7-1 Graphing Exponential Functions

Graph each function. State the domain and range.

1. \( f(x) = 2^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0.25</td>
</tr>
<tr>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Domain = \( \{ \text{all real numbers} \} \); Range = \( \{ f(x) \mid f(x) > 0 \} \)

**ANSWER:**

\[ f(x) = 5^x \]

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0.008</td>
</tr>
<tr>
<td>-2</td>
<td>0.04</td>
</tr>
<tr>
<td>-1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>125</td>
</tr>
</tbody>
</table>

Domain = \( \{ \text{all real numbers} \} \); Range = \( \{ f(x) \mid f(x) > 0 \} \)

**ANSWER:**
3. \( f(x) = 3^x - 2 + 4 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>4.004</td>
</tr>
<tr>
<td>-2</td>
<td>4.012</td>
</tr>
<tr>
<td>-1</td>
<td>4.037</td>
</tr>
<tr>
<td>0</td>
<td>4.111</td>
</tr>
<tr>
<td>1</td>
<td>4.333</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > 4 \)\}

**ANSWER:**

4. \( f(x) = 2^x + 1 + 3 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3.25</td>
</tr>
<tr>
<td>-2</td>
<td>3.5</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > 3 \)\}

**ANSWER:**
7-1 Graphing Exponential Functions

5. $f(x) = 0.25(4)^x - 6$

*Solution:*
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5.984</td>
</tr>
<tr>
<td>-1</td>
<td>-5.938</td>
</tr>
<tr>
<td>0</td>
<td>-5.75</td>
</tr>
<tr>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

Domain = {all real numbers}; Range = {$f(x) \mid f(x) > -6$}

*Answer:*

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>8.375</td>
</tr>
<tr>
<td>-2</td>
<td>8.75</td>
</tr>
<tr>
<td>-1</td>
<td>9.5</td>
</tr>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

Domain = {all real numbers}; Range = {$f(x) \mid f(x) > 8$}

*Answer:*

6. $f(x) = 3(2)^x + 8$

*Solution:*
Make a table of values. Then plot the points and sketch the graph.
7. CCSS SENSE-MAKING A virus spreads through a network of computers such that each minute, 25% more computers are infected. If the virus began at only one computer, graph the function for the first hour of the spread of the virus.

**SOLUTION:**

\[ a = 1 \text{ and } r = 0.25 \]
So, the equation that represents the situation is \( y = 1.25^x \).

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( y = 1.25^{10} \approx 9 )</td>
</tr>
<tr>
<td>20</td>
<td>( y = 1.25^{20} \approx 87 )</td>
</tr>
<tr>
<td>30</td>
<td>( y = 1.25^{30} \approx 808 )</td>
</tr>
<tr>
<td>40</td>
<td>( y = 1.25^{40} \approx 7523 )</td>
</tr>
<tr>
<td>50</td>
<td>( y = 1.25^{50} \approx 70065 )</td>
</tr>
<tr>
<td>60</td>
<td>( y = 1.25^{60} \approx 652530 )</td>
</tr>
</tbody>
</table>

**ANSWER:**

Graph each function. State the domain and range.

8. \( f(x) = 2 \left( \frac{2}{3} \right)^{x-3} - 4 \)

**SOLUTION:**

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6.125</td>
</tr>
<tr>
<td>0</td>
<td>2.75</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>6</td>
<td>-3.407</td>
</tr>
<tr>
<td>9</td>
<td>-3.824</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \( \{ f(x) \mid f(x) > -4 \} \)

**ANSWER:**

\( D = \{ \text{all real numbers} \}; R = \{ f(x) \mid f(x) > -4 \} \)
9. \( f(x) = -\frac{1}{2} \left( \frac{3}{4} \right)^{x+1} + 5 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>1.254</td>
</tr>
<tr>
<td>-4</td>
<td>3.815</td>
</tr>
<tr>
<td>0</td>
<td>4.625</td>
</tr>
<tr>
<td>4</td>
<td>4.881</td>
</tr>
<tr>
<td>8</td>
<td>4.963</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \) | \( f(x) < 5 \)\}

**ANSWER:**

\[ f(x) = -\frac{1}{2} \left( \frac{3}{4} \right)^{x+1} + 5 \]

D = \{all real numbers\}; R = \{\( f(x) \) | \( f(x) < 5 \)\}

10. \( f(x) = -\left( \frac{4}{5} \right)^{x-4} + 3 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>-1.851</td>
</tr>
<tr>
<td>-4</td>
<td>1.013</td>
</tr>
<tr>
<td>0</td>
<td>2.186</td>
</tr>
<tr>
<td>2</td>
<td>2.479</td>
</tr>
<tr>
<td>4</td>
<td>2.667</td>
</tr>
<tr>
<td>8</td>
<td>2.864</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \) | \( f(x) < 3 \)\}

**ANSWER:**

\[ f(x) = -\left( \frac{4}{5} \right)^{x-4} + 3 \]

D = \{all real numbers\}; R = \{\( f(x) \) | \( f(x) < 3 \)\}
11. \( f(x) = \frac{1}{8} \left( \frac{1}{4} \right)^{x+6} + 7 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>9</td>
</tr>
<tr>
<td>-4</td>
<td>7.008</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > 7 \}\}

**ANSWER:**

Graph of the SUV’s value for the first 20 years after the initial purchase:

12. **FINANCIAL LITERACY** A new SUV depreciates in value each year by a factor of 15%.

Draw a graph of the SUV’s value for the first 20 years after the initial purchase.
Graph each function. State the domain and range.

13. \( f(x) = 2(3)^x \)

**SOLUTION:**

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0.22</td>
</tr>
<tr>
<td>-1</td>
<td>0.67</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>54</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{ \( f(x) \mid f(x) > 0 \) \}

**ANSWER:**

D = \{all real numbers\}; R = \{ \( f(x) \mid f(x) > 0 \) \}
7-1 Graphing Exponential Functions

14. \( f(x) = -2(4)^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-0.03</td>
</tr>
<tr>
<td>-2</td>
<td>-0.125</td>
</tr>
<tr>
<td>-1</td>
<td>-0.5</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-8</td>
</tr>
<tr>
<td>2</td>
<td>-32</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{ \( f(x) \mid f(x) < 0 \) \}

**ANSWER:**

\[ \text{D} = \{ \text{all real numbers} \}; \text{R} = \{ f(x) \mid f(x) < 0 \} \]

15. \( f(x) = 4^{x + 1} - 5 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-4.94</td>
</tr>
<tr>
<td>-2</td>
<td>-4.75</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>59</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{ \( f(x) \mid f(x) > -5 \) \}

**ANSWER:**

\[ \text{D} = \{ \text{all real numbers} \}; \text{R} = \{ f(x) \mid f(x) > -5 \} \]
16. \( f(x) = 3^{2x} + 1 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>1.012</td>
</tr>
<tr>
<td>-1</td>
<td>1.111</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>82</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > 1 \}\}

**ANSWER:**

17. \( f(x) = -0.4(3)^x + 2 + 4 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3.867</td>
</tr>
<tr>
<td>-2</td>
<td>3.6</td>
</tr>
<tr>
<td>-1</td>
<td>2.8</td>
</tr>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>-6.8</td>
</tr>
<tr>
<td>2</td>
<td>-28.4</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) < 4 \}\}

**ANSWER:**
7-1 Graphing Exponential Functions

18. \( f(x) = 1.5(2)^x + 6 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>6.375</td>
</tr>
<tr>
<td>-1</td>
<td>6.75</td>
</tr>
<tr>
<td>0</td>
<td>7.5</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{f(x) | f(x) > 6\}

**ANSWER:**

D = \{all real numbers\}; R = \{f(x) | f(x) > 6\}

19. **SCIENCE** The population of a colony of beetles grows 30% each week for 10 weeks. If the initial population is 65 beetles, graph the function that represents the situation.

**SOLUTION:**
\( a = 65 \) and \( r = 0.3 \).
So, the equation that represents the situation is \( y = 65(1.3)^t \).
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 65(1.3)^0 = 65 )</td>
</tr>
<tr>
<td>2</td>
<td>( 65(1.3)^2 \approx 110 )</td>
</tr>
<tr>
<td>4</td>
<td>( 65(1.3)^4 \approx 186 )</td>
</tr>
<tr>
<td>6</td>
<td>( 65(1.3)^6 \approx 314 )</td>
</tr>
<tr>
<td>8</td>
<td>( 65(1.3)^8 \approx 530 )</td>
</tr>
<tr>
<td>10</td>
<td>( 65(1.3)^{10} \approx 896 )</td>
</tr>
</tbody>
</table>
20. \( f(x) = -4\left( \frac{3}{5} \right)^{x+4} + 3 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>1.56</td>
</tr>
<tr>
<td>0</td>
<td>2.481</td>
</tr>
<tr>
<td>2</td>
<td>2.813</td>
</tr>
<tr>
<td>4</td>
<td>2.932</td>
</tr>
<tr>
<td>6</td>
<td>2.976</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) < 3 \)\}

**ANSWER:**

\( f(x) = 3\left( \frac{2}{5} \right)^{x-3} - 6 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>-4.8</td>
</tr>
<tr>
<td>6</td>
<td>-5.81</td>
</tr>
<tr>
<td>8</td>
<td>-5.97</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > -6 \)\}

**ANSWER:**

D = \{all real numbers\}; R = \{\( f(x) \mid f(x) > -6 \)\}
7-1 Graphing Exponential Functions

22. \( f(x) = \frac{1}{2}(\frac{1}{5})^{x+5} + 8 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>70.5</td>
</tr>
<tr>
<td>-6</td>
<td>10.5</td>
</tr>
<tr>
<td>-4</td>
<td>8.1</td>
</tr>
<tr>
<td>-2</td>
<td>8.004</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > 8 \)\}

**ANSWER:**

23. \( f(x) = \frac{3}{4}(\frac{2}{3})^{x+4} - 2 \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>1.80</td>
</tr>
<tr>
<td>-6</td>
<td>-0.3125</td>
</tr>
<tr>
<td>-4</td>
<td>-1.25</td>
</tr>
<tr>
<td>-2</td>
<td>-1.67</td>
</tr>
<tr>
<td>0</td>
<td>-1.851</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) > -2 \)\}

**ANSWER:**
24. \( f(x) = \frac{-1}{2} \left( \frac{3}{8} \right)^{x+2} + 9 \)

**SOLUTION:**

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-16.28</td>
</tr>
<tr>
<td>-4</td>
<td>5.44</td>
</tr>
<tr>
<td>-2</td>
<td>8.5</td>
</tr>
<tr>
<td>0</td>
<td>8.93</td>
</tr>
<tr>
<td>2</td>
<td>8.99</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) < 9 \)\}

**ANSWER:**

25. \( f(x) = -\frac{5}{4} \left( \frac{4}{5} \right)^{x+4} + 2 \)

**SOLUTION:**

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9</td>
<td>-1.81</td>
</tr>
<tr>
<td>-6</td>
<td>0.047</td>
</tr>
<tr>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1.488</td>
</tr>
<tr>
<td>3</td>
<td>1.74</td>
</tr>
<tr>
<td>6</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Domain = \{all real numbers\}; Range = \{\( f(x) \mid f(x) < 2 \)\}

**ANSWER:**

26. **ATTENDANCE** The attendance for a basketball team declined at a rate of 5% per game throughout a losing season. Graph the function modeling the attendance if 15 home games were played and 23,500 people were at the first game.

**SOLUTION:**
7-1 Graphing Exponential Functions

\[ a = 23,500 \text{ and } r = 0.05. \]

So, the equation that represents the situation is \( y = 23,500(0.95)^x \).

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = 23,500(0.95)^0 = 23,500 )</td>
</tr>
<tr>
<td>3</td>
<td>( y = 23,500(0.95)^3 \approx 20,148 )</td>
</tr>
<tr>
<td>6</td>
<td>( y = 23,500(0.95)^6 \approx 17,275 )</td>
</tr>
<tr>
<td>9</td>
<td>( y = 23,500(0.95)^9 \approx 14,811 )</td>
</tr>
<tr>
<td>12</td>
<td>( y = 23,500(0.95)^{12} \approx 12,698 )</td>
</tr>
<tr>
<td>15</td>
<td>( y = 23,500(0.95)^{15} \approx 10,887 )</td>
</tr>
</tbody>
</table>

**ANSWER:**

The function \( y = 23,500(0.95)^x \) represents the number of pay phones in 1999.

b. The \( P(x) \)-intercept represents the number of pay phones in 1999. The asymptote is the \( x \)-axis. The number of pay phones can approach 0, but will never equal 0. This makes sense as there will probably always be a need for some pay phones.

**ANSWER:**

a. decay; 0.9
**7-1 Graphing Exponential Functions**

The $P(x)$-intercept represents the number of pay phones in 1999. The asymptote is the $x$-axis. The number of pay phones can approach 0, but will never equal 0. This makes sense as there will probably always be a need for some pay phones.

**28. HEALTH** Each day, 10% of a certain drug dissipates from the system.

- **a.** Classify the function representing this situation as either exponential growth or decay, and identify the growth or decay factor. Then graph the function.
- **b.** How much of the original amount remains in the system after 9 days?
- **c.** If a second dose should not be taken if more than 50% of the original amount is in the system, when should the label say it is safe to redose? Design the label and explain your reasoning.

**SOLUTION:**

- **a.** Since the amount of the drug in the system is decreasing, this is an example of exponential decay.

  Use the equation form $y = a(1 - r)^x$ with $a = 100$ and $r = 0.1$ to model the amount of drug still in the system. Then the equation that represents the situation is $y = 100(0.9)^x$. The rate of decay is 0.9 because it is the base of the exponential expression.

  Make a table of values. Then plot the points and sketch the graph.

- **b.** After the 9th day a little less than 40% of the original amount remains in the system.
- **c.** From the graph, a little less than 50% of the original amount is still in the system after 7 days. So, it is safe to redose on the 7th day.

**ANSWER:**

- **a.** decay; 0.9
- **b.** a little less than 40%
- **c.** Sample answer: The 7th day; see students' work.

**29. CCSS REASONING** A sequence of numbers follows a pattern in which the next number is 125% of the previous number. The first number in the
7-1 Graphing Exponential Functions

pattern is 18.

a. Write the function that represents the situation.

b. Classify the function as either exponential growth or decay, and identify the growth or decay factor. Then graph the function for the first 10 numbers.

c. What is the value of the tenth number? Round to the nearest whole number.

SOLUTION:

a. Write an exponential function that has an initial value of 18, a base of 1.25, and an exponent of \( x - 1 \) where \( x \) is the position of the number in the list. So, the function representing this situation is \( f(x) = 18(1.25)^{x-1} \).

b. Since the numbers will be increasing, this is an example of exponential growth. The rate of growth is 1.25 because it is the base of the exponent.

Make a table of values of \( f(x) = 18(1.25)^{x-1} \). Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = 18(1.25)^0 ) ( \approx 14 )</td>
</tr>
<tr>
<td>2</td>
<td>( y = 18(1.25)^2 ) ( \approx 23 )</td>
</tr>
<tr>
<td>4</td>
<td>( y = 18(1.25)^4 ) ( \approx 35 )</td>
</tr>
<tr>
<td>6</td>
<td>( y = 18(1.25)^6 ) ( \approx 55 )</td>
</tr>
<tr>
<td>8</td>
<td>( y = 18(1.25)^8 ) ( \approx 86 )</td>
</tr>
<tr>
<td>9</td>
<td>( y = 18(1.25)^9 ) ( \approx 107 )</td>
</tr>
</tbody>
</table>

\[ f(10) = 18(1.25)^{10-1} = 18(1.25)^9 \approx 134 \]

ANSWER:

a. \( f(x) = 18(1.25)^{x-1} \)

b. growth; 1.25

c. 134

For each graph, \( f(x) \) is the parent function and \( g(x) \) is a transformation of \( f(x) \). Use the graph to determine the equation of \( g(x) \).

30. \( f(x) = 3^x \)

SOLUTION:
The graph of \( f(x) \) is translated 5 units up and 4 units right. Here, \( k = 5 \) and \( h = 4 \).

So, \( g(x) = 3^{x-4} + 5 \).

ANSWER:

\( g(x) = 3^{x-4} + 5 \)
7-1 Graphing Exponential Functions

31. \( f(x) = 2^x \)

\[ x \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\hline
\quad f(x) \quad 0.5 \quad 1 \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \\
\]

\textbf{SOLUTION:}

The graph of \( f(x) \) is compressed 4 units and translated 3 units right. Here, \( a = 4 \) and \( h = 3 \).

So, \( g(x) = 4(2)^{x-3} \) or \( g(x) = \frac{1}{2}(2^x) \).

\textbf{ANSWER:}

\( g(x) = 4(2)^{x-3} \) or \( g(x) = \frac{1}{2}(2^x) \)

32. \( f(x) = 4^x \)

\[ x \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
\hline
\quad f(x) \quad 0.25 \quad 1 \quad 4 \quad 16 \quad 64 \quad 256 \quad 1024 \\
\]

\textbf{SOLUTION:}

The graph of \( f(x) \) is reflected in the \( x \)-axis and expanded.

The graph is translated one unit left and 3 units up. Here, \( a = -2 \), \( h = -1 \) and \( k = 3 \).

So, \( g(x) = -2(4)^{x+1} + 3 \).

\textbf{ANSWER:}

\( g(x) = -2(4)^{x+1} + 3 \)

33. \textbf{MULTIPLE REPRESENTATIONS} In this problem, you will use the tables below for exponential functions \( f(x) \), \( g(x) \), and \( h(x) \).

\[ \begin{array}{cccccccc}
   x & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \hline
   a(x) & 5 & 11 & 23 & 47 & 95 & 191 & 383 \\
   \end{array} \]

\[ \begin{array}{cccccccc}
   x & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \hline
   b(x) & 3 & 2.5 & 2.25 & 2.125 & 2.0313 & 2.0056 \\
   \end{array} \]

\textbf{a. GRAPHICAL} Graph the functions for \(-1 \leq x \leq 5\) on separate graphs.

\textbf{b. LOGICAL} Which function(s) has a negative coefficient, \( a \)? Explain your reasoning.

\textbf{c. LOGICAL} Which function(s) is translated to the left?

\textbf{d. ANALYTICAL} Determine which functions are growth models and which are decay models.

\textbf{SOLUTION:}

\textbf{a.}

Plot the points given in the table and sketch the graph of \( f(x) \), \( g(x) \) and \( h(x) \).

\textbf{b.}

Sample answer: \( f(x) \); the graph of \( f(x) \) is a reflection along the \( x \)-axis and the output values in the table are negative.

\textbf{c.}

\( g(x) \) and \( h(x) \) are translated to the left.

\textbf{d.}

Sample answer: \( f(x) \) and \( g(x) \) are growth functions and \( h(x) \) is a decay function. The absolute value of the output is increasing for the growth functions and decreasing for the decay function.

\textbf{ANSWER:}

\textbf{a.}
Graph each function. State the domain and range.

1. \[ f(x) = 2x \]

SOLUTION:
Make a table of values. Then plot the points and sketch the graph.

3. Solve each equation or inequality.
43. \[ \text{SOLUTION:} \]

ANSWER: 8
44. \[ \text{SOLUTION:} \]

ANSWER: 8

35. CCSS CRITIQUE
Vince and Grady were asked to graph the following functions. Vince thinks they are the same, but Grady disagrees. Who is correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>0.0625</td>
</tr>
<tr>
<td>6</td>
<td>0.03125</td>
</tr>
</tbody>
</table>

SOLUTION:
First plot the points in the table.

Next, find and graph an equation that matches the description given: an exponential function with a rate of decay of \( \frac{1}{2} \) and an initial amount of 2.

Exponential decay can be modeled by the function \( A(t) = a(1 - r)^t \) where \( r \) is the rate of decay and \( a \) is the initial amount.

\[
A(t) = a(1 - r)^t
\]

\[
A(t) = 2 \left(1 - \frac{1}{2}\right)^t
\]

Graph this function on the coordinate plane.
37. **OPEN ENDED** Give an example of a value of \( b \) for which \( f(x) = \left( \frac{8}{b} \right)^x \) represents exponential decay.

**SOLUTION:**
Sample answer: For \( b = 10 \), the given function represents exponential decay.

**ANSWER:**
Sample answer: 10

38. **WRITING IN MATH** Write the procedure for transforming the graph of \( g(x) = b^x \) to the graph \( f(x) = ab^{x-h} + k \)

**SOLUTION:**
Sample answer: The parent function, \( g(x) = b^x \), is stretched if \( a \) is greater than 1 or compressed if \( a \) is less than 1 and greater than 0. The parent function is translated up \( k \) units if \( k \) is positive and down \(|k|\) units if \( k \) is negative. The parent function is translated \( h \) units to the right if \( h \) is positive and \(|h|\) units to the left if \( h \) is negative.

**ANSWER:**
Sample answer: The parent function, \( g(x) = b^x \), is stretched if \( a \) is greater than 1 or compressed if \( a \) is less than 1. The parent function is translated up \( k \) units if \( k \) is positive and down \(|k|\) units if \( k \) is negative. The parent function is translated \( h \) units to the right if \( h \) is positive and \(|h|\) units to the left if \( h \) is negative.
7-1 Graphing Exponential Functions

39. **GRIDDED RESPONSE** In the figure, \( PO \parallel RN \), \( ON = 12, MN = 6 \), and \( RN = 4 \). What is the length of \( PO \) ?

![Graph](image)

**SOLUTION:**
\( \Delta MRN \) and \( \Delta MPO \) are similar triangles.
Find the similarity ratio.

\[
\text{Similarity ratio} = \frac{MN}{MO} = \frac{6}{18} = \frac{1}{3}
\]

Length of \( PO \):

\[
\frac{1}{3} = \frac{RN}{OP} \quad \frac{1}{3} = \frac{4}{OP} \quad OP = 12
\]

**ANSWER:**
12

40. Ivan has enough money to buy 12 used CDs. If the cost of each CD was $0.20 less, Ivan could buy 2 more CDs. How much money does Ivan have to spend on CDs?

- **A** $16.80
- **B** $16.40
- **C** $15.80
- **D** $15.40

**SOLUTION:**
Let \( x \) be the cost of a CD.
The equation that represents the situation is

\[
12x = 14(x - 0.20)
\]

\[
12x = 14x - 2.8
\]

\[-2x = -2.8
\]

\[x = 1.4
\]

The cost of 12 CDs = \( 12 \times 1.4 \)

\[= 16.8
\]

Ivan has to spend $16.80 on CDs.
A is the correct choice.

**ANSWER:**
A
Graph each function. State the domain and range.

1. \( f(x) = 2x \)
   **SOLUTION:**
   Make a table of values. Then plot the points and sketch the graph.
   \( f(0) = 0 \), \( f(1) = 2 \), \( f(2) = 4 \), \( f(3) = 6 \), etc.
   The graph is a line with a slope of 2.
   Domain = \( \mathbb{R} \); Range = \( \mathbb{R} \).

42. **SAT/ACT** Javier mows a lawn in 2 hours. Tonya mows the same lawn in 1.5 hours. About how many minutes will it take to mow the lawn if Javier and Tonya work together?
   - A 28 minutes
   - B 42 minutes
   - C 51 minutes
   - D 1.2 hours
   - E 1.4 hours
   **SOLUTION:**
   Javier mows \( \frac{1}{2} \) the lawn in 1 hour.
   Tonya mows \( \frac{2}{3} \) the lawn in 1 hour.
   Working together, Javier and Tonya mow \( \frac{1}{2} + \frac{2}{3} = \frac{7}{6} \) lawns per hour.
   So, they mow the lawn in \( \frac{1}{\frac{7}{6}} = \frac{6}{7} \) of an hour.
   \( \frac{6}{7} \approx 0.857 \)
   Multiply by 60 to convert to minutes.
   \( 0.857 \times 60 \text{ min} = 51 \text{ min} \)
   The correct answer is C.
   **ANSWER:**
   C

Solve each equation or inequality.

43. \( \sqrt{y} + 5 = \sqrt{2y - 3} \)
   **SOLUTION:**
   \( (\sqrt{y} + 5)^2 = (\sqrt{2y - 3})^2 \)
   \( y + 5 = 2y - 3 \)
   \( 8 = y \)
   **ANSWER:**
   8
44. $\sqrt{y+1} + \sqrt{y-4} = 5$

**SOLUTION:**

$$\sqrt{y+1} + \sqrt{y-4} = 5$$

$$(\sqrt{y+1})^2 = (5 - \sqrt{y-4})^2$$

$$y + 1 = 25 + y - 4 - 10\sqrt{y-4}$$

$$(-20)^2 = (-10\sqrt{y-4})^2$$

$$400 = 100y - 400$$

$$8 = y$$

**ANSWER:**

8

45. $10 - \sqrt{2x+7} \leq 3$

**SOLUTION:**

$$10 - \sqrt{2x+7} \leq 3$$

$$-\sqrt{2x+7} \leq -7$$

$$(\sqrt{2x+7})^2 \geq 7^2$$

$$2x + 7 \geq 49$$

$$x \geq 21$$

**ANSWER:**

$x \geq 21$

46. $6 + \sqrt{3y+4} < 6$

**SOLUTION:**

$$6 + \sqrt{3y+4} < 6$$

$$\sqrt{3y+4} < 0$$

The square root of $3y + 4$ cannot be negative, so there is no solution.

**ANSWER:**

no solution

47. $\sqrt{d+3} + \sqrt{d+7} > 4$

**SOLUTION:**

$$\sqrt{d+3} + \sqrt{d+7} > 4$$

$$\left(\sqrt{d+3}\right)^2 > \left(4 - \sqrt{d+7}\right)^2$$

$$d + 3 > 16 + d + 7 - 8\sqrt{d+7}$$

$$-20 > -8\sqrt{d+7}$$

$$(-20)^2 < \left(8\sqrt{d+7}\right)^2$$

$$400 < 64(d + 7)$$

$$-48 < 64d$$

$$\frac{3}{4} < d$$

**ANSWER:**

$$d > \frac{3}{4}$$

48. $\sqrt{2x+5} - \sqrt{9+x} > 0$

**SOLUTION:**

$$\sqrt{2x+5} - \sqrt{9+x} > 0$$

$$\left(\sqrt{2x+5}\right)^2 > \left(\sqrt{9+x}\right)^2$$

$$2x + 5 > 9 + x$$

$$x > 4$$

**ANSWER:**

$x > 4$

49. $\frac{1}{\sqrt[5]{y^3}}$

**SOLUTION:**

$$\frac{1}{\sqrt[5]{y^3}} = \frac{\frac{1}{3} y^\frac{3}{5}}{\frac{3}{5} y^\frac{3}{5}} = \frac{y^5}{y}$$

**ANSWER:**

$$\frac{y^5}{y}$$
50. \(\frac{xy}{\sqrt{z}}\)

**SOLUTION:**

\[
\frac{xy}{\sqrt{z}} = \frac{xy}{\frac{1}{2}z^{\frac{1}{2}}}
\]

\[= \frac{xy \cdot z^{\frac{1}{2}}}{\frac{1}{2}z^{\frac{1}{2}}}
\]

\[= \frac{xyz^{\frac{1}{2}}}{z}
\]

**ANSWER:**

\[
\frac{2}{3}xyz^{\frac{3}{2}}
\]

51. \(\frac{3x + 4x^2}{x^2}\)

**SOLUTION:**

\[
\frac{3x + 4x^2}{x^2} = \frac{3}{x^2} + \frac{4x^2}{x^2}
\]

\[= 3(x^2 + 4x^2)
\]

\[= 3x^3 + 4x^3
\]

**ANSWER:**

\[
3x^3 + 4x^3
\]

52. \(\sqrt[3]{27x^3}\)

**SOLUTION:**

\[
\sqrt[3]{27x^3} = (3^3x^3)^{\frac{1}{3}}
\]

\[= 3x
\]

**ANSWER:**

\[
\sqrt[3]{3x}
\]

53. \(\frac{\sqrt{27}}{\sqrt{3}}\)

**SOLUTION:**

\[
\frac{\sqrt{27}}{\sqrt{3}} = \sqrt[3]{27}
\]

\[= \sqrt[3]{9}
\]

\[= (3^3)^{\frac{1}{3}}
\]

\[= \sqrt[3]{3}
\]

**ANSWER:**

\[
\sqrt[3]{3}
\]

54. \(\frac{a^{-\frac{1}{2}}}{6a^3a^{-\frac{1}{4}}}\)

**SOLUTION:**

\[
\frac{a^{-\frac{1}{2}}}{6a^3a^{-\frac{1}{4}}} = \frac{a^{-\frac{1}{2}}}{6a^{3 - \frac{1}{4}}}
\]

\[= \frac{a^{-\frac{1}{2}}}{6a^{\frac{11}{4}}}
\]

\[= \frac{a^{-\frac{1}{2}}}{6a^{\frac{11}{4}}}
\]

\[= \frac{a^{\frac{5}{12}}}{6a}
\]

**ANSWER:**

\[
\frac{a^{\frac{5}{12}}}{6a}
\]
55. **FOOTBALL** The path of a football thrown across a field is given by the equation \( y = -0.005x^2 + x + 5 \), where \( x \) represents the distance, in feet, the ball has traveled horizontally and \( y \) represents the height, in feet, of the ball above ground level. About how far has the ball traveled horizontally when it returns to ground level?

**SOLUTION:**
Substitute 0 for \( y \) in the equation and solve for \( x \).

\[-0.005x^2 + x + 5 = 0\]

Substitute these values into the Quadratic Formula and simplify.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-1 \pm \sqrt{1^2 - 4(-0.005)(5)}}{2(-0.005)}
\]

\[
= \frac{-1 \pm \sqrt{1.1}}{-0.01}
\]

\[
\approx -4.88 \text{ or } 204.88
\]

The distance cannot have negative value. So the ball has traveled about 204.88 ft horizontally.

**ANSWER:**
about 204.88 ft

56. **COMMUNITY SERVICE** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised?

**SOLUTION:**

Every two adults must bring at least one student, and there are a total of 300 seats available.

So, the maximum occupancy is 200 adults and 100 students.

Maximum amount = 200($2) + 100($1) = $500.

**ANSWER:**
$500

**Simplify. Assume that no variable equals 0.**

57. \( f^{-7} \cdot f^4 \)

**SOLUTION:**

\[ f^{-7} \cdot f^4 = f^{-7+4} = f^{-3} = \frac{1}{f^3} \]

**ANSWER:**
\[ \frac{1}{f^3} \]

58. \( (3x^3)^3 \)

**SOLUTION:**

\[ (3x^3)^3 = 3^3 \cdot (x^3)^3 = 27x^9 \]

**ANSWER:**
27\(x^9\)
59. \((2y)(4xy^3)\)

**SOLUTION:**

\((2y)(4xy^3) = 8xy^4\)

**ANSWER:**

\(8xy^4\)

60. \(\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2\)

**SOLUTION:**

\[
\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2 = \left(\frac{3}{5}c^2f\right)\left(\frac{16}{9}c^2d^2\right) = \frac{16}{15}c^4d^2f
\]

**ANSWER:**

\(\frac{16}{15}c^4d^2f\)
7-2 Solving Exponential Equations and Inequalities

Solve each equation.

