4-1 Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. \( f(x) = 3x^2 \)

**SOLUTION:**

a. Compare the function \( f(x) = 3x^2 \) with the standard form of a quadratic function.

Here, \( a = 3 \), \( b = 0 \) and \( c = 0 \).

The y-intercept is 0.

The equation of the axis of symmetry is \( x = -\frac{b}{2a} \).

Therefore, \( x = 0 \) is the axis of symmetry.

The x-coordinate of the vertex is \( \frac{b}{2a} = 0 \).

b. Substitute \(-2, -1, 0, 1\) and \(2\) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. \( y\text{-int} = 0 \); axis of symmetry: \( x = 0 \); x-coordinate = 0

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
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<td>3</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

c. 

2. \( f(x) = -6x^2 \)

**SOLUTION:**

a. Compare the function \( f(x) = -6x^2 \) with the standard form of a quadratic function.

Here, \( a = -6 \), \( b = 0 \) and \( c = 0 \).

The y-intercept is 0.

The equation of the axis of symmetry is \( x = -\frac{b}{2a} \).
Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

Equation of the axis of symmetry is \( x = -1.5 \).

The x-coordinate of the vertex is \( -1.5 \).

b. Substitute \(-2, -1, 0, 1\) and \(2\) for \(x\) and make the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-24</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
<tr>
<td>2</td>
<td>-24</td>
</tr>
</tbody>
</table>

c. Graph the function.

3. \(f(x) = x^2 - 4x\)

SOLUTION:

a. Compare the function \(f(x) = x^2 - 4x\) with the standard form of a quadratic function.

Here, \(a = 1\), \(b = -4\) and \(c = 0\).

The y-intercept is 0.

The equation of the axis of symmetry is

\[
x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2.
\]

Therefore, \(x = 2\) is the axis of symmetry.

The x-coordinate of the vertex is \(\frac{b}{2a} = 2\).

b. Substitute \(0, 1, 2, 3\) and \(4\) for \(x\) and make the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

c. Graph the function.
4-1 Graphing Quadratic Functions

ANSWER:

a. y-int = 0; axis of symmetry: $x = 2$; x-coordinate = \frac{-b}{2a} = \frac{2}{2} = 1

b. $x = 0$; axis of symmetry: $x = \frac{-b}{2a} = \frac{2}{2} = 1$

c. Graph the function.

\[ f(x) = -x^2 - 3x + 4 \]

SOLUTION:

a. Compare the function $f(x) = -x^2 - 3x + 4$ with the standard form of a quadratic function. Here, $a = -1$, $b = -3$ and $c = 4$.

The y-intercept is 4.

The equation of the axis of symmetry is $x = \frac{-b}{2a} = \frac{-3}{2(-1)} = -1.5$.

Therefore, $x = 1.5$ is the axis of symmetry.

The x-coordinate of the vertex is $\frac{-b}{2a} = 1.5$.

b. Substitute 0, –1, –1.5, –2 and –3 for $x$ and make the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>–3</td>
<td>4</td>
</tr>
<tr>
<td>–2</td>
<td>6</td>
</tr>
<tr>
<td>–1.5</td>
<td>6.25</td>
</tr>
<tr>
<td>–1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. y-int = 4; axis of symmetry: $x = -1.5$; x-coordinate = -1.5

b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>–3</td>
<td>4</td>
</tr>
<tr>
<td>–2</td>
<td>6</td>
</tr>
<tr>
<td>–1.5</td>
<td>6.25</td>
</tr>
<tr>
<td>–1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

c.
4-1 Graphing Quadratic Functions

5. \( f(x) = 4x^2 - 6x - 3 \)

**SOLUTION:**

a. Compare the function \( f(x) = 4x^2 - 6x - 3 \) with the standard form of a quadratic function. Here, \( a = 4, b = -6 \) and \( c = -3 \).

The y-intercept is \(-3\).

The equation of the axis of symmetry is 
\[ x = \frac{-b}{2a} = \frac{6}{2(4)} = 0.75. \]

Therefore, \( x = 0.75 \).

The \( x \)-coordinate of the vertex is \( \frac{b}{2a} = 0.75 \).

b. Substitute 0, -1, 0.75, 1.5 and 2.5 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
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</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>0.75</td>
<td>-5.25</td>
</tr>
<tr>
<td>1.5</td>
<td>-3</td>
</tr>
<tr>
<td>2.5</td>
<td>7</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. \( y \)-int = \(-3\); axis of symmetry: \( x = 0.75 \); \( x \)-coordinate = 0.75

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>0.75</td>
<td>-5.25</td>
</tr>
<tr>
<td>1.5</td>
<td>-3</td>
</tr>
<tr>
<td>2.5</td>
<td>7</td>
</tr>
</tbody>
</table>

c. Graph the function.

6. \( f(x) = 2x^2 - 8x + 5 \)

**SOLUTION:**

a. Compare the function \( f(x) = 2x^2 - 8x + 5 \) with the standard form of a quadratic function.

Here, \( a = 2, b = -8 \) and \( c = 5 \).

The y-intercept is 5.

The equation of the axis of symmetry is
4-1 Graphing Quadratic Functions

\[ x = -\frac{b}{2a} = -\frac{-8}{2(2)} = 2. \]

Therefore, \( x = 2 \) is the axis of symmetry.

The \( x \)-coordinate of the vertex is \( \frac{b}{2a} = 2 \).

b. Substitute 0, 1, 2, 3 and 4 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. y-int = 5; axis of symmetry: \( x = 2 \); \( x \)-coordinate = 2

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

c. 
4-1 Graphing Quadratic Functions

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

7. \( f(x) = -x^2 + 6x - 1 \)

**SOLUTION:**
Compare the function \( f(x) = -x^2 + 6x - 1 \) with the standard form of a quadratic function.

Here, \( a = -1, \ b = 6 \) and \( c = -1 \).

For this function, \( a = -1 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is
\[
\frac{-b}{2a} = \frac{-6}{2(-1)} = 3.
\]

Substitute 3 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(3) = -(3)^2 + 6(3) - 1
\]
\[
= -9 + 18 - 1
\]
\[
= 8
\]

Therefore, the maximum value of the function is 8.

The domain is all real numbers.
D = \{ all real numbers \}

The range is all real numbers less than or equal to the maximum value.
\[
R = \{ f(x) \mid f(x) \leq 8 \}
\]

**ANSWER:**
max = 8; D = \{ all real numbers \},
R = \{ f(x) \mid f(x) \leq 8 \}

8. \( f(x) = x^2 + 3x - 12 \)

**SOLUTION:**
Compare the function \( f(x) = x^2 + 3x - 12 \) with the standard form of a quadratic function.

Here, \( a = 1, \ b = 3 \) and \( c = -12 \).

For this function, \( a = 1 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is
\[
\frac{-b}{2a} = \frac{-3}{2(1)} = -1.5.
\]

Substitute \(-1.5\) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(-1.5) = (-1.5)^2 + 3(-1.5) - 12
\]
\[
= 2.25 - 4.5 - 12
\]
\[
= -14.25
\]

Therefore, the minimum value of the function is \(-14.25\).

The domain is all real numbers.
D = \{ all real numbers \}.

The range is all real numbers greater than or equal to the minimum value.
\[
R = \{ f(x) \mid f(x) \geq -14.25 \}
\]

**ANSWER:**
min = \(-14.25\); D = \{ all real numbers \},
R = \{ f(x) \mid f(x) \geq -14.25 \}
4-1 Graphing Quadratic Functions

9. \( f(x) = 3x^2 + 8x + 5 \)

**SOLUTION:**

Compare the function \( f(x) = 3x^2 + 8x + 5 \) with the standard form of a quadratic function.

Here, \( a = 3, b = 8 \) and \( c = 5 \).

or this function, \( a = 3 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is 
\[
\frac{-b}{2a} = -\frac{8}{2(3)} = -\frac{4}{3}
\]

Substitute \( -\frac{4}{3} \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f\left(-\frac{4}{3}\right) = 3\left(-\frac{4}{3}\right)^2 + 8\left(-\frac{4}{3}\right) + 5
\]
\[
= 3\left(\frac{16}{9}\right) - \frac{32}{3} + 5
\]
\[
= \frac{1}{3}
\]

Therefore, the minimum value of the function is \( -\frac{1}{3} \).

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \)

The range is all real numbers greater than or equal to the minimum value.

\[
R = \left\{ f(x) \mid f(x) \geq -\frac{1}{3} \right\}
\]

**ANSWER:**

\( \text{min} = -\frac{1}{3}; \ D = \{ \text{all real numbers} \}, \)

\[
R = \left\{ f(x) \mid f(x) \geq -\frac{1}{3} \right\}
\]

10. \( f(x) = -4x^2 + 10x - 6 \)

**SOLUTION:**

Compare the function \( f(x) = -4x^2 + 10x - 6 \) with the standard form of a quadratic function.

Here, \( a = -4, b = 10 \) and \( c = -6 \).

For this function, \( a = -4 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is 
\[
\frac{-b}{2a} = -\frac{10}{2(-4)} = 1.25.
\]

Substitute \( 1.25 \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(1.25) = -4(1.25)^2 + 10(1.25) - 6
\]
\[
= -6.25 + 12.5 - 6
\]
\[
= 0.25
\]

Therefore, the maximum value of the function is 0.25

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \).

The range is all real numbers less than or equal to the maximum value.

\[
R = \left\{ f(x) \mid f(x) \leq 0.25 \right\}
\]

**ANSWER:**

\( \text{max} = 0.25; \ D = \{ \text{all real numbers} \}, \)

\[
R = \left\{ f(x) \mid f(x) \leq 0.25 \right\}
\]
11. BUSINESS A store rents 1400 videos per week at $2.25 per video. The owner estimates that they will rent 100 fewer videos for each $0.25 increase in price. What price will maximize the income of the store?

SOLUTION:
Let $x$ be the number of increase in price and let $f(x)$ be the income.

$$f(x) = (2.25 + 0.25x)(1400 - 100x)$$

$$= 3150 - 225x + 350x - 25x^2$$

$$= 3150 + 125x - 25x^2$$

$$= -25x^2 + 125x + 3150$$

Here, $a = -25$, $b = 125$, and $c = 3150$.

$$x = \frac{-b}{2a}$$

$$= \frac{-125}{2(-25)}$$

$$= \frac{125}{50}$$

$$= 2.5$$

The function gets maximum value at 2.5.
That is, 2.5 number of increase in price will maximize the income.
$2.25 + (2.5)(0.25) \approx 2.88$

So, the price of $2.88 per video will maximize the income of the store.

**ANSWER:**
$2.88$

Complete parts $a$–$c$ for each quadratic function.

a. Find the $y$-intercept, the equation of the axis of symmetry, and the $x$-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

12. $f(x) = 4x^2$

**SOLUTION:**
Here, $a = 4$, $b = 0$ and $c = 0$.

The $y$-intercept is 0.

The equation of the axis of symmetry is $x = \frac{-b}{2a}$.

Therefore, $x = 0$ is the axis of symmetry.

The $x$-coordinate of the vertex is $\frac{b}{2a} = 0$.

b. Substitute $-2$, $-1$, $0$, $1$ and $2$ for $x$ and make the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>16</td>
</tr>
<tr>
<td>$-1$</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

c. Graph the function.
4-1 Graphing Quadratic Functions

b. 

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
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<td>-2</td>
<td>16</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

13. \( f(x) = -2x^2 \)

**SOLUTION:**

a. Compare the function \( f(x) = -2x^2 \) with the standard form of a quadratic function.

Here, \( a = -2, b = 0 \) and \( c = 0 \).

The \( y \)-intercept is 0.

The equation of the axis of symmetry is \( x = \frac{-b}{2a} \).

Therefore, \( x = 0 \) is the axis of symmetry.

The \( x \)-coordinate of the vertex is \( \frac{-b}{2a} = 0 \).

b. Substitute \(-2, -1, 0, 1\) and \(2\) for \( x \) and make the table.

c. Graph the function.

**ANSWER:**

a. \( y \)-int = 0; axis of symmetry: \( x = 0 \); \( x \)-coordinate = 0

b. 

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-8</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
</tr>
</tbody>
</table>
4-1 Graphing Quadratic Functions

14. \( f(x) = x^2 - 5 \)

\text{SOLUTION:}

\text{a.} \text{ Compare the function } f(x) = x^2 - 5 \text{ with the standard form of a quadratic function.}

Here, \( a = 1, b = 0 \) and \( c = -5. \)

The \( y \)-intercept is \(-5.\)

The equation of the axis of symmetry is \( x = \frac{-b}{2a}. \)

Therefore, \( x = 0 \) is the axis of symmetry.

The \( x \)-coordinate of the vertex is \( \frac{-b}{2a} = 0. \)

\text{b.} \text{ Substitute } -2, -1, 0, 1 \text{ and } 2 \text{ for } x \text{ and make the table.}

\begin{center}
\begin{tabular}{|c|c|}
\hline
\( x \) & \( f(x) \) \\
\hline
-2 & -1 \\
-1 & -4 \\
0 & -5 \\
1 & -4 \\
2 & -1 \\
\hline
\end{tabular}
\end{center}

\text{c.} \text{ Graph the function.}

\begin{center}
\includegraphics[width=0.5\textwidth]{graph1.png}
\end{center}

15. \( f(x) = 4x^2 - 3 \)

\text{SOLUTION:}

\text{a.} \text{ Compare the function } f(x) = 4x^2 - 3 \text{ with the standard form of a quadratic function.}

Here, \( a = 4, b = 0 \) and \( c = -3. \)

The \( y \)-intercept is \(-3.\)

The equation of the axis of symmetry is \( x = \frac{-b}{2a}. \)

Therefore, \( x = 0 \) is the axis of symmetry.

The \( x \)-coordinate of the vertex is \( \frac{-b}{2a} = 0. \)

\text{b.} \text{ Substitute } -2, -1, 0, 1 \text{ and } 2 \text{ for } x \text{ and make the table.}

\begin{center}
\begin{tabular}{|c|c|}
\hline
\( x \) & \( f(x) \) \\
\hline
-2 & \text{not shown} \\
-1 & \text{not shown} \\
0 & -3 \\
1 & -3 \\
2 & \text{not shown} \\
\hline
\end{tabular}
\end{center}

\text{ANSWER:}

\text{a.} \text{ } y\text{-int } = -5; \text{ axis of symmetry: } x = 0; \text{ } x\text{-coordinate } = 

\text{b.} \text{ } x\text{-coordinate of vertex } = \text{not shown; } y\text{-intercept } = -3.
**4-1 Graphing Quadratic Functions**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
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</tr>
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</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
</tbody>
</table>

**c. Graph the function.**

![Graph of the function](image)

**ANSWER:**

a. $y$-int = -3; axis of symmetry: $x = 0$; $x$-coordinate = 0

b.  

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>13</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
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</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
</tbody>
</table>

c.  

![Graph of the function](image)

**ANSWER:**

a. $y$-int = 3; axis of symmetry: $x = 0$; $x$-coordinate = 0

16. $f(x) = x^2 + 3$

**SOLUTION:**

a. Compare the function $f(x) = x^2 + 3$ with the standard form of a quadratic function. Here, $a = 1$, $b = 0$ and $c = 3$.

The $y$-intercept is 3.

The equation of the axis of symmetry is $x = -\frac{b}{2a}$.

Therefore, $x = 0$ is the axis of symmetry.

The $x$-coordinate of the vertex is $-\frac{b}{2a} = 0$.

b. Substitute $-2, -1, 0, 1$ and $2$ for $x$ and make the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

c.  

![Graph of the function](image)

**ANSWER:**

a. $y$-int = 3; axis of symmetry: $x = 0$; $x$-coordinate = 0
4-1 Graphing Quadratic Functions

b. Complete parts a – c for each quadratic function.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

17. $f(x) = -3x^2 + 5$

**SOLUTION:**

a. Compare the function $f(x) = -3x^2 + 5$ with the standard form of a quadratic function.

Here, $a = -3$, $b = 0$ and $c = 5$.

The y-intercept is 5.

The equation of the axis of symmetry is $x = \frac{-b}{2a}$.

Therefore, $x = 0$ is the equation of axis of symmetry.

The $x$-coordinate of the vertex is $\frac{-b}{2a} = 0$.

b. Substitute $-2$, $-1$, $0$, $1$ and $2$ for $x$ and make the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. y-int = 5; axis of symmetry: $x = 0$; $x$-coordinate = 0

b. 

c. 

ANSWER:
18. \( f(x) = x^2 - 6x + 8 \)

**SOLUTION:**

a. Compare the function \( f(x) = x^2 - 6x + 8 \) with the standard form of a quadratic function.

Here, \( a = 1, b = -6 \) and \( c = 8 \).

The y-intercept is 8.

The equation of the axis of symmetry is 
\[
x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3.
\]

Therefore, \( x = 3 \) is the equation of the axis of symmetry.

The x-coordinate of the vertex is \( -\frac{b}{2a} = 3 \).

b. Substitute 1, 2, 3, 4 and 5 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

c. Graph the function.

19. \( f(x) = x^2 - 3x - 10 \)

**SOLUTION:**

a. Compare the function \( f(x) = x^2 - 3x - 10 \) with the standard form of a quadratic function.

Here, \( a = 1, b = -3 \) and \( c = -10 \).

The y-intercept is -10.

The equation of the axis of symmetry is 
\[
x = -\frac{b}{2a} = -\frac{-3}{2(1)} = 1.5.
\]

Therefore, \( x = 1.5 \) is the equation of the axis of symmetry.

The x-coordinate of the vertex is \( -\frac{b}{2a} = 1.5 \).

b. Substitute 0, 1, 1.5, 2 and 3 for \( x \) and make the table.
20. \( f(x) = -x^2 + 4x - 6 \)

**SOLUTION:**

a. Compare the function \( f(x) = -x^2 + 4x - 6 \) with the standard form of a quadratic function. Here, \( a = -1 \), \( b = 4 \) and \( c = -6 \).

The \( y \)-intercept is \(-6\).

The equation of the axis of symmetry is 
\[
x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2.
\]

Therefore, \( x = 2 \) is the equation of the axis of symmetry.

The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = 2 \).

b. Substitute 0, 1, 2, 3 and 4 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>-6</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. \( y \)-int = \(-6\); axis of symmetry: \( x = 2 \); \( x \)-coordinate = \( \frac{1}{2} \)

b. 
21. \( f(x) = -2x^2 + 3x + 9 \)

**SOLUTION:**

- a. Compare the function \( f(x) = -2x^2 + 3x + 9 \) with the standard form of a quadratic function. Here, \( a = -2, b = 3 \) and \( c = 9 \). The y-intercept is 9.

The equation of the axis of symmetry is:

\[
x = -\frac{b}{2a} = -\frac{3}{2(-2)} = 0.75.
\]

The equation of the axis of symmetry is \( x = 0.75 \).

The x-coordinate of the vertex is \( -\frac{b}{2a} = 0.75 \).

- b. Substitute \(-1, 0, 0.75, 1.5\) and \(2.5\) for \( x \) and make the table.

\[
\begin{array}{c|c}
  x & f(x) \\
  \hline
  -1 & 4 \\
  0 & 9 \\
  0.75 & 10.125 \\
  1.5 & 9 \\
  2.5 & 4 \\
\end{array}
\]

- c. Graph the function.
4-1 Graphing Quadratic Functions

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

22. \( f(x) = 5x^2 \)

**SOLUTION:**

Compare the function \( f(x) = 5x^2 \) with the standard form of a quadratic function.

Here, \( a = 5, b = 0 \) and \( c = 0 \).

For this function, \( a = 5 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is \(-\frac{b}{2a} = -\frac{0}{2(5)} = 0\).

Substitute 0 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
\begin{align*}
f(0) &= 5(0)^2 \\
&= 0
\end{align*}
\]

Therefore, the minimum value of the function is 0.

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \)

The range is all real numbers greater than or equal to the minimum value.
\( R = \{ f(x) | f(x) \geq 0 \} \)

**ANSWER:**

\( \text{min} = 0; \ D = \{ \text{all real numbers} \}, \ \ R = \{ f(x) | f(x) \geq 0 \} \)

23. \( f(x) = -x^2 - 12 \)

**SOLUTION:**

Compare the function \( f(x) = -x^2 - 12 \) with the standard form of a quadratic function.

Here, \( a = -1, b = 0 \) and \( c = -12 \).

For this function, \( a = -1 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is

\[
\begin{align*}
\frac{b}{2a} &= -\frac{0}{2(-1)} = 0.
\end{align*}
\]

Substitute 0 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
\begin{align*}
f(0) &= -(0)^2 - 12 \\
&= -12
\end{align*}
\]

Therefore, the maximum value of the function is -12.

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \)

The range is all real numbers less than or equal to the maximum value.
\( R = \{ f(x) | f(x) \leq -12 \} \)

**ANSWER:**

\( \text{max} = -12; \ D = \{ \text{all real numbers} \}, \ \ R = \{ f(x) | f(x) \leq -12 \} \)
4-1 Graphing Quadratic Functions

24. \( f(x) = x^2 - 6x + 9 \)

**SOLUTION:**
Compare the function \( f(x) = x^2 - 6x + 9 \) with the standard form of a quadratic function.

Here, \( a = 1, b = -6 \) and \( c = 9 \).

For this function, \( a = 1 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is \( \frac{b}{2a} = \frac{-6}{2(1)} = 3 \).

Substitute 3 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
\begin{align*}
f(3) &= 3^2 - 6(3) + 9 \\
&= 9 - 18 + 9 \\
&= 0
\end{align*}
\]

Therefore, the minimum value of the function is 0.

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \).

The range is all real numbers greater than or equal to the minimum value.
\( R = \{ f(x) \mid f(x) \geq 0 \} \)

**ANSWER:**
\( \min = 0; D = \{ \text{all real numbers} \}, \)
\( R = \{ f(x) \mid f(x) \geq 0 \} \)

25. \( f(x) = -x^2 - 7x + 1 \)

**SOLUTION:**
Compare the function \( f(x) = -x^2 - 7x + 1 \) with the standard form of a quadratic function.

Here, \( a = -1, b = -7 \) and \( c = 1 \).

For this function, \( a = -1 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is
\[
\frac{-b}{2a} = \frac{-(-7)}{2(-1)} = -\frac{7}{2} = -3.5.
\]

Substitute \(-3.5\) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
\begin{align*}
f(-3.5) &= -(\text{(-3.5)}^2 - 7\text{(-3.5)} + 1 \\
&= -(12.25 + 24.5 + 1) \\
&= -37.75
\end{align*}
\]

Therefore, the maximum value of the function is \( -37.75 \).

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \).

The range is all real numbers less than or equal to the maximum value.
\( R = \{ f(x) \mid f(x) \leq 13.25 \} \)

**ANSWER:**
\( \max = 13.25; D = \{ \text{all real numbers} \}, \)
\( R = \{ f(x) \mid f(x) \leq 13.25 \} \)
26. \( f(x) = 8x - 3x^2 + 2 \)

**SOLUTION:**

 Compare the function \( f(x) = 8x - 3x^2 + 2 \) with the standard form of a quadratic function. Here, \( a = -3, \ b = 8 \) and \( c = 2 \).

 For this function, \( a = -3 \), so the graph opens down and the function has a maximum value.

 The \( x \)-coordinate of the vertex is

 \[
 \frac{-b}{2a} = \frac{-8}{2(-3)} = \frac{4}{3}.
 \]

 Substitute \( \frac{4}{3} \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

 \[
 \begin{align*}
 f\left(\frac{4}{3}\right) &= 8\left(\frac{4}{3}\right) - 3\left(\frac{4}{3}\right)^2 + 2 \\
 &= \frac{32}{3} - \frac{16}{3} + 2 \\
 &= \frac{22}{3}.
\end{align*}
\]

 Therefore, the maximum value of the function is \( \frac{22}{3} \).

 The domain is all real numbers. 
\( D = \{ \text{all real numbers} \} \).

 The range is all real numbers less than or equal to the maximum value.

 \( R = \left\{ f(x) \mid f(x) \leq \frac{22}{3} \right\} \)

**ANSWER:**

\( \text{max} = \frac{22}{3}; \ D = \{ \text{all real numbers} \}, \)

\( R = \left\{ f(x) \mid f(x) \leq \frac{22}{3} \right\} \)

27. \( f(x) = 5 - 4x - 2x^2 \)

**SOLUTION:**

 Compare the function \( f(x) = 5 - 4x - 2x^2 \) with the standard form of a quadratic function.

 Here, \( a = -2, \ b = -4 \) and \( c = 5 \).

 For this function, \( a = -2 \), so the graph opens down and the function has a maximum value.

 The \( x \)-coordinate of the vertex is

 \[
 \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = \frac{1}{2}.
 \]

 Substitute \( \frac{1}{2} \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

 \[
 \begin{align*}
 f\left(\frac{1}{2}\right) &= 5 - 4\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)^2 \\
 &= 5 - 2 - \frac{1}{2} \\
 &= \frac{7}{2}.
\end{align*}
\]

 Therefore, the maximum value of the function is \( \frac{7}{2} \).

 The domain is all real numbers. 
\( D = \{ \text{all real numbers} \} \).

 The range is all real numbers less than or equal to the maximum value.

 \( R = \left\{ f(x) \mid f(x) \leq \frac{7}{2} \right\} \)

**ANSWER:**

\( \text{max} = \frac{7}{2}; \ D = \{ \text{all real numbers} \}, \)

\( R = \left\{ f(x) \mid f(x) \leq \frac{7}{2} \right\} \)
28. \( f(x) = 15 - 5x^2 \)

**SOLUTION:**

Compare the function \( f(x) = 15 - 5x^2 \) with the standard form of a quadratic function.

Here, \( a = -5, b = 0 \) and \( c = 15 \).

For this function, \( a = -5 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is
\[
-\frac{b}{2a} = -\frac{0}{2(-5)} = 0.
\]

Substitute 0 for \( x \) in the function to find the \( y \)-coordinate of the vertex.
\[
f(0) = 15 - 5(0)^2 = 15
\]

Therefore, the maximum value of the function is 15.

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \).

The range is all real numbers less than or equal to the maximum value.
\( R = \{ f(x) \mid f(x) \leq 15 \} \)

**ANSWER:**

\( \text{max} = 15; \ D = \{ \text{all real numbers} \}, \ R = \{ f(x) \mid f(x) \leq 15 \} \)

29. \( f(x) = x^2 + 12x + 27 \)

**SOLUTION:**

Compare the function \( f(x) = x^2 + 12x + 27 \) with the standard form of a quadratic function.

Here, \( a = 1, b = 12 \) and \( c = 27 \).

For this function, \( a = 1 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is
\[
-\frac{b}{2a} = -\frac{12}{2(1)} = -6.
\]

Substitute \( -6 \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.
\[
f(-6) = (-6)^2 + 12(-6) + 27 = 36 - 72 + 27 = -9
\]

Therefore, the minimum value of the function is \( -9 \).

The domain is all real numbers.
\( D = \{ \text{all real numbers} \} \).

The range is all real numbers greater than or equal to the minimum value.
\( R = \{ f(x) \mid f(x) \geq -9 \} \)

**ANSWER:**

\( \text{min} = -9; \ D = \{ \text{all real numbers} \}, \ R = \{ f(x) \mid f(x) \geq -9 \} \)
4-1 Graphing Quadratic Functions

30. \( f(x) = -x^2 + 10x + 30 \)

**SOLUTION:**

Compare the function \( f(x) = -x^2 + 10x + 30 \) with the standard form of a quadratic function.

Here, \( a = -1, \ b = 10 \) and \( c = 30 \).

For this function, \( a = -1 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is

\[
\frac{-b}{2a} = \frac{-10}{2(-1)} = 5.
\]

Substitute 5 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(5) = -(5)^2 + 10(5) + 30
\]

\[
= -25 + 50 + 30
\]

\[
= 55
\]

Therefore, the maximum value of the function is 55.

The domain is all real numbers.
\( D = \{\text{all real numbers}\} \).

The range is all real numbers less than or equal to the maximum value.
\( R = \{ f(x) | f(x) \leq 55 \} \)

**ANSWER:**

\( \text{max} = 55; \ D = \{\text{all real numbers}\}, \ R = \{ f(x) | f(x) \leq 55 \} \)

31. \( f(x) = 2x^2 - 16x - 42 \)

**SOLUTION:**

Compare the function \( f(x) = 2x^2 - 16x - 42 \) with the standard form of a quadratic function.

Here, \( a = 2, \ b = -16 \) and \( c = -42 \).

For this function, \( a = 2 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is

\[
\frac{-b}{2a} = \frac{-(-16)}{2(2)} = 4.
\]

Substitute 4 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(4) = 2(4)^2 - 16(4) - 42
\]

\[
= 32 - 64 - 42
\]

\[
= -74
\]

Therefore, the minimum value of the function is \(-74\).

The domain is all real numbers.
\( D = \{\text{all real numbers}\} \).

The range is all real numbers greater than or equal to the minimum value.
\( R = \{ f(x) | f(x) \geq -74 \} \)

**ANSWER:**

\( \text{min} = -74; \ D = \{\text{all real numbers}\}, \ R = \{ f(x) | f(x) \geq -74 \} \)
4-1 Graphing Quadratic Functions

32. **CCSS MODELING** A financial analyst determined that the cost, in thousands of dollars, of producing bicycle frames is 
\[ C = 0.000025x^2 - 0.04x + 40 \]
where \( f \) is the number of frames produced.

a. Find the number of frames that minimizes cost.

b. What is the total cost for that number of frames?

**SOLUTION:**

- **b.** The vertex is:
  \[ -\frac{b}{2a} = -\frac{-0.04}{2(0.000025)} = 800 \]
  The number of frames that minimize the cost is 800.

- **b.** Substitute 800 for \( f \) in the function and simplify.
  \[ C = 0.000025(800)^2 - 0.04(800) + 40 \]
  \[ = 16 - 32 + 40 \]
  \[ = 24 \]
  Therefore, the total cost is $24, 000.

**ANSWER:**

a. 800
b. $24,000

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

33. \( f(x) = -3x^2 - 9x + 2 \)

**SOLUTION:**

- **a.** Compare the function \( f(x) = -3x^2 - 9x + 2 \) with the standard form of a quadratic function.

Here, \( a = -3, b = -9 \) and \( c = 2 \).

The y-intercept is 2.

The equation of the axis of symmetry is

\[ x = -\frac{b}{2a} = -\frac{-9}{2(-3)} = -1.5 \]

Equation of the axis of symmetry is \( x = -1.5 \).

The x-coordinate of the vertex is \( \frac{b}{2a} = -1.5 \).

b. Substitute \(-3, -2, -1.5, -1\) and 0 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.75</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

c.
34. \( f(x) = 2x^2 - 6x - 9 \)

**SOLUTION:**

a. Compare the function \( f(x) = 2x^2 - 6x - 9 \) with the standard form of a quadratic function.

Here, \( a = 2, b = -6 \) and \( c = -9 \).

The \( y \)-intercept is \(-9\).

The equation of the axis of symmetry is
\[
x = -\frac{b}{2a} = -\frac{-6}{2(2)} = 1.5.
\]

Therefore, \( x = 1.5 \) is the axis of symmetry.

The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = 1.5 \).

b. Substitute 0, 1, 1.5, 2 and 3 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>1</td>
<td>-13</td>
</tr>
<tr>
<td>1.5</td>
<td>-13.5</td>
</tr>
<tr>
<td>2</td>
<td>-13</td>
</tr>
<tr>
<td>3</td>
<td>-9</td>
</tr>
</tbody>
</table>

c. Graph the function.

35. \( f(x) = -4x^2 + 5x \)

**SOLUTION:**

a. Compare the function \( f(x) = -4x^2 + 5x \) with the standard form of a quadratic function.

Here, \( a = -4, b = 5 \) and \( c = 0 \).

The \( y \)-intercept is 0.

The equation of the axis of symmetry is
\[
x = -\frac{b}{2a} = -\frac{5}{2(-4)} = \frac{5}{8}.
\]
4-1 Graphing Quadratic Functions

Equation of the axis of symmetry is \( x = \frac{5}{8} \).

The \( x \)-coordinate of the vertex is \( \frac{-b}{2a} = \frac{5}{8} \).

b. Substitute \(-3, -2, -1.5, -1, 0\) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{4} )</td>
<td>(-6)</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{5}{8} )</td>
<td>1.5625</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(-6)</td>
</tr>
</tbody>
</table>

c. Graph the function.

36. \( f(x) = 2x^2 + 11x \)

**SOLUTION:**

a. Compare the function \( f(x) = 2x^2 + 11x \) with the standard form of a quadratic function.

Here, \( a = 2 \), \( b = 11 \) and \( c = 0 \).

The \( y \)-intercept is 0.

The equation of the axis of symmetry is

\[ x = -\frac{b}{2a} = -\frac{11}{2(2)} = -2.75. \]

Equation of the axis of symmetry is \( x = -2.75 \).

The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = -2.75 \).

b. Substitute \(-4, -3, -2.5, -1.5 \) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4)</td>
<td>(-12)</td>
</tr>
<tr>
<td>(-3)</td>
<td>(-15)</td>
</tr>
<tr>
<td>(-2.5)</td>
<td>(-15.125)</td>
</tr>
<tr>
<td>(-1.5)</td>
<td>(-12)</td>
</tr>
</tbody>
</table>

c. Graph the function.
4-1 Graphing Quadratic Functions

ANSWER:

a. $y$-int = 0; axis of symmetry: $x = -2.75$; $x$-coordinate of vertex = $-2.75$

b.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-12</td>
</tr>
<tr>
<td>-3</td>
<td>-15</td>
</tr>
<tr>
<td>-2.75</td>
<td>-15.125</td>
</tr>
<tr>
<td>-2.5</td>
<td>-15</td>
</tr>
<tr>
<td>-1.5</td>
<td>-12</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. $y$-int = 4; axis of symmetry: $x = -6$; $x$-coordinate of vertex = $-6$

b.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>-1</td>
</tr>
<tr>
<td>-8</td>
<td>-4</td>
</tr>
<tr>
<td>-6</td>
<td>-5</td>
</tr>
<tr>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
</tbody>
</table>

c.

37. $f(x) = 0.25x^2 + 3x + 4$

SOLUTION:

a. Compare the function $f(x) = 0.25x^2 + 3x + 4$ with the standard form of a quadratic function.

Here, $a = 0.25$, $b = 3$ and $c = 4$.

The $y$-intercept is 4.

Equation of the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{3}{2(0.25)} = -6.$$
4-1 Graphing Quadratic Functions

38. \( f(x) = -0.75x^2 + 4x + 6 \)

**SOLUTION:**

a. Compare the function \( f(x) = -0.75x^2 + 4x + 6 \) with the standard form of a quadratic function.

Here, \( a = -0.75 \), \( b = 4 \) and \( c = 6 \).

The \( y \)-intercept is 6.

The equation of the axis of symmetry is

\[
x = -\frac{b}{2a} = -\frac{4}{2(-0.75)} = \frac{4}{1.5} = \frac{8}{3}.
\]

Equation of the axis of symmetry is \( x = \frac{8}{3} \).

The \( x \)-coordinate of the vertex is \( -\frac{b}{2a} = \frac{8}{3} \).

b. Substitute \( \frac{4}{3} \), \( \frac{7}{3} \), \( \frac{8}{3} \), 3 and 4 for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{4}{3} )</td>
<td>10</td>
</tr>
<tr>
<td>( \frac{7}{3} )</td>
<td>11.25</td>
</tr>
<tr>
<td>( \frac{8}{3} )</td>
<td>11(\frac{1}{3})</td>
</tr>
<tr>
<td>3</td>
<td>11.25</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

39. \( f(x) = \frac{3}{2}x^2 + 4x - \frac{5}{2} \)

**SOLUTION:**

a. Compare the function \( f(x) = \frac{3}{2}x^2 + 4x - \frac{5}{2} \) with the standard form of a quadratic function.

Here, \( a = \frac{3}{2} \), \( b = 4 \) and \( c = -\frac{5}{2} \).
4-1 Graphing Quadratic Functions

The y-intercept is $-\frac{5}{2}$ or $-2.5$.

The equation of the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{4}{2 \cdot \left(\frac{3}{2}\right)} = -\frac{4}{3}.$$

Therefore, $x = -\frac{4}{3}$ is the axis of symmetry.

The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{4}{3}$.

b. Substitute $\frac{11}{3}, \frac{8}{3}, \frac{4}{3}, 0$ and $1$ for $x$ and make the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{11}{3}$</td>
<td>3</td>
</tr>
<tr>
<td>$\frac{8}{3}$</td>
<td>$-2.5$</td>
</tr>
<tr>
<td>$\frac{4}{3}$</td>
<td>$-5\frac{1}{6}$</td>
</tr>
<tr>
<td>0</td>
<td>$-2.5$</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. $y$-int $=-2.5$; axis of symmetry: $x = -\frac{4}{3}$; $x$-coordinate of vertex $= -\frac{4}{3}$

b.

d. Substitute $-3, -2, -1.5, -1$ and $0$ for $x$ and make the table.

c. Graph the function.

ANSWER:

72. SOLUTION:

Let $\det (A) = 14$

det (A) = 14

Evaluate each determinant.

73.

Therefore, option C is the correct answer.

SOLUTION:

Is this reasonable?

Let $K$ be the number of coffee beans required to make 1 cup of coffee.

f

b.

The number of decrease is 4. Then his income is:

The vertex of the graph is $(3, -9.375)$.

The domain is all real numbers.

The maximum value.

Therefore, the cost of the coffee bean required to make 1 cup of coffee?

The discriminant is negative, there will be two complex solutions.

The function has a maximum value.

$\sqrt{0}$.

Therefore, three $0.50 increases is reasonable.

Here, $a = \frac{2}{3}$, $b = -\frac{7}{3}$ and $c = 9$.

The y-intercept is 9.

The equation of the axis of symmetry is

$$x = -\frac{b}{2a} = -\frac{(-7/3)}{2(2/3)} = 1.75.$$

Therefore, $x = 1.75$ is the axis of symmetry.

The $x$-coordinate of the vertex is $-\frac{b}{2a} = 1.75$.

b. Substitute $0.5, 1.5, 1.75, 2$ and $3$ for $x$ and make the table.
4-1 Graphing Quadratic Functions

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>1.5</td>
<td>7</td>
</tr>
<tr>
<td>1.75</td>
<td>6.25</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

c. Graph the function.

ANSWER:

a. y-int = 9; axis of symmetry: $x = 1.75$; x-coordinate of vertex = 1.75

b. Let $y = -x^2 + 6x + 475.$

$$I(x) = (9.5 + 0.5x)(50 - 2x)$$
$$= 475 - 19x + 25x - x^2$$
$$= -x^2 + 6x + 475$$

b. The function is defined in the interval [0, 25]. Therefore, $D = \{x | 0 \leq x \leq 25\}.$

The maximum value of the function is 484. Therefore, $R = \{y | 0 \leq y \leq 484\}.$

c. $\$11;$ Because the function has a maximum at $x = 3,$ it is in the domain. Therefore, three $\$0.50$ increases is reasonable.

d. The value of the function at $x = 3$ is 484. Therefore, the maximum income the club can expect to make is $\$484.$

ANSWER:

a. $I(x) = -x^2 + 6x + 475$

b. $D = \{x | 0 \leq x \leq 25\}; \ R = \{y | 0 \leq y \leq 484\}$

c. $\$11;$ Because the function has a maximum at $x = 3,$ it is in the domain. Therefore, three $\$0.50$ increases is reasonable.

d. $\$484$

41. FINANCIAL LITERACY A babysitting club sits for 50 different families. They would like to increase their current rate of $\$9.50$ per hour. After surveying the families, the club finds that the number of families will decrease by about 2 for each $\$0.50$ increase in the hourly rate.

a. Write a quadratic equation that models this situation.

b. State the domain and range of this function as it applies to the situation.

c. What hourly rate will maximize the club’s income? Is this reasonable?

d. What is the maximum income the club can expect to make?

SOLUTION:

a. Let $x$ be the number of increase.

$$I(x) = (9.5 + 0.5x)(50 - 2x)$$
$$= 475 - 19x + 25x - x^2$$
$$= -x^2 + 6x + 475$$

b. The function is defined in the interval [0, 25]. Therefore, $D = \{x | 0 \leq x \leq 25\}.$

The maximum value of the function is 484. Therefore, $R = \{y | 0 \leq y \leq 484\}.$

c. $\$11;$ Because the function has a maximum at $x = 3,$ it is in the domain. Therefore, three $\$0.50$ increases is reasonable.

d. The value of the function at $x = 3$ is 484. Therefore, the maximum income the club can expect to make is $\$484.$
4-1 Graphing Quadratic Functions

42. ACTIVITIES Last year, 300 people attended the Franklin High School Drama Club’s winter play. The ticket price was $8. The advisor estimates that 20 fewer people would attend for each $1 increase in ticket price.

   a. What ticket price would give the greatest income for the Drama Club?

   b. If the Drama Club raised its tickets to this price, how much income should it expect to bring in?

**SOLUTION:**

   a. Let \( x \) be the number of increase.

   \[
   f(x) = (8+x)(300-20x)
   \]

   \[
   = 2400 - 160 + 300x - 20x^2
   \]

   \[
   = -20x^2 + 140x + 2400
   \]

   The function gets maximum value at 3.5.

   Therefore, $11.50 will give the greatest income for the Drama Club.

   b. Substitute 3.5 for \( x \) in the function and simplify.

   \[
   f(3.5) = -20(3.5)^2 + 140(3.5) + 2400
   \]

   \[
   = -245 + 490 + 2400
   \]

   \[
   = 2645
   \]

   The Drama Club will get $2645.

   **ANSWER:**

   a. $11.50

   b. $2645

43. \( f(x) = 12x^2 - 21x + 8 \)

**SOLUTION:**

Enter \( 12x^2 - 21x + 8 \) as Y1.

**KEYSTROKES:**

\[
Y1 = 1 \quad \boxed{[1] \quad X, \quad T, \quad \theta, \quad n} \quad X^2 \quad - \quad 2
\]

\[
1 \quad \boxed{[1] \quad X, \quad T, \quad \theta, \quad n} \quad + \quad 8
\]

Fix the left and right bounds.

**KEYSTROKES:**

\[2nd \quad \boxed{[CALC]} \quad \boxed{3} \quad \boxed{\leftarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \]

\[\boxed{\leftarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \quad \boxed{\rightarrow} \]

\[\text{ENTER} \quad \text{ENTER} \]

Minimum

\[X = 0.75000273 \quad Y = 1.1875 \]

So, the maximum value of the function is \(-1.19\).

**ANSWER:**

\[\text{min} = -1.19\]
4-1 Graphing Quadratic Functions

44. \( f(x) = -9x^2 - 12x + 19 \)

**SOLUTION:**
Enter \(-9x^2 - 12x + 19\) as Y1.

**KEYSTROKES:** Y= (-) X,T,0,n X^2 - 1
2 X,T,0,n + 1 9

Fix the left and right bounds.

**KEYSTROKES:** 2nd [CALC] 4 ◄ ENTER ► ► ► ► ► ► ► ► ► ► ◄ ENTER ► ENTER

So, the maximum value of the function is 23.

**ANSWER:**
max = 23

45. \( f(x) = -8.3x^2 + 14x - 6 \)

**SOLUTION:**
Enter \(-8.3x^2 + 14x - 6\) as Y1.

**KEYSTROKES:** Y= (-) 8 . 3 X,T,0,n X^2 + 1 4 X,T,0,n - 6

Fix the left and right bounds.

**KEYSTROKES:** 2nd [CALC] 4 ◄ ENTER ► ► ► ► ► ► ► ► ► ► ◄ ENTER ► ENTER

So, the maximum value of the function is –0.01.

**ANSWER:**
max = –0.01
4-1 Graphing Quadratic Functions

46. \( f(x) = 9.7x^2 - 13x - 9 \)

**SOLUTION:**

Enter \( 9.7x^2 - 13x - 9 \) as Y1.

**KEYSTROKES:**

\[ Y = 9 \cdot 7 \begin{bmatrix} X, T, 0, n \end{bmatrix} X^2 - 1 \]
\[ 3 \begin{bmatrix} X, T, 0, n \end{bmatrix} - 9 \]

Fix the left and right bounds.

**KEYSTROKES:**

2nd [CALC] 3 ENTER ►►►►►►►► ENTER ENTER

So, the maximum value of the function is \(-13.36\).

**ANSWER:**

\( \text{min} = -13.36 \)

47. \( f(x) = 28x - 15 - 18x^2 \)

**SOLUTION:**

Enter \( 28x - 15 - 18x^2 \) as Y1.

**KEYSTROKES:**

\[ Y = 2 \begin{bmatrix} 8, X, T, 0, n \end{bmatrix} - 1 \begin{bmatrix} 5, 1 \end{bmatrix} \]
\[ 8 \begin{bmatrix} X, T, 0, n \end{bmatrix} X^2 \]

Fix the left and right bounds.

**KEYSTROKES:**

2nd [CALC] 4 ENTER ►►►►►►►► ENTER ENTER

So, the maximum value of the function is \(-4.11\).

**ANSWER:**

\( \text{max} = -4.11 \)
4-1 Graphing Quadratic Functions

48. \( f(x) = -16 - 14x - 12x^2 \)

**SOLUTION:**
Enter \(-16 - 14x - 12x^2\) as Y1.

**KEYSTROKES:**
Y= (-) 1 6 - 1

4 \[\text{X,T,0,n}\] – 1 2 \[\text{X,T,0,n}\] \(X^2\)

Fix the left and right bounds.

**KEYSTROKES:**
2nd [CALC] 4 ▲ ▲
ENTER ENTER

![Graph of quadratic function]

So, the maximum value of the function is \(-11.92\).

**ANSWER:**
max = \(-11.92\)

Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.

49. \( f(x) = -5x^2 + 4x - 8 \)

**SOLUTION:**
Compare the function \( f(x) = -5x^2 + 4x - 8 \) with the standard form of a quadratic function.

Here, \( a = -5 \), \( b = 4 \) and \( c = -8 \).

For this function, \( a = -5 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is
\[
\frac{-b}{2a} = \frac{-4}{2(-5)} = 0.4.
\]

Substitute 0.4 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(0.4) = -5(0.4)^2 + 4(0.4) - 8
= -0.8 + 1.6 - 8
= -7.2
\]

Therefore, the maximum value of the function is \(-7.2\).

The domain is all real numbers.
D = \{ all real numbers \}.

The range is all real numbers less than or equal to the maximum value.
\[
R = \{ f(x) | f(x) \leq -7.2 \}
\]

**ANSWER:**
max = \(-7.2\); D = \{ all real numbers \},
\[
R = \{ f(x) | f(x) \leq -7.2 \}
\]
50. \( f(x) = -4x^2 - 3x + 2 \)

**SOLUTION:**

Compare the function \( f(x) = -4x^2 - 3x + 2 \) with the standard form of a quadratic function. Here, \( a = -4, b = -3 \) and \( c = 2 \).

For this function, \( a = -4 \), so the graph opens down and the function has a maximum value. The \( x \)-coordinate of the vertex is

\[
\frac{-b}{2a} = \frac{-(-3)}{2(-4)} = \frac{3}{8}.
\]

Substitute \( \frac{3}{8} \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f\left(\frac{3}{8}\right) = -4\left(\frac{3}{8}\right)^2 - 3\left(\frac{3}{8}\right) + 2
\]

\[
= -4\frac{9}{64} - \frac{9}{8} + 2
\]

\[
= \frac{-36 + 9 + 128}{64}
\]

\[
= 2.5625
\]

Therefore, the maximum value of the function is 2.5625.

The domain is all real numbers.
D = \{all real numbers\}.

The range is all real numbers less than or equal to the maximum value.
\( R = \{f(x) | f(x) \leq 2.5625\} \)

**ANSWER:**
max = 2.5625; D = \{all real numbers\},
\( R = \{f(x) | f(x) \leq 2.5625\} \)

51. \( f(x) = -9 + 3x + 6x^2 \)

**SOLUTION:**

Compare the function \( f(x) = -9 + 3x + 6x^2 \) with the standard form of a quadratic function.

Here, \( a = 6, b = 3 \) and \( c = -9 \).

For this function, \( a = 6 \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is

\[
\frac{-b}{2a} = \frac{-3}{2(6)} = -0.25.
\]

Substitute \(-0.25\) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(-0.25) = -9 + 3(-0.25) + 6(-0.25)^2
\]

\[
= -9 - 0.75 + 0.375
\]

\[
= -9.375
\]

Therefore, the minimum value of the function is \(-9.375\).

The domain is all real numbers.
D = \{all real numbers\}.

The range is all real numbers greater than or equal to the minimum value.
\( R = \{f(x) | f(x) \geq -9.375\} \)

**ANSWER:**
min = \(-9.375\); D = \{all real numbers\},
\( R = \{f(x) | f(x) \geq -9.375\} \)
52. \( f(x) = 2x - 5 - 4x^2 \)

**SOLUTION:**

Compare the function \( f(x) = 2x - 5 - 4x^2 \) with the standard form of a quadratic function.

Here, \( a = -4, b = 2 \) and \( c = -5 \).

For this function, \( a = -4 \), so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is

\[
\frac{-b}{2a} = \frac{-2}{2(-4)} = 0.25.
\]

Substitute 0.25 for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(0.25) = 2(0.25) - 5 - 4(0.25)^2
\]

\[
= 0.5 - 5 - 0.25
\]

\[
= -4.75
\]

Therefore, the maximum value of the function is \(-4.75\).

The domain is all real numbers.

\( D = \{ \text{all real numbers} \} \).

The range is all real numbers less than or equal to the maximum value.

\( R = \{ f(x) | f(x) \leq -4.75 \} \)

**ANSWER:**

\( \text{max} = -4.75; \ D = \{ \text{all real numbers} \}, \ R = \{ f(x) | f(x) \leq -4.75 \} \)

53. \( f(x) = \frac{2}{3}x^2 + 6x - 10 \)

**SOLUTION:**

Compare the function \( f(x) = \frac{2}{3}x^2 + 6x - 10 \) with the standard form of a quadratic function.

Here, \( a = \frac{2}{3}, b = 6 \) and \( c = -10 \).

For this function, \( a = \frac{2}{3} \), so the graph opens up and the function has a minimum value.

The \( x \)-coordinate of the vertex is

\[
-\frac{b}{2a} = -\frac{6}{2(\frac{2}{3})} = -4.5.
\]

Substitute \(-4.5\) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(-4.5) = \frac{2}{3}(-4.5)^2 + 6(-4.5) - 10
\]

\[
= 13.5 - 27 - 10
\]

\[
= -23.5
\]

Therefore, the minimum value of the function is \(-23.5\).

The domain is all real numbers.

\( D = \{ \text{all real numbers} \} \).

The range is all real numbers greater than or equal to the minimum value.

\( R = \{ f(x) | f(x) \geq -23.5 \} \)

**ANSWER:**

\( \text{min} = -23.5; \ D = \{ \text{all real numbers} \}, \ R = \{ f(x) | f(x) \geq -23.5 \} \)
4-1 Graphing Quadratic Functions

54. \( f(x) = -\frac{3}{5}x^2 + 4x - 8 \)

**SOLUTION:**

Compare the function \( f(x) = -\frac{3}{5}x^2 + 4x - 8 \) with the standard form of a quadratic function.

Here, \( a = -\frac{3}{5}, \ b = 4 \) and \( c = -8 \).

For this function, \( a = -\frac{3}{5}, \) so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is

\[
-x = \frac{b}{2a} = \frac{4}{2\left(-\frac{3}{5}\right)} = \frac{10}{-6} = \frac{-5}{3}.
\]

Substitute \( \frac{10}{3} \) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f\left(\frac{10}{3}\right) = -\frac{3}{5} \left(\frac{10}{3}\right)^2 + 4 \left(\frac{10}{3}\right) - 8
\]

\[
= -\frac{20}{3} + \frac{40}{3} - 8
\]

\[
= -\frac{4}{3}
\]

Therefore, the maximum value of the function is \(-\frac{4}{3}\).

The domain is all real numbers.

\( D = \{ \text{all real numbers} \} \).

The range is all real numbers less than or equal to the maximum value.

\( R = \left\{ f(x) | f(x) \leq -\frac{4}{3} \right\} \)

**ANSWER:**

\( \text{max} = -\frac{4}{3}; \ D = \{ \text{all real numbers} \}, \)

\( R = \left\{ f(x) | f(x) \leq -\frac{4}{3} \right\} \)

55. **Determine the function represented by each graph.**

![Graph](image)

**SOLUTION:**

Given graph is a parabola. Therefore, the function must be in the form of \( f(x) = ax^2 + bx + c \).

Substitute the points \((0, -5)\) and \((2, -9)\) in the function.

\[
-5 = a(0)^2 + b(0) + c
\]

\( c = -5 \)

\[
-9 = a(2)^2 + b(2) - 5
\]

\( 4a + 2b = -4 \) \( \rightarrow (1) \)

\( 2a + b = -2 \) \( \rightarrow (2) \)

The vertex of the graph is \((2, -9)\).

Therefore, the \( x \)-coordinate of the vertex is \( \frac{b}{2a} = 2 \).

\( b = -4a \) \( \rightarrow (1) \)

Substitute \(-4a\) for \( b \) in the first equation and solve for \( a \).

\( 2a - 4a = -2 \)

\( -2a = -2 \)

\( a = 1 \)

Substitute 1 for \( a \) in the second equation and solve for \( b \).

\( b = -4(1) \)

\( b = -4 \)

Therefore, the required function is

\( f(x) = x^2 - 4x - 5 \).
4-1 Graphing Quadratic Functions

ANSWER:
\[ f(x) = x^2 - 4x - 5 \]

Therefore, the required function is
\[ f(x) = x^2 + 2x - 6. \]

ANSWER:
\[ f(x) = x^2 + 2x - 6 \]

SOLUTION:
Given graph is a parabola. Therefore, the function must be in the form \( f(x) = ax^2 + bx + c \).

Substitute the points \((-1, -7)\) and \((0, -6)\) in the function.

\[-6 = a(0)^2 + b(0) + c \]
\[c = -6 \]

\[-7 = a(-1)^2 + b(-1) - 6 \]
\[a - b = -1 \quad \rightarrow (1) \]

The vertex of the graph is \((-1, -7)\).

Therefore, the x-coordinate of the vertex is
\[-\frac{b}{2a} = -1.\]

\[b = 2a \quad \rightarrow (2)\]

Substitute \(2a\) for \(b\) in the first equation and solve for \(a\).

\[a - 2a = -1 \]
\[-a = -1 \]
\[a = 1 \]

Substitute 1 for \(a\) in the second equation and solve for \(b\).

\[b = 2(1) \]
\[b = 2 \]
4-1 Graphing Quadratic Functions

57. **SOLUTION:**
Given graph is a parabola. Therefore, the function must be in the form of \( f(x) = ax^2 + bx + c \).

Substitute the points \((0, 8)\) and \((3, -1)\) in the function.

\[
8 = a(0)^2 + b(0) + c
\]
\[c = 8\]

\[
-1 = a(3)^2 + b(3) + 8
\]
\[9a + 3b = -9\]
\[3a + b = 3 \quad \rightarrow (1)\]

The vertex of the graph is \((3, -1)\).

Therefore, the \(x\)-coordinate of the vertex is

\[
\frac{-b}{2a} = 3.
\]
\[b = -6a \quad \rightarrow (2)\]

Substitute \(-6a\) for \(b\) in the first equation and solve for \(a\).

\[3a - 6a = -3\]
\[-3a = -3\]
\[a = 1\]

Substitute 1 for \(a\) in the second equation and solve for \(b\).

\[b = -6(1)\]
\[b = -6\]

Therefore, the required function is

\[f(x) = x^2 - 6x + 8\]

**ANSWER:**
\[f(x) = x^2 - 6x + 8\]

58. **MULTIPLE REPRESENTATIONS** Consider \(f(x) = x^2 - 4x + 8\) and \(g(x) = 4x^2 - 4x + 8\).

a. **TABULAR** Make a table of values for \(f(x)\) and \(g(x)\) if \(-4 \leq x \leq 4\).

b. **GRAPHICAL** Graph \(f(x)\) and \(g(x)\).

c. **VERBAL** Explain the difference in the shapes of the graphs of \(f(x)\) and \(g(x)\). What value was changed to cause this difference?

d. **ANALYTICAL** Predict the appearance of the graph of \(h(x) = 0.25x^2 - 4x + 8\). Confirm your prediction by graphing all three functions if \(-10 \leq x \leq 10\).

**SOLUTION:**

\[
\begin{array}{c|cc}
\hline
x & f(x) & g(x) \\
-4 & 40 & 88 \\
-3 & 29 & 56 \\
-2 & 20 & 32 \\
-1 & 13 & 16 \\
0 & 8 & 8 \\
1 & 5 & 8 \\
2 & 4 & 16 \\
3 & 5 & 32 \\
4 & 8 & 56 \\
\hline
\end{array}
\]

b. Sample answer: \(g(x)\) is much narrower than \(f(x)\). The value of \(a\) changed from 1 to 4.

d. Sample answer: The graph of \(h(x)\) will be wider than \(f(x)\).
59. **VENDING MACHINES** Omar owns a vending machine in a bowling alley. He currently sells 600 cans of soda per week at $0.65 per can. He estimates that he will lose 100 customers for every $0.05 increase in price and gain 100 customers for every $0.05 decrease in price. (Hint: The charge must be a multiple of 5.)

a. Write and graph the related quadratic equation for a price increase.

b. If Omar lowers the price, what price should he charge in order to maximize his income?

c. What will be his income per week from the vending machine?

**SOLUTION:**

a. Let \( x \) be the number of increase.

Convert the price into cents.

\[
f(x) = (65 + 5x)(600 - 100x)
\]

\[
= 39000 - 6500x + 3000x - 500x^2
\]

\[
= -500x^2 - 3500x + 39000
\]

b. Let \( x \) be the number of decrease. Convert the price into cents.

\[
f(x) = (65 - 5x)(600 + 100x)
\]

\[
= 39000 + 6500x - 3000x - 500x^2
\]

\[
= -500x^2 + 3500x + 39000
\]

The function is maximum at 3.5.

Therefore, Omar should charge \((65 - 5(3.5) = 475)\) 45 cents or 50 cents.
4-1 Graphing Quadratic Functions

c. Suppose the number of decrease is 3. Then his income is:

\[ f(x) = (65 - 5(3))(600 + 100(3)) \]
\[ = 50 \times 900 \]
\[ = 45000 \text{ cents or } $450 \]

Suppose the number of decrease is 4. Then his income is:

\[ f(x) = (65 - 5(4))(600 + 100(4)) \]
\[ = 45 \times 1000 \]
\[ = 45000 \text{ cents or } $450 \]

Omar’s income per week from the vending machine is $450.

**ANSWER:**

a. \( f(x) = 39,000 - 3500x - 500x^2 \)

b. Omar can charge at 45 cents or 50 cents.

c. $450 per week

60. **BASEBALL** Lolita throws a baseball into the air and the height \( h \) of the ball in feet at a given time \( t \) in seconds after she releases the ball is given by the function

\[ h(t) = -16t^2 + 30t + 5. \]

a. State the domain and range for this situation.

b. Find the maximum height the ball will reach.

**SOLUTION:**

a. Time \( t \) is always positive. So, \( t \) is greater than or equal to zero. The \( t \)-intercept of the function is 2.09.

Therefore, \( D = \{t| 0 \leq t \leq 2.09\} \).

The \( x \)-coordinate of the vertex is

\[ b = \frac{-30}{2a} = \frac{-30}{2(-16)} = 0.9375 \]

The maximum of the function.

\[ h(0.9375) = -16(0.9375)^2 + 30(0.9375) + 5 \]
\[ = 19.0625 \]

Therefore, \( R = \{h(t)| 0 \leq h(t) \leq 19.0625\} \).

b. The maximum height the ball will reach is 19.0625 ft.

**ANSWER:**

a. \( D = \{t| 0 \leq t \leq 2.09\}, R = \{h(t)| 0 \leq h(t) \leq 19.0625\} \)

b. 19.0625 ft
4-1 Graphing Quadratic Functions

61. **CCSS CRITIQUE** Trent thinks that the function \( f(x) \) graphed below, and the function \( g(x) \) described next to it have the same maximum. Madison thinks that \( g(x) \) has a greater maximum. Is either of them correct? Explain your reasoning.

![Graph of f(x) and g(x)](image)

**SOLUTION:**
Sample answer: Madison. Sample answer: \( f(x) \) has a maximum of \(-2\). \( g(x) \) has a maximum of \(1\). When Trent found the \( x \)-coordinate of the vertex, he multiplied two negatives and mistakenly kept a negative.

