6-1 Operations on Functions

Find \( f + g)(x), \ f - g)(x), \ (f \cdot g)(x), \) and \( \left( \frac{f}{g} \right)(x) \) for each \( f(x) \) and \( g(x) \). Indicate any restrictions in domain or range.

1. \( f(x) = x + 2 \)
   \( g(x) = 3x - 1 \)

   **SOLUTION:**
   \[
   (f + g)(x) = f(x) + g(x) = (x + 2) + (3x - 1) = 4x + 1
   \]
   \[
   (f - g)(x) = f(x) - g(x) = (x + 2) - (3x - 1) = -2x + 3
   \]
   \[
   (f \cdot g)(x) = f(x) \cdot g(x) = (x + 2) \cdot (3x - 1) = 3x^2 + 5x - 2
   \]
   \[
   \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0
   \]
   \[
   = \frac{x + 2}{3x - 1}, \ x \neq \frac{1}{3}
   \]

   **ANSWER:**
   \[
   (f + g)(x) = 4x + 1
   \]
   \[
   (f - g)(x) = -2x + 3
   \]
   \[
   (f \cdot g)(x) = 3x^2 + 5x - 2
   \]
   \[
   \left( \frac{f}{g} \right)(x) = \frac{x + 2}{3x - 1}, \ x \neq \frac{1}{3}
   \]

2. \( f(x) = x^2 - 5 \)
   \( g(x) = -x + 8 \)

   **SOLUTION:**
   \[
   (f + g)(x) = f(x) + g(x) = (x^2 - 5) + (-x + 8) = x^2 - x + 3
   \]
   \[
   (f - g)(x) = f(x) - g(x) = (x^2 - 5) - (-x + 8) = x^2 + x - 13
   \]
   \[
   (f \cdot g)(x) = f(x) \cdot g(x) = (x^2 - 5)(-x + 8) = -x^3 + 8x^2 + 5x - 40
   \]
   \[
   \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0
   \]
   \[
   = \frac{x^2 - 5}{-x + 8}, \ x \neq 8
   \]

   **ANSWER:**
   \[
   (f + g)(x) = x^2 - x + 3
   \]
   \[
   (f - g)(x) = x^2 + x - 13
   \]
   \[
   (f \cdot g)(x) = -x^3 + 8x^2 + 5x - 40
   \]
   \[
   \left( \frac{f}{g} \right)(x) = \frac{x^2 - 5}{-x + 8}, \ x \neq 8
   \]
6-1 Operations on Functions

For each pair of functions, find \( f \circ g \) and \( g \circ f \), if they exist. State the domain and range for each composed function.

3. \( f = \{(2, 5), (6, 10), (12, 9), (7, 6)\} \)
   \( g = \{(9, 11), (6, 15), (10, 13), (5, 8)\} \)

SOLUTION:
The range of \( g(x) \) is not a subset of the domain of \( f(x) \).
So, \( f \circ g \) is undefined.

\[ [g \circ f](x) = g[f(x)] \]

Therefore:
\( g[f(2)] = g(5) = 8 \)
\( g[f(6)] = g(10) = 13 \)
\( g[f(12)] = g(9) = 11 \)
\( g[f(7)] = g(6) = 15 \)

\( g \circ f = \{(2, 8), (6, 13), (12, 11), (7, 15)\} \)

ANSWER:
\( f \circ g \) is undefined;
\( g \circ f = \{(2, 8), (6, 13), (12, 11), (7, 15)\} \)

4. \( f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\} \)
   \( g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\} \)

SOLUTION:
The range of \( g(x) \) is not a subset of the domain of \( f(x) \).
So, \( f \circ g \) is undefined.

\[ [g \circ f](x) = g[f(x)] \]

Therefore:
\( g[f(0)] = g(-3) = 2 \)
\( g \circ f = \{(0, 2)\} \)

ANSWER:
\( f \circ g \) is undefined; \( g \circ f = \{(0, 2)\} \)
Find \( [f \circ g](x) \) and \( [g \circ f](x) \) if they exist. State the domain and range for each composed function.

5. \( f(x) = -3x \)  
   \( g(x) = 5x - 6 \)

**SOLUTION:**
\[
[f \circ g](x) = f[g(x)] \\
= f[5x - 6] \\
= -3(5x - 6) \\
= -15x + 18
\]
\[
[g \circ f](x) = g[f(x)] \\
= g[-3x] \\
= 5(-3x) - 6 \\
= -15x - 6
\]

**ANSWER:**
\[
[f \circ g](x) = -15x + 18 \\
[g \circ f](x) = -15x - 6
\]

6. \( f(x) = x + 4 \)  
   \( g(x) = x^2 + 3x - 10 \)

**SOLUTION:**
\[
[f \circ g](x) = f[g(x)] \\
= f[x^2 + 3x - 10] \\
= x^2 + 3x - 10 + 4 \\
= x^2 + 3x - 6
\]
\[
[g \circ f](x) = g[f(x)] \\
= g[x + 4] \\
= (x + 4)^2 + 3(x + 4) - 10 \\
= x^2 + 11x + 18
\]

**ANSWER:**
\[
[f \circ g](x) = x^2 + 3x - 6 \\
[g \circ f](x) = x^2 + 11x + 18
\]
7. **CCSS MODELING** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora’s tax rate is 17.5%. If her pay before taxes and deductions is $950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

**SOLUTION:**
Either way, she will have $228.95 taken from her paycheck. If she takes the college savings plan deduction before taxes, $76 will go to her college plan and $152.95 will go to taxes. If she takes the college savings plan deduction after taxes, only $62.70 will go to her college plan and $166.25 will go to taxes.

**ANSWER:**
Either way, she will have $228.95 taken from her paycheck. If she takes the college savings plan deduction before taxes, $76 will go to her college plan and $152.95 will go to taxes. If she takes the college savings plan deduction after taxes, only $62.70 will go to her college plan and $166.25 will go to taxes.

---

**6-1 Operations on Functions**

Find \((f + g)(x)\), \((f - g)(x)\), \((f \cdot g)(x)\), and \(\left(\frac{f}{g}\right)(x)\) for each \(f(x)\) and \(g(x)\). Indicate any restrictions in domain or range.

8. \(f(x) = 2x\), \(g(x) = -4x + 5\)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) = (2x) + (-4x + 5) = -2x + 5
\]
\[
(f - g)(x) = f(x) - g(x) = 2x - (-4x + 5) = 6x - 5
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) = (2x)(-4x + 5) = -8x^2 + 10x
\]
\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ } g(x) \neq 0 = \frac{2x}{-4x + 5}, x \neq \frac{5}{4}
\]

**ANSWER:**
\[
(f + g)(x) = -2x + 5
\]
\[
(f - g)(x) = 6x - 5
\]
\[
(f \cdot g)(x) = -8x^2 + 10x
\]
\[
\left(\frac{f}{g}\right)(x) = \frac{2x}{-4x + 5}, x \neq \frac{5}{4}
\]
9. \( f(x) = x - 1 \)
\( g(x) = 5x - 2 \)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) \\
= (x - 1) + (5x - 2) \\
= 6x - 3
\]
\[
(f - g)(x) = f(x) - g(x) \\
= (x - 1) - (5x - 2) \\
= -4x + 1
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (x - 1) \cdot (5x - 2) \\
= 5x^2 - 7x + 2
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} \cdot g(x) \neq 0 \\
= \frac{x - 1}{5x - 2}, x \neq \frac{2}{5}
\]

**ANSWER:**
\[
(f + g)(x) = 6x - 3
\]
\[
(f - g)(x) = -4x + 1
\]
\[
(f \cdot g)(x) = 5x^2 - 7x + 2
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{x - 1}{5x - 2}, x \neq \frac{2}{5}
\]

10. \( f(x) = x^2 \)
\( g(x) = -x + 1 \)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) \\
= x^2 - x + 1
\]
\[
(f - g)(x) = f(x) - g(x) \\
= x^2 + x - 1
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (x^2) \cdot (-x + 1) \\
= -x^3 + x^2
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \\
= \frac{x^2}{-x + 1}, x \neq 1
\]

**ANSWER:**
\[
(f + g)(x) = x^2 - x + 1
\]
\[
(f - g)(x) = x^2 + x - 1
\]
\[
(f \cdot g)(x) = -x^3 + x^2
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{x^2}{-x + 1}, x \neq 1
\]
11. \( f(x) = 3x \quad g(x) = -2x + 6 \)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) \\
= 3x - 2x + 6 \\
= x + 6
\]
\[
(f - g)(x) = f(x) - g(x) \\
= 3x + 2x - 6 \\
= 5x - 6
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (3x) \cdot (-2x + 6) \\
= -6x^2 + 18x
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0 \\
= \frac{3x}{-2x + 6}, \quad x \neq 3
\]

**ANSWER:**
\[
(f + g)(x) = x + 6 \\
(f - g)(x) = 5x - 6 \\
(f \cdot g)(x) = -6x^2 + 18x \\
\left( \frac{f}{g} \right)(x) = \frac{3x}{-2x + 6}, \quad x \neq 3
\]

12. \( f(x) = x - 2 \quad g(x) = 2x - 7 \)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) \\
= x - 2 + 2x - 7 \\
= 3x - 9
\]
\[
(f - g)(x) = f(x) - g(x) \\
= x - 2 - (2x - 7) \\
= -x + 5
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (x - 2) \cdot (2x - 7) \\
= 2x^2 - 11x + 14
\]
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0 \\
= \frac{x - 2}{2x - 7}, \quad x \neq \frac{7}{2}
\]

**ANSWER:**
\[
(f + g)(x) = 3x - 9 \\
(f - g)(x) = -x + 5 \\
(f \cdot g)(x) = 2x^2 - 11x + 14 \\
\left( \frac{f}{g} \right)(x) = \frac{x - 2}{2x - 7}, \quad x \neq \frac{7}{2}
\]
6-1 Operations on Functions

13. \( f(x) = x^2 \)
   \( g(x) = x - 5 \)

**SOLUTION:**

\[ (f + g)(x) = f(x) + g(x) \]
\[ = x^2 + x - 5 \]

\[ (f - g)(x) = f(x) - g(x) \]
\[ = x^2 - x + 5 \]

\[ (f \cdot g)(x) = f(x) \cdot g(x) \]
\[ = x^3 - 5x^2 \]

\[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \]
\[ = \frac{x^2}{x - 5}, x \neq 5 \]

**ANSWER:**

\( (f + g)(x) = x^2 + x - 5 \)

\( (f - g)(x) = x^2 - x + 5 \)

\( (f \cdot g)(x) = x^3 - 5x^2 \)

\( \left( \frac{f}{g} \right)(x) = \frac{x^2}{x - 5}, x \neq 5 \)

14. \( f(x) = -x^2 + 6 \)
   \( g(x) = 2x^2 + 3x - 5 \)

**SOLUTION:**

\[ (f + g)(x) = f(x) + g(x) \]
\[ = x^2 + 3x + 1 \]

\[ (f - g)(x) = f(x) - g(x) \]
\[ = -x^2 + 6 - (2x^2 + 3x - 5) \]
\[ = -3x^2 - 3x + 11 \]

\[ (f \cdot g)(x) = f(x) \cdot g(x) \]
\[ = (-x^2 + 6)(2x^2 + 3x - 5) \]
\[ = -2x^4 - 3x^3 + 17x^2 + 18x - 30 \]

\[ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \]
\[ = \frac{-x^2 + 6}{2x^2 + 3x - 5}, 2x^2 + 3x - 5 \neq 0 \]
\[ = \frac{-x^2 + 6}{2x^2 + 3x - 5}, x \neq 1 \text{ or } \frac{-5}{2} \]

**ANSWER:**

\( (f + g)(x) = x^2 + 3x + 1 \)

\( (f - g)(x) = -3x^2 - 3x + 11 \)

\( (f \cdot g)(x) = -2x^4 - 3x^3 + 17x^2 + 18x - 30 \)

\[ \left( \frac{f}{g} \right)(x) = -\frac{x^2 + 6}{2x^2 + 3x - 5}, x \neq 1 \text{ or } \frac{-5}{2} \]
6-1 Operations on Functions

15. \( f(x) = 3x^2 - 4 \)
\( g(x) = x^2 - 8x + 4 \)

**SOLUTION:**
\[
(f + g)(x) = f(x) + g(x) = 4x^2 - 8x
\]
\[
(f - g)(x) = f(x) - g(x) = 2x^2 + 8x - 8
\]
\[
(f \cdot g)(x) = f(x) \cdot g(x)
= (3x^3 - 4)(x^2 - 8x + 4)
= 3x^5 - 24x^3 + 8x^3 + 32x - 16
\]
\[
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0
= \frac{3x^2 - 4}{x^2 - 8x + 4}, \quad x^2 - 8x + 4 \neq 0
= \frac{3x^2 - 4}{x^2 - 8x + 4}, \quad x \neq 4 \pm 2\sqrt{3}
\]

**ANSWER:**
\[
(f + g)(x) = 4x^2 - 8x
\]
\[
(f - g)(x) = 2x^2 + 8x - 8
\]
\[
(f \cdot g)(x) = 3x^4 - 24x^3 + 8x^2 + 32x - 16
\]
\[
\left(\frac{f}{g}\right)(x) = \frac{3x^2 - 4}{x^2 - 8x + 4}, \quad x \neq 4 \pm 2\sqrt{3}
\]

16. **POPULATION** In a particular county, the population of the two largest cities can be modeled by \( f(x) = 200x + 25 \) and \( g(x) = 175x - 15 \), where \( x \) is the number of years since 2000 and the population is in thousands.

**a.** What is the population of the two cities combined after any number of years?

**b.** What is the difference in the populations of the two cities?

**SOLUTION:**

**a.** The population of the cities after \( x \) years is the sum of the individual populations.
\[
(f + g)(x) = f(x) + g(x)
= 200x + 25 + 175x - 15
= 375x + 10
\]

**b.** The difference in the populations of the cities is given by:
\[
(f - g)(x) = 200x + 25 - (175x - 15)
= 25x + 40
\]

**ANSWER:**
\[
(f + g)(x) = 375x + 10
\]
\[
(f - g)(x) = 25x + 40
\]
6-1 Operations on Functions

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

17. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
   $g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

**SOLUTION:**

$[f \circ g](x) = f[g(x)]$
Therefore:

$[f \circ g](-4) = f[g(-4)]$
$= f(0)$
$= 4$

$f \circ g = \{(-4, 4)\}$

$[g \circ f](x) = g[f(x)]$
Therefore:

$[g \circ f](-8) = g[f(-8)]$
$= g(-4)$
$= 0$

$[g \circ f](0) = g[f(0)]$
$= g(4)$
$= -4$

$[g \circ f](2) = g[f(2)]$
$= g(6)$
$= -1$

$[g \circ f](-6) = g[f(-6)]$
$= g(-2)$
$= -1$

$g \circ f = \{(-8, 0), (0, -4), (2, -5), (-6, -1)\}$

**ANSWER:**

$f \circ g = \{(-4, 4)\}; g \circ f = \{(-8, 0), (0, -4), (2, -5), (-6, -1)\}$
6-1 Operations on Functions

19. \( f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\} \)
\( g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\} \)

**SOLUTION:**
The range of \( g(x) \) is not a subset of the domain of \( f(x) \).
So, \( f \circ g \) is undefined.
The range of \( f(x) \) is not a subset of the domain of \( g(x) \).
So, \( g \circ f \) is undefined.

**ANSWER:**
\( f \circ g \) is undefined; \( g \circ f \) is undefined.

20. \( f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\} \)
\( g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\} \)

**SOLUTION:**
The range of \( g(x) \) is not a subset of the domain of \( f(x) \).
So, \( f \circ g \) is undefined.
\[ [g \circ f](x) = g[f(x)] \]
\[ [g \circ f](-4) = g[f(-4)] \]
\[ = g(-14) \]
\[ = 9 \]
\[ [g \circ f](0) = g[f(0)] \]
\[ = g(-6) \]
\[ = 1 \]
\[ [g \circ f](-6) = g[f(-6)] \]
\[ = g(-18) \]
\[ = 13 \]
\[ [g \circ f](2) = g[f(2)] \]
\[ = g(-2) \]
\[ = -3 \]
\( g \circ f = \{(-4, 9), (0, 1), (-6, 13), (2, -3)\} \)

**ANSWER:**
\( f \circ g \) is undefined; \( g \circ f = \{(-4, 9), (0, 1), (-6, 13), (2, -3)\} \)
21. \( f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\} \)
\( g = \{(3, -9), (7, 2), (8, -6), (12, 0)\} \)

**SOLUTION:**

The range of \( g(x) \) is not a subset of the domain of \( f(x) \).

So, \( f \circ g \) is undefined.

\[ [g \circ f](x) = g(f(x)) \]

\[ [g \circ f](-4) = g(f(-4)) \]
\[ = g(12) \]
\[ = 0 \]

\[ [g \circ f](1) = g(f(1)) \]
\[ = g(7) \]
\[ = 2 \]

\[ g \circ f = \{(-4, 0), (1, 2)\} \]

**ANSWER:**

\( f \circ g \) is undefined; \( g \circ f = \{(-4, 0), (1, 2)\} \).

22. \( f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\} \)
\( g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\} \)

**SOLUTION:**

\[ [f \circ g](x) = f[g(x)] \]

\[ [f \circ g](-1) = f[g(-1)] \]
\[ = f(2) \]
\[ = -2 \]

\[ f \circ g = \{(-1, -2)\} \]

The range of \( f(x) \) is not a subset of the domain of \( g(x) \).

So, \( g \circ f \) is undefined.

**ANSWER:**

\( f \circ g = \{(-1, -2)\}; g \circ f \) is undefined.

23. \( f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\} \)
\( g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\} \)

**SOLUTION:**

\[ [f \circ g](x) = f[g(x)] \]

\[ [f \circ g](4) = f[g(4)] \]
\[ = f(-3) \]
\[ = 6 \]
\[ [f \circ g](3) = f[g(3)] \]
\[ = f(-7) \]
\[ = -8 \]

\[ f \circ g = \{(4,6),(3,-8)\} \]

The range of \( f(x) \) is not a subset of the domain of \( g(x) \).

So, \( g \circ f \) is undefined.

**ANSWER:**

\( f \circ g = \{(4, 6), (3, -8)\}; g \circ f \) is undefined.

24. \( f = \{(1, -1), (2, -2), (3, -3), (4, -4)\} \)
\( g = \{(1, -4), (2, -3), (3, -2), (4, -1)\} \)

**SOLUTION:**

The range of \( g(x) \) is not a subset of the domain of \( f(x) \).

So, \( f \circ g \) is undefined.

The range of \( f(x) \) is not a subset of the domain of \( g(x) \).

So, \( g \circ f \) is undefined.

**ANSWER:**

\( f \circ g \) is undefined; \( g \circ f \) is undefined.
6-1 Operations on Functions

25. \( f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\} \)
   \( g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\} \)

**SOLUTION:**

\[
\begin{align*}
[f \circ g](x) &= f\left[g\left(x\right)\right] \\
[f \circ g](3) &= f\left[g\left(3\right)\right] \\
      &= f(-4) \\
      &= -1 \\
[f \circ g](6) &= f\left[g\left(6\right)\right] \\
      &= f(4) \\
      &= 11 \\
\end{align*}
\]

\( f \circ g = \{(3, -1), (6, 11)\} \)

\[
\begin{align*}
[g \circ f](x) &= g\left[f\left(x\right)\right] \\
[g \circ f](-4) &= g\left[f\left(-4\right)\right] \\
      &= g(-1) \\
      &= 5 \\
[g \circ f](-2) &= g\left[f\left(-2\right)\right] \\
      &= g(6) \\
      &= 4 \\
[g \circ f](-1) &= g\left[f\left(-1\right)\right] \\
      &= g(10) \\
      &= 8 \\
\end{align*}
\]

\( g \circ f = \{(-4, 5), (-2, 4), (-1, 8)\} \)

**ANSWER:**

\( f \circ g = \{(3, -1), (6, 11)\}; \)

\( g \circ f = \{(-4, 5), (-2, 4), (-1, 8)\} \)

26. \( f = \{(12, -3), (9, -2), (8, -1), (6, 3)\} \)
   \( g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\} \)

**SOLUTION:**

\[
\begin{align*}
[f \circ g](x) &= f\left[g\left(x\right)\right] \\
[f \circ g](-2) &= f\left[g\left(-2\right)\right] \\
      &= f(6) \\
      &= 3 \\
[f \circ g](-4) &= f\left[g\left(-4\right)\right] \\
      &= f(8) \\
      &= -1 \\
\end{align*}
\]

\( f \circ g = \{(-2, 3), (-4, -1)\} \)

\[
\begin{align*}
[g \circ f](x) &= g\left[f\left(x\right)\right] \\
[g \circ f](12) &= g\left[f\left(12\right)\right] \\
      &= g(-3) \\
      &= -1 \\
[g \circ f](9) &= g\left[f\left(9\right)\right] \\
      &= g(-2) \\
      &= 6 \\
[g \circ f](8) &= g\left[f\left(8\right)\right] \\
      &= g(-1) \\
      &= 5 \\
\end{align*}
\]

\( g \circ f = \{(12, -1), (9, 6), (8, 5)\} \)

**ANSWER:**

\( f \circ g = \{(-2, 3), (-4, -1)\}; \)

\( g \circ f = \{(12, -1), (9, 6), (8, 5)\} \)
Find \([f \circ g](x)\) and \([g \circ f](x)\) if they exist. State the domain and range for each composed function.

27. \(f(x) = 2x\)
\(g(x) = x + 5\)

**SOLUTION:**
\[f \circ g(x) = f[g(x)]\]
\[= f(x + 5)\]
\[= 2(x + 5)\]
\[= 2x + 10\]
\[g \circ f(x) = g[f(x)]\]
\[= g(2x)\]
\[= 2x + 5\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all even numbers}\}\)
For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all odd numbers}\}\)

**ANSWER:**
\([f \circ g](x) = 2x + 10;\]
\([g \circ f](x) = 2x + 5]\)

28. \(f(x) = -3x\)
\(g(x) = -x + 8\)

**SOLUTION:**
\([f \circ g](x) = f[g(x)]\]
\[= f(-x + 8)\]
\[= -3(-x + 8)\]
\[= 3x - 24\]
\([g \circ f](x) = g[f(x)]\]
\[= g(3x)\]
\[= -(3x) + 8\]
\[= 3x + 8\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)
For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)

**ANSWER:**
\([f \circ g](x) = 3x - 24; [g \circ f](x)\]
\[= 3x + 8\]
For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)
For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)
6-1 Operations on Functions

29. \( f(x) = x + 5 \)
   \( g(x) = 3x - 7 \)

**SOLUTION:**

\[
[f \circ g](x) = f[g(x)] \\
= f(3x - 7) \\
= (3x - 7) + 5 \\
= 3x - 2 \\
\]

\[
[g \circ f](x) = g[f(x)] \\
= g(x + 5) \\
= 3(x + 5) - 7 \\
= 3x + 15 - 7 \\
= 3x + 8 \\
\]

For \([f \circ g](x), D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)

For \([g \circ f](x), D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)

**ANSWER:**

\[
[f \circ g](x) = 3x - 2; [g \circ f](x) \\
= 3x + 8 \\
\]

For \([f \circ g](x), D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)

For \([g \circ f](x), D = \{\text{all real numbers}\}, R = \{\text{all real numbers}\}\)

30. \( f(x) = x - 4 \)
   \( g(x) = x^2 - 10 \)

**SOLUTION:**

\[
[f \circ g](x) = f[g(x)] \\
= f(x^2 - 10) \\
= (x^2 - 10) - 4 \\
= x^2 - 14 \\
\]

\[
[g \circ f](x) = g[f(x)] \\
= g(x - 4) \\
= (x - 4)^2 - 10 \\
= x^2 - 8x + 6 \\
\]

For \([f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -14\}\)

For \([g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -10\}\)

**ANSWER:**

\[
[f \circ g](x) = x^2 - 14; \\
[g \circ f](x) = x^2 - 8x + 6 \\
\]

For \([f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -14\}\)

For \([g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -10\}\)
6-1 Operations on Functions

31. \( f(x) = x^2 + 6x - 2 \)
\( g(x) = x - 6 \)

**SOLUTION:**
\[
(f \circ g)(x) = f[g(x)] = f(x - 6) = (x - 6)^2 + 6(x - 6) - 2 = x^2 - 6x - 2
\]

\[
(g \circ f)(x) = g[f(x)] = g(x^2 + 6x - 2) = x^2 + 6x - 2 - 6 = x^2 + 6x - 8
\]

\(F or [f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -11\}\)
\(F or [g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -17\}\)

**ANSWER:**
\[
(f \circ g)(x) = x^2 - 6x - 2; \\
(g \circ f)(x) = x^2 + 6x - 8
\]
\(F or [f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -11\}\)
\(F or [g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq -17\}\)

32. \( f(x) = 2x^2 - x + 1 \)
\( g(x) = 4x + 3 \)

**SOLUTION:**
\[
(f \circ g)(x) = f[g(x)] = f(4x + 3) = 2(4x + 3)^2 - (4x + 3) + 1 = 32x^2 + 44x + 16
\]

\[
(g \circ f)(x) = g[f(x)] = g(2x^2 - x + 1) = 4(2x^2 - x + 1) + 3 = 8x^2 - 4x + 7
\]

\(F or [f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq 0.875\}\)
\(F or [g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq 6.5\}\)

**ANSWER:**
\[
(f \circ g)(x) = 32x^2 + 44x + 16; \\
(g \circ f)(x) = 8x^2 - 4x + 7
\]
\(F or [f \circ g](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq 0.875\}\)
\(F or [g \circ f](x), D = \{\text{all real numbers}\}, R = \{y \mid y \geq 6.5\}\)
33. \( f(x) = 4x - 1 \)
   \( g(x) = x^3 + 2 \)

**SOLUTION:**

\[
[f \circ g](x) = f[g(x)] = f(x^3 + 2)
= 4(x^3 + 2) - 1
= 4x^3 + 7
\]

\[
g \circ f (x) = g[f(x)] = g(4x - 1)
= (4x - 1)^3 + 2
= 64x^3 - 48x^2 + 12x + 1
\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{\text{all real numbers}\}\)

For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{\text{all real numbers}\}\)

**ANSWER:**

\[
[f \circ g](x) = 4x^2 + 7; [g \circ f](x) = 64x^3 - 48x^2 + 12x + 1
\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{\text{all real numbers}\}\)

For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{\text{all real numbers}\}\)

34. \( f(x) = x^2 + 3x + 1 \)
   \( g(x) = x^2 \)

**SOLUTION:**

\[
[f \circ g](x) = f[g(x)] = f(x^2)
= (x^2)^2 + 3(x^2) + 1
= x^4 + 3x^2 + 1
\]

\[
g \circ f (x) = g[f(x)] = g(x^2 + 3x + 1)
= (x^2 + 3x + 1)^2
= x^4 + 6x^3 + 11x^2 + 6x + 1
\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{y | y \geq 1\}\)

For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{y | y \geq 0\}\)

**ANSWER:**

\[
[f \circ g](x) = x^4 + 3x^3 + 1; [g \circ f](x) = x^4 + 6x^3 + 11x^2 + 6x + 1
\]

For \([f \circ g](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{y | y \geq 1\}\)

For \([g \circ f](x)\), \(D = \{\text{all real numbers}\}\), \(R = \{y | y \geq 0\}\)
6-1 Operations on Functions

35. \( f(x) = 2x^2 \)
    \( g(x) = 8x^2 + 3x \)

**SOLUTION:**

\[
(f \circ g)(x) = f(g(x))
= f(8x^2 + 3x)
= 2(8x^2 + 3x)^2
= 128x^4 + 96x^3 + 18x^2
\]

\[
g \circ f)(x) = g(f(x))
= g(2x^2)
= 8(2x^2)^2 + 3(2x^2)
= 32x^4 + 6x^2
\]

For \( f \circ g \), \( D = \{\text{all real numbers}\} \), \( R = \{y \mid y \geq 0\} \)

For \( g \circ f \), \( D = \{\text{all real numbers}\} \), \( R = \{y \mid y \geq 0\} \)

**ANSWER:**

\[
(f \circ g)(x) = 128x^4 + 96x^3 + 18x^2; \]

\[
g \circ f)(x) = 32x^4 + 6x^2
\]

For \( f \circ g \), \( D = \{\text{all real numbers}\} \), \( R = \{y \mid y \geq 0\} \)

For \( g \circ f \), \( D = \{\text{all real numbers}\} \), \( R = \{y \mid y \geq 0\} \)

36. **FINANCE** A ceramics store manufactures and sells coffee mugs. The revenue \( r(x) \) from the sale of \( x \) coffee mugs is given by \( r(x) = 6.5x \). Suppose the function for the cost of manufacturing \( x \) coffee mugs is \( c(x) = 0.75x + 1850 \).

a. Write the profit function.

b. Find the profit on 500, 1000, and 5000 coffee mugs.

**SOLUTION:**

a. The profit function \( P(x) \) is given by

\[ P(x) = r(x) - c(x) \]

where \( r(x) \) is the revenue function and \( c(x) \) is the cost function.

So:

\[ P(x) = 6.5x - (0.75x + 1850) \]

\[ = 5.75x - 1850 \]

b.

\[ P(500) = 5.75(500) - 1850 \]

\[ = $1025 \]

\[ P(1000) = 5.75(1000) - 1850 \]

\[ = 5750 - 1850 \]

\[ = $3900 \]

\[ P(5000) = 5.75(5000) - 1850 \]

\[ = 26,900 \]

**ANSWER:**

a. \( P(x) = 5.75x - 1850 \)

b. \( P(500) = $1025; P(1000) = $3900; P(5000) = $26,900 \)

37. **CCSS SENSE-MAKING** Ms. Smith wants to buy an HDTV, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.

a. Write two functions representing the price after the discount \( p(x) \) and the price after sales tax \( t(x) \).

b. Which composition of functions represents the price of the HDTV, \( (p \circ t)(x) \) or \( (t \circ p)(x) \)? Explain your reasoning.
6-1 Operations on Functions

c. How much will Ms. Smith pay for the HDTV?

**SOLUTION:**

a. Let \( p(x) \) be the price after the discount where \( x \) is the original price.
Discount = 35\%(x) = 0.35x
Therefore:
\[ p(x) = x - 0.35x = 0.65x \]
Let \( t(x) \) be the price after the sales tax.
Sales tax = 6.25\%(x) = 0.0625x
Therefore:
\[ t(x) = x + 0.0625x = 1.0625x \]

b. Since \([p \circ t](x) = [t \circ p](x)\), either function represents the price.

\[ [t \circ p](x) = t[p(x)] = t(0.65x) = 1.0625(0.65x) = 0.690625x \]

Substitute \( x = 2299 \).
\[ [t \circ p](2299) = 0.690625(2299) = 1587.75 \]

Therefore, Mr. Smith will pay $1587.75 for HDTV.

**ANSWER:**

a. \( p(x) = 0.65x \); \( t(x) = 1.0625x \)

b. Since \([p \circ t](x) = [t \circ p](x)\), either function represents the price.

c. $1587.75

---

Perform each operation if \( f(x) = x^2 + x - 12 \) and \( g(x) = x - 3 \). State the domain of the resulting function.

38. \( (f - g)(x) \)

**SOLUTION:**

\[ (f - g)(x) = f(x) - g(x) = x^2 + x - 12 - (x - 3) = x^2 - 9 \]
\( D = \{ \text{all real numbers} \} \)

**ANSWER:**

\( (f - g)(x) = x^2 - 9; D = \{ \text{all real numbers} \} \)

39. \( 2(g \cdot f)(x) \)

**SOLUTION:**

\[ 2(g \cdot f)(x) = 2 \cdot g(x) \cdot f(x) = 2(x - 3)(x^2 + x - 12) = 2x^3 - 4x^2 - 30x + 72 \]
\( D = \{ \text{all real numbers} \} \)

**ANSWER:**

\( 2(g \cdot f)(x) = 2x^3 - 4x^2 - 30x + 72; D = \{ \text{all real numbers} \} \)
40. \( \left( \frac{f}{g} \right)(x) \)

**SOLUTION:**
\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + x - 12}{x - 3}, \quad x \neq 3
\]
\[
= \frac{(x + 4)(x - 3)}{x - 3}, \quad x \neq 3
\]
\[
= x + 4, \quad x \neq 3
\]

\( D = \{ x | x \neq 3 \} \)

**ANSWER:**
\[
\left( \frac{f}{g} \right)(x) = x + 4; \quad D = \{ x | x \neq 3 \}
\]

If \( f(x) = 5x \), \( g(x) = -2x + 1 \), and \( h(x) = x^2 + 6x + 8 \), find each value.

41. \( f(g(-2)) \)

**SOLUTION:**
\[
f[g(-2)] = f[-2x + 1]
\]
\[
= 5(-2x + 1)
\]
\[
= -10x + 5
\]

Substitute \( x = -2 \). 
\[
f[g(-2)] = -10(-2) + 5
\]
\[
= 25
\]

**ANSWER:**
25

42. \( g[h(3)] \)

**SOLUTION:**
\[
g[h(x)] = g(x^2 + 6x + 8)
\]
\[
= -2(x^2 + 6x + 8) + 1
\]
\[
= -2x^2 - 12x - 15
\]
Substitute \( x = 3 \).
\[
g[h(3)] = -2(3)^2 - 12(3) - 15
\]
\[
= -69
\]

**ANSWER:**
-69

43. \( h[f(-5)] \)

**SOLUTION:**
\[
h[f(x)] = h[5x]
\]
\[
= (5x)^2 + 6(5x) + 8
\]
\[
= 25x^2 + 30x + 8
\]
Substitute \( x = -5 \).
\[
h[f(-5)] = 25(-5)^2 + 30(-5) + 8
\]
\[
= 25(25) - 150 + 8
\]
\[
= 625 - 150 + 8
\]
\[
= 483
\]

**ANSWER:**
483
6-1 Operations on Functions

44. \( h[g(2)] \)

\[
SOLUTION:
\]
\[
h[g(x)] = h[-2x + 1]
\]
\[
= (-2x + 1)^2 + 6(-2x + 1) + 8
\]
\[
= 4x^2 + 1 - 4x - 12x + 6 + 8
\]
\[
= 4x^2 - 16x + 15
\]

Substitute \( x = 2 \).

\[
h[g(2)] = 4(2)^2 - 16(2) + 15
\]
\[
= 4(4) - 16(2) + 15
\]
\[
= 16 - 32 + 15
\]
\[
= 31 - 32
\]
\[
= -1
\]

\[\text{ANSWER:} -1\]

45. \( f[h(-3)] \)

\[
SOLUTION:
\]
\[
f[h(x)] = f[x^2 + 6x + 8]
\]
\[
= 5(x^2 + 6x + 8)
\]
\[
= 5x^2 + 30x + 40
\]

Substitute \( x = -3 \).

\[
f[h(-3)] = 5(-3)^2 + 30(-3) + 40
\]
\[
= 45 - 90 + 40
\]
\[
= -5
\]

\[\text{ANSWER:} -5\]

46. \( h[f(9)] \)

\[
SOLUTION:
\]
\[
h[f(x)] = h(5x)
\]
\[
= (5x)^2 + 6(5x) + 8
\]
\[
= 25x^2 + 30x + 8
\]

Substitute \( x = 9 \).

\[
h[f(9)] = 25(9)^2 + 30(9) + 8
\]
\[
= 25(81) + 270 + 8
\]
\[
= 2025 + 270 + 8
\]
\[
= 2303
\]

\[\text{ANSWER:} 2303\]

47. \( f[g(3a)] \)

\[
SOLUTION:
\]
\[
f[g(x)] = f[-6a + 1]
\]
\[
= 5(-6a + 1)
\]
\[
= -30a + 5
\]

\[\text{ANSWER:} -30a + 5\]
6-1 Operations on Functions

48. \( f[h(a+4)] \)

**SOLUTION:**
\[
f[h(a+4)] = f[(a+4)^2 + 6(a+4) + 8]
= f[a^2 + 14a + 48]
= 5(a^2 + 14a + 48)
= 5a^2 + 70a + 240
\]

**ANSWER:**
\( 5a^2 + 70a + 240 \)

49. \( g[f(a^2 - a)] \)

**SOLUTION:**
\[
g[f(a^2 - a)] = g[5(a^2 - a)]
= g(5a^2 - 5a)
= -2(5a^2 - 5a) + 1
= -10a^2 + 10a + 1
\]

**ANSWER:**
\( -10a^2 + 10a + 1 \)

50. **MULTIPLE REPRESENTATIONS** Let \( f(x) = x^2 \) and \( g(x) = x \).

a. **TABULAR** Make a table showing values for \( f(x) \), \( g(x) \), \( (f + g)(x) \), and \( (f - g)(x) \).

b. **GRAPHICAL** Graph \( f(x) \), \( g(x) \), and \( (f + g)(x) \) on the same coordinate grid.

c. **GRAPHICAL** Graph \( f(x) \), \( g(x) \), and \( (f - g)(x) \) on the same coordinate grid.

d. Sample answer: For each value of \( x \), the vertical distance between the graph of \( g(x) \) and the \( x \)-axis is the same as the vertical distance between the graphs of \( f(x) \) and \( (f + g)(x) \) and between \( f(x) \) and \( (f - g)(x) \).

**ANSWER:**

\( x \) | \( f(x) = x^2 \) | \( g(x) = x \) | \( (f + g)(x) = x^2 + x \) | \( (f - g)(x) = x^2 - x \) \\
--- | --- | --- | --- | --- \\
-3 | 9 | -3 | 6 | 12 \\
-2 | 4 | -2 | 2 | 6 \\
-1 | 1 | -1 | 0 | 2 \\
0 | 0 | 0 | 0 | 0 \\
1 | 1 | 1 | 2 | 0 \\
2 | 4 | 2 | 6 | 2 \\
3 | 9 | 3 | 12 | 6
51. **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where \( x \) is the number of years since 1994 and \( y \) is the number of people in thousands.

- Women: \( y = 1086.4x + 56,610 \)
- Men: \( y = 999.2x + 66,450 \)

**a.** Write a function that models the total number of men and women employed in the United States during this time.

**b.** If \( f \) is the function for the number of men, and \( g \) is the function for the number of women, what does \( (f - g)(x) \) represent?

**SOLUTION:**

**a.** Add the functions.

The total number of men and women employed is given by

\[
y(x) = 1086.4x + 56,610 + 999.2x + 66,450
\]

\[
y(x) = 2085.6x + 123,060
\]

**b.** The function \( (f - g)(x) \) represents the difference in the number of men and women employed in the U.S.

**ANSWER:**

**a.** \( y = 2085.6x + 123,060 \)

**b.** The function represents the difference in the number of men and women employed in the U.S.
6-1 Operations on Functions

If \( f(x) = x + 2 \), \( g(x) = -4x + 3 \), and \( h(x) = x^2 - 2x + 1 \), find each value.

52. \((f \cdot g \cdot h)(3)\)

**SOLUTION:**
\[
(f \cdot g \cdot h)(x) = f(x) \cdot g(x) \cdot h(x)
\]
Substitute \( x = 3 \).
\[
(f \cdot g \cdot h)(3) = f(3) \cdot g(3) \cdot h(3) = (5)(-9)(4) = -180
\]
**ANSWER:**
\[-180\]

53. \(\left[(f + g) \cdot h\right](1)\)

**SOLUTION:**
\[
\left[(f + g) \cdot h\right](x) = [f + g](x) \cdot h(x)
\]
\[
= [f(x) + g(x)] \cdot h(x)
\]
\[
= f(x) \cdot h(x) + g(x) \cdot h(x)
\]
Substitute \( x = 1 \).
\[
\left[(f + g) \cdot h\right](1) = f(1) \cdot h(1) + g(1) \cdot h(1) = (3)(0) + (-1)(0) = 0
\]
**ANSWER:**
\[0\]

54. \(\left(\frac{h}{fg}\right)(-6)\)

**SOLUTION:**
\[
\left(\frac{h}{fg}\right)(-6) = \frac{h(-6)}{f(-6) \cdot g(-6)}
\]
\[
= \frac{49}{(-4)(27)} = \frac{49}{108}
\]
**ANSWER:**
\[\frac{49}{108}\]

55. \(\left[f \circ (g \cdot h)\right](2)\)

**SOLUTION:**
\[
\left[f \circ (g \cdot h)\right](2) = [f \circ g](h(2)) = [f \circ g](1) = f[g(1)] = f(-1) = 1
\]
**ANSWER:**
\[1\]
6-1 Operations on Functions

56. \([g \circ (h \circ f)](-4)\)

**SOLUTION:**

\[g \circ (h \circ f)](-4) = g(h[f(-4)])
\[= g(h(-2))
\[= g(9)
\[= -36 + 3
\[= -33

**ANSWER:**

-33

57. \([h \circ (f \circ g)](5)\)

**SOLUTION:**

\[h \circ (f \circ g)](5) = h[f(g(5))]
\[= h[f(-17)]
\[= h(-15)
\[= 256

**ANSWER:**

256

58. **MULTIPLE REPRESENTATIONS** You will explore \( (f \cdot g)(x) \) \( \left( \frac{f}{g} \right)(x) \), and \( f \circ g \), and

**[g \circ f](x)\.

**a. Tabular** Make a table showing values for

\( (f \cdot g)(x) \), \( \left( \frac{f}{g} \right)(x) \), \( f \circ g \), and \( g \circ f \).

**b. Graphical** Use a graphing calculator to graph \( (f \cdot g)(x) \), \( \left( \frac{f}{g} \right)(x) \), \( f \circ g \), and \( g \circ f \) on the same coordinate plane.

For each function \( f \) and \( g \), find \((f + g)(x)\), \((f - g)(x)\), \((f \cdot g)(x)\), \(\left( \frac{f}{g} \right)(x)\), and \((f \circ g)(x)\). Indicate any restrictions in domain or range.

**c. Verbal** Explain the relationship between \((f \cdot g)(x)\) and \(\left( \frac{f}{g} \right)(x)\).

**d. Graphical** Use a graphing calculator to graph \( f \circ g \) and \( g \circ f \) on the same coordinate plane.

**e. Verbal** Explain the relationship between \( f \circ g \) and \( g \circ f \).
6-1 Operations on Functions

[Diagram: Graph of functions (f \cdot g)(x) and (f + g)(x)]

**c.** Sample answer: When \( x \) is 2 or 4, the functions are equal.

**d.**

[Diagram: Graph of functions (f \cdot g)(x) and (g \cdot f)(x)]

**e.** Sample answer: The functions are translations of the graph of \( y = x^2 \).

**ANSWER:**

**a.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>((f \cdot g)(x))</th>
<th>((f + g)(x))</th>
<th>((f \cdot g)(x))</th>
<th>((g \cdot f)(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-60</td>
<td>-( \frac{5}{3} )</td>
<td>37</td>
<td>7</td>
</tr>
<tr>
<td>-2</td>
<td>-25</td>
<td>-1</td>
<td>26</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
<td>-( \frac{1}{2} )</td>
<td>17</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
<td>-( \frac{1}{3} )</td>
<td>10</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
<td>-1</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>-5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>undef</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

**b.**

[Diagram: Graph of function \((f \cdot g)(x)\)]

**c.** Sample answer: When \( x \) is 2 or 4, the functions are equal.

**59. OPEN ENDED** Write two functions \( f(x) \) and \( g(x) \) such that \((f \circ g)(4) = 0\).

**SOLUTION:**

Sample answer: Another way to write \((f \circ g)(x) = f(g(x))\). Write a function for \( g(x) \) first and then evaluate it. Let \( g(x) = x + 5 \). Then \( g(4) = 4 + 5 \) or 9. Next, write a function for \( f(x) \) such that when \( x = 9, f(x) = 0 \). Let \( f(x) = x - 9 \) then \( f(g(4)) = 9 - 9 \) or 0.

**ANSWER:**

Sample answer: \( f(x) = x - 9 \), \( g(x) = x + 5 \)
6-1 Operations on Functions

60. **CCSS CRITIQUE**  Chris and Tobias are finding the composition \((f \circ g)(x)\), where \(f(x) = x^2 + 2x - 8\) and \(g(x) = x^2 + 8\). Is either of them correct? Explain your reasoning.

**Chris**
\[
(f \circ g)(x) = f(g(x)) = (x^2 + 8)^2 + 2(x^2 + 8) - 8
\]
\[
= x^4 + 16x^2 + 64 + 2x^2 + 16 - 8
\]
\[
= x^4 + 16x^2 + 72
\]

**Tobias**
\[
(f \circ g)(x) = f(g(x)) = (x^2 + 8)^2 + 2(x^2 + 8) - 8
\]
\[
= x^4 + 16x^2 + 64 + 2x^2 + 16 - 8
\]
\[
= x^4 + 16x^2 + 72
\]

**SOLUTION:**
Tobias is correct. Chris did not substitute \(g(x)\) for every \(x\) in \(f(x)\).

**ANSWER:**
Tobias; Chris did not substitute \(g(x)\) for every \(x\) in \(f(x)\).

61. **CHALLENGE** Given \(f(x) = \sqrt[3]{x^3}\) and \(g(x) = \sqrt[6]{x^6}\) determine the domain for each of the following.

a. \(g(x) \cdot g(x)\)

b. \(f(x) \cdot f(x)\)

**SOLUTION:**

a. \(g(x) \cdot g(x) = \sqrt[6]{x^6} \cdot \sqrt[6]{x^6} = x^6\)

D = \{all real numbers\}

b. \(f(x) \cdot f(x) = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} = x^3\)

Since \(f(x)\) is defined for \(x \geq 0\), the domain of \(f(x) \cdot f(x)\) is \(\{x | x \geq 0\}\).

**ANSWER:**

a. D = \{all real numbers\}

b. D = \(\{x | x \geq 0\}\)
6.1 Operations on Functions

62. **REASONING** State whether each statement is sometimes, always, or never true. Explain your reasoning.

a. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $f(x)$.

b. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $g(x)$.

**SOLUTION:**

a. Always; since the range is dependent on the domain, the domain of $g[f(x)]$ is restricted by the domain of $f(x)$.

b. Sometimes; when $f(x) = 4x$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x}$, $x \geq 0$. The domain of $g(x)$ restricts the domain of $g[f(x)]$. When $f(x) = 4x^2$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x^2}$. In this case, the domain of $g(x)$ does not restrict the domain of $g[f(x)]$.

**ANSWER:**

a. Always; since the range is dependent on the domain, the domain of $g[f(x)]$ is restricted by the domain of $f(x)$.

b. Sometimes; when $f(x) = 4x$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x}$, $x \geq 0$. The domain of $g(x)$ restricts the domain of $g[f(x)]$. When $f(x) = 4x^2$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x^2}$. In this case, the domain of $g(x)$ does not restrict the domain of $g[f(x)]$.

63. **WRITING IN MATH** In the real world, why would you ever perform a composition of functions?