1. \(5^x = 27^{2x - 4}\)

   **SOLUTION:**
   \[
   5^x = 27^{2x - 4} \\
   3^5 
   \]
   \[= (3^{2x})^{2x - 4} \\
   3^5 = 3^{6x - 12} \\
   x = 12
   \]
   **ANSWER:**
   12

2. \(16^{3y - 3} = 4^{y + 1}\)

   **SOLUTION:**
   \[
   16^{2y - 3} = 4^{y + 1} \\
   (4^2)^{2y - 3} = 4^{y + 1} \\
   4^{4y - 6} = 4^{y + 1}
   \]
   Use the Property of Equality for Exponential Functions.
   \[
   4y - 6 = y + 1 \\
   3y = 7 \\
   y = \frac{7}{3}
   \]
   **ANSWER:**
   \[
   \frac{7}{3}
   \]

3. \(2^{6x} = 32^{x - 2}\)

   **SOLUTION:**
   \[
   2^{6x} = 32^{x - 2} \\
   2^{6x} = (2^5)^{x - 2} \\
   2^{6x} = 2^{5x - 10}
   \]
   Use the Property of Equality for Exponential Functions.
   \[
   6x = 5x - 10 \\
   x = -10
   \]
   **ANSWER:**
   -10

4. \(49^{x + 5} = 7^{8x - 6}\)

   **SOLUTION:**
   \[
   49^{x + 5} = 7^{8x - 6} \\
   (7^2)^{x + 5} = 7^{8x - 6} \\
   7^{2x + 10} = 7^{8x - 6}
   \]
   Use the Property of Equality for Exponential Functions.
   \[
   2x + 10 = 8x - 6 \\
   -6x = -16 \\
   x = \frac{8}{3}
   \]
   **ANSWER:**
   \[
   \frac{8}{3}
   \]
5. **SCIENCE** Mitosis is a process in which one cell divides into two. The *Escherichia coli* is one of the fastest growing bacteria. It can reproduce itself in 15 minutes.

   a. Write an exponential function to represent the number of cells *c* after *t* minutes.
   b. If you begin with one *Escherichia coli* cell, how many cells will there be in one hour?

   **SOLUTION:**
   a. The exponential function that represent the number of cells after *t* minutes is $c = 2^t$.
   b. Substitute 1 for *t* in the function and solve for *c*.
      
      $c = 2^{1(1)}$
      
      $= 16$ cells
      
      **ANSWER:**
      a. $c = 2^{15} $
      b. 16 cells

6. A certificate of deposit (CD) pays 2.25% annual interest compounded biweekly. If you deposit $500 into this CD, what will the balance be after 6 years?

   **SOLUTION:**
   Use the compound interest formula.
   Substitute $500$ for *P*, $0.0225$ for *r*, $26$ for *n* and $6$ for *t* and simplify.

   
   
   $A = P \left(1 + \frac{r}{n}\right)^{nt}$
   
   $A = 500 \left(1 + \frac{0.0225}{26}\right)^{26(6)}$
   
   $≈ 572.23$
   
   **ANSWER:**
   $572.23$

7. **Solve each inequality.**

   7. $4^{2x+6} \leq 64^{2x-4}$

   **SOLUTION:**
   
   $4^{2x+6} \leq 64^{2x-4}$
   
   $4^{2x+6} \leq (4^3)^{2x-4}$
   
   $4^{2x+6} \leq 4^{6x-12}$

   Use the Property of Inequality for Exponential Functions.

   $2x + 6 \leq 6x - 12$
   
   $-4x \leq -18$
   
   $x \geq 4.5$
   
   **ANSWER:**
   $x \geq 4.5$

8. $25^{y-3} \leq \left(\frac{1}{125}\right)^{y^2}$

   **SOLUTION:**
   
   $25^{y-3} \leq (125)^{-y-2}$
   
   $(5^2)^{y-3} \leq (5^3)^{-y-2}$

   $5^{2y-6} \leq 5^{-3y-6}$

   Use the Property of Inequality for Exponential Functions.

   $2y - 6 \leq -3y - 6$
   
   $y \leq 0$
   
   **ANSWER:**
   $\{y | y \leq 0\}$
7-2 Solving Exponential Equations and Inequalities

Solve each equation.

9. \[8^{4x + 2} = 64\]

**SOLUTION:**
\[8^{4x + 2} = 64\]
\[8^{4x + 2} = 8^2\]

Use the Property of Equality for Exponential Functions.

\[4x + 2 = 2\]
\[x = 0\]

**ANSWER:**
0

10. \[5^x - 6 = 125\]

**SOLUTION:**
\[5^x - 6 = 125\]
\[5^x - 6 = 5^3\]

Use the Property of Equality for Exponential Functions.

\[x - 6 = 3\]
\[x = 9\]

**ANSWER:**
9

11. \[81^a + 2 = 3^{3a + 1}\]

**SOLUTION:**
\[81^a + 2 = 3^{3a + 1}\]
\[\left(3^4\right)^a + 2 = 3^{3a + 1}\]
\[3^{4a + 8} = 3^{3a + 1}\]

Use the Property of Equality for Exponential Functions.

\[4a + 8 = 3a + 1\]
\[a = -7\]

**ANSWER:**
-7

12. \[256^b + 2 = 4^{2-2b}\]

**SOLUTION:**
\[256^b + 2 = 4^{2-2b}\]
\[\left(4^4\right)^b + 2 = 4^{2-2b}\]
\[4^{4b+2} = 4^{2-2b}\]

Use the Property of Equality for Exponential Functions.

\[4b + 8 = 2 - 2b\]
\[6b = -6\]
\[b = -1\]

**ANSWER:**
-1

13. \[9^{3c + 1} = 27^{3c - 1}\]

**SOLUTION:**
\[9^{3c + 1} = 27^{3c - 1}\]
\[\left(3^2\right)^{3c + 1} = \left(3^3\right)^{3c - 1}\]
\[3^{6c + 2} = 3^{3c - 3}\]

Use the Property of Equality for Exponential Functions.

\[6c + 2 = 9c - 3\]
\[-3c = -5\]
\[c = \frac{5}{3}\]

**ANSWER:**
\[\frac{5}{3}\]
14. \(8^{3y + 4} = 16^{y + 1}\)

**SOLUTION:**

\[8^{2y + 4} = 16^{y + 1}\]

\[(2^3)^{2y + 4} = (2^4)^{y + 1}\]

\[2^{6y + 12} = 2^{4y + 4}\]

Use the Property of Equality for Exponential Functions.

\[6y + 12 = 4y + 4\]

\[2y = -8\]

\[y = -4\]

**ANSWER:**

\[-4\]

---

15. **CCSS MODELING** In 2009, My-Lien received $10,000 from her grandmother. Her parents invested all of the money, and by 2021, the amount will have grown to $16,960.

**a.** Write an exponential function that could be used to model the money \(y\). Write the function in terms of \(x\), the number of years since 2009.

**b.** Assume that the amount of money continues to grow at the same rate. What would be the balance in the account in 2031?

**SOLUTION:**

**a.**

Substitute 16780 for \(y\), 10000 for \(a\) and 12 for \(x\) in the exponential function and simplify.

\[y = ab^x\]

\[16960 = 10000(b)^{12}\]

\[\sqrt[12]{16960} = \sqrt[12]{b^{12}}\]

\[1.045 \approx b\]

The exponential function that models the situation is \(y = 10000(1.045)^x\).

**b.**

Substitute 22 for \(x\) in the modeled function and solve for \(y\).

\[y = 10000(1.045)^{22}\]

\[\approx 26,336.52\]

**ANSWER:**

a. \(y = 10,000(1.045)^x\)

b. about $26,336.52
7-2 Solving Exponential Equations and Inequalities

Write an exponential function for the graph that passes through the given points.
16. (0, 6.4) and (3, 100)

**SOLUTION:**
Substitute 100 for \( y \) and 6.4 for \( a \) and 3 for \( x \) into an exponential function and determine the value of \( b \).

\[
\begin{align*}
y &= ab^x \\
100 &= 6.4b^3 \\
15.625 &= b^3 \\
\sqrt[3]{15.625} &= b \\
2.5 &\approx b
\end{align*}
\]

An exponential function that passes through the given points is \( y = 6.4(2.5)^x \).

**ANSWER:**
\( y = 6.4(2.5)^x \)

17. (0, 256) and (4, 81)

**SOLUTION:**
Substitute 81 for \( y \) and 256 for \( a \) and 4 for \( x \) into an exponential function and determine the value of \( b \).

\[
\begin{align*}
y &= ab^x \\
81 &= 256b^4 \\
0.31640625 &= b^4 \\
\sqrt[4]{0.31640625} &= b \\
0.75 &\approx b
\end{align*}
\]

An exponential function that passes through the given points is \( y = 256(0.75)^x \).

**ANSWER:**
\( y = 256(0.75)^x \)

18. (0, 128) and (5, 371,293)

**SOLUTION:**
Substitute 371,293 for \( y \) and 128 for \( a \) and 5 for \( x \) into an exponential function and determine the value of \( b \).

\[
\begin{align*}
y &= ab^x \\
371,293 &= 128b^5 \\
2900.7265625 &= b^5 \\
\sqrt[5]{2900.7265625} &= b \\
4.926 &\approx b
\end{align*}
\]

An exponential function that passes through the given points is \( y = 128(4.926)^x \).

**ANSWER:**
\( y = 128(4.926)^x \)

19. (0, 144), and (4, 21,609)

**SOLUTION:**
Substitute 21,609 for \( y \) and 144 for \( a \) and 4 for \( x \) into an exponential function and determine the value of \( b \).

\[
\begin{align*}
y &= ab^x \\
21609 &= 144b^4 \\
150.0625 &= b^4 \\
\sqrt[4]{150.0625} &= b \\
3.5 &\approx b
\end{align*}
\]

An exponential function that passes through the given points is \( y = 144(3.5)^x \).

**ANSWER:**
\( y = 144(3.5)^x \)
20. Find the balance of an account after 7 years if $700 is deposited into an account paying 4.3% interest compounded monthly.

**SOLUTION:**
Use the compound interest formula.
Substitute $700 for P, 0.043 for r, 12 for n and 7 for t and simplify.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = 700 \left(1 + \frac{0.043}{12}\right)^{12(7)} \]

\[ \approx \$945.34 \]

**ANSWER:**
$945.34

21. Determine how much is in a retirement account after 20 years if $5000 was invested at 6.05% interest compounded weekly.

**SOLUTION:**
Use the compound interest formula.
Substitute $5000 for P, 0.0605 for r, 52 for n and 20 for t and simplify.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = 5000 \left(1 + \frac{0.0605}{52}\right)^{52(20)} \]

\[ \approx \$16755.63 \]

**ANSWER:**
$16,755.63

22. A savings account offers 0.7% interest compounded bimonthly. If $110 is deposited in this account, what will the balance be after 15 years?

**SOLUTION:**
Use the compound interest formula.
Substitute $110 for P, 0.007 for r, 6 for n and 15 for t and simplify.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = 110 \left(1 + \frac{0.007}{6}\right)^{6(15)} \]

\[ \approx \$122.17 \]

**ANSWER:**
$122.17

23. A college savings account pays 13.2% annual interest compounded semiannually. What is the balance of an account after 12 years if $21,000 was initially deposited?

**SOLUTION:**
Use the compound interest formula.
Substitute $21,000 for P, 0.132 for r, 2 for n and 12 for t and simplify.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = 21000 \left(1 + \frac{0.132}{2}\right)^{12(2)} \]

\[ \approx \$97362.61 \]

**ANSWER:**
$97,362.61
Solve each inequality.

24. 625 ≥ 5^a + 8

**SOLUTION:**

625 ≥ 5^a + 8

5^4 ≥ 5^a + 8

Use the Property of Inequality for Exponential Functions.

4 ≥ a + 8

a ≤ −4

**ANSWER:**

\{a \mid a ≤ −4\}

25. 10^{5b + 2} > 1000

**SOLUTION:**

10^{5b + 2} > 1000

10^{5b + 2} > 10^3

Use the Property of Inequality for Exponential Functions.

5b + 2 > 3

5b > 1

b > \frac{1}{5}

**ANSWER:**

\{b \mid b > \frac{1}{5}\}

26. \left(\frac{1}{64}\right)^{c−2} < 32^{2c}

**SOLUTION:**

\left(\frac{1}{64}\right)^{c−2} < 32^{2c}

\left(64\right)^{−c−2} < 32^{2c}

\left(2^{6}\right)^{−c−2} < (2^4)^{2c}

2^{−6c + 12} < 2^{10c}

Use the Property of Inequality for Exponential Functions.

−6c + 12 < 10c

12 < 16c

\frac{3}{4} < c

**ANSWER:**

\{c \mid c > \frac{3}{4}\}

27. \left(\frac{1}{27}\right)^{2d−2} ≤ 81^{d+4}

**SOLUTION:**

\left(\frac{1}{27}\right)^{2d−2} ≤ 81^{d+4}

\left(3^{−3}\right)^{2d−2} ≤ \left(3^4\right)^{d+4}

3^{−6d + 6} ≤ 3^{4d + 16}

Use the Property of Inequality for Exponential Functions.

−6d + 6 ≤ 4d + 16

−10d ≤ 16

d ≥ −1

**ANSWER:**

\{d \mid d ≥ −1\}
28. \( \left( \frac{1}{9} \right)^{3x+5} \geq \left( \frac{1}{243} \right)^{x-6} \)

**SOLUTION:**
\[
\left( \frac{1}{9} \right)^{3x+5} \geq \left( \frac{1}{243} \right)^{x-6} \\
9^{-3x-5} \geq 243^{x-6} \\
(3^2)^{-3x-5} \geq (3^5)^{x-6} \\
3^{-6x-10} \geq 3^{-5x+30}
\]

Use the Property of Inequality for Exponential Functions.

\[-6t - 10 \geq -5t + 30 \]
\[-t \geq 40 \]
\[t \leq -40 \]

**ANSWER:**
\( \{ t | t \leq -40 \} \)

29. \( \left( \frac{1}{36} \right)^{w+2} < \left( \frac{1}{216} \right)^{4w} \)

**SOLUTION:**
\[
\left( \frac{1}{36} \right)^{w+2} < \left( \frac{1}{216} \right)^{4w} \\
36^{-w-2} < 216^{-4w} \\
(6^2)^{-w-2} < (6^4)^{-4w} \\
6^{-2w-4} < 6^{-12w}
\]

Use the Property of Inequality for Exponential Functions.

\[-2w - 4 < -12w \]
\[10w < 4 \]
\[w < \frac{2}{5} \]

**ANSWER:**
\( \{ w | w < \frac{2}{5} \} \)

30. **SCIENCE** A mug of hot chocolate is 90°C at time \( t = 0 \). It is surrounded by air at a constant temperature of 20°C. If stirred steadily, its temperature in Celsius after \( t \) minutes will be \( y(t) = 20 + 70(1.071)^{-t} \).

a. Find the temperature of the hot chocolate after 15 minutes.

b. Find the temperature of the hot chocolate after 30 minutes.

c. The optimum drinking temperature is 60°C. Will the mug of hot chocolate be at or below this temperature after 10 minutes?

**SOLUTION:**

a. Substitute 15 for \( t \) in the equation and simplify.

\[ y = 20 + 70(1.071)^{-15} \]
\[ \approx 45.02 \, \text{C} \]

b. Substitute 30 for \( t \) in the equation and simplify.

\[ y = 20 + 70(1.071)^{-30} \]
\[ \approx 28.94 \, \text{C} \]

c. Substitute 10 for \( t \) in the equation and simplify.

\[ y = 20 + 70(1.071)^{-10} \]
\[ \approx 55.25 \, \text{C} \]

So, temperature of the hot chocolate will be below 60°C after 10 minutes.

**ANSWER:**

a. 45.02°C

b. 28.94°C

c. below
31. **ANIMALS** Studies show that an animal will defend a territory, with area in square yards, that is directly proportional to the 1.31 power of the animal’s weight in pounds.

- **a.** If a 45-pound beaver will defend 170 square yards, write an equation for the area defended by a beaver weighing \( w \) pounds.
- **b.** Scientists believe that thousands of years ago, the beaver’s ancestors were 11 feet long and weighed 430 pounds. Use your equation to determine the area defended by these animals.

**SOLUTION:**

- **a.** Substitute 170 for \( y \), 45 for \( b \), and 1.31 for \( x \) in the exponential function.

\[
y = ab^x
\]

\[
170 = a(45^{1.31})
\]

\[
a \approx 1.16
\]

The equation for the area \( a \) defended by a beaver weighting \( w \) pounds is \( y = 1.16w^{1.31} \).

- **b.** Substitute 430 for \( w \) in the equation and solve for \( y \).

\[
y = 1.16(430)^{1.31}
\]

\[
\approx 3268 \text{ yd}^2
\]

**ANSWER:**

- **a.** \( a = 1.16w^{1.31} \)
- **b.** about 3268 yd\(^2\)

---

**Solve each equation.**

32. \( \left( \frac{1}{2} \right)^{x+1} = 8^{2x+1} \)

**SOLUTION:**

\[
\left( \frac{1}{2} \right)^{x+1} = 8^{2x+1}
\]

\[
2^{-x-1} = (2^3)^{2x+1}
\]

\[
2^{-x-1} = 2^{6x+3}
\]

Use the Property of Equality for Exponential Functions.

\[
-4x - 1 = 6x + 3
\]

\[
-10x = 4
\]

\[
x = \frac{-2}{5}
\]

**ANSWER:**

\[
\frac{-2}{5}
\]

33. \( \left( \frac{1}{5} \right)^{x-5} = 25^{3x+2} \)

**SOLUTION:**

\[
\left( \frac{1}{5} \right)^{x-5} = 25^{3x+2}
\]

\[
5^{-x+5} = (5^2)^{3x+2}
\]

\[
5^{-x+5} = 5^{6x+4}
\]

Use the Property of Equality for Exponential Functions.

\[
-x + 5 = 6x + 4
\]

\[
-7x = -1
\]

\[
x = \frac{1}{7}
\]

**ANSWER:**

\[
\frac{1}{7}
\]
Solve each equation.

1. \(35x = 272x - 4\)

**SOLUTION:**

Use the Property of Equality for Exponential Functions.

\[3 = -x - 3\]

\[x = -6\]

**ANSWER:**

-6

35. \(\frac{1}{8} = \left(\frac{1}{4}\right)^{-9x+4}\)

**SOLUTION:**

\[\left(\frac{1}{8}\right) = \left(\frac{1}{4}\right)^{-9x+4}\]

\[8^{-9x+4} = 4^{2x-4}\]

\[\left(2^3\right)^{-9x+4} = \left(2^2\right)^{2x-4}\]

\[2^{-9x+12} = 2^{4x-8}\]

Use the Property of Equality for Exponential Functions.

\[-9x + 12 = 4x - 8\]

\[-13x = 4\]

\[x = \frac{-4}{13}\]

**ANSWER:**

\[-\frac{4}{13}\]

36. \(\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{27}{8}\right)^{x-4}\)

**SOLUTION:**

\[\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{27}{8}\right)^{x-4}\]

\[\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{2}{3}\right)^{-3(x-4)}\]

\[\left(\frac{2}{3}\right)^{5x+1} = \left(\frac{2}{3}\right)^{-3x+12}\]

Use the Property of Equality for Exponential Functions.

\[5x + 1 = -3x + 12\]

\[8x = 11\]

\[x = \frac{11}{8}\]

**ANSWER:**

\[\frac{11}{8}\]

37. \(\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}\)

**SOLUTION:**

\[\left(\frac{25}{81}\right)^{2x+1} = \left(\frac{729}{125}\right)^{-3x+1}\]

\[\left(\frac{5}{9}\right)^{4x+2} = \left(\frac{5}{9}\right)^{9x-3}\]

Use the Property of Equality for Exponential Functions.

\[4x + 2 = 9x - 3\]

\[-5x = -5\]

\[x = 1\]

**ANSWER:**

1

38. **CCSS MODELING** In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion.
Solve each equation.

1. \[ 35x = 272x - 4 \]

**SOLUTION:**

Use the Property of Equality for Exponential Functions.

2. \[ 7^{x+2} = 7^{x-3} \cdot 7 \]

**SOLUTION:**

3. \[ 2x^2 - 5x + 3 = 0 \]

**SOLUTION:**

4. \[ 3x^2 - 10x + 3 = 0 \]

**SOLUTION:**

5. \[ 4x^2 - 12x + 9 = 0 \]

**SOLUTION:**

6. \[ x^2 - 5x + 6 = 0 \]

**SOLUTION:**

7. \[ x^2 - 6x + 9 = 0 \]

**SOLUTION:**

8. \[ x^2 - 10x + 25 = 0 \]

**SOLUTION:**

9. \[ x^2 - 4x + 4 = 0 \]

**SOLUTION:**

10. \[ x^2 - 2x + 1 = 0 \]

**SOLUTION:**

11. \[ x^2 + 6x + 9 = 0 \]

**SOLUTION:**

12. \[ x^2 + 4x + 4 = 0 \]

**SOLUTION:**

13. \[ 2x^2 - 12x + 18 = 0 \]

**SOLUTION:**

14. \[ 3x^2 - 6x + 3 = 0 \]

**SOLUTION:**

15. \[ 5x^2 - 10x + 5 = 0 \]

**SOLUTION:**

16. \[ 2x^2 - 8x + 6 = 0 \]

**SOLUTION:**

17. \[ 3x^2 - 9x + 3 = 0 \]

**SOLUTION:**

18. \[ x^2 - 8x + 16 = 0 \]

**SOLUTION:**

19. \[ x^2 - 10x + 25 = 0 \]

**SOLUTION:**

20. \[ x^2 - 4x + 4 = 0 \]

**SOLUTION:**

21. \[ x^2 - 6x + 9 = 0 \]

**SOLUTION:**

22. \[ x^2 - 4x + 4 = 0 \]

**SOLUTION:**

23. \[ x^2 - 4x + 1 = 0 \]

**SOLUTION:**

24. \[ x^2 - 6x + 9 = 0 \]

**SOLUTION:**

25. \[ x^2 - 8x + 16 = 0 \]

**SOLUTION:**

26. \[ x^2 - 10x + 25 = 0 \]

**SOLUTION:**

27. \[ x^2 - 4x + 4 = 0 \]

**SOLUTION:**

28. \[ x^2 - 6x + 9 = 0 \]

**SOLUTION:**

29. \[ x^2 - 4x + 1 = 0 \]

**SOLUTION:**

30. \[ x^2 - 5x + 5 = 0 \]

**SOLUTION:**

31. \[ x^2 - 4x + 3 = 0 \]

**SOLUTION:**

32. \[ x^2 - 6x + 5 = 0 \]

**SOLUTION:**

33. \[ x^2 - 7x + 12 = 0 \]

**SOLUTION:**

34. \[ x^2 - 8x + 16 = 0 \]

**SOLUTION:**

35. \[ x^2 - 10x + 25 = 0 \]

**SOLUTION:**

36. \[ x^2 - 4x + 4 = 0 \]

**SOLUTION:**

37. \[ x^2 - 6x + 9 = 0 \]

**SOLUTION:**

38. \[ x^2 - 4x + 1 = 0 \]

**SOLUTION:**

39. **TREES** The diameter of the base of a tree trunk in centimeters varies directly with the \[ \frac{3}{2} \] power of its height in meters.

**a.** A young sequoia tree is 6 meters tall, and the diameter of its base is 19.1 centimeters. Use this information to write an equation for the diameter \( d \) of the base of a sequoia tree if its height is \( h \) meters high.

**b.** The General Sherman Tree in Sequoia National Park, California, is approximately 84 meters tall. Find the diameter of the General Sherman Tree at its base.

**SOLUTION:**

**a.** The equation that represents the situation is

\[ d = 1.30h^{\frac{3}{2}}. \]

**b.** Substitute 84 for \( h \) in the equation and solve for \( d \).

\[ d = 1.30(84)^{\frac{3}{2}}; \]

\[ d \approx 1001 \]

The diameter of the General Sherman Tree at its base is about 1001 cm.

**ANSWER:**

**a.** \( d = 1.30h^{\frac{3}{2}} \)

**b.** About 1001 cm

40. **FINANCIAL LITERACY** Mrs. Jackson has two different retirement investment plans from which to choose.

**a.** Write equations for Option A and Option B given the minimum deposits.

**b.** Draw a graph to show the balances for each investment option after \( t \) years.

**c.** Explain whether Option A or Option B is the better investment choice.

**ANSWER:**

**a.** \( y = 2.556(1.0187)^x \)

**b.** 6.455 billion
SOLUTION:

a.
Use the compound interest formula.
The equation that represents Option A is \( A = 5000 \left( \frac{4.065}{4} \right)^t \).

The equation that represents Option B is \( A = 5000 \left( \left( \frac{12.042}{12} \right)^{\frac{t}{12}} + \left( \frac{52.023}{52} \right)^{\frac{t}{52}} \right) \).

b.
The graph that shows the balances for each investment option after \( t \) years:

c. Sample answer: During the first 22 years, Option B is the better choice because the total is greater than that of Option A. However, after about 22 years, the balance of Option A exceeds that of Option B, so Option A is the better choice.

ANSWER:

a. \[ A = 5000 \left( \frac{4.065}{4} \right)^t \]; \[ A = 5000 \left( \left( \frac{12.042}{12} \right)^{\frac{t}{12}} + \left( \frac{52.023}{52} \right)^{\frac{t}{52}} \right) \]

b. 

41. MULTIPLE REPRESENTATIONS In this problem, you will explore the rapid increase of an exponential function. A large sheet of paper is cut in half, and one of the resulting pieces is placed on top of the other. Then the pieces in the stack are cut in half and placed on top of each other. Suppose this procedure is repeated several times.

a. CONCRETE Perform this activity and count the number of sheets in the stack after the first cut. How many pieces will there be after the second cut? How many pieces after the third cut? How many pieces after the fourth cut?

b. TABULAR Record your results in a table.

c. SYMBOLIC Use the pattern in the table to write an equation for the number of pieces in the stack after \( x \) cuts.

d. ANALYTICAL The thickness of ordinary paper is about 0.003 inch. Write an equation for the thickness of the stack of paper after \( x \) cuts.

e. ANALYTICAL How thick will the stack of paper be after 30 cuts?

SOLUTION:

a.
There will be 2, 4, 8, 16 pieces after the first, second, third and fourth cut respectively.

b.
Solve each equation.

1. \(35x = 272x - 4\)

SOLUTION:

Use the Property of Equality for Exponential Functions.

The thickness of the stack of paper after 30 cuts is about 3221225.47 in.

ANSWER:

a. 2, 4, 8, 16

b.

\[
\begin{array}{|c|c|}
\hline
\text{Cuts} & \text{Pieces} \\
\hline
1 & 2 \\
2 & 4 \\
3 & 8 \\
4 & 16 \\
\hline
\end{array}
\]

c. \(y = 2^x\)

d. \(y = 0.003(2)^x\)

e. about 3,221,225.47 in.

42. **WRITING IN MATH** In a problem about compound interest, describe what happens as the compounding period becomes more frequent while the principal and overall time remain the same.

**SOLUTION:**

Sample answer: The more frequently interest is compounded, the higher the account balance becomes.

**ANSWER:**

Sample answer: The more frequently interest is compounded, the higher the account balance becomes.
43. **ERROR ANALYSIS** Beth and Liz are solving \( 6^{x-3} > 36^{x-1} \). Is either of them correct? Explain your reasoning.

**SOLUTION:**
Sample answer: Beth; Liz added the exponents instead of multiplying them when taking the power of a power.

**ANSWER:**
Sample answer: Beth; Liz added the exponents instead of multiplying them when taking the power of a power.

44. **CHALLENGE** Solve for \( x \): \( 16^{18} + 16^{18} + 16^{18} + 16^{18} + 16^{18} = 4^x \).

**SOLUTION:**
Simplify the exponential equation.

\[
16^4 + 16^4 + 16^4 + 16^4 + 16^4 + 16^4 = 4^x
\]

\[
5(16^4) = 4^x
\]

\[
2.36118 \times 10^{22} = 4^x
\]

Use a graphing calculator to solve the exponential equation for \( x \).
Enter \( 4x \) as \( Y1 \) and \( 2.36118 \times 10^{22} = Y2 \). Adjust the viewing window to see both graphs and the intersection.
Use the CALC function to find the intersection of the two graphs.

So, the solution to the equation is about \( x = 37.1610 \).

**ANSWER:**
37.1610

45. **OPENEnded** What would be a more beneficial change to a 5-year loan at 8% interest compounded monthly: reducing the term to 4 years or reducing the interest rate to 6.5%?

**SOLUTION:**
Reducing the term will be more beneficial. The multiplier is 1.3756 for the 4-year and 1.3828 for the 6.5%.

**ANSWER:**
Reducing the term will be more beneficial. The multiplier is 1.3756 for the 4-year and 1.3828 for the 6.5%.
46. **CCSS ARGUMENTS** Determine whether the following statements are sometimes, always, or never true. Explain your reasoning.
   a. $2^x > -8^{20x}$ for all values of $x$.
   b. The graph of an exponential growth equation is increasing.
   c. The graph of an exponential decay equation is increasing.

**SOLUTION:**
   a. Always; $2^x$ will always be positive, and $-8^{20x}$ will always be negative.
   b. Always; by definition the graph will always be increasing even if it is a small increase.
   c. Never; by definition the graph will always be decreasing even if it is a small decrease.

**ANSWER:**
   a. Always; $2^x$ will always be positive, and $-8^{20x}$ will always be negative.
   b. Always; by definition the graph will always be increasing even if it is a small increase.
   c. Never; by definition the graph will always be decreasing even if it is a small decrease.

47. **OPEN ENDED** Write an exponential inequality with a solution of $x \leq 2$.

**SOLUTION:**
   Sample answer: $4^x \leq 4^2$

**ANSWER:**
   Sample answer: $4^x \leq 4^2$

48. **PROOF** Show that $27^{2x} \cdot 81^x + 1 = 3^{2x+2} \cdot 9^{4x+1}$.

**SOLUTION:**

$$
\begin{align*}
27^{2x} \cdot 81^x + 1 &= 3^{2x+2} \cdot 9^{4x+1} \\
\left(9^x \cdot 3^2\right)^x + 1 &= \left(3^2 \cdot 3^2\right)^{x+1} \\
3^{2x+4} + 1 &= 3^{4x+4} \\
3^{2x+4} &= 3^{4x+3} \\
3x + 4 &= 4x + 3 \\
x &= 1 \\
\end{align*}
$$

**ANSWER:**

49. **WRITING IN MATH** If you were given the initial and final amounts of a radioactive substance and the amount of time that passes, how would you determine the rate at which the amount was increasing or decreasing in order to write an equation?

