0 \\
(-9)(0)(-2) &= 0 \\
(8)(0)(-9) &= 0 \\
(2)(0)(-8) &= 0 \\
(4)(0)(3) &= 0
\end{align*}
\]

Therefore, the value of the determinant is 0.

**ANSWER:**
0
### 3-7 Solving Systems of Equations Using Cramer's Rule

#### Example 34:

\[
\begin{vmatrix}
1 & 6 & 7 \\
-2 & -5 & -8 \\
4 & 4 & 9
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
1 & 6 & 1 & 6 \\
-2 & -5 & -2 & -5 \\
4 & 4 & 4 & 4
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
(1)(-5)(9) &= -45 \\
(6)(-8)(4) &= -192 \\
(7)(-2)(4) &= -56
\end{align*}
\]

Find the sum of each group.

\[
-45 - 192 - 26 = -293 \\
-140 - 32 - 108 = -280
\]

Subtract the value of the second group from the first group.

\[-293 - (-280) = -13\]

Therefore, the value of the determinant is \(-13\).

**ANSWER:**
\(-13\)

#### Example 35:

\[
\begin{vmatrix}
1 & -8 & -9 \\
6 & 5 & -6 \\
-2 & -8 & 10
\end{vmatrix}
\]

**SOLUTION:**
Rewrite the first two columns in the right of the determinant.

\[
\begin{vmatrix}
1 & -8 & 1 & -8 \\
6 & 5 & 6 & 5 \\
-2 & -8 & 10 & -2 & -8
\end{vmatrix}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
(1)(-5)(10) &= -50 \\
(-8)(-6)(-2) &= -96 \\
(-9)(6)(-8) &= 432 \\
(10)(6)(-8) &= -480
\end{align*}
\]

Find the sum of each group.

\[
50 - 96 + 432 = 386 \\
90 + 48 - 480 = -342
\]

Subtract the value of the second group from the first group.

\[386 - (-342) = 728\]

Therefore, the value of the determinant is 728.

**ANSWER:**
728
3-7 Solving Systems of Equations Using Cramer's Rule

Evaluate each determinant.

1. SOLUTION: 

\[
\begin{vmatrix}
5 & -5 & -5 \\
-8 & -3 & -2 \\
-2 & 4 & 6 \\
\end{vmatrix}
\]

ANSWER: 26

2. SOLUTION: 

\[
\begin{vmatrix}
-4 & 1 & -2 \\
10 & 12 & 9 \\
-6 & 0 & 13 \\
\end{vmatrix}
\]

ANSWER: -12

Mi-Ling ordered 6 jelly-filled doughnuts. 
ANSWER: 6

CCSS PERSEVERANCE

The salary for each of the employees in the company is $12,000. If a 40-hour week is worked, how much is the weekly salary of the employee?

Let \( x \) be the coefficient matrix of the system.

\[
\begin{align*}
-4x + 5y - z &= 6 \\
10x - 6y + 7z &= 12 \\
-6x + 9y - 13z &= 20
\end{align*}
\]

Find the sum of each group.

\[
\begin{align*}
-4 &+ 5 &- 1 &+ 6 = 12 \\
10 &- 6 &+ 7 &+ 12 = 26 \\
-6 &+ 9 &- 13 &+ 20 = 18
\end{align*}
\]

Find the product of the element of the diagonal.

\[
\begin{align*}
(-4)(5)(-1) &= -20 \\
(10)(-6)(7) &= -420 \\
(-6)(9)(-13) &= 756
\end{align*}
\]

Find the sum of the group.

\[
\begin{align*}
-20 &+ -420 &+ 756 = 136
\end{align*}
\]

Subtract the value of the second group from the first group.

\[
136 - 12 = 124
\]

Therefore, the value of the determinant is 124.

\[
\begin{align*}
\text{ANSWER:} &\quad 124
\end{align*}
\]
38. **TRAVEL** Mr. Smith’s art class took a bus trip to an art museum. The bus averaged 65 miles per hour on the highway and 25 miles per hour in the city. The art museum is 375 miles away from the school, and it took the class 7 hours to get there. Use Cramer’s Rule to find how many hours the bus was on the highway and how many hours it was driving in the city.

**SOLUTION:**
Let \(x\) and \(y\) be the number of hours driving in the highway and the city respectively. The system of equation represents the situation is:

\[
65x + 25y = 375 \\
x + y = 7
\]

Solve the above equation using Cramer’s Rule.

\[
C = \begin{vmatrix} 65 & 25 \\ 1 & 1 \end{vmatrix}
\]

\[
|C| = (65)(1) - (25)(1) = 65 - 25 = 40
\]

\[
x = \frac{375}{40} = \frac{225}{25} = 9 \\
y = \frac{73}{40} = \frac{365}{200} = 1.825
\]

The bus was driving 5 hours on the highway and 2 hours in the city.

**ANSWER:**
5 hours on the highway, 2 hours in the city.

39. **Use Cramer’s Rule to solve each system of equations.**

\[6x - 5y = 73 \\
-7x + 3y = -71\]

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{bmatrix} a & b \\ f & g \end{bmatrix}
\]

The solution of the system is

\[
x = \frac{m}{|C|} \quad \text{and} \quad y = \frac{n}{|C|} \quad \text{if} \quad C \neq 0.
\]

\[
C = \begin{vmatrix} 6 & -5 \\ -7 & 3 \end{vmatrix} = (6)(3) - (-5)(-7) = 18 - 35 = -17
\]

\[
x = \frac{-71}{-17} = 4 \quad \text{and} \quad y = \frac{365}{-17} = -21.5
\]

Therefore, the solution of the system is \((8, -5)\).

**ANSWER:**
\((8, -5)\)
3-7 Solving Systems of Equations Using Cramer's Rule

40. \(10a - 3b = -34\)
\(3a + 8b = -28\)

**SOLUTION:**

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{pmatrix}
a & b \\
f & g
\end{pmatrix}
\]

The solution of the system is

\[
x = \frac{m}{\det(C)} \quad \text{and} \quad y = \frac{n}{\det(C)} \quad \text{if} \quad \det(C) \neq 0.
\]

\[
\det(C) = (10)(8) - (-3)(3) = 80 + 9 = 89
\]

\[
\begin{bmatrix}
-34 & -3 \\
-28 & 8
\end{bmatrix}
\]

\[
x = \frac{-34 - 3}{89} = \frac{-37}{89} \quad \text{and} \quad y = \frac{10 - 34}{89} = \frac{-24}{89}
\]

\[
x = -\frac{37}{89} \quad \text{and} \quad y = -\frac{24}{89}
\]

\[
x = -4 \quad \text{and} \quad y = -2
\]

Therefore, the solution of the system is \((-4, -2)\).

**ANSWER:**

\((-4, -2)\)

41. \(-4c - 5d = -39\)
\(5c + 8d = 54\)

**SOLUTION:**

Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{pmatrix}
a & b \\
f & g
\end{pmatrix}
\]

The solution of the system is

\[
x = \frac{m}{\det(C)} \quad \text{and} \quad y = \frac{n}{\det(C)} \quad \text{if} \quad \det(C) \neq 0.
\]

\[
\det(C) = (-4)(8) - (-5)(5) = -32 + 25 = -7
\]

\[
\begin{bmatrix}
-39 & -5 \\
-42 & -7
\end{bmatrix}
\]

\[
c = \frac{-39 - 5}{-7} = \frac{-44}{-7} = 6 \quad \text{and} \quad d = \frac{-4 - 39}{-7} = \frac{-43}{-7} = 6
\]

Therefore, the solution of the system is \((6, 3)\).

**ANSWER:**

\((6, 3)\)
Evaluate each determinant.

1. SOLUTION:
   Use Cramer’s Rule.
   Let $C$ be the coefficient matrix of the system
   
   \[
   \begin{pmatrix}
   ax + by = m \\
   fx + gy = n
   \end{pmatrix} \rightarrow
   \begin{vmatrix}
   a & b \\
   f & g
   \end{vmatrix},
   \]
   The solution of the system is
   
   \[
   x = \frac{m}{|C|} \quad \text{and} \quad y = \frac{n}{|C|} \iff C \neq 0.
   \]
   
   Therefore, the solution of the system is $(5, -1)$.
   
   ANSWER:
   $(5, -1)$

42. $-6f - 8g = -22$
   $-11f + 5g = -60$

43. $9r + 4s = -55$
   $-5r - 3s = 36$
3-7 Solving Systems of Equations Using Cramer's Rule

44. \(-11v - 7v = 4\)
   \[9u + 4v = -24\]
   SOLUTION:
   Use Cramer’s Rule.
   Let \(C\) be the coefficient matrix of the system
   \[
   ax + by = m \\
   fx + gy = n \\
   \rightarrow \begin{vmatrix} a & b \\ f & g \end{vmatrix}
   \]
   The solution of the system is
   \[
   x = \frac{m}{|C|} \quad \text{and} \quad y = \frac{n}{|C|} \quad \text{if} \quad C \neq 0.
   \]
   \[
   C = \begin{vmatrix} -11 & -7 \\ 9 & 4 \end{vmatrix}
   \]
   \[
   |C| = (-11)(4) - (-7)(9) \\
   = -44 + 63 \\
   = 19
   \]
   \[
   u = \frac{4}{19} - \frac{-7}{19} \quad \text{and} \quad v = \frac{-11}{19} + \frac{4}{19}
   \]
   \[
   u = \frac{152}{19} \quad \text{and} \quad v = \frac{228}{19}
   \]
   \[
   u = -8 \quad \text{and} \quad v = 12
   \]
   Therefore, the solution of the system is \((-8, 12)\).
   ANSWER:
   \((-8, 12)\)

45. \[5x - 4y + 6z = 58\]
   \[-4x + 6y + 3z = -13\]
   \[6x + 3y + 7z = 53\]
   SOLUTION:
   Use Cramer’s Rule.
   Let \(C\) be the coefficient matrix of the system
   \[
   ax + by + cz = m \\
   fx + gy + hz = n \\
   jx + ky + lz = p \\
   \rightarrow \begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}
   \]
   The solution of the system is
   \[
   x = \frac{m}{|C|}, y = \frac{n}{|C|} \quad \text{and} \quad z = \frac{p}{|C|} \quad \text{if} \quad C \neq 0.
   \]
   \[
   C = \begin{vmatrix} 5 & -4 & 6 \\ -13 & 6 & 3 \\ 53 & 3 & 7 \end{vmatrix}
   \]
   \[
   |C| = -307
   \]
   \[
   x = \frac{58}{-307} - \frac{-4}{-307} + \frac{6}{-307}
   \]
   \[
   = \frac{-1228}{-307} = 4
   \]
   \[
   y = \frac{5}{-307} - \frac{-4}{-307} + \frac{3}{-307}
   \]
   \[
   = \frac{614}{-307} = -2
   \]
   \[
   z = \frac{5}{-307} - \frac{-4}{-307} + \frac{58}{-307}
   \]
   \[
   = \frac{-1535}{-307} = 5
   \]
   Therefore, the solution of the system is \((4, -2, 5)\).
   ANSWER:
   \((4, -2, 5)\)

46. \[8x - 4y + 7z = 34\]
   \[5x + 6y + 3z = -21\]
   \[3x + 7y - 8z = -85\]
   SOLUTION:
Evaluate each determinant.

1. SOLUTION: 
ANSWER: 26

2. SOLUTION: 
ANSWER: −12

3. Mi-Ling ordered 6 jelly-filled doughnuts. 
ANSWER: 6

4. CCSS PERSEVERANCE The salary for each of...
47. **DOUGHNUTS** Mi-Ling is ordering doughnuts for a class party. The box contains 2 dozen doughnuts, some of which are plain and some of which are jelly-filled. The plain doughnuts each cost $0.50, and the jelly-filled cost $0.60. If the total cost is $12.60, use Cramer’s Rule to find how many jelly-filled doughnuts Mi-Ling ordered.

**SOLUTION:**
Let $x$ and $y$ be the number of plain and jelly-filled doughnuts.
The system of equation that represents the situation is:

$$x + y = 24$$
$$0.5x + 0.6y = 12.6$$

Solve the above equation using Cramer’s Rule.

$$C = \begin{vmatrix} 1 & 1 \\ 0.5 & 0.6 \end{vmatrix}$$

$$|C| = (0.6)(1) - (0.5)(1)$$
$$= 0.6 - 0.5$$
$$= 0.1$$

$$y = \begin{vmatrix} 1 & 24 \\ 0.5 & 12.6 \end{vmatrix}$$

$$y = \frac{1(12.6) - 0.5(24)}{0.1}$$
$$y = \frac{12.6 - 12}{0.1}$$
$$y = 6$$

Mi-Ling ordered 6 jelly-filled doughnuts.

**ANSWER:**
6

48. **CCSS PERSEVERANCE** The salary for each of the stars of a new movie is $5 million, and the supporting actors each receive $1 million. The total amount spent for the salaries of the actors and actresses is $19 million. If the cast has 7 members, use Cramer’s Rule to find the number of stars in the movie.

**SOLUTION:**
Let $x$ and $y$ be the number of starts and the supporting actors in the movie respectively.
The system of equation that represents the situation is:

$$x + y = 7$$
$$5x + y = 19$$

Solve the above equation using Cramer’s Rule.

$$C = \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix}$$

$$|C| = (1)(1) - (5)(1)$$
$$= 1 - 5$$
$$= -4$$

$$x = \begin{vmatrix} 7 & 1 \\ 19 & 1 \end{vmatrix}$$

$$x = \frac{7(1) - 1(19)}{-4}$$
$$= \frac{7 - 19}{-4}$$
$$= 3$$

The number of stars in the movie is 3.

**ANSWER:**
3
3-7 Solving Systems of Equations Using Cramer's Rule

49. **ARCHAEOLOGY** Archaeologists found whale bones at coordinates (0, 3), (4, 7), and (5, 9). If the units of the coordinates are meters, find the area of the triangle formed by these finds.

**SOLUTION:**
The area of a triangle with vertices \((a, b), (c, d)\) and \((f, g)\) is \(|A|\), where
\[
A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ f & g & 1 \end{vmatrix}
\]
Substitute the coordinates (0, 3), (4, 7), and (5, 9) in the area formula.
\[
A = \frac{1}{2} \begin{vmatrix} 0 & 3 & 1 \\ 4 & 7 & 1 \\ 5 & 9 & 1 \end{vmatrix}
= \frac{1}{2} (4)
= 2
\]
Therefore, the area of the triangle 2 \(m^2\).