**SOLUTION:**

Sample answer: Many situations in the real world involve complex calculations in which multiple functions are used. In order to solve some problems, a composition of those functions may need to be used. For example, the product of a manufacturing plant may have to go through several processes in a particular order, in which each process is described by a function. By finding the composition, only one calculation must be made to find the solution to the problem.

**ANSWER:**

Sample answer: Many situations in the real world involve complex calculations in which multiple functions are used. In order to solve some problems, a composition of those functions may need to be used. For example, the product of a manufacturing plant may have to go through several processes in a particular order, in which each process is described by a function. By finding the composition, only one calculation must be made to find the solution to the problem.
6-1 Operations on Functions

64. What is the value of \( x \) in the equation \( 7(x - 4) = 44 - 11x \)?

A 1
B 2
C 3
D 4

**SOLUTION:**

\[
7(x - 4) = 44 - 11x \\
7x - 28 = 44 - 11x \\
18x = 72 \\
x = 4
\]

The correct choice is D.

**ANSWER:**

D

65. If \( g(x) = x^2 + 9x + 21 \) and \( h(x) = 2(x + 5)^2 \), which is an equivalent form of \( h(x) - g(x) \)?

F \( k(x) = -x^2 - 11x - 29 \)
G \( k(x) = x^2 + 11x + 29 \)
H \( k(x) = x + 4 \)
J \( k(x) = x^2 + 7x + 11 \)

**SOLUTION:**

\[
h(x) - g(x) = 2(x + 5)^2 - (x^2 + 9x + 21)
= 2(x^2 + 10x + 25) - (x^2 + 9x + 21)
= 2x^2 + 20x + 50 - x^2 - 9x - 21
= x^2 + 11x + 29
\]

The correct choice is G.

**ANSWER:**

G
6-1 Operations on Functions

66. **GRIDDED RESPONSE** In his first three years of coaching basketball at North High School, Coach Lucas’ team won 8 games the first year, 17 games the second year, and 6 games the third year. How many games does the team need to win in the fourth year so the coach’s average will be 10 wins per year?

**SOLUTION:**
Let \( x \) be the number of games to be won in the fourth year.

So:
\[
\frac{8 + 17 + 6 + x}{4} = 10
\]
\[
31 + x = 40
\]
\[
x = 9
\]

Therefore, the team should win 9 games in the fourth year so that the coach’s average will be 10 wins per year.

**ANSWER:**

9

67. **SAT/ACT** What is the value of \( f(g(6)) \) if \( f(x) = 2x + 4 \) and \( g(x) = x^2 + 5 \)?

A 38  
B 43  
C 57  
D 86  
E 261

**SOLUTION:**
\[
f(g(x)) = f(x^2 + 5)
\]
\[
= 2(x^2 + 5) + 4
\]
\[
= 2x^2 + 10 + 4
\]
\[
= 2x^2 + 14
\]

Substitute \( x = 6 \).
\[
f(g(6)) = 2(6)^2 + 14
\]
\[
= 72 + 14
\]
\[
= 86
\]

The correct choice is D.

**ANSWER:**

D
6-1 Operations on Functions

Find all rational zeros of each function.

68. \( f(x) = 2x^3 - 13x^2 + 17x + 12 \)

**SOLUTION:**

If \( \frac{p}{q} \) is a rational zero, then \( p \) is a factor of 12 and \( q \) is a factor of 2.
The possible values of \( p \): \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \)
The possible values of \( q \): \( \pm 1, \pm 2 \)
The possible values of \( \frac{p}{q} \):
\[
\begin{array}{cccc}
\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, & \pm \frac{1}{2}, & \pm \frac{3}{2} \\
3 & -13 & 17 & 12 \\
0 & 6 & -21 & -12 \\
2 & -7 & -4 & 0 \\
\end{array}
\]
The depressed polynomial is \( 2x^2 - 7x - 4 \).
Solve the quadratic equation \( 2x^2 - 7x - 4 = 0 \).
\[
\begin{align*}
2x^2 - 7x - 4 &= 0 \\
2x^2 + x - 8x - 4 &= 0 \\
x(2x + 1) - 4(2x + 1) &= 0 \\
(2x + 1)(x - 4) &= 0 \\
2x &= -1 \quad \text{or} \quad x = 4 \\
x &= -\frac{1}{2} \quad \text{or} \quad x = 4 \\
\end{align*}
\]
The zeros of the function are \( -\frac{1}{2}, 3, \text{and} 4 \).

**ANSWER:**
\(-\frac{1}{2}, 3, 4\)

69. \( f(x) = x^3 - 3x^2 - 10x + 24 \)

**SOLUTION:**

If \( \frac{p}{q} \) is a rational zero, then \( p \) is a factor of 24 and \( q \) is a factor of 1.
The possible values of \( p \):
\( \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \)
The possible values of \( q \):
\( \pm 1 \)
The possible values of \( \frac{p}{q} \):
\[
\begin{array}{cccc}
\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \\
2 & -3 & -10 & 24 \\
0 & 2 & -2 & -24 \\
1 & -1 & -12 & 0 \\
\end{array}
\]
Therefore:
\[
(x - 2)(x^2 - x - 12) = 0 \\
(x - 2)(x - 4)(x + 3) = 0 \\
x = 2, x = 4, x = -3
\]
The zeros of the function are \(-3, 2, \text{and} 4 \).

**ANSWER:**
\(-3, 2, 4\)
6-1 Operations on Functions

70. \( f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24 \)

**SOLUTION:**

If \( \frac{p}{q} \) is a rational zero, then \( p \) is a factor of 24 and \( q \) is a factor of 1.
The possible values of \( p \):
\[ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \]
The possible values of \( q \):
\[ \pm 1 \]
The possible values of \( \frac{p}{q} \):
\[ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \]

\[
\begin{array}{cccc}
1 & -4 & -7 & 34 & -24 \\
0 & 1 & -3 & -10 & 24 \\
\hline
1 & -3 & -10 & 24 & 0 \\
\end{array}
\]

\((x-1)(x^3 - 3x^2 - 10x + 24) = 0\)
The depressed polynomial is \( x^3 - 3x^2 - 10x + 24 \).
Solve \( x^3 - 3x^2 - 10x + 24 = 0 \)
\( x^3 - 3x^2 - 10x + 24 = 0 \)
\( (x-2)(x^2 - x - 12) = 0 \)
\( x = 2, x = 4, x = -3 \)
Therefore, the roots are \( x = 1, x = 2, x = 4, \) and \( x = -3 \).

**ANSWER:**
\[ 1, 2, 4, -3 \]

71. \( f(x) = 2x^3 - 5x^2 - 28x + 15 \)

**SOLUTION:**

If \( \frac{p}{q} \) is a rational zero, then \( p \) is a factor of 24 and \( q \) is a factor of 1.
The possible values of \( p \):
\[ \pm 1, \pm 2, \pm 3, \pm 5, \pm 15 \]
The possible values of \( q \):
\[ \pm 1, \pm 2 \]
The possible values of \( \frac{p}{q} \):
\[ \pm 1, \pm 2, \pm 3, \pm 5, \pm 15 \]

\[
\begin{array}{cccc}
2 & -3 & -28 & 15 \\
0 & 10 & 25 & -15 \\
\hline
2 & 5 & -3 & 0 \\
\end{array}
\]
The depressed polynomial is \( 2x^2 + 5x - 3 \).
\( (x-5)(2x^2 + 5x - 3) = 0 \)
\( (x-5)(x+3)(2x-1) = 0 \)
\[ x = 5, x = -3, x = \frac{1}{2} \]
The zeros of the function is \( -3, 5, \frac{1}{2} \).

**ANSWER:**
\[ -3, 5, \frac{1}{2} \]
6-1 Operations on Functions

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

72. \( f(x) = 2x^4 - x^3 + 5x^2 + 3x - 9 \)

**SOLUTION:**
Use Descartes’ Rule.

\[
\begin{align*}
  f(x) &= 2x^4 - x^3 + 5x^2 + 3x - 9 \\
  f(-x) &= 2x^4 + x^3 + 5x^2 - 3x - 9
\end{align*}
\]

There are three sign changes in \( f(x) \). Therefore, the number of possible positive real zeros is 3 or 1. There is only one sign change in \( f(-x) \). Therefore, the number of possible negative real zero is 1. Therefore, the number of possible imaginary zero is 2 or 0.

**ANSWER:**
3 or 1; 1; 2 or 0

73. \( f(x) = -4x^4 - x^2 - x + 1 \)

**SOLUTION:**

\[
\begin{align*}
  f(x) &= -4x^4 - x^2 - x + 1 \\
  f(-x) &= -4x^4 - x^2 + x + 1
\end{align*}
\]

There is only one sign change in \( f(x) \). Therefore, the number of possible positive real zero is 1. There is only one sign change in \( f(-x) \). Therefore, the number of possible negative real zero is 1. Therefore, the number of possible imaginary zeros is 2.

**ANSWER:**
1; 1; 2

74. \( f(x) = 3x^4 - x^3 + 8x^2 + x - 7 \)

**SOLUTION:**

\[
\begin{align*}
  f(x) &= 3x^4 - x^3 + 8x^2 + x - 7 \\
  f(-x) &= 3x^4 + x^3 + 8x^2 - x - 7
\end{align*}
\]

There are three sign changes in \( f(x) \). Therefore, the number of possible positive real zeros is 3 or 1. There is only one sign change in \( f(-x) \). Therefore, the number of possible negative real zero is 1. Therefore, the number of possible imaginary zero is 2 or 0.

**ANSWER:**
3 or 1; 1; 0 or 2

75. \( f(x) = 2x^4 - 3x^3 - 2x^2 + 3 \)

**SOLUTION:**

\[
\begin{align*}
  f(x) &= 2x^4 - 3x^3 - 2x^2 + 3 \\
  f(-x) &= 2x^4 + 3x^3 - 2x^2 + 3
\end{align*}
\]

There are two sign changes in \( f(x) \). Therefore, the number of possible positive real zeros is 2 or 0. There is only one sign change in \( f(-x) \). Therefore, the number of possible negative real zeros is 2 or 0. Therefore, the number of possible imaginary zeros is 4 or 2 or 0.

**ANSWER:**
2 or 0; 2 or 0; 4, 2, or 0
6-1 Operations on Functions

76. **MANUFACTURING** A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension?

**SOLUTION:**
Let \( x \) inches be the increase in each dimension of the box.
The new volume of the box is given by:

\[
V_2 = (12 + x)(16 + x)(18 + x)
\]

Substitute 5965 for \( V_2 \).

\[
(12 + x)(16 + x)(18 + x) = 5985
\]

\[
x^3 + 46x^2 + 696x - 2529 = 0
\]

\[
\begin{array}{ccc}
3 & 1 & 46 & 696 & -2529 \\
0 & 3 & 147 & 2529 \\
1 & 49 & 843 & 0 \\
\end{array}
\]

Therefore, 3 inches should be added to each dimension.

**ANSWER:**
3 in.

---

Solve each system of equations.

x + 4y - z = 6

77. 3x + 2y + 3z = 16

2x - y + z = 3

**SOLUTION:**

\[
\begin{array}{ccc}
6 & 4 & -1 \\
16 & 2 & 3 \\
3 & -1 & 1 \\
\end{array}
\]

\[
x =
\begin{array}{c}
1 \\
1 \\
2 \\
2 \\
3 \\
3 \\
3 \\
2 \\
\end{array}
\]

\[
= \frac{24}{24}
\]

\[
= 1
\]

\[
\begin{array}{ccc}
1 & 6 & -1 \\
3 & 16 & 3 \\
2 & 3 & 1 \\
\end{array}
\]

\[
y =
\begin{array}{c}
1 \\
1 \\
2 \\
3 \\
3 \\
3 \\
2 \\
2 \\
\end{array}
\]

\[
= \frac{48}{24}
\]

\[
= 2
\]

\[
\begin{array}{ccc}
1 & 4 & 6 \\
3 & 2 & 16 \\
2 & -1 & 3 \\
\end{array}
\]

\[
z =
\begin{array}{c}
1 \\
1 \\
3 \\
3 \\
2 \\
\end{array}
\]

\[
= \frac{72}{24}
\]

\[
= 3
\]

The solution is \((1, 2, 3)\).

**ANSWER:**

\((1, 2, 3)\)
6-1 Operations on Functions

2a + b - c = 5
78. a - b + 3c = 9
3a - 6c = 6

SOLUTION:

\[
\begin{pmatrix}
5 & 1 & -1 \\
9 & -1 & 3 \\
6 & 0 & -6
\end{pmatrix}
=\begin{pmatrix}
2 & 1 & -1 \\
1 & -1 & 3 \\
3 & 0 & -6
\end{pmatrix}
=\begin{pmatrix}
2 & 5 & -1 \\
1 & 9 & 3 \\
3 & 6 & -6
\end{pmatrix}
=\begin{pmatrix}
2 & 1 & -1 \\
1 & -1 & 3 \\
3 & 0 & -6
\end{pmatrix}
=\begin{pmatrix}
2 & 1 & 5 \\
1 & -1 & 9 \\
3 & 0 & 6
\end{pmatrix}
=\begin{pmatrix}
2 & 1 & -1 \\
1 & -1 & 3 \\
3 & 0 & -6
\end{pmatrix}
=\begin{pmatrix}
24 \\
24 \\
24
\end{pmatrix}
=4
=2
=1

y + z = 4
79. 2x + 4y - z = -3
3y = -3

SOLUTION:

\[
3y = -3
\Rightarrow y = -1
\]

Substitute \(y = -1\) in the equation \(y + z = 4\).

\[
-1 + z = 4
\Rightarrow z = 4 + 1
\Rightarrow z = 5
\]

Substitute \(y = -1\) and \(z = 5\) in the equation \(2x + 4y - z = -3\).

\[
2x + 4(-1) - 5 = -3
\Rightarrow 2x - 4 - 5 = -3
\Rightarrow 2x - 9 = -3
\Rightarrow 2x = -3 + 9
\Rightarrow 2x = 6
\Rightarrow x = 3
\]

The solution is \((3, -1, 5)\).

ANSWER:

\((3, -1, 5)\)
6-1 Operations on Functions

80. **INTERNET** A webmaster estimates that the time, in seconds, to connect to the server when \( n \) people are connecting is given by \( t(n) = 0.005n + 0.3 \). Estimate the time to connect when 50 people are connecting.

**SOLUTION:**
Replace \( n \) with 50.

\[
t(50) = 0.005(50) + 0.3 = 0.25 + 0.3 = 0.55
\]

It takes 0.55 seconds to connect 50 people.

**ANSWER:**
0.55 second

81. \( 5x - 7y = 12 \), for \( x \)

**SOLUTION:**
\[
5x = 12 + 7y
\]
\[
x = \frac{12 + 7y}{5}
\]

**ANSWER:**
\[
x = \frac{12 + 7y}{5}
\]

82. \( 3x^2 - 6xy + 1 = 4 \), for \( y \)

**SOLUTION:**
\[
-6xy = 4 - 3x^2
\]
\[
y = \frac{3 - 3x^2}{-6x} = \frac{1 - x^2}{-2x}
\]

**ANSWER:**
\[
y = \frac{1 - x^2}{-2x}
\]

83. \( 4x + 8yz = 15 \), for \( x \)

**SOLUTION:**
\[
4x = 15 - 8yz
\]
\[
x = \frac{15 - 8yz}{4}
\]

**ANSWER:**
\[
x = \frac{15 - 8yz}{4}
\]

84. \( D = mv \), for \( m \)

**SOLUTION:**
\[
\frac{D}{v} = m
\]
\[
m = \frac{D}{v}
\]

**ANSWER:**
\[
m = \frac{D}{v}
\]
6-1 Operations on Functions

85. \( A = k^2 + b \), for \( k \)

**SOLUTION:**

\[
A - b = k^2 \\
k = \pm \sqrt{A - b}
\]

**ANSWER:**

\[
k = \pm \sqrt{A - b}
\]

86. \((x + 2)^2 - (y + 5)^2 = 4\), for \( y \)

**SOLUTION:**

\[
(y + 5)^2 = (x + 2)^2 - 4
\]

Use the Square Root Property.

\[
y + 5 = \pm \sqrt{(x + 2)^2 - 4} \\
y = \pm \sqrt{(x + 2)^2 - 4} - 5
\]

**ANSWER:**

\[
y = \pm \sqrt{(x + 2)^2 - 4} - 5
\]
6-2 Inverse Functions and Relations

Find the inverse of each relation.

1. {(–9, 10), (1, –3), (8, –5)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.
The inverse of the relation is 
\( \{(10, –9), (3, 1), (5, –8)\} \).

ANSWER:
\( \{(10, –9), (3, 1), (5, –8)\} \)

2. {(-2, 9), (4, –1), (–7, 9), (7, 0)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.
The inverse of the relation is 
\( \{(9, –2), (–1, 4), (9, –7), (0, 7)\} \).

ANSWER:
\( \{(9, –2), (–1, 4), (9, –7), (0, 7)\} \)

Find the inverse of each function. Then graph the function and its inverse.

3. \( f(x) = -3x \)

SOLUTION:
Rewrite the function as an equation relating \( x \) and \( y \).
\[ y = -3x \]
Exchange \( x \) and \( y \) in the equation.
\[ x = -3y \]
Solve the equation for \( y \).
\[ y = \frac{1}{3}x \]

4. \( g(x) = 4x - 6 \)

SOLUTION:
Rewrite the function as an equation relating \( x \) and \( y \).
\[ y = 4x - 6 \]
Exchange \( x \) and \( y \) in the equation.
\[ x = 4y - 6 \]
Solve for $y$.

\[
\frac{x + 6}{4} = y
\]

Replace $y$ with $g^{-1}(x)$.

\[
g^{-1}(x) = \frac{x + 6}{4}
\]

**ANSWER:**

\[
g^{-1}(x) = \frac{x + 6}{4}
\]

5. $h(x) = x^2 - 3$

**SOLUTION:**

Rewrite the function as an equation relating $x$ and $y$.

\[
y = x^2 - 3
\]

Exchange $x$ and $y$ in the equation.

\[
x = y^2 - 3
\]

Solve for $y$.

\[
y = \pm \sqrt{x + 3}
\]

Replace $y$ with $h^{-1}(x)$.

\[
h^{-1} = \pm \sqrt{x + 3}
\]

**ANSWER:**

\[
h^{-1} = \pm \sqrt{x + 3}
\]
6-2 Inverse Functions and Relations

Determine whether each pair of functions are inverse functions. Write yes or no.

6. \( f(x) = x - 7 \)
   \( g(x) = x + 7 \)

**SOLUTION:**

The functions \( f(x) \) and \( g(x) \) are inverses if and only if \([f \circ g](x) = [g \circ f](x) = x\).

\[
[f \circ g](x) = f[g(x)]
= f[x + 7]
= (x + 7) - 7
= x
\]

\[
[g \circ f](x) = g[f(x)]
= g(x - 7)
= x - 7 + 7
= x
\]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

**ANSWER:**
Yes

\[
f(x) = \frac{1}{2}x + \frac{3}{4}
\]

\[
g(x) = 2x - \frac{4}{3}
\]

**SOLUTION:**

The functions \( f(x) \) and \( g(x) \) are inverses if and only if \([f \circ g](x) = [g \circ f](x) = x\).

\[
[f \circ g](x) = f[g(x)]
= f\left(2x - \frac{4}{3}\right)
= \frac{1}{2}\left(2x - \frac{4}{3}\right) + \frac{3}{4}
= x + \frac{1}{12}
\]

\([f \circ g](x) \neq x\)

No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No
Find the inverse of each relation.

8. \( f(x) = 2x^3 \)
\( g(x) = \frac{1}{3} \sqrt[3]{x} \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if
\[
[f \circ g](x) = [g \circ f](x) = x.
\]
\[
[f \circ g](x) = f \left[ g(\frac{1}{3} \sqrt[3]{x}) \right]
\]
\[
= f \left( \frac{1}{3} x^{\frac{3}{2}} \right)
\]
\[
= 2 \cdot \left( \frac{1}{3} x^{\frac{3}{2}} \right)^3
\]
\[
= \frac{2}{27} x^{\frac{9}{2}}
\]
\[
[f \circ g](x) \neq x
\]
No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No

Find the inverse of each relation.

9. \{(–8, 6), (6, –2), (7, –3)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is
\[
\{(6, –8), (–2, 6), (–3, 7)\}.
\]

**ANSWER:**
\{(6, –8), (–2, 6), (–3, 7)\}

10. \{(7, 7), (4, 9), (3, –7)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is
\[
\{(7, 7), (9, 4), (–7, 3)\}.
\]

**ANSWER:**
\{(7, 7), (9, 4), (–7, 3)\}

11. \{(8, –1), (–8, –1), (–2, –8), (2, 8)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is
\[
\{(–1, 8), (–1, –8), (–8, –2), (8, 2)\}.
\]

**ANSWER:**
\{(–1, 8), (–1, –8), (–8, –2), (8, 2)\}

12. \{(4, 3), (–4, –4), (–3, –5), (5, 2)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is
\[
\{(3, 4), (–4, –4), (–5, –3), (2, 5)\}.
\]

**ANSWER:**
\{(3, 4), (–4, –4), (–5, –3), (2, 5)\}
13. \{(-5, 1), (2, 6), (3, -7), (4, 8), (5, -9)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is \{(-5, 1), (6, 2), (-7, 3), (8, 4), (-9, 5)\}.

**ANSWER:** \{(-5, 1), (6, 2), (-7, 3), (8, 4), (-9, 5)\}

14. \{(3, 0), (5, 4), (7, -8), (9, 12), (11, 16)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is \{(0, 3), (4, 5), (-8, 7), (12, 9), (16, 11)\}.

**ANSWER:** \{(0, 3), (4, 5), (-8, 7), (12, 9), (16, 11)\}

**CCSS SENSE-MAKING** Find the inverse of each function. Then graph the function and its inverse.

15. \(f(x) = x + 2\)

**SOLUTION:**
Rewrite the function as an equation relating \(x\) and \(y\).

\[y = x + 2\]

Exchange \(x\) and \(y\) in the equation.

\[x = y + 2\]

Solve for \(y\).

\[y = x - 2\]

16. \(g(x) = 5x\)

**SOLUTION:**
Rewrite the function as an equation relating \(x\) and \(y\).

\[y = 5x\]

Exchange \(x\) and \(y\) in the equation.

\[x = 5y\]

Solve for \(y\).
Find the inverse of each relation.

1. \( \left\{ (-9, 10), (1, -3), (8, -5) \right\} \)

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

Therefore:
\( g^{-1}(x) = \frac{1}{5}x \)

ANSWER:
\( g^{-1}(x) = \frac{1}{5}x \)

17. \( f(x) = -2x + 1 \)

SOLUTION:
Rewrite the function with \( x \) and \( y \).

\( y = -2x + 1 \)

Interchange \( x \) and \( y \) and solve.

ANSWER:
\( y = -2x + 1 \)

18. \( h(x) = \frac{x - 4}{3} \)

SOLUTION:
Rewrite the function as an equation relating \( x \) and \( y \).
Find the inverse of each relation.

1. \(\{(-9, 10), (1, -3), (8, -5)\}\)

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

\[
y = \frac{x - 4}{3}
\]
Exchange \(x\) and \(y\) in the equation

\[
x = \frac{y - 4}{3}
\]
Solve for \(y\).

\[
y = 3x + 4
\]
Replace \(y\) with \(h^{-1}(x)\).
Therefore:
\[
h^{-1}(x) = 3x + 4
\]

ANSWER:
\[
h^{-1}(x) = 3x + 4
\]

\[19. \ f(x) = -\frac{5}{3}x - \delta\]

SOLUTION:
Rewrite using \(x\) and \(y\).

\[
y = -\frac{5}{3}x - \delta
\]
Exchange \(x\) and \(y\) in the equation and solve for \(y\).

\[
x = -\frac{5}{3}y - \delta
\]
\[
x + \frac{5}{3}y = -\delta
\]
\[
\frac{5}{3}y = -x - \delta
\]
\[
y = -\frac{3}{5}x - \frac{24}{5}
\]
\[
f^{-1}(x) = -\frac{3}{5}x - \frac{24}{5}
\]
or
\[
f^{-1}(x) = -\frac{3}{5}(x + 8)
\]

ANSWER:
\[
f^{-1}(x) = -\frac{3}{5}(x + 8)
\]
20. \( g(x) = x + 4 \)

**SOLUTION:**
Rewrite the function as an equation relating \( x \) and \( y \).

\[ y = x + 4 \]

Exchange \( x \) and \( y \) in the equation.

\[ x = y + 4 \]

Solve for \( y \).

\[ y = x - 4 \]

Replace \( y \) with \( g^{-1}(x) \). Therefore:

\[ g^{-1}(x) = x - 4 \]

**ANSWER:**
\[ g^{-1}(x) = x - 4 \]

21. \( f(x) = 4x \)

**SOLUTION:**
Rewrite the function as an equation relating \( x \) and \( y \).

\[ y = 4x \]

Exchange \( x \) and \( y \) in the equation.

\[ x = 4y \]

Solve for \( y \).

\[ y = \frac{1}{4}x \]

Replace \( y \) with \( f^{-1}(x) \). Therefore:

\[ f^{-1}(x) = \frac{1}{4}x \]

**ANSWER:**
Find the inverse of each relation.

1. \{(–9, 10), (1, –3), (8, –5)\}

SOLUTION:

To find the inverse, exchange the coordinates of the ordered pairs.

\[
\left(\begin{array}{c}
-9 \\
1 \\
8
\end{array}\right) \rightarrow \left(\begin{array}{c}
10 \\
-3 \\
-5
\end{array}\right)
\]

ANSWER:

\{\{(10, -9), (-3, 1), (-5, 8)\}\}

22. \(f(x) = -8x + 9\)

SOLUTION:

Replace \(f(x)\) with \(y\).

\[y = -8x + 9\]

Exchange \(x\) and \(y\) in the equation and solve for \(y\).

\[x = -8y + 9\]

\[x + 8y = 9\]

\[8y = -x + 9\]

\[y = \frac{-x}{8} + \frac{9}{8}\]

\[f^{-1}(x) = \frac{-x}{8} + \frac{9}{8}\]

ANSWER:

\[f^{-1}(x) = \frac{-x}{8} + \frac{9}{8}\]
23. \( f(x) = 5x^3 \)

**SOLUTION:**
Rewrite the function as an equation relating \( x \) and \( y \).

\[ y = 5x^3 \]

Exchange \( x \) and \( y \) in the equation.

\[ x = 5y^3 \]

Solve for \( y \).

\[ y = \pm \sqrt[3]{\frac{1}{5}x} \]

Replace \( y \) with \( f^{-1}(x) \).

Therefore:

\[ f^{-1}(x) = \pm \sqrt[3]{\frac{1}{5}x} \]

**ANSWER:**

\[ f^{-1}(x) = \pm \sqrt[3]{\frac{1}{5}x} \]

24. \( h(x) = x^2 + 4 \)

**SOLUTION:**
Rewrite the function as an equation relating \( x \) and \( y \).

\[ y = x^2 + 4 \]

Exchange \( x \) and \( y \) in the equation.

\[ x = y^2 + 4 \]

Solve for \( y \).

\[ y = \pm \sqrt{x - 4} \]

Replace \( y \) with \( h^{-1}(x) \).

\[ h^{-1}(x) = \pm \sqrt{x - 4} \]

**ANSWER:**
Find the inverse of each relation.

1. \{(–9, 10), (1, –3), (8, –5)\}

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

\[ h^{-1}(x) = \pm \sqrt{x - 4} \]

25. \( f(x) = \frac{1}{2}x^2 - 1 \)

**SOLUTION:**
Rewrite the function as an equation relating \(x\) and \(y\).

\[ y = \frac{1}{2}x^2 - 1 \]

Exchange \(x\) and \(y\) in the equation.

\[ x = \frac{1}{2}y^2 - 1 \]

Solve for \(y\).

\[ y = \pm \sqrt{2x + 2} \]

Replace \(y\) with \(f^{-1}(x)\).

Therefore:

\[ f^{-1}(x) = \pm \sqrt{2x + 2} \]

26. \( f(x) = (x + 1)^2 + 3 \)

**SOLUTION:**
Replace \(f(x)\) with \(y\). Then exchange \(x\) and \(y\) in the equation and solve for \(y\).

\[ f(x) = (x + 1)^2 + 3 \]

\[ y = (x + 1)^2 + 3 \]

\[ x = (y + 1)^2 + 3 \]

\[ x - 3 = (y + 1)^2 \]

\[ \pm \sqrt{x - 3} = y + 1 \]

\[ \pm \sqrt{x - 3} - 1 = y \]

\[ \pm \sqrt{x - 3} - 1 = f^{-1}(x) \]
Determine whether each pair of functions are inverse functions. Write yes or no.

27. \[ f(x) = 2x + 3 \]
   \[ g(x) = 2x - 3 \]

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if \([f \circ g](x) = [g \circ f](x) = x\). 

\[
[f \circ g](x) = f\left(g(x)\right) \\
= f\left(2x - 3\right) \\
= 2\left(2x - 3\right) + 3 \\
= 4x - 3 \\
\]

\( f(x) \neq x \)

No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No
Find the inverse of each relation.

1. \( \{(–9, 10), (1, –3), (8, –5)\} \)

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

\[
\begin{align*}
\text{Inverse:} & \quad \{(10, –9), (–3, 1), (–5, 8)\} \\
\text{Solvability:} & \quad \text{Yes, } f(x) \text{ and } g(x) \text{ are inverse functions.}
\end{align*}
\]

c. Using composition of functions, verify that these two functions are inverses.

SOLUTION:

\[
\begin{align*}
[f \circ g](x) &= f[g(x)] \\
&= f\left(\frac{x - 6}{4}\right) \\
&= \frac{x - 6}{4} + 6 \\
&= x \\
g[f](x) &= g[f(x)] \\
&= g(4x + 6) \\
&= \frac{4x + 6 - 6}{4} \\
&= x
\end{align*}
\]

Therefore:

\[
[f \circ g](x) = g[f](x) = x
\]

Yes, \(f(x)\) and \(g(x)\) are inverse functions.

ANSWER:
Yes

28. \( f(x) = 4x + 6 \)

29. \( f(x) = -\frac{1}{3}x + 3 \)

SOLUTION:
The functions \(f(x)\) and \(g(x)\) are inverses if and only if \([f \circ g](x) = [g \circ f](x) = x\).

\[
\begin{align*}
[f \circ g](x) &= f[g(x)] \\
&= f\left(-\frac{1}{3}x + 3\right) \\
&= -\frac{1}{3}\left(-\frac{1}{3}x + 3\right) + 3 \\
&= x - 3 + 3 \\
&= x \\
g[f](x) &= g[f(x)] \\
&= g\left(-\frac{1}{3}x + 3\right) \\
&= -3\left(-\frac{1}{3}x + 3\right) + 9 \\
&= x - 9 + 9 \\
&= x
\end{align*}
\]

Yes, \(f(x)\) and \(g(x)\) are inverse functions.

ANSWER:
Yes
6-2 Inverse Functions and Relations

30. \( f(x) = -6x \)
\( g(x) = \frac{1}{6}x \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if 
\[ [f \circ g](x) = [g \circ f](x) = x. \]

\[
[f \circ g](x) = f[g(x)]
= f\left(\frac{1}{6}x\right)
= -6\left(\frac{1}{6}x\right)
= -x
\]

\[ [f \circ g](x) \neq x \]

No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No

31. \( f(x) = \frac{1}{2}x + 5 \)
\( g(x) = 2x - 10 \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if 
\[ [f \circ g](x) = [g \circ f](x) = x. \]

\[
[f \circ g](x) = f[g(x)]
= f\left(\frac{1}{2}x + 5\right)
= \frac{1}{2}\left(\frac{1}{2}x + 5\right) - 10
= \frac{1}{4}x + 5 - 10
= x
\]

\[ [f \circ g](x) = [g \circ f](x) = x \]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

**ANSWER:**
Yes
Find the inverse of each relation.

1. \( \{(–9, 10), (1, –3), (8, –5)\} \)

**SOLUTION:**
To find the inverse, exchange the coordinates of the ordered pairs.

\[
\begin{align*}
\text{Solved for } x & : \quad x = \frac{y + 10}{8} \\
\text{Solved for } y & : \quad y = 8x - 10
\end{align*}
\]

**ANSWER:**
Yes, \( f(x) \) and \( g(x) \) are inverse functions.

Find the inverse of each function.

32. \( f(x) = \frac{x + 10}{8} \)
\( g(x) = 8x - 10 \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if

\[
[f \circ g](x) = [g \circ f](x) = x.
\]

\[
\begin{align*}
[f \circ g](x) &= f\left( g(x) \right) \\
&= f(8x - 10) \\
&= \frac{8x - 10 + 10}{8} \\
&= x \\
[g \circ f](x) &= g\left( f(x) \right) \\
&= g\left( \frac{x + 10}{8} \right) \\
&= 8\left( \frac{x + 10}{8} \right) - 10 \\
&= x
\end{align*}
\]

Therefore:

\[
[f \circ g](x) = [g \circ f](x) = x
\]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

33. \( f(x) = 4x^2 \)
\( g(x) = \frac{1}{2}\sqrt{x} \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if

\[
[f \circ g](x) = [g \circ f](x) = x.
\]

\[
\begin{align*}
[f \circ g](x) &= f\left( g(x) \right) \\
&= f\left( \frac{1}{2}\sqrt{x} \right) \\
&= 4\left( \frac{1}{2}\sqrt{x} \right)^2 \\
&= \frac{1}{2}\sqrt{4x^2} \\
&= x \\
[g \circ f](x) &= g\left( f(x) \right) \\
&= g(4x^2) \\
&= \frac{1}{2}\sqrt{4x^2} \\
&= x
\end{align*}
\]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

**ANSWER:**
Yes
34. \( f(x) = \frac{1}{3}x^2 + 1 \)
\( g(x) = \sqrt{3x - 3} \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if 
\[ [f \circ g](x) = [g \circ f](x) = x. \]

\[
(f \circ g)(x) = f[g(x)] \\
= f\left(\sqrt{3x - 3}\right) \\
= \frac{1}{3}\left(\sqrt{3x - 3}\right)^2 + 1 \\
= x - 1 + 1 \\
= x
\]

\[
g \circ f)(x) = g[f(x)] \\
= g\left(\frac{1}{3}x^2 + 1\right) \\
= \sqrt{\frac{1}{3}x^2 + 1} - 3 \\
= \sqrt{x^2} \\
= x
\]

\[ [f \circ g](x) = [g \circ f](x) = x \]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

**ANSWER:**
Yes

35. \( f(x) = x^2 - 9 \)
\( g(x) = x + 3 \)

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if 
\[ [f \circ g](x) = [g \circ f](x) = x. \]

\[
(f \circ g)(x) = f[g(x)] \\
= f(x + 3) \\
= (x + 3)^2 - 9 \\
= x^2 + 6x + 9 - 9 \\
= x^2 + 6x
\]

\[ [f \circ g](x) \neq x \]

No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No
6-2 Inverse Functions and Relations

\[ f(x) = \frac{2}{3}x^3 \]
\[ g(x) = \frac{2}{\sqrt{3}}x \]

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if
\[ (f \circ g)(x) = (g \circ f)(x) = x. \]

\[
(f \circ g)(x) = f(g(x)) \\
= f\left(\frac{2}{\sqrt{3}}x\right) \\
= \frac{2}{3} \left(\frac{2}{\sqrt{3}}x\right)^3 \\
= \frac{2}{3} \left(\frac{2}{\sqrt{3}}\right)^3 \]

\[ (f \circ g)(x) \neq x \]

No, \( f(x) \) and \( g(x) \) are not inverse functions.

**ANSWER:**
No

37. \[ f(x) = (x + 6)^2 \]
\[ g(x) = \sqrt{x - 6} \]

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if
\[ (f \circ g)(x) = (g \circ f)(x) = x. \]

\[
(f \circ g)(x) = f(g(x)) \\
= f(\sqrt{x - 6}) \\
= (\sqrt{x - 6} + 6)^2 \\
= x \]

\[
(g \circ f)(x) = g(f(x)) \\
= g((x + 6)^2) \\
= \sqrt{(x + 6)^2} - 6 \\
= x \]

\[ (f \circ g)(x) = (g \circ f)(x) = x \]

Yes, \( f(x) \) and \( g(x) \) are inverse functions.

**ANSWER:**
Yes
6-2 Inverse Functions and Relations

\[ f(x) = 2\sqrt{x - 5} \]

38. \[ g(x) = \frac{1}{4} x^2 - 5 \]

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if \( [f \circ g](x) = [g \circ f](x) = x \).

\[
[f \circ g](x) = f(g(x)) \\
= f\left(\frac{1}{4} x^2 - 5\right) \\
= 2\sqrt{\frac{1}{4} x^2 - 5} - 5 \\
= 2\sqrt{x^2 - 10} \\
= 2\sqrt{x^2 - 40} \\
= 2\cdot\frac{\sqrt{x^2 - 40}}{\sqrt{4}} \\
= \frac{\sqrt{x^2 - 40}}{2} \\
= x
\]

Therefore, the functions are not inverses.

**ANSWER:**
No

39. **FUEL** The average miles traveled for every gallon \( g \) of gas consumed by Leroy’s car is represented by the function \( m(g) = 28g \).

a. Find a function \( c(g) \) to represent the cost per gallon of gasoline.

b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy’s car.

**SOLUTION:**

a. Let \( g \) be the number of gallons of gasoline.

The cost per gallon of gasoline is given by \( c(g) = 2.95g \).

b. The average miles traveled for every gallon \( g \) of gas consumed by Leroy’s car is:

\[ m(g) = 28g \]

So:

\[ g = \frac{m}{28} \]

Therefore, the cost per mile is given by:

\[ c(m) = 2.95 \times \frac{m}{28} \]

\[ \approx 0.105m \]

**ANSWER:**

a. \( c(g) = 2.95g \)

b. \( c(m) \approx 0.105m \)

40. **SHOES** The shoe size for the average U.S. teen or adult male can be determined using the formula \( M(x) = 3x - 22 \), where \( x \) is length of a foot in measured inches. The shoe size for the average U.S. teen or adult female can be found by using the formula \( F(x) = 3x - 21 \).

a. Find the inverse of each function.
6-2 Inverse Functions and Relations

b. If Lucy wears a size \( \frac{7}{2} \) shoe, how long are her feet?

**SOLUTION:**

\( M(x) = 3x - 22 \)

Rewrite the function as an equation relating \( x \) and \( y \).

\( y = 3x - 22 \)

Exchange \( x \) and \( y \).

\( x = 3y - 22 \)

Solve for \( y \).

\( F^{-1}(x) = \frac{x + 22}{3} \)

\( F(x) = 3x - 21 \)

Rewrite the function as an equation relating \( x \) and \( y \).

\( y = 3x - 21 \)

Exchange \( x \) and \( y \).

\( x = 3y - 21 \)

Solve for \( y \).

\( F^{-1}(x) = \frac{x + 21}{3} \)

b. Find \( F^{-1}\left(\frac{7}{2}\right) \).

\( F^{-1}\left(\frac{7}{2}\right) = \frac{2 + 21}{3} = \frac{23}{3} \)

\( = 9 \frac{1}{2} \text{ in} \)

**ANSWER:**

\( M^{-1}(x) = \frac{x + 22}{3}; F^{-1}(x) = \frac{x + 21}{3} \)

\( 9 \frac{1}{2} \text{ in.} \)

41. GEOMETRY The formula for the area of a circle is \( A = \pi r^2 \).

a. Find the inverse of the function.

b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

**SOLUTION:**

a.

\( A = \pi r^2 \)

\( r^2 = \frac{A}{\pi} \)

\( r = \sqrt{\frac{A}{\pi}} \)

b. Substitute \( A = 36 \).

\( r = \sqrt{\frac{36}{\pi}} \approx 3.39 \text{ cm} \)

**ANSWER:**

a. \( r = \sqrt{\frac{A}{\pi}} \)

b. \( \approx 3.39 \text{ cm} \)
Use the horizontal line test to determine whether the inverse of each function is also a function.

42. \( f(x) = 2x^2 \)

**SOLUTION:**
Graph the function \( f(x) = 2x^2 \).

![Graph of f(x) = 2x^2](image)

A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

**ANSWER:**
No

43. \( f(x) = x^3 - 8 \)

**SOLUTION:**
Graph the function \( f(x) = x^3 - 8 \).

![Graph of f(x) = x^3 - 8](image)

No horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is a function.

**ANSWER:**
Yes
Find the inverse of each relation.

1. \{(–9, 10), (1, –3), (8, –5)\}

**SOLUTION:** To find the inverse, exchange the coordinates of each ordered pair:

\{(10, −9), (−3, 1), (−5, 8)\}

The inverse relation is \{(10, −9), (−3, 1), (−5, 8)\}.

c. Using composition of functions, verify that these two functions are inverses.

**SOLUTION:**

Let \(f(x)\) be the function \(g(x) = x^3 - 6x^2 + 1\), and let \(g(x)\) be the function \(h(x) = -2x^4 - x - 2\). To verify that \(f\) and \(g\) are inverses, we need to check if \(f(g(x)) = x\) and \(g(f(x)) = x\) for all \(x\) in the domain of \(f\).

For \(f(g(x)) = x\):

\[f(g(x)) = f(x^3 - 6x^2 + 1)\]

\[= (x^3 - 6x^2 + 1)^3 - 6(x^3 - 6x^2 + 1)^2 + 1\]

For \(g(f(x)) = x\):

\[g(f(x)) = g(x^3 - 6x^2 + 1)\]

\[= -2(x^3 - 6x^2 + 1)^4 - (x^3 - 6x^2 + 1) - 2\]

Since \(f(g(x)) = x\) and \(g(f(x)) = x\) for all \(x\), \(f\) and \(g\) are inverses.

---

44. \(g(x) = x^3 - 6x^2 + 1\)

**SOLUTION:**

Graph the function \(g(x) = x^3 - 6x^2 + 1\).

A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

**ANSWER:**

No

45. \(h(x) = -2x^4 - x - 2\)

**SOLUTION:**

Graph the function \(h(x) = -2x^4 - x - 2\).

A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

**ANSWER:**

No
6-2 Inverse Functions and Relations

46. \( g(x) = x^5 + x^3 - 4x \)

**SOLUTION:**
Graph the function \( g(x) = x^5 + x^3 - 4x \).

A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

**ANSWER:** No

47. \( h(x) = x^3 + x^2 - 6x + 12 \)

**SOLUTION:**
Graph the function \( h(x) = x^3 + x^2 - 6x + 12 \).

A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

**ANSWER:** No
48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid $350 in processing and registration fees. Find the selling price if Felipe paid a total of $8395.75.

**SOLUTION:**
Let \( x \) be the selling price of the car.

\[
\begin{align*}
  x + 7.25\% (x) + 350 &= 8395.75 \\
  x + 0.0725x + 350 &= 8395.75 \\
  1.0725x &= 8395.75 - 350 \\
  1.0725x &= 8045.75 \\
  x &\approx 7501.86
\end{align*}
\]

The selling price is $7501.86.

**ANSWER:**
$7501.86

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is \( F(x) = \frac{9}{5}x + 32 \).

a. Find the inverse \( F^{-1}(x) \). Show that \( F(x) \) and \( F^{-1}(x) \) are inverses.

b. Explain what purpose \( F^{-1}(x) \) serves.

**SOLUTION:**

a. Rewrite the function as an equation relating \( x \) and \( y \).

\[
y = \frac{9}{5}x + 32
\]

Exchange \( x \) and \( y \) in the equation.

\[
x = \frac{9}{5}y + 32
\]

Solve for \( y \).

\[
y = \frac{5}{9}(x - 32)
\]

Therefore:

\[
F^{-1}(x) = \frac{5}{9}(x - 32)
\]

The functions \( f(x) \) and \( g(x) \) are inverses if and only if

\[
[f \circ g](x) = [g \circ f](x) = x.
\]

\[
F[F^{-1}(x)] = F\left[\frac{5}{9}(x - 32)\right]
\]

\[
= \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32
\]

\[
= x
\]

\[
F^{-1}[F(x)] = F^{-1}\left[\frac{9}{5}x + 32\right]
\]

\[
= \frac{5}{9}\left[\frac{9}{5}x + 32 - 32\right]
\]

\[
= x
\]

Therefore, \( F(x) \) and \( F^{-1}(x) \) are inverses.

b. \( F^{-1}(x) \) can be used to convert Fahrenheit to Celsius.

**ANSWER:**

a. 

\[
F^{-1}(x) = \frac{5}{9}(x - 32);
\]

\[
F[F^{-1}(x)] = \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32
\]

\[
= -32 + 32
\]

\[
= x
\]

\[
F^{-1}[F(x)] = \frac{5}{9}\left[\frac{9}{5}x + 32 - 32\right]
\]

\[
= \frac{5}{9}\left[\frac{9}{5}x + 0\right]
\]

\[
= x
\]

b. It can be used to convert Fahrenheit to Celsius.
50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.

a. Write a function that converts nautical miles to kilometers.

b. Find the inverse of the function that converts kilometers back to nautical miles.

c. Using composition of functions, verify that these two functions are inverses.

**SOLUTION:**

a. Let \( m \) be the number of miles.

The number of kilometers in \( m \) miles is given by 

\[
K(m) = 1.852m.
\]

b. Inverse is given by 

\[
K^{-1}(m) = \frac{1}{1.852}m.
\]

c. \( K[K^{-1}(m)] = K^{-1}[K(m)] = m. \)

Therefore, the functions are inverses.

**ANSWER:**

a. \( K(m) = 1.852m \)

b. \( K^{-1}(m) = \frac{1}{1.852}m \)

c. \( K[K^{-1}(m)] = m \) and \( K^{-1}[K(m)] = m, \) so the two functions are inverses of each other.

51. **MULTIPLE REPRESENTATIONS** Consider the functions \( y = x^n \) for \( n = 0, 1, 2, \ldots \)

a. **GRAPHING** Use a graphing calculator to graph \( y = x^n \) for \( n = 0, 1, 2, 3, \) and 4.

b. **TABULAR** For which values of \( n \) is the inverse a function? Record your results in a table.

c. **ANALYTICAL** Make a conjecture about the values of \( n \) for which the inverse of \( f(x) = x^n \) is a
b. 

<table>
<thead>
<tr>
<th>Function</th>
<th>Inverse a function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^0$ or $y = 1$</td>
<td>no</td>
</tr>
<tr>
<td>$y = x^1$ or $y = x$</td>
<td>yes</td>
</tr>
<tr>
<td>$y = x^2$</td>
<td>no</td>
</tr>
<tr>
<td>$y = x^3$</td>
<td>yes</td>
</tr>
<tr>
<td>$y = x^4$</td>
<td>no</td>
</tr>
</tbody>
</table>

C. When $n$ is odd, the function passes the horizontal line test. Therefore, when $n$ is odd, the inverse of the function is a function.

**ANSWER:**

A. 

B. 