**SOLUTION:**
   Sample answer: Divide the final amount by the initial amount. If $n$ is the number of time intervals that pass, take the $n$th root of the answer.

**ANSWER:**
   Sample answer: Divide the final amount by the initial amount. If $n$ is the number of time intervals that pass, take the $n$th root of the answer.
50. \(3 \times 10^{-4} =\)
   
   A. -30,000  
   B. 0.0003  
   C. -120  
   D. 0.00003
   
   **SOLUTION:**
   \[
   3 \times 10^{-4} = \frac{3}{10^4} = \frac{3}{10000} = 0.0003
   \]
   B is the correct option.  
   
   **ANSWER:**
   B

51. Which of the following could **not** be a solution to \(5 - 3x < -3\)?
   
   F. 2.5  
   G. 3  
   H. 3.5  
   J. 4
   
   **SOLUTION:**
   Check the inequality by substituting 2.5 for \(x\).
   
   \[
   5 - 3(2.5) < -3
   \]
   \[
   -2.5 < -3 \quad \text{False}
   \]
   So, F is the correct option.  
   
   **ANSWER:**
   F

52. **GRIDDED RESPONSE** The three angles of a triangle are \(3x, x + 10, \) and \(2x - 40\). Find the measure of the smallest angle in the triangle.
   
   **SOLUTION:**
   Sum of the three angles in a triangle is 180°.
   
   \[
   3x + x + 10 + 2x - 40 = 180
   \]
   \[
   6x - 30 = 180
   \]
   \[
   6x = 210
   \]
   \[
   x = 35
   \]
   So, \(3(35) = 105\)
   \[
   35 + 10 = 45
   \]
   \[
   2(35) - 40 = 30.
   \]
   The measure of the smallest angle in the triangle is 30°.  
   
   **ANSWER:**
   30

53. **SAT/ACT** Which of the following is equivalent to \((x)(x)(x)(x)\) for all \(x\)?
   
   A. \(x + 4\)  
   B. \(4x\)  
   C. \(2x^2\)  
   D. \(4x^2\)  
   E. \(x^4\)
   
   **SOLUTION:**
   \[
   (x)(x)(x)(x) = x^{1+1+1+1} = x^4
   \]
   E is the correct choice.  
   
   **ANSWER:**
   E
Graph each function.
54. \( y = 2(3)^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>0.0027</td>
</tr>
<tr>
<td>-4</td>
<td>0.0247</td>
</tr>
<tr>
<td>-2</td>
<td>0.2222</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

**ANSWER:**

55. \( y = 5(2)^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>0.0195</td>
</tr>
<tr>
<td>-6</td>
<td>0.0781</td>
</tr>
<tr>
<td>-4</td>
<td>0.3125</td>
</tr>
<tr>
<td>-2</td>
<td>1.25</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

**ANSWER:**

7-2 Solving Exponential Equations and Inequalities
56. \( y = 4 \left( \frac{1}{3} \right)^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0.4444</td>
</tr>
<tr>
<td>4</td>
<td>0.0494</td>
</tr>
<tr>
<td>6</td>
<td>0.0055</td>
</tr>
</tbody>
</table>

**ANSWER:**

57. \( \sqrt{x + 5} - 3 = 0 \)

**SOLUTION:**

\[
\sqrt{x + 5} - 3 = 0 \\
\left( \sqrt{x + 5} \right)^2 = 3^2 \\
x + 5 = 9 \\
x = 4
\]

**ANSWER:**
4

58. \( \sqrt{3t - 5} - 3 = 4 \)

**SOLUTION:**

\[
\sqrt{3t - 5} - 3 = 4 \\
\left( \sqrt{3t - 5} \right)^2 = 7^2 \\
3t - 5 = 49 \\
3t = 54 \\
t = 18
\]

**ANSWER:**
18

59. \( \sqrt[4]{2x - 1} = 2 \)

**SOLUTION:**

\[
\sqrt[4]{2x - 1} = 2 \\
\left( \sqrt[4]{2x - 1} \right)^4 = 2^4 \\
2x - 1 = 16 \\
2x = 17 \\
x = 8.5
\]

**ANSWER:**
8.5
7-2 Solving Exponential Equations and Inequalities

60. \( \sqrt{x} - 6 - \sqrt{x} = 3 \)

**SOLUTION:**
\[
\sqrt{x} - 6 - \sqrt{x} = 3 \\
\left( \sqrt{x} - 6 \right)^2 = (3 + \sqrt{x})^2 \\
x - 6 = 9 + x + 6\sqrt{x} \\
\frac{15}{6} = \sqrt{x}
\]
The square root of \( x \) cannot be negative, so there is no solution.

**ANSWER:**
o no solution

61. \( \sqrt[3]{5m + 2} = 3 \)

**SOLUTION:**
\[
\sqrt[3]{5m + 2} = 3 \\
\left( \sqrt[3]{5m + 2} \right)^3 = 3^3 \\
5m + 2 = 27 \\
5m = 25 \\
m = 5
\]

**ANSWER:**
5

62. \( (6n - 5)^\frac{1}{3} + 3 = -2 \)

**SOLUTION:**
\[
(6n - 5)^\frac{1}{3} + 3 = -2 \\
\left( (6n - 5)^\frac{1}{3} \right)^3 = (-5)^3 \\
6n - 5 = -125 \\
6n = -120 \\
n = -20
\]

**ANSWER:**
-20

63. \( (5x + 7)^\frac{1}{3} + 3 = 5 \)

**SOLUTION:**
\[
(5x + 7)^\frac{1}{3} + 3 = 5 \\
\left( (5x + 7)^\frac{1}{3} \right)^3 = 2^3 \\
5x + 7 = 32 \\
5x = 25 \\
x = 5
\]

**ANSWER:**
5

64. \( (3x - 2)^\frac{1}{3} + 6 = 5 \)

**SOLUTION:**
\[
(3x - 2)^\frac{1}{3} + 6 = 5 \\
\left( (3x - 2)^\frac{1}{3} \right)^6 = (-1)^5 \\
3x - 2 = -1 \\
3x = 1 \\
x = \frac{1}{3}
\]

**ANSWER:**
\( \frac{1}{3} \)

65. \( (7x - 1)^\frac{1}{3} + 4 = 2 \)

**SOLUTION:**
\[
(7x - 1)^\frac{1}{3} + 4 = 2 \\
\left( (7x - 1)^\frac{1}{3} \right)^3 = (-2)^3 \\
7x - 1 = -8 \\
7x = -7 \\
x = -1
\]

**ANSWER:**
-1
66. **SALES** A salesperson earns $10 an hour plus a 10% commission on sales. Write a function to describe the salesperson’s income. If the salesperson wants to earn $1000 in a 40-hour week, what should his sales be?

**SOLUTION:**

Let \( I \) be the income of the salesperson and \( m \) be his sales.

The function that represents the situation is
\[
I(m) = 400 + 0.1m.
\]

Substitute 1000 for \( I \) in the equation and solve for \( m \).

\[
1000 = 400 + 0.1m
\]

\[
600 = 0.1m
\]

\[
m = 6000
\]

**ANSWER:**

\[
I(m) = 400 + 0.1m; \ 6000
\]

67. **STATE FAIR** A dairy makes three types of cheese—cheddar, Monterey Jack, and Swiss—and sells the cheese in three booths at the state fair. At the beginning of one day, the first booth received \( x \) pounds of each type of cheese. The second booth received \( y \) pounds of each type of cheese, and the third booth received \( z \) pounds of each type of cheese.

By the end of the day, the dairy had sold 131 pounds of cheddar, 291 pounds of Monterey Jack, and 232 pounds of Swiss. The table below shows the percent of the cheese delivered in the morning that was sold at each booth. How many pounds of cheddar cheese did each booth receive in the morning?

<table>
<thead>
<tr>
<th>Type</th>
<th>Booth 1</th>
<th>Booth 2</th>
<th>Booth 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheddar</td>
<td>40%</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>Monterey Jack</td>
<td>40%</td>
<td>90%</td>
<td>80%</td>
</tr>
<tr>
<td>Swiss</td>
<td>30%</td>
<td>70%</td>
<td>70%</td>
</tr>
</tbody>
</table>

**SOLUTION:**

The system of equations that represent the situation:
\[
4x + 3y + z = 1310 \rightarrow (1)
\]
\[
4x + 9y + 8z = 2910 \rightarrow (2)
\]
\[
3x + 7y + 7z = 2320 \rightarrow (3)
\]

Eliminate the variable \( x \) by using two pairs of equations.

Subtract (1) and (2).

\[
(-)4x + 9y + 8z = 2910
\]

\[
-6y - 7z = -1600
\]

Multiply (2) by 3 and (3) by 4 and subtract both the equations.

\[
12x + 27y + 24z = 8730
\]

\[
(-)12x + 28y + 28z = 9280
\]

\[
- y - 4z = -55
\]

Solve the system of two equations:

\[
-6y - 7z = -1600
\]

\[
(-) -6y - 24z = -3300
\]

\[
17z = 1700
\]

\[
z = 100
\]

Substitute \( z = 100 \) in the equation \(-6y - 7z = -1600\).

\[
-6y - 7(100) = -1600
\]

\[
-6y - 700 = -1600
\]

\[
-6y = -900
\]

\[
y = 150
\]

Substitute \( y = 150 \) and \( z = 100 \) in the (1) and solve for \( x \).

\[
4x + 3(150) + 100 = 1310
\]

\[
4x + 550 = 1310
\]

\[
4x = 760
\]

\[
x = 190
\]

Booth 1 has 190 lb; Booth 2 has 150 lb; Booth 3 has 100 lb.

**ANSWER:**

booth 1, 190 lb; booth 2, 150 lb; booth 3, 100 lb
Find \( [g \circ h](x) \) and \( [h \circ g](x) \).

68. \( h(x) = 2x - 1 \)
\( g(x) = 3x + 4 \)

**SOLUTION:**
\[
\begin{align*}
g(2x - 1) & = 3(2x - 1) + 4 \\
& = 6x - 3 + 4 \\
& = 6x + 1 \\
h(3x + 4) & = 2(3x + 4) - 1 \\
& = 6x + 8 - 1 \\
& = 6x + 7
\end{align*}
\]

**ANSWER:**
\( 6x + 1; 6x + 7 \)

69. \( h(x) = x^2 + 2 \)
\( g(x) = x - 3 \)

**SOLUTION:**
\[
\begin{align*}
g(x^2 + 2) & = (x^2 + 2) - 3 \\
& = x^2 - 1 \\
h(x - 3) & = (x - 3)^2 + 2 \\
& = x^2 - 6x + 2 \\
& = x^2 - 6x + 11
\end{align*}
\]

**ANSWER:**
\( x^2 - 1; x^2 - 6x + 11 \)

70. \( h(x) = x^2 + 1 \)
\( g(x) = -2x + 1 \)

**SOLUTION:**
\[
\begin{align*}
g(x^2 + 1) & = -2(x^2 + 1) + 1 \\
& = -2x^2 - 2 + 1 \\
& = -2x^2 - 1 \\
h(-2x + 1) & = (-2x + 1)^2 + 1 \\
& = 4x^2 - 4x + 2
\end{align*}
\]

**ANSWER:**
\( -2x^2 - 1; 4x^2 - 4x + 2 \)

71. \( h(x) = -5x \)
\( g(x) = 3x - 5 \)

**SOLUTION:**
\[
\begin{align*}
g(-5x) & = 3(-5x) - 5 \\
& = -15x - 5 \\
h(3x - 5) & = -5(3x - 5) \\
& = -15x + 25
\end{align*}
\]

**ANSWER:**
\( -15x - 5; -15x + 25 \)

72. \( h(x) = x^3 \)
\( g(x) = x - 2 \)

**SOLUTION:**
\[
\begin{align*}
g(x^3) & = x^3 - 2 \\
h(x - 2) & = (x - 2)^3 \\
& = x^3 - 6x^2 + 12x - 8
\end{align*}
\]

**ANSWER:**
\( x^3 - 2; x^3 - 6x^2 + 12x - 8 \)

73. \( h(x) = x + 4 \)
\( g(x) = |x| \)

**SOLUTION:**
\[
\begin{align*}
g(x + 4) & = |x + 4| \\
h(|x|) & = |x| + 4
\end{align*}
\]

**ANSWER:**
\( |x + 4|; |x| + 4 \)
Write each equation in exponential form.

1. \( \log_8 512 = 3 \)

**SOLUTION:**
\[
\log_8 512 = 3 \\
8^3 = 512 \\
\text{ANSWER:} \\
8^3 = 512
\]

2. \( \log_5 625 = 4 \)

**SOLUTION:**
\[
\log_5 625 = 4 \\
5^4 = 625 \\
\text{ANSWER:} \\
5^4 = 625
\]

Write each equation in logarithmic form.

3. \( 11^3 = 1331 \)

**SOLUTION:**
\[
11^3 = 1331 \\
\log_{11} 1331 = 3 \\
\text{ANSWER:} \\
\log_{11} 1331 = 3
\]

4. \( 16^4 = 8 \)

**SOLUTION:**
\[
16^4 = 8 \\
\log_{16} 8 = \frac{3}{4} \\
\text{ANSWER:} \\
\log_{16} 8 = \frac{3}{4}
\]

5. \( \log_{13} 169 \)

**SOLUTION:**
\[
\log_{13} 169 = \log_{13} (13^2) \\
= 2 \\
\text{ANSWER:} \\
2
\]

6. \( \log_2 \frac{1}{128} \)

**SOLUTION:**
\[
\log_2 \frac{1}{128} = \log_2 \frac{1}{2^7} \\
= \log_2 2^{-7} \\
= -7 \\
\text{ANSWER:} \\
-7
\]

7. \( \log_6 1 \)

**SOLUTION:**
\[
\log_6 1 = 0 \\
\text{ANSWER:} \\
0
\]
Graph each function. State the domain and range.

8. \( f(x) = \log_3 x \)

**SOLUTION:**
Plot the points \( \left( \frac{1}{3}, -1 \right), (1, 0), (3, 1) \) and sketch the graph.

The domain consists of all positive real numbers, and the domain consists of all real numbers.

**ANSWER:**

\[ D = \{ x \mid x > 0 \}; R = \{ \text{all real numbers} \} \]

9. \( f(x) = \log_a x \)

**SOLUTION:**
Plot the points \( (6, -1), (1, 0), \left( \frac{1}{6}, 1 \right) \) and sketch the graph.

The domain consists of all positive real numbers, and the domain consists of all real numbers.

**ANSWER:**

\[ D = \{ x \mid x > 0 \}; R = \{ \text{all real numbers} \} \]
10. \( f(x) = 4 \log_4 (x - 6) \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_4 x \).

- \( a = 4 \): The graph expands vertically.
- \( h = 6 \): The graph is translated 6 units to the right.
- \( k = 0 \): There is no vertical shift.

![Graph](image)

The domain consists of all positive real numbers greater than 6, and the domain consists of all real numbers.

**ANSWER:**

![Graph](image)

\( D = \{ x \mid x > 6 \} \); \( R = \{ \text{all real numbers} \} \)

11. \( f(x) = 2 \log_{10} (x - 5) \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{10} x \).

- \( a = 2 \): The graph expands vertically.
- \( h = 0 \): There is no horizontal shift.
- \( k = -5 \): The graph is translated 5 units down.

![Graph](image)

The domain consists of all positive real numbers, and the domain consists of all real numbers.

**ANSWER:**

![Graph](image)

\( D = \{ x \mid x > 5 \} \); \( R = \{ \text{all real numbers} \} \)
12. **SCIENCE** Use the information at the beginning of the lesson. The Palermo scale value of any object can be found using the equation \( PS = \log_{10} R \), where \( R \) is the relative risk posed by the object. Write an equation in exponential form for the inverse of the function.

**SOLUTION:**
Rewrite the equation in exponential form.
\( 10^{PS} = R \)
Interchange the variables.
\( PS = 10^R \)

**ANSWER:**
\( PS = 10^R \)

Write each equation in exponential form.

13. \( \log_2 16 = 4 \)

**SOLUTION:**
\( \log_2 16 = 4 \)
\( 2^4 = 16 \)

**ANSWER:**
\( 2^4 = 16 \)

14. \( \log_7 343 = 3 \)

**SOLUTION:**
\( \log_7 343 = 3 \)
\( 7^3 = 343 \)

**ANSWER:**
\( 7^3 = 343 \)

15. \( \log_9 \frac{1}{81} = -2 \)

**SOLUTION:**
\( \log_9 \frac{1}{81} = -2 \)
\( 9^{-2} = \frac{1}{81} \)

**ANSWER:**
\( 9^{-2} = \frac{1}{81} \)

16. \( \log_3 \frac{1}{27} = -3 \)

**SOLUTION:**
\( \log_3 \frac{1}{27} = -3 \)
\( 3^{-3} = \frac{1}{27} \)

**ANSWER:**
\( 3^{-3} = \frac{1}{27} \)

17. \( \log_{12} 144 = 2 \)

**SOLUTION:**
\( \log_{12} 144 = 2 \)
\( 12^2 = 144 \)

**ANSWER:**
\( 12^2 = 144 \)

18. \( \log_9 1 = 0 \)

**SOLUTION:**
\( \log_9 1 = 0 \)
\( 9^0 = 1 \)

**ANSWER:**
\( 9^0 = 1 \)

Write each equation in logarithmic form.

19. \( 9^{-1} = \frac{1}{9} \)

**SOLUTION:**
\( \log_9 \frac{1}{9} = -1 \)

**ANSWER:**
\( \log_9 \frac{1}{9} = -1 \)
Write each equation in exponential form.

1. \( \log_8 512 = 3 \)
   - SOLUTION: 
     \( \frac{1}{216} \)
   - ANSWER: 
     \( 8^3 = 512 \)

2. \( \log_5 625 = 4 \)
   - SOLUTION: 
   - ANSWER: 

7. \( y = -2.5(5)^x \)
   - SOLUTION: 
   - Make a table of values. Then plot the points and sketch the graph.

Evaluate each expression.

25. \( \log_3 \frac{1}{9} \)
   - SOLUTION: 
     \( \log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} \)
     \( = \log_3 3^{-2} \)
     \( = -2 \)
   - ANSWER: 
     -2

26. \( \log_4 \frac{1}{64} \)
   - SOLUTION: 
     \( \log_4 \frac{1}{64} = \log_4 \frac{1}{4^3} \)
     \( = \log_4 4^{-3} \)
     \( = -3 \)
   - ANSWER: 
     -3

27. \( \log_8 512 \)
   - SOLUTION: 
     \( \log_8 512 = \log_8 8^3 \)
     \( = 3 \)
   - ANSWER: 
     3

28. \( \log_6 216 \)
   - SOLUTION: 
     \( \log_6 216 = \log_6 6^3 \)
     \( = 3 \)
   - ANSWER: 
     3
29. \( \log_{27} 3 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
27^y &= 3 \\
3^3 &= 3^y \\
y &= 1 \\
y &= \frac{1}{3}
\end{align*}
\]

**ANSWER:** 
\( \frac{1}{3} \)

30. \( \log_{32} 2 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
32^y &= 2 \\
2^5 &= 2^y \\
y &= 1 \\
y &= \frac{1}{5}
\end{align*}
\]

**ANSWER:** 
\( \frac{1}{5} \)

31. \( \log_{9} 3 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
9^y &= 3 \\
3^2 &= 3^y \\
y &= 1 \\
y &= \frac{1}{2}
\end{align*}
\]

**ANSWER:** 
\( \frac{1}{2} \)

32. \( \log_{121} 11 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
121^y &= 11 \\
11^2 &= 11^y \\
y &= 1 \\
y &= \frac{1}{2}
\end{align*}
\]

**ANSWER:** 
\( \frac{1}{2} \)

33. \( \log_{\frac{1}{5}} 3125 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
\left( \frac{1}{5} \right)^y &= 3125 \\
5^-y &= 5^5 \\
y &= 5 \\
y &= -5
\end{align*}
\]

**ANSWER:** 
\( -5 \)

34. \( \log_{\frac{1}{8}} 512 \)

**SOLUTION:**
Let \( y \) be the unknown value.

\[
\begin{align*}
\left( \frac{1}{8} \right)^y &= 512 \\
8^-y &= 8^3 \\
y &= 3 \\
y &= -3
\end{align*}
\]

**ANSWER:** 
\( -3 \)
7-3 Logarithms and Logarithmic Functions

35. \( \log_{\frac{1}{3}} 81 \)

**SOLUTION:**
\[
\log_{\frac{1}{3}} 81 = \log_{\frac{1}{3}} \left( \frac{1}{3} \right)^4 \\
= \log_{\frac{1}{3}} \left( \frac{1}{3} \right)^4 \\
= 4
\]

**ANSWER:**
4

36. \( \log_{\frac{1}{6}} 216 \)

**SOLUTION:**
\[
\log_{\frac{1}{6}} 216 = \log_{\frac{1}{6}} \left( \frac{1}{6} \right)^3 \\
= \log_{\frac{1}{6}} \left( \frac{1}{6} \right)^3 \\
= 3
\]

**ANSWER:**
3

**CCSS PRECISION** Graph each function.
37. \( f(x) = \log_6 x \)

**SOLUTION:**
Plot the points \( \left( \frac{1}{6}, -1 \right), (1, 0), (6, 1) \) and sketch the graph.
38. \( f(x) = \log_{\frac{1}{5}} x \)

**SOLUTION:**

Plot the points \((5, -1), (1, 0), \left(\frac{1}{5}, 1\right)\) and sketch the graph.

![Graph of \( f(x) = \log_{\frac{1}{5}} x \)](image)

**ANSWER:**

![Graph of \( f(x) = \log_{\frac{1}{5}} x \)](image)

39. \( f(x) = 4 \log_2 x + 6 \)

**SOLUTION:**

The function represents a transformation of the graph of \( f(x) = \log_2 x \).

- \( a = 4 \): The graph expands vertically.
- \( h = 0 \): There is no horizontal shift.
- \( k = 6 \): The graph is translated 6 units up.

![Graph of \( f(x) = 4 \log_2 x + 6 \)](image)

**ANSWER:**

![Graph of \( f(x) = 4 \log_2 x + 6 \)](image)
40. \( f(x) = \log_9 x \)

SOLUTION:
Plot the points \((9, -1), (1, 0), \left(\frac{1}{9}, 1\right)\) and sketch the graph.

\[ \text{ANSWER:} \]

41. \( f(x) = \log_{10} x \)

SOLUTION:
Plot the points \(\left(\frac{1}{10}, -1\right), (1, 0), (10, 1)\) and sketch the graph.

\[ \text{ANSWER:} \]
42. \( f(x) = -3 \log_{\frac{1}{2}} x + 2 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{\frac{1}{2}} x \).

- \( a = -3 \): The graph is reflected across the \( x \)-axis.
- \( h = 0 \): There is no horizontal shift.
- \( k = 2 \): The graph is translated 2 units up.

**ANSWER:**

![Graph of \( f(x) = -3 \log_{\frac{1}{2}} x + 2 \)](image)

43. \( f(x) = 6 \log_{\frac{8}{10}} (x + 2) \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{\frac{8}{10}} x \).

- \( a = 6 \): The graph expands vertically.
- \( h = -2 \): The graph is translated 2 units to the left.
- \( k = 0 \): There is no vertical shift.

**ANSWER:**

![Graph of \( f(x) = 6 \log_{\frac{8}{10}} (x + 2) \)](image)
44. \( f(x) = -8 \log_3 (x - 4) \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_3 x \).

\( a = -8 \): The graph is reflected across the \( x \)-axis.
\( h = 4 \): The graph is translated 4 units to the right.
\( k = 0 \): There is no vertical shift.

**ANSWER:**

\[ f(x) = -8 \log_3 (x - 4) \]

![Graph of \( f(x) = -8 \log_3 (x - 4) \)](image)

45. \( f(x) = \log_{1/4} (x + 1) - 9 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{1/4} x \).

\( h = -1 \): The graph is translated 1 unit to the left.
\( k = -9 \): The graph is translated 9 units down.

**ANSWER:**

\[ f(x) = \log_{1/4} (x + 1) - 9 \]

![Graph of \( f(x) = \log_{1/4} (x + 1) - 9 \)](image)
7-3 Logarithms and Logarithmic Functions

46. \( f(x) = \log_5 (x - 4) - 5 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_3 x \).

\( h = 4 \): The graph is translated 4 units to the right.
\( k = -5 \): The graph is translated 5 units down.

**ANSWER:**

\( f(x) \)

\( \begin{array}{c|cccc} \\
-16 & -12 & -8 & -4 & 0 \\
4 & 8 & 12 & x & \\
\end{array} \)

47. \( f(x) = -\frac{1}{6} \log_8 (x - 3) + 4 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_3 x \).

\( a = -\frac{1}{6} \): The graph is reflected across the \( x \)-axis.
\( h = 3 \): The graph is translated 3 units to the right.
\( k = 4 \): The graph is translated 4 units up.
48. \( f(x) = -\frac{1}{3} \log_6 (x + 2) - 5 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_6 x \).

- \( a = -\frac{1}{3} \): The graph is reflected across the \( x \)-axis.
- \( h = -2 \): The graph is translated 2 units to the left.
- \( k = -5 \): The graph is translated 5 units down.

\[
\begin{align*}
\text{Graph of } f(x) & \\
& \\
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\text{Graph of } f(x) & \\
& \\
\end{align*}
\]

49. **PHOTOGRAPHY** The formula \( n = \log_2 \frac{1}{p} \) represents the change in the f-stop setting \( n \) to use in less light where \( p \) is the fraction of sunlight.

**a.** Benito’s camera is set up to take pictures in direct sunlight, but it is a cloudy day. If the amount of sunlight on a cloudy day is \( \frac{1}{4} \) as bright as direct sunlight, how many f-stop settings should he move to accommodate less light?

**b.** Graph the function.

**c.** Use the graph in part b to predict what fraction of daylight Benito is accommodating if he moves down 3 f-stop settings. Is he allowing more or less light into the camera?

**SOLUTION:**

- **a.** Substitute \( \frac{1}{4} \) for \( p \) in the formula and simplify.

\[
\begin{align*}
n & = \log_2 \frac{1}{p} \\
 & = \log_2 \frac{1}{\frac{1}{4}} \\
 & = \log_2 4 \\
 & = \log_2 2^2 \\
 & = 2
\end{align*}
\]

**b.**

\[
\begin{align*}
n & = \log_2 \frac{1}{p} \\
 & = \log_2 1 - \log_2 p \\
 & = 0 - \log_2 p \\
 & = -\log_2 p
\end{align*}
\]

The function represents a transformation of the graph of \( f(x) = \log_2 x \).

- \( a = -1 \): The graph is reflected across the \( x \)-axis.

\[
\begin{align*}
\text{Graph of } f(x) & \\
& \\
\end{align*}
\]

**c.**

Substitute 3 for \( n \) in the formula and solve for \( p \).

\[
\begin{align*}
3 & = \log_2 \frac{1}{p} \\
2^3 & = \frac{1}{p} \\
p & = \frac{1}{8}
\end{align*}
\]

As \( \frac{1}{4} > \frac{1}{8} \), he is allowing less light into the camera.
7-3 Logarithms and Logarithmic Functions

**ANSWER:**

a. 2

b. 

<table>
<thead>
<tr>
<th>x</th>
<th>y = -2.5(5)^x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.875</td>
</tr>
<tr>
<td>2</td>
<td>4.375</td>
</tr>
<tr>
<td>3</td>
<td>21.875</td>
</tr>
<tr>
<td>4</td>
<td>109.375</td>
</tr>
</tbody>
</table>

**ANSWER:**

Graph each function.

51. \( f(x) = 4 \log_2 (2x - 4) + 6 \)

**SOLUTION:**

The function represents a transformation of the graph of \( f(x) = \log_2 2x \).

\( a = 4 \): The graph expands vertically.
\( h = 4 \): The graph is translated 4 units to the right.
\( k = 6 \): The graph is translated 6 units up.

50. EDUCATION

To measure a student’s retention of knowledge, the student is tested after a given amount of time. A student’s score on an Algebra 2 test \( t \) months after the school year is over can be approximated by

\[ y(t) = 85 - 6 \log_2 (t + 1) \]

where \( y(t) \) is the student’s score as a percent.

**a.** What was the student’s score at the time the school year ended \( (t = 0) \)?

**b.** What was the student’s score after 3 months?

**c.** What was the student’s score after 15 months?

**SOLUTION:**

**a.** Substitute 0 for \( t \) in the function and simplify.

\[ y(t) = 85 - 6 \log_2 (0 + 1) \]
\[ = 85 - 6 \log_2 1 \]
\[ = 85 - 0 \]
\[ = 85 \]

**b.** Substitute 2 for \( t \) in the function and simplify.

\[ y(t) = 85 - 6 \log_2 (3 + 1) \]
\[ = 85 - 6 \log_2 4 \]
\[ = 85 - 6 \times 2 \]
\[ = 73 \]

**c.** Substitute 15 for \( t \) in the function and simplify.

\[ y(t) = 85 - 6 \log_2 (15 + 1) \]
\[ = 85 - 6 \log_2 16 \]
\[ = 85 - 6 \times 4 \]
\[ = 61 \]

**ANSWER:**

a. 85

b. 73

c. 61
7-3 Logarithms and Logarithmic Functions

52. \( f(x) = -3 \log_{12} (4x + 3) + 2 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{12} 4x \).

\( a = -3 \): The graph is reflected across the \( x \)-axis.
\( h = -3 \): The graph is translated 3 units to the left.
\( k = 2 \): The graph is translated 2 units up.

**ANSWER:**

53. \( f(x) = 15 \log_{14} (x + 1) - 9 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_{14} x \).

\( a = 15 \): The graph expands vertically.
\( h = -1 \): The graph is translated 1 unit to the left.
\( k = -9 \): The graph is translated 9 units down.
54. \( f(x) = 10 \log_5 (x - 4) - 5 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_5 x \).

- \( a = 10 \): The graph expands vertically.
- \( h = 4 \): The graph is translated 4 units to the right.
- \( k = -5 \): The graph is translated 5 units down.

**ANSWER:**

55. \( f(x) = -\frac{1}{6} \log_8 (x - 3) + 4 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_8 x \).

- \( a = -\frac{1}{6} \): The graph is reflected across the \( x \)-axis.
- \( h = 4 \): The graph is translated 4 units to the right.
- \( k = -5 \): The graph is translated 5 units down.

**ANSWER:**
56. \( f(x) = -\frac{1}{3} \log_8 (6x + 2) - 5 \)

**SOLUTION:**
The function represents a transformation of the graph of \( f(x) = \log_8 6x \).

\( a = -\frac{1}{3} \): The graph is reflected across the \( x \)-axis.
\( h = -2 \): The graph is translated 2 units to the left.
\( k = -5 \): The graph is translated 5 units down.