**ANSWER:**
Sample answer: Madison. When Trent found the \( x \)-coordinate of the vertex, he multiplied two negatives and mistakenly kept a negative.

62. **REASONING** Determine whether the following is sometimes, always, or never true. Explain your reasoning.

In a quadratic function, if two \( x \)-coordinates are equidistant from the axis of symmetry, then they will have the same \( y \)-coordinate.

**SOLUTION:**
Sample answer: Always; the coordinates of a quadratic function are symmetrical, so \( x \)-coordinates equidistant from the vertex will have the same \( y \)-coordinate.

**ANSWER:**
Sample answer: Always; the coordinates of a quadratic function are symmetrical, so \( x \)-coordinates equidistant from the vertex will have the same \( y \)-coordinate.

63. **CHALLENGE** The table at the right represents some points on the graph of a quadratic function.

### Table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>-377</td>
</tr>
<tr>
<td>c</td>
<td>-13</td>
</tr>
<tr>
<td>-5</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>22</td>
</tr>
<tr>
<td>d - 1</td>
<td>a</td>
</tr>
<tr>
<td>5</td>
<td>a - 24</td>
</tr>
<tr>
<td>7</td>
<td>-b</td>
</tr>
<tr>
<td>15</td>
<td>-202</td>
</tr>
<tr>
<td>14 - c</td>
<td>-377</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**a.** Substitute the points \((-20, -377)\), \((-5, -2)\), and \((-1, 22)\) from the table in the general quadratic function \( f(x) = ax^2 + bx + c \) to get a system of three equations in three variables.

\[
\begin{align*}
400a - 20b + c &= -377 \\
25a - 5b + c &= -2 \\
a - b + c &= 22
\end{align*}
\]

The solution of the system is \( a = -1, b = 0, \) and \( c = 23 \). So, the quadratic function is \( f(x) = -x^2 + 23 \). Substitute \((5, a - 24)\) into \( f(x) \) to find \( a \).

\[
f(x) = -x^2 + 23
\]

\[
a - 24 = -5^2 + 23
\]

\[
a = 22
\]

Substitute \((7, -b)\) into \( f(x) \) to find \( b \).
4-1 Graphing Quadratic Functions

\[ f(x) = -x^2 + 23 \]
\[-b = -7^2 + 23 \]
\[ b = 26 \]

Substitute \((c, -13)\) into \(f(x)\) to find \(c\).

\[ f(x) = -x^2 + 23 \]
\[-13 = -c^2 + 23 \]
\[ 36 = c^2 \]
\[ \pm 6 = c \]

Substitute \((d - 1, a)\) or \((d - 1, 22)\) into \(f(x)\) to find \(d\).

\[ f(x) = -x^2 + 23 \]
\[ 22 = -(d - 1)^2 + 23 \]
\[ 22 = -(d^2 - 2d + 1) + 23 \]
\[ 0 = -d^2 + 2d \]
\[ d^2 = 2d \]
\[ d = 2 \]

So, \(a = 22\), \(b = 26\), \(c = -6\), and \(d = 2\).

\textbf{b.} Because \(b = 0\), the \(x\)-coordinate of the vertex is 0.

\textbf{c.} For this function, \(a = -1\), so the graph opens down and the function has a maximum value.

\textbf{ANSWER:}
\[ a = 22; b = 26; c = -6; d = 2 \]
\[ b. \text{ maximum} \]
65. **WRITING IN MATH** Why can the discriminant be used to confirm the number and the type of solutions of a quadratic equation?

**SOLUTION:**
Sample answer: If the discriminant is positive, the Quadratic Formula will result in two real solutions because you are adding and subtracting the square root of a positive number in the numerator of the expression. If the discriminant is zero, there will be one real solution because you are adding and subtracting the square root of zero. If the discriminant is negative, there will be two complex solutions because you are adding and subtracting the square root of a negative number in the numerator of the expression.

**ANSWER:**
Sample answer: If the discriminant is positive, the Quadratic Formula will result in two real solutions because you are adding and subtracting the square root of a positive number in the numerator of the expression. If the discriminant is zero, there will be one real solution because you are adding and subtracting the square root of zero. If the discriminant is negative, there will be two complex solutions because you are adding and subtracting the square root of a negative number in the numerator of the expression.

66. Which expression is equivalent to \( \frac{8!}{5!} \)?

- **A** \( \frac{8}{5} \)
- **B** \( 8 \cdot 7 \cdot 6 \)
- **C** \( 3! \)
- **D** \( 8 \cdot 7 \cdot 6 \cdot 5 \)

**SOLUTION:**
\[
\frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6
\]
Therefore, option B is the correct answer.

**ANSWER:**
B
4-1 Graphing Quadratic Functions

67. SAT/ACT The price of coffee beans is $d$ dollars for 6 ounces, and each ounce makes $c$ cups of coffee. In terms of $c$ and $d$, what is the cost of the coffee beans required to make 1 cup of coffee?

- **F** $\frac{cd}{6}$
- **G** $\frac{6c}{d}$
- **H** $\frac{6}{cd}$
- **J** $6cd$
- **K** $\frac{d}{6c}$

**SOLUTION:**

The cost of the 1 ounce coffee beans is $\frac{d}{6}$.

$\frac{1}{c}$ ounce of coffee beans is need to make 1 cup of coffee.

Therefore, the cost of the coffee bean required to make 1 cup of coffee is $\frac{d}{6c}$.

**ANSWER:**

K

68. SHORT RESPONSE Each side of the square base of a pyramid is 20 feet, and the pyramid’s height is 90 feet. What is the volume of the pyramid?

**SOLUTION:**

Volume of a right regular pyramid is

$$V = \frac{1}{3} \text{(area of the base)} \times \text{(height)}.$$  

Base area $= 20 \times 20 = 400 \text{ ft}^2$ 

$$V = \frac{1}{3} (400)(90)$$ 

$$= 12000$$

Therefore, the volume of the pyramid is $12000 \text{ ft}^3$.

**ANSWER:**

$12,000 \text{ ft}^3$
69. Which ordered pair is the solution of the following system of equations?

\[
\begin{align*}
3x - 5y &= 11 \\
3x - 8y &= 5
\end{align*}
\]

A (2, 1)  
B (7, -2)  
C (7, 2)  
D \left( \frac{1}{3}, -2 \right)

**SOLUTION:**
Subtract the second equation from the first equation.

\[
3x - 5y - (3x - 8y) = 11 - 5
\]

\[
3y = 6
\]

\[
y = 2
\]

Substitute 2 for \( y \) in the first equation and solve for \( x \).

\[
3x - 5(2) = 11
\]

\[
3x + 10 = 11
\]

\[
x = 7
\]

The solution is (7, 2).

Therefore, option C is the correct answer.

**ANSWER:**
C

---

**Find the inverse of each matrix, if it exists.**

70. \[
\begin{bmatrix}
3 & -4 \\
2 & -1
\end{bmatrix}
\]

**SOLUTION:**

Let \( A = \begin{bmatrix} 3 & -4 \\ 2 & -1 \end{bmatrix} \).

\[
\text{det} (A) = 5
\]

\[
A^{-1} = \frac{1}{5} \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
\frac{1}{5} & \frac{4}{5} \\
\frac{2}{5} & \frac{3}{5}
\end{bmatrix}
\]

71. \[
\begin{bmatrix}
-4 & -1 \\
0 & 6
\end{bmatrix}
\]

**SOLUTION:**

Let \( A = \begin{bmatrix} -4 & -1 \\ 0 & 6 \end{bmatrix} \).

\[
\text{det} (A) = -24
\]

\[
A^{-1} = \frac{1}{-24} \begin{bmatrix} 6 & 1 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{24} \\ 0 & \frac{1}{6} \end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
\frac{1}{4} & \frac{1}{24} \\
0 & \frac{1}{6}
\end{bmatrix}
\]
72. \[
\begin{bmatrix}
2 & 8 \\
-3 & -5
\end{bmatrix}
\]

**SOLUTION:**
Let \( A = \begin{bmatrix} 2 & 8 \\ -3 & -5 \end{bmatrix} \).

\[
det(A) = 14
\]

\[
A^{-1} = \frac{1}{14} \begin{bmatrix} -5 & -8 \\ 3 & 2 \end{bmatrix}
\]

\[
= \begin{bmatrix} 5 & 4 \\ 14 & 7 \\ 3 & 1 \\ 14 & 7 \end{bmatrix}
\]

**ANSWER:**
\[
\begin{bmatrix} 5 & 4 \\ 14 & 7 \\ 3 & 1 \\ 14 & 7 \end{bmatrix}
\]

Evaluate each determinant.

73. \[
\begin{bmatrix}
6 & -3 \\
-1 & 8
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix} 6 & -3 \\ -1 & 8 \end{bmatrix} = (6)(8) - (-1)(-3)
\]

\[
= 48 - 3
\]

\[
= 45
\]

**ANSWER:**
45

74. \[
\begin{bmatrix}
-3 & -5 \\
-1 & -9
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix} -3 & -5 \\ -1 & -9 \end{bmatrix} = (-3)(-9) - (-5)(-1)
\]

\[
= 27 - 5
\]

\[
= 22
\]

**ANSWER:**
22

75. \[
\begin{bmatrix}
8 & 6 \\
4 & 3
\end{bmatrix}
\]

**SOLUTION:**
\[
\begin{bmatrix} 8 & 6 \\ 4 & 3 \end{bmatrix} = (8)(3) - (6)(4)
\]

\[
= 24 - 24
\]

\[
= 0
\]

**ANSWER:**
0

76. **MANUFACTURING** The Community Service Committee is making canvas tote bags and leather tote bags for a fundraiser. They will line both types of bags with canvas and use leather handles on both. For the canvas bags, they need 4 yards of canvas and 1 yard of leather. For the leather bags, they need 3 yards of leather and 2 yards of canvas. The committee leader purchased 56 yards of leather and 104 yards of canvas.

- **a.** Let \( c \) represent the number of canvas bags, and let \( l \) represent the number of leather bags. Write a system of inequalities for the number of bags that can be made.

- **b.** Draw the graph showing the feasible region.

- **c.** List the coordinates of the vertices of the feasible region.

- **d.** If the club plans to sell the canvas bags at a profit of $20 each and the leather bags at a profit of $35
4-1 Graphing Quadratic Functions

Each, write a function for the total profit on the bags.

e. How can the club make the maximum profit?

f. What is the maximum profit?

**SOLUTION:**

a. \( c \geq 0, \ell \geq 0, c + 3\ell \leq 56, 4c + 2\ell \leq 104 \)

b. 

![](image1.png)

c. The vertices of the solution region are \((0, 0), (26, 0), (20, 12)\) and \(\left(0, 18 \frac{2}{3}\right)\).

d. The optimal function is \(f(c, \ell) = 20c + 35\ell\).

e. Substitute the points \((0, 0), (26, 0), (20, 12)\) and \(\left(0, 18 \frac{2}{3}\right)\) in the function.

<table>
<thead>
<tr>
<th>((c, \ell))</th>
<th>(20c + 35\ell)</th>
<th>(f(c, \ell))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>(20(0) + 35(0))</td>
<td>0</td>
</tr>
<tr>
<td>((26, 0))</td>
<td>(20(26) + 35(0))</td>
<td>520</td>
</tr>
<tr>
<td>((20, 12))</td>
<td>(20(20) + 35(12))</td>
<td>820</td>
</tr>
<tr>
<td>(\left(0, 18 \frac{2}{3}\right))</td>
<td>(20(0) + 35 \left(18 \frac{2}{3}\right))</td>
<td>653.33</td>
</tr>
</tbody>
</table>

The club makes the maximum profit if they produce 20 canvas tote bags and 12 leather tote bags.

f. The maximum profit is $820.

**ANSWER:**

a. \( c \geq 0, \ell \geq 0, c + 3\ell \leq 56, 4c + 2\ell \leq 104 \)

b. 

![](image2.png)

c. \((0, 0), (26, 0), (20, 12), \left(0, 18 \frac{2}{3}\right)\)

d. \(f(c, \ell) = 20c + 35\ell\)

e. Make 20 canvas tote bags and 12 leather tote bags.

f. $820

State whether each function is a linear function. Write yes or no. Explain.

77. \(y = 4x^2 - 3x\)

**SOLUTION:**

No. It cannot be written as \(y = mx + b\).

**ANSWER:**

No; it cannot be written as \(y = mx + b\).

78. \(y = -2x - 4\)

**SOLUTION:**

Yes. It is written in \(y = mx + b\) form.

**ANSWER:**

Yes; it is written in \(y = mx + b\) form.
4-1 Graphing Quadratic Functions

79. \( y = 4 \)

**SOLUTION:**
Yes. It is written in \( y = mx + b \) form, \( m = 0 \).

**ANSWER:**
Yes; it is written in \( y = mx + b \) form, \( m = 0 \).

Evaluate each function for the given value.

80. \( f(x) = 3x^2 - 4x + 6, \) \( x = -2 \)

**SOLUTION:**
Substitute \(-2\) for \( x \) in the function and evaluate.

\[
f(-2) = 3(-2)^2 - 4(-2) + 6
= 12 + 8 + 6
= 26
\]

**ANSWER:**
26

81. \( f(x) = -2x^2 + 6x - 5, \) \( x = 4 \)

**SOLUTION:**
Substitute \( 4 \) for \( x \) in the function and evaluate.

\[
f(4) = -2(4)^2 + 6(4) - 5
= -32 + 24 - 5
= -13
\]

**ANSWER:**
\(-13\)

82. \( f(x) = 6x^2 + 18, \) \( x = -5 \)

**SOLUTION:**
Substitute \(-5\) for \( x \) in the function and evaluate.

\[
f(-5) = 6(-5)^2 + 18
= 150 + 18
= 168
\]

**ANSWER:**
168
Use the related graph of each equation to determine its solutions.

1. \(x^2 + 2x + 3 = 0\)

![Graph of \(x^2 + 2x + 3 = 0\)]

**SOLUTION:**
The graph has no \(x\)-intercepts. Thus, the equation has no real solution.

**ANSWER:**
no real solution

2. \(x^2 - 3x - 10 = 0\)

![Graph of \(x^2 - 3x - 10 = 0\)]

**SOLUTION:**
The \(x\)-intercepts of the graph are \(-2\) and \(5\). Thus, the solutions of the equation are \(-2\) and \(5\).

**ANSWER:**
\(-2, 5\)

3. \(-x^2 - 8x - 16 = 0\)

![Graph of \(-x^2 - 8x - 16 = 0\)]

**SOLUTION:**
The \(x\)-intercept of the graph is \(-4\). Thus, the solution of the equation is \(-4\).

**ANSWER:**
\(-4\)
4-2 Solving Quadratic Equations by Graphing

CCSS PRECISION Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. \( x^2 + 8x = 0 \)

**SOLUTION:**
Graph the relation function \( f(x) = x^2 + 8x \).

The \( x \)-intercepts of the graph indicate that the solutions are 0 and –8.

**ANSWER:**
0, –8

5. \( x^3 - 3x - 18 = 0 \)

**SOLUTION:**
Graph the related function \( f(x) = x^3 - 3x - 18 \).

The \( x \)-intercepts of the graph indicate that the solutions are –3 and 6.

**ANSWER:**
–3, 6
4-2 Solving Quadratic Equations by Graphing

6. $4x - x^2 + 8 = 0$

**SOLUTION:**
Graph the related function $f(x) = 4x - x^2 + 8$.

![Graph of $f(x) = 4x - x^2 + 8$.]

The $x$-intercepts of the graph indicate that one solution is between $-2$ and $-1$, and the other solution is between $5$ and $6$.

**ANSWER:**
between $-2$ and $-1$, between $5$ and $6$

7. $-12 - 5x + 3x^2 = 0$

**SOLUTION:**
Graph the related function $f(x) = -12 - 5x + 3x^2$.

![Graph of $f(x) = -12 - 5x + 3x^2$.]

The $x$-intercepts of the graph indicate that one solution is between $-2$ and $-1$, and the other solution is $3$.

**ANSWER:**
between $-2$ and $-1$, $3$
8. $x^2 - 6x + 4 = -8$

**SOLUTION:**

\[ x^2 - 6x + 4 = -8 \]
\[ x^2 - 6x - 12 = 0 \]

Graph the related function $f(x) = x^2 - 6x + 12$.

The graph has no $x$-intercepts. Thus, the equation has no real solution.

**ANSWER:**

no real solution

9. $9 - x^2 = 12$

**SOLUTION:**

\[ 9 - x^2 = 12 \]
\[ x^2 + 12 - 9 = 0 \]
\[ x^2 + 3 = 0 \]

Graph the related function $f(x) = x^2 + 3$.

The graph has no $x$-intercepts. Thus, the equation has no real solution.

**ANSWER:**

no real solution
4-2 Solving Quadratic Equations by Graphing

10. \(5x^2 + 10x - 4 = -6\)

**SOLUTION:**
\[
5x^2 + 10x - 4 + 6 = -6 + 6
\]
\[
5x^2 + 10x + 2 = 0
\]
Graph the related function \(f(x) = 5x^2 + 10x + 2\).

The \(x\)-intercepts of the graph indicate that one solution is between \(-2\) and \(-1\), and the other solution is between \(-1\) and \(0\).

**ANSWER:**

![Graph of \(f(x) = 5x^2 + 10x + 2\)](image)

between \(-2\) and \(-1\), between \(-1\) and \(0\)

11. \(x^3 - 20 = 2 + x\)

**SOLUTION:**
\[
x^3 - 20 - 2 - x = 0
\]
\[
x^2 - x - 22 = 0
\]
Graph the related function \(f(x) = x^3 - x - 22\).

The \(x\)-intercepts of the graph indicate that one solution is between \(-5\) and \(-4\), and the other solution is between \(5\) and \(6\).

**ANSWER:**

![Graph of \(f(x) = x^3 - x - 22\)](image)

between \(-5\) and \(-4\), between \(5\) and \(6\)
12. **NUMBER THEORY** Use a quadratic equation to find two real numbers with a sum of 2 and a product of −24.

**SOLUTION:**
Let \( x \) represents one of the numbers. Then \( 2 - x \) is the other number.

\[
\begin{align*}
    &x(2-x) = -24 \\
    &2x - x^2 = -24 \\
    &x^2 - 2x - 24 = 0
\end{align*}
\]

Solve the equation \( x^2 - 2x - 24 = 0 \).

\[
\begin{align*}
    &x^2 - 2x - 24 = 0 \\
    &x^2 - 6x + 4x - 24 = 0 \\
    &x(x-6) + 4(x-6) = 0 \\
    & (x-6)(x+4) = 0 \\
    & x = 6 \text{ or } x = -4
\end{align*}
\]

The two numbers are 6 and −4.

**ANSWER:**
6 and −4

13. **PHYSICS** How long will it take an object to fall from the roof of a building 400 feet above ground? Use the formula \( h(t) = -16t^2 + h_0 \), where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet.

**SOLUTION:**
Solve the equation \( -16t^2 + 400 = 0 \).

\[
\begin{align*}
    &-16t^2 = -400 \\
    &t^2 = 25 \\
    &t = 5
\end{align*}
\]

It will take 5 seconds for an object to fall.

**ANSWER:**
5 seconds
4-2 Solving Quadratic Equations by Graphing

16. \(0.5x^2 - 2x + 2 = 0\)

**SOLUTION:**
The \(x\)-intercept of the graph is 2. Thus, the solution of the equation is 2.

**ANSWER:**
2

17. \(-0.25x^2 - x - 1 = 0\)

**SOLUTION:**
The \(x\)-intercept of the graph is \(-2\). Thus, the solution of the equation is \(-2\).

**ANSWER:**
\(-2\)

18. \(x^2 - 6x + 11 = 0\)

**SOLUTION:**
The graph has no \(x\)-intercepts. Thus, the equation has no real solution.

**ANSWER:**
no real solution

19. \(-0.5x^2 + 0.5x + 6 = 0\)

**SOLUTION:**
The \(x\)-intercepts of the graph are \(-3\) and 4. Thus, the solutions of the equation are \(-3\) and 4.

**ANSWER:**
\(-3, 4\)
Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. \( x^2 = 5x \)

**SOLUTION:**

\[
x^2 = 5x
\]

\[
x^2 - 5x = 0
\]

Graph the related function \( f(x) = x^2 - 5x \).

The \( x \)-intercepts of the graph indicate that the solutions are 0 and 5.

**ANSWER:**

0, 5

21. \( -2x^2 - 4x = 0 \)

**SOLUTION:**

Graph the related function \( f(x) = -2x^2 - 4x \).

The \( x \)-intercepts of the graph indicate that the solutions are \(-2\) and 0.

**ANSWER:**

\(-2, 0\)
22. \( x^2 - 5x - 14 = 0 \)

**SOLUTION:**
Graph the related function \( x^2 - 5x - 14 = 0 \).

![Graph of \( x^2 - 5x - 14 = 0 \)]

The \( x \)-intercepts of the graph indicate that the solutions are \(-2\) and \(7\).

**ANSWER:**
\(-2, 7\)

23. \(-x^2 + 2x + 24 = 0\)

**SOLUTION:**
Graph the related function \( f(x) = -x^2 + 2x + 24 \).

![Graph of \(-x^2 + 2x + 24 = 0\)]

The \( x \)-intercepts of the graph indicate that the solutions are \(-4\) and \(6\).

**ANSWER:**
\(-4, 6\)
4-2 Solving Quadratic Equations by Graphing

24. \( x^2 - 18x = -81 \)

**SOLUTION:**

\[
\begin{align*}
  x^2 - 18x &= -81 \\
  x^2 - 18x + 81 &= -81 + 81 \\
  x^2 - 18x + 81 &= 0
\end{align*}
\]

Graph the related function \( f(x) = x^2 - 18x + 81 \).

![](image1)

The \( x \)-intercept of the graph indicates that the solution is 9.

**ANSWER:**

9


25. \( 2x^2 - 8x = -32 \)

**SOLUTION:**

\[
\begin{align*}
  2x^2 - 8x &= -32 \\
  2x^2 - 8x + 32 &= -32 + 32 \\
  2x^2 - 8x + 32 &= 0
\end{align*}
\]

Graph the related function \( f(x) = 2x^2 - 8x + 32 \).

![](image2)

The graph has no \( x \)-intercepts. Thus, the equation has no real solution.

**ANSWER:**

no real solution
26. \(2x^2 - 3x - 15 = 4\)

**SOLUTION:**

\[
2x^2 - 3x - 15 = 4 \\
x^2 - 3x - 19 = 0
\]

Graph the related function \(f(x) = 2x^2 - 3x - 19\).

The \(x\)-intercepts of the graph indicate that one solution is between \(-3\) and \(-2\), and the other solution is between \(3\) and \(4\).

**ANSWER:**

between \(-3\) and \(-2\) and between \(3\) and \(4\)

---

27. \(-3x^2 - 7 + 2x = -11\)

**SOLUTION:**

\[
-3x^2 - 7 + 2x = -11 \\
-3x^2 + 2x + 4 = 0
\]

Graph the equation \(f(x) = -3x^2 + 2x + 4\).

The \(x\)-intercepts of the graph indicate that one solution is between \(1\) and \(2\), and the other solution is between \(-1\) and \(0\).

**ANSWER:**

between \(-1\) and \(0\) and between \(1\) and \(2\)
28. \(-0.5x^2 + 3 = -5x - 2\)

**SOLUTION:**

\[-0.5x^2 + 3 = -5x - 2\]

\[-0.5x^2 + 3 + 5x + 2 = 0\]

\[-0.5x^2 + 5x + 5 = 0\]

Graph the related function \(f(x) = -0.5x^2 + 5x + 5\).

The \(x\)-intercepts of the graph indicate that one solution is between 10 and 11, and the other solution is between \(-1\) and 0.

**ANSWER:**

between \(-1\) and 0 and between 10 and 11

29. \(-2x + 12 = x^2 + 16\)

**SOLUTION:**

\[-2x + 12 = x^2 + 16\]

\[x^2 + 16 + 2x - 12 = 0\]

\[x^2 + 2x + 4 = 0\]

Graph the related function \(f(x) = x^2 + 2x + 4\).

The graph has no \(x\)-intercepts. Thus, the equation has no real solution.
4-2 Solving Quadratic Equations by Graphing

Use the tables to determine the location of the zeros of each quadratic function.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-8</td>
<td>-1</td>
<td>4</td>
<td>4</td>
<td>-1</td>
<td>-8</td>
<td>-22</td>
<td>-48</td>
</tr>
</tbody>
</table>

**SOLUTION:**
In the table, the function value changes from negative to positive between -6 and -5. So, one solution is between -6 and -5.
Similarly, the function value changes from positive to negative between -4 and -3. So, the other solution is between -4 and -3.

**ANSWER:**
between -6 and -5;
between -4 and -3

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>32</td>
<td>14</td>
<td>2</td>
<td>-3</td>
<td>-5</td>
<td>2</td>
<td>14</td>
<td>32</td>
</tr>
</tbody>
</table>

**SOLUTION:**
In the table, the function value changes from positive to negative between 0 and 1. So, one solution is between 0 and 1.
Similarly, the function value changes from negative to positive between 2 and 3. So, the other solution is between 2 and 3.

**ANSWER:**
between 0 and 1;
between 2 and 3

<table>
<thead>
<tr>
<th>$x$</th>
<th>-6</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-6</td>
<td>-1</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>-1</td>
<td>-6</td>
<td>-14</td>
</tr>
</tbody>
</table>

**SOLUTION:**
In the table, the function value changes from negative to positive between -3 and 0. So, one solution is between -3 and 0.
Similarly, the function value changes from positive to negative between 6 and 9. So, the other solution is between 6 and 9.

**ANSWER:**
between -3 and 0;
between 6 and 9

**NUMBER THEORY** Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

33. Their sum is -15, and their product is -54.

**SOLUTION:**
Let $x$ represent one of the numbers. Then $-15 - x$ is the other number.

$$x(-15 - x) = -54$$
$$-15x - x^2 = -54$$
$$x^2 + 15x - 54 = 0$$

Solve the equation $\quad x^2 + 15x - 54 = 0$.

$$x^2 + 15x - 54 = 0$$
$$x^2 - 3x + 18x - 54 = 0$$
$$x(x - 3) + 18(x - 3) = 0$$
$$(x - 3)(x + 18) = 0$$

$$x = 3 \text{ or } x = -18$$
The two numbers are 3 and -18.

**ANSWER:**
3 and -18

34. Their sum is 4, and their product is -117.

**SOLUTION:**
Let $x$ represent one of the numbers. Then $4 - x$ is the other number.

$$x(4 - x) = -117$$
$$4x - x^2 = -117$$
$$x^2 - 4x + 117 = 0$$

Solve the equation $\quad x^2 - 4x - 117 = 0$.

$$x^2 - 4x - 117 = 0$$
$$x^2 - 13x + 9x - 117 = 0$$
$$x(x - 13) + 9(x - 13) = 0$$
$$(x - 13)(x + 9) = 0$$

$$x = 13 \text{ or } x = -9$$
The two numbers are 13 and -9.

**ANSWER:**
13 and -9
35. Their sum is 12, and their product is –84.

**SOLUTION:**
Let \( x \) represent one of the numbers. Then \( 12 - x \) is the other number.

\[
x(12 - x) = -84
\]
\[
12x - x^2 = -84
\]
\[
x^2 - 12x + 84 = 0
\]

Solve the equation \( x^2 - 12x - 84 = 0 \).

\[
x = \frac{12 \pm \sqrt{144 + 336}}{2}
\]
\[
= \frac{12 \pm 22}{2}
\]
\[
= 17, -5
\]

The two numbers are about –5 and about 17.

**ANSWER:**
about –5 and 17

36. Their sum is –13, and their product is 42.

**SOLUTION:**
Let \( x \) represent one of the numbers. Then \( -13 - x \) is the other number.

\[
x(-13 - x) = 42
\]
\[
-13x - x^2 = 42
\]
\[
x^2 + 13x + 42 = 0
\]

Solve the equation \( x^2 + 13x + 42 = 0 \).

\[
x^2 + 6x + 7x + 42 = 0
\]
\[
x(x + 6) + 7(x + 6) = 0
\]
\[
(x + 6)(x + 7) = 0
\]
\[
x = -6 \text{ or } -7
\]

The two real numbers are –6 and –7.

**ANSWER:**
–6 and –7
37. Their sum is –8 and their product is –209.

**SOLUTION:**
Let \( x \) represent one of the numbers. Then \(-8 - x \) is the other number.

\[
x(-8 - x) = -209
\]
\[
-8x - x^2 = -209
\]
\[
x^2 + 8x - 209 = 0
\]

Solve the equation \( x^2 + 8x - 209 = 0 \)

\[
x^2 + 19x - 11x - 209 = 0
\]
\[
x(x + 19) - 11(x + 19) = 0
\]
\[
(x + 19)(x - 11) = 0
\]
\[
x = 11 \text{ or } -19
\]

The two real numbers are 11 and –19.

**ANSWER:**
11 and –19

**CCSS MODELING** For Exercises 38–40, use the formula \( h(t) = v_0t - 16t^2 \), where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds.

38. **BASEBALL** A baseball is hit directly upward with an initial velocity of 80 feet per second. Ignoring the height of the baseball player, how long does it take for the ball to hit the ground?

**SOLUTION:**
Substitute 80 for \( v_0 \) and 0 for \( h(t) \) in the formula \( h(t) = v_0t - 16t^2 \).

\[0 = 80t - 16t^2\]

Solve for \( t \).

\[16t^2 = 80t\]
\[t^2 = 5t\]
\[t = 5\]

The baseball takes 5 seconds to hit the ground.

**ANSWER:**
5 seconds
4-2 Solving Quadratic Equations by Graphing

39. **CANNONS** A cannonball is shot directly upward with an initial velocity of 55 feet per second. Ignoring the height of the cannon, how long does it take for the cannonball to hit the ground?

**SOLUTION:**
Substitute 55 for \( v_0 \) and 0 for \( h(t) \) in the formula \( h(t) = v_0t - 16t^2 \).

\[
0 = 55t - 16t^2
\]

Solve for \( t \).

\[
-16t^2 = -55t
\]

\[
-16t^2 + 55t = 0
\]

\[
-16t( -t) + 55t = 0
\]

\[
t = 3.4375
\]

The cannonball takes about 3.4375 seconds to hit the ground.

**ANSWER:**
about 3.4375 seconds

40. **GOLF** A golf ball is hit directly upward with an initial velocity of 100 feet per second. How long will it take for it to hit the ground?

**SOLUTION:**
Substitute 100 for \( v_0 \) and 0 for \( h(t) \) in the formula \( h(t) = v_0t - 16t^2 \).

\[
0 = 100t - 16t^2
\]

Solve for \( t \).

\[
-16t^2 = -100t
\]

\[
-16t^2 + 100t = 0
\]

\[
-16(t^2 - 6.25t) = 0
\]

\[
t = 6.25
\]

The golf ball will take about 6.25 seconds to hit the ground.

**ANSWER:**
6.25 seconds
4-2 Solving Quadratic Equations by Graphing

Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

41. \(2x^2 + x = 15\)

**SOLUTION:**
Graph the related function \(f(x) = 2x^2 + x - 15\).

The \(x\)-intercepts of the graph indicate that one solution is \(-3\), and the other solution is between 2 and 3.

**ANSWER:**
\(-3\), between 2 and 3

42. \(-5x - 12 = -2x^2\)

**SOLUTION:**
Graph the related function \(f(x) = 2x^2 - 5x - 12\).

The \(x\)-intercepts of the graph indicate that one solution is 4, and the other solution is \(-\frac{3}{2}\).

**ANSWER:**
\(-\frac{3}{2}, 4\)
4-2 Solving Quadratic Equations by Graphing

43. \(4x^2 - 15 = -4x\)

**SOLUTION:**
Graph the related function \(f(x) = 4x^2 + 4x - 15\).

![Graph of \(f(x) = 4x^2 + 4x - 15\)]

The \(x\)-intercepts of the graph indicate that one solution is between \(-3\) and \(-2\), and the other solution is between 1 and 2.

**ANSWER:**
between \(-3\) and \(-2\), between 1 and 2

44. \(-35 = -3x - 2x^2\)

**SOLUTION:**
Graph the related function \(f(x) = 2x^2 + 3x - 35\).

![Graph of \(f(x) = 2x^2 + 3x - 35\)]

The \(x\)-intercepts of the graph indicate that one solution is \(-5\), and the other solution is between 3 and 4.

**ANSWER:**
\(-5\), between 3 and 4.
4-2 Solving Quadratic Equations by Graphing

45. \(-3x^2 + 11x + 9 = 1\)

**SOLUTION:**
Graph the related function \(f(x) = -3x^2 + 11x + 8\).

The \(x\)-intercepts of the graph indicate that one solution is between \(-1\) and 0, and the other solution is between 4 and 5.

**ANSWER:**
between \(-1\) and 0, between 4 and 5

46. \(13 - 4x^2 = -3x\)

**SOLUTION:**
Graph the related function \(f(x) = -4x^2 + 3x + 13\).

The \(x\)-intercepts of the graph indicate that one solution is between \(-2\) and \(-1\), and the other solution is between 2 and 3.

**ANSWER:**
between \(-2\) and \(-1\), between 2 and 3
4-2 Solving Quadratic Equations by Graphing

47. \(-0.5x^2 + 18 = -6x + 33\)

**SOLUTION:**
Graph the related function \(f(x) = -0.5x^2 + 6x - 15\).

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & \ldots & -4 & -2 & 0 & 2 & 4 & \ldots \\
\hline
y & \ldots & -14 & -12 & -10 & -8 & -6 & \ldots \\
\hline
\end{array}
\]

The \(x\)-intercepts of the graph indicate that one solution is between 3 and 4, and the other solution is between 8 and 9.

**ANSWER:**
between 3 and 4, between 8 and 9

48. \(0.5x^2 + 0.75 = 0.25x\)

**SOLUTION:**
Graph the related function \(f(x) = 0.5x^2 - 0.25x + 0.75\).

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & \ldots & -4 & -2 & 0 & 2 & 4 & \ldots \\
\hline
y & \ldots & 8 & 6 & 4 & 2 & 0 & \ldots \\
\hline
\end{array}
\]

The graph has no \(x\)-intercepts. Thus, the equation has no real solution.

**ANSWER:**
No real solution.
4-2 Solving Quadratic Equations by Graphing

49. WATER BALLOONS Tony wants to drop a water balloon so that it splashes on his brother. Use the formula \( h(t) = -16t^2 + h_0 \), where \( t \) is the time in seconds and the initial height \( h_0 \) is in feet, to determine how far his brother should be from the target when Tony lets go of the balloon.

SOLUTION:
Substitute 60 for \( h_0 \) and 0 for \( h(t) \) in the formula \( h(t) = -16t^2 + h_0 \) and solve for \( t \).

\[
-16t^2 + 60 = 0 \\
-16t^2 = -60 \\
t^2 = 3.75 \\
t \approx 1.94
\]

Substitute 4.4 for speed and 1.94 for time taken in the distance formula and simplify.

Distance = Speed \times time taken 

\[= 4.4 \times 1.94 \approx 8.5\]

Tony’s brother should be about 8.5 ft far from the target.

ANSWER: about 8.5 ft

50. WATER HOSES A water hose can spray water at an initial velocity of 40 feet per second. Use the formula \( h(t) = v_0t - 16t^2 \), where \( h(t) \) is the height of the water in feet, \( v_0 \) is the initial velocity in feet per second, and \( t \) is the time in seconds.

a. How long will it take the water to hit the nozzle on the way down?

b. Assuming the nozzle is 5 feet up, what is the maximum height of the water?

SOLUTION:
a. Substitute 40 for \( v_0 \) and 0 for \( h(t) \) in the formula \( h(t) = v_0t - 16t^2 \).

\[40t - 16t^2 = 0\]

Solve for \( t \).

\[-16t^2 = -40t \\
\frac{-16t^2}{-16t} = \frac{-40t}{-16t} \\
t = 2.5
\]

It will take 2.5 seconds for the water to hit the nozzle on the way down.

b. The function that represents the situation is \( h(t) = -16t^2 + 40t + 5 \).

The maximum value of the function is the \( y \)-coordinate of the vertex.

The \( x \)-coordinate of the vertex is \(-\frac{40}{2(-16)}\) or \(\frac{5}{4}\).

Find the \( y \)-coordinate of the vertex by evaluating the function for \( t = \frac{5}{4} \).

\[h\left(\frac{5}{4}\right) = -16\left(\frac{5}{4}\right)^2 + 40\left(\frac{5}{4}\right) + 5\]

\[= -25 + 50 + 5 = 30\]

Thus, the maximum height of the water is 30 ft.

ANSWER:
4-2 Solving Quadratic Equations by Graphing

a. 2.5 seconds

b. 30 ft

51. SKYDIVING In 2003, John Fleming and Dan Rossi became the first two blind skydivers to be in free fall together. They jumped from an altitude of 14,000 feet and free fell to an altitude of 4000 feet before their parachutes opened. Ignoring air resistance and using the formula $h(t) = -16t^2 + h_0$, where $t$ is the time in seconds and the initial height $h_0$ is in feet, determine how long they were in free fall.

**SOLUTION:**
Substitute 14000 for $h_0$ and 4000 for $h(t)$ in the formula $h(t) = -16t^2 + h_0$ and solve for $t$.

$-16t^2 + 14000 = 4000$

$-16t^2 = -10000$

$t^2 = 625$

$t = 25$

They were in free fall for 25 seconds.

**ANSWER:**
25 seconds

52. CCSS CRITIQUE Hakeem and Tanya were asked to find the location of the roots of the quadratic function represented by the table. Is either of them correct? Explain.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x)$</td>
<td>12</td>
<td>26</td>
<td>8</td>
<td>-2</td>
<td>-4</td>
<td>2</td>
<td>16</td>
<td>38</td>
</tr>
</tbody>
</table>

**Hakeem**
The roots are between 4 and 6 because $f(x)$ stops decreasing and begins to increase between $x = 4$ and $x = 6$.

**Tanya**
The roots are between -2 and 0 because $x$ changes signs at that location.

**SOLUTION:**
Sample answer: neither are correct. Roots are located where $f(x)$ changes signs not where $x$ changes signs as Tanya states. Hakeem says that roots are found where $f(x)$ changes from decreasing to increasing. This reasoning is not true since it does not hold for all functions.

**ANSWER:**
Sample answer: No; roots are located where $f(x)$ changes signs.
53. CHALLENGE Find the value of a positive integer \( k \) such that \( f(x) = x^2 - 2kx + 55 \) has roots at \( k + 3 \) and \( k - 3 \).

**SOLUTION:**
The product of the coefficients is 55.

\[
(k + 3)(k - 3) = 55
\]

Solve for \( k \).

\[
\begin{align*}
(k + 3)(k - 3) &= 55 \\
k^2 - 3k + 3k - 9 &= 55 \\
k^2 - 9 &= 55 \\
k^2 &= 64 \\
k &= 8
\end{align*}
\]

**ANSWER:**
\( k = 8 \)

54. REASONING If a quadratic function has a minimum at \((-6, -14)\) and a root at \( x = -17 \), what is the other root? Explain your reasoning.

**SOLUTION:**
Sample answer: The other root is at \( x = 5 \) because the \( x \)-coordinates of the roots need to be equidistant from the \( x \)-value of the vertex.

**ANSWER:**
Sample answer: The other root is at \( x = 5 \) because the \( x \)-coordinates of the roots need to be equidistant from the \( x \)-value of the vertex.

55. OPEN ENDED Write a quadratic function with a maximum at \((3, 125)\) and roots at \(-2\) and \(8\).

**SOLUTION:**
\[ f(x) = -5x^2 + 30x + 80 \]

**ANSWER:**
\[ f(x) = -5x^2 + 30x + 80 \]

56. WRITING IN MATH Explain how to solve a quadratic equation by graphing its related quadratic function.

**SOLUTION:**
Sample answer: Graph the function using the axis of symmetry. Determine where the graph intersects the \( x \)-axis. The \( x \)-coordinates of those points are solutions to the quadratic equation.

**ANSWER:**
Sample answer: Graph the function using the axis of symmetry. Determine where the graph intersects the \( x \)-axis. The \( x \)-coordinates of those points are solutions to the quadratic equation.

57. SHORT RESPONSE A bag contains five different colored marbles. The colors of the marbles are black, silver, red, green, and blue. A student randomly chooses a marble. Then, without replacing it, chooses a second marble. What is the probability that the student chooses the red and then the green marble?

**SOLUTION:**
The probability of choosing the red and then the green marble without replacement is \( \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20} \).

**ANSWER:**
\( \frac{1}{20} \)
4-2 Solving Quadratic Equations by Graphing

58. Which number would be closest to zero on the number line?

A $-0.6$

B $\frac{2}{5}$

C $\sqrt{2}$

D 0.5

**SOLUTION:**
Among the choices, $\frac{2}{5}$ is the closest to zero on the number line.
So, B is the correct choice.

**ANSWER:**
B

59. SAT/ACT A salesman’s monthly gross pay consists of $3500 plus 20 percent of the dollar amount of his sales. If his gross pay for one month was $15,500, what was the dollar amount of his sales for that month?

F $12,000$

G $16,000$

H $60,000$

J $70,000$

K $77,000$

**SOLUTION:**
Let $x$ be the amount of his sale.
Write the equation for the situation and solve for $x$.

\[
3500 + \frac{20}{100}x = 15,500
\]
\[
\frac{20}{100}x = 12,000
\]
\[
x = 60,000
\]

The dollar amount of his sales for that month is $60,000.
Choice H is the correct answer.

**ANSWER:**
H
60. Find the next term in the sequence below.

\[
\frac{2x}{5}, \frac{3x}{5}, \frac{4x}{5}, \ldots
\]

A. \(x\)

B. \(5x\)

C. \(\frac{x}{5}\)

D. \(\frac{5x}{4}\)

**SOLUTION:**

The next term of the sequence is \(\frac{5x}{5}\) or \(x\).

So, A is the correct choice.

**ANSWER:**

A

---

**Determine whether each function has a maximum or minimum value, and find that value. Then state the domain and range of the function.**

61. \(f(x) = -4x^2 + 8x - 16\)

**SOLUTION:**

For the function, \(a = -4\), so the graph opens down and the function has a maximum value.

The maximum value of the function is the \(y\)-coordinate of the vertex.

The \(x\)-coordinate of the vertex is \(-\frac{8}{2(-4)} = \frac{8}{8} = 1\).

Find the \(y\)-coordinate of the vertex by evaluating the function for \(x = 1\).

\[
f(1) = -4(1)^2 + 8(1) - 16 = -4 + 8 - 16 = -12
\]

The maximum value of the function is \(-12\).

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \(\{f(x) | f(x) \leq -12\}\).

**ANSWER:**

maximum, -12; D = \{all real numbers\}, R = \(\{f(x) | f(x) \leq -12\}\).
4-2 Solving Quadratic Equations by Graphing

62. \( f(x) = 3x^2 + 12x - 18 \)

**SOLUTION:**
For the function, \( a = 3 \), so the graph opens up and the function has a minimum value.

The minimum value of the function is the \( y \)-coordinate of the vertex.

The \( x \)-coordinate of the vertex is \(- \frac{12}{2(3)} \) or \(-2\).

Find the \( y \)-coordinate of the vertex by evaluating the function for \( x = -2 \).

\[
f(x) = 3x^2 + 12x - 18 \\
f(-2) = 3(-2)^2 + 12(-2) - 18 \\
= 12 - 24 - 18 \\
= -30
\]

The minimum value of the function is \(-30\).

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{ f(x) \mid f(x) \geq -30 \} \).

**ANSWER:**
minimum, \(-30\); \( D = \{ \text{all real numbers} \} \), \( R = \{ f(x) \mid f(x) \geq -30 \} \)

63. \( f(x) = 4x + 13 - 2x^2 \)

**SOLUTION:**
For the function, \( a = -2 \), so the graph opens down and the function has a maximum value.

The maximum value of the function is the \( y \)-coordinate of the vertex.

The \( x \)-coordinate of the vertex is \(- \frac{4}{2(-2)} \) or \(1\).

Find the \( y \)-coordinate of the vertex by evaluating the function for \( x = 1 \).

\[
f(x) = 4x + 13 - 2x^2 \\
f(1) = 4(1) + 13 - 2(1)^2 \\
= 15
\]

The maximum value of the function is \(15\).

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) \mid f(x) \leq 15 \} \).

**ANSWER:**
maximum, \(15\); \( D = \{ \text{all real numbers} \} \), \( R = \{ f(x) \mid f(x) \leq 15 \} \)
Determine whether each pair of matrices are inverses of each other.

64. \[ \begin{bmatrix} 4 & -3 \\ -1 & -6 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 1 \\ 18 & 13 \end{bmatrix} \]

\[ \begin{bmatrix} 4 & -3 \\ -1 & -6 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 18 & 13 \end{bmatrix} = \begin{bmatrix} 27 & 28 \\ 26 & 117 \end{bmatrix} \]

\[ \begin{bmatrix} 27 & 28 \\ 26 & 117 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Since the multiplication of the matrices is not equal to \( I \), identity matrix, they are not inverses.

\[ \text{ANSWER: no} \]

65. \[ \begin{bmatrix} 6 & -3 \\ 4 & 8 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 1 \\ 10 & 20 \end{bmatrix} \]

\[ \begin{bmatrix} 6 & -3 \\ 4 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 10 & 20 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 20 & 15 \end{bmatrix} \]

\[ \begin{bmatrix} 6 & 6 \\ 20 & 15 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Since the multiplication of the matrices is not equal to \( I \), identity matrix, they are not inverses.

\[ \text{ANSWER: no} \]

66. \[ \begin{bmatrix} 2 & 4 \\ -3 & -2 \end{bmatrix} \text{ and } \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{4} \end{bmatrix} \]

\[ \begin{bmatrix} 2 & 4 \\ -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{3}{8} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \]

Since the multiplication of the matrices is equal to \( I \), identity matrix, the matrices are inverses of each other.

\[ \text{ANSWER: yes} \]

Solve each system of equations.

67. \[ 4x - 7y = -9 \]
\[ 5x + 2y = -22 \]

\[ \text{SOLUTION:} \]

The coefficients of \( x \)-variables are 4 and 5 and their least common multiple is 20, so multiply each equation by the value that will make the \( x \)-coefficient 20.

\[ 4x - 7y = -9 \quad \text{Multiply by } -5 \rightarrow -20x + 35y = 45 \]
\[ 5x + 2y = -22 \quad \text{Multiply by 4} \rightarrow (+) 20x + 8y = -88 \]

\[ 43y = -43 \]
\[ y = 1 \]

Substitute \(-1\) for \( y \) into either original equation and solve for \( x \).

\[ 4x - 7(-1) = -9 \]
\[ 4x + 7 = -9 \]
\[ 4x = -16 \]
\[ x = -4 \]

The solution is \((-4, -1)\).

\[ \text{ANSWER:} \]
\[ (-4, -1) \]
4-2 Solving Quadratic Equations by Graphing

68. \(3x + 8y = 24\)  
    \(-16y - 6x = 48\)

**SOLUTION:**
Multiply the equation \(3x + 8y = 24\) by 2.

\[2(3x + 8y) = 2(24)\]
\[6x + 16y = 48\]

Add the equations to eliminate one variable.

\[
\begin{align*}
6x + 16y &= 48 \\
(+ &- 6x - 16y = 48) \\
&- 0 = 96
\end{align*}
\]

Because \(0 = 96\) is not true, this system has no solution.

**ANSWER:**
no solution

69. \(8y - 2x = 38\)
\(5x - 3y = -27\)

**SOLUTION:**
The coefficients of \(x\)-variables are 2 and 5 and their least common multiple is 10, so multiply each equation by the value that will make the \(x\)-coefficient 10.

\[
\begin{align*}
-2x + 8y &= 38 \quad \text{Multiply by 5} \\
5x - 3y &= -27 \quad \text{Multiply by 2} \\
&\quad \text{→} \quad (+) 10x - 6y = -54
\end{align*}
\]

\[
\begin{align*}
34y &= 136 \\
y &= 4
\end{align*}
\]

Substitute 4 for \(y\) into either original equation and solve for \(x\).

\[
\begin{align*}
5x - 3(4) &= -27 \\
5x - 12 &= -27 \\
5x &= -15 \\
x &= -3
\end{align*}
\]

The solution is \((-3, 4)\).

**ANSWER:**
\((-3, 4)\)

70. **SALES** Alex is in charge of stocking shirts for the concession stand at the high school football game. The number of shirts needed for a regular season game is listed in the matrix. Alex plans to double the number of shirts stocked for a playoff game.

a. Write a matrix \(A\) to represent the regular season stock.

b. What scalar can be used to determine a matrix \(M\) to represent the new numbers? Find \(M\).

c. What is \(M - A\)? What does this represent in this situation?

<table>
<thead>
<tr>
<th>Size</th>
<th>small</th>
<th>medium</th>
<th>large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child</td>
<td>10</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Adult</td>
<td>25</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

**SOLUTION:**
4-2 Solving Quadratic Equations by Graphing

a. \( A = \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} \)

b. Multiply the matrix \( A \) by 2.

\[ M = 2 \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} = \begin{bmatrix} 20 & 20 & 30 \\ 50 & 70 & 90 \end{bmatrix} \]

\[ M - A = \begin{bmatrix} 20 & 20 & 30 \\ 50 & 70 & 90 \end{bmatrix} - \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} = \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} \]

\( M - A \) represents the number of shirts that Alex needs to stock additionally.

\textbf{ANSWER:} 
\[ \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} \]

72. \(-6x + 3 \geq 3x - 16\)

\textbf{SOLUTION:} 
Solve for \( x \).

\[-6x + 3 \geq 3x - 16 \]

\[-9x \geq -19\]

\[ x \leq \frac{19}{9} \]

\textbf{ANSWER:} 
\[ x \leq \frac{19}{9} \]

73. \( 6 - 4x \leq 2 \)

\textbf{SOLUTION:} 
Solve for \( x \).

\[ 6 - 4x \leq 2 \]

\[-4x \leq -4\]

\[ x \geq 1 \]

\textbf{ANSWER:} 
\[ x \geq 1 \]

Find the GCF of each set of numbers.

74. 16, 48, 128

\textbf{SOLUTION:} 
Find the prime factorization of the numbers.

\[ 16 = 2 \cdot 2 \cdot 2 \cdot 2 \]

\[ 48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \]

\[ 128 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \]

The greatest common factor of the numbers is 16.

\textbf{ANSWER:} 
16
4-2 Solving Quadratic Equations by Graphing

75. 15, 21, 49

\textbf{SOLUTION:}
Find the prime factorization of the numbers.
15 = 5 \cdot 3
21 = 7 \cdot 3
49 = 7 \cdot 7
The numbers do not have a common factor so the GCF is 1.

\textbf{ANSWER:}
1

76. 12, 28, 36

\textbf{SOLUTION:}
Find the prime factorization of the numbers.
12 = 2 \cdot 2 \cdot 3
28 = 2 \cdot 2 \cdot 7
36 = 2 \cdot 2 \cdot 3 \cdot 3
The greatest common factor of the numbers is 4.

\textbf{ANSWER:}
4
Write a quadratic equation in standard form with the given root(s).

1. \(-8, 5\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(-8\) and \(5\).

\[(x - (-8))(x - 5) = 0\]
\[(x + 8)(x - 5) = 0\]

Use the FOIL method to multiply.

\[x(x) + x(-5) + 8(x) + 8(-5) = 0\]
\[x^2 - 5x + 8x - 40 = 0\]
\[x^2 + 3x - 40 = 0\]

**ANSWER:**
\[x^2 + 3x - 40 = 0\]

---

2. \(\frac{3}{2}, \frac{1}{4}\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(\frac{3}{2}\) and \(\frac{1}{4}\).

\[\left(x - \frac{3}{2}\right)\left(x - \frac{1}{4}\right) = 0\]

Use the FOIL method to multiply.

\[x(x) + x\left(-\frac{1}{4}\right) - \frac{3}{2}(x) - \frac{3}{2}\left(-\frac{1}{4}\right) = 0\]
\[x^2 - \frac{1}{4}x - \frac{3}{2}x + \frac{3}{8} = 0\]

Multiply each side by 8.

\[8x^2 - 2x - 12x + 3 = 0\]
\[8x^2 - 14x + 3 = 0\]

**ANSWER:**
\[8x^2 - 14x + 3 = 0\]
Factor each polynomial.

4. \(35x^2 - 15x\)

**SOLUTION:**
The GCF of the two terms is 5x. Factor the GCF.

\[35x^2 - 15x = 5x(7x) - 5x(3) = 5x(7x - 3)\]

**ANSWER:**
\(5x(7x - 3)\)

5. \(18x^2 - 3x + 24x - 4\)

**SOLUTION:**
Factor 3x from the first two terms and 4 from the last two terms.

\[18x^2 - 3x + 24x - 4 = 3x(6x - 1) + 4(6x - 1)\]

Factor 6x - 1 from the two terms.

\[3x(6x - 1) + 4(6x - 1) = (6x - 1)(3x + 4)\]

Therefore,

\[18x^2 - 3x + 24x - 4 = (6x - 1)(3x + 4)\]

**ANSWER:**
\((6x - 1)(3x + 4)\)
6. \( x^2 - 12x + 32 \)

**SOLUTION:**
Find the factors of 32 whose sum is \(-12\).

<table>
<thead>
<tr>
<th>Factors of 32</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,32</td>
<td>33</td>
</tr>
<tr>
<td>(-1,\ -32)</td>
<td>(-33)</td>
</tr>
<tr>
<td>2,16</td>
<td>18</td>
</tr>
<tr>
<td>(-2,\ -16)</td>
<td>(-18)</td>
</tr>
<tr>
<td>4,8</td>
<td>12</td>
</tr>
<tr>
<td>(-4,\ -8)</td>
<td>(-12)</td>
</tr>
</tbody>
</table>

Write \(-12x\) as \((-4)x + (-8)x\).

\[ x^2 - 12x + 32 = x^2 - 4x - 8x + 32 \]

Factor \(x\) from the first two terms and \(-8\) from the last two terms.

\[ x^2 - 4x - 8x + 32 = x(x - 4) - 8(x - 4) \]

Factor \(x - 4\) from the two terms.

\[ x(x - 4) - 8(x - 4) = (x - 4)(x - 8) \]

Therefore,

\[ x^2 - 12x + 32 = (x - 4)(x - 8). \]

**ANSWER:**
\((x - 8)(x - 4)\)

7. \( x^2 - 4x - 21 \)

**SOLUTION:**
Find the factors of \(-21\) whose sum is \(-4\).

<table>
<thead>
<tr>
<th>Factors of (-21)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,\ -21</td>
<td>(-20)</td>
</tr>
<tr>
<td>(-1,\ 21)</td>
<td>20</td>
</tr>
<tr>
<td>3,\ -7</td>
<td>(-4)</td>
</tr>
<tr>
<td>(-3,\ 7)</td>
<td>4</td>
</tr>
</tbody>
</table>

Write \(-4x\) as \((-7)x + 3x\).

\[ x^2 - 4x - 21 = x^2 - 7x + 3x - 21 \]

Factor \(x\) from the first two terms and 3 from the last two terms.

\[ x^2 - 7x + 3x - 21 = x(x - 7) + 3(x - 7) \]

Factor \(x - 7\) from the two terms.

\[ x(x - 7) + 3(x - 7) = (x - 7)(x + 3) \]

Therefore,

\[ x^2 - 4x - 21 = (x - 7)(x + 3). \]

**ANSWER:**
\((x - 7)(x + 3)\)
4-3 Solving Quadratic Equations by Factoring

8. \(2x^2 + 7x - 30\)

**SOLUTION:**
Here, \(a = 2, b = 7\) and \(c = -30\).

\(ac = 2(-30) = -60\)

Find two factors of \(-60\) whose sum is 7.

\(12(-5) = -60\) and \(12 + (-5) = 7\)

Write \(7x\) as \(12x + (-5x)\).

\[2x^2 + 7x - 30 = 2x^2 + 12x - 5x - 30\]

Factor \(2x\) from the first two terms and \(-5\) from the last two terms.

\[2x^2 + 12x - 5x - 30 = 2x(x + 6) - 5(x + 6)\]

Factor \(x + 6\) from the two terms.

\[2x(x + 6) - 5(x + 6) = (x + 6)(2x - 5)\]

Therefore,

\[2x^2 + 7x - 30 = (2x - 5)(x + 6)\]

**ANSWER:**
\((2x - 5)(x + 6)\)

9. \(16x^2 - 16x + 3\)

**SOLUTION:**
Here, \(a = 16, b = -16\) and \(c = 3\).

\(ac = 16(3) = 48\)

Find two factors of \(48\) whose sum is \(-16\)

\(-12(-4) = 48\) and \(-12 + (-5) = -16\)

Write \(-16x\) as \(-12x + (-4x)\).

\[16x^2 - 16x + 3 = 16x^2 - 12x - 4x + 3\]

Factor \(4x\) from the first two terms and \(-1\) from the last two terms.

\[16x^2 - 12x - 4x + 3 = 4x(4x - 3) - 1(4x - 3)\]

Factor \(4x - 3\) from the two terms.

\[4x(4x - 3) - 1(4x - 3) = (4x - 3)(4x - 1)\]

Therefore,

\[16x^2 - 16x + 3 = (4x - 3)(4x - 1)\]

**ANSWER:**
\((4x - 3)(4x - 1)\)
4-3 Solving Quadratic Equations by Factoring

Solve each equation.

10. $x^2 - 36 = 0$

**SOLUTION:**
Use the identity $a^2 - b^2 = (a + b)(a - b)$.

Here, $a = x$ and $b = 6$.

$$x^2 - 36 = (x)^2 - (6)^2$$
$$= (x + 6)(x - 6)$$
$$x = -6, 6$$

**ANSWER:**
$-6, 6$

11. $12x^2 - 18x = 0$

**SOLUTION:**
The GCF of the two terms is $6x$. Factor the GCF.

$$12x^2 - 18x = 0$$
$$= 6x(2x) - 6x(3) = 0$$
$$= 6x(2x - 3) = 0$$
$$6x = 0 \text{ or } 2x - 3 = 0$$
$$x = 0 \text{ or } x = \frac{3}{2}$$

**ANSWER:**
$0, \frac{3}{2}$

12. $12x^2 - 2x - 2 = 0$

**SOLUTION:**
Here, $a = 12, b = -2$ and $c = -2$.

$$ac = 12(-2) = -24$$
Find two factors of $-24$ whose sum is $-2$.

$-6(4) = -24$ and $-6 + 4 = -2$

Write $-2x$ as $-6x + 4x$.

$$12x^2 - 2x - 2 = 12x^2 - 6x + 4x - 2$$

Factor $6x$ from the first two terms and $2$ from the last two terms.

$$12x^2 - 6x + 4x - 2 = 6x(2x - 1) + 2(2x - 1)$$

Factor $2x - 1$ from the two terms.

$$6x(2x - 1) + 2(2x - 1) = (2x - 1)(6x + 2)$$
$$= (2x - 1)(3x + 1)$$

Therefore,

$$12x^2 - 2x - 2 = 2(2x - 1)(3x + 1).$$

$(2x - 1) = 0 \text{ or } (3x + 1) = 0$

$2x = 1 \text{ or } 3x = -1$

$x = \frac{1}{2} \text{ or } x = -\frac{1}{3}$

**ANSWER:**
$-\frac{1}{3}, \frac{1}{2}$
13. \( x^2 - 9x = 0 \)

**SOLUTION:**

The GCF of the two terms on the left is \( x \). Factor the GCF.

\[
x(x - 9) = 0
\]

Use the Zero Product Property.

\[
x(x - 9) = 0 \implies x = 0 \quad \text{or} \quad x - 9 = 0
\]

\[
\implies x = 0 \quad \text{or} \quad x = 9
\]

Therefore, the roots are 0 and 9.

**ANSWER:**

0, 9

14. \( x^2 - 3x - 28 = 0 \)

**SOLUTION:**

Find the factors of \(-28\) whose sum is \(-3\).

\[
4(-7) = -28 \quad \text{and} \quad -7 + 4 = -3
\]

Write \(-3x\) as \(4x + (-7x)\).

\[
x^2 - 3x - 28 = 0
\]

\[
x^2 + 4x - 7x - 28 = 0
\]

Factor \( x \) from the first two terms and \(-7\) from the last two terms.

\[
x^2 + 4x - 7x - 28 = x(x + 4) - 7(x + 4) = 0
\]

Factor \( x + 4 \) from the two terms.

\[
x(x + 4) - 7(x + 4) = 0
\]

\[
(x + 4)(x - 7) = 0
\]

Use the Zero Product Property.

\[
(x + 4)(x - 7) = 0 \implies x + 4 = 0 \quad \text{or} \quad x - 7 = 0
\]

\[
\implies x = -4 \quad \text{or} \quad x = 7
\]

Therefore, the roots are \(-4\) and 7.