<table>
<thead>
<tr>
<th>Function</th>
<th>Inverse a function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = x^0$ or $y = 1$</td>
<td>no</td>
</tr>
<tr>
<td>$y = x^1$ or $y = x$</td>
<td>yes</td>
</tr>
<tr>
<td>$y = x^2$</td>
<td>no</td>
</tr>
<tr>
<td>$y = x^3$</td>
<td>yes</td>
</tr>
<tr>
<td>$y = x^4$</td>
<td>no</td>
</tr>
</tbody>
</table>

C. $n$ is odd.
52. **REASONING** If a relation is not a function, then its inverse is sometimes, always, or never a function. Explain your reasoning.

**SOLUTION:**
Sample answer: Sometimes; \( y = \pm \sqrt{x} \) is an example of a relation that is not a function, with an inverse being a function. A circle is an example of a relation that is not a function with an inverse not being a function.

**ANSWER:**
Sample answer: Sometimes; \( y = \pm \sqrt{x} \) is an example of a relation that is not a function, with an inverse being a function. A circle is an example of a relation that is not a function with an inverse not being a function.

53. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

**SOLUTION:**
Sample answer: 
\[
 f(x) = 2x, f^{-1}(x) = 0.5x; f[f^{-1}(x)] = f^{-1}[f(x)] = x
\]

**ANSWER:**
Sample answer: 
\[
 f(x) = 2x, f^{-1}(x) = 0.5x;
 f[f^{-1}(x)] = f^{-1}[f(x)] = x
\]

54. **CHALLENGE** Give an example of a function that is its own inverse.

**SOLUTION:**
Sample answer: \( f(x) = x \) and \( f^{-1}(x) = x \) or \( f(x) = -x \) and \( f^{-1}(x) = -x \)

**ANSWER:**
Sample answer: \( f(x) = x \) and \( f^{-1}(x) = x \) or \( f(x) = -x \) and \( f^{-1}(x) = -x \)

55. **CCSS ARGUMENTS** Show that the inverse of a linear function \( y = mx + b \), where \( m \neq 0 \) and \( x \neq b \), is also a linear function.

**SOLUTION:**
\[
y = mx + b
\]

To find the inverse, exchange \( x \) and \( y \), and solve for \( y \).

\[
x = my + b
\]

\[
x - b = my
\]

\[
\frac{x - b}{m} = y
\]

\[
y = \frac{x - b}{m}
\]

\[
y = \frac{1}{m}x - \frac{b}{m}
\]

**ANSWER:**
The inverse function is \( y = \frac{1}{m}x - \frac{b}{m} \).
56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for \( x \), why is the result always 5?

**SOLUTION:**
Sample answer: One of the functions carries out an operation on 5. Then the second function that is an inverse of the first function reverse the operation on 5. Thus, the result is 5.

**ANSWER:**
Sample answer: One of the functions carries out an operation on 5. Then the second function that is an inverse of the first function reverse the operation on 5. Thus, the result is 5.

57. **SHORT RESPONSE** If the length of a rectangular television screen is 24 inches and its height is 18 inches, what is the length of its diagonal in inches?

**SOLUTION:**
Let \( x \) be the length of the diagonal.

\[
x = \sqrt{24^2 + 18^2}
\]

\[
= \sqrt{900}
\]

\[
= 30
\]

The length of the diagonal is 30 inches.

**ANSWER:**
30 in.

58. **GEOMETRY** If the base of a triangle is represented by \( 2x + 5 \) and the height is represented by \( 4x \), which expression represents the area of the triangle?

- **A** \((2x + 5) + (4x)\)
- **B** \((2x + 5)(4x)\)
- **C** \(\frac{1}{2}(2x + 5) + (4x)\)
- **D** \(\frac{1}{2}(2x + 5)(4x)\)

**SOLUTION:**
The area of a triangle is given by \( A = \frac{1}{2}bh \), where \( b \) is the base and \( h \) is the height. \( A = \frac{1}{2}(2x + 5)(4x) \)

The correct choice is **D**.

**ANSWER:**
D
59. Which expression represents \( f [g(x)] \) if \( f(x) = x^2 + 3 \) and \( g(x) = -x + 1 \)?

\[
\begin{align*}
F & \quad x^2 - x + 2 \\
G & \quad -x^2 - 2 \\
H & \quad -x^3 + x^2 - 3x + 3 \\
J & \quad x^2 - 2x + 4 \\
\end{align*}
\]

**SOLUTION:**

\[
\begin{align*}
 f [g(x)] &= f(-x + 1) \\
&= (-x + 1)^2 + 3 \\
&= x^2 - 2x + 1 - 2x + 3 \\
&= x^2 - 2x + 4
\end{align*}
\]

The correct choice is **J**.

**ANSWER:**

J

60. SAT/ACT Which of the following is the inverse of \( f(x) = \frac{3x - 5}{2} \)?

\[
\begin{align*}
A & \quad g(x) = \frac{2x + 5}{3} \\
B & \quad g(x) = \frac{2x - 5}{3} \\
C & \quad g(x) = \frac{3x + 5}{2} \\
D & \quad g(x) = 2x + 5 \\
E & \quad g(x) = \frac{3x - 5}{2}
\end{align*}
\]

**SOLUTION:**

Rewrite the function as an equation relating \( x \) and \( y \).

\[
y = \frac{3x - 5}{2}
\]

Exchange \( x \) and \( y \) in the equation.

\[
x = \frac{3y - 5}{2}
\]

Solve for \( y \).

\[
y = \frac{2x + 5}{3}
\]

Therefore:

\[
g(x) = \frac{2x + 5}{3}
\]

The correct choice is **A**.

**ANSWER:**

A
6-2 Inverse Functions and Relations

If \( f(x) = 3x + 5 \), \( g(x) = x - 2 \), and \( h(x) = x^2 - 1 \), find each value.

61. \( g[f(3)] \)

\[ g[f(3)] = g(3x + 5) = 3x + 5 - 2 = 3x + 3 \]

Substitute \( x = 3 \).

\[ g[f(3)] = 3(3) + 3 = 12 \]

**ANSWER:** 12

62. \( f[h(-2)] \)

\[ f[h(-2)] = f(x^2 - 1) = 3(x^2 - 1) + 5 = 3x^2 + 2 \]

Substitute \( x = -2 \).

\[ f[h(-2)] = 3(-2)^2 + 2 = 14 \]

**ANSWER:** 14

63. \( h[g(1)] \)

\[ h[g(x)] = h(x - 2) = (x - 2)^2 - 1 = x^2 - 4x + 3 \]

Substitute \( x = 1 \).

\[ h[g(1)] = 0 \]

**ANSWER:** 0

64. CONSTRUCTION A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base, and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet?

**SOLUTION:**

Let \( x \) be the length of the shorter base.

Therefore, the length of the longer base is \( 3x + 8 \) and height is \( 3x + 1 \).

The area of a trapezoid is given by

\[ A = \frac{1}{2}h(b_1 + b_2). \]

\[ 4104 = \frac{1}{2}(3x + 1)(x + 3x + 8) \]

\[ 8208 = (3x + 1)(4x + 8) \]

\[ 8208 = 12x^2 + 24x + 4x + 8 \]

\[ 8208 = 12x^2 + 28x + 8 \]

\[ 12x^2 + 28x - 8200 = 0 \]

Use the Quadratic formula to solve for \( x \).
6-2 Inverse Functions and Relations

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

65. \( x^2 + 34x + c \)

\[ \text{SOLUTION:} \]
To find the value of c, divide the coefficient of x by 2, and square the result.

\[ c = \left( \frac{34}{2} \right)^2 \]
\[ = 289 \]
\[ x^2 + 34x + 289 = (x + 17)^2 \]

\[ \text{ANSWER:} \]
289; \((x + 17)^2\)

66. \( x^2 - 11x + c \)

\[ \text{SOLUTION:} \]
To find the value of c, divide the coefficient of x by 2, and square the result.

Therefore:

\[ c = \left( \frac{-11}{2} \right)^2 \]
\[ = \frac{121}{4} \]

\[ x^2 - 11 + \frac{121}{4} = \left( x - \frac{11}{2} \right)^2 \]

\[ \text{ANSWER:} \]
\[ \frac{121}{4}; \left( x - \frac{11}{2} \right)^2 \]

\[ x = \frac{-28 \pm \sqrt{28^2 - 4(12)(-8200)}}{2(12)} \]
\[ = \frac{-28 \pm \sqrt{784 + 393600}}{24} \]
\[ = \frac{-28 \pm \sqrt{394384}}{24} \]
\[ = \frac{-28 \pm 628}{24} \]
\[ x = \frac{-28 + 628}{24} \text{ or } x = \frac{-28 - 628}{24} \]
\[ x = \frac{600}{24} \text{ or } x = \frac{-656}{24} \]
\[ x = 25 \text{ or } x = -\frac{656}{24} \]

Since \( x \) is the length of the shorter base, it cannot be negative. Therefore, \( x = 25 \) feet.
The length of the longer base is 83 feet, and the height is 76 feet.

\[ \text{ANSWER:} \]
\( b_1 = 25 \text{ ft, } b_2 = 83 \text{ ft, } h = 76 \text{ ft} \]
6-2 Inverse Functions and Relations

Simplify.

67. \((3 + 4i)(5 - 2i)\)

**SOLUTION:**
\[
(3 + 4i)(5 - 2i) = 15 - 6i + 20i + 8
\]
\[
= 23 + 14i
\]

**ANSWER:**

23 + 14i

68. \((\sqrt{6} + i)(\sqrt{6} - i)\)

**SOLUTION:**
\[
(\sqrt{6} + i)(\sqrt{6} - i) = (\sqrt{6})^2 - i^2
\]
\[
= 6 + 1
\]
\[
= 7
\]

**ANSWER:**

7

69. \(\frac{1 + i}{1 - i}\)

**SOLUTION:**
\[
\frac{1 + i}{1 - i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}
\]
\[
= \frac{2i}{2}
\]
\[
= i
\]

**ANSWER:**

i

70. \(\frac{4 - 3i}{1 + 2i}\)

**SOLUTION:**
\[
\frac{4 - 3i}{1 + 2i} = \frac{4 - 3i}{1 + 2i} \times \frac{1 - 2i}{1 - 2i}
\]
\[
= \frac{4 - 8i - 3i - 6}{1 + 4}
\]
\[
= \frac{-2 - 11i}{5}
\]
\[
= \frac{-2}{5} - \frac{11}{5}i
\]

**ANSWER:**

\(-\frac{2}{5} - \frac{11}{5}i\)
Determine the rate of change of each graph.

71. 

**SOLUTION:**
The graph passes through the points (0, -2) and (4, 0).

\[
\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{0 - (-2)}{4 - 0} = \frac{2}{4} = \frac{1}{2}
\]

So, the rate of change is \(\frac{1}{2}\).

**ANSWER:**
\(\frac{1}{2}\)
6-2 Inverse Functions and Relations

SOLUTION:
The graph passes through the points (2, 0) and (6, 6).

Rate of change = \frac{\text{change in } y}{\text{change in } x}
\hspace{1cm} = \frac{6 - 0}{6 - 2}
\hspace{1cm} = \frac{5}{4}
\hspace{1cm} = \frac{3}{2}

So, the rate of change is \frac{3}{2}.

ANSWER:
\frac{3}{2}

Graph each inequality.

74. \( y > \frac{3}{4}x - 2 \)

SOLUTION:
Graph the boundary line \( y = \frac{3}{4}x - 2 \). Since the inequality symbol is \( > \), the boundary line should be dashed.

Test the point (0,0).

\[ 0 > \frac{3}{4}(0) - 2 \]
\[ 0 > -2 \checkmark \]

Shade the region that contains the point (0, 0).
75. \( y \leq -3x + 2 \)

**SOLUTION:**
Graph the boundary line \( y = -3x + 2 \).

Since the inequality symbol is \( \leq \), the boundary line should be solid.

Test the point \((0, 0)\).

\[
0 \leq -3(0) + 2 \\
0 \leq 2 \checkmark
\]

Shade the region that contains \((0, 0)\).

**ANSWER:**

76. \( y < -x - 4 \)

**SOLUTION:**
Graph the boundary line \( y = -x - 4 \).

Since the inequality symbol is \(<\), the boundary line should be dashed.

Test the point \((0, 0)\).

\[
0 < 0 - 4 \\
0 < -4 \times
\]

Therefore, shade the region that does not include \((0, 0)\).
Identify the domain and range of each function.

1. \( f(x) = \sqrt{4x} \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x | x \geq 0 \} \\
R = \{ f(x) | f(x) \geq 0 \}
\]

**ANSWER:**
\( D = \{ x | x \geq 0 \} \); \( R = \{ f(x) | f(x) \geq 0 \} \)

2. \( f(x) = \sqrt{x - 5} \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x | x \geq 5 \} \\
R = \{ f(x) | f(x) \geq 0 \}
\]

**ANSWER:**
\( D = \{ x | x \geq 5 \} \); \( R = \{ f(x) | f(x) \geq 0 \} \)

3. \( f(x) = \sqrt{x + 8} - 2 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

So:
\[
x + 8 \geq 0 \\
x \geq -8
\]

\[
D = \{ x | x \geq -8 \}
\]

Find \( f(-8) \) to determine the lower limit of the range.

\[
f(-2) = \sqrt{-8 + 8} - 2 \\
= -2
\]

\[
R = \{ f(x) | f(x) \geq -2 \}
\]

**ANSWER:**
\( D = \{ x | x \geq -8 \} \); \( R = \{ f(x) | f(x) \geq -2 \} \)

Graph each function. State the domain and range.
4. \( f(x) = \sqrt{x} - 2 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ D = \{ x \mid x \geq 0 \} \]

Find \( f(0) \) to determine the lower limit of the range.

\[ f(0) = -2 \]

Therefore:

\[ R = \{ f(x) \mid f(x) \geq -2 \} \]

**ANSWER:**

\[ D = \{ x \mid x \geq 0 \} ; R = \{ f(x) \mid f(x) \geq -2 \} \]

5. \( f(x) = 3\sqrt{x} - 1 \)
6-3 Square Root Functions and Inequalities

6. \( f(x) = \frac{1}{2} \sqrt{x + 4} - 1 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

So:
\[ x + 4 \geq 0 \]
\[ x \geq -4 \]

\[ D = \{ x | x \geq -4 \} \]

Find \( f(-4) \) to determine the lower limit of the range.

\[ f(-4) = \frac{1}{2} \sqrt{-4 + 4} - 1 \]
\[ = -1 \]

\[ R = \{ f(x) | f(x) \geq -1 \} \]

**ANSWER:**
\[ D = \{ x | x \geq -4 \} ; R = \{ f(x) | f(x) \geq -1 \} \]

7. \( f(x) = -\sqrt{3x - 5} + 5 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ 3x - 5 \geq 0 \]
\[ x \geq \frac{5}{3} \]

\[ D = \{ x | x \geq \frac{5}{3} \} \]

Find \( f\left(\frac{5}{3}\right) \) to determine the upper limit of the range.

\[ R = \{ f(x) | f(x) \leq 5 \} \]

**ANSWER:**
Identify the domain and range of each function.

1. SOLUTION: The domain of a square root function only includes values for which the radicand is nonnegative. The domain is \( \left\{ x \mid x \geq \frac{5}{3} \right\} \); \( R = \{ f(x) \mid f(x) \leq 5 \} \).

8. **OCEAN** The speed that a tsunami, or tidal wave, can travel is modeled by the equation \( v = 356\sqrt{d} \), where \( v \) is the speed in kilometers per hour and \( d \) is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth of a kilometer.

**SOLUTION:**
\[ v = 356\sqrt{d} \]
Substitute \( v = 145 \) and find \( d \).
\[ 145 = 356\sqrt{d} \]
\[ \sqrt{d} = \frac{145}{356} \]
\[ d \approx 0.17 \text{ km} \]

The average depth of the water is about 0.17 km.

**ANSWER:**
0.17 km

Graph each inequality.

9. \( f(x) \geq \sqrt{x} + 4 \)

**SOLUTION:**
\( D = \{ x \mid x \geq 0 \} \)
\( R = \{ y \mid y \geq 4 \} \)

Graph the \( f(x) \geq \sqrt{x} + 4 \).
6-3 Square Root Functions and Inequalities

10. \( f(x) \leq \sqrt{x - 6} + 2 \)

\textbf{SOLUTION:}
\[ D = \{ x | x \geq 6 \} \]
\[ R = \{ y | y \geq 2 \} \]

Graph the inequality \( f(x) \leq \sqrt{x - 6} + 2 \).

\textbf{ANSWER:}

11. \( f(x) < -2\sqrt{x + 3} \)

\textbf{SOLUTION:}
\[ D = \{ x | x \geq -3 \} \]
\[ R = \{ y | y < 0 \} \]

Graph the inequality \( f(x) < -2\sqrt{x + 3} \).

\textbf{ANSWER:}
Identify the domain and range of each function.

12. \( f(x) > \sqrt{2x-1} - 3 \)

SOLUTION:

\[
D = \left\{ x \mid x \geq \frac{1}{2} \right\}
\]

\[
R = \{ y \mid y > -3 \}
\]

13. \( f(x) = -\sqrt{2x} + 2 \)

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x \mid x \geq 0 \}
\]

Find \( f(0) \) to determine the upper limit of the range.

\[
R = \{ f(x) \mid f(x) \leq 2 \}
\]

ANSWER:

\[
D = \{ x \mid x \geq 0 \}; \quad R = \{ f(x) \mid f(x) \leq 2 \}
\]

14. \( f(x) = \sqrt{x} - 6 \)

SOLUTION:

The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x \mid x \geq 0 \}
\]

\[
R = \{ f(x) \mid f(x) \geq -6 \}
\]

ANSWER:

\[
D = \{ x \mid x \geq 0 \}; \quad R = \{ f(x) \mid f(x) \geq -6 \}
\]
15. \( f(x) = 4\sqrt{x-2} - 8 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ D = \{ x | x \geq 2 \} \]

Find \( f(2) \) to determine the lower limit of the range.

\[ f(2) = -8 \]

\[ R = \{ f(x) | f(x) \geq -8 \} \]

**ANSWER:**
\[ D = \{ x | x \geq 2 \} ; R = \{ f(x) | f(x) \geq -8 \} \]

16. \( f(x) = \sqrt{x+2} + 5 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ D = \{ x | x \geq -2 \} \]

Find \( f(-2) \) to determine the lower limit of the range.

\[ f(-2) = 5 \]

\[ R = \{ f(x) | f(x) \geq 5 \} \]

**ANSWER:**
\[ D = \{ x | x \geq -2 \} ; R = \{ f(x) | f(x) \geq 5 \} \]

17. \( f(x) = \sqrt{x-4} - 6 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ D = \{ x | x \geq 4 \} \]

Find \( f(4) \) to determine the lower limit of the range.

\[ R = \{ f(x) | f(x) \geq -6 \} \]

**ANSWER:**
\[ D = \{ x | x \geq 4 \} ; R = \{ f(x) | f(x) \geq -6 \} \]

18. \( f(x) = -\sqrt{x-6} + 5 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.

\[ D = \{ x | x \geq 6 \} \]

Find \( f(6) \) to determine the upper limit of the range.

\[ R = \{ f(x) | f(x) \leq 5 \} \]

**ANSWER:**
\[ D = \{ x | x \geq 6 \} ; R = \{ f(x) | f(x) \leq 5 \} \]
6-3 Square Root Functions and Inequalities

Graph each function. State the domain and range.

19. \( f(x) = \sqrt{6x} \)

**SOLUTION:**

\[ \text{D} = \{ x \mid x \geq 0 \} \]
\[ \text{R} = \{ f(x) \mid f(x) \geq 0 \} \]

**ANSWER:**

\[ \text{D} = \{ x \mid x \geq 0 \}; \text{R} = \{ f(x) \mid f(x) \geq 0 \} \]

20. \( f(x) = -\sqrt{5x} \)

**SOLUTION:**

The domain of a square root function only includes values for which the radicand is nonnegative.

\[ \text{D} = \{ x \mid x \geq 0 \} \]
\[ \text{R} = \{ f(x) \mid f(x) \leq 0 \} \]

**ANSWER:**

\[ \text{D} = \{ x \mid x \geq 0 \}; \text{R} = \{ f(x) \mid f(x) \leq 0 \} \]
21. \( f(x) = \sqrt{x - 8} \)

**SOLUTION:**

The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x \mid x \geq 8 \} \\
R = \{ f(x) \mid f(x) \geq 0 \}
\]

![Graph of \( f(x) = \sqrt{x - 8} \)](image)

**ANSWER:**

\[
D = \{ x \mid x \geq 8 \} ; R = \{ f(x) \mid f(x) \geq 0 \}
\]

22. \( f(x) = \sqrt{x + 1} \)

**SOLUTION:**

The domain of a square root function only includes values for which the radicand is nonnegative.

\[
D = \{ x \mid x \geq -1 \} \\
R = \{ f(x) \mid f(x) \geq 0 \}
\]

![Graph of \( f(x) = \sqrt{x + 1} \)](image)

**ANSWER:**

\[
D = \{ x \mid x \geq -1 \} ; R = \{ f(x) \mid f(x) \geq 0 \}
\]
6-3 Square Root Functions and Inequalities

23. \( f(x) = \sqrt{x + 3} + 2 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.
\[ D = \{ x | x \geq -3 \} \]

Find \( f(-3) \) to determine the lower limit of the range.
\[ f(-3) = 2 \]
\[ R = \{ f(x) | f(x) \geq 2 \} \]

**ANSWER:**

\[ D = \{ x | x \geq -3 \} \quad R = \{ f(x) | f(x) \geq 2 \} \]

24. \( f(x) = \sqrt{x - 4} - 10 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.
\[ D = \{ x | x \geq 4 \} \]

Find \( f(4) \) to determine the lower limit of the range.
\[ f(4) = -10 \]
\[ R = \{ f(x) | f(x) \geq -10 \} \]

**ANSWER:**

\[ D = \{ x | x \geq 4 \} \quad R = \{ f(x) | f(x) \geq -10 \} \]
25. \( f(x) = 2\sqrt{x-5} - 6 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative. 
\[ D = \{x|x \geq 5\} \]

Find \( f(5) \) to determine the lower limit of the range. 
\[ f(4) = -6 \]
\[ R = \{f(x)|f(x) \geq -6\} \]

**ANSWER:**
\[ D = \{x|x \geq 5\} \quad R = \{f(x)|f(x) \geq -6\} \]

26. \( f(x) = \frac{3}{4}\sqrt{x + 12} + 3 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative. 
\[ D = \{x|x \geq -12\} \]

Find \( f(-12) \) to determine the lower limit of the range. 
\[ f(4) = 3 \]
\[ R = \{f(x)|f(x) \geq 3\} \]

**ANSWER:**
\[ D = \{x|x \geq -12\} \quad R = \{f(x)|f(x) \geq 3\} \]
27. \( f(x) = \frac{1}{5} \sqrt{x-1} - 4 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.
\[ D = \{ x | x \geq 1 \} \]

Find \( f(1) \) to determine the upper limit of the range.
\[ f(4) = -4 \]
\[ R = \{ f(x) | f(x) \leq -4 \} \]

**ANSWER:**
\[ D = \{ x | x \geq 1 \} \]
\[ R = \{ f(x) | f(x) \leq -4 \} \]

28. \( f(x) = -3\sqrt{x} + 7 + 9 \)

**SOLUTION:**
The domain of a square root function only includes values for which the radicand is nonnegative.
\[ D = \{ x | x \geq -7 \} \]

Find \( f(1) \) to determine the upper limit of the range.
\[ f(4) = 9 \]
\[ R = \{ f(x) | f(x) \leq 9 \} \]

**ANSWER:**
\[ D = \{ x | x \geq -7 \} \]
\[ R = \{ f(x) | f(x) \leq 9 \} \]
29. **SKYDIVING** The approximate time \( t \) in seconds that it takes an object to fall a distance of \( d \) feet is given by \( t = \sqrt{\frac{d}{16}} \). Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time?

**SOLUTION:**
Substitute \( t = 11 \) in the equation and find \( d \).

\[
11 = \sqrt{\frac{d}{16}}
\]

\[
121 = \frac{d}{16}
\]

\[
d = 121(16)
\]

\[
d = 1936
\]

The parachutist falls 1936 feet.

**ANSWER:**
1936 ft

30. **CCSS MODELING** The velocity of a roller coaster as it moves down a hill is \( V = \sqrt{v^2 + 64h} \) where \( v \) is the initial velocity in feet per second and \( h \) is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.

a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation.

**SOLUTION:**

a. Substitute \( v = 10 \) and \( V = 90 \).

\[
90 = \sqrt{100 + 64h}
\]

b. Solve the equation \( 90 = \sqrt{100 + 64h} \) for \( h \).

\[
90 = \sqrt{100 + 64h}
\]

\[
8100 = 100 + 64h
\]

\[
64h = 8000
\]

\[
h = 125
\]

The designer should make the hill at a height of 125 feet.

**ANSWER:**

a. \( 90 = \sqrt{100 + 64h} \)

b. 125 ft
6-3 Square Root Functions and Inequalities

Graph each inequality.

31. \( y < \sqrt{x - 5} \)

**SOLUTION:**

Graph the boundary \( y = \sqrt{x - 5} \). Since the inequality symbol is \(<\), the boundary should be dashed.

The domain is \( \{x | x \geq 5\} \). Because \( y \) is less than, the shaded region should be below the boundary and within the domain.

32. \( y > \sqrt{x + 6} \)

**SOLUTION:**

Graph the boundary \( y = \sqrt{x + 6} \). Since the inequality symbol is \(>\), the boundary should be dashed.

The domain is \( \{x | x \geq -6\} \). Because \( y \) is greater than, the shaded region should be above the boundary and within the domain.
33. \( y \geq -4\sqrt{x} + 3 \)

**SOLUTION:**
Graph the boundary \( y = -4\sqrt{x} + 3 \). Since the inequality symbol is \( \geq \), the boundary line should be solid.

The domain is \( \{ x \mid x \geq -3 \} \). Because \( y \) is greater than, the shaded region should be above the boundary and within the domain.

\[ \text{ANSWER:} \]

34. \( y \leq -2\sqrt{x} - 6 \)

**SOLUTION:**
Graph the boundary \( y = -2\sqrt{x} - 6 \). Since the inequality symbol is \( \leq \), the boundary line should be solid.

The domain is \( \{ x \mid x \geq 6 \} \). Because \( y \) is less than, the shaded region should be below the boundary and within the domain.

\[ \text{ANSWER:} \]
Identify the domain and range of each function.

35. \( y > 2\sqrt{x + 7} - 5 \)

**SOLUTION:**

Graph the boundary \( y = 2\sqrt{x + 7} - 5 \). Since the inequality symbol is \( > \), the boundary line should be dashed.

The domain is \( \{x|x \geq -7\} \). Because \( y \) is greater than, the shaded region should be above the boundary and within the range.

![Graph of \( y > 2\sqrt{x + 7} - 5 \)](image)

**ANSWER:**

36. \( y \geq 4\sqrt{x - 2} - 12 \)

**SOLUTION:**

Graph the boundary \( y = 4\sqrt{x - 2} - 12 \). Since the inequality symbol is \( \geq \), the boundary line should be solid.

The domain is \( \{x|x \geq 2\} \). Because \( y \) is greater than, the shaded region should be above the boundary and within the range.

![Graph of \( y \geq 4\sqrt{x - 2} - 12 \)](image)

**ANSWER:**
6-3 Square Root Functions and Inequalities

37. \( y \leq 6 - 3\sqrt{x - 4} \)

**SOLUTION:**
Graph the boundary \( y = 6 - 3\sqrt{x - 4} \). Since the inequality symbol is \( \leq \), the boundary line should be solid.

The domain is \( \{x|x \geq 4\} \). Because \( y \) is less than, the shaded region should be below the boundary and within the range.

**ANSWER:**

38. \( y < \sqrt{4x - 12} + 8 \)

**SOLUTION:**
Graph the boundary \( y = \sqrt{4x - 12} + 8 \). Since the inequality symbol is \( < \), the boundary line should be dashed.

The domain is \( \{x|x \geq 3\} \). Because \( y \) is less than, the shaded region should be below the boundary and within the range.

**ANSWER:**

39. **PHYSICS** The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules, is \( E = 0.5mv^2 \), where \( m \) is the mass in kilograms and \( v \) is the velocity of the object in meters per second.

a. Solve the above formula for \( v \).
b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?

c. Escape velocity is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object.

Suppose a 100,000-kilogram ship must have a kinetic energy of \(3.624 \times 10^{14}\) joules to escape the gravitational field of Jupiter. Estimate the escape velocity of Jupiter.

**SOLUTION:**

a. \(E = 0.5mv^2\)

Solve for \(v\).

\[
\frac{2E}{m} = v^2
\]

\[
\sqrt{\frac{2E}{m}} = v
\]

b. Substitute \(E = 1,000,000\) and \(m = 1500\). Find \(v\).

\[
v = \sqrt{\frac{2,000,000}{1500}}
\approx 36.5 \text{ m/s}
\]

c. Substitute \(E = 3.624 \times 10^{14}\) and \(m = 100,000\). Find \(v\).

\[
v = \sqrt{\frac{7.248 \times 10^{14}}{100,000}}
\approx 85135 \text{ m/s}
\]

**ANSWER:**

a. \(v = \sqrt{\frac{2E}{m}}\)

b. about 36.5 m/s

c. about 85,135 m/s

40. **CCSS REASONING** After an accident, police can determine how fast a car was traveling before the driver put on his or her brakes by using the equation \(v = \sqrt{30fd}\). In this equation, \(v\) represents the speed in miles per hour, \(f\) represents the coefficient of friction, and \(d\) represents the length of the skid marks in feet. The coefficient of friction varies depending on road conditions. Assume that \(f = 0.6\).

a. Find the speed of a car that skids 25 feet.

b. If your car is going 35 miles per hour, how many feet would it take you to stop?

c. If the speed of a car is doubled, will the skid be twice as long? Explain.

**SOLUTION:**

a. Substitute \(f = 0.6\) and \(d = 25\). Find \(v\).

\[
v = \sqrt{30 \times 0.6 \times 25}
\approx 21.2 \text{ mph}
\]

b. \(v^2 = 30fd\)

\[
d = \frac{v^2}{30f}
\]

Substitute \(v = 35\) and \(f = 0.6\).

\[
d = \frac{35^2}{30 \times 0.6}
\approx 68 \text{ ft}
\]

c. No; it is not a linear function. The skid will be 4 times as long.

**ANSWER:**

a. about 21.2 mph

b. about 68 ft

c. No; it is not a linear function. The skid will be 4 times as long.
Write the square root function represented by each graph.

41.

**SOLUTION:**
\[ D = \{ x \mid x \geq 4 \} \]
\[ R = \{ y \mid y \geq -6 \} \]

The graph represents the function \( y = \sqrt{x - 4} - 6 \).

**ANSWER:**
\[ y = \sqrt{x - 4} - 6 \]

42.

**SOLUTION:**
\[ D = \{ x \mid x \geq -2 \} \]
\[ R = \{ y \mid y \geq 4 \} \]

The graph represents the function \( y = \sqrt{x + 2} + 4 \).

**ANSWER:**
\[ y = \sqrt{x + 2} + 4 \]
6-3 Square Root Functions and Inequalities

b. \( f(x): 6 \) units to the right, \( 3 \) units up; \( g(x): \frac{1}{16} \) to the left, \( 6 \) units down; \( h(x): 3 \) units to the left, \( 2 \) units up.

c. Sample answer: \( f(x) \) and \( g(x) \) appear to be stretched because the graph increases much more quickly than the parent graph.

d. Sample answer: They are stretched by the same magnitude because \( 4 = \sqrt{16} \).

e.

The rate of change decreases as the \( x \)-values increase for square root functions of the form \( a\sqrt{x-b} + c \), where \( a > 0 \).

45. PENDULUMS The period of a pendulum can be represented by \( T = 2\pi \sqrt{\frac{L}{g}} \), where \( T \) is the time in seconds, \( L \) is the length in feet, and \( g \) is gravity, 32 feet per second squared.

a. Graph the function for \( 0 \leq L \leq 10 \).

b. What is the period for lengths of 2, 5, and 8 feet?

SOLUTION:

a. Substitute 32 for \( g \) in \( T = 2\pi \sqrt{\frac{L}{8}} \).
6-3 Square Root Functions and Inequalities

\[ T = 2\pi \sqrt{\frac{L}{32}} \]

**Graph 1:**

**Graph 2:**

**b.** \[ T = 2\pi \sqrt{\frac{L}{g}} \]

Substitute \( L = 2 \) and \( g = 32 \).

\[ T = 2\pi \sqrt{\frac{2}{32}} \]

\[ \approx 1.57 \]

Substitute \( L = 5 \) and \( g = 32 \).

\[ T = 2\pi \sqrt{\frac{5}{32}} \]

\[ \approx 2.48 \]

Substitute \( L = 8 \) and \( g = 32 \).

\[ T = 2\pi \sqrt{\frac{8}{32}} \]

\[ \approx 3.14 \]

**ANSWER:**

**a.**

46. **PHYSICS** Using the function \( m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \),

Einstein’s theory of relativity states that the apparent mass \( m \) of a particle depends on its velocity \( v \). An object that is traveling extremely fast, close to the speed of light \( c \), will appear to have more mass compared to its mass at rest, \( m_0 \).

**a.** Use a graphing calculator to graph the function for a 10,000-kilogram ship for the domain \( 0 \leq v \leq 300,000,000 \). Use 300 million meters per second for the speed of light.

**b.** What viewing window did you use to view the graph?

**c.** Determine the apparent mass \( m \) of the ship for speeds of 100 million, 200 million, and 299 million meters per second.

**SOLUTION:**

**a.**
6-3 Square Root Functions and Inequalities

b. \{0, 400,000,000\} scl: 50,000,000 by \{0, 100,000\} scl: 20,000

c. (100 million, 10,607)(200 million, 13,416)(299 million, 122,577)

ANSWER:

47. CHALLENGE Write an equation for a square root function with a domain of \(\{x \mid x \geq -4\}\) a range of \(\{y \mid y \leq 6\}\) and that passes through \((5, 3)\).

SOLUTION:
Sample answer: Analyze the domain first. Since \(x > -4\), let the radical expression be \(\sqrt{x + 4}\). According to the range given, the maximum y-value is 6. When \(x = -4\), the radical expression is 0. Add 6 to this so that the range will be \(y \leq 6\). Since the range states that the maximum value for \(y\) is 6 and the domain states that the minimum value for \(x\) is -4, the radical expression must be negative in order to satisfy these constraints.

\[y = -\sqrt{x + 4} + 6\]

Check the point \((5, 3)\).

\[
\begin{align*}
y &= -\sqrt{5 + 4} + 6 \\
3 &\neq -\sqrt{9} + 6 \\
3 &\neq -3 + 6 \\
3 &= 3
\end{align*}
\]

ANSWER:
Sample answer: \(y = -\sqrt{x + 4} + 6\)

48. REASONING For what positive values of \(a\) are the domain and range of \(f(x) = \sqrt{x}\) the set of real numbers?

SOLUTION:
All positive odd numbers; the set of even numbers would result in a range of nonnegative values.

ANSWER:
all positive odd numbers
6-3 Square Root Functions and Inequalities

49. **OPEN ENDED** Write a square root function for which the domain is \( \{ x \mid x \geq 8 \} \) and the range is \( \{ y \mid y \leq 14 \} \).

**SOLUTION:**
Sample answer: Analyze the domain first. Since \( x > 8 \), let the radical expression be \( \sqrt{x - 8} \). According to the range given, the maximum \( y \)-value is 14. When \( x = 4 \), the radical expression is 0. Add 14 to this so that the range will be \( y \leq 14 \). Since the range states that the maximum value for \( y \) is 14 and the domain states that the minimum value for \( x \) is 8, the radical expression must be negative in order to satisfy these constraints.

\[
y = -\sqrt{x - 8} + 14
\]

**ANSWER:**
Sample answer: \( y = -\sqrt{x - 8} + 14 \)

50. **WRITING IN MATH** Explain why there are limitations on the domain and range of square root functions.

**SOLUTION:**
Sample answer: The domain is limited because square roots of negative numbers are imaginary. The range is limited due to the limitation of the domain.

**ANSWER:**
Sample answer: The domain is limited because square roots of negative numbers are imaginary. The range is limited due to the limitation of the domain.

51. **CCSS CRITIQUE** Cleveland thinks that the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.

![Graph](image)

**SOLUTION:**
\( y = \sqrt{5x + 10} \) has an \( x \)-intercept of -2 and would be to the right of the given graph. So, Molly is correct.

**ANSWER:**
Molly; \( y = \sqrt{5x + 10} \) has an \( x \)-intercept of -2 and would be to the right of the given graph.

52. **WRITING IN MATH** Explain why \( y = \pm \sqrt{x} \) is not a function.

**SOLUTION:**
To be a function, for every \( x \)-value there must be exactly one \( y \)-value. For every \( x \) in this equation there are two \( y \)-values, one that is negative and one that is positive. Also, the graph of \( y = \pm \sqrt{x} \) does not pass the vertical line test.

**ANSWER:**
To be a function, for every \( x \)-value there must be exactly one \( y \)-value. For every \( x \) in this equation there are two \( y \)-values, one that is negative and one that is positive. Also, the graph of \( y = \pm \sqrt{x} \) does not pass the vertical line test.
53. OPEN ENDED Write an equation of a relation that contains a radical and its inverse such that:

a. the original relation is a function, and its inverse is not a function

b. the original relation is not a function, and its inverse is a function.

**SOLUTION:**
a. Sample answer: The original is \( y = x^2 + 2 \) and inverse is \( y = \pm \sqrt{x-2} \).

b. Sample answer: The original is \( y = \pm \sqrt{x} + 4 \) and inverse is \( y = (x - 4)^2 \).

**ANSWER:**
a. Sample answer: The original is \( y = x^2 + 2 \) and inverse is \( y = \pm \sqrt{x-2} \).

b. Sample answer: The original is \( y = \pm \sqrt{x} + 4 \) and inverse is \( y = (x - 4)^2 \).

54. The expression \( \frac{-64x^6}{8x^3}, x \neq 0 \), is equivalent to

| A | \( 8x^2 \) |
| B | \( 8x^3 \) |
| C | \( -8x^2 \) |
| D | \( -8x^3 \) |

**SOLUTION:**
\[
\frac{-64x^6}{8x^3} = \frac{-8x^{6-3}}{8x^3} = -8x^3
\]

The correct choice is D.

**ANSWER:**
D
55. **PROBABILITY** For a game, Patricia must roll a standard die and draw a card from a deck of 26 cards, each card having a letter of the alphabet on it. What is the probability that Patricia will roll an odd number and draw a letter in her name?

   \[ F \quad \frac{2}{3} \]

   \[ G \quad \frac{3}{26} \]

   \[ H \quad \frac{1}{13} \]

   \[ J \quad \frac{1}{26} \]

   **SOLUTION:**
   Let \( A \) be the event getting an odd number on the die, and \( B \) be the event drawing a letter in her name.

   \[ P(A) = \frac{1}{2} \]

   \[ P(B) = \frac{6}{26} \]

   \[ P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{6}{26} = \frac{3}{26} \]

   The correct choice is G.

   **ANSWER:**
   G

56. **SHORT RESPONSE** What is the product of \((d + 6) \text{ and } (d - 3)\)?

   **SOLUTION:**
   \[ (d + 6)(d - 3) = d^2 - 3d + 6d - 18 = d^2 + 3d - 18 \]

   **ANSWER:**
   \(d^2 + 3d - 18\)
57. SAT/ACT Given the graph of the square root function below, which must be true?

I. The domain is all real numbers.

II. The function is \( y = \sqrt{x} + 3.5 \).

III. The range is about \( \{ y \mid y \geq 3.5 \} \).

**SOLUTION:**

The domain of the given function appears to be \( x \geq 1 \).

So statement I cannot be true.

Statement II can also not be true; the domain of \( y = \sqrt{x} + 3.5 \) is \( x \geq 0 \).

Statement II is true. So, the correct answer is E.

**ANSWER:**

E

---

Determine whether each pair of functions are inverse functions. Write yes or no.

\[ f(x) = 2x \]

\[ g(x) = \frac{1}{2}x \]

**SOLUTION:**

The functions \( f(x) \) and \( g(x) \) are inverses if and only if

\[ f \circ g\)(x) = \[ g \circ f\)(x) = x. \]

\[ f \circ g\)(x) = \[ f\][g(x)]

\[ = f\left(\frac{1}{2}x\right) \]

\[ = 2\left(\frac{1}{2}x\right) \]

\[ = x \]

\[ g \circ f\)(x) = \[ g \circ f\)(x) \]

\[ = g\left(2x\right) \]

\[ = \frac{1}{2}(2x) \]

\[ = x \]

Therefore, \( f \) and \( g \) are inverses.

**ANSWER:**

Yes

---

\[ f(x) = 3x - 7 \]

\[ g(x) = \frac{1}{3}x - \frac{7}{16} \]

**SOLUTION:**

The functions \( f(x) \) and \( g(x) \) are inverses if and only if

\[ f \circ g\)(x) = \[ g \circ f\)(x) = x. \]

\[ f \circ g\)(x) = \[ f\][g(x)]

\[ = f\left(\frac{1}{3}x - \frac{7}{16}\right) \]

\[ = 3\left(\frac{1}{3}x - \frac{7}{16}\right) - 7 \]

Since \[ f \circ g\)(x) \neq x, \( f \) and \( g \) are not inverses.

**ANSWER:**

No
6-3 Square Root Functions and Inequalities

\[ f(x) = \frac{3x + 2}{5} \]
\[ g(x) = \frac{5x - 2}{3} \]

**SOLUTION:**
The functions \( f(x) \) and \( g(x) \) are inverses if and only if
\[ (f \circ g)(x) = (g \circ f)(x) = x. \]
\[ (f \circ g)(x) = f(g(x)) \]
\[ = f \left( \frac{5x - 2}{3} \right) \]
\[ = 3 \left( \frac{5x - 2}{3} \right) + 2 \]
\[ = \frac{5x - 2}{3} \]
\[ = x \]
\[ (g \circ f)(x) = g(f(x)) \]
\[ = g \left( \frac{3x + 2}{5} \right) \]
\[ = 5 \left( \frac{3x + 2}{5} \right) - 2 \]
\[ = \frac{3x + 2}{3} \]
\[ = x \]

Therefore, \( f \) and \( g \) are inverses.

**ANSWER:**
Yes

61. **TIME** The formula \( h = \frac{m}{60} \) converts minutes \( m \) to hours \( h \), and \( d = \frac{h}{24} \) converts hours \( h \) to days \( d \).

Write a function that converts minutes to days.

**SOLUTION:**
\[ d(h) = \frac{h}{24} \]
\[ [d \circ h](m) = d \left( \frac{m}{60} \right) \]
\[ = \frac{m}{1440} \]

**ANSWER:**
\[ [d \circ h](m) = \frac{m}{1440} \]

62. **CABLE TV** The number of households in the United States with cable TV after 1985 can be modeled by the function \( C(t) = -43.2t^2 + 1343t + 790 \), where \( t \) represents the number of years since 1985.

a. Graph this equation for the years 1985 to 2005.

b. Describe the turning points of the graph and its end behavior.

c. What is the domain of the function? Use the graph to estimate the range for the function.

d. What trends in households with cable TV does the graph suggest? Is it reasonable to assume that the trend will continue indefinitely?

**SOLUTION:**
a.
b. rel. max. between \( t = 15 \) and \( t = 16 \), and no rel. min.;
\[
C(t) \rightarrow -\infty \text{ as } t \rightarrow -\infty, \quad C(t) \rightarrow -\infty \text{ as } t \rightarrow +\infty
\]

c. \( D = \{ \text{all real numbers} \}; \quad R = \{ y \mid y \leq 11,225 \} \)

d. The number of cable TV systems rose steadily from 1985 to 2000. Then the number began to decline. The trend may continue for some years, but the number of cable TV systems cannot decline at this rate indefinitely. The number cannot fall below 0. It is not likely that the number would come close to 0 for the foreseeable future; there is no reason to believe that cable TV systems will not be in use.

**ANSWER:**

63. **6.34**

**SOLUTION:**

6.34 can be written in the form \( \frac{p}{q}, \quad q \neq 0 \), for example, as follows:

\[
\frac{634}{100}
\]

Therefore, it is a rational number.

**ANSWER:**

rational

64. **3.787887888…**

**SOLUTION:**

3.787887888… neither terminates nor repeats. Therefore, it is irrational.

**ANSWER:**

irrational
6-3 Square Root Functions and Inequalities

65. 5.333…

**SOLUTION:**

The number can be written in the form \( \frac{p}{q} \), for example, as follows:

\[
\frac{16}{3}
\]

Therefore, it is a rational number.

**ANSWER:**

rational

66. 1.25

**SOLUTION:**

The number can be written in the form \( \frac{p}{q} \), for example, as follows:

\[
\frac{5}{4}
\]

Therefore, it is a rational number.

**ANSWER:**

rational
6-4 nth Roots

Simplify.

1. $\pm \sqrt[6]{100y^8}$

**SOLUTION:**

$$\pm \sqrt[6]{100y^8} = \pm \sqrt[6]{10^2(y^4)^2} = \pm 10y^4$$

**ANSWER:**

$\pm 10y^4$

2. $-\sqrt[12]{49u^8v^{12}}$

**SOLUTION:**

$$-\sqrt[12]{49u^8v^{12}} = -\sqrt[12]{7^2(u^4)^2(v^6)^2} = -7u^2v^6$$

**ANSWER:**

$-7u^2v^6$

3. $\sqrt[8]{(y-6)^8}$

**SOLUTION:**

$$\sqrt[8]{(y-6)^8} = \sqrt[8]{((y-6)^4)^2} = (y-6)^4$$

**ANSWER:**

$(y-6)^4$

4. $\sqrt[24]{16g^{16}h^{24}}$

**SOLUTION:**

$$\sqrt[24]{16g^{16}h^{24}} = \sqrt[24]{2^4(g^4)^4(h^6)^4} = 2g^4h^6$$

**ANSWER:**

$2g^4h^6$

5. $\sqrt{-16y^4}$

**SOLUTION:**

$$\sqrt{-16y^4} = \pm 4iy^2.$$  

**ANSWER:**

$\pm 4iy^2$

6. $\sqrt[18]{64(2y+1)^{18}}$

**SOLUTION:**

$$\sqrt[18]{64(2y+1)^{18}} = \sqrt[6]{2^6((2y+1)^3)^6}$$

Since the index 6 is even and the exponent 3 is odd, use absolute value.

$$\sqrt[6]{64(2y+1)^{18}} = 2(2y+1)^3$$

**ANSWER:**

$2(2y+1)^3$
Simplify.