**ANSWER:**

57. **CCSS MODELING** In general, the more money a company spends on advertising, the higher the sales. The amount of money in sales for a company, in thousands, can be modeled by the equation \( S(a) = 10 + 20 \log_4 (a + 1) \), where \( a \) is the amount of money spent on advertising in thousands, when \( a \geq 0 \).

a. The value of \( S(0) \approx 10 \), which means that if $10 is spent on advertising, $10,000 is returned in sales. Find the values of \( S(3), S(15), \) and \( S(63) \).

b. Interpret the meaning of each function value in the context of the problem.

c. Graph the function.

d. Use the graph in part c and your answers from part a to explain why the money spent in advertising becomes less “efficient” as it is used in larger amounts.

**SOLUTION:**
a.
Substitute 3 for \( a \) in the equation and simplify.

\[ s(3) = 10 + 20 \log_4 (3 + 1) = 30 \]

Substitute 15 for \( a \) in the equation and simplify.

\[ s(15) = 10 + 20 \log_4 (15 + 1) = 50 \]

Substitute 63 for \( a \) in the equation and simplify.

\[ s(63) = 10 + 20 \log_4 (63 + 1) = 70 \]

b. If $3000 is spent on advertising, $30,000 is returned in sales. If $15,000 is spent on advertising, $50,000 is returned in sales. If $63,000 is spent on advertising, $70,000 is returned in sales.

c.
The function represents a transformation of the graph of \( f(x) = \log_4 x \).

\( a = 20 \): The graph is expanded vertically.
\( h = -1 \): The graph is translated 1 unit to the left.
\( k = 10 \): The graph is translated 10 units up.
Because eventually the graph plateaus and no matter how much money you spend you are still returning about the same in sales.

**ANSWER:**

a. S(3) = 30, S(15) = 50, S(63) = 70

b. If $3000 is spent on advertising, $30,000 is returned in sales. If $15,000 is spent on advertising, $50,000 is returned in sales. If $63,000 is spent on advertising, $70,000 is returned in sales.

c.

d. Because eventually the graph plateaus and no matter how much money you spend you are still returning about the same in sales.

58. **BIOLOGY** The generation time for bacteria is the time that it takes for the population to double. The generation time $G$ for a specific type of bacteria can be found using experimental data and the formula 

$$G = \frac{t}{3.3 \log_b f},$$

where $t$ is the time period, $b$ is the number of bacteria at the beginning of the experiment, and $f$ is the number of bacteria at the end of the experiment.

a. The generation time for mycobacterium tuberculosis is 16 hours. How long will it take four of these bacteria to multiply into 1024 bacteria?

b. An experiment involving rats that had been exposed to salmonella showed that the generation time for the salmonella was 5 hours. After how long would 20 of these bacteria multiply into 8000?

c. E. coli are fast growing bacteria. If 6 e. coli can grow to 1296 in 4.4 hours, what is the generation time of e. coli?

**SOLUTION:**

a. Substitute $G = 16$, $b = 4$, and $f = 1024$ into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$

$$16 = \frac{t}{3.3 \log_4 1024}$$

$$52.8 \log_4 1024 = t$$

$$52.8 \cdot 5 = t$$

$$264 = t$$

Therefore, $t = 264$ hours or 11 days.

b. Substitute $G = 5$, $b = 20$, and $f = 8000$ into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$

$$5 = \frac{t}{3.3 \log_{20} 8000}$$

$$16.5 \log_{20} 8000 = t$$

$$16.5 \cdot 3 = t$$

$$49.5 = t$$

Therefore, $t = 49.5$ hours or about 2 days 1.5 hours.

c. Substitute $t = 4.4$, $b = 6$, and $f = 1296$ into the bacterial growth formula.

$$G = \frac{t}{3.3 \log_b f}$$

$$= \frac{4.4}{3.3 \log_6 1296}$$

$$= \frac{4.4}{3.3 \cdot 4}$$

$$\approx 0.333$$

Therefore, $G = \frac{1}{3}$ hour or 20 minutes.

**ANSWER:**

a. 264 h or 11 days
b. 49.5 h or about 2 days 1.5 h
c. $\frac{1}{3}$ h or 20 min
59. FINANCIAL LITERACY Jacy has spent $2000 on a credit card. The credit card company charges 24% interest, compounded monthly. The credit card company uses \( \log \left( \frac{A}{2000} \right) = 12t \) to determine how much time it will be until Jacy’s debt reaches a certain amount, if \( A \) is the amount of debt after a period of time, and \( t \) is time in years.

a. Graph the function for Jacy’s debt.

b. Approximately how long will it take Jacy’s debt to double?

c. Approximately how long will it be until Jacy’s debt triples?

**SOLUTION:**

a. Start by solving the given equation for \( A \) to obtain the function for Lacy’s debt.

\[
\log \left( \frac{1 + 0.24}{12} \right) \frac{A}{2000} = 12t \\
\log 1.02 \frac{A}{2000} = 12t \\
\frac{A}{2000} = 1.02^{12t} \\
A = 2000 \cdot 1.02^{12t}
\]

Make a table of values. Then plot the points, and sketch the graph.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( A = 2000 \cdot 1.02^{12t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( A = 2000 \cdot 1.02^{0} = 2000 )</td>
</tr>
<tr>
<td>2</td>
<td>( A = 2000 \cdot 1.02^{24} \approx 3217 )</td>
</tr>
<tr>
<td>4</td>
<td>( A = 2000 \cdot 1.02^{48} \approx 5174 )</td>
</tr>
<tr>
<td>6</td>
<td>( A = 2000 \cdot 1.02^{72} \approx 8322 )</td>
</tr>
<tr>
<td>8</td>
<td>( A = 2000 \cdot 1.02^{96} \approx 13396 )</td>
</tr>
<tr>
<td>10</td>
<td>( A = 2000 \cdot 1.02^{120} \approx 21530 )</td>
</tr>
</tbody>
</table>

b. From the graph, \( A = 4000 \) at about \( t = 3 \). So, it will take approximately 3 years for the debt to double.

c. From the graph, \( A = 6000 \) at about \( t = 4.5 \). So, it will take approximately 4.5 years for the debt to triple.

**ANSWER:**

b. \( \approx 3 \) years

c. \( \approx 4.5 \) years

60. WRITING IN MATH What should you consider when using exponential and logarithmic models to make decisions?

**SOLUTION:**

Sample answer: Exponential and logarithmic models can grow without bound, which is usually not the case of the situation that is being modeled. For instance, a population cannot grow without bound due to space and food constraints. Therefore, when using a model to make decisions, the situation that is being modeled should be carefully considered.

**ANSWER:**

Sample answer: Exponential and logarithmic models can grow without bound, which is usually not the case of the situation that is being modeled. For instance, a population cannot grow without bound due to space and food constraints. Therefore, when using a model to make decisions, the situation that is being modeled should be carefully considered.
61. **CCSS ARGUMENTS** Consider \( y = \log_b x \) in which \( b, x, \) and \( y \) are real numbers. Zero can be in the domain *sometimes, always* or *never*. Justify your answer.

**SOLUTION:**
Never; if zero were in the domain, the equation would be \( y = \log_b 0 \). Then \( b^y = 0 \). However, for any real number \( b \), there is no real power that would let \( b^y = 0 \)

**ANSWER:**
Never; if zero were in the domain, the equation would be \( y = \log_b 0 \). Then \( b^y = 0 \). However, for any real number \( b \), there is no real power that would let \( b^y = 0 \)

62. **ERROR ANALYSIS** Betsy says that the graphs of all logarithmic functions cross the \( y \)-axis at \((0, 1)\) because any number to the zero power equals 1. Tyrone disagrees. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Tyrone; sample answer: The graphs of logarithmic functions pass through \((1, 0)\) not \((0, 1)\).

**ANSWER:**
Tyrone; sample answer: The graphs of logarithmic functions pass through \((1, 0)\) not \((0, 1)\).

63. **REASONING** Without using a calculator, compare \( \log_7 51 \), \( \log_8 61 \), and \( \log_9 71 \). Which of these is the greatest? Explain your reasoning.

**SOLUTION:**
\( \log_7 51 \); Sample answer: \( \log_7 51 \) equals a little more than 2. \( \log_8 61 \) equals a little less than 2. \( \log_9 71 \) equals a little less than 2. Therefore, \( \log_7 51 \) is the greatest.

**ANSWER:**
\( \log_7 51 \); sample answer: \( \log_7 51 \) equals a little more than 2. \( \log_8 61 \) equals a little less than 2. \( \log_9 71 \) equals a little less than 2. Therefore, \( \log_7 51 \) is the greatest.

64. **OPEN ENDED** Write a logarithmic expression of the form \( y = \log_b x \) for each of the following conditions.

a. \( y \) is equal to 25.
b. \( y \) is negative.
c. \( y \) is between 0 and 1.
d. \( x \) is 1.
e. \( x \) is 0.

**SOLUTION:**
Sample answers:
a. \( \log_2 3,554,432 = 25 \);
b. \( \log_4 \frac{1}{64} = -3 \);
c. \( \log_2 \sqrt{2} = \frac{1}{2} \)
d. \( \log_7 1 = 0 \);
e. There is no possible solution; this is the empty set.

**ANSWER:**
Sample answers:
a. \( \log_2 3,554,432 = 25 \);
b. \( \log_4 \frac{1}{64} = -3 \);
c. \( \log_2 \sqrt{2} = \frac{1}{2} \)
d. \( \log_7 1 = 0 \);
e. There is no possible solution; this is the empty set.
65. **FIND THE ERROR** Elisa and Matthew are evaluating \( \log_7 49 \). Is either of them correct? Explain your reasoning.

**Elisa**

\[
\log_7 49 = y \\
7^y = 49 \\
(7^{-1})^y = 7^2 \\
7^y = 7^2 \\
y = 2
\]

**Matthew**

\[
\log_7 49 = y \\
49^y = \frac{7}{7} \\
(7^2)^y = (7)^{-1} \\
7^2y = (7)^{-1} \\
2y = -1 \\
y = -\frac{1}{2}
\]

**SOLUTION:**
No; Elisa was closer. She should have \(-y = 2\) or \(y = -2\) instead of \(y = 2\). Matthew used the definition of logarithms incorrectly.

**ANSWER:**
No; Elisa was closer. She should have \(-y = 2\) or \(y = -2\) instead of \(y = 2\). Matthew used the definition of logarithms incorrectly.

66. **WRITING IN MATH** A transformation of \( \log_{10} x \) is \( g(x) = a \log_{10} (x - h) + k \). Explain the process of graphing this transformation.

**SOLUTION:**
Sample answer: In \( g(x) = a \log_{10} (x - h) + k \), the value of \( k \) is a vertical translation and the graph will shift up \( k \) units if \( k \) is positive and down \( |k| \) units if \( k \) is negative. The value of \( h \) is a horizontal translation and the graph will shift \( h \) units to the right if \( h \) is positive and \( |h| \) units to the left if \( h \) is negative. If \( a < 0 \), the graph will be reflected across the \( x \)-axis. If \( |a| > 1 \), the graph will be expanded vertically and if \( 0 < |a| < 1 \), then the graph will be compressed vertically.

**ANSWER:**
Sample answer: In \( g(x) = a \log_{10} (x - h) + k \), the value of \( k \) is a vertical translation and the graph will shift up \( k \) units if \( k \) is positive and down \( |k| \) units if \( k \) is negative. The value of \( h \) is a horizontal translation and the graph will shift \( h \) units to the right if \( h \) is positive and \( |h| \) units to the left if \( h \) is negative. If \( a < 0 \), the graph will be reflected across the \( x \)-axis. If \( |a| > 1 \), the graph will be expanded vertically and if \( 0 < |a| < 1 \), then the graph will be compressed vertically.

67. A rectangle is twice as long as it is wide. If the width of the rectangle is 3 inches, what is the area of the rectangle in square inches?

A 9 
B 12 
C 15 
D 18

**SOLUTION:**
Length of the rectangle = \( 2 \times 3 = 6 \) inches. 
Area of the rectangle = \( 6 \times 3 = 18 \) square inches. 
D is the correct option.

**ANSWER:**
D
68. **SAT/ACT** Ichiro has some pizza. He sold 40% more slices than he ate. If he sold 70 slices of pizza, how many did he eat?

F 25
G 50
H 75
J 98
K 100

**SOLUTION:**
Let \( x \) be the number of pizza slices Ichiro ate. The equation that represents the situation is:

\[
\begin{align*}
x + 0.4x &= 70 \\
1.4x &= 70 \\
x &= 50
\end{align*}
\]

G is the correct answer.

**ANSWER:**
G

69. **SHORT RESPONSE** In the figure \( AB = BC, CD = BD, \) and \( \angle CAD = 70^\circ \). What is the measure of \( \angle ADC? \)

**SOLUTION:**
\( \triangle ABC \) and \( \triangle DBC \) are isosceles triangles.
In \( \triangle ABC \), \( \angle BCA = 70^\circ \) and \( \angle ABC = 40^\circ \).
In \( \triangle DBC \), \( \angle DBC = 40^\circ \) and \( \angle BCD = 40^\circ \).
So, \( \angle ACD = 30^\circ \).
Thus, \( \angle ADC = 80^\circ \).

**ANSWER:**
80

70. If \( 6x - 3y = 30 \) and \( 4x = 2 - y \) then find \( x + y \).

A -4
B -2
C 2
D 4

**SOLUTION:**

\[
\begin{align*}
6x - 3y &= 30 \quad \text{(1)} \\
4x &= 2 - y \quad \text{(2)}
\end{align*}
\]

Solve (2) for \( y \).

\[
\begin{align*}
4x &= 2 - y \\
4x - 2 &= -y \\
y &= -4x + 2
\end{align*}
\]

Substitute \( y = -4x + 2 \) in (1) and solve for \( x \).

\[
\begin{align*}
6x - 3(-4x + 2) &= 30 \\
6x + 12x - 6 &= 30 \\
18x &= 36 \\
x &= 2
\end{align*}
\]

Substitute \( x = 2 \) in \( y = -4x + 2 \) and simplify.

\[
\begin{align*}
y &= -4(2) + 2 \\
&= -6
\end{align*}
\]

Thus, \( x + y = -4 \).

A is the correct answer.

**ANSWER:**
A

71. \( 3^n - 2 > 27 \)

**SOLUTION:**

\[
\begin{align*}
3^n - 2 &> 27 \\
3^n &> 3^3 \\
n &> 3 \\
n &> 5
\end{align*}
\]

**ANSWER:**
\( \{n | n > 5 \} \)
7-3 Logarithms and Logarithmic Functions

72. $2^{2n} \leq \frac{1}{16}$

**SOLUTION:**

\[2^{2n} \leq \frac{1}{16}\]

\[2^{2n} \leq 2^{-4}\]

\[2n \leq -4\]

\[n \leq -2\]

**ANSWER:**

\[\{n| n \leq -2\}\]

73. $16^n < 8^n + 1$

**SOLUTION:**

\[16^n < 8^n + 1\]

\[2^{4n} < 2^{3n+3}\]

\[4n < 3n + 3\]

\[n < 3\]

**ANSWER:**

\[\{n| n < 3\}\]

74. $32^{5p} + 2 \geq 16^{5p}$

**SOLUTION:**

\[32^{5p+2} \geq 16^{5p}\]

\[2^{5p+10} \geq 2^{20p}\]

\[5p + 10 \geq 20p\]

\[5p \geq -10\]

\[p \geq -2\]

**ANSWER:**

\[\{p| p \geq -2\}\]

---

**Graph each function.**

75. $y = -\left(\frac{1}{5}\right)^x$

**SOLUTION:**

Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-0.04</td>
</tr>
<tr>
<td>4</td>
<td>-0.0016</td>
</tr>
<tr>
<td>6</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

---

**eSolutions Manual - Powered by Cognero**
7-3 Logarithms and Logarithmic Functions

76. \( y = -2.5(5)^x \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-6 & -0.002 \\
-4 & -0.004 \\
-1 & -0.5 \\
0 & -2.5 \\
1 & -12.5 \\
\hline
\end{array}
\]

**ANSWER:**

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-1 & 30 \\
0 & 1 \\
2 & 0.0011 \\
4 & 0 \\
6 & 0 \\
\hline
\end{array}
\]

77. \( y = 30^{-x} \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.
7-3 Logarithms and Logarithmic Functions

78. \( y = 0.2(5)^{-x} \)

**SOLUTION:**
Make a table of values. Then plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>25</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.0080</td>
</tr>
<tr>
<td>4</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

**ANSWER:**

79. **GEOMETRY** The area of a triangle with sides of length \( a, b, \) and \( c \) is given by
\[
\sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{1}{2}(a+b+c)
\]
If the lengths of the sides of a triangle are 6, 9, and 12 feet, what is the area of the triangle expressed in radical form?

**SOLUTION:**
\[
s = \frac{1}{2}(a+b+c)
= \frac{1}{2}(6+9+12)
= \frac{27}{2}
\]
Area of the triangle:
\[
\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{27}{2} \cdot \frac{6}{2} \cdot \frac{9}{2} \cdot \frac{12}{2}}
= \frac{27\sqrt{15}}{4} \text{ ft}^2
\]

**ANSWER:**
\[
\frac{27\sqrt{15}}{4} \text{ ft}^2
\]
7-3 Logarithms and Logarithmic Functions

80. GEOMETRY The volume of a rectangular box can be written as \(6x^3 + 31x^2 + 53x + 30\) when the height is \(x + 2\).

a. What are the width and length of the box?

b. Will the ratio of the dimensions of the box always be the same regardless of the value of \(x\)? Explain.

**SOLUTION:**

a. Divide \(6x^3 + 31x^2 + 53x + 30\) by \(x + 2\).

\[
\begin{array}{c|cccc}
-2 & 6 & 31 & 53 & 30 \\
0 & -12 & -38 & -30 & \\
6 & 19 & 15 & |0 & \\
\end{array}
\]

\[6x^3 + 31x^2 + 53x + 30 = (x + 2)(6x^2 + 19x + 15)\]

\[= (x + 2)(2x + 3)(3x + 5)\]

So, the width and length of the rectangular box are \(2x + 3\) and \(3x + 5\).

b. No; for example, if \(x = 1\), the ratio is 3:5:8, but if \(x = 2\), the ratio is 4:7:11. The ratios are not equivalent.

**ANSWER:**

a. \(2x + 3\) and \(3x + 5\)

b. No; for example, if \(x = 1\), the ratio is 3:5:8, but if \(x = 2\), the ratio is 4:7:11. The ratios are not equivalent.

81. AUTO MECHANICS Shandra is inventory manager for a local repair shop. She orders 6 batteries, 5 cases of spark plugs, and two dozen pairs of wiper blades and pays $830. She orders 3 batteries, 7 cases of spark plugs, and four dozen pairs of wiper blades and pays $820. The batteries are $22 less than twice the price of a dozen wiper blades. Use augmented matrices to determine what the cost of each item on her order is.

**SOLUTION:**

The augmented matrix that represents the situation is:

\[
\begin{bmatrix}
6 & 5 & 2 & 830 \\
3 & 7 & 4 & 820 \\
1 & 0 & -2 & -22
\end{bmatrix}
\]

Use the graphing calculator to solve the system.

**KEYSTROKES:** 2ND [MATRIX] ► ► ENTER 3 ENTER 4 ENTER 6 ENTER 5 ENTER 2 ENTER 830 ENTER 3 ENTER 7 ENTER 4 ENTER 820 ENTER 1 ENTER 0 ENTER (-) 2 ENTER (-) 22 ENTER

Find the reduced row echelon form (rref).

**KEYSTROKES:** 2ND [QUIT] 2ND [MATRIX] ► ALPHA [B] 2ND [MATRIX] ENTER ) ENTER

\[
\begin{bmatrix}
1 & 0 & 0 & 74 \\
0 & 1 & 0 & 58 \\
0 & 0 & 1 & 48
\end{bmatrix}
\]

The first three columns are the same as a 3 × 3 identity matrix. Thus, batteries cost $74, spark plugs costs $58 and wiper blades costs $48.

**ANSWER:**
batteries, $74; spark plugs, $58; wiper blades, $48
7-3 Logarithms and Logarithmic Functions

Solve each equation or inequality. Check your solution.

82. \(9^x = \frac{1}{81}\)

**SOLUTION:**

\(9^x = \frac{1}{81}\)

\(9^x = 9^{-2}\)

\(x = -2\)

**ANSWER:**

\(-2\)

83. \(2^{6x} = 4^{5x} + 2\)

**SOLUTION:**

\(2^{6x} = 4^{5x} + 2\)

\(2^{6x} = 2^{10x + 4}\)

\(6x = 10x + 4\)

\(-4x = 4\)

\(x = -1\)

**ANSWER:**

\(-1\)

84. \(49^{3p} + 1 = 7^{2p} - 5\)

**SOLUTION:**

\(49^{3p} + 1 = 7^{2p} - 5\)

\(7^{6p + 2} = 7^{2p} - 5\)

\(6p + 2 = 2p - 5\)

\(4p = -7\)

\(p = -\frac{7}{4}\)

**ANSWER:**

\(-\frac{7}{4}\)

85. \(9^x \leq 27^{x^2 - 2}\)

**SOLUTION:**

\(9^x \leq 27^{x^2 - 2}\)

\(3^{2x} \leq 3^{3x^2 - 6}\)

\(2x \leq 3x^2 - 6\)

\(x^2 - 6\)

\(x \leq \pm\sqrt{6}\)

**ANSWER:**

\(\{x | x \leq -\sqrt{6} \text{ or } x \geq \sqrt{6}\}\)
Solve each equation.

1. \( \log_8 x = \frac{4}{3} \)

**SOLUTION:**

\[ \log_8 x = \frac{4}{3} \]

\[ x = 8^{\frac{4}{3}} \]

\[ = (2^3)^{\frac{4}{3}} \]

\[ = 2^4 \]

\[ = 16 \]

**ANSWER:** 16

2. \( \log_{10} x = \frac{3}{4} \)

**SOLUTION:**

\[ \log_{10} x = \frac{3}{4} \]

\[ x = 16^{\frac{3}{4}} \]

\[ = (2^4)^{\frac{3}{4}} \]

\[ = 2^3 \]

\[ = 8 \]

**ANSWER:** 8

3. **MULTIPLE CHOICE** Solve \( \log_5 (x^2 - 10) = \log_5 3x \)

   **A** 10  
   **B** 2  
   **C** 5  
   **D** 2, 5

**SOLUTION:**

\[ \log_5 (x^2 - 10) = \log_5 3x \]

\[ x^2 - 10 = 3x \]

\[ x^2 - 3x - 10 = 0 \]

\[ (x - 5)(x + 2) = 0 \]

\[ x - 5 = 0 \quad \text{or} \quad x + 2 = 0 \]

\[ x = 5 \quad x = -2 \]

Substitute each value into the original equation.

\[ \begin{array}{c|c}
   & \hline \\
   x = 5 & x = -2 \\
   \log_5 (5^2 - 10) = & \log_5 (3 \cdot 5) \\
   \log_5 (5^2 - 10) = & \log_5 (3 \cdot (-2)) \\
   \log_5 15 = & \log_5 15 \\
   \log_5 - 6 = & \log_5 - 6 \\
   \end{array} \]

The domain of a logarithmic function cannot be 0, so \( \log_5 (-6) \) is undefined and -2 is an extraneous solution.

**C** is the correct option.

**ANSWER:**  

C

Solve each inequality.

4. \( \log_5 x > 3 \)

**SOLUTION:**

\[ \log_5 x > 3 \]

\[ x > 5^3 \]

\[ x > 125 \]

Thus, solution set is \( \{ x \mid x > 125 \} \).

**ANSWER:**  

\( \{ x \mid x > 125 \} \)
5. \( \log_{8} x \leq -2 \)

**SOLUTION:**
\[
\log_{8} x \leq -2 \\
x \leq 8^{-2} \\
x \leq \frac{1}{64}
\]

Thus, solution set is \( \{ x \mid 0 < x \leq \frac{1}{64} \} \).

**ANSWER:**
\( \{ x \mid 0 < x \leq \frac{1}{64} \} \)

6. \( \log_{4} (2x + 5) \leq \log_{4} (4x - 3) \)

**SOLUTION:**
\[
\log_{4} (2x + 5) \leq \log_{4} (4x - 3) \\
2x + 5 \leq 4x - 3 \\
2x \geq 8 \\
x \geq 4
\]

Thus, solution set is \( \{ x \mid x \geq 4 \} \).

**ANSWER:**
\( \{ x \mid x \geq 4 \} \)

7. \( \log_{8} (2x) > \log_{8} (6x - 8) \)

**SOLUTION:**
\[
\log_{8} (2x) > \log_{8} (6x - 8) \\
2x > 6x - 8 \\
4x < 8 \\
x < 2
\]

Exclude all values of \( x \) for which \( 2x \leq 0 \) or \( 6x - 8 \leq 0 \).

So, \( x > 0 \), \( x > \frac{4}{3} \) and \( x < 2 \).

Thus, solution set is \( \{ x \mid 2 > x > \frac{4}{3} \} \).

**ANSWER:**
\( \{ x \mid 2 > x > \frac{4}{3} \} \)

**CCSS STRUCTURE** Solve each equation.

8. \( \log_{81} x = \frac{3}{4} \)

**SOLUTION:**
\[
\log_{81} x = \frac{3}{4} \\
x = 81^{\frac{3}{4}} \\
= (3^{4})^{\frac{3}{4}} \\
= 3^{3} \\
= 27
\]

**ANSWER:**
27
9. \( \log_{25} x = \frac{5}{2} \)

**SOLUTION:**

\[
\log_{25} x = \frac{5}{2} \\
x = 25^{\frac{5}{2}} \\
= (5^2)^{\frac{5}{2}} \\
= 5^5 \\
= 3125
\]

**ANSWER:** 3125

10. \( \log_{8} \frac{1}{2} = x \)

**SOLUTION:**

\[
\log_{8} \frac{1}{2} = x \\
8^x = \frac{1}{2} \\
2^{3x} = 2^{-1} \\
3x = -1 \\
x = -\frac{1}{3}
\]

**ANSWER:** \(-\frac{1}{3}\)

11. \( \log_{9} \frac{1}{36} = x \)

**SOLUTION:**

\[
\log_{9} \frac{1}{36} = x \\
6^x = \frac{1}{36} \\
6^x = 6^{-2} \\
x = -2
\]

**ANSWER:** \(-2\)

12. \( \log_{x} 32 = \frac{5}{2} \)

**SOLUTION:**

\[
\log_{x} 32 = \frac{5}{2} \\
x^\frac{5}{2} = 32 \\
\left( x^2 \right)^{\frac{5}{2}} = 2^5 \\
\frac{1}{x^3} = 2 \\
x = 4
\]

**ANSWER:** 4

13. \( \log_{x} 27 = \frac{3}{2} \)

**SOLUTION:**

\[
\log_{x} 27 = \frac{3}{2} \\
x^\frac{3}{2} = 27 \\
\left( x^2 \right)^{\frac{3}{2}} = 3^3 \\
\frac{1}{x^3} = 3 \\
x = 9
\]

**ANSWER:** 9

---

**7-4 Solving Logarithmic Equations and Inequalities**
7-4 Solving Logarithmic Equations and Inequalities

14. \( \log_3 (3x + 8) = \log_3 (x^2 + x) \)

**SOLUTION:**
\[
\begin{align*}
\log_3 (3x + 8) & = \log_3 (x^2 + x) \\
3x + 8 & = x^2 + x \\
x^2 - 2x - 8 & = 0 \\
(x - 4)(x + 2) & = 0
\end{align*}
\]
\[
x - 4 = 0 \quad \text{or} \quad x + 2 = 0
\]
\[
x = 4 \quad \text{or} \quad x = -2
\]
Substitute each value into the original equation.
\[
\begin{array}{c|c}
\text{x} & \text{log}_{3}(3.4+8) = \log_{3}(4^2+4) \quad \text{log}_{3}(3.4+8) = \log_{3}((-2)^2+(-2)) \\
4 & \log_{20} 20 = \log_{20} 20 \\
-2 & \log_{20} 2 = \log_{20} 2
\end{array}
\]
Thus, \( x = -2 \) or 4.

**ANSWER:**
-2 or 4

15. \( \log_{12} (x^2 - 7) = \log_{12} (x + 5) \)

**SOLUTION:**
\[
\begin{align*}
\log_{12} (x^2 - 7) & = \log_{12} (x + 5) \\
x^2 - 7 & = x + 5 \\
x^2 - x - 12 & = 0 \\
(x - 4)(x + 3) & = 0
\end{align*}
\]
\[
x - 4 = 0 \quad \text{or} \quad x + 3 = 0
\]
\[
x = 4 \quad \text{or} \quad x = -3
\]
Substitute each value into the original equation.
\[
\begin{array}{c|c}
\text{x} & \text{log}_{12}(4^2-7) = \log_{12}(4+5) \quad \text{log}_{12}(4^2-7) = \log_{12}((-3)^2-7) \\
4 & \log_{12}9 = \log_{12}9 \\
-3 & \log_{12}2 = \log_{12}2
\end{array}
\]
Thus, \( x = -3 \) or 4.

**ANSWER:**
4 or -3

16. \( \log_6 (x^2 - 6x) = \log_6 (-8) \)

**SOLUTION:**
\[
\begin{align*}
\log_6 (x^2 - 6x) & = \log_6 (-8) \\
x^2 - 6x & = -8 \\
x^2 - 6x + 8 & = 0 \\
(x - 4)(x - 2) & = 0
\end{align*}
\]
\[
x - 4 = 0 \quad \text{or} \quad x - 2 = 0
\]
\[
x = 4 \quad \text{or} \quad x = 2
\]
Substitute each value into the original equation.
\[
\begin{array}{c|c}
\text{x} & \text{log}_{6}(-8) \quad \text{log}_{6}(-8) \quad \text{log}_{6}(-8) \\
4 & \log_{6}(-8) = \log_{6}(-8) \\
2 & \log_{6}(-8) = \log_{6}(-8)
\end{array}
\]
\( \log_6 (-8) \) is undefined, so 4 and 2 are extraneous solutions.
Thus, no solution.

**ANSWER:**
no solution
17. \( \log_9 (x^2 - 4x) = \log_9 (3x - 10) \)

**SOLUTION:**
\[
\begin{align*}
\log_9 (x^2 - 4x) &= \log_9 (3x - 10) \\
x^2 - 4x &= 3x - 10 \\
x^2 - 7x + 10 &= 0 \\
(x - 5)(x - 2) &= 0
\end{align*}
\]
\[x - 5 = 0 \quad \text{or} \quad x - 2 = 0 \]
\[x = 5 \quad \text{or} \quad x = 2 \]

Substitute each value into the original equation.

\[
\begin{align*}
\log_9 (5^2 - 4 \cdot 5) &= \log_9 (3 \cdot 2 - 10) \\
\log_9 (25 - 20) &= \log_9 (6 - 10) \\
\log_9 5 &= \log_9 5 \\
\log_9 (2^2 - 4 \cdot 2) &= \log_9 (3^2 - 10) \\
\log_9 4 &= \log_9 9 \\
\log_9 (x - 5)(x - 2) &= \log_9 (x - 5)(x - 2)
\end{align*}
\]

\( \log_9 (-4) \) is undefined and 2 is extraneous solution.
Thus, \( x = 5 \).