**ANSWER:**

\(-4, 7\)
15. \(2x^2 - 24x = -72\)

**SOLUTION:**
Divide each side of the equation by 2.
\[x^2 - 12x = -36\]
Write the equation with the right side equals zero.
\[x^2 - 12x + 36 = 0\]
Use the identity \((a - b)^2 = a^2 - 2ab + b^2\) to factor the left side of the equation.
Here, \(a = x\) and \(b = 6\).
\[x^2 - 12x + 36 = (x)^2 - 2(x)(6) + (6)^2 = (x - 6)^2\]
So, \((x - 6)^2 = 0\).
Use the Zero Product Property.
\[(x - 6)^2 = 0 \Rightarrow x - 6 = 0 \Rightarrow x = 6\]
Therefore, the root is 6 and it is a repeated root.

**ANSWER:**
6

16. **CCSS SENSE-MAKING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then?

\[
\begin{array}{c|c}
9 \text{ m} & 8 \text{ m} \\
\end{array}
\]

**SOLUTION:**
The area of a rectangle of length \(l\) and width \(w\) is \(l \times w\). So, the area of the garden is \(9(6) = 54\) m\(^2\).
Let \(x\) be the amount in length and width that has to be increased to double the area. Then,
\[(9 + x)(6 + x) = 2(54)\]
\[(x + 9)(x + 6) = 108\]
Use the FOIL method to multiply the left.
\[x(x) + x(6) + 9(x) + 9(6) = 108\]
\[x^2 + 6x + 9x + 54 = 108\]
\[x^2 + 15x - 54 = 0\]
Find the factors of –54 whose sum is 15.
\[18(-3) = -54\] and \(-3 + 18 = 15\)
Write 15 as \(-3x + 18x\).
\[x^2 + 15x - 54 = 0\]
\[x^2 - 3x + 18x - 54 = 0\]
Factor \(x\) from the first two terms and 18 from the last two terms.
\[x^2 - 3x + 18x - 54 = x(x - 3) + 18(x - 3) = 0\]
Factor \(x - 3\) from the two terms.
\[x(x - 3) + 18(x - 3) = (x - 3)(x + 18) = 0\]
\[x^2 + 15x - 54 = (x - 3)(x + 18) = 0\]
Use the Zero Product Property.
\[(x - 3)(x + 18) = 0 \Rightarrow x - 3 = 0 \text{ or } x + 18 = 0\]
\[\Rightarrow x = 3 \text{ or } x = -18\]
So, the roots are 3 and –18.
But \(x\) is a length, so it cannot be negative.
So, \(x = 3\).
Therefore, 3 m should be added to the length and width to double the area. The new dimensions of
Write a quadratic equation in standard form with the given root(s).

1. –8, 5

**SOLUTION:**

Write the pattern.

Replace \( p \) and \( q \) with -5 and \( \frac{1}{2} \).

\((x - (-5))(x - \frac{1}{2}) = 0\)

\((x + 5)\left(x - \frac{1}{2}\right) = 0\)

Use the FOIL method to multiply.

\[x(x) + x\left(-\frac{1}{2}\right) + 5(x) + 5\left(-\frac{1}{2}\right) = 0\]

\[x^2 - \frac{1}{2}x + 5x - \frac{5}{2} = 0\]

Multiply each side by 2.

\[2x^2 - x + 10x - 5 = 0\]

\[2x^2 + 9x - 5 = 0\]

**ANSWER:**

\[2x^2 + 9x - 5 = 0\]
4-3 Solving Quadratic Equations by Factoring

19. \( \frac{1}{5}, 6 \)

**SOLUTION:**
Write the pattern.

\((x - p)(x - q) = 0\)

Replace \( p \) and \( q \) with \( \frac{1}{5} \) and 6.

\( \left(x - \frac{1}{5}\right)(x - 6) = 0\)

Use the FOIL method to multiply.

\[x(x) + x(-6) - \frac{1}{5}(x) - \frac{1}{5}(-6) = 0\]

\[x^2 - 6x - \frac{1}{5}x + \frac{6}{5} = 0\]

Multiply each side by 5.

\[5x^2 - 30x - x + 6 = 0\]
\[5x^2 - 31x + 6 = 0\]

**ANSWER:**

\[5x^2 - 31x + 6 = 0\]

Factor each polynomial.

20. \( 40a^2 - 32a \)

**SOLUTION:**
The GCF of the two terms is \( 8a \). Factor the GCF.

\[40a^2 - 32a = 8a(5a) - 8a(4)\]
\[= 8a(5a - 4)\]

**ANSWER:**

\[8a(5a - 4)\]

21. \( 51c^3 - 34c \)

**SOLUTION:**
The GCF of the two terms is \( 17c \). Factor the GCF.

\[51c^3 - 34c = 17c(3c^2) - 17c(2)\]
\[= 17c(3c^2 - 2)\]

**ANSWER:**

\[17c(3c^2 - 2)\]

22. \( 32xy + 40bx - 12ay - 15ab \)

**SOLUTION:**
Factor \( 8x \) from the first two terms and \(-3a\) from the last two terms.

\[32xy + 40bx - 12ay - 15ab\]
\[= 8x(4y + 5b) - 3a(4y + 5b)\]

Factor \( 4y + 5b \) from the two terms.

\[8x(4y + 5b) - 3a(4y + 5b)\]
\[= (4y + 5b)(8x - 3a)\]

Therefore,

\[32xy + 40bx - 12ay - 15ab\]
\[= (4y + 5b)(8x - 3a)\]

**ANSWER:**

\[(8x - 3a)(4y + 5b)\]
23. \(3x^2 - 12\)

**SOLUTION:**
Factor out 3.

\[3x^2 - 12 = 3(x^2 - 4)\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 4\).

\[x^2 - 4 = (x + 2)(x - 2)\]

Therefore,

\[3x^2 - 12 = 3(x + 2)(x - 2)\]

**ANSWER:**
3(\(x + 2\))(\(x - 2\))

24. \(15y^2 - 240\)

**SOLUTION:**
Factor out 15.

\[15y^2 - 240 = 15(y^2 - 16)\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(y^2 - 16\).

\[y^2 - 16 = (y + 4)(y - 4)\]

Therefore,

\[15y^2 - 240 = 15(y + 4)(y - 4)\]

**ANSWER:**
15(\(y + 4\))(\(y - 4\))

25. \(48cg + 36cf - 4dg - 3df\)

**SOLUTION:**
Factor 12c from the first two terms and \(-d\) from the last two terms.

\[48cg + 36cf - 4dg - 3df = 12c(4g + 3f) - d(4g + 3f)\]

Factor 4g + 3f from the two terms.

\[12c(4g + 3f) - d(4g + 3f) = (4g + 3f)(12c - d)\]

Therefore,

\[48cg + 36cf - 4dg - 3df = (4g + 3f)(12c - d)\]

**ANSWER:**
(12c - d)(4g + 3f)
26. \( x^2 + 13x + 40 \)

**SOLUTION:**
Find the factors of 40 whose sum is 13.

5(8) = 40 and 5 + 8 = 13

Write 13\(x\) as 5\(x\) + 8\(x\).

\[ x^2 + 13x + 40 = x^2 + 5x + 8x + 40 \]

Factor \(x\) from the first two terms and 8 from the last two terms.

\[ x(x + 5) + 8(x + 5) = (x + 5)(x + 8) \]

Therefore,

\[ x^2 + 13x + 40 = (x + 5)(x + 8) \]

**ANSWER:**
\((x + 5)(x + 8)\)

27. \( x^2 - 9x - 22 \)

**SOLUTION:**
Find the factors of \(-22\) whose sum is \(-9\).

\(2(-11) = -22\) and \(2 + (-11) = -9\)

Write \(-9x\) as \(2x - 11x\).

\[ x^2 - 9x - 22 = x^2 + 2x - 11x - 22 \]

Factor \(x\) from the first two terms and \(-11\) from the last two terms.

\[ x^2 + 2x - 11x - 22 = x(x + 2) - 11(x + 2) \]

Factor \(x + 2\) from the two terms.

\[ x(x + 2) - 11(x + 2) = (x + 2)(x - 11) \]

Therefore,

\[ x^2 - 9x - 22 = (x + 2)(x - 11) \]

**ANSWER:**
\((x - 11)(x + 2)\)
28. \(3x^2 + 12x - 36\)

**SOLUTION:**
Here, \(a = 3\), \(b = 12\) and \(c = -36\).

\[ac = 3(-36) = -108\]

Find two factors of \(-108\) whose sum is 12.

\(-6(18) = -108\) and \(-6 + 18 = 12\)

Write \(12x\) as \(-6x + 18x\).

\[3x^2 + 12x - 36 = 3x^2 - 6x + 18x - 36\]

Factor \(3x\) from the first two terms and 18 from the last two terms.

\[3x^2 - 6x + 18x - 36 = 3x(x - 2) + 18(x - 2)\]

Factor \(x - 2\) from the two terms.

\[3x(x - 2) + 18(x - 2) = (x - 2)(3x + 18)\]
\[= (x - 2)(3)(x + 6)\]
\[= 3(x - 2)(x + 6)\]

Therefore,

\[3x^2 + 12x - 36 = 3(x - 2)(x + 6)\]

**ANSWER:**
\(3(x + 6)(x - 2)\)

29. \(15x^2 + 7x - 2\)

**SOLUTION:**
Here, \(a = 15\), \(b = 7\) and \(c = -2\).

\[ac = 15(-2) = -30\]

Find two factors of \(-30\) whose sum is 7.

\(10(-3) = -30\) and \(10 + (-3) = 7\)

Write \(7x\) as \(10x - 3x\).

\[15x^2 + 7x - 2 = 15x^2 + 10x - 3x - 2\]

Factor \(5x\) from the first two terms and \(-1\) from the last two terms.

\[15x^2 + 10x - 3x - 2 = 5x(3x + 2) - 1(3x + 2)\]

Factor \(3x + 2\) from the two terms.

\[5x(3x + 2) - 1(3x + 2) = (3x + 2)(5x - 1)\]

Therefore,

\[15x^2 + 7x - 2 = (3x + 2)(5x - 1)\]

**ANSWER:**
\((5x - 1)(3x + 2)\)
4-3 Solving Quadratic Equations by Factoring

30. \(4x^2 + 29x + 30\)

**SOLUTION:**

Here, \(a = 4, b = 29\) and \(c = 30\).

\[ac = 4(30) = 120\]

Find two factors of 120 whose sum is 29.

\[5(24) = 120\text{ and }5 + 24 = 29\]

Write \(29x\) as \(5x + 24x\).

\[4x^2 + 29x + 30 = 4x^2 + 5x + 24x + 30\]

Factor \(x\) from the first two terms and 6 from the last two terms.

\[4x^2 + 5x + 24x + 30 = x(4x + 5) + 6(4x + 5)\]

Factor \(4x + 5\) from the two terms.

\[x(4x + 5) + 6(4x + 5) = (4x + 5)(x + 6)\]

Therefore,

\[4x^2 + 29x + 30 = (4x + 5)(x + 6)\]

**ANSWER:**

\((4x + 5)(x + 6)\)

31. \(18x^2 + 15x - 12\)

**SOLUTION:**

Here, \(a = 18, b = 15\) and \(c = -12\).

\[ac = 18(-12) = -216\]

Find two factors of \(-216\) whose sum is 15.

\[24(-9) = -216\text{ and }24 + (-9) = 15\]

Write \(15x\) as \(24x + (-9)x\).

\[18x^2 + 15x - 12 = 18x^2 + 24x - 9x - 12\]

Factor \(6x\) from the first two terms and \(-3\) from the last two terms.

\[18x^2 + 24x - 9x - 12 = 6x(3x + 4) - 3(3x + 4)\]

Factor \(3x + 4\) from the two terms.

\[6x(3x + 4) - 3(3x + 4) = (6x - 3)(3x + 4)\]

\[= 3(2x - 1)(3x + 4)\]

Therefore,

\[18x^2 + 15x - 12 = 3(2x - 1)(3x + 4)\]

**ANSWER:**

\(3(2x - 1)(3x + 4)\)

32. \(8x^2z^2 - 4xz^2 - 12z^2\)

**SOLUTION:**

Factor \(z^2\) from all the three terms.

\[8x^2z^2 - 4xz^2 - 12z^2\]

\[= z^2(8x^2 - 4x - 12)\]

Factor \(8x^2 - 4x - 12\).

Here, \(a = 8, b = -4\) and \(c = -12\).
Write a quadratic equation in standard form with the given root(s).

1. –8, 5

**SOLUTION:**

Write the pattern. 

Replace p and q with 

Use the FOIL method to multiply. 

Multiply each side by 56.

Find factors of 4(9) = 36 whose sum is 12.

Find factors of 512 whose sum is 2.

Therefore, the roots are 6 and 

Use the identity 

**SOLUTION:**

Therefore, 

**ANSWER:**

Factor 

Find the factors of 

**ANSWER:**

Therefore, 

**ANSWER:**

Use the identity 

**SOLUTION:**

Therefore, 

**ANSWER:**

Factor 

Factor 2 from the two terms. 

Factor 6 from the two terms. 

The GCF of the two terms on the left is 

The GCF of the two terms is 6 

**ANSWER:**

Therefore, 

**ANSWER:**

Find factors of 30 whose sum is 15. 

The area of a rectangle of length 

Here, 

SOLUTION:

Now factor 3 

Write 12 

Find factors of 152 whose sum is 30. 

Therefore, 

**ANSWER:**

Factor 

Now factor 2 

Find two factors of 

ac = 8(–12) = –96

Find two factors of –96 whose sum is –4.

–12(8) = –96 and –12 + 8 = –4

Write –4x as –12x + 8x.

8x^2 – 4x – 12
= 8x^2 – 12x + 8x – 12

Factor 4x from the first two terms and 4 from the last two terms.

8x^2 – 12x + 8x – 12
= 4x(2x – 3) + 4(2x – 3)

Factor 2x – 3 from the two terms.

4x(2x – 3) + 4(2x – 3)
= (2x – 3)(4x + 4)
= (2x – 3)(4)(x + 1)
= 4(x + 1)(2x – 3)

Therefore,

8x^2z^2 – 4xz^2 – 12z^2
= 4z^2(x + 1)(2x – 3).

**ANSWER:**

4z^2(2x – 3)(x + 1)

33. 9x^2 – 25

**SOLUTION:**

Use the identity a^2 – b^2 = (a + b)(a – b)

9x^2 – 25 = (3x)^2 – (5)^2
= (3x + 5)(3x – 5)

Therefore,

9x^2 – 25 = (3x + 5)(3x – 5).

**ANSWER:**

(3x + 5)(3x – 5)

34. 18x^2y^2 – 24xy^2 + 36y^2

**SOLUTION:**

The GCF of the three terms is 6y^2. Factor the GCF.

18x^2y^2 – 24xy^2 + 36y^2
= 6y^2(3x^2) – 6y^2(4x) + 6y^2(6)
= 6y^2(3x^2 – 4x + 6)

**ANSWER:**

6y^2(3x^2 – 4x + 6)
35. $15x^2 - 84x - 36$

**SOLUTION:**
Factor 3 from all the three terms.

$15x^2 - 84x - 36 = 3(5x^2 - 28x - 12)$

Factor $5x^2 - 28x - 12$.

Here, $a = 5$, $b = -28$ and $c = -12$.

$ac = 5(-12) = -60$

Find two factors of $-60$ whose sum is $-28$.

$-30(2) = -60$ and $-30 + 2 = -28$

Write $-28x$ as $-30x + 2x$.

$5x^2 - 28x - 12 = 5x^2 - 30x + 2x - 12$

Factor $5x$ from the first two terms and $2$ from the last two terms.

$5x^2 - 30x + 2x - 12 = 5x(x - 6) + 2(x - 6)$

Factor $x - 6$ from the two terms.

$5x(x - 6) + 2(x - 6) = (5x + 2)(x - 6)$

Therefore,

$15x^2 - 84x - 36 = 3(5x + 2)(x - 6)$

**ANSWER:**
$3(5x + 2)(x - 6)$

36. $12x^2 + 13x - 14$

**SOLUTION:**
Here, $a = 12$, $b = 13$ and $c = -14$.

$ac = 12(-14) = -168$

Find two factors of $-168$ whose sum is $13$.

$-8(21) = -168$ and $-8 + 21 = 13$

Write $13x$ as $-8x + 21x$.

$12x^2 + 13x - 14 = 12x^2 - 8x + 21x - 14$

Factor $4x$ from the first two terms and $7$ from the last two terms.

$12x^2 - 8x + 21x - 14$

$= 4x(3x - 2) + 7(3x - 2)$

Factor $3x - 2$ from the two terms.

$4x(3x - 2) + 7(3x - 2) = (3x - 2)(4x + 7)$

Therefore,

$12x^2 + 13x - 14 = (3x - 2)(4x + 7)$

**ANSWER:**
$(4x + 7)(3x - 2)$
37. \(12xy^2 - 108x\)

**SOLUTION:**
Factor out the GCF, 12x.

\[12xy^2 - 108x = 12x(y^2 - 9)\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(y^2 - 9\).
\[y^2 - 9 = (y + 3)(y - 3)\]

Therefore,

\[12xy^2 - 108x = 12x(y + 3)(y - 3).\]

**ANSWER:**

\[12x(y + 3)(y - 3)\]

---

38. \(x^2 + 4x - 45 = 0\)

**SOLUTION:**
Find the factors of \(-45\) whose sum is 4.
\[9(-5) = -45\] and \(-5 + 9 = 4\)

Write 4x as \(-5x + 9x\).
\[x^2 + 4x - 45 = x^2 - 5x + 9x - 45 = 0\]

Factor \(x\) from the first two terms and 9 from the last two terms.
\[x^2 + 5x - 9x - 45 = 0\]
\[x(x + 5) - 9(x + 5) = 0\]

Factor \(x + 5\) from the two terms.
\[(x + 5)(x - 9) = 0\]

Use the Zero Product Property.
\[(x + 5)(x - 9) = 0 \Rightarrow x + 5 = 0 \quad \text{or} \quad x - 9 = 0\]
\[\Rightarrow x = -5 \quad \text{or} \quad x = 9\]

Therefore, the roots are \(-5\) and 9.

**ANSWER:**

\[5, -9\]
4-3 Solving Quadratic Equations by Factoring

39. \( x^2 - 5x - 24 = 0 \)

**SOLUTION:**
Find the factors of \(-24\) whose sum is \(-5\).

\[3(-8) = -24 \quad \text{and} \quad 3 + (-8) = -5\]

Write \(-5x\) as \(3x + (-5x)\).

\[
x^2 - 5x - 24 = 0
\]
\[
x^2 + 3x - 8x - 24 = 0
\]

Factor \(x\) from the first two terms and \(-8\) from the last two terms.

\[
x^2 + 3x - 8x - 24 = 0
\]
\[
x(x + 3) - 8(x + 3) = 0
\]

Factor \(x + 3\) from the two terms.

\[
x(x + 3) - 8(x + 3) = 0
\]
\[
(x + 3)(x - 8) = 0
\]

Use the Zero Product Property.

\[
(x + 3)(x - 8) = 0 \Rightarrow x + 3 = 0 \quad \text{or} \quad x - 8 = 0
\]
\[
\Rightarrow x = -3 \quad \text{or} \quad x = 8
\]

Therefore, the roots are \(-3\) and \(8\).

**ANSWER:**
8, \(-3\)

40. \( x^2 = 121 \)

**SOLUTION:**
Write the equation with right side equal to zero.

\[x^2 - 121 = 0\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 121\).

\[
x^2 - 121 = 0
\]
\[
(x + 11)(x - 11) = 0
\]

Use the Zero Product Property.

\[
(x + 11)(x - 11) = 0
\]
\[
\Rightarrow x + 11 = 0 \quad \text{or} \quad x - 11 = 0
\]
\[
\Rightarrow x = -11 \quad \text{or} \quad x = 11
\]

Therefore, the roots are \(-11\) and \(11\).

**ANSWER:**
11, \(-11\)
4-3 Solving Quadratic Equations by Factoring

41. \( x^2 + 13 = 17 \)

**SOLUTION:**
Write the equation with right side equal to zero.
\[
x^2 + 13 - 17 = 0
\]
\[
x^2 - 4 = 0
\]
Use the identity \( a^2 - b^2 = (a + b)(a - b) \) to factor \( x^2 - 4 \).
\[
x^2 - 4 = (x + 2)(x - 2) = 0
\]
Use the Zero Product Property.
\[
(x + 2)(x - 2) = 0 \Rightarrow x + 2 = 0 \text{ or } x - 2 = 0
\]
\[
\Rightarrow x = -2 \text{ or } x = 2
\]
Therefore, the roots are −2 and 2.

**ANSWER:**
2, −2

42. \( −3x^2 - 10x + 8 = 0 \)

**SOLUTION:**
Factor out −1.
\[
-1(3x^2 + 10x - 8) = 0
\]
\[
3x^2 + 10x - 8 = 0
\]
Now factor \( 3x^2 + 10x - 8 \).
Here, \( a = 3, b = 10 \) and \( c = -8 \).
\[
ac = 3(-8) = -24
\]
Find two factors of −24 whose sum is 10.
\[
12(-2) = -24 \text{ and } 12 + (-2) = 10
\]
Write \( 10x \) as \( 12x + (-2x) \).
\[
3x^2 + 10x - 8 = 3x^2 + 12x - 2x - 8
\]
Factor 3x from the first two terms and −2 from the last two terms.
\[
3x(x + 4) - 2(x + 4) = 0
\]
\[
(x + 4)(3x - 2) = 0
\]
Use the Zero Product Property.
\[
(x + 4)(3x - 2) = 0 \Rightarrow x + 4 = 0 \text{ or } 3x - 2 = 0
\]
\[
\Rightarrow x = -4 \text{ or } x = \frac{2}{3}
\]
Therefore, the roots are −4 and \( \frac{2}{3} \).

**ANSWER:**
\[ -4, \frac{2}{3} \]

43. \( -8x^2 + 46x - 30 = 0 \)

**SOLUTION:**
Factor out −1.
\[
-1(8x^2 - 46x + 30) = 0
\]
\[
8x^2 - 46x + 30 = 0
\]
Now factor \( 8x^2 - 46x + 30 \).
Here, \( a = 8, b = -46 \) and \( c = 30 \).
\[
ac = 8(30) = 240
\]
Find two factors of 240 whose sum is −46.
4-3 Solving Quadratic Equations by Factoring

-40(-6) = 240 and -40 + (-6) = -46

Write -46x as -40x + (-6x).

\[8x^2 - 46x + 30 = 8x^2 - 40x - 6x + 30\]

Factor 8x from the first two terms and -6 from the last two terms.

\[8x^2 - 40x - 6x + 30 = 0\]

\[8x(x - 5) - 6(x - 5) = 0\]

Factor \(x - 5\) from the two terms.

\[(x - 5)(8x - 6) = 0\]

Use the Zero Product Property.

\[(x - 5)(8x - 6) = 0 \Rightarrow x - 5 = 0 \text{ or } 8x - 6 = 0\]

\[\Rightarrow x = 5 \text{ or } x = \frac{6}{8} = \frac{3}{4}\]

Therefore, the roots are 5 and \(\frac{3}{4}\).

**ANSWER:**

5, \(\frac{3}{4}\)

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.

**SOLUTION:**

Let \(x\) be the length of the one of the legs. Then the length of the hypotenuse is \(3x + 4\) and that of the other leg is \(3x + 3\).

By the Pythagorean Theorem, the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.
4-3 Solving Quadratic Equations by Factoring

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.

**SOLUTION:**
Let the numbers be \(2n\) and \(2(n + 1)\).

Their product is 624.

\[
2n(2(n + 1)) = 624
\]

\[
4n^2 + 4n - 624 = 0
\]

Here, \(a = 4\), \(b = 4\) and \(c = 624\).

\[
ac = 4(624) = 2496
\]

Find two factors of 2496 whose sum is 4.

52(-48) = 2496 and 52 + (-48) = 4

Write 4\(n\) as 52\(n - 48n\).

\[
4n^2 + 4n - 624 = 0
\]

\[
4n^2 + 52n - 48n - 624 = 0
\]

Factor 4\(n\) from the first two terms and -48 from the last two terms.

\[
4n^2 + 52n - 48n - 624 = 0
\]

\[
4n(n + 13) - 48(n + 13) = 0
\]

Factor \(n + 13\) from the two terms.

\[
(4n - 48)(n + 13) = 0
\]

Use the Zero Product Property.

\[
(4n - 48)(n + 13) = 0 \Rightarrow 4n - 48 = 0 \text{ or } n + 13 = 0
\]

\[
\Rightarrow n = 12 \text{ or } n = -13
\]

When \(n = 12\), the numbers are 24 and 26.

When \(n = -13\), the numbers are -24 and -26.

**ANSWER:**
24 and 26 or -24 and -26

---

46. **GEOMETRY** Find \(x\) and the dimensions of each rectangle.

**SOLUTION:**
The area of a rectangle of length \(l\) and width \(w\) is \(l \times w\).

Here, \(l = x + 2\), \(w = x - 2\), and area = 96.

\[
(x + 2)(x - 2) = 96
\]

\[
x^2 - 4 = 96
\]

\[
x^2 = 100
\]

\[
x = \pm 10
\]

When \(x = -10\), the dimensions of the rectangle becomes negative. So, \(x = 10\).

The length of the rectangle is 12 ft and width is 8 ft.

**ANSWER:**
\(x = 10; 8 \text{ ft by } 12 \text{ ft}\)
4-3 Solving Quadratic Equations by Factoring

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = x + 4 \), \( w = x - 2 \), and area = 432.

\[
(x + 4)(x - 2) = 432
\]
\[
x^2 - 2x + 4x - 8 = 432
\]
\[
x^2 + 2x - 440 = 0
\]

Find factors of \(-440\) whose sum is 2.

\(-20(22) = -440 \text{ and } -20 + 22 = 2\)
\[
x^2 - 20x + 22x - 440 = 0
\]
\[
x(x - 20) + 22(x - 20) = 0
\]
\[
(x + 22)(x - 20) = 0
\]

\(\Rightarrow x = -22 \text{ or } x = 20\)

When \(x = -22\), the dimensions of the rectangle becomes negative. So, \(x = 20\).

The length of the rectangle is 24 ft and width is 18 ft.

**ANSWER:**
\(x = 20\); 24 in. by 18 in.

---

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = 3x - 4 \), \( w = x + 2 \), and area = 448.

\[
(3x - 4)(x + 2) = 448
\]
\[
3x^2 + 6x - 4x - 8 = 448
\]
\[
3x^2 + 2x - 456 = 0
\]

Find factors of \(3(-456) = -1368\) whose sum is 2.

\(-36(38) = -1368 \text{ and } -36 + 38 = 2\)
\[
3x^2 - 36x + 38x - 456 = 0
\]
\[
3x(x - 12) + 38(x - 12) = 0
\]
\[
(x - 12)(3x + 38) = 0
\]

\(\Rightarrow x = 12 \text{ or } x = \frac{-38}{3}\)

When \(x = \frac{-38}{3}\), the dimensions of the rectangle become negative. So, \(x = 12\).

The length of the rectangle is 32 ft and width is 14 ft.

**ANSWER:**
\(x = 12\); 14 ft by 32 ft
4-3 Solving Quadratic Equations by Factoring

Solve each equation by factoring.

49. \(12x^2 - 4x = 5\)

**SOLUTION:**
Write the equation with right side equal to zero.
\[12x^2 - 4x - 5 = 0\]
Find factors of 12\((-5) = -60\) whose sum is \(-4\).
\(-10(6) = -60\) and \(-10 + 6 = -4\)

\[12x^2 - 10x + 6x - 5 = 0\]
\[2x(6x - 5) + 1(6x - 5) = 0\]
\[(6x - 5)(2x + 1) = 0\]
\[\Rightarrow 6x - 5 = 0 \text{ or } 2x + 1 = 0\]
\[\Rightarrow x = \frac{5}{6} \quad \text{or} \quad x = -\frac{1}{2}\]

Therefore, the roots are \(\frac{5}{6}\) and \(-\frac{1}{2}\).

**ANSWER:**
\(\frac{5}{6}, -\frac{1}{2}\)

50. \(5x^2 = 15x\)

**SOLUTION:**
Write the equation with right side equal to zero.
\[5x^2 - 15x = 0\]
Factor out the GCF of the left side, \(5x\).
\[5x(x - 3) = 0\]

Use the Zero Product Property.
\[5x(x - 3) = 0 \Rightarrow 5x = 0 \quad \text{or} \quad x - 3 = 0\]
\[\Rightarrow x = 0 \quad \text{or} \quad x = 3\]

Therefore, the roots are 0 and 3.

**ANSWER:**
0, 3
4-3 Solving Quadratic Equations by Factoring

51. \(16x^2 + 36 = -48x\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[16x^2 + 48x + 36 = 0\]

Divide each side of the equation by 4.

\[4x^2 + 12x + 9 = 0\]

Find factors of 4(9) = 36 whose sum is 12.

\[6(6) = 36 \text{ and } 6 + 6 = 12\]

\[
\begin{aligned}
4x^2 + 6x + 6x + 9 &= 0 \\
2x(2x + 3) + 3(2x + 3) &= 0 \\
(2x + 3)^2 &= 0 \\
\Rightarrow 2x + 3 &= 0 \\
\Rightarrow x &= -\frac{3}{2}
\end{aligned}
\]

Therefore, the only repeated root is \(-\frac{3}{2}\).

**ANSWER:**
\[-\frac{3}{2}\]

52. \(75x^2 - 60x = -12\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[75x^2 - 60x + 12 = 0\]

Divide each side of the equation by 3.

\[25x^2 - 20x + 4 = 0\]

Find factors of 25(4) = 100 whose sum is -20.

\[-10(-10) = 100 \text{ and } -10 + (-10) = -20\]

\[
\begin{aligned}
25x^2 - 10x - 10x + 4 &= 0 \\
5x(5x - 2) - 2(5x - 2) &= 0 \\
(5x - 2)^2 &= 0 \\
\Rightarrow 5x - 2 &= 0 \\
\Rightarrow x &= \frac{2}{5}
\end{aligned}
\]

Therefore, the only repeated root is \(\frac{2}{5}\).

**ANSWER:**
\[\frac{2}{5}\]
53. \(4x^2 - 144 = 0\)

**SOLUTION:**
Factor out the GCF of the left side, 4.

\[4(x^2 - 36) = 0\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 36\).

\[x^2 - 36 = (x + 6)(x - 6)\]

Use the Zero Product Property.

\[4(x + 6)(x - 6) = 0 \Rightarrow x + 6 = 0 \quad \text{or} \quad x - 6 = 0\]

\[\Rightarrow x = -6 \quad \text{or} \quad x = 6\]

Therefore, the roots are 6 and -6.

**ANSWER:**
6, -6

54. \(-7x + 6 = 20x^2\)

**SOLUTION:**
Write the equation with right side equal to zero.

\[20x^2 + 7x - 6 = 0\]

Find factors of \(20(-6) = -120\) whose sum is 7.

\(15(-8) = -120\) and \(15 + (-8) = 7\)

\[20x^2 + 15x - 8x - 6 = 0\]

\[5x(4x + 3) - 2(4x + 3) = 0\]

\[(5x - 2)(4x + 3) = 0\]

\[\Rightarrow 5x - 2 = 0 \quad \text{or} \quad 4x + 3 = 0\]

\[\Rightarrow x = \frac{2}{5} \quad \text{or} \quad x = -\frac{3}{4}\]

Therefore, the roots are \(\frac{2}{5}\) and \(-\frac{3}{4}\).

**ANSWER:**
\[
\begin{align*}
\frac{2}{5}, & \quad -\frac{3}{4}
\end{align*}
\]
55. **MOVIE THEATER** A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was \( P(x) = -x^2 + 48x - 512 \), where \( x \) is the number of movie screens, and \( P(x) \) is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

**SOLUTION:**
For the company not to lose money, the profit should be non-negative. That is, at least zero.

\[-x^2 + 48x - 512 = 0\]

Factor out \(-1\).

\[-1(x^2 - 48x + 512) = 0\]

Factor \(x^2 - 48x + 512\).

Find factors of 512 whose sum is \(-48\).

\[-16(-32) = 512 \text{ and } -16 + (-32) = -48\]

\[x^2 - 16x - 32x + 512 = 0\]

\[x(x - 16) - 32(x - 16) = 0\]

\[(x - 16)(x - 32) = 0\]

\[\Rightarrow x - 16 = 0 \text{ or } x - 32 = 0\]

\[\Rightarrow x = 16 \text{ or } x = 32\]

A total of 16 to 32 screens will guarantee that company will not lose money.

**ANSWER:**
16 to 32 screens

---

56. Write a quadratic equation in standard form with the given root(s).

\[\frac{3}{7}, \frac{3}{8}\]

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(-\frac{4}{7}\) and \(\frac{3}{8}\).

\[\left(x - \left(-\frac{4}{7}\right)\right)\left(x - \frac{3}{8}\right) = \left(x + \frac{4}{7}\right)\left(x - \frac{3}{8}\right) = 0\]

Use the FOIL method to multiply.

\[x(x) + x\left(-\frac{3}{8}\right) + \frac{4}{7}(x) + \frac{4}{7}\left(-\frac{3}{8}\right) = 0\]

\[x^2 - \frac{3}{8}x + \frac{4}{7}x - \frac{12}{56} = 0\]

Multiply each side by 56.

\[56x^2 - 21x + 32x - 12 = 0\]

\[56x^2 + 11x - 12 = 0\]

**ANSWER:**
\[56x^2 + 11x - 12 = 0\]
57. 3.4, 0.6

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with 3.4 and 0.6.

\[(x - 3.4)(x - 0.6) = 0\]

Use the FOIL method to multiply.

\[x(x) + x(-0.6) - 3.4(x) - 3.4(-0.6) = 0\]
\[x^2 - 0.6x - 3.4x + 2.04 = 0\]
\[x^2 - 4x + 2.04 = 0\]

Multiply each side by 25.

\[25x^2 - 100x + 51 = 0\]

**ANSWER:**
\[25x^2 - 100x + 51 = 0\]

58. \(\frac{2}{11}, \frac{5}{9}\)

**SOLUTION:**
Write the pattern.

\[(x - p)(x - q) = 0\]

Replace \(p\) and \(q\) with \(\frac{2}{11}\) and \(\frac{5}{9}\).

\[\left(x - \frac{2}{11}\right)\left(x - \frac{5}{9}\right) = 0\]

Use the FOIL method to multiply.

\[x(x) + x\left(-\frac{5}{9}\right) - \frac{2}{11}(x) - \frac{2}{11}\left(-\frac{5}{9}\right) = 0\]
\[x^2 - \frac{5}{9}x - \frac{2}{11}x + \frac{10}{99} = 0\]

Multiply each side by 99.

\[99x^2 - 55x - 18x + 10 = 0\]
\[99x^2 - 73x + 10 = 0\]

**ANSWER:**
\[99x^2 - 73x + 10 = 0\]
4-3 Solving Quadratic Equations by Factoring

Solve each equation by factoring.

59. \(10x^2 + 25x = 15\)

**SOLUTION:**
Write the equation with right side equal to zero.

\(10x^2 + 25x - 15 = 0\)

Divide each side by 5.

\(2x^2 + 5x - 3 = 0\)

Find factors of 2(−3) = −6 whose sum is 5.

\(-1(6) = -6\) and \(-1 + 6 = 5\)

\(2x^2 - x + 6x + 3 = 0\)

\(x(2x - 1) + 3(2x - 1) = 0\)

\((2x - 1)(x + 3) = 0\)

⇒ \(2x - 1 = 0\) or \(x + 3 = 0\)

⇒ \(x = \frac{1}{2}\) or \(x = -3\)

Therefore, the roots are \(-3\) and \(\frac{1}{2}\).

**ANSWER:**
\(-3, \frac{1}{2}\)

60. \(27x^2 + 5 = 48x\)

**SOLUTION:**
Write the equation with right side equal to zero.

\(27x^2 - 48x + 5 = 0\)

Find factors of 27(5) = 135 whose sum is −48.

\(-45(-3) = 135\) and \(-45 + (-3) = -48\)

\(27x^2 - 45x - 3x + 5 = 0\)

\(9x(3x - 5) - 1(3x - 5) = 0\)

\((3x - 5)(9x - 1) = 0\)

⇒ \(3x - 5 = 0\) or \(9x - 1 = 0\)

⇒ \(x = \frac{5}{3}\) or \(x = \frac{1}{9}\)

Therefore, the roots are \(\frac{5}{3}\) and \(\frac{1}{9}\).

**ANSWER:**
\(\frac{5}{3}, \frac{1}{9}\)
4-3 Solving Quadratic Equations by Factoring

61. \( x^2 + 0.25x = 1.25 \)

\[ x^2 + 0.25x - 1.25 = 0 \]

Multiply each side by 4.

\[ 4x^2 + x - 5 = 0 \]

Find factors of 4(\(-5\)) = –20 whose sum is 1.

5(\(-4\)) = –20 and 5 + (\(-4\)) = 1

\[ 4x^2 - 4x + 5x - 5 = 0 \]

\[ 4x(x - 1) + 5(x - 1) = 0 \]

\[ (4x + 5)(x - 1) = 0 \]

\[ \Rightarrow 4x + 1 = 0 \text{ or } x - 1 = 0 \]

\[ \Rightarrow x = -\frac{1}{4} \text{ or } x = 1 \]

Therefore, the roots are 1 and \(-\frac{5}{4}\).

ANSWER:

1, \(-\frac{5}{4}\)

62. \( 48x^2 - 15 = -22x \)

\[ 48x^2 + 22x - 15 = 0 \]

Find factors of 48(\(-15\)) = –720 whose sum is 22.

40(\(-18\)) = 720 and 40 + (\(-18\)) = 22

\[ 48x^2 + 40x - 18x - 15 = 0 \]

\[ 8x(6x + 5) - 3(6x + 5) = 0 \]

\[ (6x + 5)(8x - 3) = 0 \]

\[ \Rightarrow 6x + 5 = 0 \text{ or } 8x - 3 = 0 \]

\[ \Rightarrow x = -\frac{5}{6} \quad \text{or} \quad x = \frac{3}{8} \]

Therefore, the roots are \(-\frac{5}{6}\) and \(\frac{3}{8}\).

ANSWER:

\[ \frac{3}{8}, -\frac{5}{6} \]
63. $3x^2 + 2x = 3.75$

**SOLUTION:**
Write the equation with right side equal to zero.

$3x^2 + 2x - 3.75 = 0$

Multiply each side by 4.

$12x^2 + 8x - 15 = 0$

Find factors of $12(-15) = -180$ whose sum is 8.

$18(-10) = 8$ and $18 + (-10) = 8$

$12x^2 + 18x - 10x - 15 = 0$

$6x(2x + 3) - 5(2x + 3) = 0$

$(6x - 5)(2x + 3) = 0$

$\Rightarrow 6x - 5 = 0$ or $2x + 3 = 0$

$\Rightarrow x = \frac{5}{6}$ or $x = -\frac{3}{2}$

Therefore, the roots are $\frac{5}{6}$ and $-\frac{3}{2}$.

**ANSWER:**

\[
\begin{array}{c}
\frac{3}{2} \\
\frac{5}{6}
\end{array}
\]

64. $-32x^3 + 56x = 12$

**SOLUTION:**
Write the equation with right side equal to zero.

$32x^3 - 56x + 12 = 0$

Divide each side by 4.

$8x^3 - 14x + 3 = 0$

Find factors of $8(3) = 24$ whose sum is $-14$.

$-12(-2) = 24$ and $-12 + (-2) = -14$

$8x^2 - 12x - 2x + 3 = 0$

$4x(2x - 3) - 1(2x - 3) = 0$

$(4x - 1)(2x - 3) = 0$

$\Rightarrow 4x - 1 = 0$ or $2x - 3 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{3}{2}$

Therefore, the roots are $\frac{1}{4}$ and $\frac{3}{2}$.

**ANSWER:**

\[
\begin{array}{c}
\frac{1}{4} \\
\frac{3}{2}
\end{array}
\]
65. **DESIGN** A square is cut out of the figure at the right. Write an expression for the area of the figure that remains, and then factor the expression.

![Figure](image)

**SOLUTION:**
The area of the figure that remains is the difference between the areas of the square with side \(x\) units and that of the square with side 6 units.

\[x^2 - (6)^2\]

Use the identity \(a^2 - b^2 = (a + b)(a - b)\) to factor \(x^2 - 36\).

\[x^2 - (6)^2 = (x + 6)(x - 6)\]

**ANSWER:**
\(x^2 - 6^2; (x + 6)(x - 6)\)

66. **CCSS PERSEVERANCE** After analyzing the market, a company that sells Web sites determined the profitability of their product was modeled by \(P(x) = -16x^2 + 368x - 2035\), where \(x\) is the price of each Web site and \(P(x)\) is the company’s profit. Determine the price range of the Web sites that will be profitable for the company.

**SOLUTION:**
For the company not to lose money, the profit should be non-negative. That is, at least zero.

\[-16x^2 + 368x - 2035 = 0\]

Factor out \(-1\).

\[-1(16x^2 - 368x + 2035) = 0\]

\[16x^2 - 368x + 2035 = 0\]

Factor \(16x^2 - 368x + 2035\).

Find factors of 16(2035) = 32560 whose sum is \(-368\).

\[-220(-148) = 32560\] and \(-220 + (-148) = -368\)

\[16x^2 - 220x - 148x + 2035 = 0\]

\[4x(4x - 55) - 37(4x - 55) = 0\]

\[(4x - 55)(4x - 37) = 0\]

\(4x - 55 = 0\) or \(4x - 37 = 0\)

\[x = \frac{55}{4} = 13\frac{3}{4}\] or \[x = \frac{37}{4} = 9\frac{1}{4}\]

A price range of $9.25 to $13.75 will be profitable for the company.

**ANSWER:**
$9.25 to $13.75

67. **PAINTINGS** Enola wants to add a border to her painting, distributed evenly, that has the same area as the painting itself. What are the dimensions of the painting with the border included?
4-3 Solving Quadratic Equations by Factoring

**SOLUTION:**

The area of a rectangle of length \( l \) and width \( w \) is \( l \times w \).

Here, \( l = 15 \) in, \( w = 10 \) in. so, area = 150 in\(^2\).

The area including the border will be double the area of the painting. So, the total area will be 300 in\(^2\).

Let \( x \) be the amount in length and width of the border. Then,

\[
(x + 15)(x + 10) = 300
\]

Use the FOIL method to multiply the left.

\[
x(x) + x(10) + 15(x) + 15(10) = 300
\]

\[
x^2 + 10x + 15x + 150 = 300
\]

\[
x^2 + 25x - 150 = 0
\]

Find the factors of \(-150\) whose sum is 25.

\(-5(30) = -150\) and \(-5 + 30 = 25\)

Write \(25x\) as \(-5x + 30x\).

\[
x^2 - 5x + 30x - 150 = 0
\]

\[
x(x - 5) + 30(x - 5) = 0
\]

\[
(x - 5)(x + 30) = 0
\]

Use the Zero Product Property.

\[
(x - 5)(x + 30) = 0 \Rightarrow x - 5 = 0 \quad \text{or} \quad x + 30 = 0
\]

\[
\Rightarrow x = 5 \quad \text{or} \quad x = -30
\]

So, the roots are 5 and \(-30\).

But \(x\) is a length, so it cannot be negative.

So, \(x = 5\).

Therefore, the dimensions of the painting, including the border are 20 in by 15 in.

**ANSWER:**

20 in. by 15 in.

68. **MULTIPLE REPRESENTATIONS** In this problem, you will consider \(a(x - p)(x - q) = 0\).

a. **GRAPHICAL** Graph the related function for \(a = 1, p = 2,\) and \(q = -3\).

b. **ANALYTICAL** What are the solutions of the equation?

c. **GRAPHICAL** Graph the related functions for \(a = 4, -3,\) and \(\frac{1}{2}\) on the same graph.

d. **VERBAL** What are the similarities and differences between the graphs?

e. **VERBAL** What conclusion can you make about the relationship between the factored form of a quadratic equation and its solutions?

**SOLUTION:**

a. When \(a = 1, p = 2,\) and \(q = -3,\) the function becomes,

\[
(x - 2)(x + 3) = 0.
\]

That is, \(x^2 + x - 6 = 0.\)

Graph the function on a coordinate plane.
4-3 Solving Quadratic Equations by Factoring

b. The graph intersects the x-axis at –3 and 2. So, the solutions are \( x = -3 \) and \( x = 2 \).

c. When \( a = 4 \), the function is \( 4x^2 + 4x - 24 = 0 \).

When \( a = -3 \), it is \( -3x^2 - 3x + 18 = 0 \) and when \( a = \frac{1}{2} \), it is \( \frac{1}{2}x^2 + \frac{1}{2}x - 3 = 0 \).

Draw the four graphs on the same coordinate plane.

d. Sample answer: They all have the same roots, \( p \) and \( q \) which are 2 and –3 respectively. Therefore, they all have the same solutions. The graphs are shaped differently due to the value of \( a \). The graph with \( a = -3 \) is flipped due to the negative.

e. When quadratic equations have the same factors, they will have the same solutions, regardless of the value of \( a \), which only affects the shape of the graphs.

ANSWER:

a. 2 and –3

c. When \( a = 3 \), the function is \( 3x^2 - 3x + 18 = 0 \) and when \( a = \frac{1}{2} \), it is \( \frac{1}{2}x^2 + \frac{1}{2}x - 3 = 0 \).

Draw the four graphs on the same coordinate plane.

d. Sample answer: They all have the same roots, \( p \) and \( q \) which are 2 and –3 respectively. Therefore, they all have the same solutions. The graphs are shaped differently due to the value of \( a \). The graph with \( a = -3 \) is flipped due to the negative.

e. When quadratic equations have the same factors, they will have the same solutions, regardless of the value of \( a \), which only affects the shape of the graphs.

69. GEOMETRY The area of the triangle is 26 square centimeters. Find the length of the base.

\[ \text{SOLUTION:} \]

The area of a triangle of base \( b \) and height \( h \) is given by the formula \( \frac{1}{2}bh \).

Here, \( b = x + 7 \), \( h = x - 2 \), and area = 26 cm\(^2\).

\[ \frac{1}{2}(x+7)(x-2) = 26 \]
\[ (x+7)(x-2) = 52 \]
\[ x^2 + 5x - 14 = 52 \]
\[ x^2 + 5x - 66 = 0 \]

Find the factors of –66 whose sum is +5.

\((-6)(11) = -66 \) and \((-6) + (11) = 5 \)

Write \( 5x \) as \((-6x) + (11x) \).
4-3 Solving Quadratic Equations by Factoring

\[ x^2 - 6x + 11x - 66 = 0 \]
\[ x(x - 6) + 11(x - 6) = 0 \]
\[ (x + 11)(x - 6) = 0 \]

Use the Zero Product Property.

\[ (x + 11)(x - 6) = 0 \Rightarrow x + 11 = 0 \quad \text{or} \quad x - 6 = 0 \]
\[ \Rightarrow x = -11 \quad \text{or} \quad x = 6 \]

So, the roots are \(-11\) and \(6\).

But when \(x = -11\), the height of the triangle becomes negative. So, \(x = 6\).

Therefore, the length of the base of the triangle is 13 cm.

**ANSWER:**
13 cm

70. **SOCCER** When a ball is kicked in the air, its height in meters above the ground can be modeled by \(h(t) = -4.9t^2 + 14.7t\) and the distance it travels can be modeled by \(d(t) = 16t\), where \(t\) is the time in seconds.

a. How long was the ball in the air?

b. How far did it travel before it hit the ground? (Hint: Ignore air resistance.)

c. What was the maximum height of the ball?

**SOLUTION:**

a. When the ball hits the ground, the height will be zero.

That is, \(h(t) = 0\).

\[-4.9t^2 + 14.7t = 0\]

Solve for \(t\) to find the time that the ball was in air.

\[ t(-4.9t + 14.7) = 0 \]
\[ \Rightarrow t = 0 \quad \text{or} \quad -4.9t + 14.7 = 0 \]
\[ \Rightarrow t = 0 \quad \text{or} \quad t = \frac{14.7}{4.9} = 3 \]

The solution \(t = 0\) is not valid as the ball has already been hit. Therefore, the ball was in air for 3 seconds.

b. Substitute \(t = 3\) in the formula to find the distance traveled by the ball.

\[ d(3) = 16(3) \]
\[ = 48 \]

Therefore, the ball will travel 48 m before it hits the ground.

c. The maximum height is the \(y\)-coordinate of the vertex of the parabola formed by the equation \(h(t) = -4.9t^2 + 14.7t\). The \(y\)-coordinate of the vertex of a parabola is given by \(-\frac{b^2 - 4ac}{4a}\).

Here, \(a = -4.9\), \(b = 14.7\) and \(c = 0\). So, the \(y\)-coordinate of the vertex is

\[ \frac{(14.7)^2 - 4(-4.9)(0)}{4(-4.9)} = \frac{216.09}{19.6} = 11.025. \]

Therefore, the maximum height of the ball is 11.025 m.

**ANSWER:**

a. 3 seconds
b. 48 m
c. 11.025 m
4-3 Solving Quadratic Equations by Factoring

Factor each polynomial.

71. $18a - 24ay + 48b - 64by$

**SOLUTION:**
Factor $3a$ from the first two terms and $8b$ from the last two terms.

$$18a - 24ay + 48b - 64by = 3a(6 - 8y) + 8b(6 - 8y)$$

Factor $6 - 8y$ from the two terms.

$$3a(6 - 8y) + 8b(6 - 8y) = (6 - 8y)(3a + 8b) = 2(3 - 4y)(3a + 8b)$$

**ANSWER:**
$2(3 - 4y)(3a + 8b)$

72. $3x^2 + 2xy + 10y + 15x$

**SOLUTION:**
Factor $x$ from the first two terms and $5$ from the last two terms.

$$3x^2 + 2xy + 10y + 15x = x(3x + 2y) + 5(2y + 3x)$$

$$= x(3x + 2y) + 5(3x + 2y)$$

Factor $3x + 2y$ from the two terms.

$$x(3x + 2y) + 5(3x + 2y) = (3x + 2y)(x + 5)$$

**ANSWER:**
$(3x + 2y)(x + 5)$

73. $6a^2b^2 - 12ab^2 - 18b^3$

**SOLUTION:**
Factor $6b^2$ from all the three terms.

$$6a^2b^2 - 12ab^2 - 18b^3 = 6b^2(a^2) - 6b^2(2a) - 6b^2(3b)$$

$$= 6b^2(a^2 - 2a - 3b)$$

**ANSWER:**
$6b^2(a^2 - 2a - 3b)$

74. $12a^2 - 18ab + 30ab^3$

**SOLUTION:**
Factor $6a$ from all the three terms.

$$12a^2 - 18ab + 30ab^3 = 6a(2a) - 6a(3b) + 6a(5b^3)$$

$$= 6a(2a - 3b + 5b^3)$$

**ANSWER:**
$6a(2a - 3b + 5b^3)$
4-3 Solving Quadratic Equations by Factoring

75. \(32ax + 12bx - 48ay - 18by\)

**SOLUTION:**
Factor 4x from the first two terms and -6y from the last two terms.

\[
32ax + 12bx - 48ay - 18by = 4x(8a + 3b) - 6y(8a + 3b)
\]

Factor 8a + 3b from the two terms.

\[
4x(8a + 3b) - 6y(8a + 3b) = (8a + 3b)(4x - 6y)
\]

\[
= (8a + 3b)(2)(2x - 3y)
\]

**ANSWER:**
\(2(2x - 3y)(8a + 3b)\)

76. \(30ac + 80bd + 40ad + 60bc\)

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

\[
30ac + 80bd + 40ad + 60bc = 30ac + 40ad + 60bc + 80bd
\]

Factor 10a from the first two terms and 20b from the last two terms.

\[
30ac + 40ad + 60bc + 80bd = 10a(3c + 4d) + 20b(3c + 4d)
\]

Factor 3c + 4d from the two terms.

\[
10a(3c + 4d) + 20b(3c + 4d) = (3c + 4d)(10a + 20b)
\]

\[
= (3c + 4d)(10)(a + 2b)
\]

**ANSWER:**
\(10(a + 2b)(3c + 4d)\)
4-3 Solving Quadratic Equations by Factoring

77. \(5ax^2 - 2by^2 - 5ay^2 + 2bx^2\)

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

\[
5ax^2 - 2by^2 - 5ay^2 + 2bx^2 = 5ax^2 - 5ay^2 + 2bx^2 - 2by^2
\]

Factor \(5a\) from the first two terms and \(2b\) from the last two terms.

\[
5ax^2 - 5ay^2 + 2bx^2 - 2by^2 = 5a(x^2 - y^2) + 2b(x^2 - y^2)
\]

Factor \(x^2 - y^2\) from the two terms.

\[
5a(x^2 - y^2) + 2b(x^2 - y^2) = (x^2 - y^2)(5a + 2b)
\]

\[
= (x + y)(x - y)(5a + 2b)
\]

**ANSWER:**
\((x + y)(x - y)(5a + 2b)\)

78. \(12c^2x + 4d^2y - 3d^2x - 16c^2y\)

**SOLUTION:**
Rearrange the terms to group the terms with common factors.

\[
12c^2x + 4d^2y - 3d^2x - 16c^2y = 12c^2x - 16c^2y - 3d^2x + 4d^2y
\]

Factor \(4c^2\) from the first two terms and \(-d^2\) from the last two terms.

\[
12c^2x - 16c^2y - 3d^2x + 4d^2y = 4c^2(3x - 4y) - d^2(3x - 4y)
\]

Factor \(3x - 4y\) from the two terms.

\[
4c^2(3x - 4y) - d^2(3x - 4y) = (3x - 4y)(4c^2 - d^2)
\]

\[
= (3x - 4y)(2c + d)(2c - d)
\]

**ANSWER:**
\((2c + d)(2c - d)(3x - 4y)\)
4-3 Solving Quadratic Equations by Factoring

79. **ERROR ANALYSIS** Gwen and Morgan are solving \(-12x^2 + 5x + 2 = 0\). Is either of them correct? Explain your reasoning.

**Gwen**

\[
\begin{align*}
-12x^2 + 5x + 2 &= 0 \\
-12x^2 + 8x - 3x + 2 &= 0 \\
4x(-3x + 2) - (3x + 2) &= 0 \\
(4x - 1)(3x + 2) &= 0 \\
x &= \frac{1}{4} \text{ or } -\frac{2}{3}
\end{align*}
\]

**Morgan**

\[
\begin{align*}
-12x^2 + 5x + 2 &= 0 \\
-12x^2 + 8x - 3x + 2 &= 0 \\
4x(-3x + 2) + (-3x + 2) &= 0 \\
(4x + 1)(-3x + 2) &= 0 \\
x &= -\frac{1}{4} \text{ or } -\frac{2}{3}
\end{align*}
\]

**SOLUTION:**
Morgan is correct. In step 3, Gwen did not have like terms in the parentheses in the third line.

**ANSWER:**
Sample answer: Morgan; Gwen did not have like terms in the parentheses in the third line.

80. **CHALLENGE** Solve \(3x^6 - 39x^4 + 108x^2 = 0\) by factoring.

**SOLUTION:**
Substitute \(x^2 = X\). Then the equation becomes

\[
3X^3 - 39X^2 + 108X = 0.
\]

Factor out \(X\) from the three terms.

\[
X \left(3X^2 - 39X + 108\right) = 0
\]

By the Zero Product Property, either \(X = 0\) or \(3X^2 - 39X + 108 = 0\).

\[
x = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0
\]

Solve the equation \(3X^2 - 39X + 108 = 0\).

Find factors of 3(108) = 324 whose sum is -39.

\[
-12(-27) = 324 \text{ and } -12 + (-27) = -39
\]

\[
3X^2 - 12X - 27X + 108 = 0
\]

\[
3X(X - 4) - 27(X - 4) = 0
\]

\[
(3X - 27)(X - 4) = 0
\]

\[
\Rightarrow 3X - 27 = 0 \text{ or } X - 4 = 0
\]

\[
\Rightarrow X = 9 \quad \text{or} \quad X = 4
\]

\[
x = 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm3
\]

\[
x = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm2
\]

Therefore, the roots are 0, 3, -3, 2, or -2.

**ANSWER:**
0, 3, -3, 2, or -2
4-3 Solving Quadratic Equations by Factoring

81. **CHALLENGE** The rule for factoring a difference of cubes is shown below. Use this rule to factor \(40x^5 - 135x^2y^3\).

\[
a^3 - b^3 = (a - b)(a^2 + ab + b^2)
\]

**SOLUTION:**
First factor out the GCF \(5x^2\) from the two terms.

\[
40x^5 - 135x^2y^3 = 5x^2(8x^3 - 27y^3)
\]

\[
40x^5 - 135x^2y^3 = 5x^2(8x^3 - 27y^3)
\]

\[
= 5x^2\left((2x)^3 - (3y)^3\right)
\]

Here, \(a = 2x\) and \(b = 3y\). Use the rule to factor \((2x)^3 - (3y)^3\).

\[
(2x)^3 - (3y)^3
\]

\[
= (2x - 3y)(4x^2 + 6xy + 9y^2)
\]

Therefore,

\[
40x^5 - 135x^2y^3
\]

\[
= 5x^2(2x - 3y)(4x^2 + 6xy + 9y^2)
\]

**ANSWER:**

\[5x^2(2x - 3y)(4x^2 + 6xy + 9y^2)\]

82. **OPEN ENDED** Choose two integers. Then write an equation in standard form with those roots. How would the equation change if the signs of the two roots were switched?

**SOLUTION:**
Sample answer: \(3\) and \(6\) \(\rightarrow\) \(x^2 - 9x + 18 = 0\). \(-3\) and \(-6\) \(\rightarrow\) \(x^2 + 9x + 18 = 0\). The linear term changes sign.

**ANSWER:**
Sample answer: \(3\) and \(6\) \(\rightarrow\) \(x^2 - 9x + 18 = 0\). \(-3\) and \(-6\) \(\rightarrow\) \(x^2 + 9x + 18 = 0\). The linear term changes sign.

83. **CHALLENGE** For a quadratic equation of the form \((x - p)(x - q) = 0\), show that the axis of symmetry of the related quadratic function is located halfway between the \(x\)-intercepts \(p\) and \(q\).

**SOLUTION:**
Sample answer:

Original equation is \((x - p)(x - q) = 0\).

Multiply.

\[x^2 - px - qx + pq = 0\]

Simplify.

\[x^2 - (p + q)x + pq = 0\]

The formula for axis of symmetry is \(x = \frac{-b}{2a}\).

We have \(a = 1\) and \(b = -(p + q)\).

\[x = \frac{-(-(p + q))}{2(1)}\]

Simplify.

\[x = \frac{p + q}{2}\]
By the definition of midpoint, \( x \) is midway between \( p \) and \( q \).

**ANSWER:**
Sample answer:

\[
(x - p)(x - q) = 0 \\
\text{Original equation}
\]

\[
x^2 - px - qx + pq = 0 \\
\text{Multiply}
\]

\[
x^2 - (p + q)x + pq = 0 \\
\text{Simplify.}
\]

\[
x = -\frac{b}{2a} \\
\text{Formula for axis of symmetry}
\]

\[
x = -\frac{-(p + q)}{2(1)} \\
a = 1 \text{ and } b = -
\]

\[
x = \frac{p + q}{2} \\
\text{Simplify.}
\]

\( x \) is midway between \( p \) and \( q \). **Definition of midpoint**

84. **WRITE A QUESTION** A classmate is using the guess and check strategy to factor trinomials of the form \( x^2 + bx + c \). Write a question to help him think of a way to use that strategy for \( ax^2 + bx + c \).

**SOLUTION:**
Sample answer: What do you know about \( a \cdot c \) to use guess and check to factor trinomials of the form \( ax^2 + bx + c \)?

**ANSWER:**
Sample answer: What do you know about \( a \cdot c \) to use guess and check to factor trinomials of the form \( ax^2 + bx + c \)?

85. **CCSS ARGUMENTS** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

In a quadratic equation in standard form where \( a, b, \) and \( c \) are integers, if \( b \) is odd, then the quadratic cannot be a perfect square trinomial.