1. SOLUTION:
   \[ \sqrt[3]{58} \approx 7.61577310 \approx 7.616 \]
   ANSWER: 7.616

2. SOLUTION:
   \[ \sqrt[3]{76} \approx -8.7177978870 \approx -8.718 \]
   ANSWER: -8.718

3. SOLUTION:
   \[ \sqrt[3]{-43} = \sqrt[3]{(-1)^3 \cdot 43} = -\sqrt[3]{43} \approx -2.122 \]
   ANSWER: -2.122

10. \( \sqrt[3]{71} \)
   SOLUTION:
   \[ \sqrt[3]{71} \approx 2.90278310 \approx 2.903 \]
   ANSWER: 2.903

11. CCSS PERSEVERANCE The radius \( r \) of the orbit of a television satellite is given by \[ \sqrt[3]{\frac{GM^2}{4\pi^2}} \], where \( G \) is the universal gravitational constant, \( M \) is the mass of Earth, and \( t \) is the time it takes the satellite to complete one orbit. Find the radius of the satellite’s orbit if \( G \) is \( 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2 \), \( M \) is \( 5.98 \times 10^{24} \text{kg} \), and \( t \) is \( 2.6 \times 10^6 \text{seconds} \).
   SOLUTION:
   Substitute the value of \( G, M, \) and \( t \) in the given expression.
   \[
   \sqrt[3]{\frac{GM^2}{4\pi^2}} = \sqrt[3]{\left(6.67 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)\left(2.6 \times 10^6\right)^2}
   = 4.088 \times 10^8
   
   The radius of the satellite’s orbit is about \( 4.088 \times 10^8 \text{m} \).
6-4 nth Roots

Simplify.

12. $\pm \sqrt[16]{121x^4y^{16}}$

**SOLUTION:**

$$\pm \sqrt[16]{121x^4y^{16}} = \pm 11x^2y^8$$

**ANSWER:**

$\pm 11x^2y^8$

15. $-\sqrt[16]{16c^4d^2}$

**SOLUTION:**

$$-\sqrt[16]{16c^4d^2} = \sqrt[16]{(c^2)^2 d^2} = -4c^2 |d|$$

**ANSWER:**

$-4c^2 |d|$

13. $\pm \sqrt[36]{225a^{16}b^{36}}$

**SOLUTION:**

$$\pm \sqrt[36]{225a^{16}b^{36}} = \pm 15a^8b^{18}$$

**ANSWER:**

$\pm 15a^8b^{18}$

16. $-\sqrt[12]{81a^{16}b^{20}c^{12}}$

**SOLUTION:**

$$-\sqrt[12]{81a^{16}b^{20}c^{12}} = -\sqrt[12]{9\left(\frac{a^8b^{10}}{c^6}\right)^2} = -9a^8b^{10}c^6$$

**ANSWER:**

$-9a^8b^{10}c^6$

14. $\pm \sqrt[4]{49x^4}$

**SOLUTION:**

$$\pm \sqrt[4]{49x^4} = \pm \sqrt[4]{7^2 (x^2)^2} = \pm 7x^2$$

**ANSWER:**

$\pm 7x^2$

17. $-\sqrt[20]{400x^{32}y^{40}}$

**SOLUTION:**

$$-\sqrt[20]{400x^{32}y^{40}} = -\sqrt[20]{20^2 (x^{16})^2 (y^{20})^2} = -20x^{16}y^{20}$$

**ANSWER:**

$-20x^{16}y^{20}$
6-4 nth Roots

18. \(\sqrt{(x+15)^4}\)

**SOLUTION:**
\[
\sqrt{(x+15)^4} = \sqrt{((x+15)^2)^2} = (x+15)^2
\]

**ANSWER:**
\((x+15)^2\)

19. \(\sqrt{(x^2 + 6)^{16}}\)

**SOLUTION:**
\[
\sqrt{(x^2 + 6)^{16}} = \sqrt{((x^2 + 6)^8)^2} = (x^2 + 6)^8
\]

**ANSWER:**
\((x^2 + 6)^8\)

20. \(\sqrt{(a^2 + 4a)^{12}}\)

**SOLUTION:**
\[
\sqrt{(a^2 + 4a)^{12}} = \sqrt{((a^2 + 4a)^6)^2} = (a^2 + 4a)^6
\]

**ANSWER:**
\((a^2 + 4a)^6\)

21. \(\sqrt[3]{8a^6b^{12}}\)

**SOLUTION:**
\[
\sqrt[3]{8a^6b^{12}} = \sqrt[3]{2^3(a^2)^3(b^4)^3} = 2a^2b^4
\]

**ANSWER:**
\(2a^2b^4\)

22. \(\frac{1}{6}(d^{24}x^{36})\)

**SOLUTION:**
\[
\frac{1}{6}(d^{24}x^{36}) = \left(\frac{1}{6}\right)(d^4)^6(x^6)^6 = d^4x^6
\]

**ANSWER:**
\(d^4x^6\)

23. \(\sqrt[3]{27b^{18}c^{12}}\)

**SOLUTION:**
\[
\sqrt[3]{27b^{18}c^{12}} = \sqrt[3]{3^3(b^6)^3(c^4)^3} = 3b^6c^4
\]

**ANSWER:**
\(3b^6c^4\)
6-4 nth Roots

24. \(-\sqrt[6]{(2x+1)^6}\)

**SOLUTION:**

\[-\sqrt[6]{(2x+1)^6} = -\sqrt[6]{(2x+1)^3}\]

Since the index 2 is even and the exponent 3 is odd, use absolute value.

\[-\sqrt[6]{(2x+1)^6} = -\left| (2x+1)^3 \right|\]

**ANSWER:**

\[-\left| (2x+1)^3 \right|\]

25. \(\sqrt[8]{(x+2)^8}\)

**SOLUTION:**

\[\sqrt[8]{(x+2)^8} = \sqrt[8]{-1(x+2)^8} \]

\[= \sqrt[8]{-1} \cdot \sqrt[8]{(x+2)^8} \]

\[= i(x+2)^4 \]

**ANSWER:**

\[i(x+2)^4\]

26. \(\sqrt[9]{(y-9)^9}\)

**SOLUTION:**

\[\sqrt[9]{(y-9)^9} = \sqrt[9]{-1} \cdot \sqrt[9]{(y-9)^9} \]

\[= -(y-9)^3 \]

**ANSWER:**

\[-(y-9)^3\]

27. \(\sqrt[6]{x^{18}}\)

**SOLUTION:**

\[\sqrt[6]{x^{18}} = \sqrt[6]{x^3} \]

Since the index 6 is even and the exponent 3 is odd, use absolute value.

\[\sqrt[6]{x^{18}} = |x^3| \]

**ANSWER:**

\[|x^3|\]
6-4 nth Roots

28. \( \sqrt[4]{a^{12}} \)

**SOLUTION:**

\( \sqrt[4]{a^{12}} = \sqrt[4]{(a^3)^4} \)

Since the index 4 is even and the exponent 3 is odd, use absolute value.

\( \sqrt[4]{a^{12}} = |a^3| \)

**ANSWER:**

\( |a^3| \)

29. \( \sqrt[3]{a^{12}} \)

**SOLUTION:**

\( \sqrt[3]{a^{12}} = \sqrt[3]{(a^4)^3} \)

\( = a^4 \)

**ANSWER:**

\( a^4 \)

30. \( \sqrt[4]{81(x+4)^4} \)

**SOLUTION:**

\( \sqrt[4]{81(x+4)^4} = \sqrt[4]{3^4(x+4)^4} \)

Since the index 4 is even and the exponent 1 is odd, use absolute value.

\( \sqrt[4]{81(x+4)^4} = 3|\(x+4|\) \)

**ANSWER:**

\( 3|\(x+4|\) \)

31. \( \sqrt[3]{(4x-7)^{24}} \)

**SOLUTION:**

\( \sqrt[3]{(4x-7)^{24}} = \sqrt[3]{((4x-7)^8)^3} \)

\( = (4x-7)^8 \)

**ANSWER:**

\( (4x-7)^8 \)

32. \( \sqrt[3]{(y^3+5)^{18}} \)

**SOLUTION:**

\( \sqrt[3]{(y^3+5)^{18}} = \sqrt[3]{(y^3+5)^6} \)

\( = (y^3+5)^6 \)

**ANSWER:**

\( (y^3+5)^6 \)

33. \( \sqrt{256(5x-2)^{12}} \)

**SOLUTION:**

\( \sqrt{256(5x-2)^{12}} = \sqrt{4^4((5x-2)^3)^4} \)

\( = 4|(5x-2)^3| \)

**ANSWER:**

\( 4|(5x-2)^3| \)
34. \(\frac{8}{\sqrt[8]{x^{16}y^8}}\)

**SOLUTION:**

\[
\frac{8}{\sqrt[8]{x^{16}y^8}} = \frac{8}{(x^2)^8y^8} = \frac{8}{x^2y}
\]

**ANSWER:**

\(x^2y\)

35. \(\sqrt[5]{32a^{15}b^{10}}\)

**SOLUTION:**

\[
\sqrt[5]{32a^{15}b^{10}} = 2^5 \frac{a^3}{(b^2)^3} = 2a^3b^2
\]

**ANSWER:**

\(2a^3b^2\)

36. **SHIPPING** An online book store wants to increase the size of the boxes it uses to ship orders. The new volume \(N\) is equal to the old volume \(V\) times the scale factor \(F\) cubed, or \(N = V \cdot F^3\). What is the scale factor if the old volume was 0.8 cubic feet and the new volume is 21.6 cubic feet?

**SOLUTION:**

Substitute 21.6 for \(N\) and 0.8 for \(V\) in the given equation and solve for \(F\).

\[
21.6 = 0.8 \cdot F^3
\]

\[
27 = F^3
\]

\[
\sqrt[3]{27} = F
\]

\(3 = F\)

**ANSWER:**

3

37. **GEOMETRY** The side length of a cube is determined by \(r = \sqrt[3]{V}\), where \(V\) is the volume in cubic units. Determine the side length of a cube with a volume of 512 cm\(^3\).

**SOLUTION:**

Substitute 512 for \(V\) in the equation.

\(r = \sqrt[3]{512} = 8\)

The side length of the cube is 8 cm.

**ANSWER:**

8 cm

38. \(\sqrt{92}\)

**SOLUTION:**

9.592

**ANSWER:**

9.592

39. \(-\sqrt{150}\)

**SOLUTION:**

-12.247

**ANSWER:**

-12.247
40. \( \sqrt[4]{0.43} \)  

SOLUTION: 0.656  

ANSWER: 0.656

41. \( \sqrt[4]{0.62} \)  

SOLUTION: 0.787  

ANSWER: 0.787

42. \( \sqrt[4]{168} \)  

SOLUTION: 5.518  

ANSWER: 5.518

43. \( \sqrt[4]{-4382} \)  

SOLUTION: -5.350  

ANSWER: -5.350

44. \( \sqrt[6]{(8912)^2} \)  

SOLUTION: 20.733  

ANSWER: 20.733

45. \( \sqrt[6]{(4756)^2} \)  

SOLUTION: 29.573  

ANSWER: 29.573

46. **GEOMETRY** The radius \( r \) of a sphere with volume \( V \) can be found using the formula \( r = \frac{3V}{4\pi} \).

![Sphere diagram](image)

a. Determine the radius for volumes of 1000 cm\(^3\), 8000 cm\(^3\), and 64,000 cm\(^3\).

b. How does the volume of the sphere change if the radius is doubled? Explain.

SOLUTION:
a. Substitute 1000 for \( V \) in the formula and solve for \( r \).
6-4 nth Roots

\[ r = \sqrt[3]{\frac{3(1000)}{4(3.14)}} \]
\[ = \sqrt[3]{\frac{3000}{12.56}} \]
\[ \approx 6.2 \]

Substitute 8000 for V in the formula and solve for r.
\[ r = \sqrt[3]{\frac{3(8000)}{4(3.14)}} \]
\[ = \sqrt[3]{\frac{24000}{12.56}} \]
\[ \approx 12.4 \]

Substitute 64000 for V in the formula and solve for r.
\[ r = \sqrt[3]{\frac{3(64000)}{4(3.14)}} \]
\[ = \sqrt[3]{\frac{192000}{12.56}} \]
\[ \approx 24.8 \]

**b. Sample answer: As r doubles, the volume increases by a factor of \(2^3\) or 8.**

**ANSWER:**

a. (1000, 6.2), (8000, 12.4), (64,000, 24.8)

b. Sample answer: As r doubles, the volume increases by a factor of \(2^3\) or 8.

**ANSWER:**

**47.** Simplify.

\[ \sqrt[6]{196c^6d^4} \]

**SOLUTION:**

\[ \sqrt[6]{196c^6d^4} = \sqrt[6]{14^2(c^3)^2(d^2)^2} \]
\[ = 14|c^3|d^2 \]

**ANSWER:**

\[ 14|c^3|d^2 \]

**48.** Simplify.

\[ \sqrt[8]{-64y^8z^6} \]

**SOLUTION:**

\[ \sqrt[8]{-64y^8z^6} = \sqrt[8]{(-1)(64)y^8z^6} \]
\[ = \sqrt[-1]{8^2 \cdot (y^4)^2 \cdot (z^3)^2} \]
\[ = i \cdot 8 \cdot y^4 \cdot z^3 \text{ or } 8iy^4z^3 \]

**ANSWER:**

\[ 8iy^4z^3 \]

**49.** Simplify.

\[ \sqrt[3]{-27a^5b^9} \]

**SOLUTION:**

\[ \sqrt[3]{-27a^5b^9} = \sqrt[3]{(-3)^3(a^5)^3(b^3)^3} \]
\[ = -3a^2b^3 \]

**ANSWER:**

\[ -3a^2b^3 \]
6-4 nth Roots

50. \( \sqrt[4]{-16x^{16}y^8} \)

**SOLUTION:**

\[
\sqrt[4]{-16x^{16}y^8} = \sqrt[4]{(-1)(2x^4)^4(y^2)^4} \\
= \sqrt[4]{-1} \cdot \sqrt[4]{(2x^4)^4 \cdot (y^2)^4} \\
= 2x^4 \sqrt[4]{y^2} \cdot \sqrt[4]{-1} \\
= 2x^4y^2 \sqrt[4]{-1}
\]

(While you know that \( \sqrt{-1} = i \), the simplification of \( \sqrt[4]{-1} \) will be addressed in our Precalculus text. For use in this course, leave the answer written using the radical notation.)

**ANSWER:**

\( 2x^4y^2 \sqrt[4]{-1} \)

51. \( \sqrt{400x^{16}y^6} \)

**SOLUTION:**

\[
\sqrt{400x^{16}y^6} = \sqrt{20^2(x^8)^2(y^3)^2} \\
= 20x^8|y^3|
\]

**ANSWER:**

\( 20x^8|y^3| \)

52. \( \sqrt[3]{8c^2d^{12}} \)

**SOLUTION:**

\[
\sqrt[3]{8c^2d^{12}} = \sqrt[3]{2^3c^2(d^4)^3} \\
= 2cd^4
\]

**ANSWER:**

\( 2cd^4 \)

53. \( \sqrt[3]{64(x + y)^6} \)

**SOLUTION:**

\[
\sqrt[3]{64(x + y)^6} = \sqrt[3]{4^3((x + y)^2)^3} \\
= 4(x + y)^2
\]

**ANSWER:**

\( 4(x + y)^2 \)

54. \( \sqrt[5]{-(y-z)^{15}} \)

**SOLUTION:**

\[
\sqrt[5]{-(y-z)^{15}} = \sqrt[5]{(-1)^5((y-z)^3)^5} \\
= -(y-z)^3
\]

**ANSWER:**

\( -(y-z)^3 \)
55. **PHYSICS** Johannes Kepler developed the formula
\[ d = \sqrt[3]{6r^2}, \]
where \( d \) is the distance of a planet from the Sun in millions of miles and \( t \) is the number of Earth-days that it takes for the planet to orbit the Sun. If the length of a year on Mars is 687 Earth-days, how far from the Sun is Mars?

**SOLUTION:**
Substitute 687 for \( t \) in the formula and simplify.

\[ d = \sqrt[3]{6(687)^2} \]
\[ = \sqrt[3]{2831814} \]
\[ \approx 141 \]
The distance of the Mars from the Sun is about 141 million miles.

**ANSWER:**
about 141 million mi

56. **CCSS SENSE-MAKING** All matter is composed of atoms. The nucleus of an atom is the center portion of the atom that contains most of the mass of the atom. A theoretical formula for the radius \( r \) of the nucleus of an atom is
\[ r = \sqrt[3]{\frac{A}{6}} \text{ meters}, \]
where \( A \) is the mass number of the nucleus. Find the radius of the nucleus for each atom in the table.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Mass Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>carbon</td>
<td>6</td>
</tr>
<tr>
<td>oxygen</td>
<td>8</td>
</tr>
<tr>
<td>sodium</td>
<td>11</td>
</tr>
<tr>
<td>aluminum</td>
<td>13</td>
</tr>
<tr>
<td>chlorine</td>
<td>17</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Substitute 6 for \( A \) in the equation and solve for \( r \).

\[ r = \sqrt[3]{\frac{6}{6}} \]
\[ \approx 2.36 \times 10^{-15} \]

57. **BIOLOGY** Kleiber’s Law, \( P = 73.34 \sqrt{m^3} \), shows the relationship between the mass \( m \) in kilograms of an organism and its metabolism \( P \) in Calories per day. Determine the metabolism for each of the animals listed at the right.
6-4 nth Roots

<table>
<thead>
<tr>
<th>Animal</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bald eagle</td>
<td>4.5</td>
</tr>
<tr>
<td>golden retriever</td>
<td>30</td>
</tr>
<tr>
<td>komodo dragon</td>
<td>72</td>
</tr>
<tr>
<td>bottlenose dolphin</td>
<td>156</td>
</tr>
<tr>
<td>Asian elephant</td>
<td>2300</td>
</tr>
</tbody>
</table>

SOLUTION:
Substitute 4.5 for \( m \) in the formula and simplify.

\[
P = 73.3\sqrt[4]{(4.5)^3}
= 73.3\sqrt[4]{27000}
\approx 939.6
\]

The metabolism of a bald eagle is about 939.6 Cal/d.
Substitute 30 for \( m \) in the formula and simplify.

\[
P = 73.3\sqrt[4]{30^3}
= 73.3\sqrt[4]{27000}
\approx 939.6
\]

The metabolism of a golden retriever is about 939.6 Cal/d.
Substitute 72 for \( m \) in the formula and simplify.

\[
P = 73.3\sqrt[4]{72^3}
= 73.3\sqrt[4]{373248}
\approx 1811.8
\]

The metabolism of a komodo dragon is about 1811.8 Cal/d.
Substitute 156 for \( m \) in the formula and simplify.

\[
P = 73.3\sqrt[4]{156^3}
= 73.3\sqrt[4]{3796416}
\approx 3235.5
\]

The metabolism of a bottlenose dolphin is about 3235.5 Cal/d.
Substitute 2300 for \( m \) in the formula and simplify.

\[
P = 73.3\sqrt[4]{2300^3}
= 73.3\sqrt[4]{12167000000}
\approx 24344.4
\]

The metabolism of an Asian elephant is about 24344.4 Cal/d.

ANSWER:
bald eagle: \( \approx 226.5 \) Cal/d; golden retriever: \( \approx 939.6 \) Cal/d; komodo dragon: \( \approx 1811.8 \) Cal/d; bottlenose dolphin: \( \approx 3235.5 \) Cal/d; Asian elephant: \( \approx 24344.4 \) Cal/d

58. MULTIPLE REPRESENTATIONS
In this problem, you will use \( f(x) = x^n \) and \( g(x) = \sqrt[n]{x} \) to explore inverses.

a. TABULAR Make tables for \( f(x) = x^n \) and \( g(x) = \sqrt[n]{x} \) to explore inverses.

b. GRAPHICAL Graph the equations.

c. ANALYTICAL Which equations are functions? Which functions are one-to-one?

d. ANALYTICAL For what values of \( n \) are \( g(x) \) and \( f(x) \) inverses of each other?

e. VERBAL What conclusions can you make about \( g(x) = \sqrt[n]{x} \) and \( f(x) = x^n \) for all positive even values of \( n \)? for all positive odd values of \( n \)?

SOLUTION:
a. \( n = 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-125</td>
<td>-64</td>
<td>-27</td>
<td>-8</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( f^{-1}(x) )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccccc}
 x & -5 & -3 & -2 & -1 & 0 \\
 f(x) & 625 & 256 & 81 & 16 & 0 \\
 f^{-1}(x) & 1 & 2 & 3 & 4 & 5
\end{array}
\]

\( n = 3 \)
6-4 nth Roots

Simplify.

1. SOLUTION:  
ANSWER:  

2. SOLUTION:  
ANSWER:  

The axis of symmetry is x = 2 and the graph opens upwards.

ANSWER:  

For all positive even values of $n$, $f(x)$ and $g(x)$ are inverse functions. For all positive odd values of $n$, $f(x)$ and $g(x)$ are inverse functions only if the range of $f(x)$ and the domain of $g(x)$ are restricted to positive values.

ANSWER:

a. $n = 3$

b. $n = 3$

b. $n = 3$

b. $n = 3$

b. $n = 3$
6-4 nth Roots

59. CCSS CRITIQUE Ashley and Kimi are simplifying $\sqrt[4]{16x^4y^8}$. Is either of them correct? Explain your reasoning.

**SOLUTION:**

Kimi is correct; Ashley’s error was keeping the $y^2$ inside the absolute value symbol.

**ANSWER:**

Kimi; Ashley’s error was keeping the $y^2$ inside the absolute value symbol.

60. CHALLENGE Under what conditions is $\sqrt{x^2 + y^2} = x + y$ true?

**SOLUTION:**

Case #1: $x = 0$ and $y \geq 0$

Case #2: $y = 0$ and $x \geq 0$

**ANSWER:**

Case #1: $x = 0$ and $y \geq 0$

Case #2: $y = 0$ and $x \geq 0$
61. **REASONING** Determine whether the statement \( \sqrt[4]{(-x)^4} = x \) is sometimes, always, or never true.

**SOLUTION:**
Sample answer: Sometimes; when
\[ x = -3, \sqrt[4]{(-x)^4} = \left| (-x) \right| = 3. \]
When \( x = 3, \sqrt[4]{(-x)^4} = \left| (-x) \right| = 3. \)

**ANSWER:**
Sample answer: Sometimes; when
\[ x = -3, \sqrt[4]{(-x)^4} = \left| (-x) \right| = 3. \]
When \( x = 3, \sqrt[4]{(-x)^4} = \left| (-x) \right| = 3. \)

62. **CHALLENGE** For what real values of \( x \) is \( \sqrt[3]{x} > x \) ?

**SOLUTION:**
\[ 0 < x < 1, \quad x < -1 \]

**ANSWER:**
\[ 0 < x < 1, \quad x < -1 \]

63. **OPEN ENDED** Write a number for which the principal square root and cube root are both integers.

**SOLUTION:**
Sample answers: 1, 64

**ANSWER:**
Sample answers: 1, 64

64. **WRITING IN MATH** Explain when and why absolute value symbols are needed when taking an \( n \)th root.

**SOLUTION:**
Sample answer: They are needed to ensure that the answer is not a negative number. When we take any odd root of a number, we find that there is just one answer. If the number is positive, the root is positive. If the number is negative, the root is negative. Every positive real number has two \( n \)th roots when \( n \) is even; one of these roots is positive and one is negative. Negative real numbers do not have \( n \)th roots when \( n \) is even. Absolute value signs are never needed when finding odd roots. When finding even \( n \)th roots, absolute value signs are sometimes necessary, as with square roots.

**ANSWER:**
Sample answer: They are needed to ensure that the answer is not a negative number. When we take any odd root of a number, we find that there is just one answer. If the number is positive, the root is positive. If the number is negative, the root is negative. Every positive real number has two \( n \)th roots when \( n \) is even; one of these roots is positive and one is negative. Negative real numbers do not have \( n \)th roots when \( n \) is even. Absolute value signs are never needed when finding odd roots. When finding even \( n \)th roots, absolute value signs are sometimes necessary, as with square roots.

65. **CHALLENGE** Write an equivalent expression in for \( \sqrt[3]{2x} \cdot \sqrt[3]{8y} \). Simplify the radical.

**SOLUTION:**
\[ \sqrt[3]{2x} \cdot \sqrt[3]{8y} = \sqrt[3]{2x8y} = \sqrt[3]{2 \cdot 2^3xy} = 2 \sqrt[3]{2xy} \]

**ANSWER:**
\[ 2 \sqrt[3]{2xy} \]
6-4 nth Roots

CHALLENGE Simplify each expression.

66. \(\sqrt[4]{0.0016}\)

**SOLUTION:**

\[
\sqrt[4]{0.0016} = (0.0016)^{\frac{1}{4}} = \sqrt[4]{0.0016} = 0.2
\]

**ANSWER:**

0.2

67. \(\sqrt[6]{-0.0000001}\)

**SOLUTION:**

\[
\sqrt[6]{-0.0000001} = (-0.0000001)^{\frac{1}{6}} = \sqrt[6]{(-1)^7 (0.0000001)}
\]

**ANSWER:**

−0.1

68. \(\frac{\sqrt[3]{-0.00032}}{\sqrt[3]{-0.027}}\)

**SOLUTION:**

\[
\frac{\sqrt[3]{-0.00032}}{\sqrt[3]{-0.027}} = \frac{-0.2}{-0.3} = \frac{2}{3}
\]

**ANSWER:**

\(\frac{2}{3}\)

69. CHALLENGE Solve \(\frac{-5}{\sqrt[5]{a}} = -125\) for \(a\).

**SOLUTION:**

Multiply both sides by \(\sqrt[5]{a}\).

\[
\sqrt[5]{a} \left(\frac{-5}{\sqrt[5]{a}}\right) = \sqrt[5]{a} (-125)
\]

\[
-5 = \sqrt[5]{a} (-125)
\]

\[
-5 = (-125 \sqrt[5]{a})
\]

\[
(-5)^2 = (-125 \sqrt[5]{a})^2
\]

\[
\frac{25}{15625} = a
\]

\[
\frac{1}{625} = a
\]

**ANSWER:**

\(\frac{1}{625}\)
6-4 \textbf{nth Roots}

70. What is the value of \(w\) in the equation \(\frac{1}{2}(4w + 36) = 3(4w - 3)\)?

\begin{align*}
A & \quad 2 \\
B & \quad 2.7 \\
C & \quad 27 \\
D & \quad 36
\end{align*}

\textbf{SOLUTION:}

\[
\frac{1}{2}(4w + 36) = 3(4w - 3) \\
2w + 18 = 12w - 9 \\
-10w = -27 \\
w = 2.7
\]

B is the correct choice.

\textbf{ANSWER:}

B

71. What is the product of the complex numbers \((5 + i)\) and \((5 - i)\)?

\begin{align*}
F & \quad 24 \\
G & \quad 26 \\
H & \quad 25 - i \\
J & \quad 26 - 10i
\end{align*}

\textbf{SOLUTION:}

\[
(5 + i)(5 - i) = 5^2 - i^2 \\
= 25 + 1 \\
= 26
\]

G is the correct choice.

\textbf{ANSWER:}

G
6-4 \textit{nth Roots}

72. \textbf{EXTENDED RESPONSE} A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Tate plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.

\begin{enumerate}
  \item a. Tate plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with the cans?

  \textbf{SOLUTION:}

  \begin{enumerate}
    \item a. Subtract the volume of 18 cans from the volume of the cylindrical cooler.

    \[ \pi (4.5)^2 (11) - 18 \pi (1.25)^2 (4.75) = \pi (99.15625) - 18 \pi (133.59375) = \pi (28.1) \approx 280.1 \]

    So, the volume of the space that will not be filled with the cans is about 280.1 in$^3$.
  
  \item b. \[ \frac{\pi (4.5)^2 (11)}{18 \pi (1.25)^2 (4.75)} = \frac{222.75}{133.59375} \approx 1.7 \]

    The ratio of the volume of the cooler to the volume of the cans in part a is about 1.7.

  \textbf{ANSWER:}

  \begin{enumerate}
    \item a. 280.1 in$^3$
    \item b. about 1.7
  \end{enumerate}

  \end{enumerate}

\end{enumerate}

73. \textbf{SAT/ACT} Which of the following is closest to $\sqrt[3]{7.32}$?

\begin{enumerate}
  \item A 1.8
  \item B 1.9
  \item C 2.0
  \item D 2.1
  \item E 2.2
\end{enumerate}

\textbf{SOLUTION:}

1.9 is closest to $\sqrt[3]{7.32}$. B is the correct choice.

\textbf{ANSWER:}

B
Simplify.

1.

SOLUTION:

ANSWER:

2.

SOLUTION:

ANSWER:

3. Graph is (2, –2). The axis of symmetry is x = 2 and the graph opens upwards.

ANSWER:

74. \( y = \sqrt{x - 5} \)

SOLUTION:

ANSWER:

75. \( y = \sqrt{x - 2} \)

SOLUTION:

ANSWER:
76. \( y = 3\sqrt{x} + 4 \)

**SOLUTION:**

\[ y = 3\sqrt{x} + 4 \]

**ANSWER:**

\[ y = 3\sqrt{x} + 4 \]

---

77. **HEALTH** The average weight of a baby born at a certain hospital is \( \frac{7}{2} \) pounds and the average length is 19.5 inches. One kilogram is about 2.2 pounds and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters.

**SOLUTION:**
The average weight of the baby in kilograms:

\[ \frac{7.5}{2.2} \approx 3.41 \text{ kg} \]

The average length of the baby in centimeters:

\[ \frac{19.5}{0.3937} \approx 49.53 \text{ cm} \]

**ANSWER:**
3.41 kg and 49.53 cm

**Simplify.**

78. \( (4c - 5) - (c + 11) + (-6c + 17) \)

**SOLUTION:**

\[ (4c - 5) - (c + 11) + (-6c + 17) \]
\[ = 4c - 5 - c - 11 - 6c + 17 \]
\[ = -3c + 1 \]

**ANSWER:**

\(-3c + 1\)
6-4 nth Roots

79. \((11x^2 + 13x - 15) - (7x^2 - 9x + 19)\)

**SOLUTION:**

\[
(11x^2 + 13x - 15) - (7x^2 - 9x + 19)
= 11x^2 + 13x - 15 - 7x^2 + 9x - 19
= 4x^2 + 22x - 34
\]

**ANSWER:**

\(4x^2 + 22x - 34\)

80. \((d - 5)(d + 3)\)

**SOLUTION:**

\[
(d - 5)(d + 3) = d^2 + 3d - 5d - 15
= d^2 - 2d - 15
\]

**ANSWER:**

\(d^2 - 2d - 15\)

81. \((2a^2 + 6)^2\)

**SOLUTION:**

\[
(2a^2 + 6)^2 = (2a^2)^2 + 2 \cdot 2a^2 \cdot 6 + 6^2
= 4a^4 + 24a^2 + 36
\]

**ANSWER:**

\(4a^4 + 24a^2 + 36\)

82. **GAS MILEAGE** The gas mileage \(y\) in miles per gallon for a certain vehicle is given by the equation

\[y = 10 + 0.9x - 0.01x^2\]

where \(x\) is the speed of the vehicle between 10 and 75 miles per hour. Find the range of speeds that would give a gas mileage of at least 25 miles per gallon.

**SOLUTION:**

The inequality that represents the situation is

\[-0.01x^2 + 0.9x + 10 \geq 25.\]

Use the Quadratic Formula and solve the related equation.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
= \frac{-0.9 \pm \sqrt{(0.9)^2 - 4(-0.01)(-15)}}{2(-0.01)}
= \frac{-0.9 \pm \sqrt{0.21}}{-0.02}
\approx \frac{-0.9 \pm 0.4583}{-0.02}
\approx 22.087 \text{ or } 67.91
\]

The two numbers divide the number line into three regions \(x \leq 22.087, 22.087 \leq x \leq 67.91\) and \(x \geq 67.91\). Test a value from each interval to see if it satisfies the original inequality. The points \(x = 20\) and \(x = 70\) are not included in the solution.

Therefore, the solution set is \(\{x \mid 22.087 \leq x \leq 67.91\}\).

**ANSWER:**

\(22.087 \leq x \leq 67.91\) mph
6-4 nth Roots

Write each equation in vertex form, if not already in that form. Identify the vertex, axis of symmetry, and direction of opening. Then graph the function.

83. \( y = -6(x + 2)^2 + 3 \)

**SOLUTION:**
The vertex of the graph is \((-2, 3)\). The axis of symmetry is \(x = -2\) and the graph opens downwards.

**ANSWER:**
\((-2, 3); x = -2; \text{down}\)

84. \( y = -\frac{1}{3}x^2 + 8x \)

**SOLUTION:**
The equation in vertex form:

\[
\begin{align*}
y &= -\frac{1}{3}x^2 + 8x \\
&= -\frac{1}{3} \left(x^2 - 24x\right) \\
&= -\frac{1}{3} \left(x^2 - 24x + 144 - 144\right) \\
&= -\frac{1}{3} \left(x - 12\right)^2 + 48
\end{align*}
\]

The vertex of the graph is \((12, 48)\). The axis of symmetry is \(x = 12\) and the graph opens downwards.

**ANSWER:**
\(y = -\frac{1}{3}(x - 12)^2 + 48; (12, 48); x = 12; \text{down}\)
85. \( y = (x - 2)^2 - 2 \)

**SOLUTION:**
The vertex of the graph is \((2, -2)\). The axis of symmetry is \(x = 2\) and the graph opens upwards.

**ANSWER:**
\((2, -2); x = 2; \text{ up}\)

86. \( y = 2x^2 + 8x + 10 \)

**SOLUTION:**
\[
\begin{align*}
y &= 2x^2 + 8x + 10 \\
&= 2(x^2 + 4x) + 10 \\
&= 2(x^2 + 4x + 4) + 10 - 8 \\
&= 2(x + 2)^2 + 2
\end{align*}
\]
The vertex of the graph is \((-2, 2)\). The axis of symmetry is \(x = -2\) and the graph opens upwards.

**ANSWER:**
\(-2, 2); x = -2; \text{ up}\)
Find each product.

86. \((x + 4)(x + 5)\)

**SOLUTION:**
\[(x + 4)(x + 5) = x^2 + 5x + 4x + 20 = x^2 + 9x + 20\]

**ANSWER:**
\[x^2 + 9x + 20\]

87. \((y - 3)(y + 4)\)

**SOLUTION:**
\[(y - 3)(y + 4) = y^2 + 4y - 3y - 12 = y^2 + y - 12\]

**ANSWER:**
\[y^2 + y - 12\]

88. \((a + 2)(a - 9)\)

**SOLUTION:**
\[(a + 2)(a - 9) = a^2 - 9a + 2a - 18 = a^2 - 7a - 18\]

**ANSWER:**
\[a^2 - 7a - 18\]

89. \((a - b)(a - 3b)\)

**SOLUTION:**
\[(a - b)(a - 3b) = a^2 - 3ab - ab + 3b^2 = a^2 - 4ab + 3b^2\]

**ANSWER:**
\[a^2 - 4ab + 3b^2\]

90. \((a - b)(a - 3b)\)

**SOLUTION:**
\[(x + 2y)(x - y) = x^2 - xy + 2xy - 2y^2 = x^2 + xy - 2y^2\]

**ANSWER:**
\[x^2 + xy - 2y^2\]

91. \((x + y)(x - y)\)

**SOLUTION:**
\[2(w + z)(w - 4z) = 2(w^2 - 4wz + wz - 4z^2) = 2(w^2 - 3wz - 4z^2) = 2w^2 - 6wz - 8z^2\]

**ANSWER:**
\[2w^2 - 6wz - 8z^2\]
6-5 Operations with Radical Expressions

CCSS PRECISION Simplify.

1. \(\sqrt{36ab^4c^5}\)

SOLUTION:

\[
\sqrt{36ab^4c^5} = \sqrt{6^2a(b^2)^2(c^2)^2c} = 6b^2c^2\sqrt{ac}
\]

ANSWER:

\(6b^2c^2\sqrt{ac}\)

2. \(\sqrt{144x^7y^5}\)

SOLUTION:

\[
\sqrt{144x^7y^5} = \sqrt{12^2(x^2)^3x(y^2)^2y} = 12x^3y^2\sqrt{xy}
\]

ANSWER:

\(12x^3y^2\sqrt{xy}\)

3. \(\sqrt{c^5}\)

SOLUTION:

\[
\frac{\sqrt{c^5}}{\sqrt{d^9}} = \frac{\sqrt{c^2}}{\sqrt{d^4}} \cdot \frac{c}{d^2} = \frac{c^2}{d^4 \sqrt{d}} = \frac{c^2 \sqrt{cd}}{d^3}
\]

ANSWER:

\(c^2 \sqrt{cd} \div d^3\)

4. \(\frac{\sqrt[4]{5x}}{\sqrt[4]{8y}}\)

SOLUTION:

\[
\frac{\sqrt[4]{5x}}{\sqrt[4]{8y}} = \frac{\sqrt[4]{5x}}{\sqrt[4]{2^3y}} = \frac{\sqrt[4]{5} \sqrt[4]{x}}{\sqrt[4]{2^3} \sqrt[4]{y}} = \frac{\sqrt[4]{10} \sqrt[4]{xy^3}}{2\sqrt[4]{y}} = \frac{\sqrt[4]{10} \sqrt[4]{xy^3}}{2y}
\]

ANSWER:

\(\frac{\sqrt{10} \sqrt[4]{xy^3}}{2y}\)
6-5 Operations with Radical Expressions

5. \(5\sqrt{2x} \cdot 3\sqrt{8x}\)

**SOLUTION:**

\[
5\sqrt{2x} \cdot 3\sqrt{8x} = 5 \cdot 3 \cdot \sqrt{2x \cdot 8x} \\
= 15\sqrt{16x^2} \\
= 15 \cdot 4x \\
= 60x
\]

**ANSWER:**

60x

6. \(4\sqrt{5a^5} \cdot \sqrt{125a^3}\)

**SOLUTION:**

\[
4\sqrt{5a^5} \cdot \sqrt{125a^3} = 4 \cdot \sqrt{5^2 \cdot a^2 \cdot a^3} \\
= 4 \cdot 5 \cdot a^3 \cdot \sqrt{a} \\
= 100a^4
\]

**ANSWER:**

100a^4

7. \(\sqrt[3]{36xy} \cdot 2\sqrt[6]{6x^2y^2}\)

**SOLUTION:**

\[
\sqrt[3]{36xy} \cdot 2\sqrt[6]{6x^2y^2} = 3 \cdot 2 \cdot \sqrt[3]{6} \cdot 6 \cdot x \cdot x^2 \cdot y \cdot y^2 \\
= 6 \cdot \sqrt[3]{6^3 \cdot x^3 \cdot y^3} \\
= 6 \cdot 6xy \\
= 36xy
\]

**ANSWER:**

36xy

8. \(\sqrt[3]{3x^3y^2} \cdot \sqrt[6]{27xy^2}\)

**SOLUTION:**

\[
\sqrt[3]{3x^3y^2} \cdot \sqrt[6]{27xy^2} = 3 \cdot x \cdot y^2 \cdot \sqrt[3]{3} \cdot x \cdot y^2 \\
= 3x|y|
\]

**ANSWER:**

3x|y|
9. \(5\sqrt{32} + \sqrt{27} + 2\sqrt{75}\)

**SOLUTION:**
\[
5\sqrt{32} + \sqrt{27} + 2\sqrt{75} \\
= 5\sqrt{16 \cdot 2} + \sqrt{9 \cdot 3} + 2\sqrt{25 \cdot 3} \\
= 5\sqrt{4^2 \cdot 2} + \sqrt{3^2 \cdot 3} + 2\sqrt{5^2 \cdot 3} \\
= 20\sqrt{2} + 3\sqrt{3} + 10\sqrt{3} \\
= 20\sqrt{2} + 13\sqrt{3}
\]

**ANSWER:**
\(20\sqrt{2} + 13\sqrt{3}\)

10. \(4\sqrt{40} + 3\sqrt{28} - \sqrt{200}\)

**SOLUTION:**
\[
4\sqrt{40} + 3\sqrt{28} - \sqrt{200} \\
= 4\sqrt{4 \cdot 10} + 3\sqrt{4 \cdot 7} - \sqrt{2 \cdot 100} \\
= 4\sqrt{2^2 \cdot 10} + 3\sqrt{2^2 \cdot 7} - \sqrt{2 \cdot 10^2} \\
= 8\sqrt{10} + 6\sqrt{7} - 10\sqrt{2}
\]

**ANSWER:**
\(8\sqrt{10} + 6\sqrt{7} - 10\sqrt{2}\)

11. \((4 + 2\sqrt{5})(3\sqrt{3} + 4\sqrt{5})\)

**SOLUTION:**
\[
(4 + 2\sqrt{5})(3\sqrt{3} + 4\sqrt{5}) \\
= 12\sqrt{3} + 16\sqrt{5} + 6\sqrt{15} + 40 \\
= 12\sqrt{3} + 16\sqrt{5} + 40 + 6\sqrt{15}
\]

**ANSWER:**
\(12\sqrt{3} + 16\sqrt{5} + 40 + 6\sqrt{15}\)

12. \((8\sqrt{3} - 2\sqrt{2})(8\sqrt{3} + 2\sqrt{2})\)

**SOLUTION:**
\[
(8\sqrt{3} - 2\sqrt{2})(8\sqrt{3} + 2\sqrt{2}) \\
= 64 \cdot 3 - 4 \cdot 2 \cdot 2 \\
= 192 - 16 \\
= 184
\]

**ANSWER:**
184
13. \[ \frac{5}{\sqrt{2} + 3} \]

**SOLUTION:**
\[
\frac{5}{\sqrt{2} + 3} = \frac{5}{\sqrt{2} + 3} \cdot \frac{\sqrt{2} - 3}{\sqrt{2} - 3} = \frac{5(\sqrt{2} - 3)}{(\sqrt{2})^2 - 3^2} = \frac{5\sqrt{2} - 15}{2 - 9} = \frac{15 - 5\sqrt{2}}{-7}
\]

**ANSWER:**
\[15 - 5\sqrt{2} \over 7\]

14. \[ \frac{8}{\sqrt{6} - 5} \]

**SOLUTION:**
\[
\frac{8}{\sqrt{6} - 5} = \frac{8}{\sqrt{6} - 5} \cdot \frac{\sqrt{6} + 5}{\sqrt{6} + 5} = \frac{8(\sqrt{6} + 5)}{(\sqrt{6})^2 - 5^2} = \frac{8\sqrt{6} + 40}{6 - 25} = \frac{-40 - 8\sqrt{6}}{19}
\]

**ANSWER:**
\[-40 - 8\sqrt{6} \over 19\]

15. \[ \frac{4 + \sqrt{2}}{\sqrt{2} - 3} \]

**SOLUTION:**
\[
\frac{4 + \sqrt{2}}{\sqrt{2} - 3} = \frac{4 + \sqrt{2}}{\sqrt{2} - 3} \cdot \frac{\sqrt{2} + 3}{\sqrt{2} + 3} = \frac{(4 + \sqrt{2})(\sqrt{2} + 3)}{(\sqrt{2})^2 - 3^2} = \frac{4\sqrt{2} + 12 + 2 + 3\sqrt{2}}{2 - 9} = \frac{7\sqrt{2} + 14}{-7} = \frac{-7 \sqrt{2}}{-7} = -2 - \sqrt{2}
\]

**ANSWER:**
\[-2 - \sqrt{2}\]

16. \[ \frac{6 - \sqrt{3}}{\sqrt{3} + 4} \]

**SOLUTION:**
\[
\frac{6 - \sqrt{3}}{\sqrt{3} + 4} = \frac{6 - \sqrt{3}}{\sqrt{3} + 4} \cdot \frac{\sqrt{3} - 4}{\sqrt{3} - 4} = \frac{(6 - \sqrt{3})(\sqrt{3} - 4)}{3 - 16} = \frac{6\sqrt{3} - 24 - 3 + 4\sqrt{3}}{-13} = \frac{10\sqrt{3} - 27}{-13} = \frac{27 - 10\sqrt{3}}{13}
\]

**ANSWER:**
\[27 - 10\sqrt{3} \over 13\]
17. GEOMETRY Find the altitude of the triangle if the area is $189 + 4\sqrt{3}$ square centimeters.

SOLUTION: Let $h$ be the altitude of the triangle.

$$189 + 4\sqrt{3} = \frac{1}{2}(12 + \sqrt{3})h$$

Solve for $h$.

$$h = \frac{2(189 + 4\sqrt{3})}{12 + \sqrt{3}}$$

$$= \frac{378 + 8\sqrt{3}}{12 + \sqrt{3}} \cdot \frac{12 - \sqrt{3}}{12 - \sqrt{3}}$$

$$= \frac{4512 - 282\sqrt{3}}{144 - 3}$$

$$= \frac{4512 - 282\sqrt{3}}{141}$$

$$= 32 - 2\sqrt{3}$$

Therefore, the altitude of the triangle is $32 - 2\sqrt{3}$ cm.