**ANSWER:**

\[
5
\]

18. \( \log_4 (2x^2 + 1) = \log_4 (10x - 7) \)

**SOLUTION:**
\[
\begin{align*}
\log_4 (2x^2 + 1) &= \log_4 (10x - 7) \\
2x^2 + 1 &= 10x - 7 \\
2x^2 - 10x + 8 &= 0 \\
(x - 4)(x - 1) &= 0
\end{align*}
\]
\[x - 4 = 0 \quad \text{or} \quad x - 1 = 0 \\
\]
\[x = 4 \quad \text{or} \quad x = 1 \]

Substitute each value into the original equation.

\[
\begin{align*}
\log_4 (2(4)^2 + 1) &= \log_4 (10(4) - 7) \\
\log_4 33 &= \log_4 33 \\
\log_4 (2(1)^2 + 1) &= \log_4 (10(1) - 7) \\
\log_4 3 &= \log_4 3
\end{align*}
\]

Thus, \( x = 1 \) or 4.

**ANSWER:**

1 or 4

19. \( \log_7 (x^2 - 4) = \log_7 (-x + 2) \)

**SOLUTION:**
\[
\begin{align*}
\log_7 (x^2 - 4) &= \log_7 (-x + 2) \\
x^2 - 4 &= -x + 2 \\
x^2 + x - 6 &= 0 \\
(x + 3)(x - 2) &= 0
\end{align*}
\]
\[x + 3 = 0 \quad \text{or} \quad x - 2 = 0 \\
\]
\[x = -3 \quad \text{or} \quad x = 2 \]

Substitute each value into the original equation.

\[
\begin{align*}
\log_7 (-3)^2 - 4) &= \log_7 (-(-3) + 2) \\
\log_7 5 &= \log_7 5 \\
\log_7 (2^2 - 4) &= \log_7 (-2 + 2) \\
\log_7 0 &= \log_7 0
\end{align*}
\]

Since you can not have a log of 0, \( x = -3 \) is the solution.

**ANSWER:**

\(-3\)

20. Write this equation in exponential form.

**SOLUTION:**
\[
\begin{align*}
w &= 93 \log_{10} d + 65 \\
w - 65 &= 93 \log_{10} d \\
\frac{w - 65}{93} &= \log_{10} d \\
\log_{10} \left( \frac{w - 65}{93} \right) &= d \\
d &= 10^{\frac{w - 65}{93}}
\end{align*}
\]

**ANSWER:**

\[
\frac{w - 65}{93}
\]

\[
d = 10^{\frac{w - 65}{93}}
\]
7-4 Solving Logarithmic Equations and Inequalities

21. In May of 1999, a tornado devastated Oklahoma City with the fastest wind speed ever recorded. If the tornado traveled 525 miles, estimate the wind speed near the center of the tornado.

**SOLUTION:**
Substitute 525 for \( d \) in the equation and simplify.

\[
w = 93 \log_{0.5} 525 + 65
\]

\[
\approx 318 \text{ mph}
\]

**ANSWER:**
318 mph

**Solve each inequality.**

22. \( \log_6 x < -3 \)

**SOLUTION:**

\[
\log_6 x < -3
\]

\[
x < 6^{-3}
\]

\[
x < \frac{1}{216}
\]

The solution set is \( \{ x \mid 0 < x < \frac{1}{216} \} \).

**ANSWER:**
\( \{ x \mid 0 < x < \frac{1}{216} \} \)

23. \( \log_4 x \geq 4 \)

**SOLUTION:**

\[
\log_4 x \geq 4
\]

\[
x \geq 4^4
\]

\[
x \geq 256
\]

The solution set is \( \{ x \mid x \geq 256 \} \).

**ANSWER:**
\( \{ x \mid x \geq 256 \} \)

24. \( \log_3 x \geq -4 \)

**SOLUTION:**

\[
\log_3 x \geq -4
\]

\[
x \geq 3^{-4}
\]

\[
x \geq \frac{1}{81}
\]

The solution set is \( \{ x \mid x \geq \frac{1}{81} \} \).

**ANSWER:**
\( \{ x \mid x \geq \frac{1}{81} \} \)

25. \( \log_2 x \leq -2 \)

**SOLUTION:**

\[
\log_2 x \leq -2
\]

\[
x \leq 2^{-2}
\]

\[
x \leq \frac{1}{4}
\]

The solution set is \( \{ x \mid 0 < x \leq \frac{1}{4} \} \).

**ANSWER:**
\( \{ x \mid 0 < x \leq \frac{1}{4} \} \)

26. \( \log_5 x > 2 \)

**SOLUTION:**

\[
\log_5 x > 2
\]

\[
x > 5^2
\]

\[
x > 25
\]

The solution set is \( \{ x \mid x > 25 \} \).

**ANSWER:**
\( \{ x \mid x > 25 \} \)
27. \( \log_7 x < -1 \)

**SOLUTION:**

\[
\log_7 x < -1 \\
\log_7 x < 7^{-1} \\
\frac{x}{7} < 1 \\
x < \frac{7}{1} \\
x < 7
\]

The solution set is \( \{ x \mid 0 < x < \frac{7}{1} \} \).

**ANSWER:**

\( \{ x \mid 0 < x < \frac{7}{1} \} \)

28. \( \log_2 (4x - 6) > \log_2 (2x + 8) \)

**SOLUTION:**

\[
\log_2 (4x - 6) > \log_2 (2x + 8) \\
4x - 6 > 2x + 8 \\
2x > 14 \\
x > 7
\]

The solution set is \( \{ x \mid x > 7 \} \).

**ANSWER:**

\( \{ x \mid x > 7 \} \)

29. \( \log_7 (x + 2) \geq \log_7 (6x - 3) \)

**SOLUTION:**

\[
\log_7 (x + 2) \geq \log_7 (6x - 3) \\
x + 2 \geq 6x - 3 \\
-5x \geq -5 \\
x \leq 1
\]

Exclude all values of \( x \) for which \( x + 2 \leq 0 \) or \( 6x - 3 \leq 0 \).

So, \( x > -2, x > \frac{1}{2} \) and \( x \leq 1 \).

The solution set is \( \{ x \mid \frac{1}{2} < x \leq 1 \} \).

**ANSWER:**

\( \{ x \mid \frac{1}{2} < x \leq 1 \} \)

30. \( \log_3 (7x - 6) < \log_3 (4x + 9) \)

**SOLUTION:**

\[
\log_3 (7x - 6) < \log_3 (4x + 9) \\
7x - 6 < 4x + 9 \\
3x < 15 \\
x < 5
\]

Exclude all values of \( x \) for which \( 7x - 6 \leq 0 \) or \( 4x + 9 \leq 0 \).

So, \( x > \frac{6}{7}, x > -\frac{9}{4} \) and \( x < 5 \).

The solution set is \( \{ x \mid \frac{6}{7} < x < 5 \} \).

**ANSWER:**

\( \{ x \mid \frac{6}{7} < x < 5 \} \)
31. \( \log_5 (12x + 5) \leq \log_5 (8x + 9) \)

**SOLUTION:**

\[
\log_5 (12x + 5) \leq \log_5 (8x + 9) \\
12x + 5 \leq 8x + 9 \\
4x \leq 4 \\
x \leq 1
\]

Exclude all values of \( x \) for which \( 12x + 5 \leq 0 \) or \( 8x + 9 \leq 0 \).

So, \( x > - \frac{5}{12}, x > - \frac{9}{8} \) and \( x \leq 1 \).

The solution set is \( \left\{ x \mid - \frac{5}{12} < x \leq 1 \right\} \).

**ANSWER:**

\( \left\{ x \mid - \frac{5}{12} < x \leq 1 \right\} \)

32. \( \log_{11} (3x - 24) \geq \log_{11} (-5x - 8) \)

**SOLUTION:**

\[
\log_{11} (3x - 24) \geq \log_{11} (-5x - 8) \\
3x - 24 \geq -5x - 8 \\
8x \geq 16 \\
x \geq 2
\]

The solution set is \( \{ x \mid x \geq 2 \} \).

**ANSWER:**

\( \{ x \mid x \geq 2 \} \)

33. \( \log_9 (9x + 4) \leq \log_9 (11x - 12) \)

**SOLUTION:**

\[
\log_9 (9x + 4) \leq \log_9 (11x - 12) \\
9x + 4 \leq 11x - 12 \\
-2x \leq -16 \\
x \geq 8
\]

The solution set is \( \{ x \mid x \geq 8 \} \).

**ANSWER:**

\( \{ x \mid x \geq 8 \} \)

34. **CCSS MODELING** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude \( M \) is given by \( M = \log_{10} x \), where \( x \) represents the amplitude of the seismic wave causing ground motion.

**a.** How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 8 as an aftershock with a Richter scale rating of 5?

**b.** In 1906, San Francisco was almost completely destroyed by a 7.8 magnitude earthquake. In 1911, an earthquake estimated at magnitude 8.1 occurred along the New Madrid fault in the Mississippi River Valley. How many times greater was the New Madrid earthquake than the San Francisco earthquake?

**SOLUTION:**

**a.**

The amplitude of the seismic wave with a Richter scale rating of 8 and 5 are \( 10^8 \) and \( 10^5 \) respectively. Divide \( 10^8 \) by \( 10^5 \).

\[
\frac{10^8}{10^5} = 10^{8-5} = 10^3
\]

The scale rating of 8 is \( 10^3 \) or 1000 times greater than the scale rating of 5.

**b.**

The amplitudes of San Francisco earthquake and New Madrid earthquake were \( 10^{7.8} \) and \( 10^{8.1} \) respectively.

Divide \( 10^{8.1} \) by \( 10^{7.8} \).

\[
\frac{10^{8.1}}{10^{7.8}} = 10^{8.1-7.8} = 10^{0.3}
\]

The New Madrid earthquake was \( 10^{0.3} \) or about 2 times greater than the San Francisco earthquake.

**ANSWER:**

**a.** \( 10^3 \) or 1000 times as great

**b.** \( 10^{0.3} \) or about 2 times as great
35. **MUSIC** The first key on a piano keyboard corresponds to a pitch with a frequency of 27.5 cycles per second. With every successive key, going up the black and white keys, the pitch multiplies by a constant. The formula for the frequency of the pitch sounded when the \( n \)th note up the keyboard is played is given by \( n = 1 + 12 \log_2 \frac{f}{27.5} \).

**a.** A note has a frequency of 220 cycles per second. How many notes up the piano keyboard is this?

**b.** Another pitch on the keyboard has a frequency of 880 cycles per second. After how many notes up the keyboard will this be found?

**SOLUTION:**

**a.**
Substitute 220 for \( f \) in the formula and solve for \( n \).

\[
n = 1 + 12 \log_2 \frac{220}{27.5}
\]

\[
= 1 + 12 \log_2 2^3
\]

\[
= 1 + 12(3)
\]

\[
= 37
\]

**b.**
Substitute 880 for \( f \) in the formula and solve for \( n \).

\[
n = 1 + 12 \log_2 \frac{880}{27.5}
\]

\[
= 1 + 12 \log_2 2^5
\]

\[
= 1 + 12(5)
\]

\[
= 61
\]

**ANSWER:**

**a.** 37

**b.** 61

36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the graphs shown: \( y = \log_4 x \) and \( y = \log \frac{x}{4} \).

**a.** **ANALYTICAL** How do the shapes of the graphs compare? How do the asymptotes and the \( x \)-intercepts of the graphs compare?

**b.** **VERBAL** Describe the relationship between the graphs.

**c.** **GRAPHICAL** Use what you know about transformations of graphs to compare and contrast the graph of each function and the graph of \( y = \log_4 x \).

1. \( y = \log_4 x + 2 \)
2. \( y = \log_4 (x + 2) \)
3. \( y = 3 \log_4 x \)

**d.** **ANALYTICAL** Describe the relationship between \( y = \log_4 x \) and \( y = -1(\log_4 x) \). What are a reasonable domain and range for each function?

**e.** **ANALYTICAL** Write an equation for a function for which the graph is the graph of \( y = \log_3 x \) translated 4 units left and 1 unit up.

**SOLUTION:**

**a.**
The shapes of the graphs are the same. The asymptote for each graph is the \( y \)-axis and the \( x \)-intercept for each graph is 1.

**b.**
The graphs are reflections of each other over the \( x \)-axis.

**c.**
1. The second graph is the same as the first, except it is shifted horizontally to the left 2 units.

2. The second graph is the same as the first, except it is shifted vertically up 2 units.
3. Each point on the second graph has a y-coordinate 3 times that of the corresponding point on the first graph.

4. The graphs are reflections of each other over the x-axis.

D = \{ x \mid x > 0 \}; R = \{ \text{all real numbers} \}

\[ f(x) = a \log(x - h) + k \] where \( h \) is the horizontal shift and \( k \) is the vertical shift. Since there is a horizontal shift of 4 and vertical shift of 1, \( h = 4 \) and \( k = 1 \).

\[ y = \log_3(x + 4) + 1 \]

ANSWER:

a. The shapes of the graphs are the same. The asymptote for each graph is the y-axis and the x-intercept for each graph is 1.

b. The graphs are reflections of each other over the x-axis.

c. 1. The second graph is the same as the first, except it is shifted vertically to the left 2 units.

d. The graphs are reflections of each other over the x-axis.

D = \{ x \mid x > 0 \}; R = \{ \text{all real numbers} \}

\[ y = \log_3(x + 4) + 1 \]

37. SOUND The relationship between the intensity of sound \( I \) and the number of decibels \( \beta \) is

\[ \beta = 10 \log_{10} \left( \frac{I}{10^{-12}} \right) \], where \( I \) is the intensity of sound in watts per square meter.
Solve each equation.

1. SOLUTION: 
   ANSWER: 16

2. SOLUTION: 
   ANSWER: 8

3. MULTIPLE CHOICE 
   SOLUTION: 
   ANSWER: 9b6c4

66. SOLUTION: 
   ANSWER: x3y4

68. SOLUTION: 
   ANSWER: 1

38. CCSS CRITIQUE Ryan and Heather are solving log₃ 𝑥 ≥ −3. Is either of them correct? Explain your reasoning.

Ryan

\[
\log_3 x \geq -3 \\
x \geq 3^{-3} \\
x \geq \frac{1}{27}
\]

Heather

\[
\log_3 x \geq -3 \\
x \geq 3^{-3} \\
0 < x \leq \frac{1}{27}
\]

SOLUTION:
Sample answer: Ryan; Heather did not need to switch the inequality symbol when raising to a negative power.

ANSWER:
Sample answer: Ryan; Heather did not need to switch the inequality symbol when raising to a negative power.

39. CHALLENGE Find \(\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27\).

SOLUTION:
\[
\log_3 27 + \log_9 27 + \log_{27} 27 + \log_{81} 27 + \log_{243} 27 \\
= 3 + \frac{\log_{27} 27}{\log_{9} 27} + 1 + \frac{\log_{27} 27}{\log_{81} 27} + \frac{\log_{27} 27}{\log_{243} 27} \\
= 3 + \frac{3}{2} + 1 + \frac{3}{4} + \frac{3}{5} \\
= \frac{17}{20}
\]

ANSWER:
\[\frac{17}{20}\]
40. **REASONING** The Property of Inequality for Logarithmic Functions states that when \( b > 1 \), \( \log_b x > \log_b y \) if and only if \( x > y \). What is the case for when \( 0 < b < 1 \)? Explain your reasoning.

**SOLUTION:**
Sample answer: When \( 0 < b < 1 \), \( \log_b x > \log_b y \) if and only if \( x < y \). The inequality symbol is switched because a fraction that is less than 1 becomes smaller when it is taken to a greater power.

**ANSWER:**
Sample answer: When \( 0 < b < 1 \), \( \log_b x > \log_b y \) if and only if \( x < y \). The inequality symbol is switched because a fraction that is less than 1 becomes smaller when it is taken to a greater power.

41. **WRITING IN MATH** Explain how the domain and range of logarithmic functions are related to the domain and range of exponential functions.

**SOLUTION:**
The logarithmic function of the form \( y = \log_b x \) is the inverse of the exponential function of the form \( y = b^x \). The domain of one of the two inverse functions is the range of the other. The range of one of the two inverse functions is the domain of the other.

**ANSWER:**
The logarithmic function of the form \( y = \log_b x \) is the inverse of the exponential function of the form \( y = b^x \). The domain of one of the two inverse functions is the range of the other. The range of one of the two inverse functions is the domain of the other.

42. **OPEN ENDED** Give an example of a logarithmic equation that has no solution.

**SOLUTION:**
Sample answer: \( \log_3 (x + 4) = \log_3 (2x + 12) \)

**ANSWER:**
Sample answer: \( \log_3 (x + 4) = \log_3 (2x + 12) \)

43. **REASONING** Choose the appropriate term. Explain your reasoning. All logarithmic equations are of the form \( y = \log_b x \).

- a. If the base of a logarithmic equation is greater than 1 and the value of \( x \) is between 0 and 1, then the value for \( y \) is (less than, greater than, equal to) 0.
- b. If the base of a logarithmic equation is between 0 and 1 and the value of \( x \) is greater than 1, then the value of \( y \) is (less than, greater than, equal to) 0.
- c. There is/are (no, one, infinitely many) solution(s) for \( b \) in the equation \( y = \log_b 0 \).
- d. There is/are (no, one, infinitely many) solution(s) for \( b \) in the equation \( y = \log_b 1 \).

**SOLUTION:**
a. less than
b. less than
c. no
d. infinitely many

44. **WRITING IN MATH** Explain why any logarithmic function of the form \( y = \log_b x \) has an \( x \)-intercept of \((1, 0)\) and no \( y \)-intercept.

**SOLUTION:**
The \( y \)-intercept of the exponential function \( y = b^x \) is \((0, 1)\). When the \( x \) and \( y \) coordinates are switched, the \( y \)-intercept is transformed to the \( x \)-intercept of \((1, 0)\). There was no \( x \)-intercept \((1, 0)\) in the exponential function of the form \( y = b^x \). So when the \( x \) and \( y \) coordinates are switched there would be no point on the inverse of \((0, 1)\), and there is no \( y \)-intercept.

**ANSWER:**
The \( y \)-intercept of the exponential function \( y = b^x \) is \((0, 1)\). When the \( x \) and \( y \) coordinates are switched, the \( y \)-intercept is transformed to the \( x \)-intercept of \((1, 0)\). There was no \( x \)-intercept \((1, 0)\) in the exponential function of the form \( y = b^x \). So when the \( x \) and \( y \) coordinates are switched there would be no point on the inverse of \((0, 1)\), and there is no \( y \)-intercept.
45. Find \( x \) if \( \frac{6.4}{x} = \frac{4}{7} \).

\[ 6.4 = \frac{4}{7}x \]

\[ 44.8 = 4x \]

\[ x = 11.2 \]

C is the correct choice.

\textbf{ANSWER:} C

46. The monthly precipitation in Houston for part of a year is shown.

<table>
<thead>
<tr>
<th>Month</th>
<th>Precipitation (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>3.60</td>
</tr>
<tr>
<td>May</td>
<td>5.15</td>
</tr>
<tr>
<td>June</td>
<td>5.35</td>
</tr>
<tr>
<td>July</td>
<td>3.18</td>
</tr>
<tr>
<td>August</td>
<td>3.83</td>
</tr>
</tbody>
</table>

Find the median precipitation.

\[ F \text{ 3.60 in.} \]
\[ G \text{ 4.22 in.} \]
\[ H \text{ 3.83 in.} \]
\[ J \text{ 4.25 in.} \]

\textbf{SOLUTION:}
Arrange the data in ascending order.
3.18, 3.60, 3.83, 5.15, 5.35
The median is the middle value. So, 3.83 is the median precipitation.
H is the correct choice.

\textbf{ANSWER:} H

47. Clara received a 10% raise each year for 3 consecutive years. What was her salary after the three raises if her starting salary was $12,000 per year?

\[ \text{A} \text{ } $14,520 \]
\[ \text{B} \text{ } $15,972 \]
\[ \text{C} \text{ } $16,248 \]
\[ \text{D} \text{ } $16,410 \]

\textbf{SOLUTION:}
Use the compound interest formula.
Substitute $12,000 for \( P \), 0.10 for \( r \), 1 for \( n \) and 3 for \( t \) and simplify.

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ A = 12000 \left( 1 + \frac{0.10}{1} \right)^{3(1)} \]

\[ A \approx 15,972 \]

B is the correct choice.

\textbf{ANSWER:} B
48. SAT/ACT A vendor has 14 helium balloons for sale: 9 are yellow, 3 are red, and 2 are green. A balloon is selected at random and sold. If the balloon sold is yellow, what is the probability that the next balloon, selected at random, is also yellow?

\[
\begin{align*}
\text{F} & : \frac{1}{9} \\
\text{G} & : \frac{1}{8} \\
\text{H} & : \frac{36}{91} \\
\text{J} & : \frac{13}{9} \\
\text{K} & : \frac{9}{14}
\end{align*}
\]

\textit{SOLUTION:}
The probability of selecting an yellow balloon next is:

\[
\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{8}{13}
\]

So, the correct answer choice is J.

\textit{ANSWER:} 

J

\textit{Evaluate each expression.}

49. \( \log_4 256 \)

\textit{SOLUTION:}

\[
\log_4 256 = \log_4 4^4 = 4
\]

\textit{ANSWER:} 

4

50. \( \log_2 \frac{1}{8} \)

\textit{SOLUTION:}

\[
\log_2 \frac{1}{8} = \log_2 2^{-3} = -3
\]

\textit{ANSWER:} 

-3

51. \( \log_6 216 \)

\textit{SOLUTION:}

\[
\log_6 216 = \log_6 6^3 = 3
\]

\textit{ANSWER:} 

3

52. \( \log_3 27 \)

\textit{SOLUTION:}

\[
\log_3 27 = \log_3 3^3 = 3
\]

\textit{ANSWER:} 

3

53. \( \log_5 \frac{1}{125} \)

\textit{SOLUTION:}

\[
\log_5 \frac{1}{125} = \log_5 5^{-3} = -3
\]

\textit{ANSWER:} 

-3

54. \( \log_7 2401 \)

\textit{SOLUTION:}

\[
\log_7 2401 = \log_7 7^4 = 4
\]

\textit{ANSWER:} 

4
Solve each equation or inequality. Check your solution.

55. \(5^{2x+3} \leq 125\)

**SOLUTION:**

\[
5^{2x+3} \leq 125 \\
5^{2x+3} \leq 5^3 \\
2x + 3 \leq 3 \\
x \leq 0
\]

**ANSWER:**

\(\{x \mid x \leq 0\}\)

56. \(3^{x-2} > 81\)

**SOLUTION:**

\[
3^{x-2} > 81 \\
3^{x-2} > 3^4 \\
3x - 2 > 4 \\
x > 2
\]

**ANSWER:**

\(\{x \mid x > 2\}\)

57. \(4^{4a} + 6 \leq 16^a\)

**SOLUTION:**

\[
4^{4a} + 6 \leq 16^a \\
4^{4a} + 6 \leq 4^2 \\
4a + 6 \leq 2a \\
2a \leq -6 \\
a \leq -3
\]

**ANSWER:**

\(\{a \mid a \leq -3\}\)

58. \(11^{2x/7} = 121^{3x}\)

**SOLUTION:**

\[
11^{2x/7} = 121^{3x} \\
11^{2x/7} = (11)^{2(3x)} \\
2x + 1 = 2(3x) \\
2x + 1 = 6x \\
1 = 4x \\
0.25 = x
\]

**ANSWER:**

\(x = 0.25\)

59. \(3^{4x-7} = 27^{2x+3}\)

**SOLUTION:**

\[
3^{4x-7} = 27^{2x+3} \\
3^{4x-7} = 3^{3(2x+3)} \\
4x - 7 = 3(2x + 3) \\
4x - 7 = 6x + 9 \\
-7 = 2x + 9 \\
-16 = 2x \\
-8 = x
\]

**ANSWER:**

\(x = -8\)

60. \(8^{x-4} \leq 2^{4-x}\)

**SOLUTION:**

\[
8^{x-4} \leq 2^{4-x} \\
2^{3(x-4)} \leq 2^{4-x} \\
(3)(x-4) \leq 4 - x \\
3x - 12 \leq 4 - x \\
4x \leq 16 \\
x \leq 4
\]

**ANSWER:**

\(\{x \mid x \leq 4\}\)
61. **SHIPPING** The height of a shipping cylinder is 4 feet more than the radius. If the volume of the cylinder is $5\pi$ cubic feet, how tall is it? Use the formula $V = \pi r^2 h$.

**SOLUTION:**
Substitute $5\pi$ for $V$ and $r + 4$ for $h$ in the formula and simplify.

\[
5\pi = \pi r^2 (r + 4)
\]

\[
r^3 + 4r^2 - 5 = 0
\]

The equation has one real root $r = 1$. Thus, the height of the shipping cylinder is $1 + 4 = 5$ ft.

**ANSWER:**
5 ft

62. **NUMBER THEORY** Two complex conjugate numbers have a sum of 12 and a product of 40. Find the two numbers.

**SOLUTION:**
The equations that represent the situation are:

\[
(a + bi) + (a - bi) = 12 \rightarrow (1)
\]

\[
(a + bi)(a - bi) = 40 \rightarrow (2)
\]

Solve equation (1).

\[
(a + bi) + (a - bi) = 12
\]

\[
2a = 12
\]

\[
a = 6
\]

Solve equation (2).

\[
(a + bi)(a - bi) = 40
\]

\[
a^2 + b^2 = 40 \quad \text{Substitute 6 for a}
\]

\[
36 + b^2 = 40
\]

\[
b = \pm 2
\]

Thus, the two numbers are $6 + 2i$ and $6 - 2i$.

**ANSWER:**
$6 + 2i, 6 - 2i$

63. **Simplify. Assume that no variable equals zero.**

\[
x^5 \cdot x^3
\]

**SOLUTION:**

\[
x^5 \cdot x^3 = x^{5+3} = x^8
\]

**ANSWER:**
$x^8$

64. **$a^2 \cdot a^6$**

**SOLUTION:**

\[
a^2 \cdot a^6 = a^{2+6} = a^8
\]

**ANSWER:**
$a^8$

65. **$(2p^2 n)^3$**

**SOLUTION:**

\[
(2p^2 n)^3 = 8p^6 n^3
\]

**ANSWER:**
$8p^6 n^3$

66. **$(3b^3 c^2)^2$**

**SOLUTION:**

\[
(3b^3 c^2)^2 = 9b^6 c^4
\]

**ANSWER:**
$9b^6 c^4$

67. **$\frac{x^4 y^6}{xy^2}$**

**SOLUTION:**

\[
\frac{x^4 y^6}{xy^2} = x^3 y^4
\]

**ANSWER:**
$x^3 y^4$
7-4 Solving Logarithmic Equations and Inequalities

68. \( \left( \frac{c^0}{d^0} \right) \)

**SOLUTION:**

\[ \left( \frac{c^0}{d^0} \right) = 1 \]

**ANSWER:**

1
7-5 Properties of Logarithms

Use \( \log_4 3 \approx 0.7925 \) and \( \log_4 5 \approx 1.1610 \) to approximate the value of each expression.

1. \( \log_4 18 \)

\[
\text{SOLUTION:} \\
\log_4 18 = \log_4 (2 \cdot 3 \cdot 3) \\
= \log_4 2 + \log_4 3 + \log_4 3 \\
\approx 0.7925 + 0.7925 + 0.7925 \\
= 2.3775 \\
\text{ANSWER:} \\
\text{about 2.3775}
\]

2. \( \log_4 15 \)

\[
\text{SOLUTION:} \\
\log_4 15 = \log_4 (3 \cdot 5) \\
= \log_4 3 + \log_4 5 \\
\approx 0.7925 + 1.1610 \\
= 1.9535 \\
\text{ANSWER:} \\
\text{about 1.9535}
\]

3. \( \log_4 \frac{5}{3} \)

\[
\text{SOLUTION:} \\
\log_4 \left( \frac{5}{3} \right) = \log_4 5 - \log_4 3 \\
\approx 1.1610 - 0.7925 \\
= 0.3685 \\
\text{ANSWER:} \\
\text{about 0.3685}
\]

4. \( \log_4 \frac{3}{4} \)

\[
\text{SOLUTION:} \\
\log_4 \left( \frac{3}{4} \right) = \log_4 3 - \log_4 4 \\
\approx 0.7925 - 1 \\
= -0.2075 \\
\text{ANSWER:} \\
\text{about -0.2075}
\]

5. **MOUNTAIN CLIMBING** As elevation increases, the atmospheric air pressure decreases. The formula for pressure based on elevation is \( a = 15,500(5 - \log_{10} P) \), where \( a \) is the altitude in meters and \( P \) is the pressure in pascals (1 psi \( \approx 6900 \) pascals). What is the air pressure at the summit in pascals for each mountain listed in the table at the right?

<table>
<thead>
<tr>
<th>Mountain</th>
<th>Country</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Everest</td>
<td>Nepal/Tibet</td>
<td>8850</td>
</tr>
<tr>
<td>Trisuli</td>
<td>India</td>
<td>7074</td>
</tr>
<tr>
<td>Bonete</td>
<td>Argentina/Chile</td>
<td>6872</td>
</tr>
<tr>
<td>McKinley</td>
<td>United States</td>
<td>6194</td>
</tr>
<tr>
<td>Logan</td>
<td>Canada</td>
<td>5959</td>
</tr>
</tbody>
</table>

\[
\text{SOLUTION:} \\
\text{Substitute 8850 for } a, \text{ then evaluate } P. \\
\quad a = 15,500(5 - \log_{10} P) \]

\[
8850 = 15,500(5 - \log_{10} P) \\
(5 - \log_{10} P) = \frac{8850}{15,500} \\
\log_{10} P = 5 - \frac{8850}{15,500} \\
P = 10^\left(5 - \frac{8850}{15,500}\right) \\
\approx 26,855.44 \\
\]

The air pressure at the summit of Mt. Everest is about 26,855.44 pascals.

Substitute 7074 for \( a \), then evaluate \( P \).

\[
\quad a = 15,500(5 - \log_{10} P) \]

\[
7074 = 15,500(5 - \log_{10} P) \\
(5 - \log_{10} P) = \frac{7074}{15,500} \\
\log_{10} P = 5 - \frac{7074}{15,500} \\
P = 10^\left(5 - \frac{7074}{15,500}\right) \\
\approx 34,963.34 \\
\]

The air pressure at the summit of Mt. Trisuli is about 34,963.34 pascals.