**SOLUTION:**
Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term, \( bx \), must be a multiple of 2, or even.

**ANSWER:**
Sample answer: Always; in order to factor using perfect square trinomials, the coefficient of the linear term, \( bx \), must be a multiple of 2, or even.
86. **WRITING IN MATH** Explain how to factor a trinomial in standard form with \( a > 1 \).

**SOLUTION:**
Sample answer: In standard form, we have \( ax^2 + bx + c \). Multiply \( a \) and \( c \).

Then find a pair of integers, \( g \) and \( h \), that multiply to equal \( ac \) and add to equal \( b \).

Then write out the quadratic, turning the middle term, \( bx \), into \( gx + hx \).

We now have \( ax^2 + gx + hx + c \).

Now factor the GCF from the first two terms and then factor the GCF from the second two terms.

So we now have \( \text{GCF}(x - q) + \text{GCF}_2(x - q) \).

Simplifying, we get \( (\text{GCF} + \text{GCF}_2)(x - q) \) or \( (x - p)(x - q) \).

**ANSWER:**
Sample answer: In standard form, we have \( ax^2 + bx + c \). Multiply \( a \) and \( c \).

Then find a pair of integers, \( g \) and \( h \), that multiply to equal \( ac \) and add to equal \( b \).

Then write out the quadratic, turning the middle term, \( bx \), into \( gx + hx \).

We now have \( ax^2 + gx + hx + c \).

Now factor the GCF from the first two terms and then factor the GCF from the second two terms.

So we now have \( \text{GCF}(x - q) + \text{GCF}_2(x - q) \).

Simplifying, we get \( (\text{GCF} + \text{GCF}_2)(x - q) \) or \( (x - p)(x - q) \).

87. **SHORT RESPONSE** If \( ABCD \) is transformed by \( (x,y) \rightarrow (3x,4y) \), determine the area of \( A'B'C'D' \).

**SOLUTION:**
Here, \( A'B'C'D' \) will be a rectangle.

Determine the length and width of \( A'B'C'D' \).

The coordinates of \( A' \) will remain \((0,0)\) and the coordinates of \( B' \) will be \((3,4),(0)\)=(12,0).

So, the width of the transformed rectangle will be 12.

Now, the coordinates of \( C' \) will be \((3,4),(4)\)=(12,16).

So, the length of the transformed rectangle will be 16.

Therefore, the area of \( A'B'C'D' \) is \( 12 \times 16 = 192 \) square units.

**ANSWER:**
192 square units

88. For \( y = 2|6 - 3x| + 4 \), which set describes \( x \) when \( y < 6 \)?

A \( \left\{ x \mid \frac{5}{3} < x < \frac{7}{3} \right\} \)

B \( \left\{ x \mid x < \frac{5}{3} \text{ or } x > \frac{7}{3} \right\} \)
C \left\{ x \mid x < \frac{5}{3} \right\}

D \left\{ x \mid x > \frac{7}{3} \right\}

**SOLUTION:**

When \( y < 6 \), \( 2|6 - 3x| + 4 < 6 \).

Subtract 4 from each side of the inequality and divide by 2.

\[
2|6 - 3x| + 4 < 6 \\
2|6 - 3x| < 2 \\
|6 - 3x| < 1
\]

When \( |a| < b \), then \(-b < a < b\). So, \(-1 < 6 - 3x < 1\).

Subtract 6 from each side.

\[-7 < -3x < 5\]

Divide each part by \(-3\). When you divide each side of an inequality by a negative number, the inequality sign should be reversed.

\[
\frac{7}{3} > x > \frac{5}{3} \quad \text{or} \quad \frac{5}{3} < x < \frac{7}{3}
\]

So, the solution set is \( \left\{ x \mid \frac{5}{3} < x < \frac{7}{3} \right\} \).

Therefore, the correct choice is A.

**ANSWER:**

A

89. **PROBABILITY** A 5-character password can contain the numbers 0 through 9 and 26 letters of the alphabet. None of the characters can be repeated. What is the probability that the password begins with a consonant?

**F** \( \frac{21}{26} \)

**G** \( \frac{21}{35} \)

**H** \( \frac{21}{36} \)

**J** \( \frac{5}{36} \)

**SOLUTION:**

There are 26 + 10 = 36 possible choices for the first character, and there are 21 consonants. So, the total number of outcomes is 36 and the number of favorable outcomes is 21.

\[
P = \frac{21}{36}
\]

Therefore, the correct choice is H.

**ANSWER:**

H
4-3 Solving Quadratic Equations by Factoring

90. SAT/ACT If \( c = \frac{8a^3}{b} \), what happens to the value of \( c \) when both \( a \) and \( b \) are doubled?

A \( c \) is unchanged.
B \( c \) is halved.
C \( c \) is doubled.
D \( c \) is multiplied by 4.
E \( c \) is multiplied by 8.

**SOLUTION:**
When \( a \) is doubled, the value of \( a^3 \) becomes 8 times the original value of \( a \). So, when the values of both \( a \) and \( b \) are doubled, the value of \( c \) gets multiplied by \( \frac{8}{2} = 4 \). Therefore, the correct choice is D.

**ANSWER:**
D

91. \( x^2 - 2x - 8 = 0 \)

Use the related graph of each equation to determine its solutions.

**SOLUTION:**
The graph intersects the \( x \)-axis at \(-2\) and \(4\). Therefore, the roots of the equation are \(-2\) and \(4\).

**ANSWER:**
\(-2, 4\)

92. \( x^2 + 4x = 12 \)

**SOLUTION:**
The graph intersects the \( x \)-axis at \(-6\) and \(2\). Therefore, the roots of the equation are \(-6\) and \(2\).

**ANSWER:**
\(-6, 2\)
Graph each function.

93. \( x^2 + 4x + 4 = 0 \)

\[
\begin{array}{c|c}
  x & f(x) \\
  \hline
  0 & 2 \\
  2 & -6 \\
  3 & -7 \\
  4 & -6 \\
  6 & 2 \\
\end{array}
\]

SOLUTION:
The graph intersects the x-axis at \(-2\). Therefore, the root of the equation is \(-2\) and it is a repeated root.

ANSWER:
\(-2\)

94. \( f(x) = x^2 - 6x + 2 \)

SOLUTION:
Make a table of values and then graph the function.
4-3 Solving Quadratic Equations by Factoring

95. \( f(x) = -2x^2 + 4x + 1 \)

**SOLUTION:**

Make a table of values and then graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
</tr>
</tbody>
</table>

**ANSWER:**

96. \( f(x) = (x - 3)(x + 4) \)

**SOLUTION:**

Multiply the expression to get \( f(x) = x^2 + x - 12 \). Then make a table of values and then graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>2</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**ANSWER:**

97. **FUNDRAISING** Lawrence High School sold wrapping paper and boxed cards for their fundraising event. The school gets $1.00 for each roll of wrapping paper sold and $0.50 for each box of cards sold.
4-3 Solving Quadratic Equations by Factoring

<table>
<thead>
<tr>
<th>Total Amounts for Each Class</th>
<th>Wrapping Paper</th>
<th>Cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshmen</td>
<td>72</td>
<td>49</td>
</tr>
<tr>
<td>Sophomores</td>
<td>68</td>
<td>63</td>
</tr>
<tr>
<td>Juniors</td>
<td>90</td>
<td>56</td>
</tr>
<tr>
<td>Seniors</td>
<td>86</td>
<td>62</td>
</tr>
</tbody>
</table>

a. Write a matrix that represents the amounts sold for each class and a matrix that represents the amount of money the school earns for each item sold.

b. Write a matrix that shows how much each class earned.

c. Which class earned the most money?

d. What is the total amount of money the school made from the fundraiser?

**SOLUTION:**

a. Let the first column represent the number of wrapping papers sold by each class and the second column represent the number of cards. Then the matrix can be written as

\[
\begin{bmatrix}
72 & 49 \\
68 & 63 \\
90 & 56 \\
86 & 62
\end{bmatrix}
\]

The school gets $1.00 for each roll of wrapping paper sold and $0.50 for each box of cards sold. Then the matrix can be written as:

\[
\begin{bmatrix}
1.00 \\
0.50
\end{bmatrix}
\]

b. Multiply the matrices to find the matrix that shows how much each class earned.

c. The “juniors” class earned the most money.

d. Add the money earned by each class to find the total money earned by the school.

\[
\begin{bmatrix}
72 (1.00) + 49 (0.50) \\
68 (1.00) + 63 (0.50) \\
90 (1.00) + 56 (0.50) \\
86 (1.00) + 62 (0.50)
\end{bmatrix}
\]

\[
\begin{bmatrix}
96.50 \\
99.50 \\
118.00 \\
117.00
\end{bmatrix}
\]

The school made the total amount of $431 from the fundraiser.

**ANSWER:**

\[
\begin{align*}
\text{a.} & \quad \begin{bmatrix} 72 & 49 \end{bmatrix} \\
\text{b.} & \quad \begin{bmatrix} 96.50 \\
\text{c.} & \quad \text{juniors} \\
\text{d.} & \quad \text{$431}
\end{bmatrix}
\end{align*}
\]
4-3 Solving Quadratic Equations by Factoring

Simplify.

98. \( \sqrt{5} \cdot \sqrt{15} \)

**SOLUTION:**
\[
\sqrt{5} \cdot \sqrt{15} = \sqrt{5 \cdot 15} = \sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}
\]

**ANSWER:**
5\( \sqrt{3} \)

99. \( \sqrt{8} \cdot \sqrt{32} \)

**SOLUTION:**
\[
\sqrt{8} \cdot \sqrt{32} = \sqrt{8 \cdot 32} = \sqrt{256} = \sqrt{2^8} = 2^4 = 16
\]

**ANSWER:**
16

100. \( 2\sqrt{3} \cdot \sqrt{27} \)

**SOLUTION:**
\[
2\sqrt{3} \cdot \sqrt{27} = 2\sqrt{3 \cdot 3 \cdot 3} = 2\sqrt{3 \cdot 3 \cdot \sqrt{3}} = 2 \cdot 3 \cdot \sqrt{3} = 18
\]

**ANSWER:**
18
4-4 Complex Numbers

Simplify.

1. \( \sqrt{-81} \)

**SOLUTION:**
\[
\sqrt{-81} = \sqrt{-1 \cdot 9 \cdot 9} \\
= \sqrt{-1} \cdot \sqrt{9^2} \\
= 9i
\]

**ANSWER:**
9i

2. \( \sqrt{-32} \)

**SOLUTION:**
\[
\sqrt{-32} = \sqrt{-1 \cdot 2^2 \cdot 2 \cdot 2 \cdot 2} \\
= \sqrt{-1} \cdot 2^2 \cdot \sqrt{2} \\
= 4i \sqrt{2}
\]

**ANSWER:**
4i \( \sqrt{2} \)

3. \( (4i)(-3i) \)

**SOLUTION:**
\[
(4i)(-3i) = -12i^2 \\
= -12(-1) \\
= 12
\]

**ANSWER:**
12

4. \( 3\sqrt{-24} \cdot 2\sqrt{-18} \)

**SOLUTION:**
\[
3\sqrt{-24} \cdot 2\sqrt{-18} = 3 \cdot \sqrt{-1} \cdot 2 \cdot \sqrt{2} \cdot \sqrt{-1} \cdot 2 \cdot \sqrt{3} \\
= 3 \cdot \sqrt{-1} \cdot 2 \cdot \sqrt{2} \cdot \sqrt{3} \\
= 72 \cdot i \sqrt{6} \\
= -72 \cdot i \sqrt{3}
\]

**ANSWER:**
-72 \( \sqrt{3} \)

5. \( i^{40} \)

**SOLUTION:**
\[
i^{40} = (i^2)^{20} = (-1)^{20} = 1
\]

**ANSWER:**
1

6. \( i^{63} \)

**SOLUTION:**
\[
i^{63} = i^{62} \cdot i = (i^2)^{31} \cdot i = -1 \cdot i = -i
\]

**ANSWER:**
- \( i \)
4-4 Complex Numbers

Solve each equation.

7. \(4x^2 + 32 = 0\)

**SOLUTION:**

\[4x^2 + 32 = 0\]

\[4x^2 = -32\]

\[x^2 = -8\]

\[x = \pm \sqrt{-8}\]

\[x = \pm \sqrt{1 \cdot 2 \cdot 2 \cdot 2}\]

\[x = \pm 2i\sqrt{2}\]

**ANSWER:**

\[\pm 2i\sqrt{2}\]

8. \(x^2 + 1 = 0\)

**SOLUTION:**

\[x^2 + 1 = 0\]

\[x^2 = -1\]

\[x = \pm \sqrt{-1}\]

\[x = \pm i\sqrt{1}\]

\[x = \pm i\]

**ANSWER:**

\[\pm i\]

Find the values of \(a\) and \(b\) that make each equation true.

9. \(3a + (4b + 2)i = 9 - 6i\)

**SOLUTION:**

Set the real parts equal to each other.

\[3a = 9\]

\[a = 3\]

Set the imaginary parts equal to each other.

\[4b + 2 = -6\]

\[4b = -8\]

\[b = -2\]

**ANSWER:**

\[3, -2\]

10. \(4b - 5 + (-a - 3)i = 7 - 8i\)

**SOLUTION:**

Set the real parts equal to each other.

\[4b - 5 = 7\]

\[4b = 12\]

\[b = 3\]

Set the imaginary parts equal to each other.

\[-a - 3 = -8\]

\[-a = -5\]

\[a = 5\]

**ANSWER:**

\[5, 3\]
4-4 Complex Numbers

Simplify.

11. 
\(-1 + 5i) + (-2 - 3i)\)

**SOLUTION:**
\((-1 + 5i) + (-2 - 3i) = (-1 - 2) + (5i - 3i) = -3 + 2i\)

**ANSWER:**
\(-3 + 2i\)

12. 
\((7 + 4i) - (1 + 2i)\)

**SOLUTION:**
\((7 + 4i) - (1 + 2i) = 7 + 4i - 1 - 2i = 6 + 2i\)

**ANSWER:**
\(6 + 2i\)

13. 
\((6 - 8i)(9 + 2i)\)

**SOLUTION:**
\((6 - 8i)(9 + 2i) = 6(9) + 6(2i) - 8i(9) - 8i(2i) = 54 + 12i - 72i - 16i^2 = 54 + 12i - 72i - 16(-1) = 54 + 12i - 72i + 16 = 70 - 60i\)

**ANSWER:**
\(70 - 60i\)

14. 
\((3 + 2i)(-2 + 4i)\)

**SOLUTION:**
\((3 + 2i)(-2 + 4i) = 3(-2) + 3(4i) + 2i(-2) + 2i(4i) = -6 + 12i - 4i + 8i^2 = -6 + 12i - 4i + 8(-1) = -6 + 12i - 4i - 8 = -14 + 8i\)

**ANSWER:**
\(-14 + 8i\)

15. 
\(\frac{3 - i}{4 + 2i}\)

**SOLUTION:**
\[
\frac{3 - i}{4 + 2i} = \frac{3 - i}{4 + 2i} \cdot \frac{4 - 2i}{4 - 2i} = \frac{(3 - i)(4 - 2i)}{(4 + 2i)(4 - 2i)} = \frac{12 - 6i - 4i + 2i^2}{16 - 4i^2} = \frac{12 - 6i - 4i - 2}{16 - 4(-1)} = \frac{10 - 10i}{20} = \frac{10(1 - i)}{2 \cdot 10} = \frac{1 - i}{2} = \frac{1}{2} \cdot \frac{1 - i}{2}\]

**ANSWER:**
\(\frac{1}{2} \cdot \frac{1 - i}{2}\)
4-4 Complex Numbers

16. \( \frac{2 + i}{5 + 6i} \)

**SOLUTION:**
\[
\frac{2 + i}{5 + 6i} = \frac{(2 + i)(5 - 6i)}{(5 + 6i)(5 - 6i)} = \frac{10 - 12i + 5i - 6i^2}{25 - 36i^2} = \frac{10 - 12i + 5i + 6}{25 + 36} = \frac{16 - 7i}{61} = \frac{16}{61} - \frac{7}{61}i
\]

**ANSWER:**
\[
\frac{16}{61} - \frac{7}{61}i
\]

CCSS STRUCTURE  Simplify.

18. \( \sqrt{-121} \)

**SOLUTION:**
\[
\sqrt{-121} = \sqrt{-1 \cdot 11^2} = 11i
\]

**ANSWER:**
11i

19. \( \sqrt{-169} \)

**SOLUTION:**
\[
\sqrt{-169} = \sqrt{-1 \cdot 13^2} = 13i
\]

**ANSWER:**
13i

20. \( \sqrt{-100} \)

**SOLUTION:**
\[
\sqrt{-100} = \sqrt{-1 \cdot 10 \cdot 10} = \sqrt{1 \cdot 10^2} = 10i
\]

**ANSWER:**
10i

17. **ELECTRICITY** The current in one part of a series circuit is \(5 - 3i\) amps. The current in another part of the circuit is \(7 + 9i\) amps. Add these complex numbers to find the total current in the circuit.

**SOLUTION:**
Total current = \((5 - 3i) + (7 + 9i)\)
\[
= 5 - 3i + 7 + 9i = 12 + 6i \text{ amps}
\]

**ANSWER:**
12 + 6i amps

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21. \( \sqrt{-81} \)

**SOLUTION:**
\[ \sqrt{-81} = \sqrt{-1 \cdot 9 \cdot 9} \]
\[ = \sqrt{-1} \cdot \sqrt{9^2} \]
\[ = 9i \]

**ANSWER:**
9i

22. \((-3i)(-7i)(2i)\)

**SOLUTION:**
\[ (-3i)(-7i)(2i) = (-3 \cdot -7 \cdot 2)(i \cdot i \cdot i) \]
\[ = (-3 \cdot -7 \cdot 2)(-1 \cdot i) \]
\[ = -42i \]

**ANSWER:**
-42i

23. \(4i(-6i)^2\)

**SOLUTION:**
\[ 4i(-6i)^2 = (4i)(36i^2) \]
\[ = (-144)(i) \]
\[ = -144i \]

**ANSWER:**
-144i

24. \(i^{11}\)

**SOLUTION:**
\[ i^{11} = i^{10} \cdot i \]
\[ = (i^2)^5 \cdot i \]
\[ = -1 \cdot i \]
\[ = -i \]

**ANSWER:**
-i

25. \(i^{25}\)

**SOLUTION:**
\[ i^{25} = i^{24} \cdot i \]
\[ = (i^2)^{12} \cdot i \]
\[ = 1 \cdot i \]
\[ = i \]

**ANSWER:**
i

26. \((10 - 7i) + (6 + 9i)\)

**SOLUTION:**
\[ (10 - 7i) + (6 + 9i) = (10 + 6) + (-7i + 9i) \]
\[ = 16 + 2i \]

**ANSWER:**
16 + 2i
27. \((−3 + i) + (−4 − i)\)

**SOLUTION:**
\[
(−3 + i) + (−4 − i) = (−3 − 4) + (i − i)
\]
\[
= −7
\]

**ANSWER:**

28. \((12 + 5i) − (9 − 2i)\)

**SOLUTION:**
\[
(12 + 5i) − (9 − 2i) = 12 + 5i − 9 + 2i
\]
\[
= 3 + 7i
\]

**ANSWER:**

29. \((11 − 8i) − (2 − 8i)\)

**SOLUTION:**
\[
(11 − 8i) − (2 − 8i) = 11 − 8i − 2 + 8i
\]
\[
= 9
\]

**ANSWER:**

30. \((1 + 2i)(1 − 2i)\)

**SOLUTION:**
\[
(1 + 2i)(1 − 2i) = (1)(1) + (1)(−2i) + 2i(1) + 2i(−2i)
\]
\[
= 1 − 2i + 2i − 4i^2
\]
\[
= 1 − 2i + 2i − 4(−1)
\]
\[
= 1 + 4
\]
\[
= 5
\]

**ANSWER:**

31. \((3 + 5i)(5 − 3i)\)

**SOLUTION:**
\[
(3 + 5i)(5 − 3i) = 3(5) + 3(−3i) + 5i(5) + 5i(−3i)
\]
\[
= 15 − 9i + 25i − 15i^2
\]
\[
= 15 − 9i + 25i + 15
\]
\[
= 30 + 16i
\]

**ANSWER:**

32. \((4 − i)(6 − 6i)\)

**SOLUTION:**
\[
(4 − i)(6 − 6i) = 4(6) + 4(−6i) − i(6) − i(−6i)
\]
\[
= 24 − 24i − 6i + 6i^2
\]
\[
= 24 − 24i − 6i + 6
\]
\[
= 18 − 30i
\]

**ANSWER:**
4-4 Complex Numbers

33. \( \frac{2i}{1+i} \)

**SOLUTION:**

\[
\frac{2i}{1+i} = \frac{2i(1-i)}{(1+i)(1-i)} = \frac{2i - 2i^2}{1-i^2} = \frac{2i + 2}{1+1} = \frac{2i + 2}{2} = 1 + i
\]

**ANSWER:**

\( 1 + i \)

34. \( \frac{5}{2+4i} \)

**SOLUTION:**

\[
\frac{5}{2+4i} = \frac{5}{2+4i} \cdot \frac{2-4i}{2-4i} = \frac{5(2-4i)}{(2+4i)(2-4i)} = \frac{10 - 20i}{4 - 16i^2} = \frac{10 - 20i}{4 + 16} = \frac{10 - 20i}{20} = \frac{1}{2} - i
\]

**ANSWER:**

\( \frac{1}{2} - i \)

35. \( \frac{5+i}{3i} \)

**SOLUTION:**

\[
\frac{5+i}{3i} = \frac{3i(5+i)}{9i^2} = \frac{15i + 3i^2}{9(-1)} = \frac{15i - 3}{-9} = \frac{1}{3} - \frac{5}{3}i
\]

**ANSWER:**

\( \frac{1}{3} - \frac{5}{3}i \)

Solve each equation.

36. \( 4x^2 + 4 = 0 \)

**SOLUTION:**

\[4x^2 + 4 = 0 \]

\[4x^2 = -4 \]

\[x^2 = -1 \]

\[x = \pm \sqrt{-1} \]

\[x = \pm i \]

**ANSWER:**

\( \pm i \)
4-4 Complex Numbers

37. \( 3x^2 + 48 = 0 \)

**SOLUTION:**

\[
3x^2 + 48 = 0 \\
3x^2 = -48 \\
x^2 = -16 \\
x = \pm \sqrt{-16} \\
x = \pm 4i
\]

**ANSWER:**

\( \pm 4i \)

38. \( 2x^2 + 50 = 0 \)

**SOLUTION:**

\[
2x^2 + 50 = 0 \\
2x^2 = -50 \\
x^2 = -25 \\
x = \pm \sqrt{-25} \\
x = \pm 5i
\]

**ANSWER:**

\( \pm 5i \)

39. \( 2x^2 + 10 = 0 \)

**SOLUTION:**

\[
2x^2 + 10 = 0 \\
2x^2 = -10 \\
x^2 = -5 \\
x = \pm \sqrt{-5} \\
x = \pm i\sqrt{5}
\]

**ANSWER:**

\( \pm i\sqrt{5} \)

40. \( 6x^2 + 108 = 0 \)

**SOLUTION:**

\[
6x^2 + 108 = 0 \\
6x^2 = -108 \\
x^2 = -18 \\
x = \pm \sqrt{-18} \\
x = \pm 3i\sqrt{2}
\]

**ANSWER:**

\( \pm 3i\sqrt{2} \)

41. \( 8x^2 + 128 = 0 \)

**SOLUTION:**

\[
8x^2 + 128 = 0 \\
8x^2 = -128 \\
x^2 = -16 \\
x = \pm \sqrt{-16} \\
x = \pm 4i
\]

**ANSWER:**

\( \pm 4i \)
Find the values of x and y that make each equation true.

42. $9 + 12i = 3x + 4yi$

**SOLUTION:**
Set the real parts equal to each other.
$9 = 3x$
$3 = x$
Set the imaginary parts equal to each other.
$12 = 4y$
$3 = y$

**ANSWER:**
3, 3

43. $x + 1 + 2yi = 3 - 6i$

**SOLUTION:**
Set the real parts equal to each other.
$x + 1 = 3$
$x = 3 - 1$
$x = 2$
Set the imaginary parts equal to each other.
$2y = -6$
$y = -3$

**ANSWER:**
2, -3

44. $2x + 7 + (3 - y)i = -4 + 6i$

**SOLUTION:**
Set the real parts equal to each other.
$2x + 7 = -4$
$2x + 7 - 7 = -4 - 7$
$2x = -11$
$x = -\frac{11}{2}$
Set the imaginary parts equal to each other.
$3 - y = 6$
$y = -3$

**ANSWER:**
$-\frac{11}{2}, -3$

45. $5 + y + (3x - 7)i = 9 - 3i$

**SOLUTION:**
Set the real parts equal to each other.
$5 + y = 9$
$y = 4$
Set the imaginary parts equal to each other.
$3x - 7 = -3$
$3x - 7 + 7 = -3 + 7$
$3x = 4$
$x = \frac{4}{3}$

**ANSWER:**
$\frac{4}{3}, 4$
4.4 Complex Numbers

46. \( a + 3b + (3a - b)i = 6 + 6i \)

**SOLUTION:**
Set the real parts equal to each other.
\( a + 3b = 6 \rightarrow (1) \)
Set the imaginary parts equal to each other.
\( 3a - b = 6 \rightarrow (2) \)
Multiply the second equation by 3 and add the resulting equation to (1).
\( a + 3b = 6 \)
\( 9a - 3b = 18 \quad (+) \)
\( 10a = 24 \)
\( a = \frac{24}{10} \)
\( a = \frac{12}{5} \)

Substitute \( a = \frac{12}{5} \) in (1).
\( \frac{12}{5} + 3b = 6 \)
\( \frac{12 + 15b}{5} = 6 \)
\( 12 + 15b = 30 \)
\( 15b = 18 \)
\( b = \frac{18}{15} \)
\( b = \frac{6}{5} \)

**ANSWER:**
\( \frac{12}{5}, \frac{6}{5} \)

47. \( (2a - 4b)i + a + 5b = 15 + 58i \)

**SOLUTION:**
Set the real parts equal to each other.
\( a + 5b = 15 \rightarrow (1) \)
Set the imaginary parts equal to each other.
\( 2a - 4b = 58 \rightarrow (2) \)
Multiply the first equation by 2 and subtract the second equation from the resulting equation.
\( 2a + 10b = 30 \)
\( 2a - 4b = 58 \quad (-) \)
\( 14b = -28 \)
\( b = -2 \)
Substitute \( b = -2 \) in (1).
\( a + 5(-2) = 15 \)
\( a - 10 = 15 \)
\( a = 25 \)

**ANSWER:**
\( 25, -2 \)

Simplify.

48. \( \sqrt{10} \cdot \sqrt{-24} \)

**SOLUTION:**
\( \sqrt{-10} \cdot \sqrt{-24} = \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{-3} \)
\( = \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{3} \)
\( = i \cdot 2 \cdot \sqrt{15} \cdot i \cdot 2 \)
\( = -4\sqrt{15} \)

**ANSWER:**
\(-4\sqrt{15}\)
4-4 Complex Numbers

49. \(4i \left(\frac{1}{2}i\right)^2 (-2i)^2\)

**SOLUTION:**
\[
4i \left(\frac{1}{2}i\right)^2 (-2i)^2 = 4i \left(\frac{1}{2}\right)^2 i^2 (-2i)^2 i^2
\]
\[
= 4i \left(\frac{1}{4}\right)(-1)(4)(-1)
\]
\[
= 4i
\]

**ANSWER:**
\(4i\)

50. \(i^{41}\)

**SOLUTION:**
\[
i^{41} = i^{40} \cdot i
\]
\[
= (i^2)^{20} \cdot i
\]
\[
= 1 \cdot i
\]
\[
= i
\]

**ANSWER:**
\(i\)

51. \((4 - 6i) + (4 + 6i)\)

**SOLUTION:**
\[
(4 - 6i) + (4 + 6i) = 4 + 4 - 6i + 6i
\]
\[
= 8
\]

**ANSWER:**
\(8\)

52. \((8 - 5i) - (7 + i)\)

**SOLUTION:**
\[
(8 - 5i) - (7 + i) = 8 - 5i - 7 - i
\]
\[
= 1 - 6i
\]

**ANSWER:**
\(1 - 6i\)

53. \((-6 - i)(3 - 3i)\)

**SOLUTION:**
\[
(-6 - i)(3 - 3i) = -6(3) - 6(-3i) - i(3) - i(-3i)
\]
\[
= -18 + 18i - 3i - 3
\]
\[
= -21 + 15i
\]

**ANSWER:**
\(-21 + 15i\)
4-4 Complex Numbers

54. \( \frac{(5 + i)^2}{3 - i} \)

\[ \text{SOLUTION:} \]
\[ \frac{(5 + i)^2}{3 - i} = \frac{(5 + i)^2}{3 - i} \cdot \frac{3 + i}{3 + i} \]
\[ = \frac{(5 + i)^2 (3 + i)}{(3 - i)(3 + i)} \]
\[ = \frac{(25 - 1 + 10i)(3 + i)}{9 + 1} \]
\[ = \frac{(24 + 10i)(3 + i)}{10} \]
\[ = \frac{72 + 30i + 24i + 10i^2}{10} \]
\[ = \frac{72 + 30i + 24i - 10}{10} \]
\[ = \frac{62 + 54i}{10} \]
\[ = \frac{31 + 27i}{5} \]

\[ \text{ANSWER:} \]
\[ \frac{31}{5} + \frac{27}{5}i \]

55. \( \frac{6 - i}{2 - 3i} \)

\[ \text{SOLUTION:} \]
\[ \frac{6 - i}{2 - 3i} = \frac{6 - i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} \]
\[ = \frac{(6 - i)(2 + 3i)}{(2 - 3i)(2 + 3i)} \]
\[ = \frac{12 + 18i - 2i - 3i^2}{4 + 9} \]
\[ = \frac{12 + 16i}{13} \]
\[ = \frac{15 + 16i}{13} \]
\[ = \frac{15}{13} + \frac{16}{13}i \]

\[ \text{ANSWER:} \]
\[ \frac{15}{13} + \frac{16}{13}i \]

56. \((4 + 6i)(2 - i)(3 + 7i)\)

\[ \text{SOLUTION:} \]
\[ (4 + 6i)(2 - i)(3 + 7i) \]
\[ = (4(2) + 4(-i) + 6i(2) + 6i(-i))(3 + 7i) \]
\[ = (-8 + 4i + 12i + 6)(3 + 7i) \]
\[ = (-2 + 16i)(3 + 7i) \]
\[ = -2(3) - 2(7i) + 16i(3) + 16i(7i) \]
\[ = -6 - 14i + 48i - 112 \]
\[ = -118 + 34i \]

\[ \text{ANSWER:} \]
\[ -118 + 34i \]
4-4 Complex Numbers

57. \((1 + i)(2 + 3i)(4 - 3i)\)

**SOLUTION:**

\[
(1 + i)(2 + 3i)(4 - 3i) \\
= (1(2) + 1(3i) + i(2) + i(3i))(4 - 3i) \\
= (2 + 3i + 2i - 3i)(4 - 3i) \\
= (-1 + 5i)(4 - 3i) \\
= -4 + 3i + 20i + 15 \\
= 11 + 23i
\]

**ANSWER:**

11 + 23i

58. \(\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}}\)

**SOLUTION:**

\[
\frac{4 - i\sqrt{2}}{4 + i\sqrt{2}} = \frac{4 - i\sqrt{2}}{4 + i\sqrt{2}} \cdot \frac{4 - i\sqrt{2}}{4 - i\sqrt{2}} \\
= \frac{(4 - i\sqrt{2})(4 - i\sqrt{2})}{(4 + i\sqrt{2})(4 - i\sqrt{2})} \\
= \frac{16 - 2i\sqrt{2} - 8i\sqrt{2} + 2}{16 + 2} \\
= \frac{16 - 10i\sqrt{2}}{16} \\
= \frac{16 - 10i\sqrt{2}}{16} \\
= \frac{7}{9} - \frac{4i\sqrt{2}}{9}
\]

**ANSWER:**

\(\frac{7}{9} - \frac{4i\sqrt{2}}{9}\)

59. \(\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}}\)

**SOLUTION:**

\[
\frac{2 - i\sqrt{3}}{2 + i\sqrt{3}} = \frac{2 - i\sqrt{3}}{2 + i\sqrt{3}} \cdot \frac{2 - i\sqrt{3}}{2 - i\sqrt{3}} \\
= \frac{(2 - i\sqrt{3})(2 - i\sqrt{3})}{(2 + i\sqrt{3})(2 - i\sqrt{3})} \\
= \frac{4 - 3 - 4i\sqrt{3}}{1} \\
= \frac{4}{7} - \frac{4i\sqrt{3}}{7}
\]

**ANSWER:**

\(\frac{1}{7} - \frac{4i\sqrt{3}}{7}\)

60. **ELECTRICITY** The impedance in one part of a series circuit is \(7 + 8j\) ohms, and the impedance in another part of the circuit is \(13 - 4j\) ohms. Add these complex numbers to find the total impedance in the circuit.

**SOLUTION:**

Total impedance = \(7 + 8j + 13 - 4j\) = \(20 + 4j\) ohms

**ANSWER:**

\(20 + 4j\) ohms
62. The voltage in a circuit is $20 - 12j$ volts, and the impedance is $6 - 4j$ ohms. What is the current?

**SOLUTION:**
We know that voltage can be calculated by $V = C \cdot I$. 
$V =$ Voltage 
$C =$ current 
$I =$ impedance 
$20 - 12j = I(6 - 4j)$
\[
I = \frac{20 - 12j}{6 - 4j} \\
= \frac{20 - 12j}{6 - 4j} \cdot \frac{6 + 4j}{6 + 4j} \\
= \frac{(20 - 12j)(6 + 4j)}{(6 - 4j)(6 + 4j)} \\
= \frac{120 + 80j - 72j - 48}{36 + 16} \\
= \frac{168 + 8j}{52} \\
= \frac{42}{13} + \frac{2}{13}j \\
\]
Therefore, the current is $\frac{42}{13} + \frac{2}{13}j$ Amps.

**ANSWER:**
$\frac{42}{13} + \frac{2}{13}j$ Amps

63. Find the sum of $ix^2 - (4 + 5i)x + 7$ and $3x^2 + (2 + 6i)x - 8i$.

**SOLUTION:**
\[
ix^2 - (4 + 5i)x + 7 + 3x^2 + (2 + 6i)x - 8i \\
= (3 + i)x^2 - 5ix - 4x + 2x + 6ix + 7 - 8i \\
= (3 + i)x^2 + (2 + 6i)x - 8i \\
= (3 + i)x^2 + (-2 + i)x - 8i \\
\]
**ANSWER:**
$(3 + i)x^2 + (-2 + i)x - 8i + 7$
4-4 Complex Numbers

64. Simplify \((2 + i)x^2 - ix + 5 + i\) – \((-3 + 4i)x^2 + (5 - 5i)x - 6\).

**SOLUTION:**

\[
(2 + i)x^2 - ix + 5 + i - (-3 + 4i)x^2 + (5 - 5i)x - 6
= [(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 - (5 - 5i)x + 6]
= 2x^2 + ix^2 - ix + 5 + i + 3x^2 - 4ix^2 - 5x + 5ix + 6
= 5x^2 - 3ix^2 + i - 5x + 4ix + 11
= (5 - 3i)x^2 + (-5 + 4i)x + i + 11
\]

**ANSWER:**

\((5 - 3i)x^2 + (-5 + 4i)x + i + 11\)

c. Sample answer: \(x^2 - 4x + 5 = 0\)

d. Sample answer: \(A\) quadratic equation will have only complex solutions when the graph of the related function has no \(x\)-intercepts.

**ANSWER:**

a. Sample answer: \(x^2 + 9 = 0\)

b. Sample answer: \(x^2 - 4x + 5 = 0\)

d.
4-4 Complex Numbers

e. Sample answer: A quadratic equation will have only complex solutions when the graph of the related function has no x-intercepts.

66. CCSS CRITIQUE Joe and Sue are simplifying $(2i)(3i)(4i)$. Is either of them correct? Explain your reasoning.

Joe
$24i^3 = -24$

Sue
$24i^3 = -24i$

**SOLUTION:**
Sue; $i^3 = -i$, not $-1$.

**ANSWER:**
Sue; $i^3 = -i$, not $-1$.

67. CHALLENGE Simplify $(1 + 2i)^3$.

**SOLUTION:**
\[(1 + 2i)^3 = (1 + 2i)(1 + 2i)(1 + 2i) = (1 - 4i)(1 + 2i) = (-3 + 4i)(1 + 2i) = -3 - 6i + 4i - 8 = -11 - 2i\]

**ANSWER:**
$-11 - 2i$

68. REASONING Determine whether the following statement is always, sometimes, or never true. Explain your reasoning.

*Every complex number has both a real part and an imaginary part.*

**SOLUTION:**
Sample answer: Always. The value of $5$ can be represented by $5 + 0i$, and the value of $3i$ can be represented by $0 + 3i$.

**ANSWER:**
Sample answer: Always. The value of $5$ can be represented by $5 + 0i$, and the value of $3i$ can be represented by $0 + 3i$.

69. OPEN ENDED Write two complex numbers with a product of $20$.

**SOLUTION:**
Sample answer: $(4 + 2i)(4 - 2i)$

**ANSWER:**
Sample answer: $(4 + 2i)(4 - 2i)$
70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.

**SOLUTION:**
Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

**ANSWER:**
Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

71. **EXTENDED RESPONSE** Refer to the figure to answer the following.

[Diagram of triangles with labeled points D, E, A, B, and C.]

a. Name two congruent triangles with vertices in correct order.

b. Explain why the triangles are congruent.

c. What is the length of $EC$? Explain your procedure.

**SOLUTION:**

a. $\triangle CBE \cong \triangle ADE$

b. $\angle AED \cong \angle CEB$ (Vertical angles)

$DE \cong BE$ (Both have length $x$.)

$\angle ADE \cong \angle CBE$ (Given)

Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.

c. $EC \cong EA$ by CPCTC (corresponding parts of congruent triangles are congruent.) $EA = 7$, so $EC = 7$.

**ANSWER:**

a. $\triangle CBE \cong \triangle ADE$

b. $\angle AED \cong \angle CEB$ (Vertical angles)

$DE \cong BE$ (Both have length $x$.)

$\angle ADE \cong \angle CBE$ (Given) Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.

c. $EC \cong EA$ by CPCTC (corresponding parts of congruent triangles are congruent.) $EA = 7$, so $EC = 7$. 
4-4 Complex Numbers

72. \((3 + 6)^2 = \)

A \(2 \times 3 + 2 \times 6\

B \(9^2\

C \(3^2 + 6^2\

D \(3^2 \times 6^2\

\text{\textbf{SOLUTION:}}\
\( (3 + 6)^2 = 9^2 \)

So, the correct option is B.

\text{\textbf{ANSWER:}}

B

73. SAT/ACT A store charges $49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store’s cost.
How much would it cost an employee to purchase the pants after the sale?

F $10.50

G $12.50

H $13.72

J $24.50

K $35.00

\text{\textbf{SOLUTION:}}

Let \(x\) be the original amount of the pants.

\(\$49 = 0.4x + x\)

\(\$49 = 1.4x\)

\(x = \$35\)

\(\$35 \cdot \frac{30}{100} = \$10.50\)

\(\$35 - \$10.50 = \$24.50\)

So, the correct option is J.

\text{\textbf{ANSWER:}}

J
74. What are the values of \( x \) and \( y \) when \((5 + 4i) - (x + yi) = (-1 - 3i)\)?

**SOLUTION:**
Set the real parts equal to each other.
\[ 5 - x = -1 \]
\[ x = 6 \]

Set the imaginary parts equal to each other.
\[ 4 - y = -3 \]
\[ y = 7 \]

So, the correct option is A.

**ANSWER:**
A

---

**Solve each equation by factoring.**

75. \(2x^2 + 7x = 15\)

**SOLUTION:**
Write the equation with right side equal to zero.
\[ 2x^2 + 7x - 15 = 0 \]

Find factors of 2(–15) = –30 whose sum is 7. 10(–3) = –30 and 10 + (–3) = 7
\[ 2x^2 + 10x - 3x - 15 = 0 \]
\[ 2x(x + 5) - 3(x + 5) = 0 \]
\[ (x + 5)(2x - 3) = 0 \]
\[ \Rightarrow x + 5 = 0 \quad \text{or} \quad 2x - 3 = 0 \]
\[ \Rightarrow x = -5 \quad \text{or} \quad x = \frac{3}{2} \]

Therefore, the roots are \(-5\) and \(\frac{3}{2}\).

**ANSWER:**
\(-5, \frac{3}{2}\)
4-4 Complex Numbers

76. \(4x^2 - 12 = 22x\)

**SOLUTION:**
Write the equation with right side equal to zero.
\(4x^2 - 22x - 12 = 0\)
Find factors of 4(-12) = -48 whose sum is -22.
\(-24(2) = -48\) and 2 + (-24) = -22
\(4x^2 - 24x + 2x - 12 = 0\)
\(4x(x - 6) + 2(x - 6) = 0\)
\((x - 6)(4x + 2) = 0\)
\(\Rightarrow x - 6 = 0 \text{ or } 4x + 2 = 0\)
\(\Rightarrow x = 6 \text{ or } x = -\frac{1}{2}\)
Therefore, the roots are \(-\frac{1}{2}\) and 6.

**ANSWER:**
\(-\frac{1}{2}, 6\)

77. \(6x^2 = 5x + 4\)

**SOLUTION:**
Write the equation with right side equal to zero.
\(6x^2 - 5x - 4 = 0\)
Find factors of 6(-4) = -24 whose sum is -5.
\(-8(3) = -24\) and 3 + (-8) = -5
\(6x^2 - 8x + 3x - 4 = 0\)
\(2x(3x - 4) + 1(3x - 4) = 0\)
\((2x + 1)(3x - 4) = 0\)
\(\Rightarrow 2x + 1 = 0 \text{ or } 3x - 4 = 0\)
\(\Rightarrow x = -\frac{1}{2} \text{ or } x = \frac{4}{3}\)
Therefore, the roots are \(-\frac{1}{2}\) and \(\frac{4}{3}\).

**ANSWER:**
\(-\frac{1}{2}, \frac{4}{3}\)

**NUMBER THEORY** Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

78. Their sum is -3, and their product is -40.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of -3 and a product of -40 is
\(x^2 - 3x - 40 = 0\).
Solve the equation.
The two real numbers are 5 and -8.

**ANSWER:**
-8, 5

79. Their sum is 19, and their product is 48.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of 19 and a product of 48 is
\(x^2 + 19x + 48 = 0\).
Solve the equation.
The two real numbers are 3 and 16.

**ANSWER:**
3, 16

80. Their sum is -15, and their product is 56.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of -15 and a product of 56 is
\(x^2 - 15x + 56 = 0\).
Solve the equation.
The two real numbers are -7 and -8.

**ANSWER:**
-7 and -8
81. Their sum is –21, and their product is 108.

**SOLUTION:**
The quadratic equation to find the two real numbers with a sum of –21 and a product of 108 is $x^2 - 21x + 108 = 0$.
Solve the equation.
The two real numbers are –9 and –12.

**ANSWER:**
–9, –12

82. **RECREATION** Refer to the table.

a. Write a matrix that represents the cost of admission for residents and a matrix that represents the cost of admission for nonresidents.

b. Write the matrix that represents the additional cost for nonresidents.

c. Write a matrix that represents the difference in cost if a child or adult goes after 6:00 P.M. instead of before 6:00 P.M.

<table>
<thead>
<tr>
<th>Daily Admission Fees</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Residents</strong></td>
</tr>
<tr>
<td>Time of day</td>
</tr>
<tr>
<td>Before 6:00 P.M.</td>
</tr>
<tr>
<td>After 6:00 P.M.</td>
</tr>
<tr>
<td><strong>Nonresidents</strong></td>
</tr>
<tr>
<td>Time of day</td>
</tr>
<tr>
<td>Before 6:00 P.M.</td>
</tr>
<tr>
<td>After 6:00 P.M.</td>
</tr>
</tbody>
</table>

**SOLUTION:**
a. 

\[
\begin{bmatrix}
3.00 & 4.50 \\
2.00 & 3.50
\end{bmatrix}
\]

b. 

\[
\begin{bmatrix}
4.50 & 6.75 \\
3.00 & 5.25
\end{bmatrix}
\]
83. **PART-TIME JOBS** Terrell makes $10 an hour cutting grass and $12 an hour for raking leaves. He cannot work more than 15 hours per week. Graph two inequalities that Terrell can use to determine how many hours he needs to work at each job if he wants to earn at least $120 per week.

**SOLUTION:**
Let \( x \) be the hours spent cutting grass and \( y \) be the hours spent raking leaves. Terrell earns $10 per hour cutting grass and $12 per hour for raking leaves. He cannot work more than 15 hours per week and he wants to earn at least $120 per week. Write an inequality that represents the hours Terrell can work.
\[ x + y \leq 15 \]
Write an inequality that represents his earnings for a week.
\[ 10x + 12y \geq 120 \]

Graph the related equations and shade in the solution to the system of inequalities.

**Determine whether each trinomial is a perfect square trinomial. Write yes or no.**

84. \( x^2 + 16x + 64 \)

**SOLUTION:**
\( x^2 + 16x + 64 \) can be written as \((x + 8)^2\).
So, \( x^2 + 16x + 64 \) is a perfect square trinomial. The answer is “yes”.

**ANSWER:**
yes

85. \( x^2 - 12x + 36 \)

**SOLUTION:**
\( x^2 - 12x + 36 \) can be written as \((x - 6)^2\).
So, \( x^2 - 12x + 36 \) is a perfect square trinomial. The answer is “yes”.

**ANSWER:**
yes

86. \( x^2 + 8x - 16 \)

**SOLUTION:**
We cannot write the given trinomial as the perfect square format. So, the answer is “no”.

**ANSWER:**
no
4-4 Complex Numbers

87. \( x^2 - 14x - 49 \)

**SOLUTION:**
We cannot write the given trinomial as the perfect square format. So, the answer is “no”.

**ANSWER:**
no

88. \( x^2 + x + 0.25 \)

**SOLUTION:**
\( x^2 + x + 0.25 \) can be written as \( (x + 0.5)^2 \).
So, \( x^2 + x + 0.25 \) is a perfect square trinomial. The answer is “yes”.

**ANSWER:**
yes

89. \( x^2 + 5x + 6.25 \)

**SOLUTION:**
\( x^2 + 5x + 6.25 \) can be written as \( (x + 2.5)^2 \).
So, \( x^2 + 5x + 6.25 \) is a perfect square trinomial. The answer is “yes”.

**ANSWER:**
yes
4-5 Completing the Square

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

1. \( x^2 + 12x + 36 = 6 \)

**SOLUTION:**

\[ x^2 + 12x + 36 = 6 \]

Factor the perfect square trinomial.

\[ (x + 6)^2 = 6 \]

Use the Square Root Property.

\[ x + 6 = \pm \sqrt{6} \]

\[ x = -6 \pm \sqrt{6} \]

\[ x = -6 + \sqrt{6} \text{ or } x = -6 - \sqrt{6} \]

\[ x \approx -3.55 \text{ or } x \approx -8.45 \]

The solution set is \{-8.45, -3.55\}.

**ANSWER:**

\{-8.45, -3.55\}

2. \( x^2 - 8x + 16 = 13 \)

**SOLUTION:**

\[ x^2 - 8x + 16 = 13 \]

Factor the perfect square trinomial.

\[ (x - 4)^2 = 13 \]

Use the Square Root Property.

\[ x - 4 = \pm \sqrt{13} \]

\[ x = 4 \pm \sqrt{13} \]

\[ x = 4 + \sqrt{13} \text{ or } x = 4 - \sqrt{13} \]

\[ x \approx 7.61 \text{ or } x \approx 0.39 \]

The solution set is \{0.39, 7.61\}.

**ANSWER:**

\{0.39, 7.61\}
4-5 Completing the Square

3. \( x^2 + 18x + 81 = 15 \)

**SOLUTION:**

\( x^2 + 18x + 81 = 15 \)

Factor the perfect square trinomial.

\((x + 9)^2 = 15\)

Use the Square Root Property.

\(x + 9 = \pm \sqrt{15}\)

\(x = -9 \pm \sqrt{15}\) or \(x = -9 - \sqrt{15}\)

\(x \approx -5.13\) or \(x \approx -12.87\)

The solution set is \{-12.87, -5.13\}.

**ANSWER:**

\{-12.87, -5.13\}

4. \( 9x^2 + 30x + 25 = 11 \)

**SOLUTION:**

\( 9x^2 + 30x + 25 = 11 \)

Factor the perfect square trinomial.

\((3x + 5)^2 = 11\)

Use the Square Root Property.

\(3x + 5 = \pm \sqrt{11}\)

\(3x = -5 \pm \sqrt{11}\)

\(x = \frac{-5 \pm \sqrt{11}}{3}\)

\(x \approx -0.56\) or \(x \approx -2.77\)

The solution set is \{-2.77, -0.56\}.

**ANSWER:**

\{-2.77, -0.56\}
5. **LASER LIGHT SHOW** The area \( A \) in square feet of a projected laser light show is given by \( A = 0.16d^2 \), where \( d \) is the distance from the laser to the screen in feet. At what distance will the projected laser light show have an area of 100 square feet?

**SOLUTION:**
Substitute 100 for \( A \), and find the value of \( d \).

\[
100 = 0.16d^2 \\
625 = d^2
\]

Use the Square Root Property.

\[ d = \pm 25 \]

\( d \) cannot be negative.

So, the distance is 25 feet.

**ANSWER:**
25 ft

---

6. **Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.**

6. \( x^2 - 10x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

So:
\[
c = 25
\]

Substitute 25 for \( c \) in the trinomial.

\[
x^2 - 10x + 25 = (x - 5)^2
\]

**ANSWER:**
25; \((x - 5)^2\)

---

7. \( x^2 - 5x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square the result.

So:
\[
c = 6.25
\]

Substitute 6.25 for \( c \) in the trinomial.

\[
x^2 - 5x + 6.25 = (x - 2.5)^2
\]

**ANSWER:**
6.25; \((x - 2.5)^2\)

---

8. \( x^2 + 2x - 8 = 0 \)

**SOLUTION:**

\[
x^2 + 2x - 8 = 0 \\
x^2 + 2x = 8 \\
x^2 + 2x + 1 = 8 + 1 \\
(x + 1)^2 = 9
\]

Use the Square Root Property.

\[
x + 1 = \pm 3
\]

\[
x = 3 - 1 \quad \text{or} \quad x = -3 - 1 \\
x = 2 \quad \text{or} \quad x = -4
\]

The solution set is \{-4, 2\}.

**ANSWER:**
\{-4, 2\}
4-5 Completing the Square

9. \( x^2 - 4x + 9 = 0 \)

**SOLUTION:**
\[
\begin{align*}
& x^2 - 4x + 9 = 0 \\
& x^2 - 4x = -9 \\
& x^2 - 4x + 4 = -9 + 4 \\
& (x - 2)^2 = -5
\end{align*}
\]
Use the Square Root Property.
\[
x - 2 = \pm \sqrt{-5}
\]
\[
x = \sqrt{-5} + 2 \quad \text{or} \quad x = -\sqrt{-5} + 2
\]
\[
x = 2 + i\sqrt{5} \quad \text{or} \quad x = 2 - i\sqrt{5}
\]
The solution set is \( \{2 - i\sqrt{5}, \ 2 + i\sqrt{5}\} \).

**ANSWER:**
\( \{2 - i\sqrt{5}, \ 2 + i\sqrt{5}\} \)

10. \( 2x^2 - 3x - 3 = 0 \)

**SOLUTION:**
\[
\begin{align*}
& 2x^2 - 3x - 3 = 0 \\
& x^2 - \frac{3}{2}x - \frac{3}{2} = 0 \\
& x^2 - \frac{3}{2}x = \frac{3}{2} \\
& x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{3}{2} + \left(\frac{3}{4}\right)^2
\end{align*}
\]
\[
\begin{align*}
& \left(x - \frac{3}{4}\right)^2 = \frac{33}{16}
\end{align*}
\]
Use the Square Root Property.
\[
x - \frac{3}{4} = \pm \sqrt{\frac{33}{16}}
\]
\[
x = \frac{3}{4} + \sqrt{\frac{33}{16}} \quad \text{or} \quad x = \frac{3}{4} - \sqrt{\frac{33}{16}}
\]
\[
x \approx 2.19 \quad \text{or} \quad x \approx -0.69
\]
The solution set is \( \{-0.69, \ 2.19\} \).

**ANSWER:**
\( \{-0.69, \ 2.19\} \)
4-5 Completing the Square

11. \(2x^2 + 6x - 12 = 0\)

**SOLUTION:**

\[
\begin{align*}
2x^2 + 6x - 12 &= 0 \\
x^2 + 3x - 6 &= 0 \\
x^2 + 3x &= 6 \\
x^2 + 3x + \left(\frac{3}{2}\right)^2 &= 6 + \left(\frac{3}{2}\right)^2 \\
\left(x + \frac{3}{2}\right)^2 &= \frac{33}{4}
\end{align*}
\]

Use the Square Root Property.

\[
\begin{align*}
x + \frac{3}{2} &= \pm \sqrt{\frac{33}{4}} \\
x &= -\frac{3}{2} \pm \frac{\sqrt{33}}{2}
\end{align*}
\]

\[
\text{ANSWER:} \quad \{-4.37, 1.37\}
\]

The solution set is \{-4.37, 1.37\}.

12. \(x^2 + 4x + 6 = 0\)

**SOLUTION:**

\[
\begin{align*}
x^2 + 4x &= -6 \\
x^2 + 4x + 2^2 &= -6 + 2^2 \\
(x + 2)^2 &= -2
\end{align*}
\]

Use the Square Root Property.

\[
\begin{align*}
x + 2 &= \pm i\sqrt{2} \\
x &= -2 \pm i\sqrt{2} \quad \text{or} \quad x = -2 - i\sqrt{2}
\end{align*}
\]

The solution set is \{-2 - i\sqrt{2}, -2 + i\sqrt{2}\}

**ANSWER:**

\{-2 - i\sqrt{2}, -2 + i\sqrt{2}\}

13. \(x^2 + 8x + 10 = 0\)

**SOLUTION:**

\[
\begin{align*}
x^2 + 8x + 10 &= 0 \\
x^2 + 8x &= -10 \\
x^2 + 8x + 4^2 &= -10 + 4^2 \\
(x + 4)^2 &= 6
\end{align*}
\]

Use the Square Root Property.

\[
\begin{align*}
x + 4 &= \pm \sqrt{6} \\
x &= -4 \pm \sqrt{6} \quad \text{or} \quad x = -4 - \sqrt{6} \\
x \approx -1.55 \quad \text{or} \quad x \approx -6.45
\end{align*}
\]

**ANSWER:**

\{-6.45, -1.55\}
4-5 Completing the Square

Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.

14. \( x^2 + 4x + 4 = 10 \)

**SOLUTION:**
\[ x^2 + 4x + 4 = 10 \]
\[ (x + 2)^2 = 10 \]

Use the Square Root Property.
\[ x + 2 = \pm \sqrt{10} \]
\[ x = -2 + \sqrt{10} \quad \text{or} \quad x = -2 - \sqrt{10} \]
\[ x \approx 1.16 \quad \text{or} \quad x \approx -5.16 \]

The solution set is \{ -5.16, 1.16 \}.

**ANSWER:**
\{ -5.16, 1.16 \}

16. \( x^2 + 8x + 16 = 18 \)

**SOLUTION:**
\[ x^2 + 8x + 16 = 18 \]
\[ (x + 4)^2 = 18 \]

Use the Square Root Property.
\[ x + 4 = \pm \sqrt{18} \]
\[ x = -4 + \sqrt{18} \quad \text{or} \quad x = -4 - \sqrt{18} \]
\[ x \approx 0.24 \quad \text{or} \quad x \approx -8.24 \]

The solution set is \{ -8.24, 0.24 \}.

**ANSWER:**
\{ -8.24, 0.24 \}

17. \( x^2 + 10x + 25 = 7 \)

**SOLUTION:**
\[ x^2 + 10x + 25 = 7 \]
\[ (x + 5)^2 = 7 \]

\[ (x + 5)^2 = 7 \]
\[ x + 5 = \pm \sqrt{7} \]
\[ x = -5 + \sqrt{7} \quad \text{or} \quad x = -5 - \sqrt{7} \]
\[ x \approx -2.35 \quad \text{or} \quad x \approx -7.65 \]

The solution set is \{ -7.65, -2.35 \}.

**ANSWER:**
\{ -7.65, -2.35 \}

18. \( 2x^2 - 4x + 2 = 0 \)

**SOLUTION:**
\[ 2x^2 - 4x + 2 = 0 \]
\[ x^2 - 2x + 1 = 0 \]
\[ (x - 1)^2 = 0 \]

\[ x - 1 = 0 \]
\[ x = 1 \]

**ANSWER:**
\{ 1 \}
4-5 Completing the Square

18. \( x^2 + 12x + 36 = 5 \)

**SOLUTION:**
\[
x^2 + 12x + 36 = 5
\]
\[
(x + 6)^2 = 5
\]
Use the Square Root Property.

\[
x + 6 = \pm \sqrt{5}
\]
\[
x = -6 + \sqrt{5} \quad \text{or} \quad x = -6 - \sqrt{5}
\]
\[
x \approx -3.76 \quad \text{or} \quad x \approx -8.24
\]
The solution set is \{-8.24, -3.76\}.

**ANSWER:**
\{-8.24, -3.76\}

19. \( x^2 - 2x + 1 = 4 \)

**SOLUTION:**
\[
x^2 - 2x + 1 = 4
\]
\[
(x - 1)^2 = 4
\]
Use the Square Root Property.

\[
x - 1 = \pm 2
\]
\[
x = 1 + 2 \quad \text{or} \quad x = 1 - 2
\]
\[
x = 3 \quad \text{or} \quad x = -1
\]
The solution set is \{-1, 3\}.

**ANSWER:**
\{-1, 3\}

20. \( x^3 - 5x + 6.25 = 4 \)

**SOLUTION:**
\[
x^3 - 5x + 6.25 = 4
\]
\[
(x - 2.5)^2 = 4
\]
Use the Square Root Property.

\[
x - 2.5 = \pm 2
\]
\[
x = 2.5 + 2 \quad \text{or} \quad x = 2.5 - 2
\]
\[
x = 4.5 \quad \text{or} \quad x = 0.5
\]
The solution set is \{0.5, 4.5\}.

**ANSWER:**
\{0.5, 4.5\}

21. \( x^2 - 15x + 56.25 = 8 \)

**SOLUTION:**
\[
x^2 - 15x + 56.25 = 8
\]
\[
x^2 - 15x + (7.5)^2 = 8
\]
\[
(x - 7.5)^2 = 8
\]
Use the Square Root Property.

\[
x - 7.5 = \pm \sqrt{8}
\]
\[
x = 7.5 + \sqrt{8} \quad \text{or} \quad x = 7.5 - \sqrt{8}
\]
\[
x \approx 10.33 \quad \text{or} \quad x \approx 4.67
\]
The solution set is \{4.67, 10.33\}.

**ANSWER:**
\{4.67, 10.33\}
4-5 Completing the Square

22. $x^2 + 32x + 256 = 1$

**SOLUTION:**

$x^2 + 32x + 256 = 1$

$$\left(x + 16\right)^2 = 1$$

Use the Square Root Property.

$x + 16 = \pm 1$

$x = -15$ or $x = -17$

The solution set is {$-17, -15$}.

**ANSWER:**

{$-17, -15$}

23. $x^2 - 3x + \frac{9}{4} = 6$

**SOLUTION:**

$x^2 - 3x + \frac{9}{4} = 6$

$$\left(x - \frac{3}{2}\right)^2 = 6$$

Use the Square Root Property.

$$x - \frac{3}{2} = \pm \sqrt{6}$$

$x \approx 3.95$ or $x \approx -0.95$

The solution set is {$-0.95, 3.95$}.

**ANSWER:**

{$-0.95, 3.95$}

24. $x^2 + 7x + \frac{49}{4} = 4$

**SOLUTION:**

$x^2 + 7x + \frac{49}{4} = 4$

$$\left(x + \frac{7}{2}\right)^2 = 4$$

Use the Square Root Property.

$$x + \frac{7}{2} = \pm 2$$

$x = -1.5$ or $x = -5.5$

The solution set is {$-5.5, -1.5$}.