**ANSWER:**
2 \(m^2\)

**Use Cramer's Rule to solve each system of equations.**

50. \(6a - 7b = -55\)
\(2a + 4b - 3c = 35\)
\(-5a - 3b + 7c = -37\)

**SOLUTION:**
Use Cramer's Rule.

Let \(C\) be the coefficient matrix of the system
\[
\begin{align*}
ax + by + cz &= m \\
f\xi + gy + hz &= n \\
j\xi + ky + lz &= p
\end{align*}
\]

The solution of the system is
\[
\begin{align*}
x &= \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{|C|}, \\
y &= \frac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{|C|}, \\
z &= \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{|C|} \text{ if } C \neq 0.
\]

\[
C = \begin{vmatrix} 6 & -7 & 0 \\ 2 & 4 & -3 \\ -5 & -3 & 7 \end{vmatrix}
\]

\[
|C| = 107
\]
\[
a = \frac{\begin{vmatrix} -7 & 0 & 0 \\ 4 & -3 & 0 \\ -5 & -3 & 7 \end{vmatrix}}{107}
= \frac{749}{107}
= 7
\]
\[
b = \frac{\begin{vmatrix} 6 & -7 & -55 \\ 2 & 4 & 35 \\ -5 & -3 & -37 \end{vmatrix}}{107}
= \frac{-321}{107}
= -3
\]
Therefore, the solution of the system is \((-1, 7, -3)\).

**ANSWER:**
\((-1, 7, -3)\)

51. \(3a - 5b - 9c = 17\)
\(4a - 3c = 31\)
\(-5a - 4b - 2c = -42\)

**SOLUTION:**
Use Cramer's Rule.
3-7 Solving Systems of Equations Using Cramer's Rule

Let $C$ be the coefficient matrix of the system

\[
\begin{align*}
ax + by + cz &= m \\
fx + gy + hz &= n \\
ix + ky + lz &= p
\end{align*}
\]

The solution of the system is

\[
\begin{align*}
x &= \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{|C|}, \\
y &= \frac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{|C|}, \\
z &= \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{|C|}
\end{align*}
\]

if $C \neq 0$.

\[
C = \begin{vmatrix} 3 & -5 & -9 \\ 4 & 0 & -3 \\ -5 & -4 & -2 \end{vmatrix}
\]

\[
|C| = -7
\]

\[
a = \begin{vmatrix} 17 & -5 & -9 \\ 31 & 0 & -3 \\ -42 & -4 & -2 \end{vmatrix} = -28
\]

\[
a = \frac{-28}{-7} = 4
\]

\[
b = \begin{vmatrix} 3 & 17 & -9 \\ 4 & 31 & -3 \\ -5 & -4 & -2 \end{vmatrix} = -56
\]

\[
b = \frac{-56}{-7} = 8
\]

\[
c = \begin{vmatrix} 3 & -5 & 17 \\ 4 & 0 & 31 \\ -5 & -4 & -42 \end{vmatrix} = 35
\]

\[
c = \frac{35}{-7} = -5
\]

Therefore, the solution of the system is $(4, 8, -5)$.

ANSWER:

$(4, 8, -5)$
3-7 Solving Systems of Equations Using Cramer's Rule

\[ x = \frac{\begin{vmatrix} -2 & -5 & 0 \\ 47 & 0 & 3 \\ -63 & 8 & -5 \end{vmatrix}}{-271} = \frac{-2168}{-271} = 8 \]

\[ y = \frac{\begin{vmatrix} 4 & -2 & 0 \\ 7 & -47 & 3 \\ 0 & -63 & -5 \end{vmatrix}}{-271} = \frac{1626}{-271} = 6 \]

\[ z = \frac{\begin{vmatrix} 4 & -5 & -2 \\ 7 & 0 & -47 \\ 0 & 8 & -63 \end{vmatrix}}{-271} = \frac{-813}{-271} = 3 \]

Therefore, the solution of the system is \((-8, -6, 3)\).

**ANSWER:**
\((-8, -6, 3)\)

53. \(7x + 8y + 9z = -149\)
\(-6x + 7y - 5z = 54\)
\(4x + 5y - 2z = -44\)

**SOLUTION:**
Use Cramer’s Rule.

Let \(C\) be the coefficient matrix of the system

\[
\begin{vmatrix}
ax + by + cz = m \\
fx + gy + hz = n \\
x + ky + lz = p
\end{vmatrix}
\begin{vmatrix}
a & b & c \\
f & g & h \\
j & k & l
\end{vmatrix}
\]

The solution of the system is

\[
\begin{vmatrix}
a & b & c \\
f & g & h \\
j & k & l
\end{vmatrix}
\]

\[
\begin{vmatrix}
a & b & m \\
f & g & n \\
j & k & p
\end{vmatrix}
\]

\[ if \ C \neq 0. \]

\[
\begin{vmatrix}
7 & 8 & 9 \\
-6 & 7 & -5 \\
4 & 5 & -2
\end{vmatrix} = 701
\]

\[
\begin{vmatrix}
7 & -149 & 9 \\
-6 & 54 & -5 \\
4 & -44 & -2
\end{vmatrix} = 701
\]

\[
\begin{vmatrix}
7 & 8 & -149 \\
-6 & 7 & 54 \\
4 & 5 & -44
\end{vmatrix} = 701
\]

Therefore, the solution of the system is

\[
\left( \frac{6187}{701}, \frac{-2904}{701}, \frac{-4212}{701} \right)
\]

**ANSWER:**

\[
\left( \frac{6187}{701}, \frac{-2904}{701}, \frac{-4212}{701} \right)
\]
3-7 Solving Systems of Equations Using Cramer's Rule

54. **GARDENING** Rob wants to build a triangular flower garden. To plan out his garden he uses a coordinate grid where each of the squares represents one square foot. The coordinates for the vertices of his garden are (−1, 7), (2, 6), and (4, −3). Find the area of the garden.

**SOLUTION:**
The area of a triangle with vertices \((a, b), (c, d)\) and \((f, g)\) is \(\left| A \right|\), where

\[
A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ f & g & 1 \end{vmatrix}
\]

Substitute the coordinates (−1, 7), (2, 6), and (4, −3) in the area formula.

\[
A = \frac{1}{2} \begin{vmatrix} -1 & 7 & 1 \\ 2 & 6 & 1 \\ 4 & -3 & 1 \end{vmatrix}
\]

\[
= \frac{1}{2}(-25)
\]

\[
= -12.5
\]

\[
\left| A \right| = 12.5
\]

Therefore, the area of the garden is 12.5 ft\(^2\).

**ANSWER:**
12.5 ft\(^2\)

55. **FINANCIAL LITERACY** A vendor sells small drinks for $1.15, medium drinks for $1.75, and large drinks for $2.25. During a week in which he sold twice as many small drinks as medium drinks, his total sales were $2,238.75 for 1385 drinks.

**a.** Use Cramer’s Rule to determine how many of each drink were sold.

**b.** The vendor decided to increase the price for small drinks to $1.25 the next week. The next week, he sold 140 fewer small drinks, 125 more medium drinks, and 35 more large drinks. Calculate his sales for that week.

**c.** Was raising the price of the small drink a good business move for the vendor? Explain your reasoning.

**SOLUTION:**

**a.** Let \(x, y\) and \(z\) be the number of small, medium and large drinks.
The system of equation that represents the situation is:

\[
1.15x + 1.75y + 2.25z = 2238.75
\]
\[
x + y + z = 1385
\]
\[
x = 2y
\]

Solve the above equation using Cramer’s Rule.

\[
C = \begin{vmatrix} 1.15 & 1.75 & 2.25 \\ 1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}
\]

\[
\left| C \right| = -2.7
\]

\[
x = \frac{2238.75 \times 1.75 \times 2.25 - 1385 \times 1 \times 2.25 - 1385 \times 1 \times 1.75}{-2.7}
\]

\[
= -1755
\]

\[
= -2.7
\]

\[
= 650
\]

\[
y = \frac{1.15 \times 2238.75 \times 2.25 - 1 \times 1385 \times 2.25 - 1 \times 1385 \times 1.75}{-2.7}
\]

\[
= -877.5
\]

\[
= -2.7
\]

\[
= 325
\]

\[
z = \frac{1.15 \times 1 \times 1385 + 1 \times 1 \times 2.25 - 1 \times 1385 \times 1.75}{-2.7}
\]

\[
= -1107
\]

\[
= -2.7
\]

\[
= 410
\]

The vendor sells 650 small drinks, 325 medium drinks and 410 large drinks.

**b.** After increasing the price, the vendor sells (650 – 140) 510 small drinks, (325+125) 450 medium drinks and (410 + 35) 445 large drinks.
Total sales for that week is \( 510 \times 1.25 + 450 \times 1.75 + 445 \times 2.25 = 2426.25 \)

c. It seems like it was a good move for the vendor. Although he sold fewer small drinks, he sold more medium and large drinks and on the whole, made more money this week than in the previous week.

**ANSWER:**

a. small: 650; medium: 325; large: 410
b. \( \$2,426.25 \)
c. It seems like it was a good move for the vendor. Although he sold fewer small drinks, he sold more medium and large drinks and on the whole, made more money this week than in the previous week.

66. **REASONING** Some systems of equations cannot be solved by using Cramer’s Rule.

a. Find the value of \( \begin{vmatrix} a & b \\ f & g \end{vmatrix} \). When is the value 0?

b. Choose values for \( a, b, f, \) and \( g \) to make the determinant of the coefficient matrix 0. What type of system is formed?

**SOLUTION:**

a. \( \begin{vmatrix} a & b \\ f & g \end{vmatrix} = ag - bf \)

If \( ag - bf = 0 \), then \( ag = bf \).

b. Sample answer: \( x + 3y = 8 \) and \( 2x + 6y = 12 \); The system is dependent or inconsistent depending on the values of \( m \) and \( n \).

**ANSWER:**

a. \( \begin{vmatrix} a & b \\ f & g \end{vmatrix} = ag - bf \); If \( ag - bf = 0 \), then \( ag = bf \).

b. Sample answer: \( x + 3y = 8 \) and \( 2x + 6y = 12 \); The system is dependent or inconsistent depending on the values of \( m \) and \( n \).

57. **REASONING** What can you determine about the solution of a system of linear equations if the determinant of the coefficients is 0?

**SOLUTION:**

Sample answer: There is no unique solution of the system. There are either infinite or no solutions.

**ANSWER:**

Sample answer: There is no unique solution of the system. There are either infinite or no solutions.

58. **CCSS CRITIQUE** James and Amber are finding the value of \( \begin{vmatrix} 8 & 3 \\ -5 & 2 \end{vmatrix} \).

**James**

\[
\begin{vmatrix} 8 & 3 \\ -5 & 2 \end{vmatrix} = 16 - (-15) = 16 + 15 = 31
\]

**Amber**

\[
\begin{vmatrix} 8 & 3 \\ -5 & 2 \end{vmatrix} = 16 - 15 = 1
\]

**SOLUTION:**

James; to find the determinant of \( \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \)

James solved this correctly because \( 3(-5) = -15. \)

**ANSWER:**

James; because \( 3(-5) = -15 \)
59. **CHALLENGE** Find the determinant of a $3 \times 3$ matrix defined by

$$a_{nn} = \begin{cases} 0 & \text{if } m+n \text{ is even} \\ m+n & \text{if } m+n \text{ is odd} \end{cases}$$

**SOLUTION:**

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 5 & 0 & 7 \\ 0 & 9 & 0 \end{bmatrix}$$

$$\text{det}(A) = \begin{vmatrix} 0 & 3 & 0 \\ 5 & 0 & 7 \\ 0 & 9 & 0 \end{vmatrix}$$

$$= 0$$

**ANSWER:**

0

60. **OPEN ENDED** Write a $2 \times 2$ matrix with each of the following characteristics.

a. The determinant equals 0.

b. The determinant equals 25.

c. The elements are all negative numbers and the determinant equals $-32$

**SOLUTION:**

a. Sample answer:

$$\begin{bmatrix} 6 & 7 \\ 6 & 7 \end{bmatrix}$$

b. Sample answer:

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$$

c. Sample answer:

$$\begin{bmatrix} -4 & -6 \\ -8 & -4 \end{bmatrix}$$

**ANSWER:**

a. Sample answer:

$$\begin{bmatrix} 6 & 7 \\ 6 & 7 \end{bmatrix}$$

b. Sample answer:

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$$

c. Sample answer:

$$\begin{bmatrix} -4 & -6 \\ -8 & -4 \end{bmatrix}$$

61. **WRITING IN MATH** Describe the possible graphical representations of a $2 \times 2$ system of linear equations if the determinant of the matrix of coefficients is 0.

**SOLUTION:**

Sample answer: Given a $2 \times 2$ system of linear equations, if the determinant of the matrix of coefficients is 0, then the system does not have a unique solution. The system may have no solution and the graphical representation shows two parallel lines. The system may have infinitely many solutions in which the graphical representation will be the same line.

**ANSWER:**

Sample answer: Given a $2 \times 2$ system of linear equations, if the determinant of the matrix of coefficients is 0, then the system does not have a unique solution. The system may have no solution and the graphical representation shows two parallel lines. The system may have infinitely many solutions in which the graphical representation will be the same line.
62. Tyler paid $25.25 to play three games of miniature golf and two rides on go-karts. Brent paid $25.75 for four games of miniature golf and one ride on the go-karts. How much does one game of miniature golf cost?

A $4.25  
B $4.75  
C $5.25  
D $5.75

**SOLUTION:**

Let $x$ and $y$ be the cost for miniature golf and go-karts.

The system of equation that represents this situation is:

\[ 3x + 2y = 25.25 \]
\[ 4x + y = 25.75 \]

Solve the above equation using Cramer’s Rule.

\[
\begin{vmatrix}
3 & 2 \\
4 & 1 \\
\end{vmatrix}
\]

\[
| C | = (3)(1) - (2)(4) \\
= 3 - 8 \\
= -5 \\
\]

\[
x = \frac{2}{-5} \\
y = \frac{25.25}{-5} \\
x = 5.25 \\
\]

Therefore, option C is the correct answer.

**ANSWER:**

C

63. Use the table to determine the expression that best represents the number of faces of any prism having a base with $n$ sides.

<table>
<thead>
<tr>
<th>Base</th>
<th>Sides of Base</th>
<th>Faces of Prisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>quadrilateral</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>pentagon</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>hexagon</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>heptagon</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>octagon</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

F $2(n - 1)$  
G $2(n + 1)$  
H $n + 2$  
J $2n$

**SOLUTION:**

The number of faces of any prism is 2 more than the number of sides. Therefore, option H is the correct answer.

**ANSWER:**

H

64. SHORT RESPONSE A right circular cone has radius 4 inches and height 6 inches. What is the lateral area of the cone? (lateral area of cone = $\pi r \ell$, where $\ell$ = slant height)

[Diagram of a right circular cone]

**SOLUTION:**

\[
\ell = \sqrt{4^2 + 6^2} \\
= \sqrt{52} \\
= \sqrt{4 \cdot 13} \\
= 2\sqrt{13} \\
\]

The slant height of the cone is $2\sqrt{13}$ inches.