Substitute 6872 for \( a \), then evaluate \( P \).
### 7-5 Properties of Logarithms

Given \( \log_3 5 \approx 1.465 \) and \( \log_5 7 \approx 1.2091 \), approximate the value of each expression.

6. \( \log_3 25 \)

**SOLUTION:**

\[
\log_3 25 = \log_3 5^2 \\
= 2 \log_3 5 \\
= 2(1.465) \\
= 2.93
\]

**ANSWER:**

2.93

7. \( \log_5 49 \)

**SOLUTION:**

\[
\log_5 49 = \log_5 7^2 \\
= 2 \log_5 7 \\
= 2(1.2091) \\
= 2.4182
\]

**ANSWER:**

2.4182

Solve each equation. Check your solutions.

8. \( \log_4 48 - \log_4 n = \log_4 6 \)

**SOLUTION:**

\[
\log_4 48 - \log_4 n = \log_4 6 \\
\log_4 \left(4 \cdot 3 \right) - \log_4 n = \log_4 (3 \cdot 2) \\
\log_4 4 + \log_4 3 - \log_4 n = \log_4 3 + \log_4 2 \\
1 + \log_4 3 - \log_4 n = \log_4 3 + 0.5 \\
\log_4 n = 1.5 \\
\log_4 4^{1.5} \\
= 8
\]

**ANSWER:**

8

---

The air pressure at the summit of Mt. Bonete is about 36028.42 pascals. Substitute 6194 for \( a \), then evaluate \( P \).

\[
a = 15500(5 - \log_{10} P) \\
6194 = 15500(5 - \log_{10} P) \\
(5 - \log_{10} P) = \frac{6194}{15500} \\
\log_{10} P = 5 - \frac{6194}{15500} \\
P = 10^{\frac{6194}{15500}} \\
\approx 36028.42
\]

**ANSWER:**

The air pressure at the summit of Mt. McKinley is about 39846.22 pascals. Substitute 5959 for \( a \), then evaluate \( P \).

\[
a = 15500(5 - \log_{10} P) \\
5959 = 15500(5 - \log_{10} P) \\
(5 - \log_{10} P) = \frac{5959}{15500} \\
\log_{10} P = 5 - \frac{5959}{15500} \\
P = 10^{\frac{5959}{15500}} \\
\approx 39846.22
\]

**ANSWER:**

The air pressure at the summit of Mt. Logan is 41261.82 pascals.

**ANSWER:**

Mt. Everest: 26,855.44 pascals; Mt. Trisuli: 34,963.34 pascals; Mt. Bonete: 36,028.42 pascals; Mt. McKinley: 39,846.22 pascals; Mt. Logan: 41,261.82 pascals
9. \( \log_3 2x + \log_3 7 = \log_3 28 \)

**SOLUTION:**

\[
\begin{align*}
\log_3 2x + \log_3 7 &= \log_3 28 \\
\log_3 (2x \cdot 7) &= \log_3 28 \\
2 + \log_3 x &= \log_3 2 + \log_3 7 \\
\log_3 x &= \log_3 2 - \log_3 14 \\
x &= \frac{2}{14}
\end{align*}
\]

**ANSWER:**

2

10. \( 3 \log_2 x = \log_2 8 \)

**SOLUTION:**

\[
\begin{align*}
3 \log_2 x &= \log_2 8 \\
\log_2 x^3 &= \log_2 8 \\
x^3 &= 8 \\
x &= 2
\end{align*}
\]

**ANSWER:**

2

11. \( \log_{10} a + \log_{10} (a - 6) = 2 \)

**SOLUTION:**

\[
\begin{align*}
\log_{10} a + \log_{10} (a - 6) &= 2 \\
\log_{10} a(a - 6) &= 2 \\
a(a - 6) &= 10^2 \\
a^2 - 6a &= 100 \\
\end{align*}
\]

By quadratic formula:

\[
a = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-100)}}{2(1)}
\]

\[
= \frac{13.4403 \text{ or } -7.4403}{2(1)}
\]

The logarithm is not defined for negative values. Therefore, the solution is 13.4403.

**ANSWER:**

13.4403

---

Use \( \log_4 2 = 0.5, \log_4 3 \approx 0.7925 \) and \( \log_4 5 = 1.1610 \) to approximate the value of each expression.

12. \( \log_4 30 \)

**SOLUTION:**

\[
\begin{align*}
\log_4 30 &= \log_4 (2 \cdot 15) \\
&= \log_4 2 + \log_4 3 + \log_4 5 \\
&\approx 0.5 + 0.7925 + 1.1610 \\
&= 2.4535
\end{align*}
\]

**ANSWER:**

2.4535

13. \( \log_4 20 \)

**SOLUTION:**

\[
\begin{align*}
\log_4 20 &= \log_4 2 \cdot 2 \cdot 5 \\
&= \log_4 2 + \log_4 2 + \log_4 5 \\
&\approx 0.5 + 0.5 + 1.1610 \\
&= 2.1610
\end{align*}
\]

**ANSWER:**

2.1610

14. \( \log_4 \frac{2}{3} \)

**SOLUTION:**

\[
\begin{align*}
\log_4 \frac{2}{3} &= \log_4 2 - \log_4 3 \\
&\approx 0.5 - 0.7925 \\
&= -0.2925
\end{align*}
\]

**ANSWER:**

-0.2925
15. \( \log_4 \frac{4}{3} \)

**SOLUTION:**

\[
\log_4 \frac{4}{3} = \log_4 \frac{2 \cdot 2}{3} \\
= \log_4 2 + \log_4 2 - \log_4 3 \\
\approx 0.5 + 0.5 - 0.7925 \\
= 0.2075
\]

**ANSWER:** 0.2075

16. \( \log_4 9 \)

**SOLUTION:**

\[
\log_4 9 = \log_4 3^2 \\
= 2 \log_4 3 \\
\approx 2(0.7925) \\
= 1.585
\]

**ANSWER:** 1.585

17. \( \log_4 8 \)

**SOLUTION:**

\[
\log_4 8 = \log_4 2^3 \\
= 3 \log_4 2 \\
= 3(0.5) \\
= 1.5
\]

**ANSWER:** 1.5

18. **SCIENCE** In 2007, an earthquake near San Francisco registered approximately 5.6 on the Richter scale. The famous San Francisco earthquake of 1906 measured 8.3 in magnitude.

**a.** How much more intense was the 1906 earthquake than the 2007 earthquake?

**b.** Richter himself classified the 1906 earthquake as having a magnitude of 8.3. More recent research indicates it was most likely a 7.9. What is the difference in intensities?

---

### Table: Earthquakes

<table>
<thead>
<tr>
<th>Year</th>
<th>Location</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1906</td>
<td>San Francisco</td>
<td>8.3</td>
</tr>
<tr>
<td>1923</td>
<td>Tokyo, Japan</td>
<td>8.3</td>
</tr>
<tr>
<td>1932</td>
<td>Gansu, China</td>
<td>7.6</td>
</tr>
<tr>
<td>1960</td>
<td>Chile</td>
<td>9.5</td>
</tr>
<tr>
<td>1964</td>
<td>Alaska</td>
<td>9.2</td>
</tr>
<tr>
<td>2007</td>
<td>San Francisco</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Source: TLC

**SOLUTION:**

**a.** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude \( M \) is given by \( M = \log_{10} x \), where \( x \) represents the amplitude of the seismic wave causing ground motion. Substitute 8.3 and 5.6 for \( M \), then evaluate the corresponding values of \( x \).

\[
M = \log_{10} x \\
1906: \\
8.3 = \log_{10} x \\
10^{8.3} = x \\
2007: \\
5.6 = \log_{10} x \\
10^{5.6} = x
\]

The ratio between the magnitudes is \( \frac{10^{8.3}}{10^{5.6}} = 10^{2.7} \).

The 1906 earthquake was \( 10^{2.7} \) or about 500 times as intense as the 2007 earthquake.

**b.** Substitute 8.3 and 7.9 for \( M \) then evaluate the corresponding values of \( x \).

\[
M = \log_{10} x \\
8.3 = \log_{10} x \\
10^{8.3} = x \\
7.9 = \log_{10} x \\
10^{7.9} = x
\]

The ratio between the magnitudes is \( \frac{10^{8.3}}{10^{7.9}} = 10^{0.4} \).
7-5 Properties of Logarithms

Richter thought the earthquake was $10^{0.4}$ or about $2\frac{1}{2}$ times more intense than it actually was.

**ANSWER:**
- a. $10^{2.7}$ or about 500 times as great.
- b. Richter thought the earthquake was $10^{0.4}$ or about $2\frac{1}{2}$ times greater than it actually was.

**Given** $\log_6 8 \approx 1.1606$ and $\log_7 9 \approx 1.1292$,
**approximate the value of each expression.**

19. $\log_6 48$

**SOLUTION:**
\[ \log_6 48 = \log_6 (6 \cdot 8) = \log_6 6 + \log_6 8 \approx 1 + 1.1606 = 2.1606 \]

**ANSWER:**
about 2.1606

20. $\log_7 81$

**SOLUTION:**
\[ \log_7 81 = \log_7 9^3 = 3 \log_7 9 \approx 3 (1.1292) = 3.3876 \]

**ANSWER:**
about 3.3876

**CCSS PERSEVERANCE**  Solve each equation. Check your solutions.

21. $\log_6 512$

**SOLUTION:**
\[ \log_6 512 = \log_6 (8^3) = 3 \log_6 8 \approx 3 (1.1606) = 3.4818 \]

**ANSWER:**
about 3.4818

22. $\log_7 729$

**SOLUTION:**
\[ \log_7 729 = \log_7 (9^3) = 3 \log_7 9 \approx 3 (1.1292) = 3.3876 \]

**ANSWER:**
about 3.3876

23. $\log_3 56 - \log_3 n = \log_3 7$

**SOLUTION:**
\[ \log_3 56 - \log_3 n = \log_3 7 \]
\[ \log_3 (56 \cdot 7) - \log_3 n = \log_3 7 \]
\[ \log_3 392 - \log_3 n = \log_3 7 \]
\[ \log_3 392 = \log_3 n + 7 \]
\[ n = 8 \]

**ANSWER:**
8
24. \( \log_2 (4x) + \log_2 5 = \log_2 40 \)

\[
\begin{align*}
\log_2 4x + \log_2 5 &= \log_2 40 \\
\log_2 4 + \log_2 x + \log_2 5 &= \log_2 (4 \cdot 5) \\
\log_2 4 + \log_2 x - \log_2 5 &= \log_2 4 + \log_2 5 + \log_2 2 \\
\log_2 x &= \log_2 2 \\
x &= 2
\end{align*}
\]

**ANSWER:**
2

25. \( 5 \log_2 x = \log_2 32 \)

\[
\begin{align*}
5 \log_2 x &= \log_2 32 \\
\log_2 x &= \log_2 2^5 \\
\log_2 x &= 5 \log_2 2 \\
x &= 2
\end{align*}
\]

**ANSWER:**
2

26. \( \log_{10} a + \log_{10} (a + 21) = 2 \)

\[
\begin{align*}
\log_{10} a + \log_{10} (a + 21) &= 2 \\
\log_{10} a(a + 21) &= 10^2 \\
a(a + 21) &= 10^2 \\
a^2 + 21a - 100 &= 0 \\
(a + 25)(a - 4) &= 0
\end{align*}
\]

By the Zero Product Property:

\( a + 25 = 0 \quad \text{or} \quad a - 4 = 0 \)

\( a = -25 \quad \text{or} \quad a = 4 \)

The logarithm is not defined for negative values. Therefore, the solution is 4.

**ANSWER:**
4

27. **PROBABILITY** In the 1930s, Dr. Frank Benford demonstrated a way to determine whether a set of numbers has been randomly chosen or manually chosen. If the sets of numbers were not randomly chosen, then the Benford formula, \( P = \log_{10} \left( 1 + \frac{1}{d} \right) \), predicts the probability of a digit \( d \) being the first digit of the set. For example, there is a 4.6% probability that the first digit is 9.

a. Rewrite the formula to solve for the digit if given the probability.

b. Find the digit that has a 9.7% probability of being selected.

c. Find the probability that the first digit is 1 (log\(_{10} 2 \approx 0.30103\)).

**SOLUTION:**

a. Rewrite the function \( d \) in terms of \( P \).

\[
P = \log_{10} \left( 1 + \frac{1}{d} \right)
\]

\[
10^p = 1 + \frac{1}{d}
\]

\[
\frac{1}{d} = 10^p - 1
\]

\[
d = \frac{1}{10^p - 1}
\]

b. Substitute 0.097 for \( P \) and evaluate.

\[
d = \frac{1}{10^{0.097} - 1} \\
\approx 4
\]

c. Substitute 1 for \( d \) in the formula

\[
P = \log_{10} \left( 1 + \frac{1}{d} \right) \text{ and evaluate.}
\]

\[
P = \log_{10} \left( 1 + \frac{1}{1} \right) = \log_{10} 2 \\
\approx 0.30103 = 30.1%
\]

**ANSWER:**

a. \( d = \frac{1}{10^P - 1} \)

b. 4

c. 30.1%
7-5 Properties of Logarithms

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to approximate the value of each expression.

28. $\log_5 40$

**SOLUTION:**

\[
\log_5 40 = \log_5 (2 \cdot 4 \cdot 5) \\
= \log_5 2 + \log_5 4 + \log_5 5 \\
\approx \frac{1}{2} \log_5 4 + 0.6826 + 1 \\
\approx 0.4307 + 1.8614 \\
= 2.2921
\]

**ANSWER:**
about 2.2921

29. $\log_5 30$

**SOLUTION:**

\[
\log_5 30 = \log_5 (2 \cdot 3 \cdot 5) \\
= \log_5 2 + \log_5 3 + \log_5 5 \\
= \frac{1}{2} \log_5 4 + 0.6826 + 1 \\
\approx 0.4307 + 1.8626 \\
= 2.1133
\]

**ANSWER:**
about 2.1133

30. $\log_5 \frac{3}{4}$

**SOLUTION:**

\[
\log_5 \frac{3}{4} = \log_5 3 - \log_5 4 \\
\approx 0.6826 - 0.8614 \\
= -0.1788
\]

**ANSWER:**
about -0.1788

31. $\log_5 \frac{4}{3}$

**SOLUTION:**

\[
\log_5 \frac{4}{3} = \log_5 4 - \log_5 3 \\
\approx 0.8614 - 0.6826 \\
= 0.1788
\]

**ANSWER:**
about 0.1788

32. $\log_5 9$

**SOLUTION:**

\[
\log_5 9 = \log_5 3^2 \\
= 2 \log_5 3 \\
\approx 2 (0.6826) \\
= 1.3652
\]

**ANSWER:**
about 1.3652

33. $\log_5 16$

**SOLUTION:**

\[
\log_5 16 = \log_5 4^2 \\
= 2 \log_5 4 \\
\approx 2 (0.8614) \\
= 1.7228
\]

**ANSWER:**
about 1.7228

34. $\log_5 12$

**SOLUTION:**

\[
\log_5 12 = \log_5 (4 \cdot 3) \\
= \log_5 4 + \log_5 3 \\
\approx 0.8614 + 0.6826 \\
= 1.544
\]

**ANSWER:**
about 1.5440
7-5 Properties of Logarithms

35. \( \log_5 27 \)

**SOLUTION:**
\[
\log_5 27 = \log_5 3^3 \\
= 3 \log_5 3 \\
= 3 \left( 0.6826 \right) \\
= 2.0478
\]

**ANSWER:**
about 2.0478

Solve each equation. Check your solutions.

36. \( \log_3 6 + \log_3 x = \log_3 12 \)

**SOLUTION:**
\[
\log_3 6 + \log_3 x = \log_3 (6 \cdot 2) \\
\log_3 x = \log_3 2 \\
x = 2
\]

**ANSWER:**
2

37. \( \log_4 a + \log_4 8 = \log_4 24 \)

**SOLUTION:**
\[
\log_4 a + \log_4 8 = \log_4 24 \\
\log_4 a + \log_4 8 = \log_4 \left( 8 \cdot 3 \right) \\
\log_4 a + \log_4 8 = \log_4 24 \\
\log_4 a = \log_4 3 \\
a = 3
\]

**ANSWER:**
3

38. \( \log_{10} 18 - \log_{10} 3x = \log_{10} 2 \)

**SOLUTION:**
\[
\log_{10} 18 - \log_{10} 3x = \log_{10} 2 \\
\log_{10} \frac{18}{3x} = \log_{10} 2 \\
\frac{18}{3x} = 2 \\
x = 3
\]

**ANSWER:**
3

39. \( \log_7 100 - \log_7 (y + 5) = \log_7 10 \)

**SOLUTION:**
\[
\log_7 100 - \log_7 (y + 5) = \log_7 10 \\
\log_7 \frac{100}{y + 5} = \log_7 10 \\
\frac{100}{y + 5} = 10 \\
y + 5 = 10 \\
y = 5
\]

**ANSWER:**
5

40. \( \log_2 n = \frac{1}{3} \log_2 27 + \log_2 36 \)

**SOLUTION:**
\[
\log_2 n = \frac{1}{3} \log_2 27 + \log_2 36 \\
\log_2 n = \log_2 \left( 27^{\frac{1}{3}} \right) + \log_2 36 \\
\log_2 n = \log_2 3 + \log_2 36 \\
\log_2 n = \log_2 \left( 3 \cdot 36 \right) \\
\log_2 n = \log_2 108 \\
n = 108
\]

**ANSWER:**
108
7-5 Properties of Logarithms

41. $3 \log_{10} 8 - \frac{1}{2} \log_{10} 36 = \log_{10} x$

**SOLUTION:**

$3 \log_{10} 8 - \frac{1}{2} \log_{10} 36 = \log_{10} x$

$\log_{10} 8^3 - \log_{10} 36^{1/2} = \log_{10} x$

$\log_{10} 512 - \log_{10} 6 = \log_{10} x$

$\log_{10} \frac{512}{6} = \log_{10} x$

$x = \frac{512}{6} = 85 \frac{1}{3}$

**ANSWER:** $85 \frac{1}{3}$

**Solve for n.**

42. $\log_a 6n - 3 \log_a x = \log_a x$

**SOLUTION:**

$\log_a 6n - 3 \log_a x = \log_a x$

$\log_a 6n = \log_a x + 3 \log_a x$

$\log_a 6n = \log_a x + \log_a x^3$

$\log_a 6n = \log_a (x \cdot x^3)$

$6n = x^4$

$n = \frac{x^4}{6}$

**ANSWER:** $\frac{x^4}{6}$

43. $2 \log_b 16 + 6 \log_b n = \log_b (x - 2)$

**SOLUTION:**

$2 \log_b 16 + 6 \log_b n = \log_b (x - 2)$

$\log_b 16^2 + \log_b n^6 = \log_b (x - 2)$

$\log_b 256 n^6 = \log_b (x - 2)$

$256 n^6 = x - 2$

$n^6 = \frac{x - 2}{256}$

$n = \left(\frac{x - 2}{256}\right)^{\frac{1}{6}}$

**ANSWER:** $\left(\frac{x - 2}{256}\right)^{\frac{1}{6}}$

**Solve each equation. Check your solutions.**

44. $\log_{10} z + \log_{10} (z + 9) = 1$

**SOLUTION:**

$\log_{10} z + \log_{10} (z + 9) = 1$

$\log_{10} (z(z + 9)) = 1$

$z(z + 9) = 10^1$

$z^2 + 9z = 10$

$z^2 + 9z - 10 = 0$

$(z + 10)(z - 1) = 0$

$z = -10 \text{ or } 1$

**ANSWER:** $1$
45. \( \log_3 (a^2 + 3) + \log_3 3 = 3 \)

\[ \text{SOLUTION:} \]
\[ \log_3 (a^2 + 3) + 1 = 3 \]
\[ \log_3 (a^2 + 3) = 2 \]
\[ a^2 + 3 = 9 \]
\[ a^2 = 6 \]
\[ a = \pm \sqrt{6} \]

\[ \text{ANSWER:} \]
\[ \sqrt{6}, -\sqrt{6} \]

46. \( \log_2 (15b - 15) - \log_2 (-b^2 + 1) = 1 \)

\[ \text{SOLUTION:} \]
\[ \log_2 \left( \frac{15b - 15}{-b^2 + 1} \right) = 1 \]
\[ \frac{15b - 15}{-b^2 + 1} = 2^1 \]
\[ 15b - 15 = 2(-b^2 + 1) \]
\[ 2b^2 + 15b - 17 = 0 \]
\[ (b - 1)(2b + 17) = 0 \]
\[ b = 1 \text{ or } -8.5 \]

Substitute each value into the original equation.

\[ b = -8.5 \]
\[ \log_2 0, \log_2 (-142.5), \text{ and } \log_2 (-71.25) \]

\[ \text{log}_2 0, \log_2 (-142.5), \text{ and } \log_2 (-71.25) \]

\[ \text{are undefined, so 1 and -8.5 are extraneous solutions. Therefore, the equation has no solution.} \]

\[ \text{ANSWER:} \]
\[ \text{no solution} \]

47. \( \log_4 (2y + 2) - \log_4 (y - 2) = 1 \)

\[ \text{SOLUTION:} \]
\[ \log_4 \left( \frac{2y + 2}{y - 2} \right) = 1 \]
\[ \frac{2y + 2}{y - 2} = 4^1 \]
\[ 2y + 2 = 4(y - 2) \]
\[ 2y = 10 \]
\[ y = 5 \]

\[ \text{ANSWER:} \]
\[ 5 \]

48. \( \log_6 0.1 + 2 \log_6 x = \log_6 2 + \log_6 5 \)

\[ \text{SOLUTION:} \]
\[ \log_6 0.1 + 2 \log_6 x = \log_6 2 + \log_6 5 \]
\[ \log_6 0.1 + \log_6 x^2 = \log_6 (10) \]
\[ 0.1x^2 = 10 \]
\[ x^2 = 100 \]
\[ x = \pm 10 \]

Logarithms are not defined for negative values. Therefore, the solution is 10.

\[ \text{ANSWER:} \]
\[ 10 \]

49. \( \log_7 64 - \log_7 \frac{8}{3} + \log_7 2 = \log_7 4p \)

\[ \text{SOLUTION:} \]
\[ \log_7 64 - \log_7 \frac{8}{3} + \log_7 2 = \log_7 4p \]
\[ \log_7 \left( \frac{64}{\frac{8}{3}} \right) = \log_7 4p \]
\[ \frac{4p}{3} = 48 \]
\[ p = 12 \]

\[ \text{ANSWER:} \]
\[ 12 \]
50. **CCSS REASONING** The humpback whale is an endangered species. Suppose there are 5000 humpback whales in existence today, and the population decreases at a rate of 4% per year.

a. Write a logarithmic function for the time in years based upon population.

b. After how long will the population drop below 1000? Round your answer to the nearest year.

**SOLUTION:**

a. It’s more natural to write an exponential function of population as a function of time. That would be: 

\[ p = 5000e^{0.06t} \]

Rewrite this as a log function.

\[ t = \log_{0.96} \left( \frac{p}{5000} \right) \]

b. Substitute 1000 for \( p \).

\[ t = \log_{0.96} \left( \frac{1000}{5000} \right) \]

\[ t = \log_{0.96} (0.2) \]

\[ t \approx 39 \]

It will take about 39 years for the population to drop below 1000.

**ANSWER:**

a. \( t = \log_{0.96} \left( \frac{p}{5000} \right) \)

b. 40 yr

State whether each equation is **true** or **false**.

51. \( \log_8 (x - 3) = \log_8 x - \log_8 3 \)

**SOLUTION:**

\[ \log_8 (x - 3) \neq \log_8 x - \log_8 3 \]

Therefore, the equation is false.

**ANSWER:**

false

52. \( \log_5 22x = \log_5 22 + \log_5 x \)

**SOLUTION:**

\[ \log_5 22x = \log_5 22 + \log_5 x \]

Therefore, the equation is true.

**ANSWER:**

true

53. \( \log_{10} 19k = 19 \log_{10} k \)

**SOLUTION:**

\[ \log_{10} 19k \neq 19 \log_{10} k \]

Therefore, the equation is false.

**ANSWER:**

false

54. \( \log_2 y^5 = 5 \log_2 y \)

**SOLUTION:**

\[ \log_2 y^5 = 5 \log_2 y \]

Therefore, the equation is true.

**ANSWER:**

true

55. \( \log_7 \frac{x}{3} = \log_7 x - \log_7 3 \)

**SOLUTION:**

\[ \log_7 \frac{x}{3} = \log_7 x - \log_7 3 \]

Therefore, the equation is true.

**ANSWER:**

true
56. \[ \log_4(z + 2) = \log_4 z + \log_4 2 \]

**SOLUTION:**
\[ \log_4(z + 2) \neq \log_4 z + \log_4 2 \]

Therefore, the equation is false.

**ANSWER:** false

57. \[ \log_8 p^4 = (\log_8 p)^4 \]

**SOLUTION:**
\[ \log_8 p^4 \neq (\log_8 p)^4 \]

Therefore, the equation is false.

**ANSWER:** false

58. \[ \log_9 \frac{x^2 y^3}{z^4} = 2 \log_9 x + 3 \log_9 y - 4 \log_9 z \]

**SOLUTION:**
\[ \log_9 \frac{x^2 y^3}{z^4} = \log_9 x^2 + \log_9 y^3 - \log_9 z^4 \]
\[ = 2 \log_9 x + 3 \log_9 y - 4 \log_9 z \]

Therefore, the equation is true.

**ANSWER:** true

59. **PARADE** An equation for loudness \( L \), in decibels, is \( L = 10 \log_{10} R \), where \( R \) is the relative intensity of the sound.

   a. Solve \( 120 = 10 \log_{10} R \) to find the relative intensity of the Macy’s Thanksgiving Day Parade with a loudness of 120 decibels depending on how close you are.

   b. Some parents with young children want the decibel level lowered to 80. How many times less intense would this be? In other words, find the ratio of their intensities.

**SOLUTION:**

   a. Solve for \( R \).
\[ 120 = 10 \log_{10} R \]
\[ 12 = \log_{10} R \]
\[ 10^{12} = R \]

   b. Substitute 80 for \( L \) and solve for \( R \).
\[ 80 = 10 \log_{10} R \]
\[ 8 = \log_{10} R \]
\[ 10^8 = R \]

The ratio of their intensities is \( \frac{10^{12}}{10^8} = 10^4 \).

Therefore, the ratio is \( 10^4 \) or about 10,000 times.

**ANSWER:**

   a. \( 10^{12} \)
   b. \( 10^4 \) or about 10,000 times

60. **FINANCIAL LITERACY** The average American carries a credit card debt of approximately $8600 with an annual percentage rate (APR) of 18.3%.

   The formula \( m = \frac{b \left( \frac{r}{n} \right)}{1 - \left(1 + \frac{r}{n} \right)^n} \) can be used to compute the monthly payment \( m \) that is necessary to pay off a credit card balance \( b \) in a given number of years \( t \), where \( r \) is the annual percentage rate and \( n \) is the number of payments per year.

   a. What monthly payment should be made in order to pay off the debt in exactly three years? What is the total amount paid?
b. The equation \( t = \frac{\log \left( \frac{1 - \frac{br}{nn}}{n} \right)}{-n \log \left( 1 + \frac{r}{n} \right)} \) can be used to calculate the number of years necessary for a given payment schedule. Copy and complete the table.

c. Graph the information in the table from part b.

d. If you could only afford to pay $100 a month, will you be able to pay off the debt? If so, how long will it take? If not, why not?

e. What is the minimum monthly payment that will work toward paying off the debt?

**SOLUTION:**

a. Substitute 8600, 0.183, 3 and 12 for \( b, r, t \) and \( n \) respectively then evaluate.

\[
m = \frac{8600 \left( \frac{0.183}{12} \right)}{1 - \left( 1 + \frac{0.183}{12} \right)^{2 \cdot 12}}
\]

\[
= 312.21
\]

The monthly payment should be $312.21.
The total amount paid is $312.21 \times 36 = $11,239.56.

b. Substitute 50, 100, 150, 200, 250 and 300 for \( m \) then solve for \( t \).

\[
\begin{array}{c|c|c}
\text{Payment (} m \text{)} & \text{Years (} t \text{)} & \text{Status} \\
\hline
50 & \text{non-real} & \\
100 & \text{non-real} & \\
150 & 11.42 & \\
200 & 5.87 & \\
250 & 4.09 & \\
300 & 3.16 & \\
\end{array}
\]

c. Graph the information in the table from part b.

d. Logarithm is not defined for negative values.

So, \( 1 - \frac{br}{nn} > 0 \)

\[
1 > \frac{br}{nn}
\]

\[
m > \frac{br}{n}
\]

\[
m > \frac{8600 \times 0.183}{12} = 131.15
\]

No. The monthly interest is $131.15, so the payments do not even cover the interest.

e. Since \( m > 131.15 \), the minimum monthly payment should be $131.16.

**ANSWER:**

a. $312.21; $11,239.56

b. 

\[
\begin{array}{c|c|c|c}
\text{Payment (} m \text{)} & \text{Years (} t \text{)} & \text{Status} \\
\hline
50 & \text{non-real} & \\
100 & \text{non-real} & \\
150 & 11.42 & \\
200 & 5.87 & \\
250 & 4.09 & \\
300 & 3.16 & \\
\end{array}
\]

c.
62. **CCSS ARGUMENTS** Use the properties of exponents to prove the Power Property of Logarithms.

**SOLUTION:**
\[
m^P = m^P \\
\left(b^{\log_b m}\right)^P = b^{P \log_b (m^P)} \\
\log_b m^P = \log_b (m^P) \\
p \log_b m = \log_b (m^P)
\]

**ANSWER:**
\[
m^P = m^P \\
\left(b^{\log_b m}\right)^P = b^{P \log_b (m^P)} \\
\log_b m^P = \log_b (m^P) \\
p \log_b m = \log_b (m^P)
\]
64. **CHALLENGE** Simplify \( \log_{\sqrt{a}} \left( a^2 \right) \) to find an exact numerical value.

**SOLUTION:**

\[
\log_{\sqrt{a}} (a^2) = x \\
\left( \sqrt{a} \right)^x = a^2 \\
(a^{1/2})^x = a^2 \\
\frac{x}{2} = 2 \\
x = 4
\]

**ANSWER:**

\[
\log_{\sqrt{a}} (a^2) = x \\
\left( \sqrt{a} \right)^x = a^2 \\
(a^{1/2})^x = a^2 \\
\frac{x}{2} = 2 \\
x = 4
\]

65. **WHICH ONE DOESN'T BELONG?** Find the expression that does not belong. Explain.

\[
\begin{align*}
\log_b 24 &= \log_b 2 + \log_b 12 \\
\log_b 24 &= \log_b 8 + \log_b 3 \\
\log_b 24 &= \log_b 20 + \log_b 4 \\
\log_b 24 &= \log_b 4 + \log_b 6
\end{align*}
\]

**SOLUTION:**

\[
\log_b 24 \neq \log_b 20 + \log_b 4
\]

All other choices are equal to \( \log_b 24 \).