ANSWER: $32 - 2\sqrt{3}$ cm
6-5 Operations with Radical Expressions

21. \( \sqrt{18a^6b^3c^5} \)

**SOLUTION:**

\[
\sqrt{18a^6b^3c^5} \\
= \sqrt{9 \cdot 2 \cdot (a^3)^2 \cdot b^2 \cdot (c^2)^2 \cdot c} \\
= \sqrt{3^2 \cdot 2 \cdot (a^3)^2 \cdot b^2 \cdot (c^2)^2 \cdot c} \\
= \sqrt{3^2 \cdot \sqrt{2} \cdot \sqrt{(a^3)^2} \cdot \sqrt{b^2} \cdot \sqrt{(c^2)^2} \cdot \sqrt{c}} \\
= 3a^3bc^2 \sqrt{2bc}
\]

**ANSWER:**

\( 3a^3bc^2 \sqrt{2bc} \)

22. \( \frac{\sqrt{5a^5}}{\sqrt{b^{13}}} \)

**SOLUTION:**

\[
\frac{\sqrt{5a^5}}{\sqrt{b^{13}}} = \frac{\sqrt{5(a^2)^2 \cdot a}}{\sqrt{(b^6)^2 \cdot b}} \\
= \frac{a^2 \sqrt{5a}}{b^6 \sqrt{b}} \\
= \frac{a^2 \sqrt{5a}}{b^6 \sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} \\
= \frac{a^2 \sqrt{5ab}}{b^7}
\]

**ANSWER:**

\( \frac{a^2 \sqrt{5ab}}{b^7} \)

23. \( \sqrt{\frac{7x}{10y^3}} \)

**SOLUTION:**

\[
\sqrt{\frac{7x}{10y^3}} = \frac{\sqrt{7x}}{y \sqrt{10y}} \\
= \frac{\sqrt{7x}}{y \sqrt{10y}} \cdot \frac{\sqrt{10y}}{\sqrt{10y}} \\
= \frac{\sqrt{70xy}}{10y^2}
\]

**ANSWER:**

\( \frac{\sqrt{70xy}}{10y^2} \)

24. \( \frac{\sqrt[3]{6x^2}}{\sqrt[5]{y}} \)

**SOLUTION:**

\[
\frac{\sqrt[3]{6x^2}}{\sqrt[5]{y}} = \frac{\sqrt[3]{6x^2}}{\sqrt[5]{y}} \cdot \frac{\sqrt[5]{y^2}}{\sqrt[5]{y^2}} \\
= \frac{\sqrt[3]{150x^2y^2}}{\sqrt[5]{5y^3}} \\
= \frac{\sqrt[3]{150x^2y^2}}{5y}
\]

**ANSWER:**

\( \frac{\sqrt[3]{150x^2y^2}}{5y} \)
25. \( \frac{\sqrt[4]{7x^3}}{\sqrt[4]{4b^2}} \)

**SOLUTION:**
\[
\frac{\sqrt[4]{7x^3}}{\sqrt[4]{4b^2}} = \frac{\sqrt[4]{7x^3}}{\sqrt[4]{2^2b^2}} \cdot \frac{\sqrt[4]{2^2b^2}}{\sqrt[4]{2^2b^2}} \\
= \frac{\sqrt[4]{28x^3b^2}}{\sqrt[4]{2^4b^4}} \\
= \frac{\sqrt[4]{28x^3b^2}}{2|b|}
\]

**ANSWER:**
\( \frac{\sqrt[4]{28x^3b^2}}{2|b|} \)

26. \( 3\sqrt{5y} \cdot 8\sqrt{10yz} \)

**SOLUTION:**
\[
3\sqrt{5y} \cdot 8\sqrt{10yz} = 24\sqrt{50y^2z} \\
= 24\sqrt{2 \cdot 5^2 \cdot y^2 \cdot z} \\
= 24 \cdot 5 \cdot y \sqrt{2z} \\
= 120y \sqrt{2z}
\]

**ANSWER:**
120y \( \sqrt{2z} \)

27. \( 2\sqrt{32a^3b^5} \cdot \sqrt{8a^7b^2} \)

**SOLUTION:**

**ANSWER:**

28. \( 6\sqrt{3ab} \cdot 4\sqrt{24ab^3} \)

**SOLUTION:**
\[
6\sqrt{3ab} \cdot 4\sqrt{24ab^3} = 24\sqrt{72a^2b^4} \\
= 24 \sqrt{6^2 \cdot 2 \cdot a^2 \cdot (b^2)^3} \\
= 144|a|b^2 \sqrt{2}
\]

**ANSWER:**
144|a|b^2 \( \sqrt{2} \)

29. \( 5\sqrt{x^8y^3} \cdot 5\sqrt{2x^5y^4} \)

**SOLUTION:**
\[
5\sqrt{x^8y^3} \cdot 5\sqrt{2x^5y^4} = 25\sqrt{2x^{13}y^7} \\
= 25 \sqrt{2 \cdot (x^3)^2 \cdot x \cdot (y^3)^2 \cdot y} \\
= 25x^6y^3 \sqrt{2xy}
\]

**ANSWER:**
25 \( x^6y^3 \sqrt{2xy} \)

30. \( 3\sqrt{90} + 4\sqrt{20} + \sqrt{162} \)

**SOLUTION:**
\[
3\sqrt{90} + 4\sqrt{20} + \sqrt{162} \\
= 3\sqrt{2 \cdot 5^2 \cdot 2 \cdot 5^2 \cdot 2} + 4\sqrt{2^2 \cdot 5^2 \cdot 2} \\
= 9\sqrt{10} + 8\sqrt{5} + 9\sqrt{2}
\]

**ANSWER:**
9\( \sqrt{10} + 8\sqrt{5} + 9\sqrt{2} \)
31. \( 9\sqrt{12} + 5\sqrt{32} - \sqrt{72} \)

**SOLUTION:**

\[
9\sqrt{12} + 5\sqrt{32} - \sqrt{72} \\
= 9\sqrt{2^2 \cdot 3} + 5\sqrt{4^2 \cdot 2} - \sqrt{6^2 \cdot 2} \\
= 18\sqrt{3} + 20\sqrt{2} - 6\sqrt{2} \\
= 18\sqrt{3} + 14\sqrt{2}
\]

**ANSWER:**

\( 18\sqrt{3} + 14\sqrt{2} \)

32. \( 4\sqrt{28} - 8\sqrt{810} + \sqrt{44} \)

**SOLUTION:**

\[
4\sqrt{28} - 8\sqrt{810} + \sqrt{44} \\
= 4\sqrt{2^2 \cdot 7} - 8\sqrt{9^2 \cdot 10} + \sqrt{2^2 \cdot 11} \\
= 8\sqrt{7} - 72\sqrt{10} + 2\sqrt{11}
\]

**ANSWER:**

\( 8\sqrt{7} - 72\sqrt{10} + 2\sqrt{11} \)

33. \( 3\sqrt{54} + 6\sqrt{288} - \sqrt{147} \)

**SOLUTION:**

\[
3\sqrt{54} + 6\sqrt{288} - \sqrt{147} \\
= 3\sqrt{3^2 \cdot 6} + 6\sqrt{12^2 \cdot 2} - \sqrt{7^2 \cdot 3} \\
= 9\sqrt{6} + 72\sqrt{2} - 7\sqrt{3}
\]

**ANSWER:**

\( 9\sqrt{6} + 72\sqrt{2} - 7\sqrt{3} \)

34. **GEOMETRY** Find the perimeter of the rectangle.

The perimeter of a rectangle of length \( l \) and width \( w \) is \( P = 2l + 2w \).

\[
P = 2(8 + \sqrt{3}) + 2\sqrt{6} \\
= 16 + 2\sqrt{3} + 2\sqrt{6}
\]

Therefore, the perimeter of the rectangle is \( 16 + 2\sqrt{3} + 2\sqrt{6} \) ft.

**ANSWER:**

\( 16 + 2\sqrt{3} + 2\sqrt{6} \) ft
6-5 Operations with Radical Expressions

35. **GEOMETRY** Find the area of the rectangle.

\[
\begin{array}{c}
8 + \sqrt{3} \text{ ft} \\
\sqrt{6} \text{ ft}
\end{array}
\]

**SOLUTION:**
The area of a rectangle of length \( l \) and width \( w \) is \( A = lw \).

\[
A = (8 + \sqrt{3}) \sqrt{6}
\]
\[
= 8\sqrt{6} + \sqrt{3} \sqrt{6}
\]
\[
= 8\sqrt{6} + \sqrt{18}
\]
\[
= 8\sqrt{6} + 3\sqrt{2}
\]

Therefore, the area of the rectangle is \( 8\sqrt{6} + 3\sqrt{2} \text{ ft}^2 \).

**ANSWER:**
\( 8\sqrt{6} + 3\sqrt{2} \text{ ft}^2 \)

36. **GEOMETRY** Find the exact surface area of a sphere with radius of \( 4 + \sqrt{5} \) inches.

**SOLUTION:**
The surface area of a sphere of radius \( r \) is \( S = 4\pi r^2 \).

\[
S = 4\pi \left( 4 + \sqrt{5} \right)^2
\]
\[
= 4\pi \left( 16 + 5 + 8\sqrt{5} \right)
\]
\[
= 4\pi \left( 21 + 8\sqrt{5} \right)
\]
\[
= \left( 84 + 32\sqrt{5} \right) \pi
\]

The surface area of the sphere is \( \left( 84 + 32\sqrt{5} \right) \pi \) square inches.

**ANSWER:**
\( \left( 84 + 32\sqrt{5} \right) \pi \text{ in}^2 \)

Simplify.

37. \( (7\sqrt{2} - 3\sqrt{3})(4\sqrt{6} + 3\sqrt{12}) \)

**SOLUTION:**
\[
(7\sqrt{2} - 3\sqrt{3})(4\sqrt{6} + 3\sqrt{12})
\]
\[
= 28\sqrt{12} + 21\sqrt{2} \cdot 6 - 12\sqrt{3} \cdot 2 - 9\sqrt{36}
\]
\[
= 28\sqrt{2^2 \cdot 3} + 21\sqrt{2^2 \cdot 6} - 12\sqrt{3^2 \cdot 2} - 9\sqrt{6^2}
\]
\[
= 56\sqrt{3} + 42\sqrt{6} - 36\sqrt{2} - 54
\]

**ANSWER:**
\( 56\sqrt{3} + 42\sqrt{6} - 36\sqrt{2} - 54 \)
6-5 Operations with Radical Expressions

38. \((8\sqrt{5} - 6\sqrt{3})(8\sqrt{5} + 6\sqrt{3})\)

**SOLUTION:**
\[
(8\sqrt{5} - 6\sqrt{3})(8\sqrt{5} + 6\sqrt{3}) = (8\sqrt{5})^2 - (6\sqrt{3})^2
\]
\[
= 64(5) - 36(3)
\]
\[
= 320 - 108
\]
\[
= 212
\]

**ANSWER:** 212

39. \((12\sqrt{10} - 6\sqrt{5})(12\sqrt{10} + 6\sqrt{5})\)

**SOLUTION:**
\[
(12\sqrt{10} - 6\sqrt{5})(12\sqrt{10} + 6\sqrt{5}) = (12\sqrt{10})^2 - (6\sqrt{5})^2
\]
\[
= 144(10) - 36(5)
\]
\[
= 1440 - 180
\]
\[
= 1260
\]

**ANSWER:** 1260

40. \((6\sqrt{3} + 5\sqrt{2})(2\sqrt{6} + 3\sqrt{8})\)

**SOLUTION:**
\[
(6\sqrt{3} + 5\sqrt{2})(2\sqrt{6} + 3\sqrt{8}) = 12\sqrt{18} + 18\sqrt{24} + 10\sqrt{12} + 15\sqrt{16}
\]
\[
= 12\sqrt{2^2 \cdot 3^2} + 18\sqrt{2^2 \cdot 2^2} + 10\sqrt{2^2 \cdot 3} + 15\sqrt{2^2 \cdot 4^2}
\]
\[
= 36\sqrt{2} + 36\sqrt{6} + 20\sqrt{3} + 60
\]

**ANSWER:**

36\sqrt{2} + 36\sqrt{6} + 20\sqrt{3} + 60

41. \(\frac{6}{\sqrt{3} - \sqrt{2}}\)

**SOLUTION:**
\[
\frac{6}{\sqrt{3} - \sqrt{2}} = \frac{6}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}
\]
\[
= \frac{6(\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2}
\]
\[
= \frac{6\sqrt{3} + 6\sqrt{2}}{3 - 2}
\]
\[
= 6\sqrt{3} + 6\sqrt{2}
\]

**ANSWER:** 6\sqrt{3} + 6\sqrt{2}

42. \(\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}}\)

**SOLUTION:**
\[
\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}} = \frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}
\]
\[
= \frac{\sqrt{2} (\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}
\]
\[
= \frac{\sqrt{10} + \sqrt{6}}{5 - 3}
\]
\[
= \frac{\sqrt{10} + \sqrt{6}}{2}
\]

**ANSWER:** \(\frac{\sqrt{10} + \sqrt{6}}{2}\)
6-5 Operations with Radical Expressions

43. \( \frac{9 - 2\sqrt{3}}{\sqrt{3} + 6} \)

**SOLUTION:**
\[
\frac{9 - 2\sqrt{3}}{\sqrt{3} + 6} = \frac{9 - 2\sqrt{3}}{\sqrt{3} + 6} \cdot \frac{\sqrt{3} - 6}{\sqrt{3} - 6} \\
= \frac{9\sqrt{3} - 54 - 6 + 12\sqrt{3}}{(\sqrt{3})^2 - 6^2} \\
= \frac{21\sqrt{3} - 60}{-36} \\
= \frac{60 - 21\sqrt{3}}{36} \\
= \frac{20 - 7\sqrt{3}}{11}
\]

**ANSWER:**
\[
\frac{20 - 7\sqrt{3}}{11}
\]

44. \( \frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}} \)

**SOLUTION:**
\[
\frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}} = \frac{2\sqrt{2} + 2\sqrt{5}}{\sqrt{5} + \sqrt{2}} \cdot \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} \\
= \frac{2\sqrt{10} - 4 + 10 - 2\sqrt{10}}{(\sqrt{5})^2 - (\sqrt{2})^2} \\
= \frac{6}{5 - 2} \\
= 2
\]

**ANSWER:**
2

45. \( \sqrt[3]{16y^4} \cdot z^{12} \)

**SOLUTION:**
\[
\sqrt[3]{16y^4} \cdot z^{12} = \sqrt[3]{2^4 \cdot y^3 \cdot z^4} \\
= 2yz \sqrt[3]{2y}
\]

**ANSWER:**
\[
2yz \sqrt[3]{2y}
\]

46. \( \sqrt[3]{-54x^6y^{11}} \)

**SOLUTION:**
\[
\sqrt[3]{-54x^6y^{11}} = \sqrt[3]{(-3)^3 \cdot 2 \cdot x^2 \cdot (y^3)^3 \cdot y^2} \\
= -3x^2y^3 \sqrt[3]{2y^2}
\]

**ANSWER:**
\[
-3x^2y^3 \sqrt[3]{2y^2}
\]

47. \( \sqrt[3]{162a^6b^{13}c} \)

**SOLUTION:**
\[
\sqrt[3]{162a^6b^{13}c} = \sqrt[3]{2^3 \cdot 3^2 \cdot a^6 \cdot b^{13} \cdot c} \\
= 3a^2b^4 \sqrt[3]{2a^3bc}
\]

**ANSWER:**
\[
3a^2b^4 \sqrt[3]{2a^3bc}
\]
48. \( \sqrt[4]{48a^9b^3c^{16}} \)

**SOLUTION:**

\[
\sqrt[4]{48a^9b^3c^{16}} = \left(2^4 \cdot (a^2)^4 \cdot a \cdot b^3 \cdot (c^4)^4\right) = 2a^2c^4 \sqrt[3]{3ab^3}
\]

**ANSWER:**

\( 2a^2c^4 \sqrt[3]{3ab^3} \)

49. \( \sqrt[4]{12x^3y^2} \)

**SOLUTION:**

\[
\sqrt[4]{12x^3y^2} = \sqrt[4]{12x^3y^2 \cdot \frac{3}{5}a^3b^3} = \sqrt[4]{1500x^3y^2a^3b^3} = \frac{\sqrt[4]{1500x^3y^2a^3b^3}}{5|a|b}
\]

**ANSWER:**

\( \frac{\sqrt[4]{1500x^3y^2a^3b^3}}{5|a|b} \)

50. \( \frac{\sqrt[3]{36xy^2}}{\sqrt[10]{10xz}} \)

**SOLUTION:**

\[
\frac{\sqrt[3]{36xy^2}}{\sqrt[10]{10xz}} = \frac{\sqrt[3]{36xy^2}}{\sqrt[10]{10xz}} \cdot \frac{\sqrt[3]{100x^2y^2}}{\sqrt[3]{100x^2y^2}} = \frac{\sqrt[3]{3600x^3y^2z^2}}{\sqrt[3]{10^3x^3z^3}} = \frac{\sqrt[3]{2^3 \cdot 450 \cdot x^3y^2 \cdot z^2}}{10xz} = \frac{2x\sqrt[3]{450y^2\cdot z^2}}{10xz} = \frac{\sqrt[3]{450y^2\cdot z^2}}{5z}
\]

**ANSWER:**

\( \frac{\sqrt[3]{450y^2\cdot z^2}}{5z} \)

51. \( x + 1 / \sqrt{x - 1} \)

**SOLUTION:**

\[
\frac{x + 1}{\sqrt{x - 1}} = \frac{x + 1}{\sqrt{x - 1}} \cdot \frac{\sqrt{x + 1}}{\sqrt{x + 1}} = \frac{(x + 1)(\sqrt{x + 1})}{(\sqrt{x})^2 - 1^2} = \frac{x\sqrt{x} + x + \sqrt{x} + 1}{x - 1}
\]

**ANSWER:**

\( \frac{(x + 1)(\sqrt{x} + 1)}{x - 1} \) or \( \frac{x\sqrt{x} + \sqrt{x} + x + 1}{x - 1} \)
54. **APPLES** The diameter of an apple is related to its weight and can be modeled by the formula 
\[ d = \sqrt[3]{3w}, \] where \( d \) is the diameter in inches and \( w \) is the weight in ounces. Find the diameter of an apple that weighs 6.47 ounces.

**SOLUTION:**
Substitute 6.47 for \( w \) and simplify.

\[
 d = \sqrt[3]{3(6.47)}
 = \sqrt[3]{19.41}
 \approx 2.69
\]

The diameter of the apple is about 2.69 inches.

**ANSWER:**
2.69 in.

Simplify each expression if \( b \) is an even number.

55. \( \sqrt[b]{a^b} \)

**SOLUTION:**
\[
 \sqrt[b]{a^b} = (a^b)^{1/b} \\
 = |a|
\]

**ANSWER:**
\( |a| \)
6-5 Operations with Radical Expressions

56. \( \sqrt[b]{a^{4b}} \)

**SOLUTION:**
\[
\sqrt[b]{a^{4b}} = (a^{4b})^{1/b} = a^4
\]

**ANSWER:**

\( a^4 \)

57. \( \sqrt[b]{a^{2b}} \)

**SOLUTION:**
\[
\sqrt[b]{a^{2b}} = (a^{2b})^{1/b} = a^2
\]

**ANSWER:**

\( a^2 \)

58. \( \sqrt[b]{a^{3b}} \)

**SOLUTION:**
\[
\sqrt[b]{a^{3b}} = (a^{3b})^{1/b} = |a^3|
\]

**ANSWER:**

\( |a^3| \)

59. **MULTIPLE REPRESENTATIONS** In this problem, you will explore operations with like radicals.

a. **NUMERICAL** Copy the diagram at the right on dot paper. Use the Pythagorean Theorem to prove that the length of the red segment is \( \sqrt{2} \) units.

b. **GRAPHICAL** Extend the segment to represent \( \sqrt{2} + \sqrt{2} \).

c. **ANALYTICAL** Use your drawing to show that \( \sqrt{2} + \sqrt{2} \neq \sqrt{2} + 2 \) or 2.

d. **GRAPHICAL** Use the dot paper to draw a square with side lengths \( \sqrt{2} \) units.

e. **NUMERICAL** Prove that the area of the square is \( \sqrt{2} \times \sqrt{2} = 2 \) square units.

**SOLUTION:**

a. 
\[
a^2 + b^2 = c^2
\]
\[
1^2 + 1^2 = c^2
\]
\[
2 = c^2
\]
\[
c = \sqrt{2}
\]

c. \( \sqrt{2} + \sqrt{2} \) units is the length of the hypotenuse of an isosceles right triangle with legs of length 2 units. Therefore, \( \sqrt{2} + \sqrt{2} > 2 \).

d. 

![Diagram](image)

e. The square creates 4 triangles with a base of 1 and a height of 1.
Therefore the area of each triangle is
\[
\frac{1}{2} bh = \frac{1}{2} (1)(1).
\]

The area of the square is 2, so \(\sqrt{2} \cdot \sqrt{2} = 2\).

**ANSWER:**

\[
a. \ a^2 + b^2 = c^2
\]
\[
1^2 + 1^2 = c^2
\]
\[
2 = c^2
\]
\[
c = \sqrt{2}
\]

\[
b.
\]

\[
\text{c. } \sqrt{2} + \sqrt{2} \text{ units is the length of the hypotenuse of an isosceles right triangle with legs of length 2 units. Therefore, } \sqrt{2} + \sqrt{2} > 2.
\]

\[
d.
\]

\[
\text{e. The square creates 4 triangles with a base of 1 and a height of 1. Therefore the area of each triangle is}
\]
\[
\frac{1}{2} bh = \frac{1}{2} (1)(1).
\]

The area of the square is 2, so \(\sqrt{2} \cdot \sqrt{2} = 2\).

60. **ERROR ANALYSIS** Twyla and Ben are simplifying \(4\sqrt{32} + 6\sqrt{18}\). Is either of them correct? Explain your reasoning.

**SOLUTION:**

Ben’s mistakes were multiplying the 4 by 16 instead of 4 and multiplying the 6 by 9 instead of 3.

**ANSWER:**

Twyla; Ben’s mistakes were multiplying the 4 by 16 instead of 4 and multiplying the 6 by 9 instead of 3.
61. CHALLENGE Show that \( \frac{-1 - i \sqrt{3}}{2} \) is a cube root of 1.

**SOLUTION:**
\[
\left( \frac{-1 - i \sqrt{3}}{2} \right)^3
= \left( \frac{-1 - i \sqrt{3}}{2} \right) \cdot \left( \frac{-1 - i \sqrt{3}}{2} \right) \cdot \left( \frac{-1 - i \sqrt{3}}{2} \right)
= \frac{(-1-i\sqrt{3})(-1-i\sqrt{3})(-1-i\sqrt{3})}{8}
= \frac{(1+i\sqrt{3}+i\sqrt{3}+3)(-1-i\sqrt{3})}{8}
= \frac{(2\sqrt{3}-2)(-1-i\sqrt{3})}{8}
= \frac{-2\sqrt{3}+2+2i\sqrt{3}}{8}
= \frac{-6i^2+2}{8} = \frac{8}{8} \text{ or } 1
\]

**ANSWER:**
\[
\left( \frac{-1 - i \sqrt{3}}{2} \right)^3
= \frac{(-1-i\sqrt{3})(-1-i\sqrt{3})(-1-i\sqrt{3})}{8}
= \frac{(1+i\sqrt{3}+i\sqrt{3}+3)(-1-i\sqrt{3})}{8}
= \frac{(2\sqrt{3}-2)(-1-i\sqrt{3})}{8}
= \frac{-2\sqrt{3}+2+2i\sqrt{3}}{8}
= \frac{-6i^2+2}{8} = \frac{8}{8} \text{ or } 1
\]

62. CCSS ARGUMENTS For what values of \( a \) is \( \sqrt{a} \cdot \sqrt{-a} \) a real number? Explain.

**SOLUTION:**
0 is the only possible value for \( a \) since \( \sqrt{a} \) is defined for \( a \geq 0 \), and \( \sqrt{-a} \) is defined for \( a \leq 0 \).

**ANSWER:**
0 is the only possible value for \( a \) since \( \sqrt{a} \) is defined for \( a \geq 0 \), and \( \sqrt{-a} \) is defined for \( a \leq 0 \).

63. CHALLENGE Find four combinations of whole numbers that satisfy \( \sqrt{256} = b \).

**SOLUTION:**
\[
\begin{align*}
1\sqrt{256} &= 256 & a = 1, b = 256 \\
2\sqrt{256} &= 16 & a = 2, b = 16 \\
4\sqrt{256} &= 4 & a = 4, b = 4 \\
8\sqrt{256} &= 2 & a = 8, b = 2
\end{align*}
\]

**ANSWER:**
\[
\begin{align*}
a &= 1, b = 256; & a &= 2, b = 16; & a &= 4, b = 4; & a &= 8, b = 2
\end{align*}
\]
6-5 Operations with Radical Expressions

64. OPEN ENDED Find a number other than 1 that has a positive whole number for a square root, cube root, and fourth root.

**SOLUTION:**
The LCM of 2, 3 and 4 is 12.

\[ 2^{12} = 4096 \]
\[ \sqrt{4096} = 64 \]
\[ \sqrt[3]{4096} = 16 \]
\[ \sqrt[4]{4096} = 8 \]

Sample answer: 4096

**ANSWER:**
Sample answer: 4096

65. WRITING IN MATH Explain why absolute values may be unnecessary when an \( n \)th root of an even power results in an odd power.

**SOLUTION:**
Sample answer: It is only necessary to use absolute values when it is possible that \( n \) could be odd or even and still be defined. It is when the radicand must be nonnegative in order for the root to be defined that the absolute values are not necessary.

**ANSWER:**
Sample answer: It is only necessary to use absolute values when it is possible that \( n \) could be odd or even and still be defined. It is when the radicand must be nonnegative in order for the root to be defined that the absolute values are not necessary.

66. PROBABILITY A six-sided number cube has faces with the numbers 1 through 6 marked on it. What is the probability that a number less than 4 will occur on one toss of the number cube?

A. \( \frac{1}{2} \)

B. \( \frac{1}{3} \)

C. \( \frac{1}{4} \)

D. \( \frac{1}{5} \)

**SOLUTION:**
There are 3 chances to get a number less than 4.

The probability of getting a number less than 4 is \( \frac{3}{6} \) or \( \frac{1}{2} \).

Option A is the correct answer.

**ANSWER:**
A
6-5 Operations with Radical Expressions

67. When the number of a year is divisible by 4, the year is a leap year. However, when the year is divisible by 100, the year is not a leap year, unless the year is divisible by 400. Which is not a leap year?

F 1884
G 1900
H 1904
J 1940

**SOLUTION:**
1900 is divisible by 100 and 1900 is not divisible by 400. Therefore, 1900 is not a leap year.

Option G is the correct answer.

**ANSWER:**
G

68. **SHORT RESPONSE** Which property is illustrated by $4x + 0 = 4x$?

**SOLUTION:**
Additive Identity Property

**ANSWER:**
Additive Identity Property

69. **SAT/ACT** The expression $\sqrt[8]{180a^2b^8}$ is equivalent to which of the following?

A $3\sqrt[10]{|a|b^4}$
B $5\sqrt[6]{|a|b^4}$
C $6\sqrt[5]{|a|b^4}$
D $18\sqrt[10]{|a|b^4}$
E $36\sqrt[5]{|a|b^4}$

**SOLUTION:**

$$\sqrt{180a^2b^8} = \sqrt{36 \cdot 5 \cdot a^2 \cdot (b^4)^2}$$
$$= \sqrt{6^2 \cdot 5 \cdot a^2 \cdot (b^4)^2}$$
$$= 6\sqrt[5]{|a|b^4}$$

Therefore, option C is the correct answer.

**ANSWER:**
C

70. $\sqrt{81x^6}$

**SOLUTION:**

$$\sqrt{81x^6} = \sqrt{9^2(x^3)^2}$$
$$= 9|x^3|$$

**ANSWER:**
$9|x^3|$
6-5 Operations with Radical Expressions

71. \( \sqrt[3]{729a^3b^9} \)

**SOLUTION:**

\[
\sqrt[3]{729a^3b^9} = \sqrt[3]{9^3a^3(b^3)^3} = 9ab^3
\]

**ANSWER:**

\( 9ab^3 \)

72. \( \sqrt{(g+5)^2} \)

**SOLUTION:**

\[
\sqrt{(g+5)^2} = |g+5|
\]

**ANSWER:**

\( |g+5| \)

73. Graph \( y \leq \sqrt{x-2} \).

**SOLUTION:**

Graph the inequality \( y \leq \sqrt{x-2} \).

![Graph of the inequality \( y \leq \sqrt{x-2} \)]
6-5 Operations with Radical Expressions

Solve each equation.

74. \(x^4 - 34x^2 + 225 = 0\)

**SOLUTION:**
\[x^4 - 34x^2 + 225 = 0\]
Let \(y = x^2\).
\[y^2 - 34y + 225 = 0\]
\[(y - 25)(y - 9) = 0\]
By the Zero Product Property:
\[
\begin{align*}
y - 25 &= 0 &\quad\text{or}\quad y - 9 &= 0 \\
y &= 25 &\quad\text{or}\quad y &= 9 \\
x^2 &= 25 &\quad\text{or}\quad x^2 &= 9 \\
x &= \pm 5 &\quad\text{or}\quad x &= \pm 3
\end{align*}
\]
The solutions are \(-5, -3, 3\) and \(5\).

**ANSWER:**
\(-5, -3, 3, 5\)

75. \(x^4 - 15x^2 - 16 = 0\)

**SOLUTION:**
\[x^4 - 15x^2 - 16 = 0\]
Let \(y = x^2\).
\[y^2 - 15y - 16 = 0\]
\[(y - 16)(y + 1) = 0\]
By the Zero Product Property:
\[
\begin{align*}
y - 16 &= 0 &\quad\text{or}\quad y + 1 &= 0 \\
y &= 16 &\quad\text{or}\quad y &= -1 \\
x^2 &= 16 &\quad\text{or}\quad x^2 &= -1 \\
x &= \pm 4 &\quad\text{or}\quad x &= \pm i
\end{align*}
\]
The solutions are \(-4, 4, -i, i\).

**ANSWER:**
\(-4, 4, -i, i\)
6-5 Operations with Radical Expressions

76. \( x^4 + 6x^2 - 27 = 0 \)

\[ \text{SOLUTION:} \]
\[ x^4 + 6x^2 - 27 = 0 \]

Let \( y = x^2 \).

\[ y^2 + 6y - 27 = 0 \]
\[ (y + 9)(y - 3) = 0 \]

By the Zero Product Property:

\[ y - 3 = 0 \quad \text{or} \quad y + 9 = 0 \]

\[ y = 3 \quad \text{or} \quad y = -9 \]

\[ x^2 = 3 \quad \text{or} \quad x^2 = -9 \]

\[ x = \pm \sqrt{3} \quad \text{or} \quad x = \pm 3i \]

The solutions are \( \pm \sqrt{3}, -3i \) and \( 3i \).

\[ \text{ANSWER:} \]
\[ -\sqrt{3}, \sqrt{3}, -3i, 3i \]

77. \( x^3 + 64 = 0 \)

\[ \text{SOLUTION:} \]
\[ x^3 + 64 = 0 \]
\[ x^3 + 4^3 = 0 \]
\[ (x + 4)(x^2 - 4x + 16) = 0 \]

By the Zero Product Property:

\[ x + 4 = 0 \]

\[ x = -4 \]

or

\[ x^2 - 4x + 16 = 0 \]

\[ x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)} \]
\[ = \frac{4 \pm \sqrt{16 - 64}}{2} \]
\[ = \frac{4 \pm \sqrt{-48}}{2} \]
\[ = \frac{4 \pm 4i\sqrt{3}}{2} \]
\[ = 2 \pm 2i\sqrt{3} \]

The solutions are \( 2 \pm 2i\sqrt{3} \) and \(-4\).

\[ \text{ANSWER:} \]
\[ -4, 2 + 2i\sqrt{3}, 2 - 2i\sqrt{3} \]
6-5 Operations with Radical Expressions

78. \(27x^3 + 1 = 0\)

**SOLUTION:**

\[
27x^3 + 1 = 0 \\
(3x)^3 + 1^3 = 0 \\
(3x + 1)(9x^2 - 3x + 1) = 0
\]

By the Zero Product Property:

\[
x + 1 = 0 \quad \text{or} \quad 9x^2 - 3x + 1 = 0
\]

\[
x = -1 \quad \text{or} \quad x = \frac{-3 \pm \sqrt{(-3)^2 - 4(9)(1)}}{2(9)}
\]

\[
x = -1 \quad \text{or} \quad x = \frac{3 \pm \sqrt{9 - 36}}{18}
\]

\[
x = \frac{1 + i\sqrt{3}}{6} \quad \text{and} \quad x = -\frac{1}{3}
\]

**ANSWER:**

\[-\frac{1}{3}, \frac{1 + i\sqrt{3}}{6}, \frac{1 - i\sqrt{3}}{6}\]

79. \(8x^3 - 27 = 0\)

**SOLUTION:**

\[
8x^3 - 27 = 0 \\
(2x)^3 - 3^3 = 0 \\
(2x - 3)(4x^2 + 6x + 9) = 0
\]

By the Zero Product Property:

\[
x - 3 = 0 \quad \text{or} \quad 4x^2 + 6x + 9 = 0
\]

\[
x = 3 \quad \text{or} \quad x = \frac{-6 \pm \sqrt{36 - 4(4)(9)}}{2(4)}
\]

\[
x = \frac{3}{2} \quad \text{or} \quad x = \frac{-6 \pm \sqrt{36 - 144}}{8}
\]

\[
x = \frac{3}{2} \quad \text{or} \quad x = \frac{-6 \pm \sqrt{-108}}{8}
\]

\[
x = \frac{-6 + 6i\sqrt{3}}{8} \quad \text{or} \quad x = \frac{-3 + 3i\sqrt{3}}{4}
\]

The solutions are \(-\frac{3}{2} \pm 3i\sqrt{3}\) and \(\frac{3}{2}\).

**ANSWER:**

\[-\frac{3}{2} - 3i\sqrt{3}, -\frac{3}{2} + 3i\sqrt{3}\]
80. **MODELS** A model car builder is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could be the width of the table?

**SOLUTION:**
Let $w$ and $l$ be the width and the length of the table. The equation and the inequality represent this situation is:

$2l + 2w = 26$
$wl \leq 30$

$w(13 - w) \leq 30$
$13x - w^2 \leq 30$
$-w^2 + 13w - 30 \leq 0$

Solve the corresponding quadratic equation
$-w^2 + 13w - 30 = 0$

$-w^2 + 13w - 30 = 0$
$w^2 - 13w + 30 = 0$
$(w - 10)(w - 3) = 0$

$w = 10 \quad \text{or} \quad w = 3$

Therefore, the solution region of the inequality $-w^2 + 13w - 30 \leq 0$ is $\{ w | w \leq 3 \text{ or } w \geq 10 \}$.

Since $w$ is the width of the table, it cannot be negative. Also, the perimeter of the table to be 26 feet; therefore, the width of the table could be between 0 and 3 ft or between 10 and 13 feet.

**ANSWER:**
between 0 and 3 ft or between 10 and 13 ft

81. **CONSTRUCTION** Cho charges $1500 to build a small deck and $2500 to build a large deck. During the spring and summer, she built 5 more small decks than large decks. If she earned $23,500 how many of each type of deck did she build?

**SOLUTION:**
Let $s$ and $l$ be the number of small and large desk respectively.

The system of equations representing this situation is:

$\begin{align*}
1500s + 2500l &= 23500 \\
\quad s + 5 &= l
\end{align*}$

Substitute $l + 5$ for $s$ in the first equation and solve for $l$.

$\begin{align*}
1500(l + 5) + 2500l &= 23500 \\
1500l + 7500 + 2500l &= 23500 \\
4000l &= 16000 \\
\quad l &= 4
\end{align*}$

Substitute 4 for $l$ in the second equation and solve for $s$.

$s = 4 + 5 = 9$

She built 9 small decks and 4 large decks.

**ANSWER:**
9 small, 4 large
82. **FOOD** The Hot Dog Grille offers the lunch combinations shown. Assume that the price of a combo meal is the same price as purchasing each item separately. Find the prices for a hot dog, a soda, and a bag of potato chips.

<table>
<thead>
<tr>
<th>Lunch Combo Meals</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Two hot dogs, one soda</td>
<td>$5.40</td>
<td></td>
</tr>
<tr>
<td>2. One hot dog, potato chips, one soda</td>
<td>$4.35</td>
<td></td>
</tr>
<tr>
<td>3. Two hot dogs, two bags of chips</td>
<td>$5.70</td>
<td></td>
</tr>
</tbody>
</table>

**SOLUTION:**
Let \(x, y\) and \(z\) be the price for a hot dog, a soda and a bag of potato chips.

The system of equations representing this situation is:

\[
\begin{align*}
2x + y &= 5.40 \\
x + y + z &= 4.35 \\
2x + 2z &= 5.70
\end{align*}
\]

Use Cramer’s Rule to find the values of \(x, y\), and \(z\).

The solutions are (1.95, 1.50, 0.90).

The price for a hot dog is $1.95.
The price for a soda is $1.50.
The price for a bag of potato chips is $0.90.

**ANSWER:**
hot dog, $1.95; soda, $1.50; potato chips, $0.90

---

**Evaluate each expression.**

83. \(2 \left( \frac{1}{6} \right)\)

**SOLUTION:**
\[
2 \left( \frac{1}{6} \right) = \frac{1}{3}
\]

**ANSWER:**
\[
\frac{1}{3}
\]

84. \(3 \left( \frac{1}{8} \right)\)

**SOLUTION:**
\[
3 \left( \frac{1}{8} \right) = \frac{3}{8}
\]

**ANSWER:**
\[
\frac{3}{8}
\]
Use Cramer’s Rule to find the values of $x$, $y$, and $z$.

The solutions are $(1.95, 1.50, 0.90)$.

The price for a hot dog is $1.95.
The price for a soda is $1.50.
The price for a bag of potato chips is $0.90.

ANSWER:
hot dog, $1.95; soda, $1.50; potato chips, $0.90

Evaluate each expression.

83.

SOLUTION:

ANSWER:

84.

SOLUTION:

ANSWER:

85.

SOLUTION:

ANSWER:

86.

SOLUTION:

ANSWER:

87.

SOLUTION:

ANSWER:

88. $rac{5}{6} - rac{2}{5}$

SOLUTION:

ANSWER:
Write each expression in radical form, or write each radical in exponential form.

1. \( \frac{1}{10^4} \)

**SOLUTION:**

\[
\frac{1}{10^4} = \frac{1}{\sqrt[4]{10}}
\]

**ANSWER:**

\( \frac{1}{\sqrt[4]{10}} \)

2. \( x^\frac{3}{5} \)

**SOLUTION:**

\[
x^\frac{3}{5} = (x^3)^\frac{1}{5} = \sqrt[5]{x^3}
\]

**ANSWER:**

\( \sqrt[5]{x^3} \)

3. \( \sqrt[3]{15} \)

**SOLUTION:**

\[
\sqrt[3]{15} = 15^{\frac{1}{3}}
\]

**ANSWER:**

\( 15^{\frac{1}{3}} \)

4. \( \sqrt[6]{7x^6y^9} \)

**SOLUTION:**

\[
\sqrt[6]{7x^6y^9} = (7x^6y^9)^{\frac{1}{6}} = 7^{\frac{1}{6}}x^{\frac{6}{6}}y^{\frac{9}{6}} = 7^{\frac{1}{3}}x^1y^{\frac{3}{2}} = 7^{\frac{1}{3}}x^1y^{\frac{3}{2}}
\]

**ANSWER:**

\( 7^{\frac{1}{3}}x^1y^{\frac{3}{2}} \)

5. \( \frac{1}{343^3} \)

**SOLUTION:**

\[
\frac{1}{343^3} = \left(7^3\right)^{\frac{1}{3}} = 7^{3 \cdot \frac{1}{3}} = 7
\]

**ANSWER:**

7

6. \( \frac{2}{125^3} \)

**SOLUTION:**

\[
\frac{2}{125^3} = \left(5^3\right)^{\frac{2}{3}} = 5^{3 \cdot \frac{2}{3}} = 5^2 = 25
\]

**ANSWER:**

25
6-6 Rational Exponents

7. \(32^{-\frac{1}{5}}\)

**SOLUTION:**
\[
32^{-\frac{1}{5}} = \frac{1}{32^{\frac{1}{5}}} = \frac{1}{(2^5)^{\frac{1}{5}}} = \frac{1}{2}
\]

**ANSWER:**
\(\frac{1}{2}\)

8. \(\frac{24^3}{4^2}\)

**SOLUTION:**
\[
\frac{24^3}{4^2} = \frac{24^3}{(2^2)^3} = \frac{24}{2^3} = \frac{24}{8} = 3
\]

**ANSWER:**
3

9. **GARDENING** If the area \(A\) of a square is known, then the lengths of its sides \(\ell\) can be computed using \(\ell = A^{\frac{1}{2}}\). You have purchased a 169 ft\(^2\) share in a community garden for the season. What is the length of one side of your square garden?

**SOLUTION:**
Substitute 169 for \(A\) in the given equation and simplify.
\[
\ell = 169^{\frac{1}{2}} = (13^2)^{\frac{1}{2}} = 13
\]

The length of one side of the square garden is 13 ft.

**ANSWER:**
13 ft

CCSS PRECISION  Simplify each expression.

10. \(a^\frac{3}{4} \cdot a^\frac{1}{2}\)

**SOLUTION:**
\[
a^\frac{3}{4} \cdot a^\frac{1}{2} = a^{\frac{3}{4} + \frac{1}{2}} = a^{\frac{3}{4} + \frac{2}{4}} = a^{\frac{5}{4}}
\]

**ANSWER:**
\(a^{\frac{5}{4}}\)
11. \( \frac{x^5}{x^5} \)

**SOLUTION:**
\[
\frac{x^5}{x^5} = x^{5-5} = x^0 = 1
\]

**ANSWER:**

\[
x^0
\]

12. \( \frac{b^3}{c^2 \cdot b^3} \)

**SOLUTION:**
\[
\frac{b^3}{c^2 \cdot b^3} = b^{3-3} \cdot c^{-2} = \frac{b^0}{c^2} = \frac{1}{c^2}
\]

**ANSWER:**

\[
\frac{1}{c^2}
\]

13. \( \frac{4}{9g^2} \)

**SOLUTION:**
\[
\frac{4}{9g^2} = \left(\frac{2}{3g}\right)^2
\]

**ANSWER:**

\[
\frac{2}{3g}
\]

14. \( \frac{\sqrt[3]{64}}{\sqrt[4]{4}} \)

**SOLUTION:**
\[
\frac{\sqrt[3]{64}}{\sqrt[4]{4}} = \left(\frac{2^6}{2^2}\right)^{\frac{1}{3}} \cdot \left(\frac{2^2}{2^2}\right)^{\frac{1}{4}} = 2^{\frac{6}{3}} \cdot 2^{\frac{2}{4}} = 2^2 \cdot 2^{\frac{1}{2}} = 2^2 \sqrt{2} = 4 \sqrt{2}
\]

**ANSWER:**

\[
4 \sqrt{2} \text{ or } \sqrt{16}
\]
6-6 Rational Exponents

15. \(\frac{g^2 - 1}{g^2 + 1}\)

**SOLUTION:**

\[
\frac{1}{\sqrt{g^2 - 1}} = \frac{\sqrt{g - 1}}{\sqrt{g + 1}}
\]

\[
= \frac{\sqrt{g - 1} \cdot \sqrt{g - 1}}{\sqrt{g + 1} \cdot \sqrt{g - 1}}
\]

\[
= \frac{\left(\sqrt{g - 1}\right)^2}{\left(\sqrt{g}\right)^2 - 1^2}
\]

\[
= \frac{g + 1 - 2\sqrt{g}}{g - 1}
\]

\[
= \frac{g + 1 - 2g\frac{1}{2}}{g - 1}
\]

**ANSWER:**

\(\frac{g - 2g\frac{1}{2} + 1}{g - 1}\)

17. \(4^\frac{2}{7}\)

**SOLUTION:**

\(4^\frac{2}{7} = (4^2)^\frac{1}{7} = 7\sqrt[7]{4^2} = \sqrt[7]{16}\)

**ANSWER:**

\(\sqrt[7]{16}\)

18. \(a^\frac{3}{4}\)

**SOLUTION:**

\(a^\frac{3}{4} = \left(a^3\right)^\frac{1}{4} = \sqrt[4]{a^3}\)

**ANSWER:**

\(\sqrt[4]{a^3}\)

19. \(\left(x^3\right)^\frac{3}{2}\)

**SOLUTION:**

\(\left(x^3\right)^\frac{3}{2} = \left(x^9\right)^\frac{1}{2} = \sqrt{x^9}\)

**ANSWER:**

\(\sqrt{x^9}\)

Write each expression in radical form, or write each radical in exponential form.

16. \(8^\frac{1}{5}\)

**SOLUTION:**

\(8^\frac{1}{5} = \sqrt[5]{8}\)

**ANSWER:**

\(\sqrt[5]{8}\)
20. \( \sqrt{17} \)

**SOLUTION:**
\[
\sqrt{17} = 17^{\frac{1}{2}}
\]

**ANSWER:**
\[
17^{\frac{1}{2}}
\]

21. \( \frac{1}{\sqrt{63}} \)

**SOLUTION:**
\[
\frac{1}{\sqrt{63}} = 63^{-\frac{1}{2}}
\]

**ANSWER:**
\[
63^{-\frac{1}{2}}
\]

22. \( \frac{1}{\sqrt[3]{5xy^2}} \)

**SOLUTION:**
\[
\frac{1}{\sqrt[3]{5xy^2}} = (5xy^2)^{-\frac{1}{3}} = 5^{\frac{1}{3}}x^{\frac{1}{3}}y^{-\frac{2}{3}}
\]

**ANSWER:**
\[
5^{\frac{1}{3}}x^{\frac{1}{3}}y^{-\frac{2}{3}}
\]

23. \( \sqrt[4]{625x^2} \)

**SOLUTION:**
\[
\sqrt[4]{625x^2} = (625x^2)^{\frac{1}{4}} = (5^4x^2)^{\frac{1}{4}} = 5x
\]

**ANSWER:**
\[
5x
\]

24. \( 27^3 \)

**SOLUTION:**
\[
27^3 = (3^3)^3 = 3
\]

**ANSWER:**
\[
3
\]

25. \( 256^\frac{1}{4} \)

**SOLUTION:**
\[
256^\frac{1}{4} = (4^4)^{\frac{1}{4}} = 4
\]

**ANSWER:**
\[
4
\]
26. $16^{\frac{1}{2}}$

**SOLUTION:**

$$16^{\frac{1}{2}} = \frac{1}{\sqrt{16}}$$

$$= \frac{1}{4}$$

**ANSWER:**

$$\frac{1}{4}$$

27. $81^{\frac{1}{4}}$

**SOLUTION:**

$$81^{\frac{1}{4}} = \frac{1}{\sqrt[4]{81}}$$

$$= \frac{1}{3}$$

**ANSWER:**

$$\frac{1}{3}$$

28. CCSS SENSE-MAKING A women’s regulation-sized basketball is slightly smaller than a men’s basketball. The radius $r$ of the ball that holds $V$ cubic units of air is $\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.

- **a.** Find the radius of a women’s basketball.
- **b.** Find the radius of a men’s basketball.

**SOLUTION:**

**a.** Substitute 413 for $V$ in the expression and simplify.

$$r = \left(\frac{3(413)}{4\pi}\right)^{\frac{1}{3}}$$

$$\approx 4.62$$

The radius of a women’s basketball is about 4.62 inches.

**b.** Substitute 455 for $V$ in the expression and simplify.

$$r = \left(\frac{3(455)}{4\pi}\right)^{\frac{1}{3}}$$

$$\approx 4.77$$

The radius of a men’s basketball is about 4.77 inches.

**ANSWER:**

**a.** about 4.62 in.

**b.** about 4.77 in.
29. **GEOMETRY** The radius \( r \) of a sphere with volume \( V \) is given by \( r = \left( \frac{3V}{4\pi} \right)^{\frac{1}{3}} \). Find the radius of a ball with a volume of 77 cm\(^3\).

**SOLUTION:**
Substitute 77 for \( V \) and simplify.

\[
r = \left( \frac{3(77)}{4\pi} \right)^{\frac{1}{3}}
\]
\[
\approx 2.64
\]

The radius of the sphere is about 2.64 cm.

**ANSWER:**
about 2.64 cm

Simplify each expression.