Lateral area = $\pi (4)\left(2\sqrt{13}\right) = 8\sqrt{13}\pi$ in$^2$.

**ANSWER:**

$8\sqrt{13}\pi$ in$^2$
65. SAT/ACT Find the area of \( \triangle ABC \).

\[
\begin{array}{ccc}
A & B & C \\
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}
\]

A 10 units \(^2\)
B 12 units \(^2\)
C 13 units \(^2\)
D 14 units \(^2\)
E 16 units \(^2\)

**SOLUTION:**
The coordinates of the vertices of the triangle are \((2, 3), (1, -3)\) and \((-3, 1)\).
The area of a triangle with vertices \((a, b), (c, d)\) and \((f, g)\) is \(|A|\), where \(A = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ c & d & 1 \\ f & g & 1 \end{vmatrix}\).

\[
\det(A) = \frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ 1 & -3 & 1 \\ -3 & 1 & 1 \end{vmatrix}
= \frac{1}{2} (-28)
= -14
|A| = 14
\]

The area of the garden is 14 ft \(^2\).

Therefore, option D is the correct answer.

**ANSWER:**
D

---

67. Determine whether each matrix product is defined. If so, state the dimensions of the product.

66. \(A_{4 \times 2} \cdot B_{2 \times 6}\)

**SOLUTION:**
The inner dimensions are equal, so the product is defined.
Its dimensions are \(4 \times 6\).

**ANSWER:**
yes; \(4 \times 6\)

67. \(C_{5 \times 4} \cdot D_{5 \times 3}\)

**SOLUTION:**
The inner dimensions are not equal, so the product is not defined.

**ANSWER:**
no

68. \(E_{2 \times 7} \cdot F_{7 \times 1}\)

**SOLUTION:**
The inner dimensions are equal, so the product is defined.
Its dimensions are \(2 \times 1\).

**ANSWER:**
yes; \(2 \times 1\)

69. BUSINESS The table lists the prices at the Sandwich Shoppe.

<table>
<thead>
<tr>
<th>Sandwich</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>ham</td>
<td>$4.50</td>
<td>$6.75</td>
<td>$9.50</td>
</tr>
<tr>
<td>salami</td>
<td>$4.50</td>
<td>$6.75</td>
<td>$9.50</td>
</tr>
<tr>
<td>veggie</td>
<td>$4.00</td>
<td>$6.25</td>
<td>$8.75</td>
</tr>
<tr>
<td>meatball</td>
<td>$4.75</td>
<td>$7.50</td>
<td>$10.25</td>
</tr>
</tbody>
</table>

a. List the prices in a \(4 \times 3\) matrix.
b. The manager decides to cut the prices of every item by 20%. List this new set of data in a \(4 \times 3\) matrix.
c. Subtract the second matrix from the first and determine the savings to the customer for each sandwich.
3-7 Solving Systems of Equations Using Cramer's Rule

**SOLUTION:**

a. Write the prices in a $4 \times 3$ matrix.

\[
\begin{bmatrix}
$4.50 & $6.75 & $9.50 \\
$4.50 & $6.75 & $9.50 \\
$4.00 & $6.25 & $8.75 \\
$4.75 & $7.50 & $10.25 \\
\end{bmatrix}
\]

b. Write a new matrix providing the prices after a 20% discount.

\[
\begin{bmatrix}
$4.50 & $6.75 & $9.50 \\
$4.50 & $6.75 & $9.50 \\
$4.00 & $6.25 & $8.75 \\
$4.75 & $7.50 & $10.25 \\
\end{bmatrix}
\]

c. Subtract the second matrix from the first.

\[
\begin{bmatrix}
$4.50 & $6.75 & $9.50 \\
$4.50 & $6.75 & $9.50 \\
$4.00 & $6.25 & $8.75 \\
$4.75 & $7.50 & $10.25 \\
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
$4.50 & $6.75 & $9.50 \\
$4.50 & $6.75 & $9.50 \\
$4.00 & $6.25 & $8.75 \\
$4.75 & $7.50 & $10.25 \\
\end{bmatrix}
\]

**Graph each function.**

70. $f(x) = 2|x - 3| - 4$

**SOLUTION:**

\[f(x) = 2|x - 3| - 4\]

**ANSWER:**

71. $f(x) = -3|2x| + 4$

**SOLUTION:**

\[f(x) = -3|2x| + 4\]

**ANSWER:**
3-7 Solving Systems of Equations Using Cramer's Rule

\\[ f(x) = |3x - 1| + 2 \]

**SOLUTION:**

![Graph of f(x) = |3x - 1| + 2](image)

**ANSWER:**

![Graph of f(x) = |3x - 1| + 2](image)

---

**Solve each system of equations.**

72.  
\[ \begin{align*} 
2x - 5y &= -26 \\
5x + 3y &= -34 
\end{align*} \]

**SOLUTION:**

Multiply the first equation by 3 and the second equation by 5 and then add.

\[ \begin{align*} 
(1)\times 3 & \quad 6x - 15y = -78 \\
(2)\times 5 & \quad 25x + 15y = -170 \\
& \quad 31x = -248 \\
& \quad x = -8 
\end{align*} \]

Substitute \(-8\) for \(x\) in the first equation and solve for \(y\).

\[ \begin{align*} 
2(-8) - 5y &= -26 \\
-16 - 5y &= -26 \\
5y &= 10 \\
y &= 2 
\end{align*} \]

The solution is \((-8, 2)\).

**ANSWER:**

\((-8, 2)\)
3-7 Solving Systems of Equations Using Cramer's Rule

74. \(4y + 6x = 10\)
\(2x - 7y = 22\)

**SOLUTION:**
Multiply the second equation by \(-3\) and with the first equation.

\[
\begin{align*}
(2) \times -3 & \quad -6x + 21y = -66 \\
(1) & \quad 6x + 4y = 10 \\
\end{align*}
\]

\[
\begin{align*}
25y &= -56 \\
y &= \frac{-56}{25}
\end{align*}
\]

Substitute \(-\frac{56}{25}\) for \(y\) in the second equation and solve for \(x\).

\[
\begin{align*}
2x - 7\left(-\frac{56}{25}\right) &= 22 \\
2x + \frac{392}{25} &= 22 \\
2x &= 22 - \frac{392}{25} \\
2x &= \frac{158}{25} \\
x &= \frac{79}{25}
\end{align*}
\]

The solution is \(\left(\frac{79}{25}, -\frac{56}{25}\right)\).

**ANSWER:**
\(\left(\frac{79}{25}, -\frac{56}{25}\right)\)

75. \(-3x - 2y = 17\)
\(-4x + 5y = -8\)

**SOLUTION:**
Multiply the first equation by \(5\) and the second equation by \(2\) and then add.

\[
\begin{align*}
(1) \times 5 & \quad -15x - 10y = 85 \\
(2) \times 2 & \quad -8x + 10y = -16 \quad -23x = 69 \\
\quad & \quad x = -3 \\
\end{align*}
\]

Substitute \(-3\) for \(x\) in the second equation and solve for \(y\).

\[
\begin{align*}
-4(-3) + 5y &= -8 \\
12 + 5y &= -8 \\
5y &= -20 \\
y &= -4
\end{align*}
\]

The solution is \((-3, -4)\).

**ANSWER:**
\((-3, -4)\)
3-8 Solving Systems of Equations Using Inverse Matrices

Determine whether the matrices in each pair are inverses.

1. \( A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \)

**SOLUTION:**
If \( A \) and \( B \) are inverses, then \( A \cdot B = B \cdot A = I \).

\[
A \cdot B = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+1 \\ -1+0 & -2+0 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -1 & -2 \end{bmatrix}
\]

Since \( A \cdot B \neq I \), they are not inverses.

**ANSWER:**
no

2. \( C = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix} \)

**SOLUTION:**
If \( C \) and \( D \) are inverses, then \( C \cdot D = D \cdot C = I \).

\[
C \cdot D = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 4+5 & 2-3 \\ 10+15 & 5-9 \end{bmatrix} = \begin{bmatrix} 9 & -1 \\ 25 & -4 \end{bmatrix}
\]

Since \( C \cdot D \neq I \), they are not inverses.

**ANSWER:**
no

3. \( F = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \)

**SOLUTION:**
If \( F \) and \( G \) are inverses, then \( F \cdot G = G \cdot F = I \).

\[
F \cdot G = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1+0 & 1-1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
G \cdot F = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1+0 & -1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Since \( F \cdot G = G \cdot F = I \), \( F \) and \( G \) are inverses.

**ANSWER:**
yes

4. \( H = \begin{bmatrix} -3 & -1 \\ -4 & -2 \end{bmatrix} \)

**SOLUTION:**
If \( H \) and \( I \) are inverses, then \( H \cdot I = I \cdot H = I \).

\[
H \cdot I = \begin{bmatrix} -3 & -1 \\ -4 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 3-3 & -6+4 \\ 4-4 & -8+8 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}
\]

Since \( H \cdot I \neq I \), they are not inverses.

**ANSWER:**
no
3-8 Solving Systems of Equations Using Inverse Matrices

Find the inverse of each matrix, if it exists.

5. \[
\begin{bmatrix}
6 & -3 \\
-1 & 0
\end{bmatrix}
\]

SOLUTION:
\[
\begin{bmatrix}
6 & -3 \\
-1 & 0
\end{bmatrix} = 0 \cdot 3 \\
= -3
\]

Since the determinant does not equal 0, the inverse exists.

Inverse \( = \frac{1}{\text{ad} - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

Substitute \( a = 6, b = -3, c = -1, \) and \( d = 0 \).

Inverse \( = \frac{1}{0 - 3} \begin{bmatrix} 0 & 3 \\ -3 & 1 \end{bmatrix} \)

\( = \begin{bmatrix} 1 & 0 \\ -3 & 6 \end{bmatrix} \)

\( = \begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix} \)

ANSWER:
\[
\begin{bmatrix}
0 & -1 \\
\frac{1}{3} & -2
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
2 & -4 \\
-3 & 0
\end{bmatrix}
\]

SOLUTION:
\[
\begin{bmatrix}
2 & -4 \\
-3 & 0
\end{bmatrix} = 0 - 12 \\
= -12
\]

Since the determinant does not equal 0, the inverse exists.

Inverse \( = \frac{1}{\text{ad} - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

Substitute \( a = 2, b = -4, c = -3, \) and \( d = 0 \).

Inverse \( = \frac{1}{0 - 12} \begin{bmatrix} 0 & 4 \\ 3 & 2 \end{bmatrix} \)

\( = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix} \)

\( = \begin{bmatrix} 0 & 1 \\ -3 & -6 \end{bmatrix} \)

ANSWER:
\[
\begin{bmatrix}
0 & -1 \\
\frac{1}{3} & 1 \\
\frac{1}{4} & -6
\end{bmatrix}
\]
7. \[
\begin{bmatrix}
-3 & 0 \\
5 & 2
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-3 & 0 \\
5 & 2
\end{bmatrix} = -6 - 0 \\
= -6
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \( a = -3, b = 0, c = 5, \) and \( d = 2 \).

\[
\begin{bmatrix} 1 & 0 \\ -6 & 5 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix} 1/3 \\ 5/6 \end{bmatrix}
\]

8. \[
\begin{bmatrix}
2 & 4 \\
1 & 2
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} = 4 - 4 \\
= 0
\]

Since the determinant is equal to 0, the inverse does not exist.

**ANSWER:**
does not exist

**Use a matrix equation to solve each system of equations.**

9. \[-2x + y = 9 \\
x + y = 3\]

**SOLUTION:**

The matrix equation is
\[
\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}
\]

Find the inverse of the coefficient matrix.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \( a = -2, b = 1, c = 1, \) and \( d = 1 \).

\[
\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}
\]

\[
\begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.

\[
\begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}
\]

The solution is \((-2, 5)\).

**ANSWER:**
\((-2, 5)\)
Determine whether the matrices in each pair are inverses.

1. SOLUTION:

If A and B are inverses, then .

Find the inverse of the coefficient matrix.

Substitute \( a = 4, b = -2, c = 6, \) and \( d = 9. \)

Multiply each side of the matrix equation by the inverse matrix.

The solution is (4, –3).

ANSWER: (4, –3)

10. \( 4x - 2y = 22 \)

\( 6x + 9y = -3 \)

SOLUTION:

The matrix equation is \[ \begin{bmatrix} 4 & -2 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ -3 \end{bmatrix}. \]

Find the inverse of the coefficient matrix.

Substitute \( a = 4, b = -2, c = 6, \) and \( d = 9. \)

Multiply each side of the matrix equation by the inverse matrix.

The solution is (1, –2).

ANSWER: (1, –2)

11. \( -2x + y = -4 \)

\( 3x + y = 1 \)

SOLUTION:

The matrix equation is \[ \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}. \]

Find the inverse of the coefficient matrix.

Substitute \( a = -2, b = 1, c = 3, \) and \( d = 1. \)

Multiply each side of the matrix equation by the inverse matrix.

The solution is (1, –2).

ANSWER: (1, –2)
12. **MONEY** Kevin had 25 quarters and dimes. The total value of all the coins was $4. How many quarters and dimes did Kevin have?

**SOLUTION:**
Let \( x \) and \( y \) be the number of quarters and dimes respectively.
\[
\begin{align*}
  x + y &= 25 \\
  0.25x + 0.10y &= 4
\end{align*}
\]
The matrix equation is
\[
\begin{bmatrix}
  1 & 1 \\
  0.25 & 0.10
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
= 
\begin{bmatrix}
  25 \\
  4
\end{bmatrix}.
\]
Find the inverse of the coefficient matrix.
\[
\text{Inverse } = \frac{1}{ad - bc}
\begin{bmatrix}
  d & -b \\
  -c & a
\end{bmatrix}
\]
Substitute \( a = 1, b = 1, c = 0.25, \) and \( d = 0.10.\)
\[
\begin{align*}
\text{Inverse } &= \frac{1}{0.1 - 0.25}
\begin{bmatrix}
  1 & 1 \\
  0.25 & 0.1
\end{bmatrix} \\
&= \frac{1}{-0.15}
\begin{bmatrix}
  1 & -1 \\
  -0.25 & 1
\end{bmatrix} \\
&= 
\begin{bmatrix}
  2 & 20 \\
  3 & 3 \\
  5 & 20 \\
  3 & 3
\end{bmatrix}
\]
Multiply each side of the matrix equation by the inverse matrix.
\[
\begin{bmatrix}
  2 & 20 \\
  3 & 3 \\
  5 & 20 \\
  3 & 3
\end{bmatrix}
\begin{bmatrix}
  1 & 1 \\
  0.25 & 0.1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
= 
\begin{bmatrix}
  2 & 20 \\
  3 & 3 \\
  5 & 20 \\
  3 & 3
\end{bmatrix}
\begin{bmatrix}
  25 \\
  4
\end{bmatrix}.
\]
Kevin have 10 quarters and 15 dimes.