**ANSWER:**

\[
\log_b 24 \neq \log_b 20 + \log_b 4; \text{ all other choices are equal to } \log_b 24.
\]

66. **REASONING** Use the properties of logarithms to prove that \( \log_x \frac{1}{x} = -\log_x x \)

**SOLUTION:**

\[
\begin{align*}
\log_x \frac{1}{x} &= -\log_x x & \text{Original equation} \\
\log_x x^{-1} &= -\log_x x & \text{Definition of negative exponents} \\
\log_x \frac{1}{x} &= (-1) \log_x x & \text{Power Property of Logarithms} \\
\log_x \frac{1}{x} &= -\log_x x & \text{Simplify}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\log_x \frac{1}{x} &= -\log_x x & \text{Original equation} \\
\log_x x^{-1} &= -\log_x x & \text{Definition of negative exponents} \\
\log_x \frac{1}{x} &= (-1) \log_x x & \text{Power Property of Logarithms} \\
\log_x \frac{1}{x} &= -\log_x x & \text{Simplify}
\end{align*}
\]
67. Simplify \( x^{3 \log_5 2} - \log_3 5 \) to find an exact numerical value.

**SOLUTION:**

\[
x^{3 \log_5 2} - \log_3 5 = x^{\log_5 2^3 - \log_3 5} = x^{\log_5 8 - \log_3 5} = x^{\log_5 \frac{8}{5}} = \frac{8}{5}
\]

**ANSWER:**

\( x^{3 \log_5 2} - \log_3 5 = \frac{8}{5} \)

68. **WRITING IN MATH** Explain how the properties of exponents and logarithms are related. Include examples like the one shown at the beginning of the lesson illustrating the Product Property, but with the Quotient Property and Power Property of Logarithms.

**SOLUTION:**

Since logarithms are exponents, the properties of logarithms are similar to the properties of exponents. The Product Property states that to multiply two powers that have the same base, add the exponents. Similarly, the logarithm of a product is the sum of the logarithms of its factors.

The Quotient Property states that to divide two powers that have the same base, subtract their exponents. Similarly the logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.

The Power Property states that to find the power of a power, multiply the exponents. Similarly, the logarithm of a power is the product of the logarithm and the exponent. Answers should include the following.

- The Product of Powers Property and Product Property of Logarithms both involve the addition of exponents, since logarithms are exponents.

**ANSWER:**

Since logarithms are exponents, the properties of logarithms are similar to the properties of exponents. The Product Property states that to multiply two powers that have the same base, add the exponents. Similarly, the logarithm of a product is the sum of the logarithms of its factors. The Quotient Property states that to divide two powers that have the same base, subtract their exponents. Similarly the logarithm of a quotient is the difference of the logarithms of the numerator and the denominator. The Power Property states that to find the power of a power, multiply the exponents. Similarly, the logarithm of a power is the...
Use \( \log_4 3 \approx 0.7925 \) and \( \log_4 5 \approx 1.1610 \) to approximate the value of each expression.

1. \( \log_4 18 \)

2. \( \log_5 (4x - 1) = \log_5 (3x + 2) \)

SOLUTION:

The solution is 3.

ANSWER:
3

7-5 Properties of Logarithms

The product of the logarithm and the exponent. Answers should include the following.

- Quotient Property:
  \[
  \log_2 \left( \frac{32}{8} \right) = \log_2 \left( \frac{2^5}{2^3} \right) \quad \text{Replace 32 with } 2^5 \\
  = \log_2 2^{(5-3)} \quad \text{Quotient of Powers} \\
  = 5 - 3 \text{ or } 2 \quad \text{Inverse Property of Exponents and Logarithms}
  \]

So, \( \log_2 \left( \frac{32}{8} \right) = \log_2 32 - \log_2 8 \)

Power Property: \( \log_3 9^4 = \log_3 (3^2)^4 \) Replace 9 with \( 3^2 \).

\[
= \log_3 3^{(2 \cdot 4)} \quad \text{Power of a Power} \\
= 2 \cdot 4 \text{ or } 8 \quad \text{Inverse Property of Exponents and Logarithms}
\]

So, \( \log_3 9^4 = 4 \log_3 9 \).

The Product of Powers Property and Product Property of Logarithms both involve the addition of exponents, since logarithms are exponents.

69. Find the mode of the data. 22, 11, 12, 23, 7, 6, 17, 15, 21, 19
   A 11
   B 15
   C 16
   D There is no mode.

SOLUTION:

None of the data are repeated more than once. Therefore, option D is correct.

ANSWER:
D

70. SAT/ACT What is the effect on the graph of \( y = 4x^2 \) when the equation is changed to \( y = 2x^2 \)?
   A The graph is rotated 90 degrees about the origin.
   B The graph is narrower.
   C The graph is wider.
   D The graph is unchanged.

SOLUTION:

The graph is wider. Option C is the correct answer.

ANSWER:
C

71. SHORT RESPONSE In \( y = 6.5(1.07)^x \), \( x \)
   represents the number of years since 2000, and \( y \)
   represents the approximate number of millions of Americans 7 years of age and older who went camping two or more times that year. Describe how the number of millions of Americans who go camping is changing over time.

SOLUTION:

Since \( x \) represents the number of years since 2000, the value of the exponent is always positive. Therefore, it is growing exponentially.

ANSWER:

growing exponentially
72. What are the x-intercepts of the graph of \( y = 4x^2 - 3x - 1 \)?

A. \(-\frac{1}{4}\) and \(-\frac{1}{4}\)
B. \(-1\) and \(-\frac{1}{4}\)
C. \(-1\) and 1
D. 1 and \(-\frac{1}{4}\)

**SOLUTION:**
Substitute 0 for \( y \) and solve for \( x \).

\[
0 = 4x^2 - 3x - 1
\]
\[
0 = 4x^2 - 4x + x - 1
\]
\[
0 = 4x(x-1) + 1(x-1)
\]
\[
0 = (4x+1)(x-1)
\]

By the Zero Product Property:
\[
4x + 1 = 0 \quad \text{or} \quad x - 1 = 0
\]
\[
x = -\frac{1}{4} \quad \text{or} \quad x = 1
\]

Therefore, option D is the correct answer.

**ANSWER:**
D

**Solve each equation. Check your solutions.**

73. \( \log_5 (3x - 1) = \log_5 (2x^2) \)

**SOLUTION:**

\[
\log_5 (3x - 1) = \log_5 (2x^2)
\]
\[
3x - 1 = 2x^2
\]
\[
2x^2 - 3x + 1 = 0
\]
\[
2x^2 - 2x - x + 1 = 0
\]
\[
2x(x - 1) - 1(x - 1) = 0
\]
\[
(x - 1)(2x - 1) = 0
\]

By the Zero Product Property:
\[
x - 1 = 0 \quad \text{or} \quad x - 1 = 0
\]
\[
x = \frac{1}{2} \quad \text{or} \quad x = 1
\]

Therefore, the solutions are 1 and \( \frac{1}{2} \).

**ANSWER:**
\( \frac{1}{2}, 1 \)

74. \( \log_{10} (x^2 + 1) = 1 \)

**SOLUTION:**

\[
\log_{10} (x^2 + 1) = 1
\]
\[
x^2 + 1 = 10^1
\]
\[
x^2 = 9
\]
\[
x = \pm 3
\]

Therefore, the solutions are \( \pm 3 \).

**ANSWER:**
\( \pm 3 \)
7.5 Properties of Logarithms

75. \( \log_{10} (x^2 - 10x) = \log_{10} (-21) \)

**SOLUTION:**

\( \log_{10} (-21) \) is not defined.
Therefore, there is no solution.

**ANSWER:**

no solution

**Evaluate each expression.**

76. \( \log_{10} 0.001 \)

**SOLUTION:**

\[ \log_{10} 0.001 = \log_{10} 10^{-3} \]
\[ = -3 \log_{10} 10 \]
\[ = -3(1) \]
\[ = -3 \]

**ANSWER:**

-3

77. \( \log_4 16^x \)

**SOLUTION:**

\[ \log_4 16^x = \log_4 (4^2)^x \]
\[ = \log_4 4^{2x} \]
\[ = 2x \log_4 4 \]
\[ = 2x(1) \]
\[ = 2x \]

**ANSWER:**

2x

78. \( \log_3 27^x \)

**SOLUTION:**

\[ \log_3 27^x = \log_3 (3^3)^x \]
\[ = \log_3 3^{3x} \]
\[ = 3x \log_3 3 \]
\[ = 3x(1) \]
\[ = 3x \]

**ANSWER:**

3x

79. **ELECTRICITY** The amount of current in amperes \( I \) that an appliance uses can be calculated using the formula \( I = \left( \frac{P}{R} \right)^{\frac{1}{2}} \), where \( P \) is the power in watts and \( R \) is the resistance in ohms. How much current does an appliance use if \( P = 120 \) watts and \( R = 3 \) ohms? Round to the nearest tenth.

**SOLUTION:**

Substitute 120 and 3 for \( P \) and \( R \) and evaluate.

\[ I = \left( \frac{120}{3} \right)^{\frac{1}{2}} \]
\[ = 40^{\frac{1}{2}} \]
\[ \approx 6.3 \]

**ANSWER:**

6.3 amps

**Determine whether each pair of functions are inverse functions. Write yes or no.**

80. \( f(x) = x + 73 \)

\( g(x) = x - 73 \)

**SOLUTION:**

\[ [f \circ g](x) = f(g(x)) \]
\[ = f(x - 73) \]
\[ = (x - 73) + 73 \]
\[ = x \]

\[ [g \circ f](x) = g(f(x)) \]
\[ = g(x + 73) \]
\[ = (x + 73) - 73 \]
\[ = x \]

Since \([f \circ g](x) = [g \circ f](x) = x\), they are inverse functions.

**ANSWER:**

Yes
g(x) = 7x – 11
81. 
h(x) = \frac{1}{7}x + 11

**SOLUTION:**

\[
[h \circ g](x) = h(g(x))
= h(7x - 11)
= \frac{1}{7}(7x - 11) + 11
= x - \frac{11}{7} + 11
= x + \frac{66}{7}
\]

\[
[g \circ h](x) = g(h(x))
= g\left(\frac{1}{7}x + 11\right)
= 7\left(\frac{1}{7}x + 11\right) - 11
= x + 77 - 11
= x + 66
\]

Since \([f \circ g](x) \neq [g \circ f](x)\), they are not inverse functions.

**ANSWER:**
No

### 82. SCULPTING

Antonio is preparing to make an ice sculpture. He has a block of ice that he wants to reduce in size by shaving off the same amount from the length, width, and height. He wants to reduce the volume of the ice block to 24 cubic feet.

![Image of an ice sculpture](image)

**a.** Write a polynomial equation to model this situation.

**b.** How much should he take from each dimension?

**SOLUTION:**

**a.** The dimensions of the ice block is 3 ft, 4 ft and 5 ft.

Let \(x\) be the shaving off the amount of ice in a side. The equation representing this situation is:

\[(3 - x)(4 - x)(5 - x) = 24\]

**b.** Solve the above equation.

\[(3 - x)(4 - x)(5 - x) = 24\]
\[(12 - 7x + x^2)(5 - x) = 24\]
\[-x^2 + 12x^2 - 47x + 60 = 24\]
\[-x^3 + 12x^2 - 47x + 36 = 0\]
\[(x - 1)(-x^2 + 11x - 36) = 0\]

By the Zero Product Property:

\(x - 1 = 0\) or \(-x^2 + 11x - 36 = 0\)

The expression \(-x^2 + 11x - 36\) is a prime.

So, \(x = 1\).

He should take 1 ft from each dimension.

**ANSWER:**

a. \((3-x)(4-x)(5-x) = 24\)

b. 1 ft
7-5 Properties of Logarithms

Solve each equation or inequality. Check your solution.

83. \(3^{4x} = 3^{3-x}\)

**SOLUTION:**
\[
3^{4x} = 3^{3-x} \\
4x = 3 - x \\
5x = 3 \\
x = \frac{3}{5}
\]
The solution is \(\frac{3}{5}\).

**ANSWER:**
\[
\frac{3}{5}
\]

84. \(3^{2n} \leq \frac{1}{9}\)

**SOLUTION:**
\[
3^{2n} \leq \frac{1}{9} \\
3^{2n} \leq \frac{1}{3^2} \\
2n \leq -2 \\
n \leq -1
\]
The solution region is \(n \leq -1\).

**ANSWER:**
\[
\{n \mid n \leq -1\}
\]

85. \(3^{5x} \cdot 81^{1-x} = 9^{x-3}\)

**SOLUTION:**
\[
3^{5x} \cdot 81^{1-x} = 9^{x-3} \\
3^{5x} \cdot (3^4)^{1-x} = (3^2)^{x-3} \\
3^{5x} \cdot 3^{4-4x} = 3^{2x-6} \\
3^{x+4} = 3^{2x-6} \\
x + 4 = 2x - 6 \\
x = 10
\]
The solution is 10.

**ANSWER:**
10

86. \(49^x = 7^{x^2-15}\)

**SOLUTION:**
\[
49^x = 7^{x^2-15} \\
(7^2)^x = 7^{x^2-15} \\
7^{2x} = 7^{x^2-15} \\
2x = x^2 - 15
\]
\[
x^2 - 2x - 15 = 0 \\
(x - 5)(x + 3) = 0
\]
By the Zero Product Property:
\[
x + 3 = 0 \quad \text{or} \quad x - 5 = 0 \\
x = -3 \quad \text{or} \quad x = 5
\]
Therefore, the solutions are -3 and 5.

**ANSWER:**
\(-3, 5\)
7-5 Properties of Logarithms

87. \( \log_2 (x + 6) > 5 \)

**SOLUTION:**

\[
\log_2 (x + 6) > 5 \\
x + 6 > 2^5 \\
x + 6 > 32 \\
x > 26
\]

The solution region is \( x > 26 \).

**ANSWER:**

\( \{ x \mid x > 26 \} \)

88. \( \log_5 (4x - 1) = \log_5 (3x + 2) \)

**SOLUTION:**

\[
\log_5 (4x - 1) = \log_5 (3x + 2) \\
4x - 1 = 3x + 2 \\
x = 3
\]

The solution is 3.

**ANSWER:**

3
Use a calculator to evaluate each expression to the nearest ten-thousandth.

1. \( \log 5 \)

**SOLUTION:**

KEYSTROKES: LOG 5 ENTER 0.698970043

\[ \log 5 \approx 0.6990 \]

**ANSWER:**

about 0.6990

2. \( \log 21 \)

**SOLUTION:**

KEYSTROKES: LOG 21 ENTER 1.322192947

\[ \log 21 \approx 1.3222 \]

**ANSWER:**

about 1.3222

3. \( \log 0.4 \)

**SOLUTION:**

KEYSTROKES: LOG 0.4 ENTER - 0.39794000867

\[ \log 0.4 \approx -0.3979 \]

**ANSWER:**

about -0.3979

4. \( \log 0.7 \)

**SOLUTION:**

KEYSTROKES: LOG 0.7 ENTER - 0.1549019599857

\[ \log 0.7 \approx -0.1549 \]

**ANSWER:**

about -0.1549

5. **SCIENCE** The amount of energy \( E \) in ergs that an earthquake releases is related to its Richter scale magnitude \( M \) by the equation \( \log E = 11.8 + 1.5M \). Use the equation to find the amount of energy released by the 1960 Chilean earthquake, which measured 8.5 on the Richter scale.

**SOLUTION:**

Substitute 8.5 for \( M \) in the equation and evaluate.

\[ \log E = 11.8 + 1.5M \]

\[ = 11.8 + 1.5(8.5) \]

\[ \log E = 24.55 \]

\[ E = 10^{24.55} \]

\[ E = 3.55 \times 10^{24} \]

The amount of energy released by the 1960 Chilean earthquake is \( 3.55 \times 10^{24} \) ergs.

**ANSWER:**

\( 3.55 \times 10^{24} \) ergs

Solve each equation. Round to the nearest ten-thousandth.

6. 4. \( 6^x = 40 \)

**SOLUTION:**

\[ 6^x = 40 \]

\[ \log 6^x = \log 40 \]

\[ x \log 6 = \log 40 \]

\[ x = \frac{\log 40}{\log 6} \approx 2.0588 \]

The solution is about 2.0588.

**ANSWER:**

about 2.0588
7-6 Common Logarithms

7. \(2.1^{a+2} = 8.25\)

**SOLUTION:**

\[
\begin{align*}
2.1^{a+2} &= 8.25 \\
\log 2.1^{a+2} &= \log 8.25 \\
(a + 2)\log 2.1 &= \log 8.25 \\
a + 2 &= \frac{\log 8.25}{\log 2.1} \\
a &= \frac{\log 8.25}{\log 2.1} - 2 \\
&\approx 0.8442
\end{align*}
\]

The solution is about 0.8442.

**ANSWER:**

about 0.8442

8. \(7^x = 20.42\)

**SOLUTION:**

\[
\begin{align*}
7^x &= 20.42 \\
\log 7^x &= \log 20.42 \\
x \log 7 &= \log 20.42 \\
x &= \frac{\log 20.42}{\log 7} \\
&\approx \pm 1.2451
\end{align*}
\]

The solution is about \(\pm 1.2451\).

**ANSWER:**

about \(\pm 1.2451\)

9. \(11^{b-3} = 5^b\)

**SOLUTION:**

\[
\begin{align*}
11^{b-3} &= 5^b \\
\log 11^{b-3} &= \log 5^b \\
(b - 3)\log 11 &= b \log 5 \\
b - 3 &= \frac{b \log 5}{\log 11} \\
1 - \frac{3}{b} &= \frac{\log 5}{\log 11} \\
\frac{3}{b} &= 1 - \frac{\log 5}{\log 11} \\
&\approx 0.3288 \\
b &\approx 9.1237
\end{align*}
\]

The solution is about 9.1237.

**ANSWER:**

about 9.1237

Solve each inequality. Round to the nearest tenthousandth.

10. \(5^{4n} > 33\)

**SOLUTION:**

\[
\begin{align*}
5^{4n} &> 33 \\
\log 5^{4n} &> \log 33 \\
4n \log 5 &> \log 33 \\
n &> \frac{\log 33}{4 \log 5} \\
&> 0.5431
\end{align*}
\]

The solution region is \(\{n \mid n > 0.5431\}\).

**ANSWER:**

\(\{n \mid n > 0.5431\}\)
11. $6^{p-1} \leq 4^p$

**SOLUTION:**

\[
6^{p-1} \leq 4^p \\
\log 6^{p-1} \leq \log 4^p \\
(p-1)\log 6 \leq p\log 4 \\
\frac{p-1}{p} \leq \frac{\log 4}{\log 6} \\
1 - \frac{1}{p} \leq \frac{\log 4}{\log 6} \\
\frac{1}{p} \geq 1 - \frac{\log 4}{\log 6} \\
\frac{1}{p} \geq \frac{\log 6 - \log 4}{\log 6} \\
p \leq \frac{\log 6 - \log 4}{\log 6} \\
1 \geq \frac{\log 6 - \log 4}{\log 6} \cdot p \\
\frac{1}{\log 6 - \log 4} \geq p \\
\log 6 \geq 4.4190 \\
p \leq 4.4190
\]

The solution region is \( \{ p \mid p \leq 4.4190 \} \).

**ANSWER:**

\( \{ p \mid p \leq 4.4190 \} \)

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

12. \( \log_3 7 \)

**SOLUTION:**

\[
\log_3 7 = \frac{\log 7}{\log 3} \\
\approx 1.7712
\]

**ANSWER:**

\( \frac{\log 7}{\log 3} \approx 1.7712 \)

13. \( \log_4 23 \)

**SOLUTION:**

\[
\log_4 23 = \frac{\log 23}{\log 4} \\
\approx 2.2618
\]

**ANSWER:**

\( \frac{\log 23}{\log 4} \approx 2.2618 \)

14. \( \log_9 13 \)

**SOLUTION:**

\[
\log_9 13 = \frac{\log 13}{\log 9} \\
\approx 1.1674
\]

**ANSWER:**

\( \frac{\log 13}{\log 9} \approx 1.1674 \)

15. \( \log_2 5 \)

**SOLUTION:**

\[
\log_2 5 = \frac{\log 5}{\log 2} \\
\approx 2.3219
\]

**ANSWER:**

\( \frac{\log 5}{\log 2} \approx 2.3219 \)

Use a calculator to evaluate each expression to the nearest ten-thousandth.

16. \( \log 3 \)

**SOLUTION:**

KEYSTROKES: LOG 3 ENTER 0.4771212547

\[ \log 3 \approx 0.4771 \]

**ANSWER:**

about 0.4771
17. log 11

*SOLUTION:*

KEystrokes: LOG 11 ENTER 1.041392685

log11 ≈ 1.0414

*ANSWER:*

about 1.0414

18. log 3.2

*SOLUTION:*

KEystrokes: LOG 3 . 2 ENTER 0.50514998

log 3.2 ≈ 0.5051

*ANSWER:*

about 0.5051

19. log 8.2

*SOLUTION:*

KEystrokes: LOG 8 . 2 ENTER 0.913813852

log 8.2 ≈ 0.9138

*ANSWER:*

about 0.9138

20. log 0.9

*SOLUTION:*

KEystrokes: LOG 0 . 9 ENTER – 0.045757491

log 0.9 ≈ –0.0458

*ANSWER:*

about –0.0458

21. log 0.04

*SOLUTION:*

KEystrokes: LOG 0 . 0 4 ENTER – 1.39794001

log 0.04 ≈ –1.3979

*ANSWER:*

about –1.3979

22. **CCSS SENSE-MAKING** Loretta had a new muffler installed on her car. The noise level of the engine dropped from 85 decibels to 73 decibels.

a. How many times the minimum intensity of sound detectable by the human ear was the car with the old muffler, if $m$ is defined to be 1?

b. How many times the minimum intensity of sound detectable by the human ear was the car with the new muffler? Find the percent of decrease of the intensity of the sound with the new muffler.

*SOLUTION:*

a.

\[ 85 = 10 \log \frac{I}{1} \]
\[ 8.5 = \log I \]
\[ I = 10^{8.5} \]
\[ \approx 316,227,766 \]

The old muffler was about 316 million times louder than the minimum intensity detectable by the human ear.

b.

\[ 73 = 10 \log \frac{I}{1} \]
\[ 7.3 = \log I \]
\[ I = 10^{7.3} \]
\[ \approx 19,952,623 \]

The new muffler is about 20 million times louder than the minimum intensity detectable by the human ear.

\[ \frac{19,952,623}{316,227,766} \approx 0.063 \]

The percent of decrease is about $100 - 6.3 = 93.7\%$.

*ANSWER:*

a. about 316,227,766 times

b. about 19,952,623 times; about 93.7%
Solve each equation. Round to the nearest ten-thousandth.

23. \(8^x = 40\)

**SOLUTION:**

\[
\begin{align*}
8^x &= 40 \\
\log 8^x &= \log 40 \\
x \log 8 &= \log 40 \\
x &= \frac{\log 40}{\log 8} \\
\approx 1.7740
\end{align*}
\]

The solution is about 1.7740.

**ANSWER:**
about 1.7740

24. \(5^x = 55\)

**SOLUTION:**

\[
\begin{align*}
5^x &= 55 \\
\log 5^x &= \log 55 \\
x \log 5 &= \log 55 \\
x &= \frac{\log 55}{\log 5} \\
\approx 2.4899
\end{align*}
\]

The solution is about 2.4899.

**ANSWER:**
about 2.4899

25. \(2.9^{a-4} = 8.1\)

**SOLUTION:**

\[
\begin{align*}
2.9^{a-4} &= 8.1 \\
\log 2.9^{a-4} &= \log 8.1 \\
(a-4) \log 2.9 &= \log 8.1 \\
a-4 &= \frac{\log 8.1}{\log 2.9} \\
a &= \frac{\log 8.1}{\log 2.9} + 4 \\
\approx 5.9647
\end{align*}
\]

The solution is about 5.9647.

**ANSWER:**
about 5.9647

26. \(9^{b-1} = 7^b\)

**SOLUTION:**

\[
\begin{align*}
9^{b-1} &= 7^b \\
\log 9^{b-1} &= \log 7^b \\
(b-1) \log 9 &= b \log 7 \\
\frac{b-1}{b} &= \frac{\log 7}{\log 9} \\
1 - \frac{1}{b} &= \frac{\log 7}{\log 9} \\
\frac{1}{b} &= 1 - \frac{\log 7}{\log 9} \\
b &= \frac{1}{1 - \frac{\log 7}{\log 9}} \\
\approx 8.7429
\end{align*}
\]

The solution is about 8.7429.

**ANSWER:**
about 8.7429
27. $13^x = 33.3$

**SOLUTION:**

\[
13^x = 33.3 \\
\log_{13} x^2 = \log 33.3 \\
x^2 \log_{13} = \log 33.3 \\
x^2 = \frac{\log 33.3}{\log 13} \\
x = \pm \sqrt{\frac{\log 33.3}{\log 13}} \\
\approx \pm 1.1691
\]

The solution is about 1.1691.

**ANSWER:**

about ±1.1691

28. $15^x = 110$

**SOLUTION:**

\[
15^x = 110 \\
\log_{15} x^2 = \log 110 \\
x^2 \log_{15} = \log 110 \\
x^2 = \frac{\log 110}{\log 15} \\
x = \pm \sqrt{\frac{\log 110}{\log 15}} \\
\approx \pm 1.3175
\]

The solution is about ±1.3175.

**ANSWER:**

about ±1.3175

29. $6^{3n} > 36$

**SOLUTION:**

\[
6^{3n} > 36 \\
\log 6^{3n} > \log 36 \\
3n \log 6 > \log 36 \\
n > \frac{\log 36}{3 \log 6} \\
> 0.6667
\]

The solution region is $\{n | n > 0.6667\}$.

**ANSWER:**

$\{n | n > 0.6667\}$

30. $2^{4x} \leq 20$

**SOLUTION:**

\[
2^{4x} \leq 20 \\
\log 2^{4x} \leq \log 20 \\
4x \log 2 \leq \log 20 \\
x \leq \frac{\log 20}{4 \log 2} \\
\leq 1.0805
\]

The solution region is $\{x | x \leq 1.0805\}$.

**ANSWER:**

$\{x | x \leq 1.0805\}$
31. \( 3^y - 1 \leq 4^p \)

**SOLUTION:**

\[
y^p - 1 \leq 4^p
\]

\[
\log_3 (y^p - 1) \leq \log_4 4^p
\]

\[
(y - 1) \log_3 3 \leq y \log_4 4
\]

\[
\frac{y - 1}{y} \leq \frac{\log_4 4}{\log_3 3}
\]

\[
1 - \frac{1}{y} \leq \frac{\log_4 4}{\log_3 3}
\]

\[
\frac{1}{y} \geq 1 - \frac{\log_4 4}{\log_3 3}
\]

\[
y \geq \frac{1}{1 - \frac{\log_4 4}{\log_3 3}}
\]

\[
\geq -3.8188
\]

The solution region is \( \{y \mid y \geq -3.8188\} \).

**ANSWER:**

\( \{y \mid y \geq -3.8188\} \)

32. \( 5^{n^2} \geq 2^n \)

**SOLUTION:**

\[
5^{n^2} \geq 2^n
\]

\[
\log 5^{n^2} \geq \log 2^n
\]

\[
(p - 2) \log 5 \geq p \log 2
\]

\[
\frac{p - 2}{p} \geq \frac{\log 2}{\log 5}
\]

\[
1 - \frac{2}{p} \geq \frac{\log 2}{\log 5}
\]

\[
\frac{2}{p} \leq 1 - \frac{\log 2}{\log 5}
\]

\[
p \geq \frac{2}{1 - \frac{\log 2}{\log 5}}
\]

\[
\geq 3.5129
\]

The solution region is \( \{p \mid p \geq 3.5129\} \).

**ANSWER:**

\( \{p \mid p \geq 3.5129\} \)

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

33. \( \log_7 18 \)

**SOLUTION:**

\[
\log_7 8 = \frac{\log 18}{\log 7}
\]

\[
\approx 1.4854
\]

**ANSWER:**

\[
\frac{\log 18}{\log 7} \approx 1.4854
\]

34. \( \log_5 31 \)

**SOLUTION:**

\[
\log_5 31 = \frac{\log 31}{\log 5}
\]

\[
\approx 2.1337
\]

**ANSWER:**

\[
\frac{\log 31}{\log 5} \approx 2.1337
\]

35. \( \log_2 16 \)

**SOLUTION:**

\[
\log_2 16 = \frac{\log 16}{\log 2}
\]

\[
= 4
\]

**ANSWER:**

\[
\frac{\log 16}{\log 2} = 4
\]

36. \( \log_4 9 \)

**SOLUTION:**

\[
\log_4 9 = \frac{\log 9}{\log 4}
\]

\[
\approx 1.5850
\]

**ANSWER:**

\[
\frac{\log 9}{\log 4} \approx 1.5850
\]
37. \( \log_3 11 \)

**SOLUTION:**
\[
\log_3 11 = \frac{\log 11}{\log 3} \\
\approx 2.1827
\]

**ANSWER:**
\[
\frac{\log 11}{\log 3} \approx 2.1827
\]

38. \( \log_6 33 \)

**SOLUTION:**
\[
\log_6 33 = \frac{\log 33}{\log 6} \\
\approx 1.9514
\]

**ANSWER:**
\[
\frac{\log 33}{\log 6} \approx 1.9514
\]

39. **PETS** The number \( n \) of pet owners in thousands after \( t \) years can be modeled by \( n = 35[\log_4 (t + 2)] \). Let \( t = 0 \) represent 2000. Use the Change of Base Formula to solve the following questions.

a. How many pet owners were there in 2010?

b. How long until there are 80,000 pet owners? When will this occur?

**SOLUTION:**

a. The value of \( t \) at 2010 is 10.

Substitute 10 for \( t \) in the equation and evaluate.

\[
n = 35\log_4 (t + 2) \\
= 35\log_4 (10 + 2) \\
= 35\log_4 12 \\
= 35 \cdot \frac{\log 12}{\log 4} \\
\approx 62.737
\]

There will be 62,737 pet owners in 2010.

b. Substitute 80 for \( n \) and solve for \( t \).

\[
80 = 35\log_4 (t + 2) \\
\frac{80}{35} = \log_4 (t + 2) \\
\frac{80}{35} = \frac{\log (t + 2)}{\log 4} \\
\frac{80}{35} \log 4 = \log (t + 2) \\
80 \approx 10^{\frac{80}{35} \log 4} \\
t + 2 = 10^{\frac{80}{35} \log 4} \\
t = 10^{\frac{80}{35} \log 4} - 2 \\
\approx 22
\]

In 2022, there will be 80,000 pet owners.