30. \( \frac{1}{x^3} \cdot x^5 \)

**SOLUTION:**
\[
\frac{1}{x^3} \cdot x^5 = x^{5-3}
\]
\[
= x^2
\]

**ANSWER:**
\( x^2 \)

31. \( a^9 \cdot a^4 \)

**SOLUTION:**
\[
a^9 \cdot a^4 = a^{9+4}
\]
\[
= a^{13}
\]

**ANSWER:**
\( a^{13} \)

32. \( b^{-\frac{3}{4}} \)

**SOLUTION:**
\[
b^{-\frac{3}{4}} = \frac{1}{b^{\frac{3}{4}}}
\]
\[
= \frac{1}{b} \cdot \frac{1}{b^{\frac{3}{4}}}
\]
\[
= \frac{1}{b}
\]

**ANSWER:**
\( \frac{1}{b} \)
33. \( y^{-\frac{4}{5}} \)

**SOLUTION:**

\[
\frac{y^{-\frac{4}{5}}}{y^{\frac{1}{5}}} = \frac{1}{y^{\frac{1}{5}}} y^{\frac{1}{5}} = 1
\]

**ANSWER:**

\( \frac{1}{y} \)

34. \( \frac{\sqrt[6]{81}}{\sqrt[3]{3}} \)

**SOLUTION:**

\[
\frac{\sqrt[6]{81}}{\sqrt[3]{3}} = \frac{3^{\frac{1}{6}}}{3^{\frac{1}{3}}} = 3^{\frac{1}{2}} = \sqrt{3}
\]

**ANSWER:**

\( \sqrt{3} \)

35. \( \frac{\sqrt[4]{27}}{\sqrt[3]{3}} \)

**SOLUTION:**

\[
\frac{\sqrt[4]{27}}{\sqrt[3]{3}} = \frac{(3^3)^{\frac{1}{4}}}{3^{\frac{1}{3}}} = 3^{\frac{3}{4}} \cdot 3^{-\frac{1}{3}} = 3^{\frac{1}{12}}
\]

**ANSWER:**

\( \sqrt[4]{3} \)

36. \( \sqrt[3]{25x^2} \)

**SOLUTION:**

\[
\sqrt[3]{25x^2} = (5^2 x^2)^{\frac{1}{3}} = 5^{\frac{1}{3}} x^\frac{2}{3} = \sqrt[3]{5} x^{\frac{2}{3}}
\]

**ANSWER:**

\( \sqrt[3]{5} x^{\frac{2}{3}} \)
6-6 Rational Exponents

37. \( \sqrt[6]{81g^3} \)

**SOLUTION:**

\[
\sqrt[6]{81g^3} = (3^4 \cdot g^3)^{\frac{1}{6}} = \frac{2}{3} \cdot \sqrt[3]{\sqrt[3]{g}} = \sqrt[3]{\sqrt[3]{g}}
\]

**ANSWER:**

\( \sqrt[3]{\sqrt[3]{g}} \)

38. \( \frac{1}{h^2 + 1} \)

**SOLUTION:**

\[
\frac{1}{h^2 + 1} = \frac{\sqrt{h} + 1}{\sqrt{h} - 1} = \frac{\sqrt{h} + 1}{\sqrt{h} - 1} \cdot \frac{\sqrt{h} - 1}{\sqrt{h} - 1} = \frac{\sqrt{h} + 1}{\sqrt{h} - 1} \cdot \frac{\sqrt{h} - 1}{\sqrt{h} - 1} = \frac{h + 2\sqrt{h}}{h - 1} = \frac{h + 2\sqrt{h}}{h - 1}
\]

**ANSWER:**

\( \frac{h + 2\sqrt{h}}{h - 1} \)

39. \( \frac{1}{x^3 + 2} \)

**SOLUTION:**

\[
\frac{1}{x^3 + 2} = \frac{\sqrt[3]{\sqrt[3]{x^3}}}{\sqrt[3]{\sqrt[3]{x^3}}} = \frac{\sqrt[3]{\sqrt[3]{x^3}}}{\sqrt[3]{\sqrt[3]{x^3}}} = \frac{\sqrt[3]{\sqrt[3]{x^3}}}{\sqrt[3]{\sqrt[3]{x^3}}} = \frac{x + 4\sqrt[3]{x} + 16 + 4\sqrt[3]{x^3} + 16\sqrt[3]{x^3}}{x - 16} = \frac{\sqrt[3]{x^3} + 16}{\sqrt[3]{x^3} + 16}
\]

**ANSWER:**

\( \frac{\sqrt[3]{x^3} + 16}{\sqrt[3]{x^3} + 16} \)
GEOMETRY Find the area of each figure.

40.

SOLUTION:
The area of a triangle of base \( b \) and height \( h \) is
\[
A = \frac{1}{2}bh.
\]
Substitute \( 3r^2w^4 \) and \( 4r^4w^2 \) for \( b \) and \( h \) and simplify.

The area of the given triangle is \( 6r^4w^4 \) unit\(^2\).

ANSWER:
\[
6r^4w^4 \text{ units}^2
\]

41.

SOLUTION:
The area of a circle is \( A = \pi r^2 \).
Substitute \( 3x^3y^5z^2 \) for \( r \) and simplify.
\[
A = \pi \left( \frac{2}{3x^3y^5z^2} \right)^2
= \pi \left( \frac{4}{9x^3y^5z^4} \right)
\approx 28.27x^3y^5z^4
\]
The area of the given circle is about
\[
28.27x^3y^5z^4 \text{ unit}^2.
\]

ANSWER:
about \( 28.27x^3y^5z^4 \) unit\(^2\)

42. Find the simplified form of \( 18^2 + 2^2 - 32^2 \).

SOLUTION:
\[
18^2 + 2^2 - 32^2 = (3^2 - 2)^2 + 2^2 - (2^5)^2
= 3\sqrt{2} + \sqrt{2} - 2^2\sqrt{2}
= 3\sqrt{2} + \sqrt{2} - 4\sqrt{2}
= 0
\]

ANSWER:
0
6-6 Rational Exponents

43. What is the simplified form of $64^3 - 32^3 + 8^3$?

**SOLUTION:**

$$64^3 - 32^3 + 8^3 = (4^3)^1 - (2^5)^{1/3} + (2^3)^{1/3}$$

$$= 4 - 2(2^5)^{1/3} + 2$$

$$= 6 - 2(2)^{5/3}$$

$$= 6 - 2(2^{5/3})^{1/3}$$

$$= 6 - 2 \cdot 4^3$$

**ANSWER:**

$$6 - 2 \cdot 4^3$$

Simplify each expression.

44. $a^4 \cdot a^3$

**SOLUTION:**

$$a^4 \cdot a^3 = a^{4+3}$$

$$= a^7$$

**ANSWER:**

$$a^7$$

45. $x^{3/2} \cdot x^{3/2}$

**SOLUTION:**

$$\frac{2}{x^3} \cdot \frac{8}{x^3} = \frac{2}{x^{3/3}}$$

$$= x^{10}$$

**ANSWER:**

$$x^{10}$$

46. $\left( \frac{3}{b^4} \right)^{1/3}$

**SOLUTION:**

$$\left( \frac{3}{b^4} \right)^{1/3} = \frac{1}{b^4}$$

**ANSWER:**

$$\frac{1}{b^4}$$

47. $\left( \frac{3}{y^{-5}} \right)^{1/4}$

**SOLUTION:**

$$\left( \frac{3}{y^{-5}} \right)^{1/4} = y^{(-5)^{1/4}}$$

$$= y^{3}$$

**ANSWER:**

$$y^{3}$$

48. $\left( \frac{3}{y^{20}} \right)^{1/4}$

**SOLUTION:**

$$\left( \frac{3}{y^{20}} \right)^{1/4} = y^{(-20)^{1/4}}$$

$$= y^{3}$$

**ANSWER:**

$$y^{3}$$
48. $\sqrt[4]{64}$

**SOLUTION:**

$\sqrt[4]{64} = \sqrt[4]{2^6}$

$= (2^6)^{\frac{1}{4}}$

$= 2^{6 \cdot \frac{1}{4}}$

$= 2^{\frac{3}{2}}$

$= 2\sqrt{2}$

**ANSWER:** $2\sqrt{2}$

49. $\sqrt[3]{216}$

**SOLUTION:**

$\sqrt[3]{216} = \sqrt[3]{6^3}$

$= (6^3)^{\frac{1}{3}}$

$= 6^{3 \cdot \frac{1}{3}}$

$= 6$

**ANSWER:** $6$

50. $d^\frac{5}{6}$

**SOLUTION:**

$d^\frac{5}{6} = \frac{1}{5} \cdot d^\frac{1}{6}$

$= \frac{d^\frac{1}{6}}{d}$

**ANSWER:** $\frac{1}{d}$

51. $w^\frac{7}{8}$

**SOLUTION:**

$w^\frac{7}{8} = \frac{1}{7} \cdot \frac{w^\frac{1}{8}}{w^\frac{1}{8}}$

$= \frac{w^\frac{1}{8}}{w}$

**ANSWER:** $\frac{w^\frac{1}{8}}{w}$

52. **WILDLIFE** A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. Then the number $D$ of deer after $t$ years is given by $D = 100 \cdot 2^t$.

a. How many deer will there be after $4\frac{1}{2}$ years?

b. Make a table that charts the population of deer every year for the next five years.
6-6 Rational Exponents

c. Make a graph using your table.

d. Using your table and graph, decide whether this is a reasonable trend over the long term. Explain.

**SOLUTION:**

a. Substitute 4.5 for \( t \) and simplify.

\[
D = 100 \cdot 2^{4.5} \\
\approx 100 \cdot 4.75 \\
= 475
\]

There will be about 475 deer after \( \frac{1}{2} \) years.

b. Substitute 0, 1, 2, 3, 4 and 5 for \( t \) and make the table.

<table>
<thead>
<tr>
<th>Year (t)</th>
<th>Deer (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>141</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>282</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>565</td>
</tr>
</tbody>
</table>

c. Plot the points on a coordinate plane and connect them as shown.

d. Sample answer: No; it is not reasonable to think that the population will continue to grow without bounds. This does not take into account the death rate of the deer.

**ANSWER:**

<table>
<thead>
<tr>
<th>Year (t)</th>
<th>Deer (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>141</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>282</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>565</td>
</tr>
</tbody>
</table>
6-6 Rational Exponents

Simplify each expression.

53. \( \frac{f^{-\frac{1}{4}}}{4f^{-\frac{1}{3}} \cdot f^{-\frac{1}{1}}} \)

SOLUTION:

\[
\begin{align*}
\frac{f^{-\frac{1}{4}}}{4f^{-\frac{1}{3}} \cdot f^{-\frac{1}{1}}} &= \frac{f^{\frac{1}{3}}}{4f^{\frac{1}{4}}} \\
&= \frac{f^{\frac{1}{3}} \cdot f^{\frac{1}{4}}}{4f^{\frac{1}{4}}} \\
&= \frac{f^{\frac{7}{12}}}{4f} \\
&= \frac{f^{\frac{7}{12}}}{4f} 
\end{align*}
\]

ANSWER:

\( \frac{f^{\frac{7}{12}}}{4f} \)

54. \( \frac{g^{\frac{5}{1}}}{g^{\frac{2}{1}} + 2} \)

SOLUTION:

\[
\begin{align*}
\frac{g^{\frac{5}{1}}}{g^{\frac{2}{1}} + 2} &= \frac{g^{\frac{5}{1}} \cdot g^{\frac{1}{1}}}{g^{\frac{2}{1}} + 2 \cdot g^{\frac{1}{1}}} \\
&= \frac{g^{\frac{6}{2}}}{g^{\frac{4}{2}} + 2} \\
&= \frac{g^{3}}{g^{2} + 2} \\
&= \frac{g^{3} - 2g^{2}}{g - 4} \\
\end{align*}
\]

ANSWER:

\( \frac{g^{3} - 2g^{2}}{g - 4} \)
6-6 Rational Exponents

55. \( \frac{c^3}{c^6} \)

SOLUTION:

\[
\frac{c^3}{c^6} = \frac{c^3 \cdot c^6}{c^6} = \frac{9}{c} = \frac{3}{c} = \frac{c \cdot c^6}{c} = \frac{c^2}{1} = c^2
\]

ANSWER:

\( \frac{1}{c^2} \)

56. \( \frac{z^5}{z^2} \)

SOLUTION:

\[
\frac{z^5}{z^2} = \frac{z^5 \cdot z^2}{z^2} = \frac{z^{10}}{z} = \frac{z^3}{z} = z^{10}
\]

ANSWER:

\( \frac{3}{z^{10}} \)

57. \( \sqrt{23} \cdot \sqrt[3]{23^2} \)

SOLUTION:

\[
\sqrt{23} \cdot \sqrt[3]{23^2} = (23)^{\frac{1}{2}} (23)^{\frac{2}{3}} = 23^{\frac{7}{3}} = 23 \cdot 23^{\frac{1}{3}} = 23\sqrt[3]{23}
\]

ANSWER:

\( 23\sqrt[3]{23} \)

58. \( \frac{\sqrt[8]{36h^4j^4}}{1} \)

SOLUTION:

\[
\frac{\sqrt[8]{36h^4j^4}}{1} = (6^2h^4j^4)^{\frac{1}{8}} = 6^4h^2j^2
\]

ANSWER:

\( 6^4h^2j^2 \)
6-6 Rational Exponents

59. \[ \sqrt[4]{81} \]

\textbf{SOLUTION:}

\[ \sqrt[4]{81} = \left( \sqrt[4]{81} \right)^{\frac{1}{4}} \]

\[ = \left( 81^{\frac{1}{2}} \right)^{\frac{1}{4}} \]

\[ = 81^{\frac{1}{4}} \]

\[ = \left( 3^4 \right)^{\frac{1}{4}} \]

\[ = 3 \]

\textbf{ANSWER:}

3

60. \[ \frac{1}{2} \sqrt{256} \]

\textbf{SOLUTION:}

\[ \frac{1}{2} \sqrt{256} = \left( \sqrt{256} \right)^{\frac{1}{2}} \]

\[ = \left( 256^{\frac{1}{2}} \right)^{\frac{1}{2}} \]

\[ = 256^{\frac{1}{4}} \]

\[ = \left( 2^8 \right)^{\frac{1}{8}} \]

\[ = 2 \]

\textbf{ANSWER:}

2

61. \[ \frac{ab}{\sqrt{c}} \]

\textbf{SOLUTION:}

\[ \frac{ab}{\sqrt{c}} = \frac{ab}{\sqrt{c}} \cdot \frac{\sqrt{c}}{\sqrt{c}} \]

\[ = \frac{ab\sqrt{c}}{c} \]

\textbf{ANSWER:}

\[ \frac{ab\sqrt{c}}{c} \]

62. \[ \frac{xy}{\sqrt[3]{z}} \]

\textbf{SOLUTION:}

\[ \frac{xy}{\sqrt[3]{z}} = \frac{xy}{\sqrt[3]{z}} \cdot \frac{\sqrt[3]{z^2}}{\sqrt[3]{z^2}} \]

\[ = \frac{xy\sqrt[3]{z^2}}{z} \]

\textbf{ANSWER:}

\[ \frac{xy\sqrt[3]{z^2}}{z} \]
6-6 Rational Exponents

63. \( \frac{\frac{1}{8^6} - \frac{1}{9^4}}{\sqrt{3} + \sqrt{2}} \)

**SOLUTION:**
\[
\frac{\frac{1}{8^6} - \frac{1}{9^4}}{\sqrt{3} + \sqrt{2}} = \frac{\left(2^3\right)^6 - \left(3^2\right)^4}{\sqrt{3} + \sqrt{2}} \cdot \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}
\]
\[
= \frac{2^6 - 3^4}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}
\]
\[
= \frac{(\sqrt{2} - \sqrt{3})(\sqrt{3} - \sqrt{2})}{3 - 2}
\]
\[
= -\left(3 + 2 - 2\sqrt{6}\right)
\]
\[
= 2\sqrt{6} - 5
\]

**ANSWER:**
\( 2\sqrt{6} - 5 \)

64. \( \frac{x^3 - x^2 z^3}{x^3 + z^3} \)

**SOLUTION:**
\[
\frac{x^3 - x^2 z^3}{x^3 + z^3} = \frac{x^2 - x^2 z^3}{x + x^3 z^3}
\]
\[
= \frac{\left(x + x^3 z^3\right)\left(x - x^3 z^3\right)}{x + x^3 z^3}
\]
\[
= x - x^3 z^3
\]

**ANSWER:**
\( x - x^3 z^3 \)

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the functions
\( f(x) = x^3 \) and \( g(x) = x^3 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. **TABULAR** Copy and complete the table to the right.

b. **GRAPHICAL** Graph \( f(x) \) and \( g(x) \).

c. **VERBAL** Explain the transformation between \( f(x) \) and \( g(x) \).
6-6 Rational Exponents

**SOLUTION:**

a. Substitute $-2, -1, 0, 1$ and $2$ for $x$ in the function $f(x)$ and $g(x)$ respectively and complete the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-8$</td>
<td>$-1.26$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$8$</td>
<td>$1.26$</td>
</tr>
</tbody>
</table>

b.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-8$</td>
<td>$-1.26$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$8$</td>
<td>$1.26$</td>
</tr>
</tbody>
</table>

c. It is a reflection of the line $y = x$.

66. **REASONING** Determine whether $-x^{-2} = (\frac{x}{x})^{-2}$ is always, sometimes, or never true. Explain your reasoning.

**SOLUTION:**

The statement is never true. The quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.

**ANSWER:**

Never; the quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.
6-6 Rational Exponents

67. CHALLENGE Consider \( \sqrt[4]{(-16)^3} \).

a. Explain why the expression is not a real number.

b. Find \( n \) such that \( n\sqrt[4]{(-16)^3} \) is a real number.

SOLUTION:

a. Sample answer: \( \sqrt[4]{(-16)^3} = \sqrt[4]{-4096} \)

There is no real number that when raised to the forth power results in a negative number.

b. Sample answer: \( \sqrt{-1} \)

This is much like multiplying \( \sqrt{-2} \cdot \sqrt{-18} \). The product rule of radicals is only defined for real numbers, so you must first rewrite each square root before multiplying as follows: \( i\sqrt{2} \cdot i\sqrt{18} \). Then you can multiply the square roots and complex numbers separately. This will result in the product \( i^2 \cdot \sqrt{36} \) or \( -6 \). From two complex numbers we obtained a product that is a real number. In a similar manner multiplying \( \sqrt[4]{(-16)^3} \) by \( \sqrt{-1} \) actually produces multiple products of which two are real number products 8 and \(-8\) and the other two are complex products \(8i\) and \(-8i\). This multiplication will be explained in a future course.

ANSWER:

a. Sample answer: \( \sqrt[4]{(-16)^3} = \sqrt[4]{-4096} \); there is no real number that when raised to the forth power results in a negative number.

b. Sample answer: \( \sqrt{-1} \)

68. OPEN ENDED Find two different expressions that equal 2 in the form \( x^{\frac{1}{4}} \).

SOLUTION:

Sample answer: Since \( x^{\frac{1}{4}} = \sqrt[4]{x} \) and the square root of 4 is 2 and the 4th root of 16 is 2: \( 4^2 \) and \( 16^{\frac{1}{4}} \) each equal 2.

ANSWER:

Sample answer: \( 4^2 \) and \( 16^{\frac{1}{4}} \)

69. WRITING IN MATH Explain how it might be easier to simplify an expression using rational exponents rather than using radicals.

SOLUTION:

Sample answer: It may be easier to simplify an expression when it has rational exponents because all the properties of exponents apply. We do not have as many properties dealing directly with radicals. However, we can convert all radicals to rational exponents, and then use the properties of exponents to simplify.

ANSWER:

Sample answer: It may be easier to simplify an expression when it has rational exponents because all the properties of exponents apply. We do not have as many properties dealing directly with radicals. However, we can convert all radicals to rational exponents, and then use the properties of exponents to simplify.
6-6 Rational Exponents

70. CCSS CRITIQUE Ayana and Kenji are simplifying \(\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\). Is either of them correct? Explain your reasoning.

\[
\text{Ayana}
\]

\[
\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3} - \frac{1}{2}} = x^{\frac{2}{6} - \frac{3}{6}} = x^{-\frac{1}{6}}
\]

\[
\text{Kenji}
\]

\[
\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3} \cdot \frac{2}{2}} = x^{\frac{2}{6}}
\]

\[
\text{SOLUTION:}
\]

No. Ayana added the exponents and Kenji divided the exponents. The exponents should have been subtracted.

\[
\text{ANSWER:}
\]

No; Ayana added the exponents and Kenji divided the exponents. The exponents should have been subtracted.

71. The expression \(\sqrt{56 - c}\) is equivalent to a positive integer when \(c\) is equal to

A 8
B -8
C 56
D 36

\[
\text{SOLUTION:}
\]

When \(c = -8\), the radicand \(56 - c\) becomes a perfect square. Therefore, option B is the correct answer.

\[
\text{ANSWER:}
\]

B
6-6 Rational Exponents

72. SAT/ACT Which of the following sentences is true about the graphs of \( y = 2(x - 3)^2 + 1 \) and \( y = 2(x + 3)^2 + 1 \)?
   
   F Their vertices are maximums.
   
   G The graphs have the same shape with different vertices.
   
   H The graphs have different shapes with the same vertices.
   
   J The graphs have different shapes with different vertices.
   
   K One graph has a vertex that is a maximum while the other graph has a vertex that is a minimum.

   **SOLUTION:**
   Compare with the general equation for a parabola in vertex form.
   
   \[ y = a(x - h) + k \]
   
   Since \( a = 2 \) in both equations, which is positive, the graph both open upwards, and vertices are both minimums.
   
   The only number which is changed is \( h \). So, the graphs have the same shape, but the \( x \)-coordinate of the vertex is different.
   
   Option G is the correct answer.

   **ANSWER:**
   G

73. GEOMETRY What is the converse of the statement? *If it is summer, then it is hot outside.*

   A If it is not hot outside, then it is not summer.
   
   B If it is not summer, then it is not hot outside.
   
   C If it is hot outside, then it is summer.
   
   D If it is hot outside, it is not summer.

   **SOLUTION:**
   The converse is produced by interchanging the hypothesis and the conclusion.
   
   Option C is the correct answer.

   **ANSWER:**
   C

74. SHORT RESPONSE If \( 3^5 \cdot p = 3^3 \), then find \( p \).

   **SOLUTION:**
   
   \[ 3^5 \cdot p = 3^3 \]
   
   \[ 3^5 \cdot 3^{-5} \cdot p = 3^3 \cdot 3^{-5} \]
   
   \[ p = 3^{-2} \]

   **ANSWER:**
   \( 3^{-2} \)
Simplify.

75. \( \sqrt{243} \)

**SOLUTION:**

\[
\sqrt{243} = \sqrt{81 \cdot 3} = 9\sqrt{3}
\]

**ANSWER:**

\( 9\sqrt{3} \)

76. \( \sqrt[6]{16y^3} \)

**SOLUTION:**

\[
\sqrt[6]{16y^3} = \sqrt[3]{2^4y^3} = 2y\sqrt[3]{2}
\]

**ANSWER:**

\( 2y\sqrt[3]{2} \)

77. \( 3\sqrt[4]{56y^6z^3} \)

**SOLUTION:**

\[
3\sqrt[4]{56y^6z^3} = 3\sqrt[4]{7 \cdot (y^3)^2 \cdot z^3}
\]

\[
= 3 \cdot \sqrt[4]{7 \cdot (y^2)^3 \cdot z^3}
\]

\[
= 3 \cdot 2 \cdot y^2 z \sqrt[4]{7}
\]

\[
= 6y^2 z \sqrt[4]{7}
\]

**ANSWER:**

\( 6y^2 z \sqrt[4]{7} \)

78. **PHYSICS** The speed of sound in a liquid is \( s = \sqrt[4]{\frac{B}{d}} \), where \( B \) is the bulk modulus of the liquid and \( d \) is its density. For water, \( B = 2.1 \times 10^9 \text{ N/m}^2 \) and \( d = 10^3 \text{ kg/m}^3 \). Find the speed of sound in water to the nearest meter per second.

**SOLUTION:**

Substitute \( 2.1 \times 10^9 \) and \( 10^3 \) for \( B \) and \( d \) and simplify.

\[
s = \sqrt[4]{\frac{2.1 \times 10^9}{10^3}} = \sqrt[4]{2.1 \times 10^6} = \sqrt[4]{2100000} \approx 1449
\]

The speed of the sound in water is 1449 m/s.

**ANSWER:**

1449 m/s

Find \( p(-4) \) and \( p(x + h) \) for each function.

79. \( p(x) = x - 2 \)

**SOLUTION:**

Substitute -4 for \( x \) and simplify.

\[ p(-4) = -4 - 2 = -6 \]

Substitute \( x + h \) for \( x \) and simplify.

\[ p(x + h) = x + h - 2 \]

**ANSWER:**

-6; \( x + h - 2 \)
80. \( p(x) = -x + 4 \)

**SOLUTION:**
Substitute \(-4\) for \(x\) and simplify.

\[ p(-4) = -(-4) + 4 \]
\[ = 4 + 4 \]
\[ = 8 \]

Substitute \(x + h\) for \(x\) and simplify.

\[ p(x + h) = -(x + h) + 4 \]
\[ = -x - h + 4 \]

**ANSWER:**
8; \(-x - h + 4\)

81. \( p(x) = 6x + 3 \)

**SOLUTION:**
Substitute \(-4\) for \(x\) and simplify.

\[ p(-4) = 6(-4) + 3 \]
\[ = -24 + 3 \]
\[ = -21 \]

Substitute \(x + h\) for \(x\) and simplify.

\[ p(x + h) = 6(x + h) + 3 \]
\[ = 6x + 6h + 3 \]

**ANSWER:**
-21; \(6x + 6h + 3\)

82. \( p(x) = x^2 + 5 \)

**SOLUTION:**
Substitute \(-4\) for \(x\) and simplify.

\[ p(-4) = (-4)^2 + 5 \]
\[ = 16 + 5 \]
\[ = 21 \]

Substitute \(x + h\) for \(x\) and simplify.

\[ p(x + h) = (x + h)^2 + 5 \]
\[ = x^2 + 2xh + h^2 + 5 \]

**ANSWER:**
21; \(x^2 + 2xh + h^2 + 5\)

83. \( p(x) = x^2 - x \)

**SOLUTION:**
Substitute \(-4\) for \(x\) and simplify.

\[ p(-4) = (-4)^2 - (-4) \]
\[ = 16 + 4 \]
\[ = 20 \]

Substitute \(x + h\) for \(x\) and simplify.

\[ p(x + h) = (x + h)^2 - (x + h) \]
\[ = x^2 + 2xh + h^2 - x - h \]

**ANSWER:**
20; \(x^2 + 2xh + h^2 - x - h\)
6-6 Rational Exponents

84. \( p(x) = 2x^3 - 1 \)

**SOLUTION:**
Substitute \(-4\) for \(x\) and simplify.

\[
p(-4) = 2(-4)^3 - 1
= 2(-64) - 1
= -128 - 1
= -129
\]

Substitute \(x + h\) for \(x\) and simplify.

\[
p(x + h) = 2(x + h)^3 - 1
= 2\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - 1
= 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 1
\]

**ANSWER:**
\(-129; 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 1\)

Solve each equation by factoring.

85. \( x^2 - 11x = 0 \)

**SOLUTION:**
\( x^2 - 11x = 0 \)
\( x(x-11) = 0 \)

\( x - 11 = 0 \) or \( x = 0 \)
\( x = 11 \) or \( x = 0 \)

The solutions are 0 and 11.

**ANSWER:**
\( \{0, 11\} \)

86. \( x^2 + 6x - 16 = 0 \)

**SOLUTION:**
\( x^2 + 6x - 16 = 0 \)
\((x + 8)(x - 2) = 0\)

\( x - 2 = 0 \) or \( x + 8 = 0 \)
\( x = 2 \) or \( x = -8 \)

The solutions are \(-8\) and 2.

**ANSWER:**
\( \{-8, 2\} \)

87. \( 4x^2 - 13x = 12 \)

**SOLUTION:**
\( 4x^2 - 13x = 12 \)
\( 4x^2 - 13x - 12 = 0 \)
\( 4x^2 - 16x + 3x - 12 = 0 \)
\( 4x(x - 4) + 3(x - 4) = 0 \)
\( (4x + 3)(x - 4) = 0 \)

\( x - 4 = 0 \) or \( 4x + 3 = 0 \)
\( x = 4 \) or \( x = -\frac{3}{4} \)

The solutions are \(-\frac{3}{4}\) and 4..
88. \( x^2 - 14x = -49 \)

\[ \begin{align*}
\text{SOLUTION: } \\
x^2 - 14x & = -49 \\
x^2 - 14x + 49 & = 0 \\
(x - 7)(x - 7) & = 0 \\
x - 7 & = 0 \\
x & = 7 
\end{align*} \]

The solution is 7.

\( \text{ANSWER: } \{7\} \)

89. \( x^2 + 9 = 6x \)

\[ \begin{align*}
\text{SOLUTION: } \\
x^2 + 9 & = 6x \\
x^2 + 9 - 6x & = 0 \\
(x - 3)^2 & = 0 \\
x - 3 & = 0 \\
x & = 3 
\end{align*} \]

The solution is 3.

\( \text{ANSWER: } \{3\} \)

90. \( x^2 - 3x = -\frac{9}{4} \)

\[ \begin{align*}
\text{SOLUTION: } \\
x^2 - 3x & = -\frac{9}{4} \\
x^2 - 3x + \frac{9}{4} & = 0 \\
\left( x - \frac{3}{2} \right) \left( x - \frac{3}{2} \right) & = 0 \\
x - \frac{3}{2} & = 0 \\
x & = \frac{3}{2} 
\end{align*} \]

The solution is \( x = \frac{3}{2} \).

\( \text{ANSWER: } \\left\{ \frac{3}{2} \right\} \)

91. \textbf{GEOMETRY} A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (Hint: Use similar triangles.)

\[ \text{SOLUTION: } \]

Draw an altitude and name the vertices. Let \( x \) and \( y \) be the width and the length of the shaded region.
6-6 Rational Exponents

Find each power.

92. \( (\sqrt{x - 3})^2 \)

**SOLUTION:**
\[
(\sqrt{x - 3})^2 = (x - 3)^2
\]
\[= x - 3
\]
**ANSWER:**
\( x - 3 \)

93. \( (\sqrt[3]{3x - 4})^3 \)

**SOLUTION:**
\[
(\sqrt[3]{3x - 4})^3 = (3x - 4)^1
\]
\[= 3x - 4
\]
**ANSWER:**
\( 3x - 4 \)

94. \( (\sqrt[4]{7x - 1})^4 \)

**SOLUTION:**
\[
(\sqrt[4]{7x - 1})^4 = (7x - 1)^1
\]
\[= 7x - 1
\]
**ANSWER:**
\( 7x - 1 \)

The triangle \( ABD \) and the triangle \( AEG \) are similar triangle. So, \( \frac{AG}{EG} = \frac{AD}{BD} \).

\[
\frac{8-x}{y} = \frac{8}{5}
\]
\[y = \frac{40 - 5x}{8}
\]

The area of the shaded region is
\[A = x \left(\frac{40 - 5x}{8}\right) = \frac{40x - 5x^2}{8}
\]

The function gets maximum at \( x = 4 \).

Substitute the 4 for \( x \) and find the value of \( y \).

\[y = \frac{40 - 5(4)}{8} = 2.5
\]

Therefore, the length of the rectangle is \( 2y = 5 \) in.

The dimensions of the rectangle with maximum area are 5 inches by 4 inches.

**ANSWER:**
5 in. by 4 in.
6-6 Rational Exponents

95. \( \left( \sqrt{x} - 4 \right)^2 \)

**SOLUTION:**
\[
\left( \sqrt{x} - 4 \right)^2 = \left( \sqrt{x} \right)^2 - 8\sqrt{x} + (4)^2 = x - 8\sqrt{x} + 16
\]

**ANSWER:**
\[ x - 8\sqrt{x} + 16 \]

96. \( \left( 2\sqrt{x} - 5 \right)^2 \)

**SOLUTION:**
\[
\left( 2\sqrt{x} - 5 \right)^2 = \left( 2\sqrt{x} \right)^2 - 20\sqrt{x} + (5)^2 = 4x - 20\sqrt{x} + 25
\]

**ANSWER:**
\[ 4x - 20\sqrt{x} + 25 \]

97. \( \left( 3\sqrt{x} + 1 \right)^2 \)

**SOLUTION:**
\[
\left( 3\sqrt{x} + 1 \right)^2 = \left( 3\sqrt{x} \right)^2 + 6\sqrt{x} + 1^2 = 9x + 6\sqrt{x} + 1
\]

**ANSWER:**
\[ 9x + 6\sqrt{x} + 1 \]
Solve each equation.

1. \( \sqrt{x - 4} + 6 = 10 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x - 4} + 6 &= 10 \\
\sqrt{x - 4} &= 4 \\
\left(\sqrt{x - 4}\right)^2 &= 4^2 \\
x - 4 &= 16 \\
x &= 20
\end{align*}
\]

**ANSWER:**
20

3. \( 8 - \sqrt{x + 12} = 3 \)

**SOLUTION:**

\[
\begin{align*}
8 - \sqrt{x + 12} &= 3 \\
\sqrt{x + 12} &= 5 \\
\left(\sqrt{x + 12}\right)^2 &= 5^2 \\
x + 12 &= 25 \\
x &= 13
\end{align*}
\]

**ANSWER:**
13

4. \( \sqrt{x - 8} + 5 = 7 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x - 8} + 5 &= 7 \\
\sqrt{x - 8} &= 2 \\
\left(\sqrt{x - 8}\right)^2 &= 2^2 \\
x - 8 &= 4 \\
x &= 12
\end{align*}
\]

**ANSWER:**
12

5. \( \sqrt[3]{x - 2} = 3 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt[3]{x - 2} &= 3 \\
\left(\sqrt[3]{x - 2}\right)^3 &= 3^3 \\
x - 2 &= 27 \\
x &= 29
\end{align*}
\]

**ANSWER:**
29
6. \((x - 5)^{\frac{1}{3}} - 4 = -2\)

\[\text{SOLUTION:}\]
\[\begin{align*}
(x - 5)^{\frac{1}{3}} &= 2 \\
\left( (x - 5)^{\frac{1}{3}} \right)^3 &= 2^3 \\
x - 5 &= 8 \\
x &= 13
\end{align*}\]

\[\text{ANSWER:} \]
13

7. \((4y)^{\frac{1}{3}} + 3 = 5\)

\[\text{SOLUTION:}\]
\[\begin{align*}
(4y)^{\frac{1}{3}} &= 2 \\
\left( (4y)^{\frac{1}{3}} \right)^3 &= 2^3 \\
4y &= 8 \\
y &= 2
\end{align*}\]

\[\text{ANSWER:} \]
2

8. \(\sqrt[3]{n + 8} - 6 = -3\)

\[\text{SOLUTION:}\]
\[\begin{align*}
\sqrt[3]{n + 8} &= 3 \\
(n + 8)^\frac{1}{3} &= 3 \\
n + 8 &= 27 \\
n &= 19
\end{align*}\]

\[\text{ANSWER:} \]
19

9. \(\sqrt{y} - 7 = 0\)

\[\text{SOLUTION:}\]
\[\begin{align*}
\sqrt{y} &= 7 \\
(y)^\frac{1}{2} &= 7 \\
y &= 49
\end{align*}\]

\[\text{ANSWER:} \]
49
10. \(2 + 4z^2 = 0\)

**SOLUTION:**

\[
\begin{align*}
2 + 4z^2 &= 0 \\
4z^2 &= -2 \\
z^2 &= -\frac{1}{2} \\
\sqrt{z} &= -\frac{1}{2} \\
z &= \frac{1}{4}
\end{align*}
\]

Check:

\[
2 + 4\left(\frac{1}{4}\right)^2 = 0
\]

\[
2 + 4 \cdot \frac{1}{2} = 0
\]

\[
2 + 2 = 0
\]

\[
4 \neq 0 \times
\]

Therefore, the equation has no solution.

**ANSWER:**

No solution

11. \(5 + \sqrt{4y - 5} = 12\)

**SOLUTION:**

\[
\begin{align*}
5 + \sqrt{4y - 5} &= 12 \\
\sqrt{4y - 5} &= 7 \\
(\sqrt{4y - 5})^2 &= 7^2 \\
4y - 5 &= 49 \\
4y &= 54 \\
y &= \frac{27}{2}
\end{align*}
\]

**ANSWER:**

\[
\frac{27}{2}
\]

12. \(\sqrt{2t - 7} = \sqrt{t + 2}\)

**SOLUTION:**

\[
\begin{align*}
\sqrt{2t - 7} &= \sqrt{t + 2} \\
(\sqrt{2t - 7})^2 &= (\sqrt{t + 2})^2 \\
2t - 7 &= t + 2 \\
t &= 9
\end{align*}
\]

**ANSWER:**

9

13. **CCSS REASONING** The time \(T\) in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula \(T = 2\pi \sqrt{\frac{L}{g}}\), where \(L\) is the length of the pendulum in feet and \(g\) is the acceleration due to gravity, 32 feet per second squared.

a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?
6-7 Solving Radical Equations and Inequalities

b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

**SOLUTION:**

a. Convert 73 feet 9.75 inches to feet.

73 feet 9.75 inches = 73 + \( \frac{9.75}{12} \) = 73.8125 ft.

Substitute 73.8125 and 32 for \( L \) and \( g \) then simplify.

\[
T = 2\pi \sqrt{\frac{73.8125}{32}}
\]

\[
\approx 9.5
\]

Therefore, the pendulum takes about 9.5 seconds to complete a swing.

b. Substitute 20 for \( T \) and 32 for \( g \).

\[
20 = 2\pi \sqrt{\frac{L}{32}}
\]

\[
\sqrt{\frac{L}{32}} = \frac{10}{\pi}
\]

\[
\left( \frac{L}{32} \right)^{\frac{1}{2}} = \left( \frac{10}{\pi} \right)^{\frac{1}{2}}
\]

\[
\frac{L}{32} = \frac{100}{\pi^2}
\]

\[
L = \frac{32 \times 100}{\pi^2}
\]

\[
\approx 324
\]

The pendulum should be about 324 ft long.

**ANSWER:**

a. about 9.5 seconds

b. about 324 ft

14. MULTIPLE CHOICE Solve \( (2y + 6)^{\frac{1}{4}} - 2 = 0 \).

A \( y = 1 \)

B \( y = 5 \)

C \( y = 11 \)

D \( y = 15 \)

**SOLUTION:**

\[
(2y + 6)^{\frac{1}{4}} = 2
\]

\[
(2y + 6) = 2^4
\]

\[
(2y + 6) = 16
\]

\[
y = 10
\]

Option B is the correct answer.

**ANSWER:**

B
6-7 Solving Radical Equations and Inequalities

Solve each equation.

1. SOLUTION: ANS \newpage ANSWER: 20

2. SOLUTION: ANS \newpage ANSWER: 23

3. SOLUTION: ANS \newpage ANSWER: 163 kg

67. CCSS ARGUMENTS Which equation does not have a solution?

SOLUTION:

Substitute 470 for kilograms can lift using the snatch and the clean and

Option F is the correct answer.

MULTIPLE CHOICE

ANSWER:

SOLUTION:

The relationship between the length and mass

Therefore, the pendulum takes about 9.5 seconds to

The pendulum should be about 324 ft long.

The formula

\[ s \leq \frac{4}{3} \leq \frac{77}{3} \]

The solution region is \( 7 \leq b \leq 43 \).

ANSWER:

7 \leq b \leq 43

16. \( \sqrt{b - 7} + 6 \leq 12 \)

SOLUTION:

Since the radicand of a square root must be greater

\( b - 7 \geq 0 \)

\( b \geq 7 \)

Solve \( \sqrt{b - 7} + 6 \leq 12 \).

\( \sqrt{b - 7} \leq 6 \)

\( (\sqrt{b - 7})^2 \leq 6^2 \)

\( b - 7 \leq 36 \)

\( b \leq 43 \)

The solution region is \( 7 \leq b \leq 43 \).

ANSWER:

7 \leq b \leq 43

15. \( \sqrt{3x + 4} - 5 \leq 4 \)

SOLUTION:

Since the radicand of a square root must be greater

\( 3x + 4 \geq 0 \)

\( 3x \geq -4 \)

\( x \geq -\frac{4}{3} \)

Solve \( \sqrt{3x + 4} - 5 \leq 4 \).

\( \sqrt{3x + 4} \leq 9 \)

\( (\sqrt{3x + 4})^2 \leq 9^2 \)

\( 3x + 4 \leq 81 \)

\( 3x \leq 77 \)

\( x \leq \frac{77}{3} \)

The solution region is \( -\frac{4}{3} \leq x \leq \frac{77}{3} \).

ANSWER:

\( -\frac{4}{3} \leq x \leq \frac{77}{3} \)
17. \(2 + \sqrt{4y - 4} \leq 6\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(4y - 4 \geq 0\).

\[
4y - 4 \geq 0 \\
4y \geq 4 \\
y \geq 1
\]

Solve \(2 + \sqrt{4y - 4} \leq 6\).

\[
2 + \sqrt{4y - 4} \leq 6 \\
\sqrt{4y - 4} \leq 4 \\
\left(\sqrt{4y - 4}\right)^2 \leq 4^2 \\
4y - 4 \leq 16 \\
4y \leq 20 \\
y \leq 5
\]

The solution region is \(1 \leq y \leq 5\).

**ANSWER:**
\(1 \leq y \leq 5\)

18. \(\sqrt{3a + 3} - 1 \leq 2\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(3a + 3 \geq 0\).

\[
3a + 3 \geq 0 \\
3a \geq -3 \\
a \geq -1
\]

Solve \(\sqrt{3a + 3} - 1 \leq 2\).

\[
\sqrt{3a + 3} - 1 \leq 2 \\
\sqrt{3a + 3} \leq 3 \\
\left(\sqrt{3a + 3}\right)^2 \leq 3^2 \\
3a + 3 \leq 9 \\
3a \leq 6 \\
a \leq 2
\]

The solution region is \(-1 \leq a \leq 2\).

**ANSWER:**
\(-1 \leq a < 2\)
19. \( 1 + \sqrt{7x - 3} > 3 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 7x - 3 \geq 0 \).

\[
7x - 3 \geq 0 \\
x \geq \frac{3}{7}
\]

Solve \( 1 + \sqrt{7x - 3} > 3 \).

\[
\begin{align*}
1 + \sqrt{7x - 3} &> 3 \\
\sqrt{7x - 3} &> 2 \\
(\sqrt{7x - 3})^2 &> 2^2 \\
7x - 3 &> 4 \\
x &> 1
\end{align*}
\]

The solution region is \( x > 1 \).

**ANSWER:**
\( x > 1 \)

20. \( \sqrt{3x + 6} + 2 \leq 5 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 3x + 6 \geq 0 \).

\[
3x + 6 \geq 0 \\
x \geq -2
\]

Solve \( \sqrt{3x + 6} + 2 \leq 5 \).

\[
\begin{align*}
\sqrt{3x + 6} &\leq 3 \\
(\sqrt{3x + 6})^2 &\leq 3^2 \\
3x + 6 &\leq 9 \\
x &\leq 1
\end{align*}
\]

The solution region is \(-2 \leq x \leq 1\).

**ANSWER:**
\(-2 \leq x \leq 1\)
Solve each equation.

1. \(20\)
2. \(23\)

21. \(-2 + \sqrt{9 - 5x} \geq 6\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(9 - 5x \geq 0\).

\[
\begin{align*}
9 - 5x & \geq 0 \\
5x & \leq 9 \\
x & \leq \frac{9}{5}
\end{align*}
\]

Solve \(-2 + \sqrt{9 - 5x} \geq 6\).

\[
\begin{align*}
-2 + \sqrt{9 - 5x} & \geq 6 \\
\sqrt{9 - 5x} & \geq 8 \\
(\sqrt{9 - 5x})^2 & \geq 8^2 \\
9 - 5x & \geq 64 \\
5x & \leq -55 \\
x & \leq -11
\end{align*}
\]

The solution region is \(x \leq -11\).

**ANSWER:**
\(x \leq -11\)

22. \(6 - \sqrt{2y + 1} < 3\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(2y + 1 \geq 0\).

\[
\begin{align*}
2y + 1 & \geq 0 \\
2y & \geq -1 \\
y & \geq -\frac{1}{2}
\end{align*}
\]

Solve \(6 - \sqrt{2y + 1} < 3\).

\[
\begin{align*}
6 - \sqrt{2y + 1} & < 3 \\
\sqrt{2y + 1} & > 3 \\
(\sqrt{2y + 1})^2 & > 3^2 \\
2y + 1 & > 9 \\
2y & > 8 \\
y & > 4
\end{align*}
\]

The solution region is \(y > 4\).

**ANSWER:**
\(y > 4\)
Solve each equation. Confirm by using a graphing calculator.

23. \( \sqrt{2x + 5} - 4 = 3 \)

**SOLUTION:**

\[
\sqrt{2x + 5} - 4 = 3 \\
\sqrt{2x + 5} = 7 \\
(\sqrt{2x + 5})^2 = 7^2 \\
2x + 5 = 49 \\
2x = 44 \\
x = 22
\]

**CHECK:**

![Image of graph showing intersection at x = 22]

**ANSWER:**

22

24. \( 6 + \sqrt{3x + 1} = 11 \)

**SOLUTION:**

\[
6 + \sqrt{3x + 1} = 11 \\
\sqrt{3x + 1} = 5 \\
(\sqrt{3x + 1})^2 = 5^2 \\
3x + 1 = 25 \\
3x = 24 \\
x = 8
\]

**CHECK:**

![Image of graph showing intersection at x = 8]

**ANSWER:**

8
25. \( \sqrt{x} + 6 = 5 - \sqrt{x} + 1 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x} + 6 &= 5 - \sqrt{x} + 1 \\
\sqrt{x} + 6 + \sqrt{x} + 1 &= 5 \\
\left(\sqrt{x} + 6 + \sqrt{x} + 1\right)^2 &= 5^2 \\
x + 6 + x + 1 + 2\sqrt{x^2 + 7x + 6} &= 25 \\
2\sqrt{x^2 + 7x + 6} + 2x &= 18 - 2x \\
\left(2\sqrt{x^2 + 7x + 6}\right)^2 &= (18 - 2x)^2 \\
4(x^2 + 7x + 6) &= 324 + 4x^2 - 72x \\
4x^2 + 28x + 24 &= 324 + 4x^2 - 72x \\
100x &= 300 \\
x &= 3
\end{align*}
\]

**CHECK:**

\[
\begin{align*}
&\text{Intersection at } x = 3 \\
&\text{Intersection at } y = 3
\end{align*}
\]

**ANSWER:**

3

26. \( \sqrt{x - 3} = \sqrt{x + 4} - 1 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x - 3} &= \sqrt{x + 4} - 1 \\
\sqrt{x - 3} - \sqrt{x + 4} &= -1 \\
\left(\sqrt{x - 3} - \sqrt{x + 4}\right)^2 &= (-1)^2 \\
(x - 3) - 2\sqrt{(x - 3)(x + 4)} - (x + 4) &= 1 \\
2\sqrt{x^2 + x - 12} &= 2x \\
\left(2\sqrt{x^2 + x - 12}\right)^2 &= (2x)^2 \\
4x^2 + 4x - 48 &= 4x^2 \\
4x &= 48 \\
x &= 12
\end{align*}
\]

**CHECK:**

\[
\begin{align*}
&\text{Intersection at } x = 12 \\
&\text{Intersection at } y = 3
\end{align*}
\]

**ANSWER:**

12
6-7 Solving Radical Equations and Inequalities

27. \( \sqrt{x-15} = 3 - \sqrt{x} \)

**SOLUTION:**
\[
\begin{align*}
\sqrt{x-15} &= 3 - \sqrt{x} \\
(\sqrt{x-15})^2 &= (3 - \sqrt{x})^2 \\
x - 15 &= 9 + x - 6\sqrt{x} \\
-6\sqrt{x} &= -24 \\
\sqrt{x} &= \frac{24}{6} \\
\sqrt{x} &= 4 \\
x &= 16
\end{align*}
\]

**Check:**
\[
\begin{align*}
\sqrt{16-15} &= 3 - \sqrt{16} \\
1 &= 3 - 4 \\
1 &= -1 \times
\end{align*}
\]

There is no real solution for the equation.