**ANSWER:**
10 quarters and 15 dimes

---

3-8 Solving Systems of Equations Using Inverse Matrices

Determine whether each pair of matrices are inverses of each other.

13. \( K = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \)

**SOLUTION:**
If \( K \) and \( L \) are inverses, then \( K \cdot L = L \cdot K = I. \)
\[
K \cdot L = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}
= \begin{bmatrix} 1+4 & 1-2 \\ 0+0 & 3+0 \end{bmatrix}
= \begin{bmatrix} 5 & -1 \\ 0 & 3 \end{bmatrix}
\]
Since \( K \cdot L \neq I, \) they are not inverses.

**ANSWER:**
no

14. \( M = \begin{bmatrix} 0 & 2 \\ 4 & 5 \end{bmatrix}, N = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \)

**SOLUTION:**
If \( M \) and \( N \) are inverses, then \( M \cdot N = N \cdot M = I. \)
\[
M \cdot N = \begin{bmatrix} 0 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}
= \begin{bmatrix} 0+0 & 0+0 \\ 4+0 & 4+0 \end{bmatrix}
= \begin{bmatrix} 0 & 0 \\ 4 & 4 \end{bmatrix}
\]
Since \( M \cdot N \neq I, \) they are not inverses.

**ANSWER:**
no
3-8 Solving Systems of Equations Using Inverse Matrices

15. \( P = \begin{bmatrix} 4 & 0 \\ 3 & 0 \end{bmatrix}, Q = \begin{bmatrix} -1 & -1 \\ 2 & 3 \end{bmatrix} \)

**SOLUTION:**

If \( P \) and \( Q \) are inverses, then \( P \cdot Q = Q \cdot P = I \).

\[
P \cdot Q = \begin{bmatrix} 4 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -4 + 0 & -4 + 0 \\ -3 + 0 & -3 + 0 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -3 & -3 \end{bmatrix}
\]

Since \( P \cdot Q \neq I \), they are not inverses.

**ANSWER:**

no

16. \( R = \begin{bmatrix} 1 & -1 \\ 4 & 1 \\ 0 & 2 \end{bmatrix}, S = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \)

**SOLUTION:**

If \( R \) and \( S \) are inverses, then \( R \cdot S = S \cdot R = I \).

\[
R \cdot S = \begin{bmatrix} 1 & -1 \\ 4 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}
\]

Since \( R \cdot S \neq I \), they are not inverses.

**ANSWER:**

no

Find the inverse of each matrix, if it exists.

17. \( \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \)

**SOLUTION:**

\[
\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = 6 \cdot 0
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \( a = 3, b = 0, c = 0, \) and \( d = 2 \).

\[
\text{Inverse} = \frac{1}{6} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]
3-8 Solving Systems of Equations Using Inverse Matrices

18. \[
\begin{bmatrix}
2 & 3 \\ 3 & 2 \\
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
2 & 3 \\ 3 & 2 \\
\end{bmatrix} = 4 - 9 \\
= -5
\]

Since the determinant does not equal 0, the inverse exists.

Inverse \(= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

Substitute \(a = 2, b = 3, c = 3, \) and \(d = 2.\)

Inverse \(= \frac{1}{4 - 9} \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \)

\(= \frac{1}{4 - 9} \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \)

\(= \begin{bmatrix} 2 & 3 \\ -5 & -3 \\ 2 & 5 \end{bmatrix} \)

\(= \begin{bmatrix} 3 & 5 \\ 2 & 5 \\ 5 & 5 \end{bmatrix} \)

**ANSWER:**
\[
\begin{bmatrix}
2 & 3 \\ 5 & 5 \\
3 & 2 \\ 5 & 5 \\
\end{bmatrix}
\]

19. \[
\begin{bmatrix}
3 & 0 \\ 5 & 1 \\
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
3 & 0 \\ 5 & 1 \\
\end{bmatrix} = 3 - 0 \\
= 3
\]

Since the determinant does not equal 0, the inverse exists.

Inverse \(= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

Substitute \(a = 3, b = 0, c = 5, \) and \(d = 1.\)

Inverse \(= \frac{1}{3 - 0} \begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix} \)

\(= \frac{1}{3 - 0} \begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix} \)

\(= \begin{bmatrix} 1 & 0 \\ 3 & 5 \\ 5 & 1 \end{bmatrix} \)

**ANSWER:**
\[
\begin{bmatrix}
1 & 0 \\ 3 & 5 \\
5 & 1 \\
\end{bmatrix}
\]
3-8 Solving Systems of Equations Using Inverse Matrices

20. \[
\begin{bmatrix}
1 & -1 \\
-6 & -1
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
1 & -1 \\
-6 & -1
\end{bmatrix} = -1 - 6 = -7
\]

Since the determinant does not equal 0, the inverse exists.

Inverse = \[\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\]

Substitute \(a = 1, b = -1, c = -6,\) and \(d = -1.\)

\[
\text{Inverse} = \frac{1}{-1 - 6}\begin{bmatrix} -1 & 1 \\ 6 & 1 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -1 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 6 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 \\
6 & 1
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
1 & 1 \\
6 & 1 \\
7 & 7
\end{bmatrix}
\]

21. \[
\begin{bmatrix}
-5 & -4 \\
4 & 2
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-5 & -4 \\
4 & 2
\end{bmatrix} = -10 + 16 = 6
\]

Since the determinant does not equal 0, the inverse exists.

Inverse = \[\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\]

Substitute \(a = -5, b = -4, c = 4,\) and \(d = 2.\)

\[
\text{Inverse} = \frac{1}{-10 + 16}\begin{bmatrix} -5 & -4 \\ 4 & 2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -5 & -4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\
2 & 5 \\
3 & 6
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
1 & 2 \\
3 & 3 \\
2 & 5 \\
3 & 6
\end{bmatrix}
\]
Determine whether the matrices in each pair are inverses.

22. \[
\begin{bmatrix}
-5 & 9 \\
4 & -8
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
-5 & 9 \\
4 & -8
\end{bmatrix} = 40 - 36 = 4
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \(a = -5, b = 9, c = 4,\) and \(d = -8\).

\[
\text{Inverse} = \frac{1}{40 - 36} \begin{bmatrix} -8 & -9 \\ 4 & -5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -8 & -9 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} -2 & -9/4 \\ 1 & -5/4 \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
-2 & -9/4 \\
1 & -5/4
\end{bmatrix}
\]

23. \[
\begin{bmatrix}
6 & -5 \\
4 & 9
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
6 & -5 \\
4 & 9
\end{bmatrix} = 54 + 20 = 74
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \(a = 6, b = -5, c = 4,\) and \(d = 9\).

\[
\text{Inverse} = \frac{1}{54 + 20} \begin{bmatrix} 9 & 5 \\ 4 & 6 \end{bmatrix} = \frac{1}{74} \begin{bmatrix} 9 & 5 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} \frac{9}{74} & \frac{5}{74} \\ \frac{2}{37} & \frac{3}{37} \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
\frac{9}{74} & \frac{5}{74} \\
\frac{2}{37} & \frac{3}{37}
\end{bmatrix}
\]
Determine whether the matrices in each pair are inverses.

24. \[ \begin{bmatrix} -4 & -2 \\ 7 & 8 \end{bmatrix} \]

**SOLUTION:**
\[
\begin{aligned}
\begin{bmatrix} -4 & -2 \\ 7 & 8 \end{bmatrix} &= -32 + 14 \\
&= -32 + 14 \\
&= -18 
\end{aligned}
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \( a = -4, b = -2, c = 7, \) and \( d = 8. \)

\[
\begin{aligned}
\text{Inverse} &= \frac{1}{-32 + 14} \begin{bmatrix} 8 & 2 \\ -7 & -4 \end{bmatrix} \\
&= \frac{1}{18} \begin{bmatrix} 8 & 2 \\ -7 & -4 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 1 \\ -9 & -9 \end{bmatrix}
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} 4 & 1 \\ -9 & -9 \end{bmatrix}
\]

25. \[ \begin{bmatrix} -6 & 8 \\ 8 & -7 \end{bmatrix} \]

**SOLUTION:**
\[
\begin{aligned}
\begin{bmatrix} -6 & 8 \\ 8 & -7 \end{bmatrix} &= 42 - 64 \\
&= 22
\end{aligned}
\]

Since the determinant does not equal 0, the inverse exists.

\[
\text{Inverse} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

Substitute \( a = -6, b = 8, c = 8, \) and \( d = -7. \)

\[
\begin{aligned}
\text{Inverse} &= \frac{1}{42 - 64} \begin{bmatrix} -7 & 8 \\ 8 & -6 \end{bmatrix} \\
&= \frac{1}{-22} \begin{bmatrix} -7 & 8 \\ 8 & -6 \end{bmatrix} \\
&= \begin{bmatrix} 7 & 4 \\ 22 & 11 \end{bmatrix}
\end{aligned}
\]

**ANSWER:**
\[
\begin{bmatrix} 7 & 4 \\ 22 & 11 \end{bmatrix}
\]

26. **BAKING** Peggy is preparing a colored frosting for a cake. For the right shade of purple, she needs 25 milliliters of a 44% concentration food coloring. The store has a 25% red and a 50% blue concentration of food coloring. How many milliliters each of blue food coloring and red food coloring should be mixed to make the necessary amount of purple food coloring?

**SOLUTION:**
Let \( x \) and \( y \) be the amount of blue food coloring and red food coloring respectively.

\[
x + y = 25 \\
0.5x + 0.25y = (0.44)25
\]

Simplify the second equation.
3-8 Solving Systems of Equations Using Inverse Matrices

0.5x + 0.25y = 11

The matrix equation is

\[
\begin{bmatrix}
1 & 1 \\
0.5 & 0.25
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
25 \\
11
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.

\[
\text{Inverse} = \frac{1}{ad-bc} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Substitute \(a=1, b=1, c=0.5,\) and \(d=0.25.\)

\[
\text{Inverse} = \frac{1}{0.25-0.5} \begin{bmatrix}
0.25 & -1 \\
-0.5 & 1
\end{bmatrix}
= \begin{bmatrix}
-1 \\
-0.25
\end{bmatrix} = \begin{bmatrix}
4 \\
-4
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.

\[
\begin{bmatrix}
-1 & 4 \\
2 & -4
\end{bmatrix}
\begin{bmatrix}
1 & 1 \\
0.5 & 0.25
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
-1 & 4 \\
2 & -4
\end{bmatrix}
\begin{bmatrix}
25 \\
11
\end{bmatrix}
= \begin{bmatrix}
19 \\
6
\end{bmatrix}
\]

She has to mix 6 milliliters of the red food coloring and 19 milliliters of the blue food coloring to get 44% concentration of purple food coloring.

**ANSWER:**

6 mL of the red food coloring and 19 mL of the blue food coloring
3-8 Solving Systems of Equations Using Inverse Matrices

28. \(-x + y = 3\)
\[-2x + y = 6\]

**SOLUTION:**
The matrix equation is
\[
\begin{bmatrix}
-1 & 1 \\
-2 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
6
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.

\[
\text{Inverse} = \frac{1}{ad - bc}
\begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Substitute \(a = -1, \ b = 1, \ c = -2, \) and \(d = 1.\)

\[
\text{Inverse} = \frac{1}{-1 + 2}
\begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.

\[
\begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
1 & -1 \\
2 & -1
\end{bmatrix}
\begin{bmatrix}
3 \\
6
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
3 \\
6
\end{bmatrix}
\]

The solution is \((-3, 0).\)

**ANSWER:**
\((-3, 0)\)

29. \(x + y = 4\)
\(-4x + y = 9\)

**SOLUTION:**
The matrix equation is
\[
\begin{bmatrix}
1 & 1 \\
-4 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
4 \\
9
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.

\[
\text{Inverse} = \frac{1}{ad - bc}
\begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

Substitute \(a = 1, \ b = 1, \ c = -4, \) and \(d = 1.\)

\[
\text{Inverse} = \frac{1}{1 + 4}
\begin{bmatrix}
1 & -1 \\
4 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & -1 \\
4 & 1
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.

\[
\begin{bmatrix}
1 & -1 \\
4 & 1
\end{bmatrix}
\begin{bmatrix}
1 & -1 \\
4 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
1 & -1 \\
4 & 1
\end{bmatrix}
\begin{bmatrix}
4 \\
9
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
4 \\
9
\end{bmatrix}
\]

The solution is \((-1, 5).\)

**ANSWER:**
\((-1, 5)\)

30. \(3x + y = 3\)
\(5x + 3y = 6\)

**SOLUTION:**
The matrix equation is
\[
\begin{bmatrix}
3 & 1 \\
5 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
6
\end{bmatrix}.
\]
3-8 Solving Systems of Equations Using Inverse Matrices

\[
\begin{bmatrix}
3 & 1 \\
5 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
6
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.

Substitute \( a = 3, b = 1, c = 5, \) and \( d = 3 \).

\[
\text{Inverse } = \frac{1}{ad - bc}
\begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

\[
= \frac{1}{9 - 5}
\begin{bmatrix}
3 & -1 \\
-5 & 3
\end{bmatrix}
\]

\[
= \frac{1}{4}
\begin{bmatrix}
3 & -1 \\
-5 & 3
\end{bmatrix}
\]

\[
= \frac{1}{4}
\begin{bmatrix}
3 & 1 \\
5 & 3
\end{bmatrix}
\]

\[
= \frac{1}{4}
\begin{bmatrix}
3 & 1 \\
5 & 3
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.

\[
\begin{bmatrix}
3 & 1 \\
4 & 4 \\
5 & 3 \\
4 & 4
\end{bmatrix}
\begin{bmatrix}
3 & 1 \\
4 & 4 \\
5 & 3 \\
4 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
3 & 1 \\
4 & 4 \\
5 & 3 \\
4 & 4
\end{bmatrix}
\begin{bmatrix}
3 \\
6
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
4
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
\frac{3}{4} \\
\frac{3}{4}
\end{bmatrix}
\]

The solution is \( \left( \frac{3}{4}, \frac{3}{4} \right) \).

\text{ANSWER:} \left( \frac{3}{4}, \frac{3}{4} \right)

31. \( y - x = 5 \)
\( 2y - 2x = 8 \)

\text{SOLUTION:}
Rewrite the given system as below.

\[
\begin{bmatrix}
-1 & 1 \\
-2 & 2
\end{bmatrix}
= -2 + 2
\]

\[
= 0
\]

Since the determinant is equal to 0, the inverse does not exist.
Therefore, the system has no solution.