**ANSWER:**

{$-5.5, -1.5$}

25. $x^2 - 9x + \frac{81}{4} = \frac{1}{4}$

**SOLUTION:**

$x^2 - 9x + \frac{81}{4} = \frac{1}{4}$

$$\left(x - \frac{9}{2}\right)^2 = \frac{1}{4}$$

Use the Square Root Property.

$$x - \frac{9}{2} = \pm \frac{1}{2}$$

$x = 5$ or $x = 4$

The solution set is {4, 5}.

**ANSWER:**

{4, 5}
4-5 Completing the Square

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

26. \( x^2 + 8x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2, and square the result.

\[
c = 4^2 = 16
\]

Substitute 16 for \( c \) in the trinomial.

\[
x^2 + 8x + 16 = (x + 4)^2
\]

**ANSWER:**
16; \((x + 4)^2\)

28. \( x^2 - 11x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2, and square it.

\[
c = \left( -\frac{11}{2} \right)^2 = \frac{121}{4}
\]

Substitute \( c = \frac{121}{4} \) in the trinomial.

\[
x^2 - 11x + \frac{121}{4} = \left( x - \frac{11}{2} \right)^2
\]

**ANSWER:**
\(\frac{121}{4}; \left(x - \frac{11}{2}\right)^2\)

27. \( x^2 + 16x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2, and square it.

\[
c = 8^2 = 64
\]

Substitute 64 for \( c \) in the trinomial.

\[
x^2 + 16x + 64 = (x + 8)^2
\]

**ANSWER:**
64; \((x + 8)^2\)

29. \( x^2 + 9x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2, and square it.

\[
c = (4.5)^2 = 20.25
\]

Substitute \( c = 20.25 \) in the trinomial.

\[
x^2 + 9x + 20.25 = (x + 4.5)^2
\]

**ANSWER:**
20.25; \((x + 4.5)^2\)
Solve each equation by completing the square.

30. \( x^2 - 4x + 12 = 0 \)

**SOLUTION:**
\[
x^2 - 4x + 12 = 0 \\
x^2 - 4x = -12 \\
x^2 - 4x + 4 = -12 + 4 \\
(x - 2)^2 = -8
\]

Use the Square Root Property.
\[
x - 2 = \pm \sqrt{-8} \\
x = 2 + 2i\sqrt{2} \text{  or  } x = 2 - 2i\sqrt{2}
\]

The solution set is \( \{2 + 2i\sqrt{2}, 2 - 2i\sqrt{2}\} \).

**ANSWER:**
\( \{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}\} \)

31. \( x^2 + 2x - 12 = 0 \)

**SOLUTION:**
\[
x^2 + 2x - 12 = 0 \\
x^2 + 2x + 1 = 13 \\
(x + 1)^2 = 13
\]

Use the Square Root Property.
\[
x + 1 = \pm \sqrt{13} \\
x = -1 + \sqrt{13} \text{  or  } x = -1 - \sqrt{13} \\
x \approx 2.61 \text{  or  } x \approx -4.61
\]

The solution set is \( \{-4.61, 2.61\} \).

**ANSWER:**
\( \{-4.61, 2.61\} \)

32. \( x^2 + 6x + 8 = 0 \)

**SOLUTION:**
\[
x^2 + 6x + 8 = 0 \\
x^2 + 6x = -8 \\
x^2 + 6x + 9 = -8 + 9 \\
(x + 3)^2 = 1
\]

Use the Square Root Property.
\[
x + 3 = \pm 1 \\
x = -2 \text{  or  } x = -4
\]

The solution set is \( \{-4, -2\} \).

**ANSWER:**
\( \{-4, -2\} \)

33. \( x^2 - 4x + 3 = 0 \)

**SOLUTION:**
\[
x^2 - 4x + 3 = 0 \\
x^2 - 4x + 4 = 1 \\
(x - 2)^2 = 1
\]

Use the Square Root Property.
\[
x - 2 = \pm 1 \\
x = 3 \text{  or  } x = 1
\]

The solution set is \( \{1, 3\} \).

**ANSWER:**
\( \{1, 3\} \)
34. \(2x^2 + x - 3 = 0\)

**SOLUTION:**

\[
\begin{align*}
2x^2 + x - 3 &= 0 \\
x^2 + \frac{1}{2}x - \frac{3}{2} &= 0 \\
x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 &= \frac{3}{2} + \left(\frac{1}{4}\right)^2 \\
\left(x + \frac{1}{4}\right)^2 &= \frac{25}{16}
\end{align*}
\]

Use the Square Root Property.

\[
x + \frac{1}{4} = \pm\frac{5}{4}
\]

\[
x = 1 \text{ or } x = -\frac{3}{2}
\]

The solution set is \(\left\{-\frac{3}{2}, 1\right\}\).

**ANSWER:**

\(\left\{-\frac{3}{2}, 1\right\}\)

35. \(2x^2 - 3x + 5 = 0\)

**SOLUTION:**

\[
\begin{align*}
2x^2 - 3x + 5 &= 0 \\
x^2 - \frac{3}{2}x + \frac{5}{2} &= 0 \\
x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 &= -\frac{5}{2} + \left(\frac{3}{4}\right)^2 \\
\left(x - \frac{3}{4}\right)^2 &= -\frac{31}{16}
\end{align*}
\]

Use the Square Root Property.

\[
x - \frac{3}{4} = \pm\frac{i\sqrt{31}}{4}
\]

\[
x = \frac{3 + i\sqrt{31}}{4} \text{ or } x = \frac{3 - i\sqrt{31}}{4}
\]

The solution set is \(\left\{\frac{3 - i\sqrt{31}}{4}, \frac{3 + i\sqrt{31}}{4}\right\}\).

**ANSWER:**

\(\left\{\frac{3 - i\sqrt{31}}{4}, \frac{3 + i\sqrt{31}}{4}\right\}\)
36. \( 2x^2 + 5x + 7 = 0 \)

**SOLUTION:**

\[
2x^2 + 5x + 7 = 0 \\
\frac{5}{2}x = -\frac{7}{2} \\
x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -\frac{7}{2} + \left(\frac{5}{4}\right)^2 \\
\left(x + \frac{5}{4}\right)^2 = -\frac{31}{16} \\
x = -\frac{5+i\sqrt{31}}{4} \text{ or } x = -\frac{5-i\sqrt{31}}{4}
\]

The solution set is \( \left\{ \frac{-5+i\sqrt{31}}{4}, \frac{-5-i\sqrt{31}}{4} \right\} \).

**ANSWER:**

\( \left\{ \frac{-5+i\sqrt{31}}{4}, \frac{-5-i\sqrt{31}}{4} \right\} \)

37. \( 3x^2 - 6x - 9 = 0 \)

**SOLUTION:**

\[
3x^2 - 6x - 9 = 0 \\
x^2 - 2x - 3 = 0 \\
x^2 - 2x + 1 = 4 \\
(x-1)^2 = 4 \\
x - 1 = \pm 2 \\
x = 3 \text{ or } x = -1
\]

The solution set is \( \{-1, 3\} \).

**ANSWER:**

\( \{-1, 3\} \)

38. \( x^2 - 2x + 3 = 0 \)

**SOLUTION:**

\[
x^2 - 2x + 3 = 0 \\
x^2 - 2x + 1 = -2 \\
(x-1)^2 = -2 \\
x - 1 = \pm i\sqrt{2} \\
x = 1 + i\sqrt{2} \text{ or } x = 1 - i\sqrt{2} 
\]

The solution set is \( \{1 - i\sqrt{2}, 1 + i\sqrt{2}\} \).

**ANSWER:**

\( \{1 - i\sqrt{2}, 1 + i\sqrt{2}\} \)

39. \( x^2 + 4x + 11 = 0 \)

**SOLUTION:**

\[
x^2 + 4x + 11 = 0 \\
x^2 + 4x + 4 = -7 \\
(x+2)^2 = -7 \\
x + 2 = \pm i\sqrt{7} \\
x = -2 + i\sqrt{7} \text{ or } x = -2 - i\sqrt{7} 
\]

The solution set is \( \{-2 + i\sqrt{7}, -2 - i\sqrt{7}\} \).

**ANSWER:**

\( \{-2 + i\sqrt{7}, -2 - i\sqrt{7}\} \)
4-5 Completing the Square

40. \(x^2 - 6x + 18 = 0\)

**SOLUTION:**

\[x^2 - 6x + 18 = 0\]
\[x^2 - 6x + 9 = -9\]
\[(x - 3)^2 = -9\]
\[x - 3 = \pm 3i\]
\[x = 3 + 3i \text{ or } x = 3 - 3i\]

The solution set is \(\{3 - 3i, 3 + 3i\}\).

**ANSWER:**

\(\{3 - 3i, 3 + 3i\}\)

41. \(x^2 - 10x + 29 = 0\)

**SOLUTION:**

\[x^2 - 10x + 29 = 0\]
\[x^2 - 10 + 25 = -4\]
\[(x - 5)^2 = -4\]
\[x - 5 = \pm 2i\]
\[x = 5 + 2i \text{ or } x = 5 - 2i\]

The solution set is \(\{5 - 2i, 5 + 2i\}\).

**ANSWER:**

\(\{5 - 2i, 5 + 2i\}\)

42. \(3x^2 - 4x = 2\)

**SOLUTION:**

\[3x^2 - 4x = 2\]
\[x^2 - \frac{4}{3}x + \left(\frac{4}{6}\right)^2 = \frac{2}{3} + \left(\frac{4}{6}\right)^2\]
\[\left(x - \frac{4}{6}\right)^2 = \frac{10}{9}\]
\[x - \frac{2}{3} = \pm \frac{\sqrt{10}}{3}\]
\[x = 1.72 \text{ or } x = -0.39\]

The solution set is \(\{-0.39, 1.72\}\).

**ANSWER:**

\(\{-0.39, 1.72\}\)
### 4-5 Completing the Square

43. \(2x^2 - 7x = -12\)

**SOLUTION:**

\[
\begin{align*}
2x^2 - 7x &= -12 \\
x^2 - \frac{7}{2}x &= -6 \\
x^2 - \frac{7}{2}x + \frac{49}{16} &= -6 + \frac{49}{16} \\
\left(x - \frac{7}{4}\right)^2 &= -\frac{47}{16}
\end{align*}
\]

Use the Square Root Property.

\[
x - \frac{7}{4} = \pm \frac{i\sqrt{47}}{4}
\]

\[
x = \frac{7 + i\sqrt{47}}{4} \quad \text{or} \quad x = \frac{7 - i\sqrt{47}}{4}
\]

The solution set is \(\left\{\frac{7 - i\sqrt{47}}{4}, \frac{7 + i\sqrt{47}}{4}\right\}\).

**ANSWER:**

\(\left\{\frac{7 - i\sqrt{47}}{4}, \frac{7 + i\sqrt{47}}{4}\right\}\)

44. \(x^3 - 2.4x = 2.2\)

**SOLUTION:**

\[
\begin{align*}
x^3 - 2.4x &= 2.2 \\
x^3 - 2.4x + (1.2)^2 &= 2.2 + (1.2)^2 \\
(x - 1.2)^2 &= 3.64 \\
x - 1.2 &= \pm \sqrt{3.64} \\
x &= 1.2 \pm \sqrt{3.64} \\
x &\approx 3.11 \quad \text{or} \quad x \approx -0.71
\end{align*}
\]

The solution set is \(-0.71, 3.11\).

**ANSWER:**

\(-0.71, 3.11\)

45. \(x^3 - 5.3x = -8.6\)

**SOLUTION:**

\[
\begin{align*}
x^3 - 5.3x &= -8.6 \\
x^3 - 5.3x + (2.65)^2 &= -8.6 + (2.65)^2 \\
(x - 2.65)^2 &= -1.5775 \\
x - 2.65 &= \pm \sqrt{-1.5775} \\
x &= 2.65 \pm \sqrt{1.5775} \quad \text{or} \quad x = 2.65 \pm i\sqrt{1.5775}
\end{align*}
\]

The solution set is \(\left\{2.65 - i\sqrt{1.5775}, 2.65 + i\sqrt{1.5775}\right\}\).

**ANSWER:**

\(\left\{2.65 - i\sqrt{1.5775}, 2.65 + i\sqrt{1.5775}\right\}\)
4-5 Completing the Square

46. \( x^2 - \frac{1}{5} x - \frac{11}{5} = 0 \)

\textbf{SOLUTION:}
\[
x^2 - \frac{1}{5} x - \frac{11}{5} = 0 \\
x^2 - \frac{1}{5} x = \frac{11}{5} \\
x^2 - \frac{1}{5} x + \left( \frac{1}{10} \right)^2 = \frac{11}{5} + \left( \frac{1}{10} \right)^2 \\
\left( x - \frac{1}{10} \right)^2 = \frac{221}{100} \\
x - 0.1 \approx \pm 1.49 \\
x \approx 1.59 \text{ or } x \approx -1.39
\]

The solution set is \{-1.39, 1.59\}.

\textbf{ANSWER:}
\{-1.39, 1.59\}

47. \( x^2 - \frac{9}{2} x - \frac{24}{5} = 0 \)

\textbf{SOLUTION:}
\[
x^2 - \frac{9}{2} x - \frac{24}{5} = 0 \\
x^2 - \frac{9}{2} x = \frac{24}{5} \\
x - \frac{9}{4} + \left( \frac{9}{4} \right)^2 = \frac{24}{5} + \left( \frac{9}{4} \right)^2 \\
\left( x - \frac{9}{4} \right)^2 = 9.8625 \\
x - 2.25 \approx \pm 3.14 \\
x \approx 5.39 \text{ or } x \approx -0.89
\]

The solution set is \{-0.89, 5.39\}.

\textbf{ANSWER:}
\{-0.89, 5.39\}

48. \textbf{CCSS MODELING} An architect’s blueprints call for a dining room measuring 13 feet by 13 feet. The customer would like the dining room to be a square, but with an area of 250 square feet. How much will this add to the dimensions of the room?

\[
\begin{array}{c}
13 \text{ ft} \\
x \text{ ft}
\end{array}
\]

\textbf{SOLUTION:}

The area of a square is given by \( A = s^2 \), where \( s \) is the side length.

Therefore:
\[
250 = (13 + x)^2
\]

Solve for \( x \).

Therefore, about 2.81 ft should be added to the dimensions of the room.

\textbf{ANSWER:}
about 2.81 ft
4-5 Completing the Square

Solve each equation. Round to the nearest hundredth if necessary.

49. \( 4x^2 - 28x + 49 = 5 \)

**SOLUTION:**
Write the equation in standard form and solve using the quadratic formula.

\( 4x^2 - 28x + 44 = 0 \)

The quadratic formula is given by:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(28) \pm \sqrt{(-28)^2 - 4(4)(44)}}{2(4)}
\]

\( x \approx 4.62 \) or \( x \approx 2.38 \)

The solution set is \( \{2.38, 4.62\} \).

**ANSWER:**
\( \{2.38, 4.62\} \)

---

50. \( 9x^2 + 30x + 25 = 11 \)

**SOLUTION:**
Write the equation in standard form and solve using the quadratic formula.

\( 9x^2 + 30x + 14 = 0 \)

The quadratic formula is given by:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-30 \pm \sqrt{(30)^2 - 4(9)(14)}}{2(9)}
\]

\( x \approx -0.56 \) or \( x \approx -2.77 \)

The solution set is \( \{-2.77, -0.56\} \).

**ANSWER:**
\( \{-2.77, -0.56\} \)
4-5 Completing the Square

51. \(x^2 + x + \frac{1}{3} = \frac{2}{3}\)

**SOLUTION:**
Write the equation in standard form and solve using the quadratic formula.

\[x^2 + x - \frac{1}{3} = 0\]

The quadratic formula is given by:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-\frac{1}{3})}}{2(1)}\]

\[x \approx 0.26 \text{ or } x \approx -1.26\]

The solution set is \(\{0.26, -1.26\}\).

**ANSWER:**
\(\{-1.26, 0.26\}\)

52. \(x^2 + 1.2x + 0.56 = 0.91\)

**SOLUTION:**
Write the equation in standard form and solve using the quadratic formula.

\[x^2 + 1.2x - 0.35 = 0\]

The quadratic formula is given by:

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-1.2 \pm \sqrt{(1.2)^2 - 4(1)(0.35)}}{2(1)}\]

\[x \approx 0.24 \text{ or } x \approx -1.44\]

The solution set is \(\{-1.44, 0.24\}\).

**ANSWER:**
\(\{-1.44, 0.24\}\)
4-5 Completing the Square

53. **FIREWORKS** A firework’s distance \( d \) meters from the ground is given by \( d = -1.5t^2 + 25t \), where \( t \) is the number of seconds after the firework has been lit.

a. How many seconds have passed since the firework was lit when the firework explodes if it explodes at the maximum height of its path?

**SOLUTION:**

- The maximum occurs at the vertex.
- Find the \( x \)-coordinate of the vertex.

The \( x \)-coordinate of the vertex is given by \( x = \frac{-b}{2a} \).

Here, \( a = -1.5 \) and \( b = 25 \).

Therefore:

\[
t = \frac{25}{3}
\]

\[
= 8 \frac{1}{3}
\]

That is \( 8 \frac{1}{3} \) seconds should have passed when the firework explodes at the maximum height.

b. Substitute \( 8 \frac{1}{3} \) for \( t \) in the equation and solve for \( d \).

\[
d = -1.5 \left( 8 \frac{1}{3} \right)^2 + 25 \left( 8 \frac{1}{3} \right)
\]

\[
\approx 104.2 \text{ ft}
\]

The height of the firework is about 104.2 ft.

**ANSWER:**

- a. \( 8 \frac{1}{3} \) seconds

- b. about 104.2 ft

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

54. \( x^2 + 0.7x + c \)

**SOLUTION:**

To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{0.7}{2} \right)^2
\]

\[
= 0.1225
\]

Substitute the value of \( c \) in the trinomial.

\[x^2 + 0.7x + 0.1225 = (x + 0.35)^2\]

**ANSWER:**

0.1225; \((x + 0.35)^2\)

55. \( x^3 - 3.2x + c \)

**SOLUTION:**

To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{-3.2}{2} \right)^2
\]

\[
= 2.56
\]

Substitute the value of \( c \) in the trinomial.

\[x^3 - 3.2x + 2.56 = (x - 1.6)^2\]

**ANSWER:**

2.56; \((x - 1.6)^2\)
56. \( x^2 - 1.8x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{-1.8}{2} \right)^2 = 0.81 = \frac{81}{100}
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 - 1.8x + \frac{81}{100} = \left( x - \frac{9}{10} \right)^2
\]

**ANSWER:**
\[
\frac{81}{100}\left( x - \frac{9}{10} \right)^2
\]

57. **MULTIPLE REPRESENTATIONS** In this problem, you will use quadratics to investigate golden rectangles and the golden ratio.

**a. GEOMETRIC**
- Draw square \( ABCD \).
- Locate the midpoint of \( CD \). Label the midpoint \( P \).
- Draw \( PB \).
- Construct an arc with a radius of \( PB \) from \( B \) clockwise past the bottom of the square.
- Extend \( CD \) until it intersects the arc. Label this point \( Q \).
- Construct rectangle \( ARQD \).

**b. ALGEBRAIC** Let \( AD = x \) and \( CQ = 1 \). Use completing the square to solve \( \frac{DQ}{AD} = \frac{QR}{CQ} \) for \( x \).

c. **TABULAR** Make a table of \( x \) and values for \( CQ = 2, 3, \) and \( 4 \).

d. **VERBAL** What do you notice about the \( x \)-values? Write an equation you could use to determine \( x \) for \( CQ = n \), where \( n \) is a nonzero real number.
4-5 Completing the Square

\[
x = \frac{1 + \sqrt{5}}{2}
\]

c.

<table>
<thead>
<tr>
<th>CQ</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 + \sqrt{5}</td>
</tr>
<tr>
<td>3</td>
<td>3 + 3\sqrt{5}</td>
</tr>
<tr>
<td>4</td>
<td>2 + 2\sqrt{5}</td>
</tr>
</tbody>
</table>

d. Sample answer: the x-values are multiples of \(\frac{1 + \sqrt{5}}{2}\);

\[
x = \frac{n(1 + \sqrt{5})}{2}
\]

ANSWER:

a.

![Diagram]

b. \(x = \frac{1 + \sqrt{5}}{2}\)

c.

<table>
<thead>
<tr>
<th>CQ</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 + \sqrt{5}</td>
</tr>
<tr>
<td>3</td>
<td>3 + 3\sqrt{5}</td>
</tr>
<tr>
<td>4</td>
<td>2 + 2\sqrt{5}</td>
</tr>
</tbody>
</table>

D. Sample answer: the x-values are multiples of \(\frac{1 + \sqrt{5}}{2}\);

\[
x = \frac{n(1 + \sqrt{5})}{2}
\]

58. ERROR ANALYSIS Alonso and Aida are solving \(x^2 + 8x - 20 = 0\) by completing the square. Is either of them correct? Explain your reasoning.

**Alonso**

\[
x^2 + 8x - 20 = 0
\]
\[
x^2 + 8x = 20
\]
\[
x^2 + 8x + 16 = 20 + 16
\]
\[
(x + 4)^2 = 36
\]
\[
x + 4 = \pm 6
\]
\[
x = -4 \pm 6
\]

**Aida**

\[
x^2 + 8x - 20 = 0
\]
\[
x^2 + 8x = 20
\]
\[
x^2 + 8x + 16 = 20
\]
\[
(x + 4)^2 = 20
\]
\[
x + 4 = \pm \sqrt{20}
\]
\[
x = -4 \pm \sqrt{20}
\]

**SOLUTION:**

Alonso: Aida did not add 16 to each side; she added it only to the left side.

**ANSWER:**

Alonso: Aida did not add 16 to each side; she added it only to the left side.
4-5 Completing the Square

59. CHALLENGE Solve \( x^2 + bx + c = 0 \) by completing the square. Your answer will be an expression for \( x \) in terms of \( b \) and \( c \).

**SOLUTION:**

\[
\begin{align*}
  x^2 + bx + c &= 0 \\
  x^2 + bx + c - c &= -c \\
  x^2 + bx &= -c \\
  x^2 + bx + \left(\frac{b}{2}\right)^2 &= -c + \left(\frac{b}{2}\right)^2 \\
  \left(x + \frac{b}{2}\right)^2 &= \frac{b^2}{4} - c
\end{align*}
\]

Use the Square Root Property.

\[
x + \frac{b}{2} = \pm \sqrt{\frac{b^2}{4} - c}
\]

\[
x = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}
\]

**ANSWER:**

\[
x = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}
\]

60. CCSS ARGUMENTS Without solving, determine how many unique solutions there are for each equation. Are they rational, real, or complex? Justify your reasoning.

a. \((x + 2)^2 = 16\)

b. \((x - 2)^2 = 16\)

c. \(-(x - 2)^2 = 16\)

d. \(36 - (x - 2)^2 = 16\)

e. \(16(x + 2)^2 = 0\)

**SOLUTION:**

a. 2; rational; 16 is a perfect square so \( x + 2 \) and \( x \) are rational.

b. 2; rational; 16 is a perfect square so \( x - 2 \) and \( x \) are rational.

c. 2; complex; If the opposite of square is positive, the square is negative. The square root of a negative number is complex.

d. 2; real; The square must equal 20. Since that is positive but not a perfect square, the solutions will be real but not rational.

e. 1; rational; The expression must be equal to 0 and only -2 makes the expression equal to 0.

f. 1; rational; The expressions \((x + 4)\) and \((x + 6)\) must either be equal or opposites. No value makes them equal, -5 makes them opposites. The only solution is -5.
4-5 Completing the Square

61. **OPEN ENDED** Write a perfect square trinomial equation in which the linear coefficient is negative and the constant term is a fraction. Then solve the equation.

**SOLUTION:**
Sample answer:

\[ x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{4} \]

The solution set is \( \left\{ \frac{5}{6}, -\frac{1}{6} \right\} \).

**ANSWER:**
Sample answer: \( x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{4}; \left\{ \frac{5}{6}, -\frac{1}{6} \right\} \)

62. **WRITING IN MATH** Explain what it means to complete the square. Include a description of the steps you would take.

**SOLUTION:**
Completing the square allows you to rewrite one side of a quadratic equation in the form of a perfect square. Once in this form, the equation can be solved by using the Square Root Property.

**ANSWER:**
Completing the square allows you to rewrite one side of a quadratic equation in the form of a perfect square. Once in this form, the equation can be solved by using the Square Root Property.

63. **SAT/ACT** If \( x^2 + y^2 = 2xy \), then \( y \) must equal

A \(-1\)
B \(0\)
C \(1\)
D \(-x\)
E \(x\)

**SOLUTION:**
\[
\begin{align*}
x^2 + y^2 &= 2xy \\
x^2 + y^2 - 2xy &= 0 \\
(x - y)^2 &= 0 \\
x &= y
\end{align*}
\]

The correct choice is E.
4-5 Completing the Square

64. GEOMETRY Find the area of the shaded region.

\[ \text{SOLUTION:} \]

The area of the outer rectangle is given by:

\[ A_1 = 10 \times 6 = 60 \text{ m}^2 \]

The area of the inner rectangle is given by:

\[ A_2 = 6 \times 3 = 18 \text{ m}^2 \]

The area of the shaded region is \( A_1 - A_2 \).

\[ A_1 - A_2 = 60 - 18 = 42 \text{ m}^2 \]

The correct choice is H.

\[ \text{ANSWER:} \text{ H} \]

65. SHORT RESPONSE What value of \( c \) should be used to solve the following equation by completing the square?

\[ 5x^2 - 50x + c = 12 + c \]

\[ \text{SOLUTION:} \]

\[ 5(x^2 - 10x + \frac{c}{5}) = 12 + c \]

Thus \( \frac{c}{5} = 25 \) and \( c = 125 \)

\[ \text{ANSWER:} \]

125
4-5 Completing the Square

66. If \(5 - 3i\) is a solution for \(x^2 + ax + b = 0\), where \(a\) and \(b\) are real numbers, what is the value of \(b^2\)?

A 10
B 14
C 34
D 40

**SOLUTION:**
Substitute \(5 - 3i\) for \(x\) in the equation.
\[
(5 - 3i)^2 + a(5 - 3i) + b = 0
\]
\[
25 - 30i - 9 + 5a - 3ai + b = 0
\]
\[
5a + b + 16 - (3a + 30)i = 0
\]
Equate the real and the imaginary parts.
\[
- (3a + 30) = 0\]
\[
a = -10
\]
And:
\[
5a + b + 16 = 0
\]
Substitute \(a = -10\).
\[
5(-10) + b + 16 = 0
\]
\[
b - 34 = 0
\]
\[
b = 34
\]
The correct choice is C.

**ANSWER:**
C

Simplify.

67. \((8 + 5i)^2\)

**SOLUTION:**
\[
(8 + 5i)^2 = 8^2 + 2(8)(5i) + (5i)^2
\]
\[
= 64 + 80i - 25
\]
\[
= 39 + 80i
\]

**ANSWER:**
39 + 80i

68. \(4(3 - i) + 6(2 - 5i)\)

**SOLUTION:**
\[
4(3 - i) + 6(2 - 5i) = 12 - 4i + 12 - 30i
\]
\[
= 24 - 34i
\]

**ANSWER:**
24 - 34i

69. \(\frac{5 - 2i}{6 + 9i}\)

**SOLUTION:**
\[
\frac{5 - 2i}{6 + 9i} \times \frac{6 - 9i}{6 + 9i}
\]
\[
= \frac{30 - 45i - 12i + 18i^2}{6^2 - (9i)^2}
\]
\[
= \frac{12 - 57i}{117}
\]
\[
= \frac{4}{39} - \frac{19}{39}i
\]

**ANSWER:**
\[\frac{4}{39} - \frac{19}{39}i\]
Write a quadratic equation in standard form with
the given root(s).

70. \( \frac{4}{5}, \frac{3}{4} \)

**SOLUTION:**

\[
\left(x - \frac{4}{5}\right)\left(x - \frac{3}{4}\right) = 0
\]

\[
x^2 - \frac{3}{4}x - \frac{4}{5}x + \frac{12}{20} = 0
\]

\[
20x^2 - 31x + 12 = 0
\]

**ANSWER:**

\( 20x^2 - 31x + 12 = 0 \)

71. \( -\frac{2}{5}, 6 \)

**SOLUTION:**

\[
\left(x - \left(-\frac{2}{5}\right)\right)(x - 6) = 0
\]

\[
\left(x + \frac{2}{5}\right)(x - 6) = 0
\]

\[
x^2 - 6x + \frac{2}{5}x - \frac{12}{5} = 0
\]

\[
5x^2 - 28x - 12 = 0
\]

**ANSWER:**

\( 5x^2 - 28x - 12 = 0 \)

72. \( -\frac{1}{4}, -\frac{6}{7} \)

**SOLUTION:**

\[
\left(x - \left(-\frac{1}{4}\right)\right)\left(x - \left(-\frac{6}{7}\right)\right) = 0
\]

\[
\left(x + \frac{1}{4}\right)\left(x + \frac{6}{7}\right) = 0
\]

\[
x^2 + \frac{6}{7}x + \frac{1}{4}x + \frac{6}{28} = 0
\]

\[
28x^2 + 31x + 6 = 0
\]

**ANSWER:**

\( 28x^2 + 31x + 6 = 0 \)

73. **TRAVEL**

Yoko is going with the Spanish Club to Costa Rica. She buys 10 traveler’s checks in
denominations of $20, $50, and $100, totaling $370.

She has twice as many $20 checks as $50 checks.

How many of each denomination of traveler’s
checks does she have?

**SOLUTION:**

Let \( x, y, \) and \( z \) represent the number of $20, $50, and
$100 checks respectively.

So: \( x + y + z = 10 \)

And: \( 20x + 50y + 100z = 370 \)

Also: \( x = 2y \)

Solve the equations.

\( x = 6, y = 3, \) and \( z = 1. \)

Therefore, Yoko has 1-$100, 3-$50, and 6-$20
checks.

**ANSWER:**

1 $100, 3 $50, and 6 $20 checks
74. SHOPPING Main St. Media sells all DVDs for one price and all books for another price. Alex bought 4 DVDs and 6 books for $170, while Matt bought 3 DVDs and 8 books for $180. What is the cost of a DVD and the cost of a book?

**SOLUTION:**
Let \( x \) and \( y \) be the cost of a DVD and a book.

\[
\begin{align*}
4x + 6y &= 170 \quad \rightarrow (1) \\
3x + 8y &= 180 \quad \rightarrow (2)
\end{align*}
\]

Multiply the first and the second equation by 3 and \(-4\) respectively then add.

\[
\begin{align*}
(1) \times 3 & \quad 12x + 18y = 510 \\
(2) \times -4 & \quad -12x - 32y = -720
\end{align*}
\]

\[
\begin{align*}
-14y &= -210 \\
y &= 15
\end{align*}
\]

Substitute 15 for \( y \) in the first equation and solve for \( x \).

\[
\begin{align*}
4x + 6(15) &= 170 \\
4x + 90 &= 170 \\
4x &= 80 \\
x &= 20
\end{align*}
\]

The cost of a DVD is $20.

The cost of a book is $15.

**ANSWER:**
DVD: $20; book: $15

---

Graph each inequality.

75. \( y \geq 4x - 3 \)

**SOLUTION:**
The boundary of the graph is the graph of \( y = 4x - 3 \). Since the inequality symbol is \( \geq \), the line will be solid. Test the point \((0, 0)\).

\[
\begin{align*}
0 &\geq 4(0) - 3 \\
0 &\geq -3 \quad \checkmark
\end{align*}
\]

Shade the region that includes \((0, 0)\).

The graph of the inequality \( y \geq 4x - 3 \) is:

---

**ANSWER:**

DVD: $20; book: $15
4-5 Completing the Square

76. \(2x - 3y < 6\)

**SOLUTION:**
The boundary of the graph is the graph of \(2x - 3y = 6\). Since the inequality symbol is <, the line will be dashed.

Test the point \((0, 0)\).

\[
2(0) - 3(0) < 6
\]

\[
0 < 6 \quad \checkmark
\]

Shade the region that contains \((0, 0)\).

The graph of the inequality is:

![Graph of inequality](image)

**ANSWER:**

77. \(5x + 2y + 3 \leq 0\)

**SOLUTION:**
The boundary of the graph is the graph of \(5x + 2y + 3 = 0\). Since the inequality symbol is \(\leq\), the line will be solid.

Test the point \((0, 0)\).

\[
5(0) + 2(0) + 3 \leq 0
\]

\[
3 \leq 0 \quad \times
\]

Shade the region that does not contain \((0, 0)\).

The graph of the inequality is:

![Graph of inequality](image)

**ANSWER:**
Write the piecewise function shown in each graph.

78.

SOLUTION:
The left portion is the graph of the constant function \( f(x) = -4 \). There is a dot at \(-2\), so the constant function is defined for \( x \leq -2 \).

The center portion is the graph of \( f(x) = -2x + 3 \). There are circles at \(-2\) and \(1\), so the function is defined for \(-2 < x < 1\).

The right portion is the graph of \( f(x) = x - 5 \). There is a dot at \(2\), so the function is defined for \( x \geq 2 \).

The piece-wise function is:

\[
    f(x) = \begin{cases} 
        -4 & \text{if } x \leq -2 \\
        -2x + 3 & \text{if } -2 < x < 1 \\
        x - 5 & \text{if } x \geq 2 
    \end{cases}
\]

ANSWER:

\[
    f(x) = \begin{cases} 
        -4 & \text{if } x \leq -2 \\
        -2x + 3 & \text{if } -2 < x < 1 \\
        x - 5 & \text{if } x \geq 2 
    \end{cases}
\]
4-5 Completing the Square

80.

SOLUTION:
The left portion is the graph of \( f(x) = x + 12 \). There is a dot at \(-6\), so the function is defined for \( x \leq -6 \).
The center portion is the graph of the constant function \( f(x) = 8 \). There are circles at \(-6\) and \(2\), so the constant function is defined for \(-6 < x < 2\).
The right portion is the graph of \( f(x) = -2.5x + 15 \). There is a dot at \(2\), so the function is defined for \( x \geq 2 \).
The piece-wise function is:

\[
 f(x) = \begin{cases} 
 x + 12 & \text{if } x \leq -6 \\
 8 & \text{if } -6 < x < 2 \\
 -2.5x + 15 & \text{if } x \geq 2 
\end{cases}
\]

ANSWER:

\[
 f(x) = \begin{cases} 
 x + 12 & \text{if } x \leq -6 \\
 8 & \text{if } -6 < x < 2 \\
 -2.5x + 15 & \text{if } x \geq 2 
\end{cases}
\]

Evaluate \( b^2 - 4ac \) for the given values of \( a, b, \) and \( c \).

81. \( a = 5, b = 6, c = 2 \)

SOLUTION:
Substitute \( a = 5, b = 6, \) and \( c = 2 \).

\[
 6^2 - 4(5)(2) = 36 - 40 = -4
\]

ANSWER:

\(-4\)

82. \( a = -2, b = -7, c = 3 \)

SOLUTION:
Substitute \( a = -2, b = -7, \) and \( c = 3 \).

\[
 (-7)^2 - 4(-2)(3) = 49 + 24 = 73
\]

ANSWER:

\(73\)

83. \( a = -5, b = -8, c = -10 \)

SOLUTION:
Substitute \( a = -5, b = -8, \) and \( c = -10 \).

\[
 (-8)^2 - 4(-5)(-10) = 64 - 200 = -136
\]

ANSWER:

\(-136\)
Solve each equation by using the Quadratic Formula.

1. \(x^2 + 12x - 9 = 0\)

**SOLUTION:**

Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).

\[x^2 + 12x - 9 = 0 \Rightarrow 1x^2 + 12x - 9 = 0\]

\(a = 1\)
\(b = 12\)
\(c = -9\)

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{-12 \pm \sqrt{(12)^2 - 4(1)(-9)}}{2(1)}\]
\[x = \frac{-12 \pm \sqrt{144}}{2}\]
\[x = \frac{-12 \pm 12\sqrt{5}}{2}\]
\[x = -6 \pm 3\sqrt{5}\]

**ANSWER:**

\(-6 \pm 3\sqrt{5}\)

---

2. \(x^2 + 8x + 5 = 0\)

**SOLUTION:**

Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).

\[x^2 + 8x + 5 = 0 \Rightarrow 1x^2 + 8x + 5 = 0\]

\(a = 1\)
\(b = 8\)
\(c = 5\)

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(5)}}{2(1)}\]
\[x = \frac{-8 \pm \sqrt{64 - 20}}{2}\]
\[x = \frac{-8 \pm \sqrt{44}}{2}\]
\[x = \frac{-8 \pm 2\sqrt{11}}{2}\]
\[x = -4 \pm \sqrt{11}\]

**ANSWER:**

\(-4 \pm \sqrt{11}\)
4-6 The Quadratic Formula and the Discriminant

3. $4x^2 - 5x - 2 = 0$

**SOLUTION:**
Identify $a$, $b$, and $c$ from the equation.

\[a = 4\]
\[b = -5\]
\[c = -2\]

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{(-5) \pm \sqrt{(-5)^2 - 4(4)(-2)}}{2(4)}\]
\[x = \frac{5 \pm \sqrt{57}}{8}\]

**ANSWER:**
\[\frac{5 \pm \sqrt{57}}{8}\]

4. $9x^2 + 6x - 4 = 0$

**SOLUTION:**
Identify $a$, $b$, and $c$ from the equation.

\[a = 9\]
\[b = 6\]
\[c = -4\]

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-6 \pm \sqrt{(6)^2 - 4(9)(-4)}}{2(9)}\]
\[x = \frac{-6 \pm \sqrt{144}}{18}\]
\[x = \frac{-6 \pm 12}{18}\]
\[x = \frac{-6 \pm 6\sqrt{5}}{18}\]
\[x = \frac{-1 \pm \sqrt{5}}{3}\]

**ANSWER:**
\[\frac{-1 \pm \sqrt{5}}{3}\]
5. $10x^2 - 3 = 13x$

**SOLUTION:**
Write the equation in the form $ax^2 + bx + c = 0$ and identify $a$, $b$, and $c$.

$10x^2 - 3 = 13x \rightarrow 10x^2 - 13x - 3 = 0$

$a = 10$

$b = -13$

$c = -3$

Substitute these values into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{(-13) \pm \sqrt{(-13)^2 - 4(10)(-3)}}{2(10)}$$

$$x = \frac{13 \pm \sqrt{289}}{20}$$

$$x = \frac{13 \pm 17}{20}$$

$$= 1.5 \text{ or } -0.2$$

**ANSWER:**
$(1.5, -0.2)$

6. $22x = 12x^2 + 6$

**SOLUTION:**
Write the equation in the form $ax^2 + bx + c = 0$ and identify $a$, $b$, and $c$.

$22x = 12x^2 + 6 \rightarrow 12x^2 - 22x + 6 = 0$

$a = 12$

$b = -22$

$c = 6$

Substitute these values into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-22) \pm \sqrt{(-22)^2 - 4(12)(6)}}{2(12)}$$

$$x = \frac{22 \pm \sqrt{196}}{24}$$

$$= \frac{22 \pm 14}{24}$$

$$= 1.5 \text{ or } \frac{1}{3}$$

**ANSWER:**
$\left(1.5, \frac{1}{3}\right)$
4-6 The Quadratic Formula and the Discriminant

7. \(-3x^2 + 4x = -8\)

**SOLUTION:**
Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).

\[-3x^2 + 4x = -8 \rightarrow -3x^2 + 4x + 8 = 0\]

\[a = -3\]
\[b = 4\]
\[c = 8\]

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{-4 \pm \sqrt{(4)^2 - 4(-3)(8)}}{2(-3)}\]
\[= \frac{-4 \pm \sqrt{112}}{-6}\]
\[= \frac{-4 \pm 4\sqrt{7}}{-6}\]
\[= \frac{2 \pm 2\sqrt{7}}{3}\]

**ANSWER:**
\[\frac{2 \pm 2\sqrt{7}}{3}\]

8. \(x^3 + 3 = -6x + 8\)

**SOLUTION:**

\[x^3 + 3 = -6x + 8 \rightarrow 1x^3 + 6x - 5 = 0\]

\[a = 1\]
\[b = 6\]
\[c = -5\]

Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).
Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-5)}}{2(1)}\]
\[= \frac{-6 \pm \sqrt{56}}{2}\]
\[= \frac{-6 \pm 2\sqrt{14}}{2}\]
\[= -3 \pm \sqrt{14}\]

**ANSWER:**
\[-3 \pm \sqrt{14}\]

9. CCSS MODELING An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is \(h = -16t^2 - 64t - 60\), where \(h\) is the height in feet and \(t\) is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet?
4-6 The Quadratic Formula and the Discriminant

**SOLUTION:**
Substitute 0 for \( h \) in the given function.

\[
h = -16t^2 - 64t + 60
\]

\( 0 = -16t^2 - 64t + 60 \)

Identify \( a \), \( b \), and \( c \) from the equation.

\[
a = -16
\]

\[
b = -64
\]

\[
c = 60
\]

Substitute these values into the Quadratic Formula and simplify.

\[
t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
t = \frac{-(-64) \pm \sqrt{(-64)^2 - 4(-16)(60)}}{2(-16)}
\]

\[
= \frac{64 \pm \sqrt{7936}}{-32}
\]

\[
= \frac{64 \pm 89.1}{-32}
\]

\[
= 2.09 \text{ or } -4.09
\]

\( t = 0.78 \) or \( -4.78 \)

\( t = 0.78 \) is reasonable.

**ANSWER:**
about 0.78 second

---

Complete parts \( a \) and \( b \) for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

10. \( 3x^2 + 8x + 2 = 0 \)

**SOLUTION:**

a. Identify \( a \), \( b \), and \( c \) from the equation.

\( a = 3 \), \( b = 8 \) and \( c = 2 \).

Substitute the values in \( b^2 - 4ac \) and simplify.

\[
8^2 - 4(3)(2) = 64 - 24 = 40
\]

b. The discriminant is not a perfect square, so there are two irrational roots.

**ANSWER:**

a. 40

b. 2 irrational roots
4-6 The Quadratic Formula and the Discriminant

11. \(2x^2 - 6x + 9 = 0\)

**SOLUTION:**

a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = 2\), \(b = -6\) and \(c = 9\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\[
(-6)^2 - 4(2)(9) = 36 - 72 \\
= -36
\]

b. The discriminant is negative, so there are two complex roots.

**ANSWER:**

a. \(-36\)

b. 2 complex roots

12. \(-16x^2 + 8x - 1 = 0\)

**SOLUTION:**

a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = -16\), \(b = 8\) and \(c = -1\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\[
8^2 - 4(-16)(-1) = 64 - 64 \\
= 0
\]

b. The discriminant is 0, so there is one rational root.

**ANSWER:**

a. 0

b. 1 rational root

13. \(5x^2 + 2x + 4 = 0\)

**SOLUTION:**

a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = 5\), \(b = 2\) and \(c = 4\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\[
2^2 - 4(5)(4) = 4 - 80 \\
= -76
\]

b. The discriminant is negative, so there are two complex roots.

**ANSWER:**

a. \(-76\)

b. 2 complex roots
Solve each equation by using the Quadratic Formula.

14. \( x^2 + 45x = -200 \)

**SOLUTION:**
Write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a, b, \) and \( c \).

\[ x^2 + 45x = -200 \rightarrow 1x^2 + 45x + 200 = 0 \]
\[ a = 1 \]
\[ b = 45 \]
\[ c = 200 \]

Substitute these values into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-45 \pm \sqrt{(45)^2 - 4(1)(200)}}{2(1)}
\]

\[
x = \frac{-45 \pm \sqrt{2025 - 800}}{2}
\]

\[
x = \frac{-45 \pm \sqrt{1225}}{2}
\]

\[
x = \frac{-45 \pm 35}{2}
\]

\[
x = -5 \text{ or } 40
\]

**ANSWER:**
\(-5, -40\)

15. \( 4x^2 - 6 = -12x \)

**SOLUTION:**
Write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a, b, \) and \( c \).

\[ 4x^2 - 6 = -12x \rightarrow 4x^2 + 12x - 6 = 0 \]
\[ a = 4 \]
\[ b = 12 \]
\[ c = -6 \]

Substitute these values into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-12 \pm \sqrt{(12)^2 - 4(4)(-6)}}{2(4)}
\]

\[
x = \frac{-12 \pm \sqrt{144 + 96}}{8}
\]

\[
x = \frac{-12 \pm \sqrt{240}}{8}
\]

\[
x = \frac{-12 \pm 4\sqrt{15}}{8}
\]

\[
x = -3 \pm \sqrt{15}
\]

**ANSWER:**
\(-3 + \sqrt{15}\)
4-6 The Quadratic Formula and the Discriminant

16. \(3x^2 - 4x - 8 = -6\)

**SOLUTION:**

Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).

\(3x^2 - 4x - 8 = -6 \rightarrow 3x^2 - 4x - 2 = 0\)

\(a = 3\)

\(b = -4\)

\(c = -2\)

Substitute these values into the Quadratic Formula and simplify.

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)} \\
&= \frac{4 \pm \sqrt{40}}{6} \\
&= \frac{4 \pm 2\sqrt{10}}{6} \\
&= \frac{2 \pm \sqrt{10}}{3}
\end{align*}
\]

**ANSWER:**

\(\frac{2 \pm \sqrt{10}}{3}\)

17. \(4x^2 - 9 = -7x - 4\)

**SOLUTION:**

Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).

\(4x^2 - 9 = -7x - 4 \rightarrow 4x^2 + 7x - 5 = 0\)

\(a = 4\)

\(b = 7\)

\(c = -5\)

Substitute these values into the Quadratic Formula and simplify.

\[
\begin{align*}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-7 \pm \sqrt{7^2 - 4(4)(-5)}}{2(4)} \\
&= \frac{-7 \pm \sqrt{129}}{8}
\end{align*}
\]

**ANSWER:**

\(\frac{-7 \pm \sqrt{129}}{8}\)
4-6 The Quadratic Formula and the Discriminant

18. \(5x^2 - 9 = 11x\)

**SOLUTION:**
Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c.\)

\(5x^2 - 9 = 11x \rightarrow 5x^2 - 11x - 9 = 0\)

\(a = 5\)

\(b = -11\)

\(c = -9\)

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[= \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-9)}}{2(5)}\]

\[= \frac{11 \pm \sqrt{301}}{10}\]

**ANSWER:**
\[\frac{11 \pm \sqrt{301}}{10}\]

19. \(12x^2 + 9x - 2 = -17\)

**SOLUTION:**
Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c.\)

\(12x^2 + 9x - 2 = -17 \rightarrow 12x^2 + 9x + 15 = 0\)

\(a = 12\)

\(b = 9\)

\(c = 15\)

Substitute these values into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[= \frac{-9 \pm \sqrt{9^2 - 4(12)(15)}}{2(12)}\]

\[= \frac{-9 \pm \sqrt{-639}}{24}\]

\[= \frac{-9 \pm 3\sqrt{71}}{24}\]

\[= \frac{-3 \pm i\sqrt{71}}{8}\]

**ANSWER:**
\[\frac{-3 \pm i\sqrt{71}}{8}\]
20. **DIVING** Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height $h$ of a diver in meters above the pool after $t$ seconds can be approximated by the equation $h = -4.9t^2 + 3t + 10$.

**a.** Determine a domain and range for which this function makes sense.

**b.** When will the diver hit the water?

**SOLUTION:**

**a.** $D: \{ t \mid 0 \leq t \leq 2 \}$, $R: \{ h \mid 0 \leq h \leq 10 \}$

**b.** Substitute 0 for $h$ in the equation $h = -4.9t^2 + 3t + 10$.

$-4.9t^2 + 3t + 10 = 0$

Identify $a$, $b$, and $c$ from the equation.

$a = -4.9$, $b = 3$ and $c = 10$.

Substitute the values of $a$, $b$, and $c$ into the Quadratic Formula and simplify.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(-4.9)(10)}}{2(-4.9)}$

$x = \frac{-3 \pm \sqrt{205}}{-9.8}$

$\approx 1.77 \text{ or } -1.16$

As the time cannot be in negative it is about 1.77 seconds.

**ANSWER:**

**a.** $D: \{ t \mid 0 \leq t \leq 2 \}$,

$R: \{ h \mid 0 \leq h \leq 10 \}$

**b.** about 1.77 seconds

**Complete parts $a$–$c$ for each quadratic equation.**

**a.** Find the value of the discriminant.

**b.** Describe the number and type of roots.

**c.** Find the exact solutions by using the Quadratic Formula.

21. $2x^2 + 3x - 3 = 0$

**SOLUTION:**

**a.** Identify $a$, $b$, and $c$ from the equation.

$a = 2$, $b = 3$ and $c = -3$.

Substitute the values in $b^2 - 4ac$ and simplify.

$3^2 - 4(2)(-3) = 9 + 24$

$= 33$

**b.** The discriminant is not a perfect square, so there are two irrational roots

**c.** Substitute the values of $a$, $b$, and $c$ into the Quadratic Formula and simplify.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{-3 \pm 33}{4}$

**ANSWER:**

**a.** 33

**b.** 2 irrational

**c.** $\frac{-3 \pm \sqrt{33}}{4}$
22. \(4x^2 - 6x + 2 = 0\)

**SOLUTION:**

a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = 4, b = -6\) and \(c = 2\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\((-6)^2 - 4(4)(2) = 36 - 32 = 4\)

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of \(a\), \(b\), and \(c\) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(2)}}{2(4)}
\]

\[
= \frac{6 \pm \sqrt{4}}{8}
\]

\[
= \frac{6 \pm 2}{8}
\]

\[
= \frac{1}{2}\text{ or } 1
\]

**ANSWER:**

a. 4
b. 2 rational
c. \(\frac{1}{2}, 1\)

---

23. \(6x^2 + 5x - 1 = 0\)

**SOLUTION:**

a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = 6, b = 5\) and \(c = -1\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\(5^2 - 4(6)(-1) = 25 + 24 = 49\)

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of \(a\), \(b\), and \(c\) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-5 \pm \sqrt{5^2 - 4(6)(-1)}}{2(6)}
\]

\[
= \frac{-5 \pm \sqrt{49}}{12}
\]

\[
= \frac{-5 \pm 7}{12}
\]

\[
= \frac{1}{6}\text{ or } -1
\]

**ANSWER:**

a. 49
b. 2 rational
c. \(\frac{1}{6}, -1\)
4-6 The Quadratic Formula and the Discriminant

24. $6x^2 - x - 5 = 0$

**SOLUTION:**

a. Identify $a$, $b$, and $c$ from the equation.

$a = 6$, $b = -1$ and $c = -5$.

Substitute the values in $b^2 - 4ac$ and simplify.

\[ (-1)^2 - 4(6)(-5) = 121 \]

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of $a$, $b$, and $c$ into the Quadratic Formula and simplify.

\[
\begin{align*}
    x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
    x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-5)}}{2(6)} \\
    &= \frac{1 \pm \sqrt{121}}{12} \\
    &= \frac{1 \pm 11}{12} \\
    &= \frac{1}{12} \quad \text{or} \quad \frac{5}{6} \\
\end{align*}
\]

**ANSWER:**

a. $121$

b. 2 rational

c. $1, -\frac{5}{6}$

25. $3x^2 - 3x + 8 = 0$

**SOLUTION:**

a. Identify $a$, $b$, and $c$ from the equation.

$a = 3$, $b = -3$ and $c = 8$.

Substitute the values in $b^2 - 4ac$ and simplify.

\[ (-3)^2 - 4(3)(8) = -87 \]

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of $a$, $b$, and $c$ into the Quadratic Formula and simplify.

\[
\begin{align*}
    x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
    x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(8)}}{2(3)} \\
    &= \frac{3 \pm i\sqrt{87}}{6} \\
\end{align*}
\]

**ANSWER:**

a. $-87$

b. 2 complex

c. $\frac{3 \pm i\sqrt{87}}{6}$
26. \(2x^2 + 4x + 7 = 0\)

**SOLUTION:**
a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = 2\), \(b = 4\) and \(c = 7\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\[4^2 - 4(2)(7) = -40\]

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of \(a\), \(b\), and \(c\) into the Quadratic Formula and simplify.

\[
x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = -\frac{4 \pm \sqrt{4^2 - 4(2)(7)}}{2(2)}
\]

\[= -\frac{4 \pm \sqrt{-40}}{4}
\]

\[= -\frac{2 \pm i\sqrt{10}}{2}
\]

**ANSWER:**
a. \(-40\)

b. 2 complex

c. \(\frac{-2 \pm i\sqrt{10}}{2}\)

27. \(-5x^2 + 4x + 1 = 0\)

**SOLUTION:**
a. Identify \(a\), \(b\), and \(c\) from the equation.

\(a = -5\), \(b = 4\) and \(c = 1\).

Substitute the values in \(b^2 - 4ac\) and simplify.

\[4^2 - 4(-5)(1) = 36\]

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of \(a\), \(b\), and \(c\) into the Quadratic Formula and simplify.

\[
x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = -\frac{4 \pm \sqrt{4^2 - 4(-5)(1)}}{2(-5)}
\]

\[= -\frac{4 \pm \sqrt{36}}{-10}
\]

\[= -\frac{4 \pm 6}{-10}
\]

\[= 1 \text{ or } -\frac{1}{5}
\]

**ANSWER:**
a. \(36\)

b. 2 rational

c. \(1, -\frac{1}{5}\)
4-6 The Quadratic Formula and the Discriminant

28. \( x^2 - 6x = -9 \)

**SOLUTION:**

a. Write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a, b, \) and \( c. \)

\[ x^2 - 6x = -9 \rightarrow 1x^2 - 6x + 9 = 0 \]

\( a = 1 \)

\( b = -6 \)

\( c = 9 \)

Substitute the values in \( b^2 - 4ac \) and simplify.

\[ (-6)^2 - 4(1)(9) = 0 \]

b. The discriminant is 0, so there is one rational root.

c. Substitute the values of \( a, b, \) and \( c \) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}
\]

\[
= \frac{6 \pm \sqrt{0}}{2}
\]

\[
= 3
\]

**ANSWER:**

a. 0

b. 1 rational

c. 3

29. \( -3x^2 - 7x + 2 = 6 \)

**SOLUTION:**

a. Write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a, b, \) and \( c. \)

\[ -3x^2 - 7x + 2 = 6 \rightarrow -3x^2 - 7x - 4 = 0 \]

\( a = -3 \)

\( b = -7 \)

\( c = -4 \)

Substitute the values in \( b^2 - 4ac \) and simplify.

\[ (-7)^2 - 4(-3)(-4) = 1 \]

b. The discriminant is 1, so there are two rational roots.

c. Substitute the values of \( a, b, \) and \( c \) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{(-7) \pm \sqrt{(-7)^2 - 4(-3)(-4)}}{2(-3)}
\]

\[
= \frac{7 \pm \sqrt{1}}{-6}
\]

\[
= \frac{7 \pm 1}{-6}
\]

\[
= -1 \text{ or } -\frac{4}{3}
\]

**ANSWER:**

a. 1

b. 2 rational

c. \(-1, -\frac{4}{3}\)
Solve each equation by using the Quadratic Formula.

30. \(-8x^2 + 5 = -4x\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c\).

\(-8x^2 + 5 = -4x \rightarrow -8x^2 + 4x + 5 = 0\)

\(a = -8\)

\(b = 4\)

\(c = 5\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\[4^2 - 4(-8)(5) = 176\]

b. The discriminant is not a perfect square, so there are two irrational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-4 \pm \sqrt{4^2 - 4(-8)(5)}}{2(-8)}\]

\[= \frac{-4 \pm \sqrt{176}}{-16}\]

\[= \frac{-4 \pm 4\sqrt{11}}{-16}\]

\[= \frac{1 \pm \sqrt{11}}{4}\]

**ANSWER:**

a. 176

b. 2 irrational

c. \(\frac{1 \pm \sqrt{11}}{4}\)

31. \(x^2 + 2x - 4 = -9\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c\).

\(x^2 + 2x - 4 = -9 \rightarrow x^2 + 2x + 5 = 0\)

\(a = 1\)

\(b = 2\)

\(c = 5\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\[2^2 - 4(1)(5) = -16\]

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}\]

\[= \frac{-2 \pm \sqrt{-16}}{2}\]

\[= \frac{-2 \pm 4i}{2}\]

\[= -1 \pm 2i\]

**ANSWER:**

a. -16

b. 2 complex

c. \(-1 \pm 2i\)
32. \(-6x^2 + 5 = -4x + 8\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c\).

\[-6x^2 + 5 = -4x + 8 \rightarrow -6x^2 + 4x - 3 = 0\]

\(a = -6\)
\(b = 4\)
\(c = -3\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\[
4^2 - 4(-6)(-3) = -56
\]

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-4 \pm \sqrt{4^2 - 4(-6)(-3)}}{2(-6)}
\]

\[
= \frac{-4 \pm \sqrt{-56}}{-12}
\]

\[
= \frac{-4 \pm 2i\sqrt{14}}{-12}
\]

\[
= \frac{2 \pm i\sqrt{14}}{6}
\]

**ANSWER:**

a. \(-56\)

b. 2 complex

33. **VIDEO GAMES** While Darnell is grounded his friend Jack brings him a video game. Darnell stands at his bedroom window, and Jack stands directly below the window. If Jack tosses a game cartridge to Darnell with an initial velocity of 35 feet per second, an equation for the height \(h\) feet of the cartridge after \(t\) seconds is \(h = -16t^2 + 35t + 5\).

a. If the window is 25 feet above the ground, will Darnell have 0, 1, or 2 chances to catch the video game cartridge?

b. If Darnell is unable to catch the video game cartridge, when will it hit the ground?

**SOLUTION:**

a. Substitute 25 for \(y\) and simplify.

\[-16t^2 + 35t + 5 = 25\]
\[-16t^2 + 35t - 20 = 0\]

\(a = -16, b = 35\) and \(c = -20.\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\[
b^2 - 4ac = 35^2 - 4(-16)(-20)
\]

\[
= -55
\]

Since the discriminant is negative, it has 0 real roots.

So, Darnell will have 0 chances to catch the video game cartridge.

b. Substitute 0 for \(h\) in the equation \(h = -16t^2 + 35t + 5\)

\[-16t^2 + 35t + 5 = 0\]

Identify \(a, b,\) and \(c\) from the equation.
4-6 The Quadratic Formula and the Discriminant

\[ a = -16, \ b = 35 \text{ and } c = 5. \]

Substitute the values of \( a, b, \) and \( c \) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-35 \pm \sqrt{35^2 - 4(-16)(5)}}{2(-16)}
\]

\[
x = \frac{-35 \pm \sqrt{1344}}{-32}
\]

\[
\approx 2.3 \text{ or } -0.05
\]

As the time cannot be in negative it is about 2.3 seconds.

**ANSWER:**

a. 0

b. about 2.3 seconds

34. **CCSS SENSE-MAKING** Civil engineers are designing a section of road that is going to dip below sea level. The road’s curve can be modeled by the equation \( y = 0.00005x^2 - 0.06x \), where \( x \) is the horizontal distance in feet between the points where the road is at sea level and \( y \) is the elevation. The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

**SOLUTION:**

Identify \( a, b, \) and \( c \) from the equation.

\[ a = 0.00005, \ b = 0.06 \text{ and } c = 0. \]

Substitute the values of \( a, b, \) and \( c \) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-0.06 \pm \sqrt{(0.06)^2 - 4(0.00005)(0)}}{2(0.00005)}
\]

\[
x = \frac{-0.06 \pm \sqrt{0.0036}}{0.0001}
\]

\[
= -0.06 \pm 0.06
\]

\[
= 0 \text{ or } 1200
\]

The engineers will place the stop signs at 0 ft and 1200 ft.