**ANSWER:**
no real solution

28. \( \sqrt{x-10} = 1 - \sqrt{x} \)

**SOLUTION:**
\[
\begin{align*}
\sqrt{x-10} &= 1 - \sqrt{x} \\
(\sqrt{x-10})^2 &= (1 - \sqrt{x})^2 \\
x - 10 &= 1 + x - 2\sqrt{x} \\
2\sqrt{x} &= 11 \\
\sqrt{x} &= \frac{11}{2} \\
x &= \frac{121}{4}
\end{align*}
\]

**Check:**
\[
\begin{align*}
\sqrt{\frac{121}{4}-10} &= 1 - \sqrt{\frac{121}{4}} \\
\sqrt{\frac{81}{4}} &= 1 - \frac{11}{2} \\
\frac{9}{2} &= -\frac{9}{2} \times
\end{align*}
\]

There is no real solution for the equation.

**ANSWER:**
no real solution
6-7 Solving Radical Equations and Inequalities

29. \( 6 + \sqrt{4x + 8} = 9 \)

SOLUTION:

\[
\begin{align*}
6 + \sqrt{4x + 8} &= 9 \\
\sqrt{4x + 8} &= 3 \\
(\sqrt{4x + 8})^2 &= 3^2 \\
4x + 8 &= 9 \\
4x &= 1 \\
x &= \frac{1}{4}
\end{align*}
\]

CHECK:

[Graph showing the solution]

ANSWER:

\( \frac{1}{4} \)

30. \( 2 + \sqrt{3y - 5} = 10 \)

SOLUTION:

\[
\begin{align*}
2 + \sqrt{3y - 5} &= 10 \\
\sqrt{3y - 5} &= 8 \\
(\sqrt{3y - 5})^2 &= 8^2 \\
3y - 5 &= 64 \\
3y &= 69 \\
y &= 23
\end{align*}
\]

CHECK:

[Graph showing the solution]

ANSWER:

23
31. \( \sqrt{x - 4} = \sqrt{2x - 13} \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x - 4} &= \sqrt{2x - 13} \\
(\sqrt{x - 4})^2 &= (\sqrt{2x - 13})^2 \\
x - 4 &= 2x - 13 \\
x &= 9
\end{align*}
\]

**CHECK:**

![Graph showing the solution](image)

**ANSWER:**

9

32. \( \sqrt{7a - 2} = \sqrt{a + 3} \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{7a - 2} &= \sqrt{a + 3} \\
(\sqrt{7a - 2})^2 &= (\sqrt{a + 3})^2 \\
7a - 2 &= a + 3 \\
6a &= 5 \\
a &= \frac{5}{6}
\end{align*}
\]

**CHECK:**

![Graph showing the solution](image)

**ANSWER:**

\( \frac{5}{6} \)
33. \( \sqrt{x - 5} - \sqrt{x} = -2 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{x - 5} - \sqrt{x} &= -2 \\
(\sqrt{x - 5} - \sqrt{x})^2 &= (-2)^2 \\
x - 5 + x - 2(\sqrt{x^2 - 5x}) &= 4 \\
2\sqrt{x^2 - 5x} &= 2x - 9 \\
\left(2\sqrt{x^2 - 5x}\right)^2 &= (2x - 9)^2 \\
4x^2 - 20x &= 4x^2 + 81 - 36x \\
16x &= 81 \\
x &= \frac{81}{16}
\end{align*}
\]

**CHECK:**

Intersection

\[
\begin{align*}
x &= 5.0625 \\
y &= -2
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
81 \\
16
\end{align*}
\]

34. \( \sqrt{b - 6} + \sqrt{b} = 3 \)

**SOLUTION:**

\[
\begin{align*}
\sqrt{b - 6} + \sqrt{b} &= 3 \\
(\sqrt{b - 6} + \sqrt{b})^2 &= 3^2 \\
b - 6 + b + 2\sqrt{b^2 - 6b} &= 9 \\
2\sqrt{b^2 - 6b} &= (-2b + 15)^2 \\
4b^2 - 24b &= 4b^2 + 225 - 60b \\
36b &= 225 \\
b &= \frac{225}{36} \\
b &= \frac{25}{4}
\end{align*}
\]

**CHECK:**

Intersection

\[
\begin{align*}
x &= 6.25 \\
y &= 3
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
25 \\
4
\end{align*}
\]
35. **CCSS SENSE-MAKING** Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula \( t = \frac{1}{4} \sqrt{d - h} \) describes the time \( t \) in seconds at which the keys are \( h \) meters above the ground and Isabel is \( d \) meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds?

**SOLUTION:**
Substitute 2 for \( t \) and 65 for \( d \).

\[
 t = \frac{1}{4} \sqrt{d - h} \\
2 = \frac{1}{4} \sqrt{65 - h} \\
\sqrt{65 - h} = 8 \\
(\sqrt{65 - h})^2 = 8^2 \\
65 - h = 64 \\
h = 1
\]

The keys will be 1 meter above the ground after 2 seconds.

**ANSWER:**
1 m
38. \((6q + 1)^\frac{1}{3} + 2 = 5\)

**SOLUTION:**

\[
(6q + 1)^\frac{1}{3} + 2 = 5 \\
(6q + 1)^\frac{1}{3} = 3 \\
\left((6q + 1)^\frac{1}{3}\right)^3 = 3^3 \\
6q + 1 = 81 \\
6q = 80 \\
q = \frac{40}{3}
\]

**ANSWER:**

\[
\frac{40}{3}
\]

39. \((3x + 7)^\frac{1}{3} - 3 = 1\)

**SOLUTION:**

\[
(3x + 7)^\frac{1}{3} - 3 = 1 \\
(3x + 7)^\frac{1}{3} = 4 \\
\left((3x + 7)^\frac{1}{3}\right)^3 = 4^3 \\
3x + 7 = 256 \\
3x = 249 \\
x = 83
\]

**ANSWER:**

83

40. \((3y - 2)^\frac{1}{3} + 5 = 6\)

**SOLUTION:**

\[
(3y - 2)^\frac{1}{3} + 5 = 6 \\
(3y - 2)^\frac{1}{3} = 1 \\
\left((3y - 2)^\frac{1}{3}\right)^3 = 1^3 \\
3y - 2 = 1 \\
3y = 3 \\
y = 1
\]

**ANSWER:**

1

41. \((4z - 1)^\frac{1}{5} - 1 = 2\)

**SOLUTION:**

\[
(4z - 1)^\frac{1}{5} - 1 = 2 \\
(4z - 1)^\frac{1}{5} = 3 \\
\left((4z - 1)^\frac{1}{5}\right)^5 = 3^5 \\
4z - 1 = 243 \\
4z = 244 \\
z = 61
\]

**ANSWER:**

61
42. \(2(x - 10)^\frac{1}{3} + 4 = 0\)

**SOLUTION:**

\[
2(x - 10)^\frac{1}{3} + 4 = 0
\]

\[
2(x - 10)^\frac{1}{3} = -4
\]

\[
\left(2(x - 10)^\frac{1}{3}\right)^3 = (-4)^3
\]

\[
8(x - 10) = -64
\]

\[
x - 10 = -8
\]

\[
x = 2
\]

**ANSWER:**

2

43. \(3(x + 5)^\frac{1}{3} - 6 = 0\)

**SOLUTION:**

\[
3(x + 5)^\frac{1}{3} - 6 = 0
\]

\[
3(x + 5)^\frac{1}{3} = 6
\]

\[
\left(3(x + 5)^\frac{1}{3}\right)^3 = 6^3
\]

\[
27(x + 5) = 216
\]

\[
x + 5 = 8
\]

\[
x = 3
\]

**ANSWER:**

3

44. \(\sqrt[3]{5x + 10} - 5 = 0\)

**SOLUTION:**

\[
\sqrt[3]{5x + 10} - 5 = 0
\]

\[
\sqrt[3]{5x + 10} = 5
\]

\[
\left(\sqrt[3]{5x + 10}\right)^3 = 5^3
\]

\[
5x + 10 = 125
\]

\[
x = 23
\]

**ANSWER:**

23

45. \(\sqrt[3]{4n - 8} - 4 = 0\)

**SOLUTION:**

\[
\sqrt[3]{4n - 8} - 4 = 0
\]

\[
\sqrt[3]{4n - 8} = 4
\]

\[
\left(\sqrt[3]{4n - 8}\right)^3 = 4^3
\]

\[
4n - 8 = 64
\]

\[
n = 18
\]

**ANSWER:**

18
6-7 Solving Radical Equations and Inequalities

46. \( \frac{1}{7} (14a)^\frac{1}{3} = 1 \)

**SOLUTION:**

\[
\frac{1}{7} (14a)^\frac{1}{3} = 1 \\
(14a)^\frac{1}{3} = 7 \\
\left( (14a)^\frac{1}{3} \right)^3 = 7^3 \\
14a = 343 \\
a = 24.5
\]

**ANSWER:**

24.5

47. \( \frac{1}{4} (32b)^\frac{1}{3} = 1 \)

**SOLUTION:**

\[
\frac{1}{4} (32b)^\frac{1}{3} = 1 \\
(32b)^\frac{1}{3} = 4 \\
\left( (32b)^\frac{1}{3} \right)^3 = 4^3 \\
32b = 64 \\
b = 2
\]

**ANSWER:**

2

48. **MULTIPLE CHOICE** Solve \( \sqrt[3]{y + 2} + 9 = 14 \).

A 23

B 53

C 123

D 623

**SOLUTION:**

\[
\sqrt[3]{y + 2} + 9 = 14 \\
\sqrt[3]{y + 2} = 5 \\
\left( \sqrt[3]{y + 2} \right)^4 = 5^4 \\
y + 2 = 625 \\
y = 623
\]

Option D is the correct answer.

**ANSWER:**

D
49. **MULTIPLE CHOICE** Solve \((2x - 1)^{\frac{1}{4}} - 2 = 1\).

**SOLUTION:**
\[
(2x - 1)^{\frac{1}{4}} - 2 = 1
\]
\[
(2x - 1)^{\frac{1}{4}} = 3
\]
\[
(2x - 1)^{\frac{1}{4}} = 3^4
\]
\[
2x - 1 = 81
\]
\[
2x = 82
\]
\[
x = 41
\]

Option F is the correct answer.

**ANSWER:**
F

---

Solve each inequality.

50. \(1 + \sqrt{5x - 2} > 4\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(5x - 2 \geq 0\).

\[
5x - 2 \geq 0
\]
\[
x \geq \frac{2}{5}
\]

Solve \(1 + \sqrt{5x - 2} > 4\).

\[
1 + \sqrt{5x - 2} > 4
\]
\[
\sqrt{5x - 2} > 3
\]
\[
(\sqrt{5x - 2})^2 > 3^2
\]
\[
5x - 2 > 9
\]
\[
x > \frac{11}{5}
\]

The solution region is \(x > \frac{11}{5}\).

**ANSWER:**
\(x > \frac{11}{5}\)
51. $\sqrt{2x+14} - 6 \geq 4$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $2x + 14 \geq 0$.

\[
2x + 14 \geq 0 \\
2x \geq -14 \\
x \geq -7
\]

Solve $\sqrt{2x+14} - 6 \geq 4$.

\[
\sqrt{2x+14} \geq 10 \\
(\sqrt{2x+14})^2 \geq 10^2 \\
2x + 14 \geq 100 \\
2x \geq 86 \\
x \geq 43
\]

The solution region is $x \geq 43$.

**ANSWER:**
$x \geq 43$

52. $10 - \sqrt{2x+7} \leq 3$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $2x + 7 \geq 0$.

\[
2x + 7 \geq 0 \\
2x \geq -7 \\
x \geq -\frac{7}{2}
\]

Solve $10 - \sqrt{2x+7} \leq 3$.

\[
10 - \sqrt{2x+7} \leq 3 \\
-\sqrt{2x+7} \leq -7 \\
\sqrt{2x+7} \geq 7 \\
(\sqrt{2x+7})^2 \geq 7^2 \\
2x + 7 \geq 49 \\
2x \geq 42 \\
x \geq 21
\]

The solution region is $x \geq 21$.

**ANSWER:**
$x \geq 21$

53. $6 + \sqrt{3y+4} < 6$

**SOLUTION:**

\[
6 + \sqrt{3y+4} < 6 \\
\sqrt{3y+4} < 0
\]

Since the value of radical is nonnegative, the inequality has no real solution.

**ANSWER:**
no real solution
6-7 Solving Radical Equations and Inequalities

54. \( \sqrt{2x+5} - \sqrt{9+x} > 0 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 2x + 5 \geq 0 \) and \( 9 + x \geq 0 \).

\[
\begin{align*}
2x + 5 & \geq 0 \\
2x & \geq -5 \\
x & \geq -\frac{5}{2} \\
9 + x & \geq 0 \\
x & \geq -9
\end{align*}
\]

Solve \( \sqrt{2x+5} - \sqrt{9+x} > 0 \).

\[
\begin{align*}
\sqrt{2x+5} - \sqrt{9+x} > 0 \\
\sqrt{2x+5} > \sqrt{9+x} \\
\left(\sqrt{2x+5}\right)^2 > \left(\sqrt{9+x}\right)^2 \\
2x + 5 > x + 9 \\
x > 4
\end{align*}
\]

The solution region is \( x > 4 \).

**ANSWER:**
\( x > 4 \)

55. \( \sqrt{d+3} + \sqrt{d+7} > 4 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( d + 3 \geq 0 \) and \( d + 7 \geq 0 \).

\[
\begin{align*}
d + 3 & \geq 0 \\
d & \geq -3 \\
d + 7 & \geq 0 \\
d & \geq -7
\end{align*}
\]

Solve \( \sqrt{d+3} + \sqrt{d+7} > 4 \).

\[
\begin{align*}
\sqrt{d+3} + \sqrt{d+7} & > 4 \\
\left(\sqrt{d+3} + \sqrt{d+7}\right)^2 & > 4^2 \\
d + 3 + d + 7 + 2\sqrt{(d+3)(d+7)} & > 16 \\
2\sqrt{d^2 + 10d + 21} & > 6 - 2d \\
(2\sqrt{d^2 + 10d + 21})^2 & > (6 - 2d)^2 \\
4d^2 + 40d + 84 & > 36 + 4d^2 - 24d \\
64d & > -48 \\
d & > -\frac{3}{4}
\end{align*}
\]

The solution region is \( d > -\frac{3}{4} \).

**ANSWER:**
\( d > -\frac{3}{4} \)
6-7 Solving Radical Equations and Inequalities

56. $\sqrt{3x + 9} - 2 < 7$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $3x + 9 \geq 0$.

\[3x + 9 \geq 0\]
\[3x \geq -9\]
\[x \geq -3\]

Solve $\sqrt{3x + 9} - 2 < 7$.

\[\sqrt{3x + 9} < 9\]
\[\left(\sqrt{3x + 9}\right)^2 < 9^2\]
\[3x + 9 < 81\]
\[3x < 72\]
\[x < 24\]

The solution region is $-3 \leq x < 24$.

**ANSWER:**
\[-3 \leq x < 24\]

57. $\sqrt{2y + 5} + 3 \leq 6$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $2y + 5 \geq 0$.

\[2y + 5 \geq 0\]
\[2y \geq -5\]
\[y \geq -\frac{5}{2}\]

Solve $\sqrt{2y + 5} + 3 \leq 6$.

\[\sqrt{2y + 5} \leq 3\]
\[\left(\sqrt{2y + 5}\right)^2 \leq 3^2\]
\[2y + 5 \leq 9\]
\[2y \leq 4\]
\[y \leq 2\]

The solution region is $-\frac{5}{2} \leq y \leq 2$.

**ANSWER:**
\[-\frac{5}{2} \leq y \leq 2\]
58. \(-2 + \sqrt{8 - 4z} \geq 8\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(8 - 4z \geq 0\).

\[
8 - 4z \geq 0 \\
4z \leq 8 \\
z \leq 2
\]

Solve \(-2 + \sqrt{8 - 4z} \geq 8\).

\[
-2 + \sqrt{8 - 4z} \geq 8 \\
\sqrt{8 - 4z} \geq 10 \\
\left(\sqrt{8 - 4z}\right)^2 \geq 10^2 \\
8 - 4z \geq 100 \\
4z \leq -92 \\
z \leq -23
\]

The solution region is \(z \leq -23\).

**ANSWER:**
\(z \leq -23\)

59. \(-3 + \sqrt{6a + 1} > 4\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(6a + 1 \geq 0\).

\[
6a + 1 \geq 0 \\
6a \geq -1 \\
a \geq -\frac{1}{6}
\]

Solve \(-3 + \sqrt{6a + 1} > 4\).

\[
-3 + \sqrt{6a + 1} > 4 \\
\sqrt{6a + 1} > 7 \\
\left(\sqrt{6a + 1}\right)^2 > 7^2 \\
6a + 1 > 49 \\
6a > 48 \\
a > 8
\]

The solution region is \(a > 8\).

**ANSWER:**
\(a > 8\)
60. $\sqrt{2} - \sqrt{b} + 6 \leq -\sqrt{b}$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $b + 6 \geq 0$ and $b \geq 0$.

\[ b + 6 \geq 0 \]
\[ b \geq -6 \]

Solve $\sqrt{2} - \sqrt{b} + 6 \leq -\sqrt{b}$.

\[ \sqrt{2} - \sqrt{b} + 6 \leq -\sqrt{b} \]
\[ \sqrt{b} - \sqrt{b} + 6 \leq -\sqrt{2} \]
\[ (\sqrt{b} - \sqrt{b} + 6)^2 \leq (-\sqrt{2})^2 \]
\[ b + b + 6 - 2\sqrt{b^2 + 6b} \leq 2 \]
\[ 2b - 2\sqrt{b^2 + 6b} \leq -4 \]
\[ b - \sqrt{b^2 + 6b} \leq -2 \]
\[ -\sqrt{b^2 + 6b} \leq b - 2 \]
\[ (-\sqrt{b^2 + 6b})^2 \leq (-b - 2)^2 \]
\[ b^2 + 6b \leq b^2 + 4 + 4b \]
\[ 2b \leq 4 \]
\[ b \leq 2 \]

The solution region is $0 \leq b \leq 2$.

**ANSWER:**
$0 \leq b \leq 2$

61. $\sqrt{c + 9} - \sqrt{c} > \sqrt{3}$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $c + 9 \geq 0$ and $c \geq 0$.

\[ c + 9 \geq 0 \]
\[ c \geq -9 \]

Solve $\sqrt{c + 9} - \sqrt{c} > \sqrt{3}$.

\[ \sqrt{c + 9} - \sqrt{c} > \sqrt{3} \]
\[ (\sqrt{c + 9} - \sqrt{c})^2 \geq (\sqrt{3})^2 \]
\[ c + 9 + c - 2\sqrt{c^2 + 9c} > 3 \]
\[ 2\sqrt{c^2 + 9c} < 2c + 6 \]
\[ 2\sqrt{c^2 + 9c} < c^2 + 9c \]
\[ c^2 + 9c < c^2 + 6c + 9 \]
\[ 3c < 9 \]
\[ c < 3 \]

The solution region is $0 \leq c < 3$.

**ANSWER:**
$0 \leq c < 3$
62. **PENDULUMS** The formula \( s = 2\pi \sqrt{\frac{l}{32}} \) represents the swing of a pendulum, where \( s \) is the time in seconds to swing back and forth, and \( l \) is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 1.5 seconds.

**SOLUTION:**

Substitute 1.5 for \( s \) and solve for \( l \).

\[
\begin{align*}
1.5 &= 2\pi \sqrt{\frac{l}{32}} \\
\frac{l}{32} &= \left(\frac{1.5}{2\pi}\right)^2 \\
l &= 32 \left(\frac{1.5}{2\pi}\right)^2 \\
\approx 1.82
\end{align*}
\]

The length of the pendulum is about 1.82 ft.

**ANSWER:**

about 1.82 ft

63. **FISH** The relationship between the length and mass of certain fish can be approximated by the equation \( L = 0.46\sqrt[3]{M} \). where \( L \) is the length in meters and \( M \) is the mass in kilograms. Solve this equation for \( M \).

**SOLUTION:**

\[
\begin{align*}
L &= 0.46\sqrt[3]{M} \\
\sqrt[3]{M} &= \frac{L}{0.46} \\
M &= \left(\frac{L}{0.46}\right)^3
\end{align*}
\]

**ANSWER:**

\[
M = \left(\frac{L}{0.46}\right)^3
\]

64. **HANG TIME** Refer to the information at the beginning of the lesson regarding hang time. Describe how the height of a jump is related to the amount of time in the air. Write a step-by-step explanation of how to determine the height of Jordan’s 0.98-second jump.

**SOLUTION:**

If the height of a person’s jump and the amount of time he or she is in the air are related by an equation involving radicals, then the hang time associated with a given height can be found by solving a radical equation.

**ANSWER:**

If the height of a person’s jump and the amount of time he or she is in the air are related by an equation involving radicals, then the hang time associated with a given height can be found by solving a radical equation.
65. **CONCERTS** The organizers of a concert are preparing for the arrival of 50,000 people in the open field where the concert will take place. Each person is allotted 5 square feet of space, so the organizers rope off a circular area of 250,000 square feet. Using the formula \( A = \pi r^2 \) where \( A \) represents the area of the circular region and \( r \) represents the radius of the region, find the radius of this region.

**SOLUTION:**
Substitute 250,000 for \( A \) and solve for \( r \).

\[
250000 = \pi r^2 \\
\frac{r^2}{\pi} = \frac{250000}{\pi} \\
\sqrt{r^2} = \sqrt{\frac{250000}{\pi}} \\
r = \frac{500}{\sqrt{\pi}} \\
\approx 282
\]

The radius of the region is about 282 ft.

**ANSWER:**
about 282 ft

66. **WEIGHTLIFTING** The formula \( M = 512 - 146,230B^{8/5} \) can be used to estimate the maximum total mass that a weightlifter of mass \( B \) kilograms can lift using the snatch and the clean and jerk. According to the formula, how much does a person weigh who can lift at most 470 kilograms?

**SOLUTION:**
Substitute 470 for \( M \) and solve for \( B \).

\[
470 = 512 - 146230B^{8/5} \\
146230B^{8/5} = 512 - 470 \\
= 42 \\
B^{8/5} = \frac{42}{146230} \\
B^5 = \frac{146230}{42} \\
\left(\frac{8}{5}\right)^{8/5} = \left(\frac{146230}{42}\right)^{5/8} \\
B \approx 163.54
\]

The person weigh 163 kg can lift at most 470 kilograms.

**ANSWER:**
163 kg
67. **CCSS ARGUMENTS** Which equation does not have a solution?

\[
\sqrt{x - 1} + 3 = 4
\]

\[
\sqrt{x - 2} + 7 = 10
\]

\[
\sqrt{x + 1} + 3 = 4
\]

\[
\sqrt{x + 2} - 7 = -10
\]

**SOLUTION:**
\[
\sqrt{x + 2} - 7 = -10
\]
\[
\sqrt{x + 2} = -3
\]

Since the value of the radical is negative, it does not have real solution.

**ANSWER:**
\[
\sqrt{x + 2} - 7 = -10
\]

68. **CHALLENGE** Lola is working to solve \((x + 5)^{\frac{1}{2}} = -4\). She said that she could tell there was no real solution without even working the problem. Is Lola correct? Explain your reasoning.

**SOLUTION:**
Yes; since \(\sqrt{x + 5} \geq 0\), the left side of the equation is nonnegative. Therefore, the left side of the equation cannot equal \(-4\). Thus the equation has no solution.

**ANSWER:**
Yes; since \(\sqrt{x + 5} \geq 0\), the left side of the equation is nonnegative. Therefore, the left side of the equation cannot equal \(-4\). Thus the equation has no solution.

69. **REASONING** Determine whether \(\frac{\sqrt{x^2}}{-x} = x\) is sometimes, always, or never true when \(x\) is a real number. Explain your reasoning.

**SOLUTION:**
\[
\frac{\sqrt{x^2}}{-x} = x
\]
\[
\frac{x^2}{-x} = x
\]
\[
x^2 = x(-x)
\]
\[
x^2 = -x^2
\]

But this is only true when \(x = 0\). And in that case, we have division by zero in the original equation. So the equation is never true.

**ANSWER:**
never;
6-7 Solving Radical Equations and Inequalities

70. **OPEN ENDED** Select a whole number. Now work backward to write two radical equations that have that whole number as solutions. Write one square root equation and one cube root equation. You may need to experiment until you find a whole number you can easily use.

**SOLUTION:**
Sample answer using 6: \( \sqrt{x-2} = 2, (x + 21)^{\frac{1}{3}} = 3 \)

\[
\begin{align*}
\sqrt{x-2} &= 2 \\
(x-2)^2 &= 2^2 \\
x - 2 &= 4 \\
x &= 6
\end{align*}
\]

\[
\begin{align*}
(x + 21)^{\frac{1}{3}} &= 3 \\
((x + 21)^{\frac{1}{3}})^3 &= 3^3 \\
x + 21 &= 27 \\
x &= 6
\end{align*}
\]

**ANSWER:**
Sample answer using 6: \( \sqrt{x-2} = 2, (x + 21)^{\frac{1}{3}} = 3 \)

71. **WRITING IN MATH** Explain the relationship between the index of the root of a variable in an equation and the power to which you raise each side of the equation to solve the equation.

**SOLUTION:**
They are the same number. For example, \( \left( \sqrt[3]{64} \right)^3 = 64 \).

**ANSWER:**
They are the same number.
6-7 Solving Radical Equations and Inequalities

72. OPEN ENDED Write an equation that can be solved by raising each side of the equation to the given power.

a. \( \frac{3}{2} \) power

b. \( \frac{5}{4} \) power

c. \( \frac{7}{8} \) power

SOLUTION:

a. Sample answer: \( 0 = 6x^3 - 5 \)
b. Sample answer: \( 0 = x^5 - 9 \)
c. Sample answer: \( 10x^7 = -1 \)

ANSWER:

73. CHALLENGE Solve \( 7^{3x-1} = 49^{x+1} \) for \( x \). (Hint: \( b^x = b^y \) if and only if \( x = y \)).

SOLUTION:

\[
7^{3x-1} = 49^{x+1} \\
7^{3x-1} = (7^2)^{x+1} \\
7^{3x-1} = 7^{2x+2}
\]

Equate the powers and solve for \( x \).

\[
3x - 1 = 2x + 2 \\
x = 3
\]

ANSWER:

3

REASONING Determine whether the following statements are sometimes, always, or never true for \( \frac{1}{x^n} = a \). Explain your reasoning.

74. If \( n \) is odd, there will be extraneous solutions.

SOLUTION:

never;
Sample answer: The radicand can be negative.

ANSWER:

never; Sample answer: The radicand can be negative.
6-7 Solving Radical Equations and Inequalities

75. If \( n \) is even, there will be extraneous solutions.

**SOLUTION:**
sometimes;
Sample answer: when the radicand is negative, then there will be extraneous roots.

**ANSWER:**
sometimes; Sample answer: when the radicand is negative, then there will be extraneous roots.

76. What is an equivalent form of \( \frac{4}{5+i} \)?

**SOLUTION:**

\[
\frac{4}{5+i} = \frac{4(5-i)}{5^2 - i^2} = \frac{20-4i}{25+1} = \frac{20-4i}{26} = \frac{10-2i}{13}
\]

Option A is the correct answer.

**ANSWER:**
A

77. Which set of points describes a function?

- **F** \( \{(3, 0), (-2, 5), (2, -1), (2, 9)\} \)
- **G** \( \{(-3, 5), (-2, 3), (-1, 5), (0, 7)\} \)
- **H** \( \{(2, 5), (2, 4), (2, 3), (2, 2)\} \)
- **J** \( \{(3, 1), (-3, 2), (3, 3), (-3, 4)\} \)

**SOLUTION:**
In option F and H, the element 2 has more than one image.

In option J, the elements 3 and -3 have more than one image. In option G, every element has a unique image. So, it describes a function.

Therefore, option G is the correct answer.

**ANSWER:**
G

78. **SHORT RESPONSE** The perimeter of an isosceles triangle is 56 inches. If one leg is 20 inches long, what is the measure of the base of the triangle?

**SOLUTION:**
Since the given triangle is an isosceles triangle, the measure of another leg should be 20 inches. Therefore, the measure of the base of the triangle is \((56 - 20 - 20)\) or 16 in.

**ANSWER:**
16 in.
6-7 Solving Radical Equations and Inequalities

79. SAT/ACT If \( \sqrt{x+5} + 1 = 4 \), what is the value of \( x \)?

\[ \begin{align*}
A & \quad 4 \\
B & \quad 10 \\
C & \quad 11 \\
D & \quad 12 \\
E & \quad 20
\end{align*} \]

**SOLUTION:**

\[ \begin{align*}
\sqrt{x+5} + 1 & = 4 \\
\sqrt{x+5} & = 3 \\
\left( \sqrt{x+5} \right)^2 & = 3^2 \\
x+5 & = 9 \\
x & = 4
\end{align*} \]

Option A is the correct answer.

**ANSWER:**

A

80. \( 27^{-\frac{2}{3}} \)

**SOLUTION:**

\[ \begin{align*}
27^{-\frac{2}{3}} & = \frac{1}{27^{\frac{2}{3}}} \\
& = \frac{1}{\left(3^3\right)^{\frac{2}{3}}} \\
& = \frac{1}{3^2} \\
& = \frac{1}{9}
\end{align*} \]

**ANSWER:**

\( \frac{1}{9} \)

81. \( 9^3 \cdot 9^3 \)

**SOLUTION:**

\[ \begin{align*}
9^3 \cdot 9^3 & = 9^{3+3} \\
& = 9^6 \\
& = 81
\end{align*} \]

**ANSWER:**

81
82. \( \left( \frac{8}{27} \right)^{-\frac{2}{3}} \)

**SOLUTION:**

\[
\left( \frac{8}{27} \right)^{-\frac{2}{3}} = \left( \frac{27}{8} \right)^{\frac{2}{3}} \\
= \left( \frac{3^3}{2^3} \right)^{\frac{2}{3}} \\
= \frac{3^2}{2^2} \\
= \frac{9}{4}
\]

**ANSWER:**

\[ \frac{9}{4} \]

---

83. **GEOMETRY** The measures of the legs of a right triangle can be represented by the expressions \(4x^2y^2\) and \(8x^2y^2\). Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

**SOLUTION:**

\[ c = \sqrt{a^2 + b^2} \]

Substitute \(4x^2y^2\) and \(8x^2y^2\) for \(a\) and \(b\) and simplify.

\[
c = \sqrt{(4x^2y^2)^2 + (8x^2y^2)^2} \\
= \sqrt{16x^4y^4 + 64x^4y^4} \\
= \sqrt{16x^4y^4 (1 + 4)} \\
= 4x^2y^2 \sqrt{5}
\]

Therefore, the measure of the hypotenuse is \(4x^2y^2 \sqrt{5}\).

**ANSWER:**

\[ 4x^2y^2 \sqrt{5} \]
6-7 Solving Radical Equations and Inequalities

Find the inverse of each function.

84. \( y = 3x - 4 \)

**SOLUTION:**

\[ y = 3x - 4 \]

Interchange \( x \) and \( y \), then solve for \( y \)

\[ x = 3y - 4 \]

\[ 3y - 4 = x \]

\[ 3y = x + 4 \]

\[ y = \frac{x + 4}{3} \]

**ANSWER:**

\[ y = \frac{x + 4}{3} \]

85. \( y = -2x - 3 \)

**SOLUTION:**

\[ y = -2x - 3 \]

Interchange \( x \) and \( y \), then solve for \( y \)

\[ x = -2y - 3 \]

\[ x + 2y = -3 \]

\[ 2y = -x - 3 \]

\[ y = \frac{-x - 3}{2} \]

**ANSWER:**

\[ y = \frac{-x - 3}{2} \]

86. \( y = x^2 \)

**SOLUTION:**

\[ y = x^2 \]

Interchange \( x \) and \( y \), then solve for \( y \)

\[ y^2 = x \]

\[ y = \pm \sqrt{x} \]

**ANSWER:**

\[ y = \pm \sqrt{x} \]

87. \( y = (2x + 3)^2 \)

**SOLUTION:**

\[ y = (2x + 3)^2 \]

Interchange \( x \) and \( y \) and solve for \( y \).

\[ (2y + 3)^2 = x \]

\[ 2y + 3 = \pm \sqrt{x} \]

\[ 2y = \pm \sqrt{x} - 3 \]

\[ y = \frac{\pm \sqrt{x} - 3}{2} \]

\[ = \pm \frac{1}{2} \sqrt{x} - \frac{3}{2} \]

**ANSWER:**

\[ y = \pm \frac{1}{2} \sqrt{x} - \frac{3}{2} \]
6-7 Solving Radical Equations and Inequalities

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeros.

\[ f(x) \]

**SOLUTION:**

a. The function tends to \( +\infty \) as \( x \) tends to \( -\infty \).
The function tends to \( -\infty \) as \( x \) tends to \( +\infty \).

b. Since the end behaviors are in opposite directions, the graph represents an odd-degree polynomial.

c. Since the graph intersects the \( x \)-axis at three points, the number of real zeros is 3.

**ANSWER:**

a. \( f(x) \to -\infty \) as \( x \to +\infty \),
\( f(x) \to +\infty \) as \( x \to -\infty \);

b. odd;

c. 3
Solve each equation. Write in simplest form.

91. \( \frac{8}{5} x = \frac{4}{15} \)

**SOLUTION:**

\[
\begin{align*}
8 & = 4\cdot 5\\
5 & = 15\cdot 8\\
\frac{1}{5} & = \frac{1}{15} \\
\end{align*}
\]

**ANSWER:**

\( \frac{1}{6} \)

92. \( \frac{27}{14} y = \frac{6}{7} \)

**SOLUTION:**

\[
\begin{align*}
27 & = 6\cdot 14\\
14 & = 7\cdot 27\\
\frac{4}{7} & = \frac{9}{27} \\
\end{align*}
\]

**ANSWER:**

\( \frac{4}{9} \)
6-7 Solving Radical Equations and Inequalities

93. \( \frac{3}{10} = \frac{12}{25}a \)

\text{SOLUTION:}
\[
\frac{3}{10} = \frac{12}{25}a \\
\Rightarrow a = \frac{3 \cdot 25}{10 \cdot 12} \\
\Rightarrow a = \frac{5}{8}
\]

\text{ANSWER:}
\[
\frac{5}{8}
\]

94. \( \frac{6}{7} = 9m \)

\text{SOLUTION:}
\[
\frac{6}{7} = 9m \\
\Rightarrow m = \frac{6 \cdot 1}{7 \cdot 9} \\
\Rightarrow m = \frac{2}{21}
\]

\text{ANSWER:}
\[
\frac{2}{21}
\]

95. \( \frac{9}{8} = 18 \)

\text{SOLUTION:}
\[
\frac{9}{8} = 18 \\
\Rightarrow 8 \cdot \frac{9}{b} = 18 \\
\Rightarrow b = \frac{8 \cdot 9}{18} \\
\Rightarrow b = 4
\]

\text{ANSWER:}
\[
4
\]

96. \( \frac{6}{7} = \frac{3}{4} \)

\text{SOLUTION:}
\[
\frac{6}{7} = \frac{3}{4} \\
\Rightarrow 7 \cdot \frac{6}{3} = 4 \\
\Rightarrow n = \frac{7 \cdot 6}{3} \\
\Rightarrow n = \frac{42}{3} \\
\Rightarrow n = \frac{7}{8}
\]

\text{ANSWER:}
\[
\frac{7}{8}
\]
97. \( \frac{1}{3} p = \frac{5}{6} \)

**SOLUTION:**

\[
\frac{1}{3} p = \frac{5}{6} \\
\times 3 \quad \times 3 \\
p = \frac{5}{2} \\
= 2 \frac{1}{2}
\]

**ANSWER:**

\( 2 \frac{1}{2} \)

98. \( \frac{2}{3} q = 7 \)

**SOLUTION:**

\[
\frac{2}{3} q = 7 \\
\times 3 \quad \times 3 \\
q = 7 \cdot \frac{3}{2} \\
= 21 \frac{3}{2} \\
= 10 \frac{1}{2}
\]

**ANSWER:**

\( 10 \frac{1}{2} \)
Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

1. \( f(x) = 3x + 8, g(x) = \frac{x - 8}{3} \)

**SOLUTION:**

\[
\begin{align*}
[f \circ g](x) &= f(g(x)) \\
&= f\left(\frac{x - 8}{3}\right) \\
&= 3\left(\frac{x - 8}{3}\right) + 8 \\
&= x - 8 + 8 \\
&= x \\
[g \circ f](x) &= g(f(x)) \\
&= g(3x + 8) \\
&= \frac{(3x + 8) - 8}{3} \\
&= \frac{3x}{3} \\
&= x
\end{align*}
\]

Since \( [f \circ g](x) = [g \circ f](x) = x \), they are inverse functions.

**ANSWER:**

Yes
3. \( f(x) = x + 7, g(x) = x - 7 \)

**SOLUTION:**
\[
[f \circ g](x) = f(g(x))
= f(x - 7)
= (x - 7) + 7
= x - 7 + 7
= x
\]
\[
g \circ f)(x) = g(f(x))
= g(x + 7)
= (x + 7) - 7
= x + 7 - 7
= x
\]

Since \([f \circ g](x) = [g \circ f](x) = x\), they are inverse functions.

**ANSWER:**
Yes

4. \( g(x) = 3x - 2, f(x) = \frac{x - 2}{3} \)

**SOLUTION:**
\[
[f \circ g](x) = f(g(x))
= f(3x - 2)
= 3x - 2 - 2 \frac{3}{3}
= x - 4
\]
\[
g \circ f)(x) = g(f(x))
= g\left(\frac{x - 2}{3}\right)
= 3\left(\frac{x - 2}{3}\right) - 2
= x - 2 - 2
= x - 4
\]

Since \([f \circ g](x) \neq [g \circ f](x)\), they are not inverse functions.

**ANSWER:**
No
5. **MULTIPLE CHOICE** Which inequality represents the graph below?

![Graph Image]

A. \( y \geq \sqrt{x+4} \)

B. \( y \leq \sqrt{x+4} \)

C. \( y \geq \sqrt{x-4} \)

D. \( y \leq \sqrt{x-4} \)

**SOLUTION:**
The graph represents the inequality \( y \leq \sqrt{x+4} \).

Option B is the correct answer.

**ANSWER:** B

If \( f(x) = 3x + 2 \) and \( g(x) = x^2 - 2x + 1 \), find each function.

6. \((f + g)(x)\)

**SOLUTION:**
\[
(f + g)(x) = 3x + 2 + x^2 - 2x + 1
\]
\[
= x^2 + x + 3
\]

**ANSWER:**
\((f + g)(x) = x^2 + x + 3\)

7. \((f \cdot g)(x)\)

**SOLUTION:**
\[
(f \cdot g)(x) = (3x + 2)(x^2 - 2x + 1)
\]
\[
= 3x^3 - 6x^2 + 3x + 2x^2 - 4x + 2
\]
\[
= 3x^3 - 4x^2 - x + 2
\]

**ANSWER:**
\((f \cdot g)(x) = 3x^3 - 4x^2 - x + 2\)

8. \((f - g)(x)\)

**SOLUTION:**
\[
(f - g)(x) = (3x + 2) - (x^2 - 2x + 1)
\]
\[
= 3x + 2 - x^2 + 2x - 1
\]
\[
= -x^2 + 5x + 1
\]

**ANSWER:**
\((f - g)(x) = -x^2 + 5x + 1\)

9. \(\left(\frac{f}{g}\right)(x)\)

**SOLUTION:**
\[
\left(\frac{f}{g}\right)(x) = \frac{3x + 2}{x^2 - 2x + 1} \text{ for } x \neq 1
\]

**ANSWER:**
\(\left(\frac{f}{g}\right)(x) = \frac{3x + 2}{x^2 - 2x + 1}, x \neq 1\)
Solve each equation.

10. \( \sqrt{a+12} = \sqrt{5a-4} \)

**SOLUTION:**

\[
\sqrt{a+12} = \sqrt{5a-4} \\
\left(\sqrt{a+12}\right)^2 = \left(\sqrt{5a-4}\right)^2 \\
a+12 = 5a-4 \\
4a = 16 \\
a = 4
\]

**ANSWER:**

4

11. \( \sqrt{3x} = \sqrt{x-2} \)

**SOLUTION:**

\[
\sqrt{3x} = \sqrt{x-2} \\
\left(\sqrt{3x}\right)^2 = \left(\sqrt{x-2}\right)^2 \\
3x = x-2 \\
2x = -2 \\
x = -1
\]

For \( x = -1 \), the radical \( \sqrt{3x} \) become undefined.

Therefore, there is no solution.

**ANSWER:**

no solution

12. \( 4\left(\sqrt[3]{3x+1}\right) - 8 = 0 \)

**SOLUTION:**

\[
4\left(\sqrt[3]{3x+1}\right) - 8 = 0 \\
4\left(\sqrt[3]{3x+1}\right) = 8 \\
\sqrt[3]{3x+1} = 2 \\
\left(\sqrt[3]{3x+1}\right)^3 = 2^3 \\
3x+1 = 16 \\
3x = 15 \\
x = 5
\]

**ANSWER:**

5

13. \( \sqrt[3]{5m+6} + 15 = 21 \)

**SOLUTION:**

\[
\sqrt[3]{5m+6} + 15 = 21 \\
\sqrt[3]{5m+6} = 6 \\
\left(\sqrt[3]{5m+6}\right)^3 = 6^3 \\
5m+6 = 216 \\
5m = 210 \\
m = 42
\]

**ANSWER:**

42
14. \( \sqrt{3x + 21} = \sqrt{5x + 27} \)

**SOLUTION:**
\[
\left( \sqrt{3x + 21} \right)^2 = \left( \sqrt{5x + 27} \right)^2
\]
\[
3x + 21 = 5x + 27
\]
\[
2x = -6
\]
\[
x = -3
\]

**ANSWER:**
-3

15. \( 1 + \sqrt{x + 11} = \sqrt{2x + 15} \)

**SOLUTION:**
\[
1 + \sqrt{x + 11} = \sqrt{2x + 15}
\]
\[
\left( 1 + \sqrt{x + 11} \right)^2 = \left( \sqrt{2x + 15} \right)^2
\]
\[
1 + x + 11 + 2\sqrt{x + 11} = 2x + 15
\]
\[
2\sqrt{x + 11} = x + 3
\]
\[
\left( 2\sqrt{x + 11} \right)^2 = (x + 3)^2
\]
\[
4(x + 11) = x^2 + 6x + 9
\]
\[
x^2 + 2x - 35 = 0
\]
\[
(x + 7)(x - 5) = 0
\]

By Zero Product Property:
\[
x + 7 = 0 \quad \text{or} \quad x - 5 = 0
\]
\[
x = -7 \quad \text{or} \quad x = 5
\]

Check:
\[
1 + \sqrt{-7 + 11} = \sqrt{2(-7) + 15}
\]
\[
1 + \sqrt{4} = \sqrt{-14 + 15}
\]
\[
1 + 2 = 1
\]
\[
3 \neq 1 \times
\]
\[
1 + \sqrt{5 + 11} = \sqrt{2(5) + 15}
\]
\[
1 + \sqrt{16} = \sqrt{10 + 15}
\]
\[
1 + 4 = 5
\]
\[
5 = 5 \checkmark
\]

The solution is 5.

**ANSWER:**
5
16. \( \sqrt{x-5} = \sqrt{2x-4} \)

**SOLUTION:**

\[
\sqrt{x-5} = \sqrt{2x-4} \\
(\sqrt{x-5})^2 = (\sqrt{2x-4})^2 \\
x - 5 = 2x - 4 \\
x = -1
\]

For \( x = -1 \), the function \( \sqrt{x-5} \) is undefined. Therefore, there is no solution.

**ANSWER:**

no solution

17. \( \sqrt{x-6} - \sqrt{x} = 3 \)

**SOLUTION:**

\[
\sqrt{x-6} - \sqrt{x} = 3 \\
(\sqrt{x-6} - \sqrt{x})^2 = 3^2 \\
x - 6 + x - 2\sqrt{x^2 - 6x} = 9 \\
2\sqrt{x^2 - 6x} = 2x - 15 \\
(2\sqrt{x^2 - 6x})^2 = (2x - 15)^2 \\
4x^2 - 24x = 4x^2 - 60x + 225 \\
36x = 225 \\
x = \frac{25}{4}
\]

Check:

\[
\sqrt{\frac{25}{4}} - 6 - \sqrt{\frac{25}{4}} = 3 \\
\sqrt{\frac{25}{4}} - 24 - \sqrt{\frac{25}{4}} = 3 \\
\sqrt{\frac{1}{4}} - \sqrt{\frac{25}{4}} = 3 \\
\frac{1}{2} - \frac{5}{2} = 3 \\
\frac{1 - 5}{2} = 3 \\
\frac{-4}{2} = 3 \\
-2 \neq 3
\]

There exists no solution for the equation.

**ANSWER:**

no solution
Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

1. Substitute 5, 7 and 10 for a, b and c.

The area of the triangle is \( \text{ft}^2 \).

ANSWER: \( \text{ft}^2 \)

---

18. MULTIPLE CHOICE Which expression is equivalent to \( 125^{\frac{1}{3}} \)?

- F 5
- G \( \frac{1}{5} \)
- H \( \frac{1}{5} \)
- J 5

SOLUTION:

\[
125^{\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}} = \frac{1}{\left(5^3\right)^{\frac{1}{3}}} = \frac{1}{5}
\]

Option H is the correct answer.