\text{ANSWER:}
no solution
3-8 Solving Systems of Equations Using Inverse Matrices

32. \(4x + 2y = 6\)
\(6x - 3y = 9\)

**SOLUTION:**
The matrix equation is
\[
\begin{bmatrix} 4 & 2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}.
\]
Find the inverse of the coefficient matrix.
Inverse = \(\frac{1}{12 - 12} \begin{bmatrix} -3 & 2 \\ 6 & 4 \end{bmatrix}\)
= \(\frac{1}{-24} \begin{bmatrix} -3 & 2 \\ 6 & 4 \end{bmatrix}\)
= \(\frac{1}{8} \begin{bmatrix} 1 & 1 \\ -1 & 6 \end{bmatrix}\)

Multiply each side of the matrix equation by the inverse matrix.
\[
\begin{bmatrix} \frac{1}{8} & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 6 \\ 9 \end{bmatrix}
\]
\[
\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}
\]
\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}
\]
The solution is \((1.5, 0)\).

**ANSWER:**
\((1.5, 0)\)

33. \(1.6y - 0.2x = 1\)
\(0.4y - 0.1x = 0.5\)

**SOLUTION:**
Rewrite the given system as below.
\[-0.2x + 1.6y = 1\]
\[-0.1x + 0.4y = 0.5\]
The matrix equation is
\[
\begin{bmatrix} -0.2 & 1.6 \\ -0.1 & 0.4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}.
\]
Find the inverse of the coefficient matrix.
Inverse = \(\frac{1}{0.08 + 0.16} \begin{bmatrix} 0.4 & -1.6 \\ -0.1 & 0.4 \end{bmatrix}\)
= \(\frac{1}{0.08} \begin{bmatrix} 0.4 & -1.6 \\ -0.1 & 0.4 \end{bmatrix}\)
= \(\begin{bmatrix} 5 & -20 \\ 1.25 & -2.5 \end{bmatrix}\)

Multiply each side of the matrix equation by the inverse matrix.
\[
\begin{bmatrix} 5 & -20 \\ 1.25 & -2.5 \end{bmatrix} \begin{bmatrix} -0.2 & 1.6 \\ -0.1 & 0.4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -20 \\ 1.25 & -2.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}
\]
\[
\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \end{bmatrix}
\]
\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \end{bmatrix}
\]
The solution is \((-5, 0)\).

**ANSWER:**
\((-5, 0)\)
3-8 Solving Systems of Equations Using Inverse Matrices

34. \( 4y - x = -2 \)
\( 3y - x = 6 \)

**SOLUTION:**
Rewrite the given system as below.
\(-x + 4y = -2 \)
\(-x + 3y = 6 \)

The matrix equation is
\[
\begin{bmatrix}
-1 & 4 \\
-1 & 3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
-2 \\
6 \\
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.
Inverse\(=\frac{1}{ad-bc}
\begin{bmatrix}
d & -b \\
-c & a \\
\end{bmatrix}
\]
Substitute \(a=-1, b=4, c=-1,\) and \(d=3.\)
Inverse
\[
=\frac{1}{-1(3) - 4(-1)}
=\begin{bmatrix}
3 & -4 \\
1 & -1 \\
\end{bmatrix}
=\begin{bmatrix}
3 & -4 \\
1 & -1 \\
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.
\[
\begin{bmatrix}
3 & -4 \\
1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
-1 & 4 \\
-1 & 3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
3 & -4 \\
1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
-2 \\
6 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
-30 \\
-8 \\
\end{bmatrix}
\]

The solution is \((-30, -8).\)

**ANSWER:**
\((-30, -8)\)

35. \(2y - 4x = 3\)
\(4x - 3y = -6\)

**SOLUTION:**
Rewrite the given system as below.
\[-4x + 2y = 3 \]
\[4x - 3y = -6 \]

The matrix equation is
\[
\begin{bmatrix}
-4 & 2 \\
4 & -3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
3 \\
-6 \\
\end{bmatrix}.
\]

Find the inverse of the coefficient matrix.
Inverse\(=\frac{1}{ad-bc}
\begin{bmatrix}
d & -b \\
-c & a \\
\end{bmatrix}
\]
Substitute \(a=-4, b=2, c=4,\) and \(d=-3.\)
Inverse
\[
=\frac{1}{(-4)(4) - 2(-3)}
=\begin{bmatrix}
3 & 1 \\
-4 & 2 \\
\end{bmatrix}
=\begin{bmatrix}
3 & 1 \\
-4 & 2 \\
\end{bmatrix}
\]

Multiply each side of the matrix equation by the inverse matrix.
\[
\begin{bmatrix}
3 & 1 \\
-4 & 2 \\
\end{bmatrix}
\begin{bmatrix}
-4 & 2 \\
4 & -3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
3 \\
-6 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix}
=
\begin{bmatrix}
3 \\
3 \\
\end{bmatrix}
\]

The solution is \(\left(\frac{3}{4}, 3\right).\)

**ANSWER:**
\(\left(\frac{3}{4}, 3\right)\)

36. **POPULATIONS** The diagram shows the annual percent migration between a city and its suburbs.
3-8 Solving Systems of Equations Using Inverse Matrices

a. Write a matrix to represent the transitions in city population and suburb population.
b. There are currently 16,275 people living in the city and 17,552 people living in the suburbs. Assuming that the trends continue, predict the number of people who will live in the suburbs next year.
c. Use the inverse of the matrix from part b to find the number of people who lived in the city last year.

**SOLUTION:**
a. 

\[
\begin{array}{ccc}
\text{From} & \text{city} & \text{suburbs} \\
\text{To city} & 0.95 & 0.03 \\
\text{Suburbs} & 0.05 & 0.97
\end{array}
\]

b. Multiply the matrices \[
\begin{bmatrix}
0.95 & 0.03 \\
0.05 & 0.97
\end{bmatrix}
\begin{bmatrix}
16275 \\
17552
\end{bmatrix}
\]
and \[
\begin{bmatrix}
16275 \\
17552
\end{bmatrix}
\begin{bmatrix}
0.95 & 0.03 \\
0.05 & 0.97
\end{bmatrix}
\]

Therefore, there will be about 17,839 people live in the suburbs next year.

c. Find the inverse of the matrix \[
\begin{bmatrix}
0.95 & 0.03 \\
0.05 & 0.97
\end{bmatrix}
\]

Let \( A = \begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix} \).

\[
A^{-1} = \frac{1}{0.92} \begin{bmatrix} 0.97 & -0.03 \\ -0.05 & 0.95 \end{bmatrix}
\]

Multiply the matrices \[
\frac{1}{0.92} \begin{bmatrix} 0.97 & -0.03 \\ -0.05 & 0.95 \end{bmatrix}
\begin{bmatrix} 16275 \\
17552
\end{bmatrix}
\]

So, about 16,587 people lived in the city last year.

**ANSWER:**

- a. To city \( 0.95 \ 0.03 \)
- Suburbs \( 0.05 \ 0.97 \)
- b. About 17,839
- c. About 16,587

37. **MUSIC** The diagram shows the trends in digital audio player and portable CD player ownership over the past five years for Central City. Every person in Central City has either a digital audio player or a portable CD player. Central City has a stable population of 25,000 people, of whom 17,252 own digital audio players and 7748 own portable CD players.

a. Write a matrix to represent the transitions in player ownership.
b. Assume that the trends continue. Predict the number of people who will own digital audio players next year.
c. Use the inverse of the matrix from part b to find the number of people who owned digital audio players last year.

**SOLUTION:**
a. 

\[
\begin{array}{ccc}
\text{From} & \text{CD} & \text{MP3} \\
\text{To CD} & 0.35 & 0.12 \\
\text{MP3} & 0.65 & 0.88
\end{array}
\]

b. Multiply the matrices \[
\begin{bmatrix}
0.35 & 0.12 \\
0.65 & 0.88
\end{bmatrix}
\begin{bmatrix}
7748 \\
17252
\end{bmatrix}
\]

- a. Write a matrix to represent the transitions in player ownership.
- b. Assume that the trends continue. Predict the number of people who will own digital audio players next year.
- c. Use the inverse of the matrix from part b to find the number of people who owned digital audio players last year.

**SOLUTION:**
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\[
\begin{bmatrix}
0.35 & 0.12 \\
0.65 & 0.88
\end{bmatrix}
\begin{bmatrix}
7748 \\
17252
\end{bmatrix}
= \begin{bmatrix}
0.35(7748) + 0.12(17252) \\
0.65(7748) + 0.88(17252)
\end{bmatrix}
= \begin{bmatrix}
4782.04 \\
20217.96
\end{bmatrix}
\]

So, about 20,218 people will own digital audio players next year.

c. Find the inverse of the matrix \( \begin{bmatrix} 0.35 & 0.12 \\ 0.65 & 0.88 \end{bmatrix} \).

Let \( A = \begin{bmatrix} 0.35 & 0.12 \\ 0.65 & 0.88 \end{bmatrix} \).

\[
A^{-1} = \frac{1}{0.23} \begin{bmatrix} 0.88 & -0.12 \\ -0.65 & 0.35 \end{bmatrix}
\]

Multiply the matrices \( \frac{1}{0.23} \begin{bmatrix} 0.88 & -0.12 \\ -0.65 & 0.35 \end{bmatrix} \) and \( \begin{bmatrix} 7748 \\ 17252 \end{bmatrix} \).

\[
\frac{1}{0.23} \begin{bmatrix} 0.88 & -0.12 \\ -0.65 & 0.35 \end{bmatrix} \begin{bmatrix} 7748 \\ 17252 \end{bmatrix}
= \frac{1}{0.23} \begin{bmatrix} 0.88(7748) - 0.12(17252) \\ -0.65(7748) + 0.35(17252) \end{bmatrix}
= \begin{bmatrix} 4782.04 \\ 20217.96 \end{bmatrix}
\]

So, about 4357 people owned digital audio players last year.

\textbf{ANSWER:}

\begin{tabular}{l|l|l}
   & CD & MP3 \\
\hline
To CD & 0.35 & 0.12 \\
MP3   & 0.65 & 0.88 \\
\end{tabular}

38. \textbf{CCSS CRITIQUE} Cody and Megan are setting up matrix equations for the system \(5x + 7y = 19\) and \(3y + 4x = 10\). Is either of them correct? Explain your reasoning.

\[
\begin{bmatrix}
5 & 7 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix} 19 \\
10 \end{bmatrix}
\]

\textbf{SOLUTION:}

In the coefficient matrix, the first column are the coefficients of the \(x\)-terms while the second column are the coefficients of the \(y\)-terms. Megan is correct; Cody put 3 for \(x\) in the second equation instead of 4.

\textbf{ANSWER:}

Megan: Cody put 3 for \(x\) in the second equation instead of 4.

39. \textbf{CHALLENGE} Describe what a matrix equation with infinite solutions looks like.

\textbf{SOLUTION:}

The system would have to consist of two equations that are the same or one equation that is a multiple of the other.

\textbf{ANSWER:}

The system would have to consist of two equations that are the same or one equation that is a multiple of the other.
3-8 Solving Systems of Equations Using Inverse Matrices

40. REASONING Determine whether the following statement is always, sometimes, or never true. Explain your reasoning.

A square matrix has a multiplicative inverse.

SOLUTION:
Sometimes; Sample answer: A square matrix has a multiplicative inverse if its determinant does not equal 0.

ANSWER:
Sometimes; Sample answer: A square matrix has a multiplicative inverse if its determinant does not equal 0.

41. OPEN ENDED Write a matrix equation that does not have a solution.

SOLUTION:
Sample answer: \[
\begin{bmatrix}
2 & 3 \\
4 & 6
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
9 \\
10
\end{bmatrix};
\]
n any matrix that has a determinant equal to 0, such as \[
\begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix}.
\]

ANSWER:
Sample answer: \[
\begin{bmatrix}
2 & 3 \\
4 & 6
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
9 \\
10
\end{bmatrix};
\]
n any matrix that has a determinant equal to 0, such as \[
\begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix}.
\]

42. WRITING IN MATH When would you prefer to solve a system of equations using algebraic methods, and when would you prefer to use matrices? Explain.

SOLUTION:
Sample answer: Some systems in two variables can be easier to solve by using algebraic methods such as combination or elimination. More complex systems can be easier to solve by using matrices.

ANSWER:
Sample answer: Some systems in two variables can be easier to solve by using algebraic methods such as combination or elimination. More complex systems can be easier to solve by using matrices.

43. The Yogurt Shoppe sells cones in three sizes: small, $0.89; medium, $1.19; and large, $1.39. One day Santos sold 52 cones. He sold seven more medium cones than small cones. If he sold $58.98 in cones, how many medium cones did he sell?

A 11
B 17
C 24
D 36

SOLUTION:
Let \( x \), \( y \), and \( z \) be the number of small size, medium size, and large size cones respectively.

\[
\begin{align*}
x + y + z &= 52 \quad (1) \\
y &= x + 7 \quad (2) \\
0.89x + 1.19y + 1.39z &= 58.98 \quad (3)
\end{align*}
\]

Substitute \( y = x + 7 \) in the first equation.

\[
x + x + 7 + z = 52 \\
2x + z = 45 \quad (4)
\]

Substitute \( y = x + 7 \) in the third equation.

\[
0.89x + 1.19(x + 7) + 1.39z = 58.98 \\
0.89x + 1.19x + 8.33 + 1.39z = 58.98 \\
2.08x + 1.39z = 50.65 \quad (5)
\]

Multiply the fourth equation by 1.39 and subtract it from 5.

\[
2.08x + 1.39z = 50.65 \\
2.78x + 1.39z = 62.55 \quad (\_\_\_) \\
\_\_\_0.7x \quad = -11.9
\]

Divide each side by \(-0.70\).

\[
x = 17
\]

Substitute \( x = 19 \) in (2).

\[
y = 17 + 7 \\
= 24
\]

The number of medium cones is 24. So, the correct option is C.

ANSWER:
C
Determine whether the matrices in each pair are inverses.

1. SOLUTION: If A and B are inverses, then.

56. SOLUTION: The graph is a horizontal line. So, it represents a constant function. ANSWER: constant.

54. SHORT RESPONSE What is the solution of the system of equations 6a + 8b = 5 and 10a − 12b = 2?

SOLUTION: Rewrite the given system as below.

6a + 8b = 5
5a − 6b = 1

The matrix equation is

\[ \begin{bmatrix} 6 & 8 \\ 5 & -6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \]

Find the inverse of the coefficient matrix.

Inverse = \( \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \)

Substitute \( a = 6, b = 8, c = 5, \) and \( d = -6 \).
46. SAT/ACT Each year at Capital High School the students vote to choose the theme of the homecoming dance. The theme “A Night Under the Stars” received 225 votes, and “The Time of My Life” received 480 votes. If 40% of girls voted for “A Night Under the Stars” and 75% of boys voted for “The Time of My Life,” how many girls and boys voted?