**ANSWER:**

0 ft and 1200 ft
4-6 The Quadratic Formula and the Discriminant

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

35. \(5x^2 + 8x = 0\)

**SOLUTION:**
a. Identify \(a, b,\) and \(c\) from the equation.

\(a = 5,\ b = 8,\) and \(c = 0.\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\(8^2 - 4(5)(0) = 64\)

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-8 \pm \sqrt{8^2 - 4(5)(0)}}{2(5)}\]

\[x = \frac{-8 \pm \sqrt{64}}{10}\]

\[x = \frac{-8 \pm 8}{10}\]

\[x = 0\ or\ -\frac{8}{5}\]

**ANSWER:**
a. 64

b. 2 rational

c. \(0, -\frac{8}{5}\)

36. \(8x^2 = -2x + 1\)

**SOLUTION:**
a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c.\)

\(8x^2 = -2x + 1 \rightarrow 8x^2 + 2x - 1 = 0\)

\(a = 8\)

\(b = 2\)

\(c = -1\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\(2^2 - 4(8)(-1) = 36\)

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-2 \pm \sqrt{2^2 - 4(8)(-1)}}{2(8)}\]

\[x = \frac{-2 \pm \sqrt{36}}{16}\]

\[x = \frac{-2 \pm 6}{16}\]

\[x = \frac{1}{4} \text{ or } -\frac{1}{2}\]

**ANSWER:**
a. 36

b. 2 rational

c. \(\frac{1}{4}, -\frac{1}{2}\)
Solve each equation by using the Quadratic Formula.

37. \(4x - 3 = -12x^2\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c.\)

\(4x - 3 = -12x^2 \Rightarrow 12x^2 + 4x - 3 = 0\)

\(a = 12\)

\(b = 4\)

\(c = -3\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\(4^2 - 4(12)(-3) = 160\)

b. The discriminant is not a perfect square, so there are two irrational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-4 \pm \sqrt{4^2 - 4(12)(-3)}}{2(12)}\]

\[= \frac{-4 \pm \sqrt{160}}{24}\]

\[= \frac{-4 \pm 4\sqrt{10}}{24}\]

\[= \frac{-1 \pm \sqrt{10}}{6}\]

**ANSWER:**

a. 160

b. 2 irrational

c. \(-\frac{1 \pm \sqrt{10}}{6}\)

38. \(0.8x^2 + 2.6x = -3.2\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c.\)

\(0.8x^2 + 2.6x = -3.2 \Rightarrow 0.8x^2 + 2.6x + 3.2 = 0\)

\(a = 0.8\)

\(b = 2.6\)

\(c = 3.2\)

Substitute the values in \(b^2 - 4ac\) and simplify.

\((2.6)^2 - 4(0.8)(3.2) = -3.48\)

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-2.6 \pm \sqrt{(2.6)^2 - 4(0.8)(3.2)}}{2(0.8)}\]

\[= \frac{-2.6 \pm \sqrt{-3.48}}{1.6}\]

\[= \frac{-2.6 \pm 2i\sqrt{0.87}}{1.6}\]

\[= -1.3 \pm i\sqrt{0.87}\]

**ANSWER:**

a. \(-3.48\)

b. 2 imaginary roots

c. \(-\frac{1.3 \pm i\sqrt{0.87}}{0.8}\)
39. \(0.6x^2 + 1.4x = 4.8\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c\).

\[0.6x^2 + 1.4x - 4.8 = 0\]

\[a = 0.6, \quad b = 1.4, \quad c = -4.8\]

Substitute the values in \(b^2 - 4ac\) and simplify.

\[(1.4)^2 - 4(0.6)(-4.8) = 13.48\]

b. The discriminant is not a perfect square, so there are two irrational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-1.4 \pm \sqrt{(1.4)^2 - 4(0.6)(-4.8)}}{2(0.6)}\]

\[= \frac{-1.4 \pm \sqrt{13.48}}{1.2}\]

\[= \frac{-1.4 \pm 2\sqrt{3.37}}{1.2}\]

\[= -0.7 \pm \sqrt{3.37}\]

\[= 0.6\]

**ANSWER:**

a. 13.48
b. 2 irrational
c. \(-0.7 \pm \sqrt{3.37}\)

40. \(-4x^2 + 12 = -6x - 8\)

**SOLUTION:**

a. Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a, b,\) and \(c\).

\[-4x^2 + 12 = -6x - 8 \rightarrow -4x^2 + 6x + 20 = 0\]

\[a = -4, \quad b = 6, \quad c = 20\]

Substitute the values in \(b^2 - 4ac\) and simplify.

\[6^2 - 4(-4)(20) = 356\]

b. The discriminant is not a perfect square, so there are two irrational roots.

c. Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-6 \pm \sqrt{6^2 - 4(-4)(20)}}{2(-4)}\]

\[= \frac{-6 \pm \sqrt{356}}{-8}\]

\[= \frac{-6 \pm 2\sqrt{89}}{-8}\]

\[= \frac{3 \pm \sqrt{89}}{4}\]

**ANSWER:**

a. 356
b. 2 irrational
c. \(\frac{3 \pm \sqrt{89}}{4}\)

41. **SMOKING** A decrease in smoking in the United States has resulted in lower death rates caused by lung cancer. The number of deaths per 100,000
people y can be approximated by \( y = -0.26x^2 - 0.55x + 91.81 \), where \( x \) represents the number of years after 2000.

a. Calculate the number of deaths per 100,000 people for 2015 and 2017.

b. Use the Quadratic Formula to solve for \( x \) when \( y = 50 \).

c. According to the quadratic function, when will the death rate be 0 per 100,000? Do you think that this prediction is reasonable? Why or why not?

<table>
<thead>
<tr>
<th>Year</th>
<th>Deaths per 100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>91.8</td>
</tr>
<tr>
<td>2002</td>
<td>89.7</td>
</tr>
<tr>
<td>2004</td>
<td>85.5</td>
</tr>
<tr>
<td>2010</td>
<td>60.3</td>
</tr>
<tr>
<td>2015</td>
<td>?</td>
</tr>
<tr>
<td>2017</td>
<td>?</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Substitute 15 for \( x \) in the equation \( y = -0.26x^2 - 0.55x + 91.81 \) and simplify.
\[
\begin{align*}
y &= \ -0.26 \times (15) ^2 \ - \ 0.55 \times (15) \ + \ 91.81 \\
&= \ -58.5 \ - \ 8.25 \ + \ 91.81 \\
&= \ 25.06
\end{align*}
\]
Substitute 17 for \( x \) in the equation \( y = -0.26x^2 - 0.55x + 91.81 \) and simplify.
\[
\begin{align*}
y &= \ -0.26 \times (17) ^2 \ - \ 0.55 \times (17) \ + \ 91.81 \\
&= \ -75.14 \ - \ 9.35 \ + \ 91.81 \\
&= \ 7.32
\end{align*}
\]

b. Substitute 50 for \( y \) in the equation and simplify.
\[
\begin{align*}
-0.26x^2 - 0.55x + 91.81 &= 50 \\
-0.26x^2 - 0.55x + 41.81 &= 0 \\
a &= \ -0.26 \ , \ b &= \ -0.55 \ , \ c &= \ 41.81
\end{align*}
\]
Substitute the values of \( a, b, \) and \( c \) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-(0.55) \pm \sqrt{(-0.55)^2 - 4(-0.26)(41.81)}}{2(-0.26)}
\]
\[
= \frac{0.55 \pm \sqrt{95.7849}}{0.52}
\]
\[
\approx -19.88 \ or \ 17.77
\]
The year at which the death will be 0 is 2018.
No, this prediction is not reasonable. The death rate from cancer will never be 0 unless a cure is found. If and when a cure will be found cannot be predicted.

**ANSWER:**

a. 25.1, 7.3

b. 11.7

c. 2018; Sample answer: no; the death rate from cancer will never be 0 unless a cure is found. If and when a cure will be found cannot be predicted.
4.6 The Quadratic Formula and the Discriminant

42. **NUMBER THEORY** The sum $S$ of consecutive integers $1, 2, 3, \ldots, n$ is given by the formula

$$S = \frac{1}{2}n(n+1).$$

How many consecutive integers, starting with 1, must be added to get a sum of 666?

**SOLUTION:**

Substitute 666 for $S$ in the formula $S = \frac{1}{2}n(n+1)$.

$$\frac{1}{2}n(n+1) = 666$$

$$n(n+1) = 1332$$

$$n^2 + n - 1332 = 0$$

$$a = 1, b = 1, c = -1332.$$ 

Substitute the values of $a$, $b$, and $c$ into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1332)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{5329}}{2}$$

$$x = \frac{-1 \pm 73}{2}$$

$$= 36 \text{ or } -37$$

So there are 36 consecutive integers.

**ANSWER:**

36

43. **CCSS CRITIQUE** Tama and Jonathan are determining the number of solutions of $3x^2 - 5x = 7$. Is either of them correct? Explain your reasoning.

**SOLUTION:**

Tama
$$3x^2 - 5x = 7$$

$$b^2 - 4ac = (-5)^2 - 4(3)(7)$$

$$= -59$$

Since the discriminant is negative, there are no real solutions.

Jonathan
$$3x^2 - 5x = 7$$

$$b^2 - 4ac = (-5)^2 - 4(3)(7)$$

$$= 109$$

Since the discriminant is positive, there are two real roots.

**SOLUTION:**

Jonathan is correct; you must first write the equation in the form $ax^2 + bx + c = 0$ to determine the values of $a$, $b$, and $c$. Therefore, the value of $c$ is $-7$, not 7.

**ANSWER:**

Jonathan is correct; you must first write the equation in the form $ax^2 + bx + c = 0$ to determine the values of $a$, $b$, and $c$. Therefore, the value of $c$ is $-7$, not 7.
4-6 The Quadratic Formula and the Discriminant

44. **CHALLENGE** Find the solutions of \(4ix^2 - 4ix + 5i = 0\) by using the Quadratic Formula.

**SOLUTION:**
Identify \(a, b,\) and \(c\) from the equation.

\[ a = 4i, \quad b = -4i, \quad c = 5i. \]

Substitute the values of \(a, b,\) and \(c\) into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-(4i) \pm \sqrt{(-4i)^2 - 4(4i)(5i)}}{2(4i)}
\]

\[
x = \frac{4i \pm \sqrt{-64i^2}}{8i}
\]

\[
x = \frac{1 \pm 2i}{2}
\]

**ANSWER:**
\[
\frac{1 \pm 2i}{2}
\]

45. **REASONING** Determine whether each statement is sometimes, always, or never true. Explain your reasoning.

a. In a quadratic equation in standard form, if \(a\) and \(c\) are different signs, then the solutions will be real.

b. If the discriminant of a quadratic equation is greater than 1, the two roots are real irrational numbers.

**SOLUTION:**

a. Always; when \(a\) and \(c\) are opposite signs, then \(ac\) will always be negative and \(-4ac\) will always be positive. Since \(b^2\) will also always be positive, then \(b^2 - 4ac\) represents the addition of two positive values, which will never be negative. Hence, the discriminant can never be negative and the solutions can never be imaginary.

b. Sometimes; the roots will only be irrational if \(b^2 - 4ac\) is not a perfect square.

**ANSWER:**

a. Sample answer: Always; when \(a\) and \(c\) are opposite signs, then \(ac\) will always be negative and \(-4ac\) will always be positive. Since \(b^2\) will also always be positive, then \(b^2 - 4ac\) represents the addition of two positive values, which will never be negative. Hence, the discriminant can never be negative and the solutions can never be imaginary.

b. Sample answer: Sometimes; the roots will only be irrational if \(b^2 - 4ac\) is not a perfect square.

46. **OPEN ENDED** Sketch the corresponding graph and state the number and type of roots for each of the following.

a. \(b^2 - 4ac = 0\)

b. A quadratic function in which \(f(x)\) never equals zero.

c. A quadratic function in which \(f(a) = 0\) and \(f(b) = 0; \ a \neq b\).
4-6 The Quadratic Formula and the Discriminant

d. The discriminant is less than zero.

e. $a$ and $b$ are both solutions and can be represented as fractions.

**SOLUTION:**

a. Sample answer: 1 rational root

b. Sample answer: 2 complex roots

c. Sample answer: 2 real roots

d. Sample answer: 2 complex roots

e. Sample answer: 2 rational roots
4-6 The Quadratic Formula and the Discriminant

e. Sample answer: 2 rational roots

47. CHALLENGE Find the value(s) of \( m \) in the quadratic equation \( x^2 + x + m + 1 = 0 \) such that it has one solution.

**SOLUTION:**
Since the equation has one solution, its discriminant value is 0.

\[
\begin{align*}
   b^2 - 4ac &= 0 \\
   1^2 - 4(1)(m+1) &= 0 \\
   1 - 4m - 4 &= 0 \\
   -4m &= 3 \\
   m &= -\frac{3}{4} \\
   m &= -0.75
\end{align*}
\]

**ANSWER:**
-0.75

48. **WRITING IN MATH** Describe three different ways to solve \( x^2 - 2x - 15 = 0 \). Which method do you prefer, and why?

**SOLUTION:**

1. Factor \( x^2 - 2x - 15 \) as \((x + 3)(x - 5)\). Then according to the Zero Product Property, either \( x + 3 = 0 \) or \( x - 5 = 0 \). Solving these equations, \( x = -3 \) or \( x = 5 \).

2. Rewrite the equation as \( x^2 - 2x = 15 \). Then add 1 to each side of the equation to complete the square on the left side.

   \[ (x - 1)^2 = 16. \]

   Taking the square root of each side, \( x - 1 = \pm 4 \).

   Therefore, \( x = 1 \pm 4 \) and \( x = -3 \) or \( x = 5 \).

3. Use the Quadratic Formula.

   \[ x = \frac{2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)} \quad \text{or} \quad x = \frac{2 \pm \sqrt{64}}{2}. \]

   Simplifying the expression, \( x = -3 \) or \( x = 5 \). See students’ preferences.

**ANSWER:**

Sample answer: (1) Factor \( x^2 - 2x - 15 \) as \((x + 3)(x - 5)\). Then according to the Zero Product Property, either \( x + 3 = 0 \) or \( x - 5 = 0 \). Solving these equations, \( x = -3 \) or \( x = 5 \).

(2) Rewrite the equation as \( x^2 - 2x = 15 \). Then add 1 to each side of the equation to complete the square on the left side. Then \((x - 1)^2 = 16\). Taking the square root of each side, \( x - 1 = \pm 4 \). Therefore, \( x = 1 \pm 4 \) and \( x = -3 \) or \( x = 5 \).

(3) Use the Quadratic Formula. Thus,

\[ x = \frac{2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)} \quad \text{or} \quad x = \frac{2 \pm \sqrt{64}}{2}. \]

Simplifying the expression, \( x = -3 \) or \( x = 5 \). See students’ preferences.
4-6 The Quadratic Formula and the Discriminant

49. A company determined that its monthly profit $P$ is given by $P = -8x^2 + 165x - 100$, where $x$ is the selling price for each unit of product. Which of the following is the best estimate of the maximum price per unit that the company can charge without losing money?

A $10
B $20
C $30
D $40

**SOLUTION:**
Substitute 0 for $P$ in the equation $P = -8x^2 + 165x - 100$ and solve it using Quadratic Formula.

\[ -8x^2 + 165x - 100 = 0 \]

\[ x = \frac{-165 \pm \sqrt{165^2 - 4(-8)(-100)}}{2(-8)} \]

\[ = \frac{-165 \pm \sqrt{24025}}{-16} \]

\[ = \frac{-165 \pm 155}{-16} \]

\[ = 20 \text{ or } 0.625 \]

Thus, the best estimate of the maximum price per unit is $20.

B is the correct option.

**ANSWER:**

B

50. SAT/ACT For which of the following sets of numbers is the mean greater than the median?

F {4, 5, 6, 7, 8}
G {4, 6, 6, 6, 8}
H {4, 5, 6, 7, 9}
J {3, 5, 6, 7, 8}
K {2, 6, 6, 6, 6}

**SOLUTION:**
Only the set {4, 5, 6, 7, 9} has the mean greater than the median. So, H is the correct option.

**ANSWER:**

H
51. **SHORT RESPONSE** In the figure below, \( P \) is the center of the circle with radius 15 inches. What is the area of \( \triangle APB \)?

**SOLUTION:**
Substitute 15 for base and 15 for height in the formula \( A = \frac{1}{2} \cdot \text{base} \cdot \text{height} \).

\[
A = \frac{1}{2}(15)(15) \\
= 112.5
\]

The area of \( \triangle APB \) is 112.5 in\(^2\).

**ANSWER:**
112.5 in\(^2\)

52. 75% of 88 is the same as 60% of what number?

A 100
B 101
C 108
D 110

**SOLUTION:**
Let the unknown number be \( x \).

\[
\frac{60}{100} \cdot x = \frac{75}{100} \cdot 88 \\
0.60x = 66 \\
x = 110
\]

So, D is the correct option.

**ANSWER:**
D
4-6 The Quadratic Formula and the Discriminant

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

53. \( x^2 + 13x + c \)

**SOLUTION:**
Find one half of 13 and square the result.

\[
\frac{13}{2} = 6.5
\]

\[
(6.5)^2 = 42.25
\]

Add the result 42.25 to \( x^2 + 13x \).

\[
x^2 + 13x + 42.25
\]

The value of the \( c \) is 42.25.

The trinomial \( x^2 + 13x + 42.25 \) can be written as \( (x + 6.5)^2 \).

**ANSWER:**
42.25; \( (x + 6.5)^2 \)

54. \( x^2 + 2.4x + c \)

**SOLUTION:**
Find one half of 2.4 and square the result.

\[
\frac{2.4}{2} = 1.2
\]

\[
(1.2)^2 = 1.44
\]

Add the result 1.44 to \( x^2 + 2.4x \).

\[
x^2 + 2.4x + 1.44
\]

The value of the \( c \) is 1.44.

The trinomial \( x^2 + 2.4x + 1.44 \) can be written as \( (x + 1.2)^2 \).

**ANSWER:**
1.44; \( (x + 1.2)^2 \)
4-6 The Quadratic Formula and the Discriminant

55. \(x^2 + \frac{4}{5}x + c\)

**SOLUTION:**

Find one half of \(\frac{4}{5}\) and square the result.

\[
\frac{4}{5} = \frac{2}{5}
\]

\[
\left(\frac{2}{5}\right)^2 = \frac{4}{25}
\]

Add the result \(\frac{4}{25}\) to \(x^2 + \frac{4}{5}x\).

\[x^2 + \frac{4}{5}x + \frac{4}{25}\]

The value of the \(c\) is \(\frac{4}{25}\).

The trinomial \(x^2 + \frac{4}{5}x + \frac{4}{25}\) can be written as

\[
\left(x + \frac{2}{5}\right)^2.
\]

**ANSWER:**

\[
\frac{4}{25} \cdot \left(x + \frac{2}{5}\right)^2
\]

56. \(i^{26}\)

**SOLUTION:**

\[i^{26} = (i^4)^6 \cdot i^2 = 1^6 \cdot i^2 = 1 \cdot i^2 = -1\]

**ANSWER:**

\(-1\)

57. \(\sqrt{-16}\)

**SOLUTION:**

\[
\sqrt{-16} = \sqrt{-1 \cdot 4^2} = \sqrt{-1} \cdot 4\]

\[= 4i\]

**ANSWER:**

\(4i\)

58. \(4\sqrt{-9} \cdot 2\sqrt{-25}\)

**SOLUTION:**

\[
4\sqrt{-9} \cdot 2\sqrt{-25} = 4\sqrt{1 \cdot 3^2} \cdot 2\sqrt{1 \cdot 5^2}
\]

\[= 4 \cdot 3 \cdot 2 \cdot i \cdot 5
\]

\[= 120i^2
\]

\[= -120
\]

**ANSWER:**

\(-120\)
59. PILOT TRAINING Evita is training for her pilot’s license. Flight instruction costs $105 per hour, and the simulator costs $45 per hour. She spent 4 more hours in airplane training than in the simulator. If Evita spent $3870, how much time did she spend training in an airplane and in a simulator?

**SOLUTION:**
Let $x$ represents the hours of flight instruction and $y$ represents the hours in the simulator.
The system of equation that represents the situation is

\[105x + 45y = 3870\]
\[x = y + 4.\]

Substitute $y + 4$ for $x$ in the equation
\[105x + 45y = 3870\]
and solve for $y$.

\[105(y + 4) + 45y = 3870\]
\[105y + 420 + 45y = 3870\]
\[150y = 3450\]
\[y = 23\]

Substitute 23 for $y$ into either of the original equation and find $x$.

\[x = y + 4\]
\[x = 23 + 4\]
\[= 27\]

27 hours of flight instruction and 23 hours in the simulator.

**ANSWER:**
27 hours of flight instruction and 23 hours in the simulator.

60. BUSINESS Ms. Larson owns three fruit farms on which she grows apples, peaches, and apricots. She sells apples for $22 a case, peaches for $25 a case, and apricots for $18 a case.

**a.** Write an inventory matrix for the number of cases for each type of fruit for each farm and a cost matrix for the price per case for each type of fruit.

**b.** Find the total income of the three fruit farms expressed as a matrix.

**c.** What is the total income from all three fruit farms?

| Number of Cases in Stock of Each Type of Fruit |
|-------------------------------|----------|----------|----------|
| Fruit        | Farm 1  | Farm 2  | Farm 3  |
| apples       | 290     | 175     | 110     |
| peaches      | 165     | 240     | 75      |
| apricots     | 210     | 190     | 0       |

**SOLUTION:**
a. Inventory matrix:
\[I = \begin{bmatrix} 290 & 165 & 210 \\ 175 & 240 & 190 \\ 110 & 75 & 0 \end{bmatrix}\]

Cost matrix:
\[C = \begin{bmatrix} 22 \\ 25 \\ 18 \end{bmatrix}\]

\[\begin{bmatrix} 290 & 165 & 210 \\ 175 & 240 & 190 \end{bmatrix} \begin{bmatrix} 22 \\ 25 \\ 18 \end{bmatrix} = \begin{bmatrix} 6380 + 4125 + 3780 \\ 3850 + 6000 + 3420 \\ 14285 \end{bmatrix}
= \begin{bmatrix} 14285 \\ 13270 \\ 4295 \end{bmatrix}\]

b. Total income from all the three fruit farms:
\[14,285 + 13,270 + 4,295 = 31,850\]

**ANSWER:**
a. $\begin{bmatrix} 290 & 165 & 210 \\ 175 & 240 & 190 \end{bmatrix}, C = \begin{bmatrix} 22 \\ 25 \\ 18 \end{bmatrix}\]

b. $14,285$
$c. $31,850$
Write an equation for each graph.

61.

**SOLUTION:**
This is a parabola with vertex at (0, 1). The equation of the given graph is \( y = x^2 + 1 \).

**ANSWER:**
\( y = x^2 + 1 \)

62.

**SOLUTION:**
This is a parabola with vertex at (0, 0), through the point (4, 4). The equation of the given graph is \( y = 0.25x^2 \).

**ANSWER:**
\( y = 0.25x^2 \)
Write each function in vertex form.

1. \( y = x^2 + 6x + 2 \)

**SOLUTION:**

\[
y = x^2 + 6x + 2 \\
= (x^2 + 6x + 9) - 9 + 2 \\
= (x + 3)^2 - 7
\]

**ANSWER:**

\( y = (x + 3)^2 - 7 \)

2. \( y = -2x^2 + 8x - 5 \)

**SOLUTION:**

\[
y = -2x^2 + 8x - 5 \\
= -2(x^2 - 4x) - 5 \\
= -2\left( (x^2 - 4x + 4) - 4 \right) - 5 \\
= -2(x - 2)^2 - 4 - 5 \\
= -2(x - 2)^2 + 8 - 5 \\
= -2(x - 2)^2 + 3
\]

**ANSWER:**

\( y = -2(x - 2)^2 + 3 \)

3. \( y = 4x^2 + 24x + 24 \)

**SOLUTION:**

\[
y = 4x^2 + 24x + 24 \\
= 4(x^2 + 6x) + 24 \\
= 4\left( (x^2 + 6x + 9) - 9 \right) + 24 \\
= 4(x + 3)^2 - 9 + 24 \\
= 4(x + 3)^2 - 36 + 24 \\
= 4(x + 3)^2 - 12
\]

**ANSWER:**

\( y = 4(x + 3)^2 - 12 \)

---

4. **MULTIPLE CHOICE** Which function is shown in the graph?

![Graph of a quadratic function](image)

A. \( y = -(x + 3)^2 + 6 \)

B. \( y = -(x - 3)^2 - 6 \)

C. \( y = -2(x + 3)^2 + 6 \)

D. \( y = -2(x - 3)^2 - 6 \)

**SOLUTION:**

From the figure, the vertex \((h, k)\) of the parabola is \((-3, 6)\). Substitute \((-5, 2)\) for \((x, y)\) in the vertex form to find \(a\).

\[
2 = a(-5 + 3)^2 + 6 \\
2 - 6 = a(4) \\
-4 = a(4) \\
\]

The graph represents the function \( y = -(x + 3)^2 + 6 \). The answer is choice A.

**ANSWER:**

A
Graph each function.

5. \( y = (x - 3)^2 - 4 \)

**SOLUTION:**
The vertex is at (3, –4) and the axis of symmetry is \( x = 3 \). Since \( a = 1 > 0 \), the graph opens up.

[Graph of \( y = (x - 3)^2 - 4 \)]

**ANSWER:**

6. \( y = -2x^2 + 5 \)

**SOLUTION:**
\[
y = -2x^2 + 5 \\
= -2(x - 0)^2 + 5
\]
The vertex is at (0, 5) and the axis of symmetry is \( x = 0 \). Since \( a = -2 < 0 \), the graph opens down.

[Graph of \( y = -2x^2 + 5 \)]
4-7 Transformations of Quadratic Graphs

7. \( y = \frac{1}{2}(x + 6)^2 - 8 \)

**SOLUTION:**
The vertex is at \((-6, -8)\) and the axis of symmetry is \(x = -6\). Since \(a = \frac{1}{2} > 0\), the graph opens up.

**ANSWER:**

![Graph of \( y = \frac{1}{2}(x + 6)^2 - 8 \)](image)

Write each function in vertex form.

8. \( y = x^2 + 9x + 8 \)

**SOLUTION:**
\[
y = x^2 + 9x + 8 \\
= \left[ x^2 + 9x + \frac{81}{4} \right] + 8 - \frac{81}{4} \\
= \left[ x^2 + 9x + \frac{81}{4} \right] - \frac{81}{4} + 8 \\
= \left( x + \frac{9}{2} \right)^2 - \frac{81}{4} + 8 \\
= \left( x + \frac{9}{2} \right)^2 - \frac{49}{4} \\
\]

**ANSWER:**
\( y = \left( x + \frac{9}{2} \right)^2 - \frac{49}{4} \)

9. \( y = x^2 - 6x + 3 \)

**SOLUTION:**
\[
y = x^2 - 6x + 3 \\
= (x^2 - 6x + 9) - 9 + 3 \\
= (x - 3)^2 - 6 \\
\]

**ANSWER:**
\( y = (x - 3)^2 - 6 \)

10. \( y = -2x^2 + 5x \)

**SOLUTION:**
\[
y = -2x^2 + 5x \\
= -2 \left( x^2 - \frac{5}{2}x \right) \\
= -2 \left( x^2 - \frac{5}{2}x + \frac{25}{4} \right) - \frac{25}{2} + \frac{25}{4} \\
= -2 \left( x - \frac{5}{4} \right)^2 + \frac{25}{8} \\
\]

**ANSWER:**
\( y = -2 \left( x - \frac{5}{4} \right)^2 + \frac{25}{8} \)

11. \( y = x^2 + 2x + 7 \)

**SOLUTION:**
\[
y = x^2 + 2x + 7 \\
= (x^2 + 2x + 1) - 1 + 7 \\
= (x + 1)^2 + 6 \\
\]

**ANSWER:**
\( y = (x + 1)^2 + 6 \)
12. \( y = -3x^2 + 12x - 10 \)

**SOLUTION:**
\[
\begin{align*}
y &= -3x^2 + 12x - 10 \\
&= -3(x^2 - 4x) - 10 \\
&= -3(x^2 - 4x + 4 - 4) - 10 \\
&= -3(x - 2)^2 + 12 - 10 \\
&= -3(x - 2)^2 + 2
\end{align*}
\]

**ANSWER:**
\( y = -3(x - 2)^2 + 2 \)

13. \( y = x^2 + 8x + 16 \)

**SOLUTION:**
\[
\begin{align*}
y &= x^2 + 8x + 16 \\
&= x^2 + 8x + 4^2 \\
&= (x + 4)^2
\end{align*}
\]

**ANSWER:**
\( y = (x + 4)^2 \)

14. \( y = 2x^2 - 4x - 3 \)

**SOLUTION:**
\[
\begin{align*}
y &= 2x^2 - 4x - 3 \\
&= 2(x^2 - 2x) - 3 \\
&= 2(x^2 - 2x + 1 - 1) - 3 \\
&= 2(x - 1)^2 - 2 - 3 \\
&= 2(x - 1)^2 - 5
\end{align*}
\]

**ANSWER:**
\( y = 2(x - 1)^2 - 5 \)

15. \( y = 3x^2 + 10x \)

**SOLUTION:**
\[
\begin{align*}
y &= 3x^2 + 10x \\
&= 3 \left( x^2 + \frac{10}{3}x \right) \\
&= 3 \left[ x^2 + 2 \left( \frac{5}{3} \right) x + \left( \frac{5}{3} \right)^2 - \left( \frac{5}{3} \right)^2 \right] \\
&= 3 \left[ x^2 + 2 \left( \frac{5}{3} \right) x + \frac{25}{9} - \frac{25}{9} \right] \\
&= 3 \left[ x^2 + \frac{10}{3} x + \frac{25}{9} \right] - 3 \left( \frac{25}{9} \right) \\
&= 3 \left( x + \frac{5}{3} \right)^2 - \frac{25}{3}
\end{align*}
\]

**ANSWER:**
\( y = 3 \left( x + \frac{5}{3} \right)^2 - \frac{25}{3} \)

16. \( y = x^2 - 4x + 9 \)

**SOLUTION:**
\[
\begin{align*}
y &= x^2 - 4x + 9 \\
&= (x^2 - 4x + 4) + 4 + 9 \\
&= (x - 2)^2 + 5
\end{align*}
\]

**ANSWER:**
\( y = (x - 2)^2 + 5 \)

17. \( y = -4x^2 - 24x - 15 \)

**SOLUTION:**
\[
\begin{align*}
y &= -4x^2 - 24x - 15 \\
&= -4(x^2 + 6x) - 15 \\
&= -4(x^2 + 6x + 9 - 9) - 15 \\
&= -4(x^2 + 6x + 9) + 36 - 15 \\
&= -4(x + 3)^2 + 36 - 15 \\
&= -4(x + 3)^2 + 21
\end{align*}
\]

**ANSWER:**
\( y = -4(x + 3)^2 + 21 \)
18.  \( y = x^2 -12x + 36 \)

**SOLUTION:**

\[
y = x^2 -12x + 36 \\
= x^2 -12x + 6^2 \\
= (x-6)^2
\]

**ANSWER:**

\( y = (x-6)^2 \)

19.  \( y = -x^2 - 4x - 1 \)

**SOLUTION:**

\[
y = -x^2 - 4x - 1 \\
= -(x^2 + 4x) - 1 \\
= -(x^2 + 4x + 4 - 4) - 1 \\
= -(x^2 + 4x + 4) + 4 - 1 \\
= -(x + 2)^2 + 4 - 1 \\
= -(x + 2)^2 + 3
\]

**ANSWER:**

\( y = -(x + 2)^2 + 3 \)

20. **FIREWORKS** During an Independence Day fireworks show, the height \( h \) in meters of a specific rocket after \( t \) seconds can be modeled by \( h = -4.9(t - 4)^2 + 80 \). Graph the function.

**SOLUTION:**

Assign the height \( h \) in \( y \)-axis and the time \( t \) in \( x \)-axis. The vertex of the graph is at \((4, 80)\) and the axis of symmetry is \( t = 4 \). Since \( a = -4.9 < 0 \), the graph opens down.

![Graph of the function](image-url)
21. **FINANCIAL LITERACY** A bicycle rental shop rents an average of 120 bicycles per week and charges $25 per day. The manager estimates that there will be 15 additional bicycles rented for each $1 reduction in the rental price. The maximum income the manager can expect can be modeled by \( y = -15x^2 + 255x + 3000 \). Write this function in vertex form. Then graph.

**SOLUTION:**

\[
y = -15x^2 + 255x + 3000 \\
= -15(x^2 - 17x) + 3000 \\
= -15 \left[ x^2 - 17x + \left( \frac{17}{2} \right)^2 - \left( \frac{17}{2} \right)^2 \right] + 3000 \\
= -15 \left[ x^2 - 17x + \left( \frac{17}{2} \right)^2 \right] + 15 \left( \frac{17}{2} \right)^2 + 3000 \\
= -15 \left( x - \frac{17}{2} \right)^2 + 15 \left( \frac{17}{2} \right)^2 + 3000 \\
= -15(x - 8.5)^2 + 4083.75 \\
\]

The vertex of the graph is at (8.5, 4083.75) and the axis of symmetry is \( x = 8.5 \). Since \( a = -15 < 0 \), the graph opens down.

**ANSWER:**

\[
y = -15(x - 8.5)^2 + 4083.75 \\
\]

22. **Graph each function.**

\[
y = (x - 5)^2 + 3 \\
\]

**SOLUTION:**

The vertex is at (5, 3) and the axis of symmetry is \( x = 5 \). Since \( a = 1 > 0 \), the graph opens up.
23. \( y = 9x^2 - 8 \)

**SOLUTION:**

\[ y = 9x^2 - 8 \]

\[ = 9(x - 0)^2 - 8 \]

The vertex is at \((0, -8)\) and the axis of symmetry is \(x = 0\). \(a = 9 > 0\). Therefore, the graph opens up.

**ANSWER:**

\[ \text{The graph opens up.} \]

25. \( y = \frac{1}{10}(x + 6)^2 + 6 \)

**SOLUTION:**

The vertex is at \((-6, 6)\) and the axis of symmetry is \(x = -6\). \(a = \frac{1}{10} > 0\). Therefore, the graph opens up.

24. \( y = -2(x - 5)^2 \)

**SOLUTION:**

The vertex is at \((5, 0)\) and the axis of symmetry is \(x = 5\). Since \(a = -2 < 0\), the graph opens down.

**ANSWER:**

26. \( y = -3(x - 5)^2 - 2 \)

**SOLUTION:**

The vertex is at \((5, -2)\) and the axis of symmetry is \(x = 5\). \(a = -3 < 0\). Therefore, the graph opens down.

**ANSWER:**
27. \( y = -\frac{1}{4}x^2 - 5 \)

**SOLUTION:**
The vertex is at \((0, -5)\) and the axis of symmetry is \(x = 0\). \(a = -\frac{1}{4} < 0\), Therefore, the graph opens down.

**ANSWER:**

28. \( y = 2x^2 + 10 \)

**SOLUTION:**
The vertex is at \((0, 10)\) and the axis of symmetry is \(x = 0\). Since \(a = 2 > 0\), the graph opens up.

**ANSWER:**

29. \( y = -(x + 3)^2 \)

**SOLUTION:**
The vertex is at \((-3, 0)\) and the axis of symmetry is \(x = -3\). \(a = -1 < 0\), Therefore, the graph opens down.

**ANSWER:**

30. \( y = \frac{1}{6}(x - 3)^2 - 10 \)

**SOLUTION:**
The vertex is at \((3, -10)\) and the axis of symmetry is \(x = 3\). \(a = \frac{1}{6} > 0\), Therefore, the graph opens up.

**ANSWER:**
31. \( y = (x - 9)^2 - 7 \)

**SOLUTION:**
The vertex is at \((9, -7)\) and the axis of symmetry is \(x = 9\). Since \(a = 1 > 0\), the graph opens up.

**ANSWER:**

32. \( y = \frac{5}{8}x^2 - 8 \)

**SOLUTION:**
The vertex is at \((0, -8)\) and the axis of symmetry is \(x = 0\). \(a = -\frac{5}{8} < 0\), Therefore, the graph opens down.

**ANSWER:**

33. \( y = -4(x - 10)^2 - 10 \)

**SOLUTION:**
The vertex is at \((10, -10)\) and the axis of symmetry is \(x = 10\). \(a = -4 < 0\), Therefore, the graph opens down.
4-7 Transformations of Quadratic Graphs

34. CCSS MODELING A sailboard manufacturer uses an automated process to manufacture the masts for its sailboards. The function \( f(x) = \frac{1}{250} x^2 + \frac{3}{5} x \) is programmed into a computer to make one such mast.

a. Write the quadratic function in vertex form. Then graph the function.

**SOLUTION:**

\[
f(x) = \frac{1}{250} x^2 + \frac{3}{5} x
\]

\[
= \frac{1}{250} \left( x^2 + 150x \right)
\]

\[
= \frac{1}{250} \left( x^2 + 150x + 5625 - 5625 \right)
\]

\[
= \frac{1}{250} (x + 75)^2 - \frac{45}{2}
\]

The vertex of the function is \((-75, -\frac{45}{2})\).

b. They can adjust the coefficient of \(x^2\).

**ANSWER:**

\[
a. \quad \frac{1}{250} (x + 75)^2 - \frac{45}{2}
\]

b. They can adjust the coefficient of \(x^2\).

---

Write an equation in vertex form for each parabola.

**35.**

**SOLUTION:**

From the figure, the vertex \((h, k)\) of the parabola is \((6, 1)\). Substitute \((7, 10)\) for \((x, y)\) in the vertex form to find \(a\).

\[
10 = a(7 - 6)^2 + 1
\]

\[
10 - 1 = a(1)
\]

\[
9 = a
\]

The graph represents the equation \(y = 9(x - 6)^2 + 1\).

**ANSWER:**

\(y = 9(x - 6)^2 + 1\)

---

**36.**

**SOLUTION:**

From the figure, the vertex \((h, k)\) of the parabola is \((-4, 3)\). Substitute \((-3, 6)\) for \((x, y)\) in the vertex form to find \(a\).

\[
6 = a(-3 + 4)^2 + 3
\]

\[
6 - 3 = a(1)
\]

\[
3 = a
\]

The graph represents the equation \(y = 3(x + 4)^2 + 3\).

**ANSWER:**

\(y = 3(x + 4)^2 + 3\)
4-7 Transformations of Quadratic Graphs

37. **SOLUTION:**
From the figure, the vertex \((h, k)\) of the parabola is \((3, 0)\). Substitute \((6, -6)\) for \((x, y)\) in the vertex form to find \(a\).
\[-6 = a(6 - 3)^2 + 0\]
\[-6 = a(9)\]
\[a = -\frac{2}{3}\]
The graph represents the equation \(y = -\frac{2}{3}(x - 3)^2\).

**ANSWER:**
\[y = -\frac{2}{3}(x - 3)^2\]

38. **SOLUTION:**
From the figure, the vertex \((h, k)\) of the parabola is \((5, 4)\). Substitute \((6, 1)\) for \((x, y)\) in the vertex form to find \(a\).
\[1 = a(6 - 5)^2 + 4\]
\[1 - 4 = a(1)\]
\[-3 = a\]
The graph represents the equation \(y = -3(x - 5)^2 + 4\).

**ANSWER:**
\[y = -3(x - 5)^2 + 4\]

39. **SOLUTION:**
From the figure, the vertex \((h, k)\) of the parabola is \((0, 5)\). Substitute \((3, 8)\) for \((x, y)\) in the vertex form to find \(a\).
\[8 = a(3 - 0)^2 + 5\]
\[8 - 5 = a(9)\]
\[3 = 9a\]
\[a = \frac{1}{3}\]
The function of the parabola is \(y = \frac{1}{3}x^2 + 5\).

**ANSWER:**
\[y = \frac{1}{3}x^2 + 5\]
Write each function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

41. \(3x^2 - 4x = 2 + y\)

**SOLUTION:**

\[
y = 3x^2 - 4x - 2
= 3\left[ x^2 - \frac{4}{3} x \right] - 2
= 3\left[ x^2 - \frac{4}{3} x + \frac{4}{9} \right] - \frac{4}{9} - 2
= 3\left( x - \frac{2}{3} \right)^2 - \frac{10}{3}
\]

**Vertex:** \(\left( \frac{2}{3}, -\frac{10}{3} \right)\)

**Axis of symmetry:** \(x = \frac{2}{3}\)

Since \(a = 3 > 0\), the graph opens up.

**ANSWER:**

\[y = 3\left( x - \frac{2}{3} \right)^2 - \frac{10}{3}, \text{ open up}\]
42. \(-2x^2 + 7x = y - 12\)

**SOLUTION:**

\(-2x^2 + 7x = y - 12\)

\[ y = -2x^2 + 7x + 12 \]

\[ = -2\left( x^2 - \frac{7}{2}x + \frac{49}{16} \right) + 12 \]

\[ = -2\left( x^2 - \frac{7}{2}x + \frac{49}{16} \right) + 2\left( \frac{49}{16} \right) + 12 \]

\[ = -2\left( x^2 - \frac{7}{2}x + \frac{49}{16} \right) + \frac{145}{8} \]

\[ = -2\left( x - \frac{7}{4} \right)^2 + \frac{145}{8} \]

Vertex: \( \left( \frac{7}{4}, \frac{145}{8} \right) \)

Axis of symmetry: \( x = \frac{7}{4} \)

Since \( a = -2 < 0 \), the graph opens down.

**ANSWER:**

\[ y = -2\left( x - \frac{7}{4} \right)^2 + \frac{145}{8}, \text{opens down} \]

43. \(-x^2 - 4.7x = y - 2.8\)

**SOLUTION:**

\(-x^2 - 4.7x = y - 2.8\)

\[ y = -x^2 - 4.7x + 2.8 \]

\[ = -(x^2 + 4.7x + 5.5225) - 5.5225 + 2.8 \]

\[ = -(x + 2.35)^2 + 5.5225 + 2.8 \]

\[ = -(x + 2.35)^2 + 8.3225 \]

Vertex: \( (-2.35, 8.3225) \)

Axis of symmetry: \( x = -2.35 \)

Since \( a = -1 < 0 \), the graph opens down.

**ANSWER:**

\[ y = -(x + 2.35)^2 + 8.3225; (-2.35, 8.3225), x = -2.35, \text{opens down} \]

44. \(x^2 + 1.4x - 1.2 = y\)

**SOLUTION:**

\(x^2 + 1.4x - 1.2 = y\)

\[ y = (x^2 + 1.4x + 0.49) - 0.49 - 1.2 \]

\[ = (x + 0.7)^2 - 1.69 \]

Vertex: \( (-0.7, -1.69) \)

Axis of symmetry: \( x = -0.7 \)

Since \( a = 1 > 0 \), the graph opens up.

**ANSWER:**

\[ y = (x + 0.7)^2 - 1.69; (-0.7, -1.69), x = -0.7, \text{opens up} \]

45. \(x^2 - \frac{2}{3}x - \frac{26}{9} = y\)

**SOLUTION:**

\[ y = x^2 - \frac{2}{3}x - \frac{26}{9} \]

\[ = \left( x^2 - \frac{2}{3}x + \frac{1}{9} \right) - \frac{1}{9} - \frac{26}{9} \]

\[ = \left( x - \frac{1}{3} \right)^2 - 3 \]

Vertex: \( \left( \frac{1}{3}, -3 \right) \)

Axis of symmetry: \( x = \frac{1}{3} \)

Since \( a = 1 > 0 \), the graph opens up.

**ANSWER:**

\[ y = \left( x - \frac{1}{3} \right)^2 - 3; \left( \frac{1}{3}, -3 \right), x = \frac{1}{3}, \text{open up} \]
4-7 Transformations of Quadratic Graphs

46. \( x^2 + 7x + \frac{49}{4} = y \)

**SOLUTION:**

\[
y = x^2 + 7x + \frac{49}{4}
\]

\[
= \left(x + \frac{7}{2}\right)^2
\]

\[
= (x + 3.5)^2
\]

Compare it with the vertex form.

Vertex: \((-3.5, 0)\)

Axis of symmetry: \(x = -3.5\)

Since \(a = 1 > 0\), the graph opens up.

**ANSWER:**

\[
y = (x + 3.5)^2 \quad ; \quad (-3.5, 0), \quad x = -3.5, \quad \text{opens up}
\]

47. **CARS** The formula \( S(t) = \frac{1}{2}at^2 + v_0t \) can be used to determine the position \( S(t) \) of an object after \( t \) seconds at a rate of acceleration \( a \) with initial velocity \( v_0 \). Valerie’s car can accelerate 0.002 miles per second squared.

**a.** Express \( S(t) \) in vertex form as she accelerates from 35 miles per hour to enter highway traffic.

**b.** How long will it take Valerie to match the average speed of highway traffic of 68 miles per hour? (Hint: Use \( \text{acceleration} \times \text{time} = \text{velocity} \).)

**c.** If the entrance ramp is 1 mile long, will Valerie have sufficient time to match the average highway speed? Explain.

**SOLUTION:**

**a.** We are given the position formula \( S(t) = 0.001(t + 17,500)^2 - 306,250 \), where \( t \) is the time in seconds. We are also given an acceleration \( a \) of 0.002 miles per second squared and an initial velocity \( v_0 \) of 35 miles per hour. Begin by rewriting the initial velocity in terms of miles per second so that the units of time are all given in seconds.

\[
35 \text{ mi} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{35 \text{ mi}}{3600 \text{ sec}} 
\]

\[
\approx 0.00972 \text{ mi/sec}
\]

Substitute these values for \( a \) and \( v_0 \) into the given formula and then write the resulting quadratic equation in vertex form.

\[
S(t) = \frac{1}{2}(0.002)t^2 + (0.009722)t
\]

\[
= 0.001t^2 + 0.009722t
\]

\[
= 0.001\left(t^2 + 9.722t\right)
\]

\[
= 0.001\left(t^2 + 9.722t + \left(\frac{9.722}{2}\right)^2\right) - 0.001\left(\frac{9.722}{2}\right)^2
\]

\[
= 0.001\left(t + 4.861\right)^2 - 0.024
\]

**b.** Valerie needs to accelerate from a velocity of 35 miles per hour to 65 miles per hour, an increase in velocity of 68 – 35 or 33 miles per hour. She is accelerating at a rate of 0.002 miles per second squared, which is \( \frac{0.002 \text{ mi}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \) or 7.2 mi/hr. Use the relationship \( \text{acceleration} \times \text{time} = \text{velocity} \).

\[
a \cdot t = v
\]

\[
7.2t = 33
\]

\[
t = \frac{33}{7.2} \quad \text{or} \quad 4.58
\]

Therefore, accelerating at a rate of 0.002 miles per second squared, it will take Valerie about 4.58 seconds to increase her speed from 35 to 68 miles per hour.

**c.** Substitute \( \frac{1}{8} \) or 0.125 for \( S(t) \) and solve for \( t \).

\[
S(t) = 0.001\left(t + 4.861\right)^2 - 0.024
\]

\[
0.125 = 0.001\left(t + 4.861\right)^2 - 0.024
\]

\[
0.149 = 0.001\left(t + 4.861\right)^2
\]

\[
149 = \left(t + 4.861\right)^2
\]

\[
\sqrt{149} = t + 4.861
\]

\[
t \approx 7.35
\]

At her a starting velocity of 35 miles per hour and accelerating at a rate of 0.002 miles per second square, Valerie will be on the ramp for about 7.35 seconds. Since it will take her 4.58 seconds to accelerate to 68 mph, she will be on the ramp long enough to accelerate to match the average expressway speed.

**ANSWER:**

**a.** \( S(t) = 0.001\left(t + 4.861\right)^2 - 0.024 \)

**b.** 4.58 seconds
4-7 Transformations of Quadratic Graphs

c. Yes; if we substitute $\frac{1}{8}$ for $S(t)$ and solve for $t$ we get 7.35 seconds. This is how long Valerie will be on the ramp. Since it will take her 4.58 seconds to accelerate to 68 mph, she will be on the ramp long enough to accelerate to match the average expressway speed.

48. OPEN ENDED Write an equation for a parabola that has been translated, compressed, and reflected in the x-axis.

**SOLUTION:**
Sample answer:
To write an equation that will translate horizontally, use a positive or negative value for $h$. An equation that is reflected in the x-axis has a negative value for $a$. And an equation that is compressed, the absolute value of the $a$ value is between 0 and 1.

$$y = -\frac{1}{2}(x-4)^2$$

**ANSWER:**
Sample answer: $y = -\frac{1}{2}(x-4)^2$

49. CHALLENGE Explain how you can find an equation of a parabola using the coordinates of three points on the graph.

**SOLUTION:**
The equation of a parabola can be written in the form $y = ax^2 + bx + c$ with $a \neq 0$. For each of the three points, substitute the value of the $x$-coordinate for $x$ in the equation and substitute the value of the $y$-coordinate for $y$ in the equation. This will produce three equations in three variables $a$, $b$, and $c$. Solve the system of equations to find the values of $a$, $b$, and $c$. These values determine the quadratic equation.

**ANSWER:**
The equation of a parabola can be written in the form $y = ax^2 + bx + c$ with $a \neq 0$. For each of the three points, substitute the value of the $x$-coordinate for $x$ in the equation and substitute the value of the $y$-coordinate for $y$ in the equation. This will produce three equations in three variables $a$, $b$, and $c$. Solve the system of equations to find the values of $a$, $b$, and $c$. These values determine the quadratic equation.

50. CHALLENGE Write the standard form of a quadratic function $ax^2 + bx + c = y$ in vertex form. Identify the vertex and the axis of symmetry.

**SOLUTION:**
$$ax^2 + bx + c = y$$
$$a\left(x^2 + \frac{b}{a}x\right) + c = y$$
$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c = y$$
$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = y$$
$$a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) = y$$

Vertex: $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

Axis of Symmetry: $x = -\frac{b}{2a}$

**ANSWER:**
$$a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) = y; x = -\frac{b}{2a}$$

51. REASONING Describe the graph of

$f(x) = a\left(x - h\right)^2 + k$ when $a = 0$. Is the graph the same as that of $g(x) = ax^2 + bx + c$ when $a = 0$? Explain.

**SOLUTION:**
Sample answer: The variable $a$ represents different values for these functions, so making $a = 0$ will have a different effect on each function. For $f(x)$, when $a = 0$, the graph will be a horizontal line, $f(x) = k$. For $g(x)$, when $a = 0$, the graph will be linear, but not necessarily horizontal, $g(x) = bx + c$.

**ANSWER:**
Sample answer: The variable $a$ represents different values for these functions, so making $a = 0$ will have a different effect on each function. For $f(x)$, when $a = 0$, the graph will be a horizontal line, $f(x) = k$. For $g(x)$, when $a = 0$, the graph will be linear, but not necessarily horizontal, $g(x) = bx + c$. 
4-7 Transformations of Quadratic Graphs

52. CCSS ARGUMENTS  Explain how the graph of \( y = x^2 \) can be used to graph any quadratic function. Include a description of the effects produced by changing \( a, h, \) and \( k \) in the equation \( y = a(x - h)^2 + k \), and a comparison of the graph of \( y = x^2 \) and the graph of \( y = a(x - h)^2 + k \) using values you choose for \( a, h, \) and \( k \).

**SOLUTION:**

All quadratic functions are transformations of the parent graph \( y = x^2 \). By identifying these transformations when a quadratic function is written in vertex form, you can redraw the graph of \( y = x^2 \) with its vertex translated to \((h, k)\), widened or narrowed as determined by \( a \), opening downward if \( a \) is negative.

**ANSWER:**

All quadratic functions are transformations of the parent graph \( y = x^2 \). By identifying these transformations when a quadratic function is written in vertex form, you can redraw the graph of \( y = x^2 \) with its vertex translated to \((h, k)\), widened or narrowed as determined by \( a \), opening downward if \( a \) is negative.

53. Flowering bushes need a mixture of 70% soil and 30% vermiculite. About how many buckets of vermiculite should you add to 20 buckets of soil?

A 6.0
B 8.0
C 14.0
D 24.0

**SOLUTION:**

Let \( x \) be the number of buckets of vermiculite.

\[
\frac{70}{30} = \frac{20}{x}
\]

\[
x = \frac{20 \times 30}{70} 
\approx 8.57
\]

It is about 8 buckets. The correct choice is B.

**ANSWER:**

B

54. SAT/ACT The sum of the integers \( x \) and \( y \) is 495. The units digit of \( x \) is 0. If \( x \) is divided by 10, the result is equal to \( y \). What is the value of \( x \)?

F 40
G 45
H 245
J 250
K 450

**SOLUTION:**

\[
\frac{x}{10} = y 
\]

From equation 2, \( x = 10y \). Substitute \( x = 10y \) in equation 1.

\[
10y + y = 495
\]

\[
11y = 495
\]

\[
y = 45
\]

\[
x = 10(45) = 450
\]

The correct choice is K.

**ANSWER:**

K

55. What is the solution set of the inequality \( |4x - 1| < 9 \)?

A \( \{x \mid 2.5 < x \text{ or } x < -2\} \)
B \( \{x \mid x < 2.5\} \)
C \( \{x \mid x > -2\} \)
D \( \{x \mid -2 < x < 2.5\} \)

**SOLUTION:**

\[
|4x - 1| < 9
\]

\[
-9 < 4x - 1 < 9
\]

\[
-9 + 1 < 4x - 1 + 1 < 9 + 1
\]

\[
-8 < 4x < 10
\]

\[
\frac{-8}{4} < x < \frac{10}{4}
\]

\[
-2 < x < 2.5
\]

The correct choice is D.

**ANSWER:**

D
56. **SHORT RESPONSE** At your store, you buy wrenches for $30.00 a dozen and sell them for $3.50 each. What is the percent markup for the wrenches?

**SOLUTION:**
The selling price of 1 dozen wrenches is:
\[3.50 \times 12 = 42\]
The percent markup for the wrenches is:
\[
\text{Percent markup} = \frac{42 - 30}{30} \times 100\%
\]
\[= \frac{12}{30} \times 100\% \]
\[= 40\% \]

**ANSWER:**
40%

**Solve each equation by using the method of your choice. Find exact solutions.**

57. \(4x^2 + 15x = 21\)

**SOLUTION:**
\[
4x^2 + 15x = 21
\]
\[
x^2 + \frac{15}{4}x = \frac{21}{4}
\]
\[
x^2 + \frac{15}{4}x + \frac{225}{64} = \frac{21}{4} + \frac{225}{64}
\]
\[
\left( x + \frac{15}{8} \right) = \frac{561}{64}
\]
\[
\left( x + \frac{15}{8} \right) = \pm \sqrt{\frac{561}{8}}
\]
\[
x = -\frac{15 \pm \sqrt{561}}{8}
\]

**ANSWER:**
\[-\frac{15 \pm \sqrt{561}}{8}\]

58. \(-3x^2 + 19 = 5x\)

**SOLUTION:**
\[-3x^2 + 19 = 5x
\]
\[3x^2 + 5x = 19
\]
\[x^2 + \frac{5}{3}x = \frac{19}{3}
\]
\[x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{19}{3} + \frac{25}{36}
\]
\[\left( x + \frac{5}{6} \right)^2 = \frac{253}{36}
\]
\[x + \frac{5}{6} = \pm \frac{\sqrt{253}}{6}
\]
\[x = -\frac{5 \pm \sqrt{253}}{6}
\]

**ANSWER:**
\[-\frac{5 \pm \sqrt{253}}{6}\]
59. \(6x - 5x^2 + 9 = 3\)

**SOLUTION:**
\[
6x - 5x^2 + 9 = 3 \\
5x^2 - 6x = 9 - 3 \\
5x^2 - 6x = 6 \\
\frac{x^2 - 6}{5} x = \frac{6}{5} \\
x^2 \frac{6}{5} x + \frac{9}{25} = \frac{6}{5} + \frac{9}{25} \\
\left(x - \frac{3}{5}\right)^2 = \frac{39}{25} \\
x - \frac{3}{5} = \pm \frac{\sqrt{39}}{5} \\
x = \frac{3}{5} \pm \frac{\sqrt{39}}{5} \\
x = \frac{3 \pm \sqrt{39}}{5}
\]

**ANSWER:**
\[
\frac{3 \pm \sqrt{39}}{5}
\]

Find the value of \(c\) that makes each trinomial a perfect square.

60. \(x^2 - 12x + c\)

**SOLUTION:**
\[
c = \left(\frac{-12}{2}\right)^2 \\
= \left(-6\right)^2 \\
= 36
\]

**ANSWER:**
36

61. \(x^2 + 0.1x + c\)

**SOLUTION:**
\[
c = \left(\frac{0.1}{2}\right)^2 \\
= 0.05^2 \\
= 0.0025
\]

**ANSWER:**
0.0025

62. \(x^2 - 0.45x + c\)

**SOLUTION:**
\[
c = \left(\frac{-0.45}{2}\right)^2 \\
= \left(-0.225\right) \\
= 0.050625
\]

**ANSWER:**
0.050625

Determine whether each function has a maximum or minimum value, and find that value.

63. \(f(x) = 6x^2 - 8x + 12\)

**SOLUTION:**

\(a = 6 > 0\). The graph opens up. Therefore, it has minimum value.

The \(x\) coordinate of the vertex is:
\[
\frac{-8}{2(6)} = \frac{2}{3}
\]

The minimum value is:
\[
f\left(\frac{2}{3}\right) = 6\left(\frac{2}{3}\right)^2 - 8\left(\frac{2}{3}\right) + 12 \\
= \frac{24}{9} - \frac{16}{3} + 12 \\
= \frac{84}{9} \\
= 9\frac{1}{3}
\]

**ANSWER:**
minimum, \(9\frac{1}{3}\)
64. \( f(x) = -4x^2 + x - 18 \)

**SOLUTION:**
\[ a = -4 < 0. \] The graph opens down. Therefore, it has *maximum* value.

The \( x \) coordinate of the function is: \[ -\frac{1}{2(-4)} = \frac{1}{8} \]

The maximum value is:
\[
\begin{align*}
  f \left( \frac{1}{8} \right) &= -4 \left( \frac{1}{8} \right)^2 + \left( \frac{1}{8} \right) - 18 \\
  &= -4 \times \frac{1}{64} + \frac{1}{8} - 18 \\
  &= -\frac{1}{16} + \frac{1}{8} - 18 \\
  &= -\frac{1}{16} - \frac{1}{2} \\
  &= -\frac{17}{16} \\
\end{align*}
\]

**ANSWER:**
maximum, \(-17\frac{15}{16}\)

65. \( f(x) = 3x^2 - 9 + 6x \)

**SOLUTION:**
\[ a = 3 > 0. \] The graph opens up. Therefore, it has *minimum* value.

The \( x \) coordinate of the function is: \[ \frac{-6}{2(3)} = -1 \]

The minimum value is:
\[
\begin{align*}
  f(-1) &= 3(-1)^2 - 9 + 6(-1) \\
  &= 3 - 9 - 6 \\
  &= -12 \\
\end{align*}
\]

**ANSWER:**
minimum, \(-12\)

66. **ARCHAEOLOGY** A coordinate grid is laid over an archaeology dig to identify the location of artifacts. Three corners of a building have been partially unearthed at \((-1, 6), (4, 5), \) and \((-1, -2).\) If each square on the grid measures one square foot, estimate the area of the floor of the building.

**SOLUTION:**
Graph the triangle.

There are about 20 complete squares inside the triangle. Therefore, the area of the floor is about 20 ft\(^2\).

**ANSWER:**
about 20 ft\(^2\)
67. HOTELS Use the costs for an overnight stay at a hotel provided at the right.
   a. Write a 3 \times 2 matrix that represents the cost of each room.
   b. Write a 2 \times 3 matrix that represents the cost of each room.

   **SOLUTION:**

<table>
<thead>
<tr>
<th>Weekday</th>
<th>Weekend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>$60.00</td>
</tr>
<tr>
<td>Double</td>
<td>$70.00</td>
</tr>
<tr>
<td>Suite</td>
<td>$75.00</td>
</tr>
</tbody>
</table>

   **ANSWER:**

   a. Weekday Weekend
      | Single | 60     | 79     |
      | Double | 70     | 89     |
      | Suite  | 75     | 95     |
   b. Weekday Weekend
      | 60     | 70     | 75     |
      | 79     | 89     | 95     |

68. Solve each system of equations by graphing.
   \begin{align*}
   y &= 3x - 4 \\
   y &= -2x + 16
   \end{align*}

   **SOLUTION:**

   Graph the equations on the same coordinate plane and find the intersecting point.

69. \begin{align*}
   2x + 5y &= 1 \\
   6y - 5x &= 16
   \end{align*}

   **SOLUTION:**

   Graph the equations on the same coordinate plane and find the intersecting point.
4-7 Transformations of Quadratic Graphs

70. \(4x + 3y = -30\)
\[3x - 2y = 3\]

**SOLUTION:**
Graph the equations on the same coordinate plane and find the intersecting point.

![Graph of equations](image)

**ANSWER:**

71. \(f(3) = x^2 - 4x + 12\)

**SOLUTION:**
\[f(3) = 3^2 - 4(3) + 12 = 9 - 12 + 12 = 9\]

**ANSWER:**
9

72. \(f(-2) = -4x^2 + x - 8\)

**SOLUTION:**
\[f(-2) = -4(-2)^2 + 2 - 8 = -16 - 2 - 8 = -26\]

**ANSWER:**
-26

73. \(f(4) = 3x^2 + x\)

**SOLUTION:**
\[f(4) = 3(4)^2 + 4 = 48 + 4 = 52\]

**ANSWER:**
52

Determine whether the given value satisfies the inequality.