ANSWER: H

---

19. \( (2 + \sqrt{5})(6 - 3\sqrt{5}) \)

SOLUTION:

\[
(2 + \sqrt{5})(6 - 3\sqrt{5}) = 12 - 6\sqrt{5} + 6\sqrt{5} - 3\sqrt{5}^2
\]

\[
= 12 - 15
\]

\[
= -3
\]

ANSWER: -3

---

20. \( (3 - 2\sqrt{2})(-7 + \sqrt{2}) \)

SOLUTION:

\[
(3 - 2\sqrt{2})(-7 + \sqrt{2}) = -21 + 17\sqrt{2} - 14\sqrt{2} + 4
\]

\[
= -21 + 17\sqrt{2} - 4
\]

\[
= -25 + 17\sqrt{2}
\]

ANSWER: \( 17\sqrt{2} - 25 \)

21. \( \frac{12}{2 - \sqrt{3}} \)

SOLUTION:

\[
\frac{12}{2 - \sqrt{3}} = \frac{12}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}}
\]

\[
= \frac{12(2 + \sqrt{3})}{4 - 3}
\]

\[
= 24 + 12\sqrt{3}
\]

ANSWER: \( 12\sqrt{3} + 24 \)
22. \(\frac{\frac{1}{m^2 - 1}}{2m^2 + 1}\)

**SOLUTION:**

\[
\frac{\frac{1}{m^2 - 1}}{2m^2 + 1} = \frac{\sqrt{m - 1}}{2\sqrt{m + 1}}
\]

\[
= \frac{\sqrt{m - 1}}{2\sqrt{m + 1}} \cdot \frac{2\sqrt{m} - 1}{2\sqrt{m} - 1}
\]

\[
= \frac{2\sqrt{m^2} - \sqrt{m} - 2\sqrt{m} + 1}{(2\sqrt{m})^2 - 1^2}
\]

\[
= \frac{2m - 3\sqrt{m} + 1}{4m - 1}
\]

\[
= \frac{2m - 3m + 1}{4m - 1}
\]

**ANSWER:**

\(\frac{2m - 3m^2 + 1}{4m - 1}\)

23. \(4\sqrt{3} - 8\sqrt{48}\)

**SOLUTION:**

\(4\sqrt{3} - 8\sqrt{48} = 4\sqrt{3} - 8\sqrt{16 \cdot 3}\)

\(= 4\sqrt{3} - 8 \cdot 4 \cdot \sqrt{3}\)

\(= 4\sqrt{3} - 32\sqrt{3}\)

\(= -28\sqrt{3}\)

**ANSWER:**

\(-28\sqrt{3}\)

24. \(\frac{2}{1} \cdot \frac{5}{5}\)

**SOLUTION:**

\(\frac{2}{1} \cdot \frac{5}{5} = \frac{2 \cdot 1 \cdot 5}{2 \cdot 1 \cdot 6}\)

\(= \frac{12}{6}\)

\(= 2\)

**ANSWER:**

\(5^2\) or 25

25. \(\sqrt[6]{729a^9b^{24}}\)

**SOLUTION:**

\(\sqrt[6]{729a^9b^{24}} = (729a^9b^{24})^{\frac{1}{6}}\)

\(= (3^6 \cdot a^6 \cdot b^6)^{\frac{1}{6}}\)

\(= 3a^2b^4\)

**ANSWER:**

\(3ab^4\sqrt[6]{a}\)
26. \( \sqrt[5]{32x^{15}y^{10}} \)

**SOLUTION:**

\[
\sqrt[5]{32x^{15}y^{10}} = (32x^{15}y^{10})^{\frac{1}{5}}
\]

\[
= (2^5(x^3)(y^2)^3)^{\frac{1}{5}}
\]

\[
= 2x^3y^2
\]

**ANSWER:**

\( 2x^3y^2 \)

27. \( \frac{4}{5} \)

**SOLUTION:**

\[
w^{-\frac{4}{5}} = \frac{1}{w^{\frac{4}{5}}}
\]

\[
= \frac{1}{\sqrt[5]{w^4}}
\]

\[
= \frac{1}{\sqrt[5]{w} \cdot \sqrt[5]{w} \cdot \sqrt[5]{w} \cdot \sqrt[5]{w} \cdot \sqrt[5]{w}}
\]

\[
= \frac{1}{w^{\frac{1}{5}}}
\]

\[
= \frac{1}{w^{\frac{5}{5}}}
\]

**ANSWER:**

\( \frac{1}{w^5} \)

28. \( \frac{2}{r^3} \)

29. \( \frac{a^{\frac{2}{3}}}{6a^3 \cdot a^{-\frac{1}{4}}} \)

**SOLUTION:**

\[
\frac{a^{\frac{2}{3}}}{6a^3 \cdot a^{-\frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^\frac{12}{4} \cdot a^{-\frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{12} \cdot a^{-\frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

\[
= \frac{a^{\frac{2}{3}}}{6a^{\frac{12}{4} + \frac{1}{4}}}
\]

**ANSWER:**

\( \frac{5}{a^{\frac{12}{4} + \frac{1}{4}}} \)

\( \frac{5}{a^{\frac{12}{4} + \frac{1}{4}}} \)
30. \[
\frac{\frac{3}{y^2}}{\frac{1}{y^2} + 2}
\]

SOLUTION:
\[
\frac{\frac{3}{y^2}}{\frac{1}{y^2} + 2} = \frac{\frac{3}{y^2}}{\frac{1}{y^2} + 2} \cdot \frac{\frac{1}{y^2} - 2}{\frac{1}{y^2} - 2}
\]
\[
= \frac{\frac{3}{y^2} - 2y^2}{(\frac{1}{y^2})^2 - 2^2}
\]
\[
= \frac{y^2 - 2y^2}{y - 4}
\]

ANSWER:
\[
\frac{y^2 - 2y^2}{y - 4}
\]

31. MULTIPLE CHOICE What is the area of the rectangle?

A \[2\sqrt{3} + 3\sqrt{2}\] units^2

B \[4 + 2\sqrt{6} + 2\sqrt{3}\] units^2

C \[2\sqrt{3} + \sqrt{6}\] units^2

D \[2\sqrt{3} + 3\] units^2

SOLUTION:
Area = \[(2 + \sqrt{6})\sqrt{3}\]
\[
= 2\sqrt{3} + \sqrt{18}
\]
\[
= 2\sqrt{3} + 3\sqrt{2}\] unit^2

Option A is the correct answer.

ANSWER:
A
Practice Test - Chapter 6

Solve each inequality.

32. \( \sqrt{4x - 3} < 5 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 4x - 3 \geq 0 \).

\[
4x - 3 \geq 0 \\
4x \geq 3 \\
x \geq \frac{3}{4}
\]

Solve \( \sqrt{4x - 3} < 5 \).

\[
\sqrt{4x - 3} < 5 \\
(\sqrt{4x - 3})^2 < 5^2 \\
4x - 3 < 25 \\
4x < 28 \\
x < 7
\]

The solution region is \( \frac{3}{4} \leq x < 7 \).

**ANSWER:**
\( \frac{3}{4} \leq x < 7 \)

33. \(-2 + \sqrt{3m - 1} < 4\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 3m - 1 \geq 0 \).

\[
3m - 1 \geq 0 \\
3m \geq 1 \\
m \geq \frac{1}{3}
\]

Solve \(-2 + \sqrt{3m - 1} < 4\).

\[
-2 + \sqrt{3m - 1} < 4 \\
\sqrt{3m - 1} < 6 \\
(\sqrt{3m - 1})^2 < 6^2 \\
3m - 1 < 36 \\
3m < 37 \\
m < \frac{37}{3}
\]

The solution region is \( \frac{1}{3} \leq m < \frac{37}{3} \).

**ANSWER:**
\( \frac{1}{3} \leq m < \frac{37}{3} \)
Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

The area of the triangle is $\text{ft}^2$.

**ANSWER:** $\text{ft}^2$

---

**Practice Test - Chapter 6**

34. $2 + \sqrt{4x - 4} \leq 6$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $4x - 4 \geq 0$.

$4x - 4 \geq 0$
$4x \geq 4$
$x \geq 1$

Solve $2 + \sqrt{4x - 4} \leq 6$.

$2 + \sqrt{4x - 4} \leq 6$
$\sqrt{4x - 4} \leq 4$
$(\sqrt{4x - 4})^2 \leq 4^2$
$4x - 4 \leq 16$
$4x \leq 20$
$x \leq 5$

The solution region is $1 \leq x \leq 5$.

**ANSWER:** $1 \leq x \leq 5$

35. $\sqrt{2x + 3} - 4 \leq 5$

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve $2x + 3 \geq 0$.

$2x + 3 \geq 0$
$2x \geq -3$
$x \geq \frac{-3}{2}$

Solve $\sqrt{2x + 3} - 4 \leq 5$.

$\sqrt{2x + 3} - 4 \leq 5$
$\sqrt{2x + 3} \leq 9$
$(\sqrt{2x + 3})^2 \leq 9^2$
$2x + 3 \leq 81$
$2x \leq 78$
$x \leq 39$

The solution region is $-\frac{3}{2} \leq x \leq 39$.

**ANSWER:** $-\frac{3}{2} \leq x \leq 39$
36. \( \sqrt{b+12} - \sqrt{b} > 2 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( b+12 \geq 0 \) and \( b \geq 0 \).

\[
\begin{align*}
b+12 & \geq 0 \quad \text{or} \quad b \geq 0 \\
b & \geq -12 \quad \text{or} \quad b \geq 0
\end{align*}
\]

Solve \( \sqrt{b+12} - \sqrt{b} > 2 \).

\[
\begin{align*}
\sqrt{b+12} - \sqrt{b} & > 2 \\
\left(\sqrt{b+12} - \sqrt{b}\right)^2 & > 2^2 \\
b + 12 + b - 2\sqrt{b^2 + 12b} & > 4 \\
\sqrt{b^2 + 12b} & < b + 4 \\
\left(\sqrt{b^2 + 12b}\right)^2 & < (b + 4)^2 \\
b^2 + 12b & < b^2 + 16 + 8b \\
4b & < 16 \\
b & < 4
\end{align*}
\]

The solution region is \( 0 \leq b < 4 \).

**ANSWER:**
\( 0 \leq b < 4 \)

37. \( \sqrt{y-7} + 5 \geq 10 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( y - 7 \geq 0 \).

\[
\begin{align*}
y - 7 & \geq 0 \\
y & \geq 7
\end{align*}
\]

Solve \( \sqrt{y-7} + 5 \geq 10 \).

\[
\begin{align*}
\sqrt{y-7} & \geq 5 \\
\left(\sqrt{y-7}\right)^2 & \geq 5^2 \\
y - 7 & \geq 25 \\
y & \geq 32
\end{align*}
\]

The solution region is \( y \geq 32 \).

**ANSWER:**
\( y \geq 32 \)
Determine whether each pair of functions are inverse functions. Write yes or no. Explain your reasoning.

1. Substitute 5, 7 and 10 for a, b and c.

The area of the triangle is ft².

ANSWER:

ft² 

39. \( \sqrt{c+5} + \sqrt{c+10} > 2 \)

SOLUTION:

c + 5 > 0

c > -5

c + 10 > 0

c > -10

Therefore, the inequality is defined for \( c > -5 \).

The domain of the function \( f(c) = \sqrt{c+5} + \sqrt{c+10} \) is \( D = \{ c | c > -5 \} \).

The range of the function \( f(c) \) is

\[ \{ f(c) | f(c) > \sqrt{5} \} \].

Since the lower limit of the range of the function itself is \( \sqrt{5} > 2 \), the solution of the inequality \( \sqrt{c+5} + \sqrt{c+10} > 2 \) is \( c > -5 \).

ANSWER:

\( c > -5 \)
Practice Test - Chapter 6

40. GEOMETRY  The area of a triangle with sides of length $a$, $b$, and $c$ is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$,

where

$$s = \frac{1}{2}(a+b+c)$$. What is the area of the triangle expressed in radical form?

SOLUTION:
Substitute 5, 7 and 10 for $a$, $b$ and $c$.

$$s = \frac{1}{2}(5 + 7 + 10)$$
$$= \frac{22}{2} = 11$$

$$A = \sqrt{11(11-5)(11-7)(11-10)}$$
$$= \sqrt{11(6)(4)(1)}$$
$$= 2\sqrt{66}$$

The area of the triangle is $2\sqrt{66}$ ft$^2$.

ANSWER:
$2\sqrt{66}$ ft$^2$
Choose a word or term that best completes each statement.

1. If both compositions result in the _____, then the functions are inverse functions.

   SOLUTION: identity function

   ANSWER: identity function

2. In a(n)_______, the results of one function are used to evaluate a second function.

   SOLUTION: composition of functions

   ANSWER: composition of functions

3. Radicals are_______ if both the index and the radicand are identical.

   SOLUTION: like radical expressions

   ANSWER: like radical expressions

4. When there is more than one real root, the nonnegative root is called the__________.

   SOLUTION: principal root

   ANSWER: principal root

5. To eliminate radicals from a denominator or fractions from a radicand, you use a process called___________.

   SOLUTION: rationalizing the denominator

   ANSWER: rationalizing the denominator

6. Equations with radicals that have variables in the radicands are called___________.

   SOLUTION: radical equations

   ANSWER: radical equations

7. Two relations are_______ if and only if one relation contains the element \((b, a)\) when the other relation contains the element \((a, b)\).

   SOLUTION: inverse relations

   ANSWER: inverse relations

8. When solving a radical equation, sometimes you will obtain a number that does not satisfy the original equation. Such a number is called a(n) __________.

   SOLUTION: extraneous solution

   ANSWER: extraneous solution
9. The square root function is a type of __________.

**SOLUTION:**
radical function

**ANSWER:**
radical function

10. Find \([f \circ g](x)\) and \([g \circ f](x)\).

\[f(x) = 2x + 1\]
\[g(x) = 4x - 5\]

**SOLUTION:**
\[f[g(x)] = 2(4x - 5) + 1\]
\[= 8x - 10 + 1\]
\[= 8x - 9\]
\[g[f(x)] = 4(2x + 1) - 5\]
\[= 8x + 4 - 5\]
\[= 8x - 1\]

**ANSWER:**
\([f \circ g](x) = 8x - 9\]
\([g \circ f](x) = 8x - 1\]

11. \(f(x) = x^2 + 1\)
\(g(x) = x - 7\)

**SOLUTION:**
\[f[g(x)] = (x - 7)^2 + 1\]
\[= x^2 - 14x + 49 + 1\]
\[= x^2 - 14x + 50\]
\[g[f(x)] = x^2 + 1 - 7\]
\[= x^2 - 6\]

**ANSWER:**
\([f \circ g](x) = x^2 - 14x + 50\]
\([g \circ f](x) = x^2 - 6\]

12. \(f(x) = x^2 + 4\)
\(g(x) = -2x + 1\)

**SOLUTION:**
\[f[g(x)] = (-2x + 1)^2 + 4\]
\[= 4x^2 - 4x + 1 + 4\]
\[= 4x^2 - 4x + 5\]
\[g[f(x)] = -2(x^2 + 4) + 1\]
\[= -2x^2 - 8 + 1\]
\[= -2x^2 - 7\]

**ANSWER:**
\([f \circ g](x) = 4x^2 - 4x + 5\]
\([g \circ f](x) = -2x^2 - 7\)
13. \( f(x) = 4x \)  
   \( g(x) = 5x - 1 \)

**SOLUTION:**

\[
 f \left[ g(x) \right] = 4(5x - 1) \\
 = 20x - 4 \\
g \left[ f(x) \right] = 5(4x) - 1 \\
 = 20x - 1
\]

**ANSWER:**

\[
[f \circ g](x) = 20x - 4 \\
g \circ f](x) = 20x - 1
\]

14. \( f(x) = x^3 \)  
   \( g(x) = x - 1 \)

**SOLUTION:**

\[
 f \left[ g(x) \right] = (x - 1)^3 \\
 = x^3 - 3x^2 + 3x - 1 \\
g \left[ f(x) \right] = x^3 - 1
\]

**ANSWER:**

\[
[f \circ g](x) = x - 3x^2 + 3x - 1 \\
g \circ f](x) = x^3 - 1
\]

15. \( f(x) = x^2 + 2x - 3 \)  
   \( g(x) = x + 1 \)

**SOLUTION:**

\[
 f \left[ g(x) \right] = (x + 1)^2 + 2(x + 1) - 3 \\
 = x^2 + 2x + 1 + 2x + 2 - 3 \\
 = x^2 + 4x \\
g \left[ f(x) \right] = x^2 + 2x - 3 + 1 \\
 = x^2 + 2x - 2
\]

**ANSWER:**

\[
[f \circ g](x) = x^2 + 4x \\
g \circ f](x) = x^2 + 2x - 2
\]

16. **MEASUREMENT** The formula \( f = 3y \) converts yards \( y \) to feet \( f \) and \( f = \frac{n}{12} \) converts inches \( n \) to feet \( f \). Write a composition of functions that converts yards to inches.

**SOLUTION:**

Substitute \( 3y \) for \( f \) into the second equation and solve for \( n \).

\[
f = \frac{n}{12}
\]

\[
3y = \frac{n}{12}
\]

\[
n = 36y
\]

**ANSWER:**

\( n = 36y \)
Find the inverse of each function. Then graph the function and its inverse.

17. \( f(x) = 5x - 6 \)

**SOLUTION:**
Rewrite \( f(x) \) as \( y = 5x - 6 \). Interchange the variables and solve for \( y \).

\[
x = 5y - 6 \\
-5y = -x - 6 \\
y = \frac{x + 6}{5}
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

\[ f^{-1}(x) = \frac{x + 6}{5} \]

18. \( f(x) = -3x - 5 \)

**SOLUTION:**
Rewrite \( f(x) \) as \( y = -3x - 5 \). Interchange the variables and solve for \( y \).

\[
x = -3y - 5 \\
3y = -x - 5 \\
y = \frac{x + 5}{-3} \\
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

\[ f^{-1}(x) = \frac{x + 5}{-3} \]
Study Guide and Review - Chapter 6

19. \( f(x) = \frac{1}{2}x + 3 \)

**SOLUTION:**

Rewrite \( f(x) \) as \( y = \frac{1}{2}x + 3 \). Interchange the variables and solve for \( y \).

\[
\begin{align*}
x &= \frac{1}{2}y + 3 \\
-\frac{1}{2}y &= -x + 3 \\
y &= 2x - 6
\end{align*}
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

**ANSWER:**

\( f^{-1}(x) = 2x - 6 \)

20. \( f(x) = \frac{4x + 1}{5} \)

**SOLUTION:**

Rewrite \( f(x) \) as \( y = \frac{4x + 1}{5} \). Interchange the variables and solve for \( y \).

\[
\begin{align*}
x &= \frac{4y + 1}{5} \\
5x &= 4y + 1 \\
-4y &= -5x + 1 \\
y &= \frac{5x - 1}{4}
\end{align*}
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

**ANSWER:**

\( f^{-1}(x) = \frac{5x - 1}{4} \)
21. \( f(x) = x^2 \)

**SOLUTION:**

Rewrite \( f(x) \) as \( y = x^2 \).

Interchange the variables and solve for \( y \).

\[
\begin{align*}
  x &= y^2 \\
  \sqrt{x} &= \sqrt{y^2} \\
  \pm \sqrt{x} &= y
\end{align*}
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

**ANSWER:**

\[ f^{-1}(x) = \pm \sqrt{x} \]

22. \( f(x) = (2x + 1)^2 \)

**SOLUTION:**

Rewrite \( f(x) \) as \( y = (2x + 1)^2 \).

Interchange the variables and solve for \( y \).

\[
\begin{align*}
  x &= (2y + 1)^2 \\
  \pm \sqrt{x} &= 2y + 1 \\
  -1 \pm \sqrt{x} &= 2y \\
  \frac{-1 \pm \sqrt{x}}{2} &= y
\end{align*}
\]

Replace \( y \) with \( f^{-1}(x) \). Graph the function and its inverse in the same coordinate plane.

**ANSWER:**

\[ f^{-1}(x) = \frac{-1 \pm \sqrt{x}}{2} \]
23. **SHOPPING** Samuel bought a computer. The sales tax rate was 6% of the sale price, and he paid $50 for shipping. Find the sale price if Samuel paid a total of $1322.

**SOLUTION:**
Let $x$ be the sale price of the computer.

The equation that represent the situation is 

$$x + 0.06x + 50 = 1322.$$ 

Solve for $x$.

$$x + 0.06x + 50 = 1322$$
$$1.06x = 1272$$
$$x = 1200$$

The sale price is $1200.

**ANSWER:** 
$1200

---

Use the horizontal line test to determine whether the inverse of each function is also a function.

24. $f(x) = 3x^2$

**SOLUTION:**
Graph the function $f(x) = 3x^2$.

![Graph of the function $f(x) = 3x^2$.](image)

The horizontal line drawn passes through more than one point. So, the inverse of this function is not a function.

**ANSWER:** 
No
25. \( h(x) = x^3 - 3 \)

**SOLUTION:**
Graph the function \( h(x) = x^3 - 3 \).

![Graph of \( h(x) = x^3 - 3 \)](image)

No horizontal line can be drawn so that it passes through more than one point. The inverse of this function is a function.

**ANSWER:**
Yes

26. \( g(x) = -3x^3 + 2x - 1 \)

**SOLUTION:**
Graph the function \( g(x) = -3x^3 + 2x - 1 \).

![Graph of \( g(x) = -3x^3 + 2x - 1 \)](image)

The horizontal line drawn passes through more than one point. So, the inverse of this function is not a function.

**ANSWER:**
No
27. \( g(x) = 4x^3 - 5x \)

**SOLUTION:**
Graph the function \( g(x) = 4x^3 - 5x \).

![Graph of \( y = 4x^3 - 5x \)](image)

The horizontal line drawn passes through more than one point. The inverse of this function is not a function.

**ANSWER:**
No

28. \( f(x) = -3x^5 + x^2 - 3 \)

**SOLUTION:**
Graph the function \( f(x) = -3x^5 + x^2 - 3 \).

![Graph of \( y = -3x^5 + x^2 - 3 \)](image)

The horizontal line \( y = -3 \) drawn passes through more than one point. So, the inverse of this function is not a function.

**ANSWER:**
No
29. \( h(x) = 4x^4 - 5x \)

\textbf{SOLUTION:}
Graph of the function \( h(x) = 4x^4 - 5x \).

The horizontal line drawn passes through more than one point. So, the inverse of this function is not a function.

\textbf{ANSWER:}
No

30. **FINANCIAL LITERACY** During the last month, Jonathan has made two deposits of $45, made a deposit of double his original balance, and has withdrawn $35 five times. His balance is now $189. Write an equation that models this problem. How much money did Jonathan have in his account at the beginning of the month?

\textbf{SOLUTION:}
Let \( x \) be the original balance in Jonathan’s account.

The equation that represents the situation is \( x + 2(45) + 2x - 5(35) = 189 \).

Solve for \( x \).

\[
\begin{align*}
  x + 2(45) + 2x - 5(35) &= 189 \\
  x + 90 + 2x - 175 &= 189 \\
  3x &= 274 \\
  x &\approx 91.33
\end{align*}
\]

The original balance in the account is about $91.33.

\textbf{ANSWER:}
\( x + 2(45) + 2x - 5(35) = 189; \) about $91.33

Graph each function. State the domain and range.

31. \( f(x) = \sqrt{3x} \)

\textbf{SOLUTION:}
Identify the domain.

Write the radicand as greater than or equal to 0.

\[
3x \geq 0 \\
x \geq 0
\]

Make a table of values for \( x \geq 0 \) and graph the function.
32. \( f(x) = -\sqrt{6x} \)

**SOLUTION:**
Identify the domain.

Write the radicand as greater than or equal to 0.

\[ 6x \geq 0 \]
\[ x \geq 0 \]

Make a table of values for \( x \geq 0 \) and graph the function.

\[ D = \{ x | x \geq 0 \}; R = \{ f(x) | f(x) \geq 0 \} \]

\[ D = \{ x | x \geq 0 \}; R = \{ f(x) | f(x) \leq 0 \} \]

33. \( f(x) = \sqrt{x - 7} \)
Choose a word or term that best completes each statement.

1. If both compositions result in the ______, then the solution
   ANSWER: no solution

72. SOLUTION:

73. SOLUTION: [0].

Write a composition of functions that converts ______(_______(______) for width in ______).

Study Guide and Review - Chapter 6

55. SOLUTION:

57. SOLUTION:

36. SOLUTION:

32. SOLUTION:

11. 3

8. 1

9. 1.41

10. 1.73

11. 2

12. 2.23

The domain is \{x \mid x \geq 7\}, and the range is \{f(x) \mid f(x) \geq 0\}.

ANSWER:
The domain is \( \{ x \mid x \geq 5 \} \), and the range is \( \{ f(x) \mid f(x) \geq 3 \} \).

**ANSWER:**

\[
D = \{ x \mid x^3 - 5 \} ; R = \{ f(x) \mid f(x)^3 - 3 \}
\]

35. \( f(x) = \frac{3}{4} \sqrt{x - 1} + 5 \)

**SOLUTION:**

Identify the domain.

Write the radicand as greater than or equal to 0.

\[
x - 1 \geq 0
\]

\[
x \geq 1
\]

Make a table of values for \( x \geq 1 \) and graph the function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5.75</td>
</tr>
<tr>
<td>3</td>
<td>6.06</td>
</tr>
<tr>
<td>4</td>
<td>6.30</td>
</tr>
<tr>
<td>5</td>
<td>6.5</td>
</tr>
<tr>
<td>6</td>
<td>6.68</td>
</tr>
</tbody>
</table>

The domain is \( \{ x \mid x \geq -1 \} \), and the range is \( \{ f(x) \mid f(x) \geq 5 \} \).

**ANSWER:**

\[
D = \{ x \mid x \geq 1 \} ; R = \{ f(x) \mid f(x) \geq 5 \}
\]

36. \( f(x) = -\frac{1}{3} \sqrt{x + 4} - 1 \)

**SOLUTION:**

Identify the domain.

Write the radicand as greater than or equal to 0.

\[
x + 4 \geq 0
\]

\[
x \geq -4
\]

Make a table of values for \( x \geq -4 \) and graph the function.
Choose a word or term that best completes each statement.

1. If both compositions result in the ______, then the solution is ______.

   **ANSWER:** no solution

72. **ANSWER:**

73. **SOLUTION:**

   What is the radius of a circle with a diameter of 18 units and a circumference of 56 units?

   **ANSWER:**

   The radius of the circle is about 9.8 in.

37. **GEOMETRY** The area of a circle is given by the formula \( A = \pi r^2 \). What is the radius of a circle with an area of 300 square inches?

   **SOLUTION:**

   Substitute 300 for \( A \) in the formula and solve for \( r \).

   \[
   \begin{align*}
   300 &= \pi r^2 \\
   \frac{300}{\pi} &= r^2 \\
   \sqrt{\frac{300}{\pi}} &= r \\
   9.8 &\approx r
   \end{align*}
   \]

   The radius of the circle is about 9.8 in.

   **ANSWER:**

   about 9.8 in.

---

### Table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-1</td>
</tr>
<tr>
<td>-3</td>
<td>-1.33</td>
</tr>
<tr>
<td>-2</td>
<td>-1.47</td>
</tr>
<tr>
<td>-1</td>
<td>-1.58</td>
</tr>
<tr>
<td>0</td>
<td>-1.67</td>
</tr>
<tr>
<td>1</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

### Graph

The domain is \( \{ x \mid x \geq -4 \} \), and the range is \( \{ f(x) \mid f(x) \leq -1 \} \).

**ANSWER:**

\[
D = \{ x \mid x \geq -4 \}; R = \{ f(x) \mid f(x) \leq -1 \}
\]
Graph each inequality.

38. \( y \geq \sqrt{x} + 3 \)

**SOLUTION:**

```
\[ y \geq \sqrt{x} + 3 \]
```

**ANSWER:**

```
\[ y \geq \sqrt{x} + 3 \]
```

39. \( y > -\sqrt{x} - 1 + 2 \)

**SOLUTION:**

```
\[ y > -\sqrt{x} - 1 + 2 \]
```

**ANSWER:**

```
\[ y > -\sqrt{x} - 1 + 2 \]
```
40. \( y < 2\sqrt{x - 5} \)

SOLUTION:

\[ y < 2\sqrt{x - 5} \]

\[ y < 2\sqrt{5} \]

\[ y < 10 \]

ANSWER: 

41. \( \pm \sqrt{121} \)

\[ \pm \sqrt{121} = \pm \sqrt{11^2} \]

\[ = \pm 11 \]

SOLUTION:

\[ \pm \sqrt{121} = \pm \sqrt{11^2} \]

\[ = \pm 11 \]

ANSWER: 

42. \( \sqrt[3]{-125} \)

\[ \sqrt[3]{-125} = \sqrt[3]{(-5)^3} \]

\[ = -5 \]

SOLUTION:

\[ \sqrt[3]{-125} = \sqrt[3]{(-5)^3} \]

\[ = -5 \]

ANSWER: 

43. \( \sqrt{(-6)^2} \)

\[ \sqrt{(-6)^2} = \sqrt{36} \]

\[ = 6 \]

SOLUTION:

\[ \sqrt{(-6)^2} = \sqrt{36} \]

\[ = 6 \]

ANSWER: 

44. \( \sqrt{-(x + 3)^4} \)

\[ \sqrt{-(x + 3)^4} = \sqrt{-1(x + 3)^4} \]

\[ = i(x + 3)^2 \]

SOLUTION:

\[ \sqrt{-(x + 3)^4} = \sqrt{-1(x + 3)^4} \]

\[ = i(x + 3)^2 \]

ANSWER: 

\[ i(x + 3)^2 \]
45. \( \sqrt[6]{(x^2 + 2)^{18}} \)

SOLUTION:
\[
\sqrt[6]{(x^2 + 2)^{18}} = (x^2 + 2)^{18/6} = (x^2 + 2)^3
\]

ANSWER: \((x^2 + 2)^3\)

46. \( \sqrt[3]{27(x + 3)^3} \)

SOLUTION:
\[
\sqrt[3]{27(x + 3)^3} = \sqrt[3]{3^3(x + 3)^3} = 3(x + 3)
\]

ANSWER: \(3(x + 3)\)

47. \( \sqrt[4]{a^8b^{12}} \)

SOLUTION:
\[
\sqrt[4]{a^8b^{12}} = \sqrt[4]{(a^2)^4(b^3)^4} = a^2 |b^3|
\]

ANSWER: \(a^2 |b^3|\)

48. \( \sqrt[5]{243x^{10}y^{25}} \)

SOLUTION:
\[
\sqrt[5]{243x^{10}y^{25}} = \sqrt[5]{3^5(x^2)^5(y^5)^5} = 3x^2y^5
\]

ANSWER: \(3x^2y^5\)

49. PHYSICS The velocity \( v \) of an object can be defined as \( v = \sqrt{\frac{2K}{m}} \), where \( m \) is the mass of an object and \( K \) is the kinetic energy in joules. Find the velocity in meters per second of an object with a mass of 17 grams and a kinetic energy of 850 joules.

SOLUTION:
Substitute 17 for \( m \) and 850 for \( K \) in the formula and solve for \( v \):
\[
v = \sqrt{\frac{2(850)}{17}} = \sqrt{100} = 10
\]

The velocity of the object is 10 m/s.

ANSWER: 10 m/s
50. $\sqrt[3]{54}$

**SOLUTION:**

$$\sqrt[3]{54} = \sqrt[3]{3^2 \cdot 2}$$

$$= 3\sqrt{2}$$

**ANSWER:**

$3\sqrt{2}$

51. $\sqrt[4]{144a^3b^5}$

**SOLUTION:**

$$\sqrt[4]{144a^3b^5} = \sqrt[4]{(12ab)^2 \cdot ab}$$

$$= 12ab^2 \sqrt{ab}$$

**ANSWER:**

$12ab^2 \sqrt{ab}$

52. $4 \sqrt[4]{6y \cdot 3 \sqrt{7x^2y}}$

**SOLUTION:**

$$4 \sqrt[4]{6y \cdot 3 \sqrt{7x^2y}} = 12 \sqrt[4]{6y \cdot (7x^2y)}$$

$$= 12 \sqrt[4]{42x^2y^2}$$

$$= 12 |x| y \sqrt{42}$$

**ANSWER:**

$12 |x| y \sqrt{42}$

53. $6\sqrt{72} + 7\sqrt{98} - \sqrt{50}$

**SOLUTION:**

$$6\sqrt{72} + 7\sqrt{98} - \sqrt{50} = 6\sqrt{36 \cdot 2} + 7\sqrt{49 \cdot 2} - \sqrt{25 \cdot 2}$$

$$= 36\sqrt{2} + 49\sqrt{2} - 5\sqrt{2}$$

$$= (36 + 49 - 5)\sqrt{2}$$

$$= 80\sqrt{2}$$

**ANSWER:**

$80\sqrt{2}$

54. $(6\sqrt{5} - 2\sqrt{2})(3\sqrt{5} + 4\sqrt{2})$

**SOLUTION:**

$$(6\sqrt{5} - 2\sqrt{2})(3\sqrt{5} + 4\sqrt{2}) = 18 \cdot 5 + 24\sqrt{10} - 6\sqrt{10} - 8 \cdot 2$$

$$= 74 + 18\sqrt{10}$$

**ANSWER:**

$74 + 18\sqrt{10}$
55. \[ \frac{\sqrt{6m^5}}{\sqrt{p^{11}}} \]

**SOLUTION:**
\[
\frac{\sqrt{6m^5}}{\sqrt{p^{11}}} = \frac{m^2 \sqrt{6m}}{p^5 \sqrt{p}} \cdot \frac{\sqrt{p}}{\sqrt{p}} = \frac{m^2 \sqrt{6mp}}{p^6}
\]

**ANSWER:**
\[ \frac{m^2 \sqrt{6mp}}{p^6} \]

56. \[ \frac{3}{5 + \sqrt{2}} \]

**SOLUTION:**
\[
\frac{3}{5 + \sqrt{2}} = \frac{3}{5 + \sqrt{2}} \cdot \frac{5 - \sqrt{2}}{5 - \sqrt{2}} = \frac{3(5 - \sqrt{2})}{25 - 2} = \frac{15 - 3\sqrt{2}}{23}
\]

**ANSWER:**
\[ \frac{15 - 3\sqrt{2}}{23} \]

57. \[ \frac{\sqrt{3}}{\sqrt{5} - \sqrt{6}} \]

**SOLUTION:**
\[
\frac{\sqrt{3}}{\sqrt{5} - \sqrt{6}} = \frac{\sqrt{3} \cdot (\sqrt{5} + \sqrt{6})}{\sqrt{5} - \sqrt{6} \cdot (\sqrt{5} + \sqrt{6})} = \frac{\sqrt{15} + \sqrt{18}}{5 - 6} = -\sqrt{15} - 3\sqrt{2}
\]

**ANSWER:**
\[ -\sqrt{15} - 3\sqrt{2} \]
58. GEOMETRY What are the perimeter and the area of the rectangle?

\[
\begin{array}{c}
6 - \sqrt{2} \\
8 + \sqrt{3}
\end{array}
\]

**SOLUTION:**
Substitute \(8 + \sqrt{3}\) for length and \(6 - \sqrt{2}\) for width in the perimeter formula and simplify.

\[
\text{Perimeter} = 2 \left(8 + \sqrt{3} + 6 - \sqrt{2}\right) \\
= 16 + 2\sqrt{3} + 12 - 2\sqrt{2} \\
= 28 + 2\sqrt{3} - 2\sqrt{2}
\]

The perimeter of the rectangle is \(28 + 2\sqrt{3} - 2\sqrt{2}\) units.

\[
\text{Area} = (8 + \sqrt{3})(6 - \sqrt{2}) \\
= 48 + 6\sqrt{3} - 8\sqrt{2} - \sqrt{6}
\]

The area of the rectangle is \(48 + 6\sqrt{3} - 8\sqrt{2} - \sqrt{6}\) square units.

**ANSWER:**
perimeter = \(28 + 2\sqrt{3} - 2\sqrt{2}\) units;
area = \(48 + 6\sqrt{3} - 8\sqrt{2} - \sqrt{6}\) units²
61. \( \frac{\frac{1}{3} d^6}{d^4} \)

\[ \text{SOLUTION:} \]
\[ \frac{\frac{1}{3} d^6}{d^4} = \frac{d^8}{d^4} \]
\[ = d^{\frac{7}{4}} \]
\[ = \frac{1}{\frac{7}{5}} \cdot \frac{d^{\frac{12}{5}}}{d^{\frac{5}{12}}} \]
\[ = \frac{\frac{5}{1}}{d} \]

\[ \text{ANSWER:} \]
\[ \frac{5}{d^{\frac{12}{5}}} \]

Simplify each expression.

62. \( \frac{1}{y^4} \)

\[ \text{SOLUTION:} \]
\[ \frac{1}{y^4} = \frac{1}{y^4} \cdot \frac{y^3}{y^3} \]
\[ = \frac{y^4}{y} \]

\[ \text{ANSWER:} \]
\[ \frac{3}{y^4} \]

63. \( \sqrt[3]{729} \)

\[ \text{SOLUTION:} \]
\[ \sqrt[3]{729} = \frac{3}{27} \]
\[ = \frac{3}{3^3} \]
\[ = 3 \]

\[ \text{ANSWER:} \]
\[ 3 \]

64. \( \frac{2}{x^3 - x^3 y^3} \)

\[ \text{SOLUTION:} \]
\[ \frac{2}{x^3 - x^3 y^3} = \frac{2}{1} \cdot \frac{\frac{1}{x^3} - \frac{1}{y^3}}{x^3} \]
\[ = \frac{1}{x^3} - \frac{2}{y^3} \]

\[ \text{ANSWER:} \]
\[ \frac{1}{x^3} - \frac{2}{y^3} \]
65. **GEOMETRY** What is the area of the circle?

\[ r = 2a^2b^2c \]

**SOLUTION:**
Substitute \( 2a^2b^2c \) for \( r \) in the area of circle formula and simplify:

\[
\text{Area} = \pi \left( \frac{1}{2} \right)^2 \left( 2a^2b^5c \right)
\]
\[
= \pi \left( \frac{1}{4} a^2b^5c^2 \right)
\]

So, the area of the circle is \( 4a^2b^5c \pi \) units\(^2\).

**ANSWER:**
\[
4a^2b^5c \pi \text{ units}^2
\]

**Solve each equation.**

66. \( \sqrt{x - 3} + 5 = 15 \)

**SOLUTION:**
\[
\sqrt{x - 3} + 5 = 15
\]
\[
\left( \sqrt{x - 3} \right)^2 = 10^2
\]
\[
x - 3 = 100
\]
\[
x = 103
\]

**ANSWER:**
103

67. \( -\sqrt{x - 11} = 3 - \sqrt{x} \)

**SOLUTION:**
\[
\left( -\sqrt{x - 11} \right)^2 = (3 - \sqrt{x})^2
\]
\[
x - 11 = 9 - 6\sqrt{x} + x
\]
\[
20 = 6\sqrt{x}
\]
\[
\left( \frac{10}{3} \right)^2 = (\sqrt{x})^2
\]
\[
\frac{100}{9} = x
\]

**ANSWER:**
\[
\frac{100}{9}
\]

68. \( 4 + \sqrt{3x - 1} = 8 \)

**SOLUTION:**
\[
4 + \sqrt{3x - 1} = 8
\]
\[
\left( \sqrt{3x - 1} \right)^2 = 4^2
\]
\[
3x - 1 = 16
\]
\[
3x = 17
\]
\[
x = \frac{17}{3}
\]

**ANSWER:**
\[
\frac{17}{3}
\]
69. \( \sqrt{m+3} = \sqrt{2m+1} \)

**SOLUTION:**
\[
\left( \sqrt{m+3} \right)^2 = \left( \sqrt{2m+1} \right)^2 \\
m + 3 = 2m + 1 \\
m = 2
\]

**ANSWER:**
2

70. \( \sqrt{2x+3} = 3 \)

**SOLUTION:**
\[
\left( \sqrt{2x+3} \right)^2 = 3^2 \\
2x + 3 = 9 \\
x = 3
\]

**ANSWER:**
3

71. \( (x+1)^\frac{1}{4} = -3 \)

**SOLUTION:**
\[
(x + 1)^\frac{1}{4} = -3 \\
\left( x + 1 \right)^\frac{1}{4} = (-3)^4 \\
x + 1 = 81 \\
x = 80
\]

**CHECK:**
\[
(x + 1)^\frac{1}{4} = -3 \\
(80 + 1)^\frac{1}{4} \neq -3 \\
\sqrt[4]{81} \neq -3 \\
3 \neq -3
\]

no solution

**ANSWER:**
no solution
72. \( a^3 - 4 = 0 \)

**SOLUTION:**

\[
\begin{align*}
\frac{1}{3} a^3 - 4 &= 0 \\
\left( \frac{1}{3} a \right)^3 &= 4^3 \\
a &= 64
\end{align*}
\]

**ANSWER:**

64

73. \( 3 \left(3x - 1\right)^{\frac{1}{3}} - 6 = 0 \)

**SOLUTION:**

\[
\begin{align*}
3 \left(3x - 1\right)^{\frac{1}{3}} &= 6 \\
\left(3x - 1\right)^{\frac{1}{3}} &= 2 \\
3x - 1 &= 8 \\
x &= 3
\end{align*}
\]

**ANSWER:**

3

74. **PHYSICS** The formula \( t = 2\pi \sqrt{\frac{\ell}{32}} \) represents the swing of a pendulum, where \( t \) is the time in seconds for the pendulum to swing back and forth and \( \ell \) is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 2.75 seconds.

**SOLUTION:**

Substitute 2.75 for \( t \) in the formula and solve.

\[
\begin{align*}
2.75 &= 2\pi \sqrt{\frac{\ell}{32}} \\
\frac{2.75}{2\pi} &= \sqrt{\frac{\ell}{32}} \\
\left(\frac{2.75}{2\pi}\right)^2 &= \frac{\ell}{32} \\
32 \left(\frac{2.75}{2\pi}\right)^2 &= \ell \\
\ell &\approx 6.13
\end{align*}
\]

The length of a pendulum is about 6.13 ft.

**ANSWER:**

about 6.13 ft
Solve each inequality.

75. \( 2 + \sqrt{3x - 1} < 5 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 3x - 1 \geq 0 \) to identify the values of \( x \) for which the left side of the inequality is defined.

\[
\begin{align*}
3x - 1 &\geq 0 \\
3x &\geq 1 \\
x &\geq \frac{1}{3}
\end{align*}
\]

Now, solve \( 2 + \sqrt{3x - 1} < 5 \).

\[
\begin{align*}
2 + \sqrt{3x - 1} &< 5 \\
\left(\sqrt{3x - 1}\right)^2 &< 3^2 \\
3x - 1 &< 9 \\
x &< \frac{10}{3}
\end{align*}
\]

The solution region is \( \frac{1}{3} \leq x < \frac{10}{3} \).

**ANSWER:**
\( \frac{1}{3} \leq x < \frac{10}{3} \)

76. \( \sqrt{3x + 13} - 5 \geq 5 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 3x + 13 \geq 0 \) to identify the values of \( x \) for which the left side of the inequality is defined.

\[
\begin{align*}
3x + 13 &\geq 0 \\
3x &\geq -13 \\
x &\geq -\frac{13}{3}
\end{align*}
\]

Now, solve \( \sqrt{3x + 13} - 5 \geq 5 \).

\[
\begin{align*}
\sqrt{3x + 13} - 5 &\geq 5 \\
\left(\sqrt{3x + 13}\right)^2 &\geq 10^2 \\
3x + 13 &\geq 100 \\
3x &\geq 87 \\
x &\geq 29
\end{align*}
\]

The solution region is \( x \geq 29 \).

**ANSWER:**
\( x \geq 29 \)
77. \( 6 - \sqrt{3x + 5} \leq 3 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( 3x + 5 \geq 0 \) to identify the values of \( x \) for which the left side of the inequality is defined.

\[
3x + 5 \geq 0 \\
3x \geq -5 \\
x \geq \frac{-5}{3}
\]

Now, solve \( 6 - \sqrt{3x + 5} \leq 3 \).

\[
6 - \sqrt{3x + 5} \leq 3 \\
\left( \sqrt{3x + 5} \right)^2 \geq 3^2 \\
3x + 5 \geq 9 \\
x \geq \frac{4}{3}
\]

The solution region is \( x \geq \frac{4}{3} \).

**ANSWER:**
\( x \geq \frac{4}{3} \)

78. \( \sqrt{-3x + 4} - 5 \geq 3 \)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \( -3x + 4 \geq 0 \) to identify the values of \( x \) for which the left side of the inequality is defined.

\[
-3x + 4 \geq 0 \\
-3x \geq -4 \\
x \leq \frac{4}{3}
\]

Now, solve \( \sqrt{-3x + 4} - 5 \geq 3 \).

\[
\sqrt{-3x + 4} - 5 \geq 3 \\
\left( \sqrt{-3x + 4} \right)^2 \geq 8^2 \\
-3x + 4 \geq 64 \\
x \leq -20
\]

The solution region is \( x \leq -20 \).

**ANSWER:**
\( x \leq -20 \)
79. \(5 + \sqrt{2y-7} < 5\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(2y - 7 \geq 0\) to identify the values of \(x\) for which the left side of the inequality is defined.

\[
2y - 7 \geq 0 \\
2y \geq 7 \\
y \geq \frac{7}{2}
\]

Now, solve \(5 + \sqrt{2y-7} < 5\).

\[
5 + \sqrt{2y-7} < 5 \\
(\sqrt{2y-7})^2 < 0 \\
2y - 7 < 0 \\
y < \frac{7}{2}
\]

Since there is no common region between the inequalities \(y \geq \frac{7}{2}\) and \(y < \frac{7}{2}\), the inequality \(5 + \sqrt{2y-7} < 5\) has no solution.

**ANSWER:**
no solution

80. \(3 + \sqrt{2x-3} \geq 3\)

**SOLUTION:**
Since the radicand of a square root must be greater than or equal to zero, first solve \(2x - 3 \geq 0\) to identify the values of \(x\) for which the left side of the inequality is defined.

\[
2x - 3 \geq 0 \\
2x \geq 3 \\
x \geq \frac{3}{2}
\]

Now, solve \(3 + \sqrt{2x-3} \geq 3\).

\[
3 + \sqrt{2x-3} \geq 3 \\
(\sqrt{2x-3})^2 \geq 0 \\
2x - 3 \geq 0 \\
x \geq \frac{3}{2}
\]

The solution region is \(x \geq \frac{3}{2}\).

**ANSWER:**
\[x \geq \frac{3}{2}\]
81. \( \sqrt{3x+1} - \sqrt{6+x} > 0 \)

\[ \begin{align*}
\text{SOLUTION:} & \\
\sqrt{3x+1} - \sqrt{6+x} & > 0 \\
(\sqrt{3x+1})^2 - (\sqrt{6+x})^2 & > 0 \\
3x + 1 - (6 + x) & > 0 \\
2x & > 5 \\
x & > \frac{5}{2}
\end{align*} \]

\[ \text{ANSWER:} \]
\[ x > \frac{5}{2} \]