A 176 boys and 351 girls
B 395 boys and 310 girls
C 380 boys and 325 girls
D 705 boys and 325 girls
E 854 boys and 176 girls

**SOLUTION:**
Let \( x \) be the number of girls who voted and let \( y \) be the number of boys who voted. Since 40% of girls voted for “A Night Under the Stars”, 60% of girls voted for “The Time of My Life”. Similarly, since 75% of boys voted for “The Time of My Life”, remaining 25% voted for “A Night Under the Stars”. Write a matrix equation to represent the situation.

\[
\begin{bmatrix}
0.4 & 0.25 \\
0.6 & 0.75
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
225 \\
480
\end{bmatrix}
\]

Solve the matrix equation.

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\frac{1}{0.15}
\begin{bmatrix}
0.75 & -0.25 \\
-0.6 & 0.4
\end{bmatrix}
\begin{bmatrix}
225 \\
480
\end{bmatrix}
= 
\frac{1}{0.15}
\begin{bmatrix}
75(225) - 25(480) \\
-6(225) + 0.4(480)
\end{bmatrix}
= 
\frac{1}{0.15}
\begin{bmatrix}
48.75 \\
57
\end{bmatrix}
\]

So, there were 325 girls and 380 boys who voted. The correct choice is C.

**ANSWER:**
C
3-8 Solving Systems of Equations Using Inverse Matrices

49. \[
\begin{bmatrix}
8 & 6 & -1 \\
-4 & 5 & 1 \\
-3 & -2 & 9
\end{bmatrix}
\]

**SOLUTION:**

Rewrite the first two columns to the right of the determinant.

\[
\begin{bmatrix}
8 & 6 & -1 & 8 & 6 \\
-4 & 5 & 1 & -4 & 5 \\
-3 & -2 & 9 & -3 & -2
\end{bmatrix}
\]

Find the products of the elements of the diagonals.

\[
\begin{align*}
8(5)(9) &= 360 \\
6(1)(-3) &= -18 \\
(-1)(-4)(-2) &= -8 \\
(9)(-4)(6) &= -216
\end{align*}
\]

Find the sum of each group.

\[
360 - 18 - 8 = 334 \\
-216 + 15 - 16 = -\]

Subtract the sum of the second group from the sum of the first group.

\[
= 334 - (-217)
\]

The value of the determinant is 551.

**ANSWER:**

551

Find each product, if possible.

50. \[
\begin{bmatrix}
4 & 2 \\
-1 & -3
\end{bmatrix}
\begin{bmatrix}
6 & 2 \\
5 & 1
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
4 & 2 \\
-1 & -3
\end{bmatrix}
\begin{bmatrix}
6 & 2 \\
5 & 1
\end{bmatrix}
= \begin{bmatrix}
24 + 10 & 8 + 2 \\
-6 - 15 & -2 - 3
\end{bmatrix}
= \begin{bmatrix}
34 & 10 \\
-21 & -5
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
34 & 10 \\
-21 & -5
\end{bmatrix}
\]

51. \[
\begin{bmatrix}
8 & -2 \\
-4 & -5
\end{bmatrix}
\begin{bmatrix}
-2 \\
3
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
8 & -2 \\
-4 & -5
\end{bmatrix}
= \begin{bmatrix}
-16 & -6 \\
-8 & -15
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
-22 \\
-7
\end{bmatrix}
\]

52. \[
\begin{bmatrix}
-3 & -6 & -8 \\
-4 & -4 & 5
\end{bmatrix}
\]

**SOLUTION:**

It is impossible to find the product of these matrices.

**ANSWER:**

impossible
53. **MILK** The Yoder Family Dairy produces at most 200 gallons of skim and whole milk each day for delivery to large bakeries and restaurants. Regular customers require at least 15 gallons of skim and 21 gallons of whole milk each day. If the profit on a gallon of skim milk is $0.82 and the profit on a gallon of whole milk is $0.75, how many gallons of each type of milk should the dairy produce each day to maximize profits?

**SOLUTION:**
Let \( x \) be the number of gallons of skim milk. Let \( y \) be the number of gallons of whole milk.

\[
\begin{align*}
  x + y &\leq 200 \\
  x &\geq 15 \\
  y &\geq 21
\end{align*}
\]

The optimize function is \( f(x, y) = 0.82x + 0.75y \).

Graph the inequalities in the same coordinate plane.

The vertices of the feasible region are (15, 185), (15, 21), and (179, 21).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y) = 0.82x + 0.75y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15, 21)</td>
<td>28.05</td>
</tr>
<tr>
<td>(15, 185)</td>
<td>151.05</td>
</tr>
<tr>
<td>(179, 21)</td>
<td>587.78</td>
</tr>
</tbody>
</table>

To maximize the profit, the dairy has to produce 179 gallons of skim milk and 21 gallons of whole milk.

**ANSWER:**
179 gal of skim and 21 gal of whole milk

54. **Identify the type of function represented by each graph.**

**SOLUTION:**
The graph represents a quadratic function.

**ANSWER:**
quadratic

55. **Identify the type of function represented by each graph.**

**SOLUTION:**
The graph is in a “V” shape. So, it represents an absolute value function.

**ANSWER:**
absolute value

56. **Identify the type of function represented by each graph.**

**SOLUTION:**
The graph is a horizontal line. So, it represents a constant function.

**ANSWER:**
constant
Solve each system of equations by using either substitution or elimination.

1. 

\[ y = x + 4 \]
\[ x + y = -12 \]

**SOLUTION:**
Substitute \( x + 4 \) for \( y \) in the second equation and solve for \( x \).

\[
\begin{align*}
  x + x + 4 &= -12 \\
  2x + 4 &= -12 \\
   2x &= -16 \\
     x &= -8
\end{align*}
\]

Substitute \(-8\) for \( x \) in the first equation and solve for \( y \).

\[
\begin{align*}
  y &= -8 + 4 \\
     &= -4
\end{align*}
\]

Therefore, the solution is \((-8, -4)\).

**ANSWER:**
\((-8, -4)\)

2. 

\[ 3x + 5y = -7 \]
\[ 6x - 4y = 0 \]

**SOLUTION:**
Multiply the first equation by \(-2\) and add with the second equation.

\[
\begin{align*}
  -6x - 10y &= 14 \\
   6x - 4y &= 0 \\
 -14y &= 14 \\
   y &= -1
\end{align*}
\]

Substitute \(-1\) for \( y \) in the second equation and solve for \( x \).

\[
\begin{align*}
  6x - 4(-1) &= 0 \\
   6x &= 4 \\
    x &= \frac{2}{3}
\end{align*}
\]

Therefore, the solution is \(\left(-\frac{2}{3}, -1\right)\).

**ANSWER:**
\(\left(-\frac{2}{3}, -1\right)\)
3. \[5x + 2y = 4\]
\[3y - 4x = -40\]

**SOLUTION:**
Multiply the first and the second equation by 4 and 5 then add.

\[
\begin{align*}
20x + 8y &= 16 \\
-20x + 15y &= -200 \\
23y &= -184 \\
y &= -8 \\
\end{align*}
\]

Substitute \(-8\) for \(y\) in the second equation and solve for \(x\).

\[
\begin{align*}
5x + 2(-8) &= 4 \\
5x - 16 &= 4 \\
5x &= 20 \\
x &= 4 \\
\end{align*}
\]

Therefore, the solution is \((4, -8)\).

**ANSWER:**
\((4, -8)\)

4. \[8x - 3y = -13\]
\[-3x + 5y = 1\]

**SOLUTION:**
Multiply the first and the second equation by 5 and 3 respectively then add.

\[
\begin{align*}
40x - 15y &= -65 \\
-9y + 15y &= 3 \\
31x &= -62 \\
x &= -2 \\
\end{align*}
\]

Substitute \(-2\) for \(x\) in the second equation and solve for \(y\).

\[
\begin{align*}
-3(-2) + 5y &= 1 \\
6 + 5y &= 1 \\
5y &= -5 \\
y &= -1 \\
\end{align*}
\]

Therefore, the solution is \((-2, -1)\).

**ANSWER:**
\((-2, -1)\)

5. **MULTIPLE CHOICE** Which graph shows the solution of the system of inequalities?

\[y \leq 2x + 3\]
\[y < \frac{1}{3}x + 5\]

**Answer:**
A

[Graph A]

B

[Graph B]

C

[Graph C]
Practice Test - Chapter 3

Solve each system of equations by using either substitution or elimination.

1. SOLUTION:

Substitute \( x + 4 \) for ... system is if .

Therefore, the solution of the system is \((4, 2, –1)\).

ANSWER: \((4, 2, −1)\)

---

Solve each system of inequalities by graphing.

6. \( x + y > 6 \)
   \( x - y < 0 \)

SOLUTION:

\[
\begin{array}{c}
\text{D} \\
\end{array}
\]

ANSWER:

\[
\begin{array}{c}
\text{B} \\
\end{array}
\]

7. \( y \geq 2x - 5 \)
   \( y \leq x + 4 \)

SOLUTION:

\[
\begin{array}{c}
\text{B} \\
\end{array}
\]

ANSWER:

8. \[3x + 4y \leq 12\]
\[6x - 3y \geq 18\]

**SOLUTION:**

Graph the system of inequalities and find the vertices of the feasible region.

**ANSWER:**

9. \[5y + 2x \leq 20\]
\[4x + 3y > 12\]

**SOLUTION:**

Graph the system of inequalities and find the vertices of the feasible region.

**ANSWER:**

10. **SALONS** Sierra King is a nail technician. She allots 20 minutes for a manicure and 45 minutes for a pedicure in her 7-hour word day. No more than 5 pedicures can be scheduled each day. The prices are $18 for a manicure and $45 for a pedicure. How many manicures and pedicures should Ms. King schedule to maximize her daily income? What is her maximum daily income?

**SOLUTION:**

Let \(m\) represent the number of manicures and \(p\) represent the number of pedicures.
Write the system of constraints for the scenario. Since she cannot do a negative procedure, both variables must be greater than or equal to 0.
\[0 \leq p \leq 5\]
\[m \geq 0\]
\[20m + 45p \leq 420\]

Graph the system of inequalities and find the vertices of the feasible region.

The vertices of the feasible region are (0, 0), (0, 5), (9.75, 5), and (21, 0).

Since the price for each manicure is $18 and the price for each pedicure is $45, the function that represents her daily income is \(I(m, p) = 185m + 45p\).
Sierra can only do a whole number of manicures and pedicures so substitute (0, 0), (0, 5), (9, 5), and (18, 0) in the income function to determine the maximum.

\[
\begin{array}{c|c|c}
(m, p) & 18m + 45p & I(m, p) \\
\hline
(0, 0) & 18(0) + 45(0) & 0 \\
(0, 5) & 18(0) + 45(5) & 225 \\
(9, 5) & 18(9) + 45(5) & 387 \\
(21, 0) & 18(21) + 45(0) & 378 \\
\end{array}
\]

The maximum income will be produced when she schedules 9 manicures and 5 pedicures.
This will produce a maximum income of $387.

**ANSWER:**
9 manicures and 5 pedicures; $387

11. **COLLEGE FOOTBALL** In a recent year, Darren McFadden of Arkansas placed second overall in the Heisman Trophy voting. Players are given 3 points for every first-place vote, 2 points for every second-place vote, and 1 point for every third-place vote. McFadden received 490 total votes for first, second, and third place, for a total of 878 points. If he had 4 more than twice as many second-place votes as third-place votes, how many votes did he receive for each place?

**SOLUTION:**
Let \( x \), \( y \) and \( z \) be the number of first, second and third place vote. The system of equations represent this situation is:

\[
\begin{align*}
  x + y + z &= 490 \quad \rightarrow (1) \\
  3x + 2y + z &= 878 \quad \rightarrow (2) \\
  y &= 2z + 4 \quad \rightarrow (3)
\end{align*}
\]

Substitute \( 2y + 4 \) for \( z \) in the first and second equation and simplify.

\[
\begin{align*}
  x + 2z + 4 + z &= 490 \\
  x + 3z &= 486 \quad \rightarrow (4) \\
  3x + 2(2z + 4) + z &= 878 \\
  3x + 4z + 8 + z &= 878 \\
  3x + 5z &= 870 \quad \rightarrow (5)
\end{align*}
\]

Multiply the fourth equation by \(-3\) and add with the fifth equation.

\[
\begin{align*}
  (4) \times -3 &-3x - 9z = -1458 \\
  (5) &3x + 5z = 870 \\
  -4z &= -588 \\
  z &= 147
\end{align*}
\]

Substitute 147 for \( z \) in the fourth equation and solve for \( x \).

\[
\begin{align*}
  x + 3(147) &= 486 \\
  x + 441 &= 486 \\
  x &= 45
\end{align*}
\]

Substitute 147 for \( z \) in the third equation and solve for \( y \).

\[
\begin{align*}
  y &= 2(147) + 4 \\
  &= 294 + 4 \\
  &= 298
\end{align*}
\]

Therefore, he received 45 first-place, 298 second-place and 147 third-place votes.

**ANSWER:**
45 first, 298 second, 147 third
Perform the indicated operations. If the matrix does not exist, write impossible.

12. \[
\begin{bmatrix}
-3 & 0 + 4 & 3 \\
-1 & -3 & 0
\end{bmatrix}
\]

SOLUTION:
Distribute the scalar.

\[
= \begin{bmatrix}
-12a - 8 \\
0 + 12 \\
9 - 4
\end{bmatrix}
= \begin{bmatrix}
-12a - 8 \\
12 \\
5
\end{bmatrix}
\]

ANSWER:
\[
\begin{bmatrix}
-12a - 8 \\
12 \\
5
\end{bmatrix}
\]

13. \[
\begin{bmatrix}
-3 & 0 & 2 & 4 \\
1 & 5 & -6 & 0
\end{bmatrix}
\]

SOLUTION:
\[
= \begin{bmatrix}
-6 & -12 \\
-28 & 4
\end{bmatrix}
\]

ANSWER:
\[
\begin{bmatrix}
-6 & -12 \\
-28 & 4
\end{bmatrix}
\]
16. **MULTIPLE CHOICE** What is the value of
\[
\begin{bmatrix}
2 & 3 & -1 \\
0 & 2 & 4 \\
-2 & 5 & 6
\end{bmatrix}
\]?

- **F** -44
- **G** 44
- **H** \(\frac{1}{44}\)
- **J** \(-\frac{1}{44}\)

**SOLUTION:**
\[
\begin{bmatrix}
2 & 3 & -1 \\
0 & 2 & 4 \\
-2 & 5 & 6
\end{bmatrix} = 2(12 - 20) - 3(0 + 8) - 1(0 + 4)
\]
\[
= 2(-8) - 3(8) - 1(4)
\]
\[
= -16 - 24 - 4
\]
\[
= -44
\]
Therefore, option F is the correct answer.