74. \(3x^2 - 5 > 6; x = 2\)

**SOLUTION:**
\[3(2)^2 - 5 > 6\]
\[12 - 5 > 6\]
\[7 > 6\] True

**ANSWER:**
yes

75. \(-2x^2 + x - 1 < 4; x = -2\)

**SOLUTION:**
\[-2(-2)^2 - 2 - 1 < 4\]
\[-8 - 3 < 4\]
\[-11 < 4\] True

**ANSWER:**
yes

76. \(4x^2 + x - 3 \leq 36; x = 3\)

**SOLUTION:**
\[4(3)^2 + 3 - 3 \leq 36\]
\[36 \leq 36\] True

**ANSWER:**
yes
Graph each inequality.

1. \( y \leq x^2 - 8x + 2 \)

**SOLUTION:**
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
\begin{align*}
y & \leq x^2 - 8x + 2 \\
0 & \leq 0^2 - 8(0) + 2 \\
0 & \leq 2
\end{align*}
\]

So, \((0, 0)\) is a solution of the inequality. Shade the region of the graph that contains \((0, 0)\).

**ANSWER:**

2. \( y > x^2 + 6x - 2 \)

**SOLUTION:**
First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

\[
\begin{align*}
y & > x^2 + 6x - 2 \\
0 & > 0^2 + 6(0) - 2 \\
0 & > -2
\end{align*}
\]

So, \((0, 0)\) is a solution of the inequality. Shade the region of the graph that contains \((0, 0)\).
4-8 Quadratic Inequalities

3. \( y \geq -x^2 + 4x + 1 \)

**SOLUTION:**

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
y \geq -x^2 + 4x + 1
1 \geq -1^2 + 4(1) + 1
1 \geq 6
\]

So, \((1, 1)\) is not a solution of the inequality. Shade the region of the graph that does not contain \((1, 1)\).

![Graph of the parabola](image)

**ANSWER:**

![Graph of the parabola](image)

---

**CCSS SENSE-MAKING** Solve each inequality by graphing.

4. \( 0 < x^2 - 5x + 4 \)

**SOLUTION:**

First, write the related equation and factor it.

\[
x^2 - 5x + 4 = 0
(x - 1)(x - 4) = 0
\]

By the Zero Product Property:

\[
x - 1 = 0 \quad \text{or} \quad x - 4 = 0
x = 1 \quad \text{or} \quad x = 4
\]

Sketch the graph of a parabola that has \(x\)-intercepts at 1 and 4. The graph should open up because \(a > 0\).

![Graph of the parabola](image)

The graph lies above the \(x\)-axis left to \(x = 1\) and right to \(x = 4\). Thus, the solution set of the inequality is \(\{x | x < 1 \text{ or } x > 4\}\).

**ANSWER:**

\(\{x | x < 1 \text{ or } x > 4\}\)
Graph each inequality.

1. SOLUTION:
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola. So, (0, 0) is a solution of the inequality. Shade the region of the graph that does not contain (0, 0).

The graph lies completely above the x-axis. Thus, the solution set of the inequality is \( \{ x \mid \text{all real numbers} \} \).

5. \( x^2 + 8x + 15 < 0 \)

SOLUTION:
First, write the related equation and factor it.

\[
x^2 + 8x + 15 = 0
\]

\[
(x + 5)(x + 3) = 0
\]

By the Zero Product Property:

\[
x + 5 = 0 \quad \text{or} \quad x + 3 = 0
\]

\[
x = -5 \quad \text{or} \quad x = -3
\]

Sketch the graph of a parabola that has x-intercepts at \(-5\) and \(-3\). The graph should open up because \( a > 0 \).

The graph lies below the x-axis between \( x = -5 \) and \( x = -3 \). Thus, the solution set of the inequality is \( \{ x \mid -5 < x < -3 \} \).

ANSWER:
\( \{ x \mid -5 < x < -3 \} \)

6. \( -2x^2 - 2x + 12 \geq 0 \)

SOLUTION:
First, write the related equation and factor it.

\[
-2x^2 - 2x + 12 = 0
\]

\[
-2(x + 3)(x - 2) = 0
\]

By the Zero Product Property:

\[
x + 3 = 0 \quad \text{or} \quad x - 2 = 0
\]

\[
x = -3 \quad \text{or} \quad x = 2
\]

Sketch the graph of a parabola that has x-intercepts at \(-3\) and \(2\). The graph should open down because \( a < 0 \).

The graph lies above the x-axis between \( x = -3 \) and \( x = 2 \) including the two endpoints. Thus, the solution set of the inequality is \( \{ x \mid -3 \leq x \leq 2 \} \).

ANSWER:
\( \{ x \mid -3 \leq x \leq 2 \} \)
Graph each inequality.

1. SOLUTION:
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}
\]
\[
x = \frac{4 \pm \sqrt{8}}{4}
\]
\[
x = 1 \pm \frac{\sqrt{2}}{2}
\]
\[
x = 1.71, 0.29
\]

Sketch the graph of a parabola that has \(x\)-intercepts at 0.29 and 1.71. The graph should open up because \(a > 0\).

The graph lies below the \(x\)-axis between \(x \approx 0.29\) and \(x \approx 1.71\) including the two end points. Thus, the solution set of the inequality is \(\{x | 0.29 \leq x \leq 1.71\}\).

ANSWER:
\(\{x | 0.29 \leq x \leq 1.71\}\)

8. SOCCER A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground \(h(t)\) at time \(t\) can be represented by \(h(t) = -0.1t^2 + 2.4t + 1.5\). If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal?

SOLUTION:
The height of the goal is 8 ft. So, the ball will enter the goal when \(h(t)\) less than or equal to 8.

\[-0.1t^2 + 2.4t + 1.5 \leq 8\]
\[-0.1t^2 + 2.4t - 6.5 \leq 0\]

First, write the related equation and solve it.

\[
t = \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(-0.1)(-6.5)}}{2(-0.1)}
\]
\[
t = \frac{-2.4 \pm \sqrt{3.16}}{-0.2}
\]
\[
\approx 12 \pm 8.89
\]
\[
\approx 3.11, 20.89
\]

That is, at about 3.11 s and 20.89 s the ball will be at the height 8 ft, so it can be inside the goal. Find the value of \(t\) for which the ball will hit the ground, that is, when the height is zero.

\[-0.1t^2 + 2.4t + 1.5 = 0\]
\[
t = \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(-0.1)(1.5)}}{2(-0.1)}
\]
\[
t = \frac{-2.4 \pm \sqrt{3.16}}{-0.2}
\]
\[
\approx 12 \pm 12.61
\]
\[
\approx -0.61, 24.61
\]

Since a negative value for time has no meaning, discard the negative solution. So, the ball hits the ground in about 24.61 seconds. The ball will be inside the goal when \(0 < t < 3.11\) or \(20.89 < t \leq 24.61\).

ANSWER:
\(\{t | 0 < t < 3.11\}\) or \(\{t | 20.89 < t \leq 24.61\}\)
4-8 Quadratic Inequalities

Solve each inequality algebraically.

9. \(x^2 + 6x - 16 < 0\)

\[\text{SOLUTION:}\]
First, write the related equation and factor it.

\[x^2 + 6x - 16 = 0\]
\[(x + 8)(x - 2) = 0\]

By the Zero Product Property:
\[x + 8 = 0 \text{ or } x - 2 = 0\]
\[x = -8 \text{ or } x = 2\]

The two numbers divide the number line into three regions: \(x \leq -8, -8 < x < 2\) and \(x \geq 2\). Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{array}{ccc}
\text{interval} & \text{test point} & \text{value of inequality} \\
\hline
x \leq -8 & 0 & -16 < 0 \\
-8 < x < 2 & -4 & 0 \\
x \geq 2 & 2 & 20 > 0
\end{array}
\]

Note that, the points \(x = -8\) and \(x = 2\) are not included in the solution. Therefore, the solution set is \(\{x \mid -8 < x < 2\}\).

\[\text{ANSWER:}\]
\[\{x \mid -8 < x < 2\}\]

10. \(x^2 - 14x > -49\)

\[\text{SOLUTION:}\]
First, write the related equation and factor it.

\[x^2 - 14x + 49 = 0\]
\[(x - 7)(x - 7) = 0\]

By the Zero Product Property:
\[x - 7 = 0\]
\[x = 7\]

The number 7 divides the number line into three regions: \(x < 7, x = 7\) and \(x > 7\). Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{array}{ccc}
\text{interval} & \text{test point} & \text{value of inequality} \\
\hline
x < 7 & 5 & -36 < 0 \\
x = 7 & 7 & 0 = 0 \\
x > 7 & 10 & 25 > 0
\end{array}
\]

Therefore, the solution set is \(\{x \mid x < 7 \text{ or } x > 7\}\).

\[\text{ANSWER:}\]
\[\{x \mid x < 7 \text{ or } x > 7\}\]
11. \(-x^2 + 12x \geq 28\)

**SOLUTION:**
First, write the related equation and factor it.

\[-x^2 + 12x - 28 = 0\]

\[
x = \frac{-12 \pm \sqrt{(12)^2 - 4(-28)(-1)}}{2(-1)}
\]

\[
= \frac{-12 \pm \sqrt{32}}{-2}
\]

\[
= 6 \pm 2\sqrt{2}
\]

\[\approx 8.83, 3.17\]

The two numbers divide the number line into three regions \(x \leq 3.17, 3.17 \leq x \leq 8.83\) and \(x \geq 8.83\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>Region</th>
<th>Test Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x \leq 3.17)</td>
<td>(-2)</td>
<td>-10 \geq 0</td>
</tr>
<tr>
<td>(3.17 \leq x \leq 8.83)</td>
<td>(4)</td>
<td>-4 \geq 0</td>
</tr>
<tr>
<td>(x \geq 8.83)</td>
<td>(8)</td>
<td>-10 \geq 0</td>
</tr>
</tbody>
</table>

Note that, the points \(x = 3.17\) and \(x = 8.83\) are also included in the solution. Therefore, the solution set is \(\{x \mid 3.17 \leq x \leq 8.83\}\).

**ANSWER:**
\(\{x \mid 3.17 \leq x \leq 8.83\}\)

12. \(x^2 - 4x \leq 21\)

**SOLUTION:**
First, write the related equation and factor it.

\[x^2 - 4x - 21 = 0\]

\[(x - 7)(x + 3) = 0\]

\[x = 7 \text{ or } x = -3\]

The two numbers divide the number line into three regions \(x \leq -3, -3 \leq x \leq 7\) and \(x \geq 7\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>Region</th>
<th>Test Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x \leq -3)</td>
<td>(-4)</td>
<td>-31 \leq 0</td>
</tr>
<tr>
<td>(-3 \leq x \leq 7)</td>
<td>(2)</td>
<td>-10 \leq 0</td>
</tr>
<tr>
<td>(x \geq 7)</td>
<td>(8)</td>
<td>17 \leq 0</td>
</tr>
</tbody>
</table>

Note that, the points \(x = -3\) and \(x = 7\) are also included in the solution. Therefore, the solution set is \(\{x \mid -3 \leq x \leq 7\}\).

**ANSWER:**
\(\{x \mid -3 \leq x \leq 7\}\)
4-8 Quadratic Inequalities

Graph each inequality.

13. \( y \geq x^2 + 5x + 6 \)

**SOLUTION:**
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
y \geq x^2 + 5x + 6 \\
0 \geq 0^2 + 5(0) + 6 \\
0 \not\geq 6
\]

So, \((0, 0)\) is not a solution of the inequality. Shade the region of the graph that does not contain \((0, 0)\).

14. \( x^2 - 2x - 8 < y \)

**SOLUTION:**
First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

\[
y > x^2 - 2x - 8 \\
0 > 0^2 - 2(0) - 8 \\
0 > -8
\]

So, \((0, 0)\) is a solution of the inequality. Shade the region of the graph that contains \((0, 0)\).
4-8 Quadratic Inequalities

15. \( y \leq -x^2 - 7x + 8 \)

**SOLUTION:**
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
y \leq -x^2 - 7x + 8
0 \leq -0^2 - 7(0) + 8
0 \leq 8
\]

So, \((0, 0)\) is a solution of the inequality. Shade the region of the graph that contains \((0, 0)\).

**ANSWER:**

16. \( -x^2 + 12x - 36 > y \)

**SOLUTION:**
First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

\[
-x^2 + 12x - 36 > y
-0^2 + 12(0) - 36 > 0
-36 \neq 0
\]

So, \((0, 0)\) is not a solution of the inequality. Shade the region of the graph that does not contain \((0, 0)\).

**ANSWER:**
Graph each inequality.

17. $y > 2x^2 - 2x - 3$

**SOLUTION:**
First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

\[
y > 2x^2 - 2x - 3
\]
\[
0 > 2(0)^2 - 2(0) - 3
\]
\[
0 > -3
\]

So, $(0, 0)$ is a solution of the inequality. Shade the region of the graph that contains $(0, 0)$.

**ANSWER:**

18. $y \geq -4x^2 + 12x - 7$

**SOLUTION:**
First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

\[
y \geq -4x^2 + 12x - 7
\]
\[
0 \geq -4(0)^2 + 12(0) - 7
\]
\[
0 \geq -7
\]

So, $(0, 0)$ is a solution of the inequality. Shade the region of the graph that contains $(0, 0)$.

**ANSWER:**
4-8 Quadratic Inequalities

Solve each inequality by graphing.

19. \( x^2 - 9x + 9 < 0 \)

**SOLUTION:**
First, write the related equation and solve it.

\[
x^2 - 9x + 9 = 0
\]
\[
x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(9)}}{2(1)}
\]
\[
x = \frac{9 \pm \sqrt{45}}{2}
\]
\[
\approx 1.1 \text{ or } 7.9
\]

Sketch the graph of a parabola that has \( x \)-intercepts at 1.1 and 7.9. The graph should open up because \( a > 0 \).

The graph lies below the \( x \)-axis between \( x = 1.1 \) and \( x = 7.9 \). Thus, the solution set of the inequality is \( \{ x | 1.1 < x < 7.9 \} \).

**ANSWER:**
\( \{ x | 1.1 < x < 7.9 \} \)

20. \( x^2 - 2x - 24 \leq 0 \)

**SOLUTION:**
First, write the related equation and factor it.

\[
x^2 - 2x - 24 = 0
\]
\[
(x - 6)(x + 4) = 0
\]
\[
x = 6 \text{ or } x = -4
\]

Sketch the graph of a parabola that has \( x \)-intercepts at \(-4\) and 6. The graph should open up because \( a > 0 \).

The graph lies below the \( x \)-axis between \( x = -4 \) and \( x = 6 \) including the two end points. Thus, the solution set of the inequality is \( \{ x | -4 \leq x \leq 6 \} \).

**ANSWER:**
\( \{ x | -4 \leq x \leq 6 \} \)
21. $x^2 + 8x + 16 \geq 0$

**SOLUTION:**
First, write the related equation and factor it.

\[ x^2 + 8x + 16 = 0 \]
\[ (x + 4)^2 = 0 \]
\[ x + 4 = 0 \]
\[ x = -4 \]

The equation has only one real root.
The graph should open up because $a > 0$.

The graph lies completely above the $x$-axis. Thus, the solution set of the inequality is $\{x \mid \text{all real numbers}\}$.

**ANSWER:**
$\{x \mid \text{all real numbers}\}$

22. $x^3 + 6x + 3 > 0$

**SOLUTION:**
First, write the related equation and solve it.

\[ x^3 + 6x + 3 = 0 \]

Sketch the graph of a parabola that has $x$-intercepts at $-5.45$ and $-0.55$. The graph should open up because $a > 0$.

The graph lies above the $x$-axis left to $x = -5.45$ and right to $x = -0.55$. Thus, the solution set of the inequality is $\{x \mid x < -5.45 \text{ or } x > -0.55\}$.

**ANSWER:**
$\{x \mid x < -5.45 \text{ or } x > -0.55\}$
23. \(0 > -x^2 + 7x + 12\)

**SOLUTION:**
First, write the related equation and solve it.

\[-x^2 + 7x + 12 = 0\]

\[x = \frac{-7 \pm \sqrt{(7)^2 - 4(-1)(12)}}{2(-1)}\]

\[x \approx -1.42 \text{ or } 8.42\]

Sketch the graph of a parabola that has \(x\)-intercepts at \(-1.42\) and \(8.42\). The graph should open down because \(a < 0\).

The graph lies below the \(x\)-axis left to \(x = -1.42\) and right to \(x = 8.42\). Thus, the solution set of the inequality is \(\{x \mid x < -1.42 \text{ or } x > 8.42\}\).

**ANSWER:**
\(\{x \mid x < -1.42 \text{ or } x > 8.42\}\)

24. \(-x^2 + 2x - 15 < 0\)

**SOLUTION:**
First, write the related equation and factor it.

\[-x^2 + 2x - 15 = 0\]

\[x = \frac{-2 \pm \sqrt{(-2)^2 - 4(-1)(-15)}}{2(-1)}\]

\[x \approx -1 + 7.48i, -1 - 7.48i\]

The equation does not have real roots. The parabola does not intersect the \(x\)-axis. The graph should open down because \(a < 0\).

The graph lies completely below the \(x\)-axis. Thus, the solution set of the inequality is \(\{x \mid \text{all real numbers}\}\).

**ANSWER:**
\(\{x \mid \text{all real numbers}\}\)
25. \(4x^2 + 12x + 10 \leq 0\)

**SOLUTION:**
First, write the related equation and factor it.

\[
4x^2 + 12x + 10 = 0
\]

\[
x = \frac{-(12) \pm \sqrt{(12)^2 - 4(4)(10)}}{2(4)}
\]

\[
= \frac{-12 \pm \sqrt{-16}}{8}
\]

\[
\approx -\frac{3}{2} + \frac{1}{2}i, \quad -\frac{3}{2} - \frac{1}{2}i
\]

The equation does not have real roots. The parabola does not intersect the \(x\)-axis. The graph should open up because \(a > 0\).

![Graph of parabola](image)

No part of the graph lies below the \(x\)-axis. Thus, the solution set of the inequality is \(\emptyset\).

**ANSWER:**
\(\emptyset\)

---

26. \(-3x^2 - 3x + 9 > 0\)

**SOLUTION:**
First, write the related equation and solve it.

\[
-3x^2 - 3x + 9 = 0
\]

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-3)(9)}}{2(-3)}
\]

\[
\approx 1.30 \text{ or } -2.30
\]

Sketch the graph of a parabola that has \(x\)-intercepts at 1.30 and −2.30. The graph should open down because \(a < 0\).

![Graph of parabola](image)

The graph lies above the \(x\)-axis between \(x \approx -2.30\) and \(x \approx 1.30\). Thus, the solution set of the inequality is \(\{x \mid -2.30 < x < 1.30\}\).

**ANSWER:**
\(\{x \mid -2.30 < x < 1.30\}\)
4-8 Quadratic Inequalities

27. \( 0 > -2x^2 + 4x + 4 \)

**SOLUTION:**
First, write the related equation and solve it.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

First, write the related equation and solve for \( x \).

\[
-2x^2 + 4x + 4 = 0
\]

\[
x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(4)}}{2(-2)}
\]

\[
= 1 \pm \sqrt{3}
\]

\[
\approx -0.73 \text{ or } 2.73
\]

Sketch the graph of a parabola that has \( x \)-intercepts at \(-0.73\) and \(2.73\). The graph should open down because \( a < 0 \).

The graph lies below the \( x \)-axis left to \( x \approx -0.73 \) and right to \( x \approx 2.73 \). Thus, the solution set of the inequality is \( \{ x \mid x < -0.73 \text{ or } x > 2.73 \} \).

**ANSWER:**
\( \{ x \mid x < -0.73 \text{ or } x > 2.73 \} \)

28. \( 3x^2 + 12x + 36 \leq 0 \)

**SOLUTION:**
First, write the related equation and solve it.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

First, write the related equation and factor it.

\[
3x^2 + 12x + 36 = 0
\]

\[
x = \frac{-12 \pm \sqrt{(12)^2 - 4(3)(36)}}{2(3)}
\]

\[
= \frac{-12 \pm 12\sqrt{2}}{6}
\]

The equation does not have any real roots. The parabola does not intersect the \( x \)-axis. The graph should open up because \( a > 0 \).

The graph lies entirely above the \( x \)-axis. Thus, the solution set of the inequality is \( \emptyset \).

**ANSWER:**
\( \emptyset \)
29. \( 0 \leq -4x^2 + 8x + 5 \)

**SOLUTION:**
First, write the related equation and solve it.

\[
-4x^2 + 8x + 5 = 0
\]

\[
x = \frac{-8 \pm \sqrt{(8)^2 - 4(-4)(5)}}{2(-4)}
\]

\[
= -0.5 \text{ or } 2.5
\]

Sketch the graph of a parabola that has \( x \)-intercepts at \(-0.5\) and 2.5. The graph should open down because \( a < 0 \).

The graph lies above the \( x \)-axis between \( x = -0.5 \) and \( x = 2.5 \) including the two endpoints. Thus, the solution set of the inequality is \{\( x \mid -0.5 \leq x \leq 2.5 \}\}.

**ANSWER:**
\{\( x \mid -0.5 \leq x \leq 2.5 \}\}

30. \( -2x^2 + 3x + 3 \leq 0 \)

**SOLUTION:**
First, write the related equation and solve it.

\[
-2x^2 + 3x + 3 = 0
\]

\[
x = \frac{-(3) \pm \sqrt{(3)^2 - 4(-2)(3)}}{2(-2)}
\]

\[
= \frac{-3 \pm \sqrt{33}}{-4}
\]

\[
\approx -0.69 \text{ or } 2.19
\]

Sketch the graph of a parabola that has \( x \)-intercepts at \(-0.69\) and 2.19. The graph should open down because \( a < 0 \).

The graph lies below the \( x \)-axis left to \( x \approx -0.69 \) and right to \( x \approx 2.19 \) including the two end points. Thus, the solution set of the inequality is \{\( x \mid x \leq -0.69 \) or \( x \leq 2.19 \}\}.

**ANSWER:**
\{\( x \mid x \leq -0.69 \) or \( x \geq 2.19 \}\}
4-8 Quadratic Inequalities

31. ARCHITECTURE An arched entry of a room is shaped like a parabola that can be represented by the equation \( f(x) = -x^2 + 6x + 1 \). How far from the sides of the arch is its height at least 7 feet?

**SOLUTION:**
The height is at least 7 feet for all the places where the value of \( f(x) \) is greater than or equal to 7. That is,

\[-x^2 + 6x + 1 \geq 7 \quad \text{or} \quad -x^2 + 6x - 6 \geq 0.\]

Write the related equation and solve it.

\[-x^2 + 6x - 6 = 0\]

\[x = \frac{-6 \pm \sqrt{(6)^2 - 4(-1)(-6)}}{2(-1)}\]

\[= \frac{-6 \pm \sqrt{12}}{-2}\]

\[\approx 1.27 \text{ or } 4.73\]

The graph of the parabola representing the equation \( y = -x^2 + 6x - 6 \) lies above the x-axis between \( x \approx 1.27 \) and \( x \approx 4.73 \) including the two end points. Thus, the solution set of the inequality is \( \{ x | 1.27 \leq x \leq 4.73 \} \).

Therefore, from 1.27 ft to 4.73 ft, the height will be at least 7 feet.

**ANSWER:**
about 1.26 ft to 4.73 ft

32. MANUFACTURING A box is formed by cutting 4-inch squares from each corner of a square piece of cardboard and then folding the sides. If \( V(x) = 4x^2 - 64x + 256 \) represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic inches?

**SOLUTION:**
Substitute 0 for \( V(x) \) and solve for \( x \).

\[4x^2 - 64x + 256 = 0\]

\[4(x^2 - 16x + 64) = 0\]

\[4(x - 8)^2 = 0\]

\[x - 8 = 0\]

\[x = 8\]

Therefore, \( x \) must be greater than 8 in.

The volume of the box should be less than or equal to 750 cu. in. That is,

\[4x^2 - 64x + 256 \leq 750 \quad \text{or} \quad 4x^2 - 64x - 494 \leq 0.\]

Write the related equation and solve it.

\[4x^2 - 64x - 494 = 0\]

\[x = \frac{-(-64) \pm \sqrt{(-64)^2 - 4(4)(-494)}}{2(4)}\]

\[= \frac{64 \pm \sqrt{12000}}{8}\]

\[\approx -5.69 \text{ or } 21.69\]

The value of \( x \) should be positive.

So, the maximum width should be 21.69 in.

The dimensions of the original card board is greater than 8 in. but no more than 21.69 in.

**ANSWER:**
greater than 8 in. but no more than 21.69 in.
Solve each inequality algebraically.

33. \(x^2 - 9x < -20\)

**SOLUTION:**
First, write the related equation and solve it.

\[
x^2 - 9x = -20
\]
\[
x^2 - 9x + 20 = 0
\]
\[
(x - 5)(x - 4) = 0
\]

By the Zero Product Property:

\[
x - 5 = 0 \quad \text{or} \quad x - 4 = 0
\]
\[
x = 5 \quad \text{or} \quad x = 4
\]

The two numbers divide the number line into three regions \(x \leq 4\), \(4 < x < 5\) and \(x \geq 5\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>(x \leq 4)</th>
<th>(4 &lt; x &lt; 5)</th>
<th>(x \geq 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test (x = 2)</td>
<td>Test (x = 4.5)</td>
<td>Test (x = 6)</td>
</tr>
<tr>
<td>(x^2 - 9x + 20 &lt; 0)</td>
<td>(x^2 - 9x + 20 &lt; 0)</td>
<td>(x^2 - 9x + 20 &lt; 0)</td>
</tr>
<tr>
<td>((4.5)^2 - 9(4.5) + 20 &lt; 0)</td>
<td>(6^2 - 9(6) + 20 &lt; 0)</td>
<td>(2^2 - 0 &lt; 0)</td>
</tr>
</tbody>
</table>

Note that the points \(x = 4\) and \(x = 5\) are not included in the solution. Therefore, the solution set is \(\{x \mid 4 < x < 5\}\).

**ANSWER:**

\(\{x \mid 4 < x < 5\}\)

---

34. \(x^2 + 7x \geq -10\)

**SOLUTION:**
First, write the related equation and solve it.

\[
x^2 + 7x = -10
\]
\[
x^2 + 7x + 10 = 0
\]
\[
(x + 5)(x + 2) = 0
\]

By the Zero Product Property:

\[
x + 5 = 0 \quad \text{or} \quad x + 2 = 0
\]
\[
x = -5 \quad \text{or} \quad x = -2
\]

The two numbers divide the number line into three regions \(x \leq -5\), \(-5 \leq x \leq -2\) and \(x \geq -2\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>(x \leq -5)</th>
<th>(-5 \leq x \leq -2)</th>
<th>(x \geq -2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test (x = -7)</td>
<td>Test (x = -3)</td>
<td>Test (x = 0)</td>
</tr>
<tr>
<td>(x^2 + 7x \geq -10)</td>
<td>(x^2 + 7x \geq -10)</td>
<td>(x^2 + 7x \geq -10)</td>
</tr>
<tr>
<td>((-7)^2 + 7(-7) \geq -10)</td>
<td>((-3)^2 + 7(-3) \geq -10)</td>
<td>(0^2 + 7(0) \geq -10)</td>
</tr>
<tr>
<td>(0 \leq -10)</td>
<td>(-12 \leq -10)</td>
<td>(0 \geq -10)</td>
</tr>
</tbody>
</table>

Therefore, the solution set is \(\{x \mid x \leq -5\ \text{or} \ x \geq -2\}\).

**ANSWER:**

\(\{x \mid x \leq -5\ \text{or} \ x \geq -2\}\)
35. \(2 > x^2 - x\)

**SOLUTION:**
First, write the related equation and solve it.

\[x^2 - x - 2 = 0\]
\[(x - 2)(x + 1) = 0\]

By the Zero Product Property:

\[x - 2 = 0 \quad \text{or} \quad x + 1 = 0\]
\[x = 2 \quad \text{or} \quad x = -1\]

The two numbers divide the number line into three regions: \(x \leq -1\), \(-1 < x < 2\) and \(x \geq 2\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>(x &lt; -1)</th>
<th>(-1 &lt; x &lt; 2)</th>
<th>(x &gt; 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test (x = -2)</td>
<td>Test (x = 1)</td>
<td>Test (x = 4)</td>
</tr>
<tr>
<td>(2 &gt; x^2 - x)</td>
<td>(2 &gt; x^2 - x)</td>
<td>(2 &gt; x^2 - x)</td>
</tr>
<tr>
<td>(2 &gt; (-2)^2 - (-2))</td>
<td>(2 &gt; 1^2 - 1)</td>
<td>(2 &gt; 4^2 - 4)</td>
</tr>
<tr>
<td>(2 \neq 6x)</td>
<td>(2 &gt; 0)</td>
<td>(2 \neq 12x)</td>
</tr>
</tbody>
</table>

Note that the points \(x = -1\) and \(x = 2\) are not included in the solution. Therefore, the solution set is \(\{x \mid -1 < x < 2\}\).

**ANSWER:**
\(\{x \mid -1 < x < 2\}\)

36. \(-3 \leq -x^2 - 4x\)

**SOLUTION:**
First, write the related equation and solve it.

\[x^2 + 4x - 3 = 0\]
\[x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}\]
\[= \frac{-4 \pm 2\sqrt{7}}{2}\]
\[= -2 \pm \sqrt{7}\]
\[\approx -4.65 \text{ or } 0.65\]

The two numbers divide the number line into three regions: \(x \leq -4.65\), \(-4.65 \leq x \leq 0.65\) and \(x \geq 0.65\). Test a value from each interval to see if it satisfies the original inequality.

<table>
<thead>
<tr>
<th>(x \leq -4.65)</th>
<th>(-4.65 \leq x \leq 0.65)</th>
<th>(x \geq 0.65)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test (x = -5)</td>
<td>Test (x = 0)</td>
<td>Test (x = 1)</td>
</tr>
<tr>
<td>(-3 \leq -x^2 - 4x)</td>
<td>(-3 \leq -x^2 - 4x)</td>
<td>(-3 \leq -x^2 - 4x)</td>
</tr>
<tr>
<td>(-3 \leq (-5)^2 - 4(-5))</td>
<td>(-3 \leq (0)^2 - 4(0))</td>
<td>(-3 \leq (1)^2 - 4(1))</td>
</tr>
<tr>
<td>(-3 \neq -5x)</td>
<td>(-3 \leq 0)</td>
<td>(-3 \neq -5x)</td>
</tr>
</tbody>
</table>

Therefore, the solution set is \(\{x \mid -4.65 \leq x \leq 0.65\}\).

**ANSWER:**
\(\{x \mid -4.65 \leq x \leq 0.65\}\)
Graph each inequality.

37. \(-x^2 + 2x \leq -10\)

**SOLUTION:**

First, write the related equation and solve it.

\[
x^2 - 2x - 10 = 0
\]

\[
x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-10)}}{2(1)}
\]

\[
= \frac{2 \pm \sqrt{44}}{2}
\]

\[
= 1 \pm \sqrt{11}
\]

\[
= -2.32 \text{ or } 4.32
\]

Therefore, the solution set is \(\{x \mid x \leq -2.32 \text{ or } x \geq 4.32\}\).

**ANSWER:**

\(\{x \mid x \leq -2.32 \text{ or } x \geq 4.32\}\)

38. \(-6 > x^2 + 4x\)

**SOLUTION:**

\(-6 > x^2 + 4x\)

\(0 > x^2 + 4x + 6\)

\(0 < -(x^2 + 4x + 6)\)

The solution is empty set, as no real value for \(x\) will satisfy the inequality.

**ANSWER:**

\(\emptyset\)

39. \(2x^2 + 4 \geq 9\)

**SOLUTION:**

First, write the related equation and solve it.

\(2x^2 - 5 = 0\)

\(2x^2 = 5\)

\(x^2 = 2.5\)

\(x = \pm \sqrt{2.5}\)

\(x \approx -1.58 \text{ or } 1.58\)

The two numbers divide the number line into three regions \(x \leq -1.58\), \(-1.58 \leq x \leq 1.58\) and \(x \geq 1.58\).

Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{array}{ccc}
  x & \leq -1.58 & -1.58 \leq x \leq 1.58 & x \geq 1.58 \\
  2x^2 + 4 & \geq 9 & 2x^2 + 4 & \geq 9 & 2x^2 + 4 & \geq 9 \\
  2(-2)^2 + 4 & \geq 9 & 2(0)^2 + 4 & \geq 9 & 2(2)^2 + 4 & \geq 9 \\
  12 & \geq 9 & 4 & \geq 9 & 12 & \geq 9
\end{array}
\]

Therefore, the solution set is \(\{x \mid x \leq -1.58 \text{ or } x \geq 1.58\}\).

**ANSWER:**

\(\{x \mid x \leq -1.58 \text{ or } x \geq 1.58\}\)
4-8 Quadratic Inequalities

40. \(3x^2 + x \geq -3\)

\textbf{SOLUTION:}

\[3x^2 + x \geq -3\]
\[3x^2 + x + 3 \geq 0\]

The solution set of the quadratic inequality is all real numbers, as all the real numbers satisfy the inequality.
Solution set: \(\{x \mid \text{all real numbers}\}\)

\textbf{ANSWER:}
\(\{x \mid \text{all real numbers}\}\)

41. \(-4x^2 + 2x < 3\)

\textbf{SOLUTION:}

\[-4x^2 + 2x < 3\]
\[-4x^2 + 2x - 3 < 0\]

The solution set of the quadratic inequality is all real numbers, as all the real numbers satisfy the inequality.
Solution set: \(\{x \mid \text{all real numbers}\}\)

\textbf{ANSWER:}
\(\{x \mid \text{all real numbers}\}\)

42. \(-11 \geq -2x^2 - 5x\)

\textbf{SOLUTION:}
First, write the related equation and solve it.

\[2x^2 + 5x - 11 = 0\]
\[x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-11)}}{2(2)}\]
\[= \frac{-5 \pm \sqrt{113}}{4}\]
\[\approx -3.91 \text{ or } 1.41\]

The two numbers divide the number line into three regions \(x \leq -3.91, -3.91 \leq x \leq 1.41\) and \(x \geq 1.41\).
Test a value from each interval to see if it satisfies the original inequality.

\begin{align*}
\text{If } x & \leq -3.91 \quad \text{If } -3.91 \leq x \leq 1.41 \quad \text{If } x \geq 1.41 \\
\text{Test } x = -4 & \quad \text{Test } x = 1.1 & \quad \text{Test } x = 2 \\
-11 \geq -2x^2 - 5x & \quad -11 \geq -2x^2 - 5x & \quad -11 \geq -2x^2 - 5x \\
-11 \geq -2(-4)^2 - 5(-4) & \quad -11 \geq -2(1.1)^2 - 5(1.1) & \quad -11 \geq -2(2)^2 - 5(2) \\
-11 \geq -12 & \quad -11 \geq -12.06 & \quad -11 \geq -18\end{align*}

Therefore, the solution set is \(\{x \mid x \leq -3.91 \text{ or } x \geq 1.41\}\).

\textbf{ANSWER:}
\(\{x \mid x \leq -3.91 \text{ or } x \geq 1.41\}\)
43. \(-12 < -5x^2 - 10x\)  

**SOLUTION:** 
First, write the related equation and solve it. 

\[
5x^2 + 10x - 12 = 0 \\
x = \frac{-10 \pm \sqrt{10^2 - 4(5)(-12)}}{2(5)} \\
= \frac{-10 \pm \sqrt{400}}{10} \\
= \frac{-10 \pm 20}{10} \\
= -1, 2 \\
\Longrightarrow x \leq -1 \text{ or } x \geq 2 
\]

The two numbers divide the number line into three regions, \(x \leq -1\), \(-1 < x < 2\), and \(x \geq 2\). Test a value from each interval to see if it satisfies the original inequality. 

\[
\begin{array}{ccc}
    x & x < -1 & x > 2 \\
    Test x = -3 & -12 < -5x^2 - 10x & -12 < -5x^2 - 10x \\
    Test x = 0 & -12 < -5x^2 - 10x & -12 < -5x^2 - 10x \\
    Test x = 1 & -12 < -5x^2 - 10x & -12 < -5x^2 - 10x \\
\end{array}
\]

Therefore, the solution set is \(\{ x \mid -2.84 < x < 2 \}\). 

**ANSWER:** 
\(\{ x \mid -2.84 < x < 2 \}\)

44. \(-3x^2 - 10x > -1\)  

**SOLUTION:** 
First, write the related equation and solve it. 

\[
3x^2 + 10x - 1 < 0 \\
x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-1)}}{2(3)} \\
= \frac{-10 \pm \sqrt{44}}{6} \\
= \frac{-10 \pm 2\sqrt{11}}{6} \\
= \frac{-5 \pm \sqrt{11}}{3} \\
\approx -3.43 \text{ or } 0.10 
\]

The two numbers divide the number line into three regions, \(x \leq -3.43\), \(-3.43 < x < 0.10\), and \(x \geq 0.10\). Test a value from each interval to see if it satisfies the original inequality. 

\[
\begin{array}{ccc}
    x & -3.43 < x < 0.10 & x \geq 0.10 \\
    Test x = -4 & -3(-4)^2 + 10(-4) > -1 & 3(-4)^2 + 10(-4) > -1 \\
    Test x = 0 & -3(0)^2 + 10(0) > -1 & 3(0)^2 + 10(0) > -1 \\
    Test x = 1 & -3(1)^2 + 10(1) > -1 & 3(1)^2 + 10(1) > -1 \\
\end{array}
\]

Therefore, the solution set is \(\{ x \mid -3.43 < x < 0.10 \}\). 

**ANSWER:** 
\(\{ x \mid -3.43 < x < 0.10 \}\)

45. **CCSS PERSEVERANCE** The Sanchez family is adding a deck along two sides of their swimming pool. The deck width will be the same on both sides and the total area of the pool and deck cannot exceed 750 square feet. 

a. Graph the quadratic inequality. 

b. Determine the maximum width of the deck.
4-8 Quadratic Inequalities

**SOLUTION:**

a. The area of the pool including the deck is \((24 + x)(12 + x)\). This should be less than or equal to 750 sq. ft.

\[
(24 + x)(12 + x) \leq 750
\]

\[
x^2 + 36x + 288 \leq 750
\]

\[
x^2 + 36x - 462 \leq 0
\]

Graph the inequality on a coordinate plane.

b. Write the related equation and factor it.

\[
x^2 + 36x - 462 = 0
\]

\[
x = \frac{-36 \pm \sqrt{36^2 - 4(-462)}}{2(1)}
\]

\[
= \frac{-36 \pm \sqrt{3144}}{2}
\]

\[
= -56, 10.04
\]

Here, \(x\) is a length, so it cannot be negative. So, the maximum width is 10.04 ft. For the same reason, the minimum length should be greater than 0.

**ANSWER:**

a.
Write a quadratic inequality for each graph.

46.

SOLUTION:
The coordinates of the vertex of the parabola is (2, –10). So, the equation of the parabola is $y = a(x - 2)^2 - 10$. Use any pair of points on the parabola to find the value of $a$.

\[-6 = a(4 - 2)^2 - 10\]
\[4 = 4a\]
\[1 = a\]

So, the equation of the parabola is

\[y = 1\left(x^2 - 4x + 4\right) - 10\]
\[y = x^2 - 4x - 6\]

The boundary line is dashed and the region above the line is shaded. So, the inequality is $y > x^2 - 4x - 6$.

ANSWER:
\[y > x^2 - 4x - 6\]

47.

SOLUTION:
The coordinates of the vertex of the parabola is (1, 7). So, the equation of the parabola is $y = a(x - 1)^2 + 7$. Use any pair of points on the parabola to find the value of $a$.

\[6 = a(2-1)^2 + 7\]
\[-1 = a\]

So, the equation of the parabola is

\[y = -1\left(x^2 - 2x + 1\right) + 7\]
\[y = -x^2 + 2x + 6\]

The boundary line is a solid line and the region below the line is shaded. So, the inequality is $y \leq -x^2 + 2x + 6$.

ANSWER:
\[y \leq -x^2 + 2x + 6\]
4-8 Quadratic Inequalities

SOLUTION:
The coordinates of the vertex of the parabola is (–8, 18). So, the equation of the parabola is \( y = a(x + 8)^2 + 18 \). Use any pair of points on the parabola to find the value of \( a \).

\[
14 = a(-4 + 8)^2 + 18
\]
\[
-4 = 16a
\]
\[
\frac{1}{4} = a
\]

So, the equation of the parabola is

\[
y = -\frac{1}{4}(x^2 + 16x + 64) + 18.
\]
\[
y = -\frac{1}{4}x^2 - 4x + 2
\]

The boundary line is dashed and the region above the line is shaded. So, the inequality is \( y > -0.25x^2 - 4x + 2 \).

ANSWER:
\( y > -0.25x^2 - 4x + 2 \)

Solve each quadratic inequality by using a graph, a table, or algebraically.

49. \(-2x^2 + 12x < -15\)

SOLUTION:
First, write the related equation and solve it.

\[
-2x^2 + 12x + 15 = 0
\]
\[
x = \frac{-12 \pm \sqrt{12^2 - 4(-2)(15)}}{2(-2)}
\]
\[
= \frac{-12 \pm \sqrt{264}}{-4}
\]
\[
\approx -1.06 \text{ or } 7.06
\]

The two numbers divide the number line into three regions \( x < -1.06, -1.06 \leq x \leq 7.06 \) and \( x > 7.06 \). Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{align*}
&x < -1.06 & &-1.06 \leq x \leq 7.06 & &x > 7.06 \\
&-2(-2)^2 + 12(-2) < -15 & &-1.06 \leq x \leq 7.06 & &-2(8) + 12(8) < -15 \\
&-2(-8) + 12(6) < -15 & &-2(0) + 12(8) < -15 & &-2(-8) + 12(6) < -15 \\
&-32 < -15 < 0 & &-15 < 0 < -15 & &-42 < -15 < 0
\end{align*}
\]

Note that, the points \( x = -1.06 \) and \( x = 7.06 \) are not included in the solution. Therefore, the solution set is \( \{ x \mid x < -1.06 \text{ or } x > 7.06 \} \).

ANSWER:
\( \{ x \mid x < -1.06 \text{ or } x > 7.06 \} \)
4-8 Quadratic Inequalities

50. \(5x^2 + x + 3 \geq 0\)

**SOLUTION:**
First, write the related equation and solve it.

\[
5x^2 + x + 3 = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-1 \pm \sqrt{1^2 - 4(5)(3)}}{2(5)}
\]

\[
x = \frac{-1 \pm \sqrt{-59}}{10}
\]

The equation does not have real roots. Graph the function.
The graph should open up because \(a > 0\).

The graph lies completely above the \(x\)-axis. Thus, the solution set of the inequality is \(\{x \mid \text{all real numbers}\}\).

**ANSWER:**
\(\{x \mid \text{all real numbers}\}\)

51. \(11 \leq 4x^2 + 7x\)

**SOLUTION:**
First, write the related equation and solve it.

\[
4x^2 + 7x - 11 = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-7 \pm \sqrt{7^2 - 4(4)(-11)}}{2(4)}
\]

\[
x = \frac{-7 \pm 15}{8}
\]

\[
x = -2.75 \text{ or } 1
\]

The two numbers divide the number line into three regions \(x \leq -2.75, -2.75 \leq x \leq 1\) and \(x \geq 1\). Test a value from each interval to see if it satisfies the original inequality.

\[
\begin{array}{ccc}
-2.75 & -2.75 & 1 \\
11 & 11 & 11 \\
\vdots & \vdots & \vdots \\
\end{array}
\]

\[
\begin{array}{ccc}
\leq & \leq & \geq \\
-3 & -1 & 2 \\
\vdots & \vdots & \vdots \\
\end{array}
\]

Therefore, the solution set is \(\{x \mid x \leq -2.75 \text{ or } x \geq 1\}\).

**ANSWER:**
\(\{x \mid x \leq -2.75 \text{ or } x \geq 1\}\)
52. \( x^2 - 4x \leq -7 \)

**SOLUTION:**
First, write the related equation and solve it. 

\[
x^2 - 4x + 7 = 0
\]

\[
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}
\]

\[
= \frac{4 \pm \sqrt{-12}}{2}
\]

The equation does not have real roots. 

The graph should open up because \( a > 0 \). 

No part of the graph lies below the \( x \)-axis. Thus, the solution set of the inequality is \( \emptyset \). 

**ANSWER:** 
\( \emptyset \)

53. \( -3x^2 + 10x < 5 \)

**SOLUTION:**
First, write the related equation and solve it. 

\[
-3x^2 + 10x - 5 = 0
\]

\[
x = \frac{-10 \pm \sqrt{(10)^2 - 4(-3)(-5)}}{2(-3)}
\]

\[
= \frac{-10 \pm \sqrt{40}}{-6}
\]

\[
\approx 0.61 \text{ or } 2.72
\]

The two numbers divide the number line into three regions: \( x < 0.61 \), \( 0.61 \leq x \leq 2.72 \) and \( x > 2.72 \). Test a value from each interval to see if it satisfies the original inequality. 

<table>
<thead>
<tr>
<th>Interval</th>
<th>(-3(0)^2 + 10(0) &lt; 5)</th>
<th>(-3(1)^2 + 10(1) &lt; 5)</th>
<th>(-3(3)^2 + 10(3) &lt; 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; x &lt; 5)</td>
<td>(0 &lt; 5)</td>
<td>(-3 + 10 &lt; 5)</td>
<td>(-27 + 30 &lt; 5)</td>
</tr>
<tr>
<td>(x \geq 5)</td>
<td>(x \geq 5)</td>
<td>(7 \geq 5x)</td>
<td>(3 &lt; 5x)</td>
</tr>
</tbody>
</table>

Note that, the points \( x = 0.61 \) and \( x = 2.72 \) are not included in the solution. Therefore, the solution set is \( \{x \mid x < 0.61 \text{ or } x > 2.72\} \). 

**ANSWER:** 
\( \{x \mid x < 0.61 \text{ or } x > 2.72\} \)
4-8 Quadratic Inequalities

54. \(-1 \geq -x^2 - 5x\)

SOLUTION:
First, write the related equation and solve it.

\[-x^2 - 5x + 1 = 0\]

\[x = \frac{(-5) \pm \sqrt{(-5)^2 - 4(-1)(1)}}{2(-1)}\]

\[= \frac{5 \pm \sqrt{29}}{-2}\]

\[\approx -5.19 \text{ or } 0.19\]

Sketch the graph of a parabola that has \(x\)-intercepts at -5.19 and 0.19. The graph should open down because \(a < 0\).

The graph lies below the \(x\)-axis left to \(x \approx -5.19\) and right to \(x \approx 0.19\). Not that: The \(x\)-intercepts are also included in the solution. Thus, the solution set of the inequality is \(\{x \mid x \leq -5.19 \text{ or } x \geq 0.19\}\).

ANSWER:
\(\{x \mid x \leq -5.19 \text{ or } x \geq 0.19\}\)

55. BUSINESS An electronics manufacturer uses the function \(P(x) = x(-27.5x + 3520) + 20,000\) to model their monthly profits when selling \(x\) thousand digital audio players.

a. Graph the quadratic inequality for a monthly profit of at least $100,000.

b. How many digital audio players must the manufacturer sell to earn a profit of at least $100,000 in a month?

c. Suppose the manufacturer has an additional monthly expense of $25,000. Explain how this affects the graph of the profit function. Then determine how many digital audio players the manufacturer needs to sell to have at least $100,000 in profits.

SOLUTION:
a. The profit is at least $100,000 if the value of \(P(x)\) is greater than or equal to 100,000. That is,

\[-27.5x^2 + 3520x + 20000 \geq 100000\]

Graph the inequality on a coordinate plane.

b. The graph lies above the \(x\)-axis from \(x = 30\) to \(x = 98.5\). So, the company should sell 30,000 to 98,000 digital audio players to earn a profit of at least $100,000 in a month.

c. If the manufacturer has an additional monthly expense of $25,000, the amount 25,000 will be subtracted from the profit function. So, the graph denoting the inequality will be shifted down 25,000 units. The manufacturer must sell from 47,000 to 81,000 digital audio players.

ANSWER:
4-8 Quadratic Inequalities

a.

Also, \( \pi (5)^2 - \pi \left( \frac{x}{2} \right)^2 \leq 42. \)

Graph the two inequalities on a coordinate plane.

b. from 30,000 to 98,000 digital audio players

c. The graph is shifted down 25,000 units. The manufacturer must sell from 47,000 to 81,000 digital audio players.

56. UTILITIES A contractor is installing drain pipes for a shopping center’s parking lot. The outer diameter of the pipe is to be 10 inches. The cross sectional area of the pipe must be at least 35 square inches and should not be more than 42 square inches.

a. Graph the quadratic inequalities.

b. What thickness of drain pipe can the contractor use?

SOLUTION:

a. The inner radius is \( \frac{x}{2} \) cm. The cross sectional area of the pipe must be at least 35 square inches. So,

\[
\pi (5)^2 - \pi \left( \frac{x}{2} \right)^2 \geq 35.
\]

b. Solve the inequality \( 35 \leq 25\pi - \left( \frac{\pi}{4} \right) x^2 \leq 42. \)

\[
35 - 25\pi \leq -\left( \frac{\pi}{4} \right) x^2 \leq 42 - 25\pi
\]

\[
-4\left( 35 - 25\pi \right) \geq x^2 \geq -4\left( 42 - 25\pi \right)
\]

\[
\frac{55.44}{\pi} \geq x^2 \geq 46.52
\]

Here, \( x \) is a length, it cannot be negative. So, take the positive square root.

\[
7.45 \geq x \geq 6.82
\]

That is, the value of \( x \) should be between 6.82 and 7.45.

When \( x \) is 6.82, the thickness of the pipe will be \( \frac{10 - 6.82}{2} = 1.59. \)

When \( x \) is 7.45, the thickness of the pipe will be \( \frac{10 - 7.45}{2} = 1.275 \approx 1.28. \)

Therefore, the thickness of the pipe should be between 1.28 in and 1.59 in.

ANSWER:
4-8 Quadratic Inequalities

a.

b. 1.28 in. to 1.59 in.

57. OPEN ENDED Write a quadratic inequality for each condition.

a. The solution set is all real numbers.

b. The solution set is the empty set.

SOLUTION:

a. Sample answer: to find an inequality with the solution set of all real numbers, choose one point on the number line such as -1 to divide the number line into two intervals. Then the write a related quadratic equation: \((x + 1)(x + 1) = 0\). Test values from each interval to determine the sign of the inequality. When \(x = -1\), the equation equals 0 so the inequality sign must be \(\geq\) or \(\leq\) or equal to. When \(x = -2\), the equation equals 1 which is greater than 0. When \(x = 1\), the equation equals 4 which is greater than 0. So the inequality is: \(x^2 + 2x + 1 \geq 0\).

b. Sample answer: to find an inequality with a solution set of the empty set, write an inequality that does not have any real solutions such as: \(x^2 - 4x + 6 < 0\).

ANSWER:

a. Sample answer: \(x^2 + 2x + 1 \geq 0\)

b. Sample answer: \(x^2 - 4x + 6 < 0\)

58. CCSS CRITIQUE  Don and Diego used a graph to solve the quadratic inequality \(x^2 - 2x - 8 > 0\). Is either of them correct? Explain.

SOLUTION:

Don graphed the inequality in two variables, so he is not correct.

The inequality is \(x^2 - 2x - 8 > 0\). So, one should consider the region of the graph of \(y = x^2 - 2x - 8\) above the x-axis to find the limits for \(x\). But Diego has marked the wrong region. So, he is not correct either.

ANSWER:

Neither; Don graphed the inequality in two variables, and Diego graphed the wrong interval.

59. REASONING Are the boundaries of the solution set of \(x^2 + 4x - 12 \leq 0\) twice the value of the boundaries of \(\frac{1}{2}x^2 + 2x - 6 \leq 0\)? Explain.

SOLUTION:

The graphs of the inequalities intersect the x-axis at the same points. So, both have the same boundaries for the solution set.

ANSWER:

No; the graphs of the inequalities intersect the x-axis at the same points.
60. **REASONING** Determine if the following statement is sometimes, always, or never true. Explain your reasoning.

The intersection of \( y \leq -ax^2 + c \) and \( y \geq ax^2 - c \) is the empty set.

**SOLUTION:**
When \( a \) is positive and \( c \) is negative, there is no intersection, hence the solution is an empty set and when \( a \) is negative and \( c \) is positive there is an intersection, and hence the solution is not an empty set. Therefore, the statement is *sometimes* true.

**ANSWER:**
Sample answer: Sometimes; when \( a \) is positive and \( c \) is negative, there is no solution and when \( a \) is negative and \( c \) is positive there is a solution set.

61. **CHALLENGE** Graph the intersection of the graphs of \( y \leq -x^2 + 4 \) and \( y \geq x^2 - 4 \).

**SOLUTION:**
Graph the two inequalities on a coordinate plane and find the intersection.

**ANSWER:**

---

62. **WRITING IN MATH** How are the techniques used when solving quadratic inequalities and quadratic equations similar? different?

**SOLUTION:**
For both quadratic and linear inequalities, you must first graph the related equation. You use the inequality symbol to determine if the line is dashed or solid. Then you use test points to determine where to shade. One difference is that one related equation is a straight line while the other related equation is a curve.

**ANSWER:**
For both quadratic and linear inequalities, you must first graph the related equation. You use the inequality symbol to determine if the line is dashed or solid. Then you use test points to determine where to shade. One difference is that one related equation is a straight line while the other related equation is a curve.
63. **GRIDDED RESPONSE** You need to seed an area that is 80 feet by 40 feet. Each bag of seed can cover 25 square yards of land. How many bags of seed will you need?

**SOLUTION:**
Each yard is equivalent to 3 feet.

So, \(80 \text{ ft} = \left(\frac{80}{3}\right)\text{ yd}\) and \(40 \text{ ft} = \left(\frac{40}{3}\right)\text{ yd}\).

The area that has to be seeded is

\[
\frac{80}{3} \times \frac{40}{3} = \frac{3200}{9} \approx 355.56.
\]

Each bag of seed can cover 25 square yards of land. Divide 355.56 by 25 to find the number of bags required to seed the area.

\[
\frac{355.56}{25} = 14.2224
\]

Therefore, 15 bags of seed is required for the area.

**ANSWER:**
15

64. **SAT/ACT** The product of two integers is between 107 and 116. Which of the following cannot be one of the integers?

A 5  
B 10  
C 12  
D 15  
E 23

**SOLUTION:**
Identify the number which does not have multiples between 107 and 116.

The numbers 5 and 10 have 110 as a multiple, and 12 has 108 as a multiple. But 15 does not have a multiple between 107 and 116. So, it cannot be one of the numbers. Therefore, the correct choice is D.

**ANSWER:**
D
4-8 Quadratic Inequalities

65. PROBABILITY  Five students are to be arranged side by side with the tallest student in the center and the two shortest students on the ends. If no two students are the same height, how many different arrangements are possible?

F 2
G 4
H 5
J 6

SOLUTION:
There are two choices for the first place, two for the second, and one choice for the middle position. So, the total number of arrangements is 4. Therefore, the correct choice is G.

ANSWER:  G

66. SHORT RESPONSE  Simplify \( \frac{5 + i}{6 - 3i} \).

SOLUTION:
\[
\frac{5 + i}{6 - 3i} = \frac{5 + i}{6 - 3i} \cdot \frac{6 + 3i}{6 + 3i}
\]
\[
= \frac{30 + 15i + 6i + 3i^2}{36 + 18i - 18i - 9i^2}
\]
\[
= \frac{30 + 21i - 3}{36 + 9}
\]
\[
= \frac{27 + 21i}{45}
\]
\[
= \frac{3}{5} + \frac{7}{15}i
\]

ANSWER:  \( \frac{3}{5} + \frac{7}{15}i \)

Write an equation in vertex form for each parabola.

67.

SOLUTION:
The vertex of the parabola is at (3, -4), so \( h = 3 \) and \( k = -4 \).
Since (5, 4) is a point on the parabola, let \( x = 5 \) and \( y = 4 \).
Substitute these values into the vertex form of the equation and solve for \( a \).

\[
y = a(x - h)^2 + k
\]
\[
4 = a(5 - 3)^2 - 4
\]
\[
4 = 4a - 4
\]
\[
8 = 4a
\]
\[
2 = a
\]

The equation of the parabola in vertex form is

\[ y = 2(x - 3)^2 - 4 \]

ANSWER:
\[ y = 2(x - 3)^2 - 4 \]
4-8 Quadratic Inequalities

**SOLUTION:**
The vertex of the parabola is at (-2, 1), so \( h = -2 \) and \( k = 1 \).
Since \((-3, -2)\) is a point on the parabola, let \( x = -3 \) and \( y = -2 \).
Substitute these values into the vertex form of the equation and solve for \( a \).

\[
y = a(x - h)^2 + k
\]

\[
-2 = a(-3 - (-2))^2 + 1
-2 = a + 1
-3 = a
\]

The equation of the parabola in vertex form is

\[
y = -3(x + 2)^2 + 1
\]

**ANSWER:**
\[
y = -3(x + 2)^2 + 1
\]

**SOLUTION:**
The vertex of the parabola is at \((-4, 3)\), so \( h = -4 \) and \( k = 3 \).
Since \((-8, 7)\) is a point on the parabola, let \( x = -8 \) and \( y = 7 \).
Substitute these values into the vertex form of the equation and solve for \( a \).

\[
y = a(x - h)^2 + k
\]

\[
7 = a(-8 - (-4))^2 + 3
7 = 16a + 3
4 = 16a
0.25 = a
\]

The equation of the parabola in vertex form is

\[
y = 0.25(x + 4)^2 + 3
\]

**ANSWER:**
\[
y = 0.25(x + 4)^2 + 3
\]
4-8 Quadratic Inequalities

Complete parts a and b for each quadratic equation

a. Find the value of the discriminant.

b. Describe the number and type of roots.

70. \(4x^2 + 7x - 3 = 0\)

\textit{SOLUTION:}
\begin{enumerate}
\item[a.] Identify \(a\), \(b\), and \(c\) from the equation.
\[a = 4, b = 7 \text{ and } c = -3.\]
\item[b.] The discriminant is not a perfect square, so there are two irrational roots.
\end{enumerate}

\textit{ANSWER:}
\(97; 2\) irrational roots

71. \(-3x^2 + 2x - 4 = 9\)

\textit{SOLUTION:}
\begin{enumerate}
\item[a.] Write the equation in the form \(ax^2 + bx + c = 0\) and identify \(a\), \(b\), and \(c\).
\[\begin{align*}
-3x^2 + 2x - 4 & = 9 \\
-3x^2 + 2x - 13 & = 0
\end{align*}\]
\[a = -3, b = 2\text{ and } c = -13.\]
\item[b.] The discriminant is a negative, so there are two complex roots.
\end{enumerate}

\textit{ANSWER:}
\(-152; 2\) complex roots
4-8 Quadratic Inequalities

72. \( 6x^2 + x - 4 = 12 \)

**SOLUTION:**

a. Write the equation in the form \( ax^2 + bx + c = 0 \) and identify \( a, b, \) and \( c. \)

\[ 6x^2 + x - 4 = 12 \]
\[ 6x^2 + x - 16 = 0 \]

\( a = 6, b = 1 \) and \( c = -16. \)

Substitute these values in \( b^2 - 4ac \) and simplify.

\[ b^2 - 4ac = 1^2 - 4(6)(-16) \]
\[ = 1 + 384 \]
\[ = 385 \]

b. The discriminant is not a perfect square, so there are two irrational roots.

**ANSWER:**

385; 2 irrational roots

Perform the indicated operation. If the matrix does not exist, write *impossible.*

73. \[
\begin{bmatrix}
3 & -6 \\
-5 & 2
\end{bmatrix} - 3 \begin{bmatrix}
4 & -1 \\
-2 & 8
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
3 & -6 \\
-5 & 2
\end{bmatrix} - 3 \begin{bmatrix}
4 & -1 \\
-2 & 8
\end{bmatrix} = \begin{bmatrix}
12 & -24 \\
-20 & 8
\end{bmatrix} - \begin{bmatrix}
12 & -3 \\
-6 & 24
\end{bmatrix}
\]
\[
= \begin{bmatrix}
0 & -21 \\
-14 & -16
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
0 & -21 \\
-14 & -16
\end{bmatrix}
\]

74. \[
\begin{bmatrix}
5 & -9 \\
5 & 11
\end{bmatrix} - 6 \begin{bmatrix}
3 & -7 \\
-5 & 8
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
5 & -9 \\
5 & 11
\end{bmatrix} - 6 \begin{bmatrix}
3 & -7 \\
-5 & 8
\end{bmatrix} = \begin{bmatrix}
-10 & 18 \\
-10 & -22
\end{bmatrix} - \begin{bmatrix}
18 & -42 \\
-30 & 48
\end{bmatrix}
\]
\[
= \begin{bmatrix}
-28 & 60 \\
20 & -70
\end{bmatrix}
\]

75. \[
\begin{bmatrix}
2 & -6 \\
-4 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-1 & 1 \\
6 & 4
\end{bmatrix}
\]

**SOLUTION:**

\[
\begin{bmatrix}
2 & -6 \\
-4 & 6
\end{bmatrix} \cdot \begin{bmatrix}
-1 & 1 \\
6 & 4
\end{bmatrix} = \begin{bmatrix}
4 & -10 \\
-14 & 20
\end{bmatrix}
\]

**ANSWER:**

\[
\begin{bmatrix}
10 & -38 & -22 \\
-14 & 40 & 20
\end{bmatrix}
\]

76. **EXERCISE** Refer to the graphic.

a. For each option, write an equation that represents the cost of belonging to the gym.

b. Graph the equations. Estimate the break-even
4-8 Quadratic Inequalities

point for the gym memberships.

c. Explain what the break-even point means.

d. If you plan to visit the gym at least once per week during the year, which option should you chose?

**SOLUTION:**

**a.** Let \( y \) be the cost of belonging to the gym. Let \( x \) be the number of visit to the gym.
For option 1, the equation that represents the cost of belonging to the gym is \( y = 400 \).
For option 2, the equation that represents the cost of belonging to the gym is \( y = 150 + 5x \).

**b.** Graph the equations in the same coordinate plane.

The break-even point for the gym memberships is (50,400).

**c.** It means that the options cost the same if you visit 50 times in a year.

**d.** Over a year’s time, the option 1: $400 per year would be more economical.

**ANSWER:**

**a.** \( y = 400 \); \( y = 150 + 5x \)

---

**b.**

Graph the equations in the same coordinate plane.

**c.** It means that the options cost the same if you visit 50 times in a year.

**d.** $400 per year

**Use the Distributive Property to find each product.**

77. \(-6(x - 4)\)

**SOLUTION:**

\[-6(x - 4) = -6x + 24\]

**ANSWER:**

\[-6x + 24\]

78. \(8(w + 3x)\)

**SOLUTION:**

\[8(w + 3x) = 8w + 24x\]

**ANSWER:**

\[8w + 24x\]
4-8 Quadratic Inequalities

79. \(-4(-2y + 3z)\)

**SOLUTION:**
\[-4(-2y + 3z) = 8y - 12z\]

**ANSWER:**
\[8y - 12z\]

80. \(-1(c - d)\)

**SOLUTION:**
\[-1(c - d) = -c + d\]
\[= d - c\]

**ANSWER:**
\[d - c\]

81. \(0.5(5x + 6y)\)

**SOLUTION:**
\[0.5(5x + 6y) = 2.5x + 3y\]

**ANSWER:**
\[2.5x + 3y\]

82. \(-3(-6y - 4z)\)

**SOLUTION:**
\[-3(-6y - 4z) = 18y + 12z\]

**ANSWER:**
\[18y + 12z\]
Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

1. \( f(x) = x^2 + 4x - 7 \)

**SOLUTION:**

a. Compare the function \( f(x) = x^2 + 4x - 7 \) with the standard form of the quadratic function.

Here, \( a = 1, b = 4 \) and \( c = -7 \).

The y-intercept is \(-7\).

The equation of the axis of symmetry is

\[
x = \frac{-b}{2a} = \frac{-4}{2(1)} = -2.
\]

Therefore, the axis of symmetry is \( x = -2 \).

The x-coordinate of the vertex is \( -\frac{b}{2a} = -2 \).

b. Substitute \(-4, -3, -2, -1\) and \(0\) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-7</td>
</tr>
<tr>
<td>-3</td>
<td>-10</td>
</tr>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>-1</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>-7</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. y-intercept: \(-7\); axis of symmetry: \( x = -2 \);

x-coordinate of vertex: \(-2\)

b.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-7</td>
</tr>
<tr>
<td>-3</td>
<td>-10</td>
</tr>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>-1</td>
<td>-10</td>
</tr>
<tr>
<td>0</td>
<td>-7</td>
</tr>
</tbody>
</table>

c.

2. \( f(x) = -2x^2 + 5x \)

**SOLUTION:**

a. Compare the function \( f(x) = -2x^2 + 5x \) with the standard form of the quadratic function.

Here, \( a = -2, b = 5 \) and \( c = 0 \).

The y-intercept is \(0\).

The equation of the axis of symmetry is

\[
x = \frac{-b}{2a} = \frac{-5}{2(-2)} = \frac{5}{4}.
\]

Therefore, the axis of symmetry is \( x = \frac{5}{4} \).

The x-coordinate of the vertex is \( -\frac{b}{2a} = \frac{5}{4} \).

b. Substitute \(0, 1, \frac{5}{4}, 2 \) and \(3\) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( \frac{5}{4} )</td>
<td>( \frac{25}{4} )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. y-intercept: \(0\); axis of symmetry: \( x = \frac{5}{4} \); x-coordinate of vertex: \( \frac{5}{4} \)
Complete parts a–c for each quadratic function.

3. \( f(x) = -x^2 - 6x - 9 \)

**SOLUTION:**

a. Compare the function \( f(x) = -x^2 - 6x - 9 \) with the standard form of the quadratic function. Here, \( a = -1, b = -6 \) and \( c = -9 \). The \( y \)-intercept is \( -9 \). The equation of the axis of symmetry is

\[
 x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -3.
\]
Therefore, the axis of symmetry is \( x = -3 \).

The \( x \)-coordinate of the vertex is \( \frac{b}{2a} = -3 \).

b. Substitute \(-5, -4, -3, -2 \) and \(-1\) for \( x \) and make the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td>(-4)</td>
</tr>
<tr>
<td>(-4)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(-3)</td>
<td>(0)</td>
</tr>
<tr>
<td>(-2)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(-1)</td>
<td>(-4)</td>
</tr>
</tbody>
</table>

c. Graph the function.

**ANSWER:**

a. \( y \)-intercept: \( -9 \); axis of symmetry: \( x = -3 \); \( x \)-coordinate of vertex: \( -3 \)

4. \( f(x) = x^2 + 10x + 25 \)

**SOLUTION:**

Compare the function \( f(x) = x^2 + 10x + 25 \) with the standard form of the quadratic function. Here, \( a = 1, b = 10 \) and \( c = 25 \). For this function, \( a > 0 \), so the graph opens up and the function has a minimum value. The \( x \)-coordinate of the vertex is

\[
 -\frac{b}{2a} = -\frac{10}{2(1)} = -5.
\]

Substitute \(-5\) for \( x \) in the function to find the \( y \)-coordinate of the vertex.

\[
f(-5) = (-5)^2 + 10(-5) + 25 = 25 - 50 + 25 = 0
\]

Therefore, the minimum value of the function is \( 0 \).

**ANSWER:**

min.; 0
5. \( f(x) = -x^2 + 6x \)

**SOLUTION:**

Compare the function \( f(x) = -x^2 + 6x \) with the standard form of the quadratic function. Here, \( a = -1, b = 6 \) and \( c = 0 \).

For this function, \( a = -1 \) so the graph opens down and the function has a maximum value.

The \( x \)-coordinate of the vertex is
\[
-\frac{b}{2a} = -\frac{6}{2(-1)} = 3.
\]

Substitute 3 for \( x \) in the function to find the \( y \)-coordinate of the vertex.
\[
f(3) = -(3)^2 + 6(3)
\]
\[
= -9 + 18
\]
\[
= 9
\]

Therefore, the maximum value of the function is 9.

**ANSWER:**

max.; 9

Solve each equation using the method of your choice. Find exact solutions.

6. \( x^2 - 8x - 9 = 0 \)

**SOLUTION:**

\[
x^2 - 8x - 9 = 0
\]
\[
(x - 9)(x + 1) = 0
\]

\[x + 1 = 0 \text{ or } x - 9 = 0\]

\[x = -1 \text{ or } x = 9\]

**ANSWER:**

-1, 9

7. \( -4.8x^2 + 1.6x + 24 = 0 \)

**SOLUTION:**

\[
x = \frac{-1.6 \pm \sqrt{(1.6)^2 - 4(-4.8)(24)}}{2(-4.8)}
\]
\[
= \frac{-1.6 \pm \sqrt{2.56 + 460.8}}{-9.6}
\]
\[
= \frac{-1.6 \pm \sqrt{463.36}}{-9.6}
\]
\[
= \frac{-1.6 \pm 21.52}{-9.6}
\]
\[
= \frac{-1.6(1 \pm \sqrt{181})}{-9.6}
\]
\[
= \frac{1 \pm \sqrt{181}}{6}
\]

**ANSWER:**

\[\frac{1 \pm \sqrt{181}}{6}\]

8. \( 12x^2 + 15x - 4 = 0 \)

**SOLUTION:**

\[
x = \frac{-15 \pm \sqrt{15^2 - 4(12)(-4)}}{2(12)}
\]
\[
= \frac{-15 \pm \sqrt{225 + 192}}{24}
\]
\[
= \frac{-15 \pm \sqrt{417}}{24}
\]

**ANSWER:**

\[\frac{-15 \pm \sqrt{417}}{24}\]
9. \( x^2 - 7x - \frac{17}{4} = 0 \)

**SOLUTION:**
\[
x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-\frac{17}{4})}}{2(1)}
\]
\[
= \frac{7 \pm \sqrt{49 + 17}}{2}
\]
\[
= \frac{7 \pm \sqrt{66}}{2}
\]
**ANSWER:**
\[
\frac{7 \pm \sqrt{66}}{2}
\]

10. \( 4x^2 + x = 3 \)

**SOLUTION:**
\[
4x^2 + x - 3 = 3 - 3
\]
\[
4x^2 + x - 3 = 0
\]
\[
x = \frac{-(1) \pm \sqrt{(1)^2 - 4(4)(-3)}}{2(4)}
\]
\[
= \frac{-1 \pm \sqrt{1 + 48}}{8}
\]
\[
= \frac{-1 \pm \sqrt{49}}{8}
\]
\[
= \frac{-1 \pm 7}{8}
\]
\[
x = -1, \frac{3}{4}
\]
**ANSWER:**
\[
-1, \frac{3}{4}
\]

11. \( -9x^2 + 40x + 84 = 0 \)

**SOLUTION:**
\[
x = \frac{-40 \pm \sqrt{40^2 - 4(-9)(84)}}{2(-9)}
\]
\[
= \frac{-40 \pm \sqrt{1600 + 3024}}{-18}
\]
\[
= \frac{-40 \pm 68}{-18}
\]
\[
x = 6, -\frac{14}{9}
\]
**ANSWER:**
\[
-\frac{14}{9}, 6
\]

12. **PHYSICAL SCIENCE** Parker throws a ball off the top of a building. The building is 350 feet high and the initial velocity of the ball is 30 feet per second. Find out how long it will take the ball to hit the ground by solving the equation \( -16t^2 - 30t + 350 = 0 \).

**SOLUTION:**
Substitute 0 for \( h(t) \) and find the roots.
\[
-16t^2 - 30t + 350 = 0
\]
\[
t = \frac{-(30) \pm \sqrt{(-30)^2 - 4(-16)(350)}}{2(-16)}
\]
\[
= \frac{30 \pm \sqrt{23300}}{-32}
\]
\[
t = 3.8 \text{ or } t = -5.71
\]
The value of \( t \) should be positive. Therefore, the ball will reach the ground in about 3.83 s.

**ANSWER:**
about 3.83 seconds
13. **MULTIPLE CHOICE** Which equation below has roots at $-6$ and $\frac{1}{5}$?

A $0 = 5x^2 - 29x - 6$
B $0 = 5x^2 + 31x + 6$
C $0 = 5x^2 + 29x - 6$
D $0 = 5x^2 - 31x + 6$

**SOLUTION:**
Reorder the roots of the equation $0 = 5x^2 - 29x - 6$ are $-\frac{1}{5}$ and $6$.
Reorder the roots of the equation $0 = 5x^2 + 31x + 6$ are $-\frac{1}{5}$ and $-6$.
Reorder the roots of the equation $0 = 5x^2 + 29x - 6$ are $\frac{1}{5}$ and $-6$.
Reorder the roots of the equation $0 = 5x^2 - 31x + 6$ are $\frac{1}{5}$ and $6$.

**ANSWER:**
C

14. **PHYSICS** A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground $t$ seconds after release is modeled by $h(t) = -16t^2 + 112t + 6$.

a. When will the ball reach 130 feet?
b. Will the ball ever reach 250 feet? Explain.
c. In how many seconds after its release will the ball hit the ground?

**SOLUTION:**
a. Substitute 130 for $h(t)$ and find the roots of the equation.

$$-16t^2 + 112t + 6 = 130$$

$$-16t^2 + 112t - 124 = 0$$

$$t = \frac{-112 \pm \sqrt{112^2 - 4(-16)(-124)}}{2(-16)}$$

$$t = \frac{-112 \pm \sqrt{4608}}{-32}$$

$$t = 5.6 \text{ or } t = 1.4$$

The ball will reach 130 feet at about 1.4 s and 5.6 s.

b. No; if you graph the function, the vertex is 202 units above the horizontal axis. So, the height will never be 250.

c. Substitute 0 for $h(t)$ and find the roots.

$$-16t^2 + 112t + 6 = 0$$

$$t = \frac{-112 \pm \sqrt{112^2 - 4(-16)(6)}}{2(-16)}$$

$$t = \frac{-112 \pm \sqrt{12928}}{-32}$$

$$t \approx 7 \text{ or } t = -0.05$$

The value of $t$ should be positive. Therefore, the ball will reach the ground in about 7 s.

**ANSWER:**
a. at about 1.4 s and 5.6 s
b. No; if you graph the function, the vertex is 202 units above the horizontal axis. So, the height will never be 250.
c. in about 7 s
15. The rectangle below has an area of 104 square inches. Find the value of \( x \) and the dimensions of the rectangle.

\[
\begin{array}{c}
\text{Area: } A = 104 \text{ in}^2 \\
\text{x-intercept: } x = 1 \\
\text{y-intercept: } x = 4
\end{array}
\]

**SOLUTION:**

Area of the given rectangle is \( (x + 4)(x - 1) \).

\[
\begin{align*}
(x - 1)(x + 4) &= 104 \\
x^2 + 3x - 4 &= 104 \\
x^2 + 3x - 108 &= 0 \\
(x + 12)(x - 9) &= 0
\end{align*}
\]

Therefore, \( x = -12 \) or \( x = 9 \).

The value of \( x \) should be positive.

Therefore, \( x = 9 \).

The dimensions of the rectangle are 8 inches by 13 inches.

**ANSWER:**

\( x = 9; \) 8 inches by 13 inches

16. \( (3 - 4i) - (9 - 5i) \)

**SOLUTION:**

\[
(3 - 4i) - (9 - 5i) = 3 - 4i - 9 + 5i
\]

\[= -6 + i\]

**ANSWER:**

\(-6 + i\)

17. \( \frac{4i}{4 - i} \)

**SOLUTION:**

\[
\begin{align*}
\frac{4i}{4 - i} &= \frac{4i(4 + i)}{(4 - i)(4 + i)} \\
&= \frac{16i + 4i^2}{4^2 - (-i)^2} \\
&= \frac{16i + 4(-1)}{16 - (-1)} \\
&= \frac{-4 + 16i}{17} \\
&= -\frac{4}{17} + \frac{16i}{17}
\end{align*}
\]

**ANSWER:**

\(-\frac{4}{17} + \frac{16i}{17}\)

18. **MULTIPLE CHOICE** Which value of \( c \) makes the trinomial \( x^2 - 12x + c \) a perfect square trinomial?

\( \text{F} \) 6 \\
\( \text{G} \) 12 \\
\( \text{H} \) 36 \\
\( \text{J} \) 144

**SOLUTION:**

To make the given trinomial as a perfect square, add the square of half of the coefficient of \( x \).

Square of half of the coefficient of \( x \) is \( \left(\frac{-12}{2}\right)^2 = 36 \)

Therefore, option \( \text{H} \) is the correct answer.

**ANSWER:**

\( \text{H} \)
Complete parts a–c for each quadratic function.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solution by using the Quadratic Formula.

19. $6x^2 + 7x = 0$

**SOLUTION:**
a. Compare the equation with the standard quadratic equation.
Here, $a = 6$, $b = 7$ and $c = 0$.

\[b^2 - 4ac = 7^2 - 4(6)(0)\]

\[= 49\]
b. The value of the discriminant is positive and a perfect square. So, the equation has two rational roots.

c.  

\[6x^2 + 7x = 0\]

\[x(6x + 7) = 0\]

\[6x + 7 = 0\] or \[x = 0\]

\[x = -\frac{7}{6}\] or \[x = 0\]

**ANSWER:**
a. 49
b. 2 rational roots
c. $-\frac{7}{6}, 0$

20. $5x^2 = -6x + 1$

**SOLUTION:**
a. 

\[5x^2 = -6x + 1\]

\[5x^2 + 6x - 1 = 0\]

Compare the equation with the standard quadratic equation.
Here, $a = 5$, $b = 6$ and $c = -1$.

\[b^2 - 4ac = 6^2 - 4(5)(-1)\]

\[= 36 + 20\]

\[= 56\]
b. The value of the discriminant is positive and not a perfect square. So, the equation has two irrational roots.

c.  

\[x = \frac{-6 \pm \sqrt{6^2 - 4(5)(-1)}}{2(5)}\]

\[= \frac{-6 \pm \sqrt{56}}{10}\]

\[= \frac{-3 \pm \sqrt{14}}{5}\]

**ANSWER:**
a. 56
b. 2 irrational roots
c. $\frac{-3 \pm \sqrt{14}}{5}$
21. \(2x^2 + 5x - 8 = -13\)

**SOLUTION:**

\[
\begin{align*}
2x^2 + 5x - 8 &= -13 \\
2x^2 + 5x + 5 &= 0 \\
\text{Compare the equation with the standard quadratic equation.} \\
\text{Here, } a = 2, b = 5 \text{ and } c = 5.
\end{align*}
\]

\[
\begin{align*}
b^2 - 4ac &= 5^2 - 4(2)(5) \\
&= 25 - 40 \\
&= -15
\end{align*}
\]

b. The value of the discriminant is negative. So, the equation has two complex roots.

c. \[
x = \frac{-5 \pm \sqrt{5^2 - 4(2)(5)}}{2(5)} \\
= \frac{-5 \pm \sqrt{-15}}{10} \\
= \frac{-5 \pm i\sqrt{15}}{10}
\]

**ANSWER:**

a. 15

b. 2 complex roots

c. \(\frac{-5 \pm i\sqrt{15}}{4}\)

---

22. \(3x^2 + 6x = 2 + y\)

**SOLUTION:**

\[
\begin{align*}
3x^2 + 6x &= 2 + y \\
3(x^2 + 2x) &= 2 + y \\
3(x^2 + 2x + 1) - 3 &= 2 + y \\
3(x + 1)^2 - 3 &= 2 + y \\
3(x + 1)^2 - 5 &= y
\end{align*}
\]

The vertex of the function is \((-1, 5)\).
The axis of the symmetry is \(x = -1\).

Since the coefficient of the \(x^2\) is positive the graph opens upwards.

**ANSWER:**

\[
y = 3(x + 1)^2 - 5; \text{ vertex: } (-1, -5); \text{ axis of symmetry: } x = -1; \text{ opens up}
\]

---

23. \(x^2 + 9x + \frac{81}{4} = y\)

**SOLUTION:**

\[
\begin{align*}
x^2 + 9x + \frac{81}{4} &= y \\
\left( x + \frac{9}{2} \right)^2 &= y
\end{align*}
\]

The vertex of the function is \(\left( -\frac{9}{2}, 0 \right)\).
The axis of the symmetry is \(x = -\frac{9}{2}\).

Since the coefficient of the term \(x^2\) is positive the graph opens upwards.

**ANSWER:**

\[
y = \left( x + \frac{9}{2} \right)^2; \text{ vertex: } \left( -\frac{9}{2}, 0 \right); \text{ axis of symmetry: } x = -\frac{9}{2}; \text{ opens up}
\]
24. Graph the quadratic inequality \(0 < -3x^2 + 4x + 10\).

**SOLUTION:**
First graph the related equation. The parabola should be dashed. Next choose a test point such as \((0, 0)\) and determine if that is a solution to the inequality.
\[
f'(x) < -3x^2 + 4x + 10
\]
\[
0 < -3(0)^2 + 4(0) + 10
\]
\[
0 < 10
\]
The solution of the inequality contains \((0, 0)\). Shade the region of the graph that contains this point.

**ANSWER:**

25. \(x^2 + 6x > -5\)

**SOLUTION:**
\[
x^2 + 6x > -5
\]
\[
x^2 + 6x + 5 \geq -5 + 5
\]
\[
x^2 + 6x + 5 > 0
\]
\[
0 < x^2 + 6x + 5
\]
Graph the inequality.

Therefore, the solution is \(\{x \mid x < -5 \text{ or } x > -1\}\).

**ANSWER:**
\[\{x \mid x < -5 \text{ or } x > -1\}\]

26. \(4x^2 - 19x \leq -12\)

**SOLUTION:**
\[
4x^2 - 19x \leq -12
\]
\[
4x^2 - 19x + 12 \leq -12 + 12
\]
\[
4x^2 - 19x + 12 \leq 0
\]
Graph the inequality.

Therefore, the solution is \(\left\{x \mid \frac{3}{4} \leq x \leq 4 \right\}\).

**ANSWER:**
\[\left\{x \mid \frac{3}{4} \leq x \leq 4 \right\}\]
State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. The factored form of a quadratic equation is \(ax^2 + bx + c = 0\) where \(a \neq 0\) and \(a, b,\) and \(c\) are integers.

**SOLUTION:**
False, standard form

**ANSWER:**
false, standard form

2. The graph of a quadratic function is called a parabola.

**SOLUTION:**
true

**ANSWER:**
true

3. The vertex form of a quadratic function is \(y = a(x - p)(x - q)\).

**SOLUTION:**
false

**ANSWER:**
false, factored form

4. The axis of symmetry will intersect a parabola in one point called the vertex.

**SOLUTION:**
true

**ANSWER:**
true

5. A method called FOIL method is used to make a quadratic expression a perfect square in order to solve the related equation.

**SOLUTION:**
false, completing the square

**ANSWER:**
false, completing the square

6. The equation \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\) is known as the discriminant.

**SOLUTION:**
False, Quadratic Formula

**ANSWER:**
false, Quadratic Formula
7. The number $6i$ is called a **pure imaginary number**.

**SOLUTION:**
true

**ANSWER:**
true

8. The two numbers $2 + 3i$ and $2 - 3i$ are called **complex conjugates**.

**SOLUTION:**
true

**ANSWER:**
true

Complete parts a–c for each quadratic function.

a. Find the $y$-intercept, the equation of the axis of symmetry, and the $x$-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

9. $f(x) = x^2 + 5x + 12$

**SOLUTION:**
a. To find the $y$-intercept, substitute $x = 0$ in the function. The $y$-intercept is 12.

$$f(x) = x^2 + 5x + 12 = \left(x + \frac{5}{2}\right)^2 + \frac{23}{4}$$

The $x$-coordinate of the vertex is $-\frac{5}{2}$.

The line of symmetry passes through the vertex. Therefore, the equation of the axis of symmetry is $x = -\frac{5}{2}$.

b. 

c. The graph of the quadratic function is:

**ANSWER:**

a. $y$-int: 12; $x = \frac{-5}{2}$, $\frac{-5}{2}$

b. 

c.
10. \( f(x) = x^2 - 7x + 15 \)

**SOLUTION:**

a. To find the y-intercept, substitute \( x = 0 \) in the function.

The y-intercept is 15.

The x-coordinate of the vertex is given by \( x = -\frac{b}{2a} \).

So:

\[
x = \frac{7}{2}
\]

The line of symmetry passes through the vertex.

Therefore, the equation of the axis of symmetry is \( x = \frac{7}{2} \).

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7/2</td>
<td>11/4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

c. The graph of the quadratic function is:

\[ f(x) = x^2 - 7x + 15 \]

11. \( f(x) = -2x^2 + 9x - 5 \)

**SOLUTION:**

a. \( f(x) = -2x^2 + 9x - 5 \)

To find the y-intercept, substitute \( x = 0 \) in the function.

The y-intercept is -5.

The x-coordinate of the vertex is given by \( x = -\frac{b}{2a} \).

Substitute the values \( a \) and \( b \).

\[
x = \frac{9}{4}
\]

The line of symmetry passes through the vertex.

Therefore, the equation of the axis of symmetry is \( x = \frac{9}{4} \).

b.
**SOLUTION:**

a. To find the y-intercept, substitute $x = 0$ in the function.

The y-intercept is $-1$.

The $x$-coordinate of the vertex is given by $x = \frac{-b}{2a}$.

\[
x = -\frac{12}{2(-3)} = 2
\]

Equation of the axis of the symmetry is $x = 2$.

b. 

\[
\begin{array}{|c|c|}
\hline
x & f(x) \\
\hline
0 & -1 \\
1 & 8 \\
2 & 11 \\
3 & 8 \\
4 & -1 \\
\hline
\end{array}
\]

c. The graph of the quadratic function is:

\[
12. \ f(x) = -3x^2 + 12x - 1
\]
Determine whether each function has a maximum or minimum value and find the maximum or minimum value. Then state the domain and range of the function.

13. \( f(x) = -x^2 + 3x - 1 \)

**SOLUTION:**
The sign of \( a \) is negative so the function has a maximum value.

\[
f(x) = -x^2 + 3x - 1
\]

\[
= -(x^2 - 3x + 1)
\]

\[
= -(x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 1)
\]

\[
= -(x^2 - 3x + \frac{9}{4}) + \frac{9}{4} - 1
\]

\[
= -(x - \frac{3}{2})^2 + \frac{5}{4}
\]

The maximum value occurs at the vertex. The \( y \)-coordinate of the vertex is 1.25.

Therefore, the maximum value is 1.25.

Domain = \{all real numbers\}
Range = \{\( f(x) \) | \( f(x) \leq 1.25 \} \}

**ANSWER:**
max; 1.25; D = \{all real numbers\};

\( R = \{f(x) | f(x) \leq 1.25 \} \)

14. \( f(x) = -3x^2 - 4x + 5 \)

**SOLUTION:**
The sign of \( a \) is negative so the function has a maximum value.

\[
f(x) = -3x^2 - 4x + 5
\]

\[
= -3\left(x^2 + \frac{4}{3}x\right) + 5
\]

\[
= -3\left(x^2 + \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) + 5
\]

\[
= -3\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) + \frac{4}{9} + 5
\]

\[
= -3\left(x + \frac{2}{3}\right)^2 + \frac{19}{3}
\]

The maximum value occurs at the vertex. Substitute \( x = -\frac{2}{3} \) in \( f(x) = -3x^2 - 4x + 5 \).

\[
f\left(-\frac{2}{3}\right) = -3\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 5
\]

\[
= -3\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 5
\]

\[
= -3\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 5
\]

\[
= -\frac{4}{3} + \frac{8}{3} + 5
\]

\[
= -\frac{4}{3} + \frac{8}{3} + 5
\]

\[
= \frac{19}{3}
\]

So, the maximum value is \( \frac{19}{3} \).

Domain = \{all real numbers\}
Range = \{\( f(x) \) | \( f(x) \leq \frac{19}{3} \} \)

**ANSWER:**
max; \( \frac{19}{3} \); D = \{all real numbers\};
**Study Guide and Review - Chapter 4**

\[ R = \left\{ f(x) \mid f(x) \leq \frac{10}{3} \right\} \]

15. **BUSINESS** Sal’s Shirt Store sells 100 T-shirts per week at a rate of $10 per shirt. Sal estimates that he will sell 5 less shirts for each $1 increase in price. What price will maximize Sal’s T-shirt income?

**SOLUTION:**
Let \( x \) be the price increase, in dollars, and let \( P(x) \) be the total income.

\[ P(x) = (100 - 5x) \cdot (10 + x) \]
\[ = -5x^2 + 50x + 1000 \]

The maximum occurs at the vertex.

Use \( x = -\frac{b}{2a} \) to find the x-coordinate of the vertex.

\[ x = \frac{-50}{2(-5)} \]
\[ = \frac{50}{10} \]
\[ = 5 \]

The price should be increased $5 to $15.

Substitute 5 for \( x \) in \( 100 - 5x \).

\[ 100 - 5(5) = 75 \]

The store should sell 75 T-shirts at $15 each to maximize the income.

**ANSWER:**
75 T-shirts at $15 each

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

16. \( x^2 - x - 20 = 0 \)

**SOLUTION:**
Graph the function \( y = x^2 - x - 20 \).

The graph intersects the x-axis at -4 and 5.

So, the roots of the equation are \( x = -4 \) and \( x = 5 \).

**ANSWER:**
\( \{-4, 5\} \)
17. \(2x^2 - x - 3 = 0\)

**SOLUTION:**
Graph the function \(y = 2x^2 - x - 3\).

The graph intersects the \(x\)-axis at \(-1\) and \(\frac{3}{2}\).

So, the roots of the equation are \(x = -1\) and \(x = \frac{3}{2}\).

**ANSWER:**
\[\{-1, \frac{3}{2}\}\]

18. \(4x^2 - 6x - 15 = 0\)

**SOLUTION:**
Graph the function.

The graph intersects the \(x\)-axis between \(-2\) and \(-1\), and between 2 and 3. So the roots of the equation are between \(-2\) and \(-1\) and between 2 and 3.

**ANSWER:**
between \(-1\) and \(-2\); between 2 and 3
19. **BASEBALL** A baseball is hit upward at 120 feet per second. Use the formula \( h(t) = v_0t - 16t^2 \), where \( h(t) \) is the height of an object in feet, \( v_0 \) is the object’s initial velocity in feet per second, and \( t \) is the time in seconds. Ignoring the height of the ball when it was hit, how long does it take for the ball to hit the ground?

**SOLUTION:**

\[ h(t) = v_0t - 16t^2 \]

Substitute \( h = 0 \) and \( v_0 = 120 \) in the equation, and solve for \( t \).

\[
120t - 16t^2 = 0
\]

\[
8t(15 - 2t) = 0
\]

\[
t = 0 \quad \text{or} \quad t = 7.5
\]

Since \( t = 0 \) represents the time the when it was hit, \( t = 0 \) is inadmissible.

Therefore, it takes 7.5 seconds for the ball to hit the ground.

**ANSWER:**

7.5 seconds

Write a quadratic equation in standard form with the given roots.

20. 5, 6

**SOLUTION:**

\[
(x - 5)(x - 6) = 0
\]

\[
x^2 - 6x - 5x + 30 = 0
\]

\[
x^2 - 11x + 30 = 0
\]

**ANSWER:**

\[ x^2 - 11x + 30 = 0 \]

21. –3, –7

**SOLUTION:**

\[
(x + 3)(x + 7) = 0
\]

\[
x^2 + 7x + 3x + 21 = 0
\]

\[
x^2 + 10x + 21 = 0
\]

**ANSWER:**

\[ x^2 + 10x + 21 = 0 \]

22. –4, 2

**SOLUTION:**

\[
(x + 4)(x - 2) = 0
\]

\[
x^2 - 2x + 4x - 8 = 0
\]

\[
x^2 + 2x - 8 = 0
\]

**ANSWER:**

\[ x^2 + 2x - 8 = 0 \]

23. \(-\frac{2}{3}, 1\)

**SOLUTION:**

\[
\left(x + \frac{2}{3}\right)(x - 1) = 0
\]

\[
x^2 - x + \frac{2}{3}x - \frac{2}{3} = 0
\]

\[
x^2 - \frac{1}{3}x - \frac{2}{3} = 0
\]

\[
3x^2 - x - 2 = 0
\]

**ANSWER:**

\[ 3x^2 - x - 2 = 0 \]
24. \( \frac{1}{6}, 5 \)

**SOLUTION:**

\[
\left( x - \frac{1}{6} \right)(x - 5) = 0
\]

\[
x^2 - \frac{5x}{6} - \frac{x}{6} + \frac{5}{6} = 0
\]

\[
6x^2 - 31x + 5 = 0
\]

**ANSWER:**

\[
6x^2 - 31x + 5 = 0
\]

25. \(-\frac{1}{4}, -1\)

**SOLUTION:**

\[
\left( x + \frac{1}{4} \right)(x + 1) = 0
\]

\[
x^2 + x + \frac{1}{4}x + \frac{1}{4} = 0
\]

\[
4x^2 + 5x + 1 = 0
\]

**ANSWER:**

\[
4x^2 + 5x + 1 = 0
\]

---

**Solve each equation by factoring.**

26. \( 2x^2 - 2x - 24 = 0 \)

**SOLUTION:**

\[
2x^2 - 2x - 24 = 0
\]

\[
2x^2 - 8x + 6x - 24 = 0
\]

\[
2x(x - 4) + 6(x - 4) = 0
\]

\[
(x - 4)(2x + 6) = 0
\]

\[
x = 4 \text{ or } x = -3
\]

The solution set is \( \{-3, 4\} \).

**ANSWER:**

\( \{-3, 4\} \)

27. \( 2x^2 - 5x - 3 = 0 \)

**SOLUTION:**

\[
2x^2 - 5x - 3 = 0
\]

\[
2x^2 - 6x + x - 3 = 0
\]

\[
2x(x - 3) + (x - 3) = 0
\]

\[
(x - 3)(2x + 1) = 0
\]

\[
x = 3 \text{ or } x = -\frac{1}{2}
\]

The solution set is \( \left\{ -\frac{1}{2}, 3 \right\} \).

**ANSWER:**

\( \left\{ -\frac{1}{2}, 3 \right\} \)
28. \(3x^2 - 16x + 5 = 0\)

**SOLUTION:**
\[
3x^2 - 16x + 5 = 0 \\
3x^2 - 15x - x + 5 = 0 \\
3x(x - 5) - (x - 5) = 0 \\
(x - 5)(3x - 1) = 0
\]
\[x = 5 \text{ or } x = \frac{1}{3}\]

The solution set is \(\left\{\frac{1}{3}, 5\right\}\).

**ANSWER:**
\(\left\{\frac{1}{3}, 5\right\}\)

29. Find \(x\) and the dimensions of the rectangle below.

![Rectangle with area 126 ft²](image)

**SOLUTION:**
The area of a rectangle is given by \(A = l \times w\) where \(l\) is the length and \(w\) is the width.

\[(x + 2)(x - 3) = 126\]
\[x^2 - x - 6 = 126\]
\[x^2 - x - 132 = 0\]
\[(x - 12)(x + 11) = 0\]
\[x = 12 \text{ or } x = -11\]

\(x\) cannot be negative. Therefore, \(x = 12\).

The dimensions of the rectangle are 14 feet by 9 feet.

**ANSWER:**
\(x = 12\); 9 feet by 14 feet

30. \(\sqrt{-8}\)

**SOLUTION:**
\[
\sqrt{-8} = \sqrt{-1 \cdot 4 \cdot 2} \\
= \sqrt{1 \cdot 4 \cdot 2} \\
= 2i\sqrt{2}
\]

**ANSWER:**
\(2i\sqrt{2}\)
31. \( (2-i) + (13+4i) \)

**SOLUTION:**
Add the real and the imaginary parts separately.

\[ (2-i) + (13+4i) = 15 + 3i \]

**ANSWER:**
15 + 3i

32. \( (6+2i) - (4-3i) \)

**SOLUTION:**
Subtract the real and the imaginary parts separately.

\[ 6 + 2i - (4 - 3i) = 2 + 5i \]

**ANSWER:**
2 + 5i

33. \( (6+5i)(3-2i) \)

**SOLUTION:**
Multiply the complex numbers.

\[ (6+5i)(3-2i) = 18 - 12i + 15i - 10i^2 \]
\[ = 18 - 12i + 15i + 10(1) \]
\[ = 28 + 3i \]

**ANSWER:**
28 + 3i

34. **ELECTRICITY** The impedance in one part of a series circuit is \( 3 + 2j \) ohms, and the impedance in the other part of the circuit is \( 4 - 3j \) ohms. Add these complex numbers to find the total impedance in the circuit.

**SOLUTION:**
Add the complex numbers.

\[ (3 + 2j) + (4 - 3j) = 7 - j \]

The total impedance in the circuit is \( 7 - j \) ohms.

**ANSWER:**
7 - j ohms

35. \( 2x^2 + 50 = 0 \)

**SOLUTION:**
Solve for \( x \).

\[ 2x^2 + 50 = 0 \]
\[ x^2 = -25 \]
\[ x = \pm \sqrt{-25} \]
\[ x = \pm 5i \]

**ANSWER:**
\( \pm 5i \)
SOLUTION:
4x^2 + 16 = 0
\[ x^2 = -4 \]
\[ x = \pm \sqrt{-4} \]
\[ x = \pm 2i \]

ANSWER:
\[ \pm 2i \]

SOLUTION:
3x^2 + 15 = 0
\[ x^2 = -5 \]
\[ x = \pm \sqrt{-5} \]
\[ x = \pm i \sqrt{5} \]

ANSWER:
\[ \pm i \sqrt{5} \]

SOLUTION:
8x^2 + 16 = 0
\[ x^2 = -2 \]
\[ x = \pm \sqrt{-2} \]
\[ x = \pm i \sqrt{2} \]

ANSWER:
\[ \pm i \sqrt{2} \]

Find the value of \( c \) that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

SOLUTION:
To find the value of \( c \), divide the coefficient of \( x \) by 2, and square it.

\[ c = \left( \frac{18}{2} \right)^2 \]
\[ = 81 \]

Substitute the value of \( c \) in the trinomial.

\[ x^2 + 18x + 81 = (x + 9)^2 \]

ANSWER:
81; \((x + 9)^2\)
41. \( x^2 - 4x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{-4}{2} \right)^2 = 4
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 - 4x + 4 = (x - 2)^2
\]

**ANSWER:**
4; \((x - 2)^2\)

42. \( x^2 - 7x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{-7}{2} \right)^2 = \frac{49}{4}
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2
\]

**ANSWER:**
\(\frac{49}{4}; \left(x - \frac{7}{2}\right)^2\)

43. \( x^2 + 2.4x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{2.4}{2} \right)^2 = 1.44
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 + 2.4x + 1.44 = (x + 1.2)^2
\]

**ANSWER:**
1.44; \((x + 1.2)^2\)

44. \( x^2 - \frac{1}{2}x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square it.

\[
c = \left( \frac{-\frac{1}{2}}{2} \right)^2 = \frac{1}{16}
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2
\]

**ANSWER:**
\(\frac{1}{16}; \left(x - \frac{1}{4}\right)^2\)
45. \( x^2 + \frac{6}{5}x + c \)

**SOLUTION:**
To find the value of \( c \), divide the coefficient of \( x \) by 2 and square the result.

\[
c = \left( \frac{6}{10} \right)^2 = \frac{36}{100} = \frac{9}{25}
\]

Substitute the value of \( c \) in the trinomial.

\[
x^2 + \frac{6}{5}x + \frac{9}{25} = \left( x + \frac{3}{5} \right)^2
\]

**ANSWER:**
\[
\frac{9}{25} \left( x + \frac{3}{5} \right)^2
\]

Solve each equation by completing the square.

46. \( x^2 - 6x - 7 = 0 \)

**SOLUTION:**
\[
x^2 - 6x - 7 = 0
\]
\[
x^2 - 6x = 7
\]
\[
x^2 - 6x + (3)^2 = 7 + (3)^2
\]
\[
(x - 3)^2 = 16
\]
\[
x - 3 = \pm 4
\]
\[
x = 7 \quad \text{or} \quad x = -1
\]

The solution set is \( \{-1, 7\} \).

**ANSWER:**
\( \{-1, 7\} \)

47. \( x^3 - 2x + 8 = 0 \)

**SOLUTION:**
\[
x^3 - 2x + 8 = 0
\]
\[
x^2 - 2x = -8
\]
\[
x^2 - 2x + (1)^2 = -8 + (1)^2
\]
\[
(x - 1)^2 = -7
\]
\[
x - 1 = \pm i\sqrt{7}
\]
\[
x = 1 \pm i\sqrt{7}
\]

The solution set is \( \{1 + i\sqrt{7}, 1 - i\sqrt{7}\} \).

**ANSWER:**
\( \{1 + i\sqrt{7}\} \)
48. \(2x^2 + 4x - 3 = 0\)

**SOLUTION:**

\[\begin{align*}
2x^2 + 4x - 3 &= 0 \\
x^2 + 2x &= \frac{3}{2} \\
x^2 + 2x + 1 &= \frac{3}{2} + 1 \\
(x + 1)^2 &= \frac{5}{2} \\
x + 1 &= \pm \sqrt{\frac{5}{2}} \\
x &= -1 \pm \sqrt{\frac{5}{2}} \\
&= -1 \pm \frac{\sqrt{10}}{2} \\
&= \frac{-2 \pm \sqrt{10}}{2} 
\end{align*}\]

The solution set is \(\left\{ \frac{-2 + \sqrt{10}}{2}, \frac{-2 - \sqrt{10}}{2} \right\}\).

**ANSWER:**

\(\left\{ \frac{-2 + \sqrt{10}}{2}, \frac{-2 - \sqrt{10}}{2} \right\}\)

49. \(2x^2 + 3x - 5 = 0\)

**SOLUTION:**

\[\begin{align*}
2x^2 + 3x - 5 &= 0 \\
x^2 + \frac{3}{2}x &= \frac{5}{2} \\
x^2 + \frac{3}{2}x + \frac{9}{16} &= \frac{5}{2} + \frac{9}{16} \\
\left(x + \frac{3}{4}\right)^2 &= \frac{49}{16} \\
x + \frac{3}{4} &= \pm \frac{7}{4} \\
&= 1 \text{ or } x = -\frac{5}{2} 
\end{align*}\]

**ANSWER:**

\(\left\{ 1, -\frac{5}{2} \right\}\)
50. **FLOOR PLAN** Mario’s living room has a length 6 feet wider than the width. The area of the living room is 280 square feet. What are the dimensions of his living room?

**SOLUTION:**
Let x feet be the width of the room.

So, the length is x + 6 feet.

The area of a rectangle is given by \( A = l \times w \), where \( l \) is length and \( w \) is the width.

Therefore:
\[
x(x + 6) = 280
\]
\[
x^2 + 6x - 280 = 0
\]
\[
(x + 20)(x - 14) = 0
\]
\[
x = -20 \text{ or } x = 14
\]

\( x \) cannot be negative.
So, \( x = 14 \).

Therefore, the width of the room is 14 feet, and the length is 20 feet.

**ANSWER:**
20 feet by 14 feet

---

51. **Complete parts a–c for each quadratic equation.**

a. **Find the value of the discriminant.**

b. **Describe the number and type of roots.**

c. **Find the exact solutions by using the Quadratic Formula.**

\[ x^2 - 10x + 25 = 0 \]

**SOLUTION:**
\[
\text{a. Discriminant } = b^2 - 4ac
\]

Substitute the values of \( a, b, \) and \( c \).
\[
(-10)^2 - 4(1)(25) = 100 - 100
\]
\[
= 0
\]

b. Since the discriminant is zero, the quadratic equation has one real root.

\[
c. \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Substitute the values of \( a, b, \) and \( c \).
\[
x = \frac{10 \pm \sqrt{100 - 100}}{2}
\]
\[
= 5
\]

The solution set is \( \{-5\} \).

**ANSWER:**

a. 0

b. 1 real rational root

c. \{5\}
52. \(x^2 + 4x - 32 = 0\)

**SOLUTION:**

a. Discriminant = \(b^2 - 4ac\)
Substitute the values of \(a, b,\) and \(c.\)
\((4)^2 - 4(1)(-32) = 16 + 128\)
\[= 144\]

b. Since the discriminant is a perfect square, the quadratic equation has 2 rational real roots.

c. \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
Substitute the values of \(a, b,\) and \(c.\)
\[x = \frac{-4 \pm \sqrt{16 + 128}}{2}\]
\[= \frac{-4 \pm 12}{2}\]
\[x = 4\text{ or } -8\]

The solution set is \(-8, 4\).

**ANSWER:**

a. 144

b. 2 rational real roots

c. \(-8, 4\)

53. \(2x^2 + 3x - 18 = 0\)

**SOLUTION:**

a. Discriminant = \(b^2 - 4ac\)
Substitute the values of \(a, b,\) and \(c.\)
\((3)^2 - 4(2)(-18) = 9 + 144\)
\[= 153\]

b. Since the discriminant is greater than zero and not a perfect square, the quadratic equation has 2 irrational real roots.

c. \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
Substitute the values of \(a, b,\) and \(c.\)
\[x = \frac{-3 \pm \sqrt{153}}{4}\]
\[= \frac{-3 \pm 3\sqrt{17}}{4}\]

The solution set is \(-\frac{3 + 3\sqrt{17}}{4}, -\frac{3 - 3\sqrt{17}}{4}\).

**ANSWER:**

a. 153

b. 2 irrational real roots

c. \(-\frac{3 + 3\sqrt{17}}{4}\)
54. $2x^2 + 19x - 33 = 0$

**SOLUTION:**

a. Discriminant $= b^2 - 4ac$
Substitute the values of $a$, $b$, and $c$.

$(19)^2 - 4(2)(-33) = 361 + 264$

$= 625$

b. Since the discriminant is a perfect square, the quadratic equation has 2 rational real roots.

c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute the values of $a$, $b$, and $c$.

$x = \frac{-19 \pm \sqrt{625}}{4}$

$x = \frac{-19 \pm 25}{4}$

$x = \frac{3}{2}$ or $x = -11$

The solution set is $\left\{ -11, \frac{3}{2} \right\}$.

**ANSWER:**

a. 625

b. 2 real rational roots

c. $\left\{ -11, \frac{3}{2} \right\}$

55. $x^3 - 2x + 9 = 0$

**SOLUTION:**

a. Discriminant $= b^2 - 4ac$
Substitute the values of $a$, $b$, and $c$.

$(-2)^2 - 4(1)(9) = -32$

b. Since the discriminant is less than zero, the quadratic equation has 2 complex roots.

c. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute the values of $a$, $b$, and $c$.

$x = \frac{2 \pm \sqrt{-32}}{2}$

$x = \frac{2 \pm 4i\sqrt{2}}{2}$

$x = 1 \pm 2i\sqrt{2}$

The solution set is $\left\{ 1 - 2i\sqrt{2}, 1 + 2i\sqrt{2} \right\}$.

**ANSWER:**

a. $-32$

b. 2 complex roots

c. $\left\{ 1 \pm 2i\sqrt{2} \right\}$
56. \(4x^2 - 4x + 1 = 0\)

**SOLUTION:**

a. Discriminant = \(b^2 - 4ac\)
Substitute the values of \(a, b,\) and \(c.\)

\((-4)^2 - 4(4)(1) = 16 - 16\)
\[= 0\]

b. Since the discriminant is zero, the quadratic equation has 1 real rational root.

c. \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
Substitute the values of \(a, b,\) and \(c.\)

\[x = \frac{4 \pm 0}{8}\]
\[= \frac{1}{2}\]

The solution set is \(\left\{ \frac{1}{2} \right\}.\)

**ANSWER:**

a. 0

b. 1 real rational root

c. \(\left\{ \frac{1}{2} \right\}\)

57. \(2x^2 + 5x + 9 = 0\)

**SOLUTION:**

a. Discriminant = \(b^2 - 4ac\)
Substitute the values of \(a, b,\) and \(c.\)

\((5)^2 - 4(2)(9) = 25 - 72\)
\[= -47\]

b. Since the discriminant is less than zero, the quadratic equation has 2 complex roots.

c. \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
Substitute the values of \(a, b,\) and \(c.\)

\[x = \frac{-5 \pm \sqrt{-47}}{4}\]
\[= \frac{-5 \pm i\sqrt{47}}{4}\]

The solution set is \(\left\{ \frac{-5 + i\sqrt{47}}{4}, \frac{-5 - i\sqrt{47}}{4} \right\}.\)

**ANSWER:**

a. \(-47\)

b. 2 complex roots

c. \(\left\{ \frac{-5 \pm i\sqrt{47}}{4} \right\}\)
58. PHYSICAL SCIENCE Lauren throws a ball with an initial velocity of 40 feet per second. The equation for the height of the ball is \( h = -16t^2 + 40t + 5 \), where \( h \) represents the height in feet and \( t \) represents the time in seconds. When will the ball hit the ground?

**SOLUTION:**

\[ h = -16t^2 + 40t + 5 \]

Substitute \( h = 0 \) in the equation and solve for \( t \).

\[ -16t^2 + 40t + 5 = 0 \]

\[ t = \frac{-40 \pm \sqrt{(40)^2 - 4(-16)(5)}}{2(-16)} \]

\[ t = \frac{-40 \pm \sqrt{1600 + 320}}{-32} \]

\[ t = \frac{-40 \pm \sqrt{1920}}{-32} \]

\[ t = 0.12 \text{ or } t = 2.62 \]

Since \( t \) cannot be negative, \( t = 2.62 \). Therefore, it takes about 2.62 seconds to hit the ground.

**ANSWER:**

about 2.62 seconds

---

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

59. \( y = -3(x - 1)^2 + 5 \)

**SOLUTION:**

The equation is already in vertex form.

\( y = -3(x - 1)^2 + 5 \)

The vertex is at \((1, 5)\).

The line of symmetry is \( x = 1 \).

Here, \( a \), the coefficient of \( x^2 \), is less than zero. So, the graph opens down.

**ANSWER:**

\( y = -3(x - 1)^2 + 5; (1, 5); x = 1; \text{ opens down} \)
60. \( y = 2x^2 + 12x - 8 \)

**SOLUTION:**
\[
\begin{align*}
y & = 2x^2 + 12x - 8 \\
& = 2(x^2 + 6x) - 8 \\
& = 2(x^2 + 6x + 9 - 9) - 8 \\
& = 2(x + 3)^2 - 26
\end{align*}
\]
The vertex is at \((-3, -26)\).
The line of symmetry is \(x = -3\).

Here, \(a\), the coefficient of \(x^2\), is greater than zero. 
So, the graph opens up.

**ANSWER:**
\[
y = 2(x + 3)^2 - 26; (-3, -26); x = -3; \text{ opens up}
\]

61. \( y = -\frac{1}{2}x^2 - 2x + 12 \)

**SOLUTION:**
\[
\begin{align*}
y & = -\frac{1}{2}x^2 - 2x + 12 \\
& = -\frac{1}{2}(x^2 + 4x) + 12 \\
& = -\frac{1}{2}(x^2 + 4x + 4 - 4) + 12 \\
& = -\frac{1}{2}(x^2 + 4x + 4) + 2 + 12 \\
& = -\frac{1}{2}(x + 2)^2 + 14
\end{align*}
\]
The vertex is at \((-2, 14)\).
The line of symmetry is \(x = -2\).

Here, \(a\), the coefficient of \(x^2\), is less than zero. 
So, the graph opens down.

**ANSWER:**
\[
y = -\frac{1}{2}(x + 2)^2 + 14; (-2, 14); x = -2; \text{ opens down}
\]
62. \( y = 3x^2 + 36x + 25 \)

**SOLUTION:**

\[
\begin{align*}
y &= 3x^2 + 36x + 25 \\
&= 3(x^2 + 12x) + 25 \\
&= 3(x^2 + 12x + 36 - 36) + 25 \\
&= 3(x + 6)^2 - 83
\end{align*}
\]

The vertex is at \((-6, -83)\).
The line of symmetry is \(x = -6\).

Here, \(a\), the coefficient of \(x^2\), is greater than zero.
So, the graph opens up.

**ANSWER:**

\( y = 3(x + 6)^2 - 83; \ (-6, -83); \ x = 6; \) opens up

63. The graph at the right shows a product of 2 numbers with a sum of 10. Find a function that models this product and use it to determine the two numbers that would give a maximum product.

**SOLUTION:**

The standard form of a quadratic function is \(f(x) = ax^2 + bx + c\).
The graph intersects the \(x\)-axis at 0 and 10.

So:
\[c = 0\) and \(100a + 10b = 0\).

The vertex is at \((5, 25)\).

Therefore:
\[25a + 5b = 25\]

Solve the equations.
\(b = 10\) and \(a = -1\).

Substitute the values of \(a\), \(b\), and \(c\) in the function.
\[f(x) = -x^2 + 10x\]

The function \(f(x)\) is maximum when \(x = 5\).
So, one number should be 5.
The sum of the numbers is 10.
Therefore, the other number is 5.

**ANSWER:**

\[f(x) = -x^2 + 10x; \ 5 \text{ and } 5\]
Graph each quadratic inequality.

64. \( y \geq x^2 + 5x + 4 \)

**SOLUTION:**
Graph the related function \( y = x^2 + 5x + 4 \).

Since the inequality symbol is \( \geq \), the parabola should be solid.

Test the point \((-2, 2)\).
\[
2 \geq (-2)^2 + 5(-2) + 4
\]
\[
2 \geq -2 \quad \checkmark
\]
Shade the region that contains \((-2, 2)\).

**ANSWER:**

65. \( y < -x^2 + 5x - 6 \)

**SOLUTION:**
Graph the related function \( y = -x^2 + 5x - 6 \).

Since the inequality symbol is \(<\), the parabola should be dashed.

Test the point \((2, -4)\).
\[
-4 < -(2)^2 + 5(2) - 6
\]
\[
-4 < -4 + 10 - 6
\]
\[
-4 < 0 \quad \checkmark
\]
Shade the region that contains the point \((2, -4)\).
66. \( y > x^2 - 6x + 8 \)

**SOLUTION:**
Graph the related function \( y = x^2 - 6x + 8 \).

Since the inequality symbol is >, the parabola should be dashed.

Test the point (3, 2).

\[
\begin{align*}
2 & > (3)^2 - 6(3) + 8 \\
2 & > 9 - 18 + 8 \\
2 & > -1 \checkmark
\end{align*}
\]

Shade the region that contains the point (3, 2).

**ANSWER:**

67. \( y \leq x^2 + 10x - 4 \)

**SOLUTION:**
Graph the related function \( y = x^2 + 10x - 4 \).

Since the inequality symbol is \( \leq \), the parabola should be solid.

Test the point (0, 0).

\[
\begin{align*}
0 & \leq (0)^2 + 10(0) - 4 \\
0 & \leq -4 \times
\end{align*}
\]

Shade the region that does not contain the point (0, 0).

The graph of the quadratic inequality is:
68. Solomon wants to put a deck along two sides of his garden. The deck width will be the same on both sides and the total area of the garden and deck cannot exceed 500 square feet. How wide can the deck be?

![Diagram of a rectangle with dimensions labeled: 20 ft by x, 15 ft by x.]

**SOLUTION:**

The area of a rectangle is given by \( A = l \times w \), where \( l \) is the length and \( w \) is the width.

So:

\[
(20 + x)(15 + x) \leq 500
\]

Solve the inequality.

\[
x^2 + 35x + 300 \leq 500
\]
\[
x^2 + 35x - 200 \leq 0
\]
\[
(x + 40)(x - 5) \leq 0
\]

Since \( x \) should be positive, the solution set of the inequality is \( \{x|0 \leq x \leq 5\} \).

Therefore, the width of the deck should be between 0 and 5 feet.

**ANSWER:**

between 0 and 5 ft

---

**Solve each inequality using a graph or algebraically.**

69. \( x^2 + 8x + 12 > 0 \)

**SOLUTION:**

\[
x^2 + 8x + 12 = 0
\]
\[
(x + 2)(x + 6) = 0
\]
\[
x = -2 \quad \text{or} \quad x = -6
\]

Test one value of \( x \) less than \(-6\), one between \(-2\) and \(-6\), and one greater than \(-2\).

Test \(-7\).

\[
(-7)^2 + 8(-7) + 12 > 0
\]
\[
5 > 0 \quad \checkmark
\]

Test \(-4\).

\[
(-4)^2 + 8(-4) + 12 > 0
\]
\[
-4 > 0 \quad \times
\]

Test 0.

\[
(0)^2 + 8(0) + 12 > 0
\]
\[
12 > 0 \quad \checkmark
\]

Therefore, the solution set is \( \{x|x < -6 \quad \text{or} \quad x > -2\} \)

**ANSWER:**

\( \{x|x < -6 \quad \text{or} \quad x > -2\} \)
70. \(6x + x^2 \geq -9\)

**SOLUTION:**
\[
\begin{align*}
6x + x^2 & \geq -9 \\
x^2 + 6x + 9 & \geq 0 \\
(x + 3)^2 & \geq 0
\end{align*}
\]

The inequality is true for all \(x\).
So, the solution is the set of real numbers.

**ANSWER:**
all real numbers

71. \(2x^2 + 3x - 20 > 0\)

**SOLUTION:**
Solve the related equation \(2x^2 + 3x - 20 = 0\).

\[
x = \frac{-3 \pm \sqrt{9 + 160}}{4} = \frac{5}{2} \text{ or } x = -4
\]

Test one value of \(x\) less than \(-4\), one between \(-4\) and \(\frac{5}{2}\), and one greater than \(\frac{5}{2}\).

Test \(-5\).
\[
2(-5)^2 + 3(-5) - 20 > 0 \\
65 > 0 \quad \checkmark
\]

Test the point 0.
\[
2(0)^2 + 3(0) - 20 > 0 \\
-20 > 0 \quad \times
\]

Test the point 3.
\[
2(3)^2 + 3(3) - 20 > 0 \\
7 > 0 \quad \checkmark
\]

Therefore, the solution set is \(\left\{ x \mid x < -4 \text{ or } x > \frac{5}{2} \right\}\).

**ANSWER:**
\(\left\{ x \mid x < -4 \text{ or } x > \frac{5}{2} \right\}\)
72. \(4x^2 - 3 < -5x\)

**SOLUTION:**
Solve the related equation \(4x^2 + 5x - 3 = 0\).

\[
x = \frac{-5 \pm \sqrt{25 + 48}}{8} \quad \text{or} \quad x = 0.44 \quad \text{or} \quad x = -1.69
\]

Test one value of \(x\) less than \(-1.69\), one between \(-1.69\) and \(0.44\), and one greater than \(0.44\).

Test the point \(-2\).
\[
4(-2)^2 - 3 < -5(-2) \quad \Rightarrow \quad 13 < 10 \quad \times
\]

Test the point \(0\).
\[
4(0)^2 - 3 < -5(0) \quad \Rightarrow \quad -3 < 0 \quad \checkmark
\]

Test the point \(1\).
\[
4(1)^2 - 3 < -5(-1) \quad \Rightarrow \quad 1 < 5 \quad \times
\]

Therefore, the solution set is \(\{x \mid -1.69 < x < 0.44\}\).

**ANSWER:**
\(\{x \mid -1.69 < x < 0.44\}\)

73. \(3x^2 + 4 > 8x\)

**SOLUTION:**
Solve the related equation \(3x^2 - 8x + 4 = 0\).

\[
x = \frac{8 \pm \sqrt{64 - 48}}{6} \quad \Rightarrow \quad x = 2 \quad \text{or} \quad x = \frac{2}{3}
\]

Test one value of \(x\) less than \(\frac{2}{3}\), one between \(\frac{2}{3}\) and \(2\), and one greater than \(2\).

Test 0.
\[
3(0)^2 + 4 > 8(0) \quad \Rightarrow \quad 4 > 0 \quad \checkmark
\]

Test the point \(1\).
\[
3(1)^2 + 4 > 8(1) \quad \Rightarrow \quad 7 > 8 \quad \times
\]

Test the point 3.
\[
3(3)^2 + 4 > 8(3) \quad \Rightarrow \quad 31 > 24 \quad \checkmark
\]

Therefore, the solution set of the inequality is \(\{x \mid x < \frac{2}{3} \quad \text{or} \quad x > 2\}\).

**ANSWER:**
\(\{x \mid x < \frac{2}{3} \quad \text{or} \quad x > 2\}\)