**ANSWER:** F

---

Find the inverse of each matrix, if it exists.

17. \[
\begin{bmatrix}
5 & 0 \\
0 & 1
\end{bmatrix}
\]

**SOLUTION:**
Let \(A = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \).
\[
det(A) = 5
\]
\[
A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}
\]
\[
= \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix}
\]

**ANSWER:**

18. \[
\begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\]

**SOLUTION:**
Let \(A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \).
\[
det(A) = -3
\]
\[
A^{-1} = \frac{1}{-3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}
\]
\[
= \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}
\]

**ANSWER:**
Practice Test - Chapter 3

19. \[
\begin{bmatrix}
6 & 3 \\
8 & 4
\end{bmatrix}
\]

**SOLUTION:**
Let \( A = \begin{bmatrix} -3 & -2 \\ 6 & 4 \end{bmatrix} \).

\( \det(A) = 0 \)

The value of the matrix is zero. So the inverse does not exist.

**ANSWER:**
No Inverse Exists

20. \[
\begin{bmatrix}
-3 & -2 \\
6 & 4
\end{bmatrix}
\]

**SOLUTION:**
Let \( A = \begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix} \).

\( \det(A) = 0 \)

The value of the matrix is zero. So the inverse does not exist.

**ANSWER:**
No Inverse Exists

Use Cramer’s Rule to solve the following system of equations.

21. \( 2x - y = -9 \)
   \( x + 2y = 8 \)

**SOLUTION:**
Cramer’s Rule.

Let \( C \) be the coefficient matrix of the system

\[
\begin{vmatrix}
ax + by = m & | & a & b \\
fz + gz = n & | & f & g
\end{vmatrix}
\]

The solution of the system is

\[
\begin{align*}
x &= \frac{m \cdot f - b \cdot n}{\det(C)} \\
y &= \frac{a \cdot n - m \cdot g}{\det(C)}
\end{align*}
\]

if \( C \neq 0 \).

\[
C = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}
\]

\[\det(C) = (2)(2) - (-1)(1) = 5\]

\[
\begin{align*}
x &= \frac{-9 \cdot 1 - 2 \cdot 8}{5} \\
y &= \frac{1 \cdot 8 - 2 \cdot 5}{5}
\end{align*}
\]

Therefore, the solution of the system is \((-2, 5)\).

**ANSWER:**
\((-2, 5)\)

22. \( x - y + 2z = 0 \)
   \( 3x + z = 11 \)
   \( -x + 2y = 0 \)

**SOLUTION:**
Cramer’s Rule.

Let \( C \) be the coefficient matrix of the system

\[
\begin{vmatrix}
x - y + 2z = 0 & | & 1 & -1 & 2 \\
3x + z = 11 & | & 3 & 0 & 1 \\
x - y + 2z = 0 & | & 1 & -1 & 2
\end{vmatrix}
\]

Therefore, the solution of the system is \( (4, 2, -1) \).

**ANSWER:**
\((4, 2, -1)\)
Practice Test - Chapter 3

\[ \begin{align*}
ax + by + cz &= m \\
fx + gy + hz &= n \\
fx + ky + lz &= p
\end{align*} \]

The solution of the system is

\[ \begin{align*}
x &= \frac{m b c}{|C|}, \\
y &= \frac{a m c}{|C|}, \\
z &= \frac{a b m}{|C|}
\end{align*} \]

if \( C \neq 0 \).

\[ C = \begin{bmatrix}
1 & -1 & 2 \\
3 & 0 & 1 \\
-1 & 2 & 0
\end{bmatrix}
\]

\[ |C| = 11
\]

\[ x = \frac{44}{11} = 4
\]

\[ y = \frac{22}{11} = 2
\]

\[ z = \frac{-11}{11} = -1
\]

Therefore, the solution of the system is \((4, 2, -1)\).

**ANSWER:**

\((4, 2, -1)\)
Choose the term from above to complete each sentence.

1. A feasible region that is open and can go on forever is called __________.
   
   SOLUTION: unbounded
   
   ANSWER: unbounded

2. To __________ means to seek the best price or profit using linear programming.
   
   SOLUTION: Optimize
   
   ANSWER: optimize

3. A matrix that contains the constants in a system of equations is called a(n) ____________.
   
   SOLUTION: constant matrix
   
   ANSWER: constant matrix

4. A matrix can be multiplied by a constant called a(n) ________________.
   
   SOLUTION: scalar
   
   ANSWER: scalar

5. The __________ of a matrix with 4 rows and 3 columns are $4 \times 3$.
   
   SOLUTION: dimensions
   
   ANSWER: dimensions

6. A system of equations is ________ if it has at least one solution.
   
   SOLUTION: consistent
   
   ANSWER: consistent

7. The __________ matrix is a square matrix that, when multiplied by another matrix, equals that same matrix.
   
   SOLUTION: identity
   
   ANSWER: identity

8. The ________ is the point at which the income equals the cost.
   
   SOLUTION: break-even point
   
   ANSWER: break-even point

9. A system of equations is ________ if it has no solutions.
   
   SOLUTION: inconsistent
   
   ANSWER: inconsistent

10. If the product of two matrices is the identity matrix, they are ______________.
    
    SOLUTION: inverses
    
    ANSWER: inverses
Solve each system of equations by graphing.

11. \[ \begin{align*}
3x + 4y &= 8 \\
x - 3y &= -6
\end{align*} \]

**SOLUTION:**
Graph both equations on the coordinate plane.

The solution of the system is (0, 2).

**ANSWER:**
(0, 2)

12. \[ \begin{align*}
x + \frac{8}{3}y &= 12 \\
\frac{1}{2}x + \frac{4}{3}y &= 6
\end{align*} \]

**SOLUTION:**
Graph both equations on the coordinate plane.

Since the graph of both the equations coincides, the system has infinitely many solutions.

**ANSWER:**
infinitely many solutions

13. \[ \begin{align*}
y - 3x &= 13 \\
y &= \frac{1}{3}x + 5
\end{align*} \]

**SOLUTION:**
Graph both equations on the coordinate plane.

The solution of the system is (−3, 4).

**ANSWER:**
(−3, 4)

14. \[ \begin{align*}
6x - 14y &= 5 \\
3x - 7y &= 5
\end{align*} \]

**SOLUTION:**
Graph both equations on the coordinate plane.

Since the graphs are parallel, the lines never intersect. So, the system has no solution.

**ANSWER:**
no solution
15. **LAWN CARE** André and Paul each mow lawns. André charges a $30 service fee and $10 per hour. Paul charges a $10 service fee and $15 per hour. After how many hours will André and Paul charge the same amount?

**SOLUTION:**
Let $x =$ number of hours mowed.

\[
\begin{align*}
30 + 10x &= 10 + 15x \\
10x - 15x &= 10 - 30 \\
-5x &= -20 \\
-5x &= \frac{-20}{-5} \\
x &= 4
\end{align*}
\]

Therefore, André and Paul will charge the same amount for 4 hours.

**ANSWER:**
4 hours

**Solve each system of equations by using either substitution or elimination.**

16. \[
\begin{align*}
x + y &= 6 \\
3x - 2y &= -2
\end{align*}
\]

**SOLUTION:**
Substitute $y = 6 - x$ in the equation $3x - 2y = -2$.

\[
\begin{align*}
3x - 2(6 - x) &= -2 \\
3x - 12 + 2x &= -2 \\
5x - 12 &= -2 \\
5x &= -2 + 12 \\
5x &= 10 \\
x &= 2
\end{align*}
\]

Substitute $x = 2$ in $y = 6 - x$.

\[
\begin{align*}
y &= 6 - 2 \\
y &= 4
\end{align*}
\]

The solution is $(2, 4)$.

**ANSWER:**
$(2, 4)$

17. \[
\begin{align*}
5x - 2y &= 4 \\
-2y + x &= 12
\end{align*}
\]

**SOLUTION:**
Substitute $-2y = -5x + 4$ in the equation $-2y + x = 12$.

\[
\begin{align*}
-2y + x &= 12 \\
-5x + 4 + x &= 12 \\
-4x + 4 &= 12 \\
-4x &= 12 - 4 \\
-4x &= 8 \\
x &= -2
\end{align*}
\]

Substitute $x = -2$ in the equation $5x - 2y = 4$.

\[
\begin{align*}
5x - 2y &= 4 \\
5(-2) - 2y &= 4 \\
-10 - 2y &= 4 \\
-2y &= 14 \\
y &= -7
\end{align*}
\]

The solution is $(-2, -7)$. 

**ANSWER:**
$(-2, -7)$
18. \( x + y = 3.5 \)  
\( x - y = 7 \)

**SOLUTION:**
Substitute \( y = x - 7 \) in the equation \( x + y = 3.5 \).

\[
\begin{align*}
x + x - 7 &= 3.5 \\
2x - 7 &= 3.5 \\
2x &= 10.5 \\
x &= 5.25
\end{align*}
\]

Substitute \( x = 5.25 \) in the equation \( x - y = 7 \).

\[
\begin{align*}
5.25 - y &= 7 \\
-y &= 7 - 5.25 \\
-y &= 1.75 \\
y &= -1.75
\end{align*}
\]

The solution is \((5.25, -1.75)\).

**ANSWER:**
\((5.25, -1.75)\)

19. \( 3y - 5x = 0 \)
\( 2y - 4x = -2 \)

**SOLUTION:**
Substitute \( y = \frac{5}{3} x \) in the equation \( 2y - 4x = -2 \).

\[
\begin{align*}
2 \left( \frac{5}{3} x \right) - 4x &= -2 \\
\frac{10}{3} x - 4x &= -2 \\
\frac{10x - 12x}{3} &= -2 \\
\frac{2}{3} x &= -2 \\
x &= \frac{6}{2} \\
x &= 3
\end{align*}
\]

Substitute \( x = 3 \) in the equation \( 3y - 5x = 0 \).

\[
\begin{align*}
3y - 5x &= 0 \\
3y - 5(3) &= 0 \\
3y - 15 &= 0 \\
3y &= 15 \\
y &= 5
\end{align*}
\]

The solution is \((3, 5)\).

**ANSWER:**
\((3, 5)\)
20. SCHOOL SUPPLIES At an office supply store, Emilio bought 3 notebooks and 5 pens for $13.75. If a notebook costs $1.25 more than a pen, how much does a notebook cost? How much does a pen cost?

**SOLUTION:**
Let \( x \) = cost of a note book. Let \( y \) = cost of a pen. The system of equations representing the situation is:

\[
3x + 5y = 13.75 \\
x = y + 1.25
\]

Substitute \( x = y + 1.25 \) in the equation \( 3x + 5y = 13.75 \).

\[
3(y + 1.25) + 5y = 13.75 \\
3y + 3.75 + 5y = 13.75 \\
8y + 3.75 = 13.75 \\
8y = 10 \\
y = \frac{10}{8} \\
y = 1.25
\]

Substitute \( y = 1.25 \) in the equation \( x = y + 1.25 \).

\[
x = 1.25 + 1.25 \\
x = 2.50
\]

Therefore, the cost of a note book is $2.50 and the cost of a pen is $1.25.

**ANSWER:**
notebook: $2.50; pen: $1.25
22. \(|y| > 2\)  
\(x > 3\)  
\(\text{SOLUTION:}\)  
Graph the system of inequalities in the same coordinate plane.

\[
\begin{align*}
&y > 2 \\
x > 3
\end{align*}
\]

\(\text{ANSWER:}\)

\[
\begin{align*}
&y > 2 \\
x > 3
\end{align*}
\]

23. \(y \geq x + 3\)  
\(2y \leq x - 5\)  
\(\text{SOLUTION:}\)  
Graph the system of inequalities in the same coordinate plane.

\[
\begin{align*}
&y \geq x + 3 \\
&2y \leq x - 5
\end{align*}
\]

\(\text{ANSWER:}\)

\[
\begin{align*}
&y \geq x + 3 \\
&2y \leq x - 5
\end{align*}
\]
24. \[ y > x + 1 \]
\[ x < -2 \]

SOLUTION:
Graph the system of inequalities in the same coordinate plane.

\[ \text{ANSWER:} \]

25. JEWELRY Payton makes jewelry to sell at her mother’s clothing store. She spends no more than 3 hours making jewelry on Saturdays. It takes her 15 minutes to set up her supplies and 25 minutes to make each bracelet. Draw a graph that represents this.

SOLUTION:
Let \( x \) = number of bracelets, and \( y \) = number of minutes.
The system of inequalities representing the situation is:

\[ y \geq 25x + 15 \]
\[ y \leq 180 \]

Graph the inequalities in the same coordinate plane.

\[ \text{ANSWER:} \]
26. **FLOWERS** A florist can make a grand arrangement in 18 minutes or a simple arrangement in 10 minutes. The florist makes at least twice as many of the simple arrangements as the grand arrangements. The florist can work only 40 hours per week. The profit on the simple arrangements is $10 and the profit on the grand arrangements is $25. Find the number and type of arrangements that the florist should produce to maximize profit.

**SOLUTION:**
Let \( x \) be the number of simple arrangement and \( y \) be the number of grand arrangements.

\[
\begin{align*}
  x &\geq 2y \\
  10x + 18y &\leq 2400
\end{align*}
\]

The optimize function is \( f(x, y) = 10x + 25y \).

Graph the inequalities in the same coordinate plane.

![Graph of inequalities](image)

The vertices of the feasible region are (0, 0), (240, 0) and (126, 63).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>0</td>
</tr>
<tr>
<td>(240, 0)</td>
<td>2400</td>
</tr>
<tr>
<td>(126, 63)</td>
<td>2835</td>
</tr>
</tbody>
</table>

So, to maximize the profit, the florist should produce 126 simple arrangements and 63 grand arrangements.

**ANSWER:**
126 simple and 63 grand

27. **MANUFACTURING** A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one and 1 hour in step two, and produces a profit of $20. Each pair of indoor shoes requires 1 hour in step one and 3 hours in step two, and produces a profit of $15. The company has 40 hours of labor available per day for step one and 60 hours available for step two. What is the combination of shoes for this profit?

**SOLUTION:**
Let \( x \) be the number of pair of outdoor shoes and \( y \) be the number of pair of indoor shoes.

\[
\begin{align*}
  2x + y &\leq 40 \\
  x + 3y &\leq 60
\end{align*}
\]

The optimize function is \( f(x, y) = 20x + 15y \).

Graph the inequalities in the same coordinate plane.

![Graph of inequalities](image)

The vertices of the feasible region are (0, 20), (20, 0) and (12, 16).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 20)</td>
<td>300</td>
</tr>
<tr>
<td>(20, 0)</td>
<td>400</td>
</tr>
<tr>
<td>(12, 16)</td>
<td>480</td>
</tr>
</tbody>
</table>

The manufacturer’s maximum profit is $480. For this profit, the manufacturer has to produce 12 outdoor and 16 indoor pair of shoes.

**ANSWER:**
$480; 12 outdoor, 16 indoor
Solve each system of equations.

28. \( a - 4b + c = 3 \)

\( b - 3c = 10 \)

\( 3b - 8c = 24 \)

**SOLUTION:**

\( b - 3c = 10 \)

\[ b = 3c + 10 \]

Substitute \((3c + 10)\) for \(b\) in the equation \(3b - 8c = 24\).

\[ 3(3c + 10) - 8c = 24 \]

\[ 9c + 30 - 8c = 24 \]

\[ c + 30 = 24 \]

\[ c = 24 - 30 \]

\[ c = -6 \]

Substitute \(c = -6\) in the equation \(b = 3c + 10\).

\[ b = 3(-6) + 10 \]

\[ = -18 + 10 \]

\[ = -8 \]

Substitute \(b = -8\) and \(c = -6\) in the equation \(a - 4b + c = 3\).

\[ a - 4(-8) + (-6) = 3 \]

\[ a + 32 - 6 = 3 \]

\[ a + 26 = 3 \]

\[ a = 3 - 26 \]

\[ a = -23 \]

The solution of the system is \((-23, -8, -6)\).

**ANSWER:**

\((-23, -8, -6)\)
30. **AMUSEMENT PARKS** Dustin, Luis, and Marci went to an amusement park. They purchased snacks from the same vendor. Their snacks and how much they paid are listed in the table. How much did each snack cost?

<table>
<thead>
<tr>
<th>Name</th>
<th>Hot Dogs</th>
<th>Popcorn</th>
<th>Soda</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dustin</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>$15.25</td>
</tr>
<tr>
<td>Luis</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>$14.00</td>
</tr>
<tr>
<td>Marci</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>$10.25</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let $x = \text{cost of a hot dog}$.
Let $y = \text{cost of a popcorn}$.
Let $z = \text{cost of a soda}$.

The system of equations representing the situations is:

$$x + 2y + 3z = 15.25 \quad \rightarrow (1)$$

$$2x + 3z = 14 \quad \rightarrow (2)$$

$$x + 2y + z = 10.25 \quad \rightarrow (3)$$

Use equation (3) in (1).
Substitute $x + 2y = 10.25 - z$ in equation (1).

$$10.25 - z + 3z = 15.25$$
$$10.25 + 2z = 15.25$$
$$2z = 5$$
$$z = 2.50$$

Substitute $z = 2.50$ in equation (2).

$$2x + 3(2.50) = 14$$
$$2x + 7.50 = 14$$
$$2x = 14 - 7.50$$
$$2x = 6.50$$
$$x = 3.25$$

Substitute $x = 3.25$ and $z = 2.50$ in the equation (1).

$$x + 2y + 3z = 15.25$$
$$3.25 + 2y + 3(2.50) = 15.25$$
$$3.25 + 2y + 7.50 = 15.25$$
$$2y + 10.75 = 15.25$$
$$2y = 15.25 - 10.75$$
$$2y = 4.50$$
$$y = 2.25$$

Therefore, the cost of a hot dog is $3.25. The cost of a popcorn is $2.25 and the cost of a soda is $2.50.

**ANSWER:**
hot dog: $3.25; popcorn: $2.25; soda: $2.50

Perform the indicated operations. If the matrix does not exist, write impossible.

31. $$3 \begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix}$$

**SOLUTION:**

$$3 \begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} -3 & 27 \\ 9 & 12 \end{bmatrix}$$

**ANSWER:**
$$\begin{bmatrix} -3 & 27 \\ 9 & 12 \end{bmatrix}$$

32. $$\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

**SOLUTION:**

$$\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 11 \\ -8 \end{bmatrix}$$

**ANSWER:**
$$\begin{bmatrix} 11 \\ -8 \end{bmatrix}$$
33. **RETAIL.** Current Fashions buys shirts, jeans and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

<table>
<thead>
<tr>
<th>Item</th>
<th>Purchase Price</th>
<th>Selling Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>shirts</td>
<td>$15</td>
<td>$35</td>
</tr>
<tr>
<td>jeans</td>
<td>$25</td>
<td>$55</td>
</tr>
<tr>
<td>shoes</td>
<td>$30</td>
<td>$85</td>
</tr>
</tbody>
</table>

a. Write a matrix for the purchase price.
b. Write a matrix for the selling price.
c. Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

**SOLUTION:**
a. Purchase price:
\[
\begin{bmatrix}
15 \\
25 \\
30
\end{bmatrix}
\]
b. Selling price:
\[
\begin{bmatrix}
35 \\
55 \\
85
\end{bmatrix}
\]
c. Subtract the matrices.
\[
\begin{bmatrix}
35 \\
55 \\
85
\end{bmatrix} - \begin{bmatrix}
15 \\
25 \\
30
\end{bmatrix} = \begin{bmatrix}
20 \\
30 \\
55
\end{bmatrix}
\]

**ANSWER:**
a. buying price: \[
\begin{bmatrix}
25 \\
30
\end{bmatrix}
\]
b. selling price: \[
\begin{bmatrix}
55 \\
85
\end{bmatrix}
\]
c. \[
\begin{bmatrix}
35 \\
55 \\
85
\end{bmatrix} - \begin{bmatrix}
15 \\
25 \\
30
\end{bmatrix} = \begin{bmatrix}
20 \\
30 \\
55
\end{bmatrix}
\]

34. \[
\begin{bmatrix}
3 \\
-7
\end{bmatrix} \cdot \begin{bmatrix}
9 \\
-5
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
3 \\
-7
\end{bmatrix} \cdot \begin{bmatrix}
9 \\
-5
\end{bmatrix} = \begin{bmatrix}
27 \\
35
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
27 \\
35
\end{bmatrix}
\]

35. \[
\begin{bmatrix}
-3 & 0 & 2 \\
6 & -1 & 5
\end{bmatrix} \cdot \begin{bmatrix}
8 & -1 \\
-4 & 3 \\
6 & 7
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
-3 & 0 & 2 \\
6 & -1 & 5
\end{bmatrix} \cdot \begin{bmatrix}
8 & -1 \\
-4 & 3 \\
6 & 7
\end{bmatrix} = \begin{bmatrix}
-12 & 17 \\
82 & 26
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
-12 & 17 \\
82 & 26
\end{bmatrix}
\]

36. \[
\begin{bmatrix}
2 & 11 \\
0 & -3 \\
-6 & 7
\end{bmatrix} \cdot \begin{bmatrix}
0 & 8 & -5 \\
12 & 0 & 9 \\
4 & -6 & 7
\end{bmatrix}
\]

**SOLUTION:**
The inner dimensions of the matrices are not equal. So, the matrices cannot be multiplied.

**ANSWER:**
undefined
37. **GROCERIES** Martin bought 1 gallon of milk, 2 apples, 4 frozen dinners, and 1 box of cereal. The following matrix shows the prices for each item respectively.

\[
\begin{bmatrix}
2.59 & 0.49 & 5.25 & 3.99
\end{bmatrix}
\]

Use matrix multiplication to find the total amount of money Martin spent at the grocery store.

**SOLUTION:**

\[
\begin{bmatrix}
2.59 & 0.49 & 5.25 & 3.99
\end{bmatrix}
\begin{bmatrix}
1 \\
2 \\
4 \\
1
\end{bmatrix} = \begin{bmatrix} 28.56 \end{bmatrix}
\]

So, Martin spent $28.56.

**ANSWER:**

$28.56

Evaluate each determinant.

38. \[
\begin{vmatrix}
2 & 4 \\
7 & -3
\end{vmatrix}
\]

**SOLUTION:**

\[
\begin{vmatrix}
2 & 4 \\
7 & -3
\end{vmatrix} = -6 - 28 = -34
\]

**ANSWER:**

-34

39. \[
\begin{vmatrix}
2 & 3 & -1 \\
0 & 2 & 4 \\
-2 & 5 & 6
\end{vmatrix}
\]

**SOLUTION:**

\[
\begin{vmatrix}
2 & 3 & -1 \\
0 & 2 & 4 \\
-2 & 5 & 6
\end{vmatrix} = 2(12 - 20) - 3(0 + 8) - 1(0 + 4)
\]

\[
= -16 - 24 - 4
\]

\[
= -44
\]

**ANSWER:**

-44

---

**Use Cramer’s Rule to solve each system of equations.**

40. \[
3x - y = 0 \\
5x + 2y = 22
\]

**SOLUTION:**

Let \[
C = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}
\]

\[
|C| = \begin{vmatrix} 3 & -1 \\ 5 & 2 \end{vmatrix} = 11
\]

\[
x = \frac{\begin{vmatrix} 0 & -1 \\ 22 & 2 \end{vmatrix}}{11} = \frac{22}{11} = 2
\]

\[
y = \frac{\begin{vmatrix} 3 & 0 \\ 5 & 22 \end{vmatrix}}{11} = \frac{66}{11} = 6
\]

Therefore, the solution is \((2, 6)\).

**ANSWER:**

\((2, 6)\)
41. \[5x + 2y = 4 \]
\[3x + 4y + 2z = 6 \]
\[7x + 3y + 4z = 29 \]

**SOLUTION:**

Let \[C = \begin{bmatrix} 5 & 2 & 0 \\ 3 & 4 & 2 \\ 7 & 3 & 4 \end{bmatrix} \]

\[|C| = 5(16 - 6) - 2(12 - 14) + 0(9 - 28) = 50 + 4 + 0 = 54 \]

\[x = \frac{4 \cdot 2 \cdot 0 - 6 \cdot 29 \cdot 4}{54} = \frac{108}{54} = 2 \]

\[y = \frac{5 \cdot 2 \cdot 0 - 3 \cdot 29 \cdot 4}{54} = \frac{162}{54} = -3 \]

\[z = \frac{5 \cdot 2 \cdot 0 - 3 \cdot 29 \cdot 4}{54} = \frac{324}{54} = 6 \]

The solution is \((2, -3, 6)\).

**ANSWER:**

\((2, -3, 6)\)

42. **JEWELRY** Alana paid $98.25 for 3 necklaces and 2 pairs of earrings. Petra paid $133.50 for 2 necklaces and 4 pairs of earrings. Use Cramer’s Rule to find out how much 1 necklace costs and how much 1 pair of earrings costs.

**SOLUTION:**

Let \(x\) be the number of necklaces and \(y\) be the number of pairs of earrings.

\[3x + 2y = 98.25 \]
\[2x + 4y = 133.50 \]

Let \[C = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} \]

\[|C| = 3 \cdot 4 - 2 \cdot 2 = 12 - 4 = 8 \]

\[x = \frac{98.25 \cdot 2 - 133.50 \cdot 4}{8} = \frac{126}{8} = 15.75 \]

\[y = \frac{3 \cdot 133.50 - 2 \cdot 98.25}{8} = \frac{204}{8} = 25.50 \]

So, the cost of 1 necklace is $15.75 and a pair of earrings is $25.50.

**ANSWER:**

necklace: $15.75; pair of earrings: $25.50
Find the inverse of each matrix, if it exists.

43. \[
\begin{bmatrix}
7 & 4 \\
3 & 2
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
7 & 4 \\
3 & 2
\end{bmatrix} = 7(2) - 3(4) \\
= 14 - 12 \\
= 2
\]

Since the determinant is non-zero, the inverse exists.

\[
A^{-1} = \frac{1}{2}\begin{bmatrix}
2 & -4 \\
-3 & 7
\end{bmatrix}
\]

**ANSWER:**
\[
\frac{1}{2}\begin{bmatrix}
2 & -4 \\
-3 & 7
\end{bmatrix}
\]

44. \[
\begin{bmatrix}
2 & 5 \\
-5 & -13
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
2 & 5 \\
-5 & -13
\end{bmatrix} = -26 + 25 \\
= -1
\]

Since the determinant is non-zero, the inverse exists.

\[
A^{-1} = \frac{1}{-1}\begin{bmatrix}
-13 & -5 \\
5 & 2
\end{bmatrix} \\
= \begin{bmatrix}
13 & 5 \\
-5 & -2
\end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix}
13 & 5 \\
-5 & -2
\end{bmatrix}
\]

45. \[
\begin{bmatrix}
6 & -3 \\
-8 & 4
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix}
6 & -3 \\
-8 & 4
\end{bmatrix} = 24 - 24 = 0
\]

Since the determinant is 0, the inverse does not exist.

**ANSWER:**
No inverse exists.

Use a matrix equation to solve each system of equations.

46. \[
\begin{bmatrix}
5 & 3 \\
3 & 2
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
4 \\
0
\end{bmatrix}
\]

**SOLUTION:**
Let \(A = \begin{bmatrix}
5 & 3 \\
3 & 2
\end{bmatrix}\).

\[
A^{-1} = \begin{bmatrix}
2 & -3 \\
-3 & 5
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 & -3 \\
-3 & 5
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
2 & -3 \\
-3 & 5
\end{bmatrix} \begin{bmatrix}
4 \\
0
\end{bmatrix} = \begin{bmatrix}
8 \\
-12
\end{bmatrix}
\]

The solution of the system is \((8, -12)\).

**ANSWER:**
(8, -12)
47. \[
\begin{bmatrix}
3 & -1 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
= 
\begin{bmatrix}
5 \\
4
\end{bmatrix}
\]

**SOLUTION:**
Let \( A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \).

\[
A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} a \\ b \end{bmatrix}
= \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 1 & 4 \end{bmatrix}
\begin{bmatrix} 5 \\ 4 \end{bmatrix}
\]

\[
\begin{bmatrix} a \\ b \end{bmatrix}
= \begin{bmatrix} 2 \\ 1 \end{bmatrix}
\]

The solution of the system is (2, 1).

**ANSWER:**
(2, 1)

48. **HEALTH FOOD** Heath sells nuts and raisins by the pound. Sonia bought 2 pounds of nuts and 2 pounds of raisins for $23.50. Drew bought 3 pounds of nuts and 1 pound of raisins for $22.25. What is the cost of 1 pound of nuts and 1 pound of raisins?

**SOLUTION:**
Let \( x \) = cost of 1 pound of nuts and \( y \) = cost of 1 pound of raisins.

\[
2x + 2y = 23.50
\]
\[
3x + y = 22.25
\]

The matrix equation is \( \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix} \).

The inverse of \( \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \) is \( -\frac{1}{4} \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} \).

\[
\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}
\begin{bmatrix} x \\ y \end{bmatrix}
= \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\begin{bmatrix} x \\ y \end{bmatrix}
= \begin{bmatrix} 5.25 \\ 6.50 \end{bmatrix}
\]

The cost of 1 pound of nuts is $5.25 and 1 pound of raisins is $6.50.

**ANSWER:**
nuts: $5.25 per pound; raisins: $6.50 per pound