**ANSWER:**

a. 62,737 owners

b. 2022

40. **CCSS PRECISION** Five years ago the grizzly bear population in a certain national park was 325. Today it is 450. Studies show that the park can support a population of 750.
7-6 Common Logarithms

a. What is the average annual rate of growth in the population if the grizzly bears reproduce once a year?
b. How many years will it take to reach the maximum population if the population growth continues at the same average rate?

**SOLUTION:**
a. Substitute 325, 450 and 5 for \( a, A(t) \) and \( t \) in the equation \( A(t) = a(1 + r)^t \).

\[
450 = 325(1 + r)^5
\]

Solve for \( r \).

\[
\frac{450}{325} = (1 + r)^5
\]

\[
\sqrt[5]{\frac{450}{325}} = 1 + r
\]

\[
r \approx 0.067
\]

The average annual rate is about 0.067 or 6.7%.

b. Substitute 750 for \( A(t) \) and evaluate.

\[
750 = 325(1 + 0.067)^t
\]

\[
\frac{750}{325} = 1.067^t
\]

\[
\log \frac{750}{325} = \log 1.067^t
\]

\[
\log 750 - \log 325 = t \log 1.067
\]

\[
t = \frac{\log 750 - \log 325}{\log 1.067}
\]

\[
\approx 13
\]

It will take 8 years to reach the maximum population.

**ANSWER:**
a. 0.067 or 6.7%
b. 8 yr

Solve each equation or inequality. Round to the nearest ten-thousandth.

41. \( 3^x = 40 \)

**SOLUTION:**

\[
3^x = 40
\]

\[
\log 3^x = \log 40
\]

\[
x \log 3 = \log 40
\]

\[
x = \frac{\log 40}{\log 3}
\]

\[
\approx 3.3578
\]

The solution is about 3.3578.

**ANSWER:**

about 3.3578

42. \( 5^p = 15 \)

**SOLUTION:**

\[
5^p = 15
\]

\[
\log 5^p = \log 15
\]

\[
p \log 5 = \log 15
\]

\[
p = \frac{\log 5}{\log 5}
\]

\[
\approx 0.5609
\]

The solution is about 0.5609.

**ANSWER:**

about 0.5609
7-6 Common Logarithms

43. \(4^n + 2 = 14.5\)

\[4^n + 2 = 14.5\]
\[\log 4^n + 2 = \log 14.5\]
\[(n + 2)\log 4 = \log 14.5\]
\[n \log 4 + 2\log 4 = \log 14.5\]
\[n \log 4 = \log 14.5 - 2\log 4\]
\[n = \frac{\log 14.5 - 2\log 4}{\log 4}\]
\[n \approx -0.0710\]

The solution is about \(-0.0710\).

\textbf{ANSWER:} about \(-0.0710\)

44. \(8^x - 4 = 6.3\)

\[8^x - 4 = 6.3\]
\[\log 8^x - 4 = \log 6.3\]
\[(x - 4)\log 8 = \log 6.3\]
\[x\log 8 - 4\log 8 = \log 6.3\]
\[x\log 8 = \log 6.3 + 4\log 8\]
\[x = \frac{\log 6.3 + 4\log 8}{\log 8}\]
\[x \approx 4.8851\]

The solution is about \(4.8851\).

\textbf{ANSWER:} about \(4.8851\)

45. \(7.4^n - 3 = 32.5\)

\[7.4^n - 3 = 32.5\]
\[\log 7.4^n - 3 = \log 32.5\]
\[(n - 3)\log 7.4 = \log 32.5\]
\[n\log 7.4 - 3\log 7.4 = \log 32.5\]
\[n = \frac{\log 32.5 + 3\log 7.4}{\log 7.4}\]
\[n \approx 4.7393\]

The solution is about \(4.7393\).

\textbf{ANSWER:} about \(4.7393\)

46. \(3.1^y - 5 = 9.2\)

\[3.1^y - 5 = 9.2\]
\[\log 3.1^y - 5 = \log 9.2\]
\[(y - 5)\log 3.1 = \log 9.2\]
\[y\log 3.1 - 5\log 3.1 = \log 9.2\]
\[y = \frac{\log 9.2 + 5\log 3.1}{\log 3.1}\]
\[y \approx 6.9615\]

The solution is about \(6.9615\).

\textbf{ANSWER:} about \(6.9615\)

47. \(5^x \geq 42\)

\[5^x \geq 42\]
\[\log 5^x \geq \log 42\]
\[x\log 5 \geq \log 42\]
\[x \geq \frac{\log 42}{\log 5}\]
\[x \geq 2.3223\]

The solution region is \(\{x | x \geq 2.3223\}\).

\textbf{ANSWER:} \(\{x | x \geq 2.3223\}\)
48. $9^{2a} < 120$

**SOLUTION:**

\[ 9^{2a} < 120 \]

\[ \log 9^{2a} < \log 120 \]

\[ 2a \log 9 < \log 120 \]

\[ 2a < \frac{\log 120}{\log 9} \]

\[ a < \frac{\log 120}{2 \log 9} \]

\[ a < 1.0894 \]

The solution region is \( \{ a \mid a < 1.0894 \} \).

**ANSWER:**

\( \{ a \mid a < 1.0894 \} \)

49. $3^{4x} \leq 72$

**SOLUTION:**

\[ 3^{4x} \leq 72 \]

\[ \log 3^{4x} \leq \log 72 \]

\[ 4x \log 3 \leq \log 72 \]

\[ 4x \leq \frac{\log 72}{\log 3} \]

\[ x \leq \frac{\log 72}{4 \log 3} \]

\[ x \leq 0.9732 \]

The solution region is \( \{ x \mid x \leq 0.9732 \} \).

**ANSWER:**

\( \{ x \mid x \leq 0.9732 \} \)

50. $7^{2n} > 52^{4n} + 3$

**SOLUTION:**

\[ 7^{2n} > 52^{4n} + 3 \]

\[ \log 7^{2n} > \log 52^{4n} + 3 \]

\[ 2n \log 7 > (4n + 3) \log 52 \]

\[ 2n \log 7 > 4n \log 52 + 3 \log 52 \]

\[ 2n \log 7 - 4n \log 52 > 3 \log 52 \]

\[ n(2 \log 7 - 4 \log 52) > 3 \log 52 \]

Note that \((2 \log 7 - 4 \log 52)\) is a negative number, so when we divide both sides by this, we need to reverse the inequality.

\[ n < \frac{3 \log 52}{2 \log 7 - 4 \log 52} \]

\[ n < -0.9950 \]

The solution region is \( \{ n \mid n < -0.9950 \} \).

**ANSWER:**

\( \{ n \mid n < -0.9950 \} \)

51. $d^5 \leq 13^{5-p}$

**SOLUTION:**

\[ d^5 \leq 13^{5-p} \]

\[ \log d^5 \leq \log 13^{5-p} \]

\[ 5 \log d \leq (5 - p) \log 13 \]

\[ 5 \log d \leq 5 \log 13 - p \log 13 \]

\[ 5 \log d + p \log 13 \leq 5 \log 13 \]

\[ p(\log d + \log 13) \leq 5 \log 13 \]

\[ p \leq \frac{5 \log 13}{\log d + \log 13} \]

\[ p \leq 2.9437 \]

The solution region is \( \{ p \mid p \leq 2.9437 \} \).

**ANSWER:**

\( \{ p \mid p \leq 2.9437 \} \)
7-6 Common Logarithms

52. $2^y + 3 \geq 8^3$

**SOLUTION:**

$$2^y + 3 \geq 8^3$$

$$\log 2^y + 3 \geq \log 8^3$$

$$(y + 3) \log 2 \geq 3y \log 8$$

$$y \log 2 + 3 \log 2 \geq 3y \log 8$$

$$y \log 2 - 3y \log 8 \geq -3 \log 2$$

$$y (\log 2 - 3 \log 8) \geq -3 \log 2$$

$$y \leq \frac{-3 \log 2}{\log 2 - 3 \log 8}$$

$$y \leq 0.3750$$

The solution region is $\{y \mid y \leq 0.3750\}$.

**ANSWER:**

$\{y \mid y \leq 0.3750\}$

Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.

53. $\log_4 12$

**SOLUTION:**

$$\log_4 12 = \frac{\log 12}{\log 4}$$

$$\approx 1.7925$$

**ANSWER:**

$$\log 12 \approx 1.7925$$

54. $\log_3 21$

**SOLUTION:**

$$\log_3 21 = \frac{\log 21}{\log 3}$$

$$\approx 2.7712$$

**ANSWER:**

$$\log 21 \approx 2.7712$$

55. $\log_8 2$

**SOLUTION:**

$$\log_8 2 = \frac{\log 2}{\log 8}$$

$$\approx 0.3333$$

**ANSWER:**

$$\frac{\log 2}{\log 8} \approx 0.3333$$

56. $\log_6 7$

**SOLUTION:**

$$\log_6 7 = \frac{\log 7}{\log 6}$$

$$\approx 1.0860$$

**ANSWER:**

$$\frac{\log 7}{\log 6} \approx 1.0860$$

57. $\log_5 (2.7)^2$

**SOLUTION:**

$$\log_5 (2.7)^2 = \frac{\log 7.29}{\log 5}$$

$$\approx 1.2343$$

**ANSWER:**

$$\frac{\log 7.29}{\log 5} \approx 1.2343$$

58. $\log_7 \sqrt{5}$

**SOLUTION:**

$$\log_7 \sqrt{5} = \frac{\log \sqrt{5}}{\log 7}$$

$$\approx \frac{\log 2.236}{\log 7}$$

$$\approx 0.4135$$

**ANSWER:**

$$\frac{\log \sqrt{5}}{\log 7} \approx 0.4135$$
59. **MUSIC** A musical cent is a unit in a logarithmic scale of relative pitch or intervals. One octave is equal to 1200 cents. The formula

\[
n = 1200 \left( \log_2 \frac{a}{b} \right)
\]

can be used to determine the difference in cents between two notes with frequencies \( a \) and \( b \).

**a.** Find the interval in cents when the frequency changes from 443 Hertz (Hz) to 415 Hz.

**b.** If the interval is 55 cents and the beginning frequency is 225 Hz, find the final frequency.

**SOLUTION:**

**a.** Substitute 443 and 415 for \( a \) and \( b \) then evaluate.

\[
n = 1200 \left( \log_2 \frac{443}{415} \right)
\]

\[
\approx 113.03
\]

The interval is 113.03 cents.

**b.** Substitute 55 and 225 for \( n \) and \( a \) then solve for \( b \).

\[
\frac{55}{1200} = \log_2 \frac{225}{b}
\]

\[
\frac{55}{2^{1200}} = \frac{225}{b}
\]

\[
b = \frac{225 \times 55}{2^{1200}}
\]

\[
\approx 218
\]

The final frequency is 218.

**ANSWER:**

**a.** 113.03 cents

**b.** about 218 Hz

---

**Solve each equation. Round to the nearest tenthousandth.**

60. \( 10^{x^2} = 60 \)

**SOLUTION:**

\[
10^{x^2} = 60
\]

\[
\log 10^{x^2} = \log 60
\]

\[
x^2 \log 10 = \log 60
\]

\[
x^2 = \frac{\log 60}{\log 10}
\]

\[
x = \pm \sqrt{\frac{\log 60}{\log 10}}
\]

\[
= \pm 1.3335
\]

The solution is about \( \pm 1.3335 \).

**ANSWER:**

about \( \pm 1.3335 \)

61. \( 4^{x^2-3} = 16 \)

**SOLUTION:**

\[
4^{x^2-3} = 16
\]

\[
\log 4^{x^2-3} = \log 16
\]

\[
\left( x^2-3 \right) \log 4 = \log 16
\]

\[
x^2 - 3 = \frac{\log 16}{\log 4}
\]

\[
x^2 - 3 = 2
\]

\[
x^2 = 5
\]

\[
x = \pm \sqrt{5} \approx \pm 2.2361
\]

The solutions are \( \pm \sqrt{5} \approx \pm 2.2361 \).

**ANSWER:**

\( \pm \sqrt{5} \approx \pm 2.2361 \)
62. \(9^{6y-2} = 3^{3y+1}\)

**SOLUTION:**

\[
\begin{align*}
9^{6y-2} &= 3^{3y+1} \\
\log 9^{6y-2} &= \log 3^{3y+1} \\
(6y-2)\log 9 &= (3y+1)\log 3 \\
3y+1 &= (6y-2)2 \\
3y+1 &= 12y-4 \\
9y &= 5 \\
y &= \frac{5}{9}
\end{align*}
\]

The solution is about 0.5556.

**ANSWER:**

about 0.5556

63. \(8^{2x-4} = 4^{x+1}\)

**SOLUTION:**

\[
\begin{align*}
8^{2x-4} &= 4^{x+1} \\
\log 8^{2x-4} &= \log 4^{x+1} \\
(2x-4)\log 8 &= (x+1)\log 4 \\
(2x-4)\frac{\log 8}{\log 4} &= (x+1) \\
(2x-4)\frac{3}{2} &= (x+1) \\
3(2x-4) &= 2(x+1) \\
6x-12 &= 2x+2 \\
4x &= 14 \\
x &= 3.5
\end{align*}
\]

The solution is 3.5.

**ANSWER:**

3.5

64. \(16^x = \sqrt{4^{x+3}}\)

**SOLUTION:**

\[
\begin{align*}
16^x &= \sqrt{4^{x+3}} \\
\log 16^x &= \log 4^{x+3} \\
x \log 16 &= (x+3)\log 4 \\
x \frac{\log 16}{\log 4} &= \frac{x+3}{2} \\
2x &= x+3 \\
x &= 3
\end{align*}
\]

The solution is 1.

**ANSWER:**

1

65. \(2^x = \sqrt{3^{x-1}}\)

**SOLUTION:**

\[
\begin{align*}
2^x &= \sqrt{3^{x-1}} \\
\log 2^x &= \log (3^{x-1})^{0.5} \\
x \log 2 &= (0.5x - 0.5)\log 3 \\
x \log 2 &= 0.5x \log 3 - 0.5 \log 3 \\
0.5x \log 3 &= 0.5 \log 3 \\
x &= \frac{\log 3}{\log 3} \\
\log 3 &= -3.8188
\end{align*}
\]

The solution is about \(-3.8188\).

**ANSWER:**

about \(-3.8188\)

66. **ENVIRONMENTAL SCIENCE**

An environmental engineer is testing drinking water wells in coastal communities for pollution, specifically unsafe levels of arsenic. The safe standard for arsenic is 0.025 parts per million (ppm). Also, the pH of the arsenic level should be less than 9.5. The formula for hydrogen ion concentration is \(pH = -\log\)
7-6 Common Logarithms

H. (Hint: 1 kilogram of water occupies approximately 1 liter. 1 ppm = 1 mg/kg.)
   a. Suppose the hydrogen ion concentration of a well is $1.25 \times 10^{-11}$. Should the environmental engineer be worried about too high an arsenic content?
   b. The environmental engineer finds 1 milligram of arsenic in a 3 liter sample, is the well safe?
   c. What is the hydrogen ion concentration that meets the troublesome pH level of 9.5?

**SOLUTION:**

a. Substitute $1.25 \times 10^{-11}$ for $H$ and evaluate.

$$pH = -\log H$$
$$= -\log(1.25 \times 10^{-11})$$
$$\approx 10.9$$

Yes. The environmental engineer be worried about too high an arsenic content, since 10.9 > 9.5.

b. 1 milligram of arsenic in a 3 liter sample is $\frac{1}{3}$ ppm.

Substitute $\frac{1}{3}$ for $H$ and evaluate.

$$pH = -\log \frac{1}{3}$$
$$\approx 0.4771$$

Since 0.4771 > 0.025, the well is not safe.

c. Substitute 9.5 for $pH$ and solve for $H$.

$$9.5 = -\log H$$
$$-9.5 = \log H$$
$$H = 10^{-9.5}$$
$$\approx 3.16 \times 10^{-10}$$

The hydrogen ion concentration is $3.16 \times 10^{-10}$.

**ANSWER:**

a. yes; 10.9 > 9.5
   b. no
   c. $3.16 \times 10^{-10}$

67. MULTIPLE REPRESENTATIONS In this problem, you will solve the exponential equation $4^x =$
7-6 Common Logarithms

**ANSWER:**

**a.** The solution is between 1.8 and 1.9.

**b.** (1.85, 13)

**c.** Yes; all methods produce the solution of 1.85. They all should produce the same result because you are starting with the same equation. If they do not, then an error was made.

68. **CCSS CRITIQUE** Sam and Rosamaria are solving \(4^p = 10\). Is either of them correct? Explain your reasoning.

### Sam

\[4^p = 10\]
\[
\log 4^p = \log 10
\]
\[
p \log 4 = \log 10
\]
\[
p = \frac{\log 10}{\log 4}
\]

### Rosamaria

\[4^p = 10\]
\[
\log 4^p = \log 10
\]
\[
3p \log 4 = \log 10
\]
\[
p = \frac{3 \log 4}{\log 10}
\]

**SOLUTION:**

Rosamaria; Sam forgot to bring the 3 down from the exponent when he took the log of each side.

**ANSWER:**

Rosamaria; Sam forgot to bring the 3 down from the exponent when he took the log of each side.

69. **CHALLENGE** Solve \(\log_a 3 = \log_a x\) for \(x\) and explain each step.

**SOLUTION:**

\[
\log_a 3 = \log_a x
\]
\[
\frac{\log_a 3}{\log_a \sqrt{a}} = \log_a x
\]
\[
\frac{3}{2} = \log_a x
\]
\[
2 \log_a 3 = \log_a x
\]
\[
\log_a 3^2 = \log_a x
\]
\[
3^2 = x
\]
\[
x = 9
\]

**ANSWER:**

Original equation

Change of Base Formula

Multiply numerator and denominator by 2.

Power Property of Logarithms

Property of equality for logarithmic functions

Simplify
7.6 Common Logarithms

70. **REASONING** Write \( \frac{\log_5 9}{\log_5 3} \) as a single logarithm.

**SOLUTION:**

\[
\frac{\log_5 9}{\log_5 3} = \frac{\log_5 9}{\log_5 3} \quad \text{Change of base formula}
\]

\[
= \frac{\log_5 3}{\log_5 3} \quad \text{Multiply by reciprocal of denominator}
\]

\[
= \log_5 9 \quad \text{Cancel common factor of } \log 5.
\]

**ANSWER:**

\[
\frac{\log_5 9}{\log_5 3} = \log_5 9
\]

71. **PROOF** Find the values of \( \log_3 27 \) and \( \log_{27} 3 \).

Make and prove a conjecture about the relationship between \( \log_a b \) and \( \log_b a \).

**SOLUTION:**

\( \log_3 27 = 3 \) and \( \log_{27} 3 = \frac{1}{3} \)

Conjecture: \( \log_a b = \frac{1}{\log_b a} \)

Proof: \( \log_a b = \frac{1}{\log_b a} \)

Original Statement

\[
\log_a b = \frac{1}{\log_b a}
\]

Change of Base Formula

\[
\frac{1}{\log_a b} = \frac{1}{\log_b a}
\]

Inverse Property of Exponents and Logarithms

**ANSWER:**

\( \log_3 27 = 3 \) and \( \log_{27} 3 = \frac{1}{3} \)

Conjecture: \( \log_a b = \frac{1}{\log_b a} \)

Proof: \( \log_a b = \frac{1}{\log_b a} \)

Original Statement

\[
\log_a b = \frac{1}{\log_b a}
\]

Change of Base Formula

\[
\frac{1}{\log_a b} = \frac{1}{\log_b a}
\]

Inverse Property of Exponents and Logarithms

72. **WRITING IN MATH** Explain how exponents and logarithms are related. Include examples like how to solve a logarithmic equation using exponents and how to solve an exponential equation using logarithms.

**SOLUTION:**

Logarithms are exponents. To solve logarithmic equations, write each side of the equation using exponents and solve by using the Inverse Property of Exponents and Logarithms. To solve exponential equations, use the Property of Equality for Logarithmic Functions and the Power Property of Logarithms.

**ANSWER:**

Logarithms are exponents. To solve logarithmic equations, write each side of the equation using exponents and solve by using the Inverse Property of Exponents and Logarithms. To solve exponential equations, use the Property of Equality for Logarithmic Functions and the Power Property of Logarithms.

73. Which expression represents \( f \left[ g(x) \right] \) if \( f(x) = x^2 + 4x + 3 \) and \( g(x) = x - 5 \)?

A \( x^2 + 4x - 2 \)

B \( x^2 - 6x + 8 \)

C \( x^2 - 9x + 23 \)

D \( x^2 - 14x + 6 \)

**SOLUTION:**

\[
f \left[ g(x) \right] = f(x - 5)
\]

\[
= (x - 5)^2 + 4(x - 5) + 3
\]

\[
= x^2 - 10x + 25 + 4x - 20 + 3
\]

\[
= x^2 - 6x + 8
\]

Option B is the correct answer.

**ANSWER:**

B
74. **EXTENDED RESPONSE** Colleen rented 3 documentaries, 2 video games, and 2 movies. The charge was $16.29. The next week, she rented 1 documentary, 3 video games, and 4 movies for a total charge of $19.84. The third week she rented 2 documentaries, 1 video game, and 1 movie for a total charge of $9.14.

a. Write a system of equations to determine the cost to rent each item.

b. What is the cost to rent each item?

**SOLUTION:**

a. Let \( d \), \( v \) and \( m \) be the number of documentaries, video games and movies. The system of equation represents this situation is:

\[
3d + 2v + 2m = 16.29 \\
d + 3v + 4m = 19.84 \\
2d + v + m = 9.14
\]

b. The solution of the above system of equation are 1.99, 2.79 and 2.37.

**ANSWER:**

a. \( 3d + 2v + 2m = 16.29, d + 3v + 4m = 19.84, 2d + v + m = 9.14 \)

b. documentaries: $1.99, video games: $2.79, movies: $2.37

75. **GEOMETRY** If the surface area of a cube is increased by a factor of 9, what is the change in the length of the sides if the cube?

F The length is 2 times the original length.

G The length is 3 times the original length.

H The length is 6 times the original length.

J The length is 9 times the original length.

**SOLUTION:**

If the surface area of a cube is increased by a factor of 9, the length is 3 times the original length. Therefore, option G is the correct answer.

**ANSWER:**

G

76. **SAT/ACT** Which of the following *most* accurately describes the translation of the graph \( y = (x + 4)^2 - 3 \) to the graph of \( y = (x - 1)^2 + 3 \)?

A down 1 and to the right 3

B down 6 and to the left 5

C up 1 and to the left 3

D up 1 and to the right 3

E up 6 and to the right 5

**SOLUTION:**

The graph move up 6 units and to the right 5. Option E is the correct answer.

**ANSWER:**

E

**Solve each equation. Check your solutions.**

77. \( \log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x \)

**SOLUTION:**

\[
\log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x \\
\log_5 7 + \log_5 4^{1/2} = \log_5 x \\
\log_5 \left( 7 \times 4^{1/2} \right) = \log_5 x \\
x = \left( 7 \times 4^{1/2} \right) \\
= 7 \times 2 \\
x = 14
\]

The solution is 14.

**ANSWER:**

14
7.6 Common Logarithms

78. \(2 \log_2 x - \log_2 (x + 3) = 2\)

**SOLUTION:**
\[
2 \log_2 x - \log_2 (x + 3) = 2 \\
\log_2 \left( \frac{x^2}{x + 3} \right) = 2 \\
2^2 = \frac{x^2}{x + 3} \\
4 = \frac{x^2}{x + 3} \\
4x + 12 = x^2 \\
x^2 - 4x - 12 = 0 \\
(x - 6)(x + 2) = 0
\]

By Zero Product Property:
\[x - 6 = 0 \text{ or } x + 3 = 0\]
\[x = 6 \text{ or } x = -3\]

Logarithms are not defined for negative values. Therefore, the solution is 6.

**ANSWER:**
6

79. \(\log_5 48 - \log_5 \frac{16}{5} + \log_5 5 = \log_5 5x\)

**SOLUTION:**
\[
\log_5 48 - \log_5 \frac{16}{5} + \log_5 5 = \log_5 5x \\
\log_5 \left( \frac{48 \cdot 5}{16} \right) = \log_5 5x \\
\frac{48}{16} \cdot 5 = 5x \\
x = 15
\]

The solution is 15.

**ANSWER:**
15

80. \(\log_{10} a + \log_{10} (a + 21) = 2\)

**SOLUTION:**
\[
\log_{10} a + \log_{10} (a + 21) = 2 \\
\log_{10} (a(a + 21)) = 2 \\
a(a + 21) = 10^2 \\
a^2 + 21a - 100 = 0 \\
(a + 25)(a - 4) = 0
\]

By the Zero Product Property:
\[a - 4 = 0 \text{ or } a + 25 = 0\]
\[a = 4 \quad \text{or} \quad a = -25\]

Logarithms are not defined for negative values. Therefore, the solution is 4.

**ANSWER:**
4

Solve each equation or inequality.

81. \(\log_4 x = \frac{1}{2}\)

**SOLUTION:**
\[
\log_4 x = \frac{1}{2} \\
x = 4^{\frac{1}{2}} \\
x = 2
\]

The solution is 2.

**ANSWER:**
2
7-6 Common Logarithms

82. \( \log_{81} 729 = x \)

**SOLUTION:**

\[
\log_{81} 729 = x \\
\frac{\log 729}{\log 81} = x \\
\frac{\log 3^6}{\log 3^4} = x \\
\frac{6 \log 3}{4 \log 3} = x \\
x = \frac{3}{2}
\]

The solution is \( \frac{3}{2} \).

**ANSWER:**

\( \frac{3}{2} \)

83. \( \log_8 (x^2 + x) = \log_8 12 \)

**SOLUTION:**

\[
\log_8 (x^2 + x) = \log_8 12 \\
x^2 + x = 12 \\
x^2 + x - 12 = 0 \\
(x + 4)(x - 3) = 0
\]

By the Zero Product Property:

\( x + 4 = 0 \) or \( x - 3 = 0 \)

\( x = -4 \) or \( x = 3 \)

The solution is \(-4, 3\).

**ANSWER:**

\(-4, 3\)

84. \( \log_8 (3y - 1) < \log_8 (y + 5) \)

**SOLUTION:**

Logarithms are defined only for positive values. So, the argument should be greater than zero.

\[
3y - 1 > 0 \quad \text{or} \quad y + 5 > 0 \\
3y > 1 \quad \text{or} \quad y > -5 \\
y > \frac{1}{3} \quad \text{or} \quad y > -5
\]

Solve the original equation.

\[
\log_8 (3y - 1) < \log_8 (y + 5) \\
3y - 1 < y + 5 \\
2y < 6 \\
y < 3
\]

The common region is the solution of the given inequality. Therefore, the solution region is \( \frac{1}{3} < y < 3 \).

**ANSWER:**

\( \left\{ y \middle| \frac{1}{3} < y < 3 \right\} \)
7-6 Common Logarithms

85. SAILING The area of a triangular sail is $16x^4 - 60x^3 - 28x^2 + 56x - 32$ square meters. The base of the triangle is $x - 4$ meters. What is the height of the sail?

*SOLUTION*

The area of a triangle is $A = \frac{1}{2}bh$.

Substitute $16x^4 - 60x^3 - 28x^2 + 56x - 32$ and $x - 4$ for $A$ and $b$ respectively.

\[16x^4 - 60x^3 - 28x^2 + 56x - 32 = \frac{1}{2}(x - 4)h\]

Solve for $h$.

\[2\left(16x^4 - 60x^3 - 28x^2 + 56x - 32\right) = h\]

\[32x^4 - 120x^3 - 56x^2 + 112x - 64 = h\]

\[32x^3 + 8x^2 - 24x + 16 = h\]

The height of the sail is $32x^3 + 8x^2 - 24x + 16$.

*ANSWER*

$32x^3 + 8x^2 - 24x + 16$

86. HOME REPAIR Mr. Turner is getting new locks installed. The locksmith charges $85 for the service call, $25 for each door, and each lock costs $30.

a. Write an equation that represents the cost for $x$ number of doors.

\[y = 85 + 25x + 30x\]

\[= 85 + 55x\]

b. Substitute 4 for $x$ and evaluate.

\[y = 85 + 55(4)\]

\[= 85 + 220\]

\[= 305\]

This will cost $305.

*ANSWER*

a. $y = 85 + 55x$

b. $305$

Write an equivalent exponential equation.

87. $\log_2 5 = x$

*SOLUTION*

$\log_2 5 = x$

$2^x = 5$

*ANSWER*

$2^x = 5$

88. $\log_4 x = 3$

*SOLUTION*

$\log_4 x = 3$

$4^3 = x$

*ANSWER*

$4^3 = x$
7-6 Common Logarithms

89. \( \log_5 25 = 2 \)

**SOLUTION:**
\[ \log_5 25 = 2 \]
\[ 5^2 = 25 \]
**ANSWER:**
\[ 5^2 = 25 \]

90. \( \log_7 10 = x \)

**SOLUTION:**
\[ \log_7 10 = x \]
\[ 7^x = 10 \]
**ANSWER:**
\[ 7^x = 10 \]

91. \( \log_6 x = 4 \)

**SOLUTION:**
\[ \log_6 4 = x \]
\[ 6^x = 4 \]
**ANSWER:**
\[ 6^4 = x \]

92. \( \log_4 64 = 3 \)

**SOLUTION:**
\[ \log_4 64 = 3 \]
\[ 4^3 = 64 \]
**ANSWER:**
\[ 4^3 = 64 \]
7-7 Base e and Natural Logarithms

Write an equivalent exponential or logarithmic function.

1. \( e^x = 30 \)
   
   **SOLUTION:**
   
   \[ e^x = 30 \]
   
   \[ \log_e 30 = x \]
   
   **ANSWER:**
   
   \[ \ln 30 = x \]

2. \( \ln x = 42 \)
   
   **SOLUTION:**
   
   \[ \ln x = 42 \]
   
   \[ \log_e x = 42 \]
   
   \[ e^{42} = x \]
   
   **ANSWER:**
   
   \[ e^{42} = x \]

3. \( e^3 = x \)
   
   **SOLUTION:**
   
   \[ e^3 = x \]
   
   \[ \log_e x = 3 \]
   
   **ANSWER:**
   
   \[ \ln x = 3 \]

4. \( \ln 18 = x \)
   
   **SOLUTION:**
   
   \[ \ln 18 = x \]
   
   \[ \log_e 18 = x \]
   
   \[ e^x = 18 \]
   
   **ANSWER:**
   
   \[ e^x = 18 \]

Write each as a single logarithm.

5. \( 3 \ln 2 + 2 \ln 4 \)
   
   **SOLUTION:**
   
   \[ 3 \ln 2 + 2 \ln 4 = \ln 2^3 + \ln 4^2 \]
   
   \[ = \ln 8 + \ln 16 \]
   
   \[ = \ln (8 \times 16) \]
   
   \[ = \ln 128 \]
   
   \[ = \ln 2^7 \]
   
   \[ = 7 \ln 2 \]
   
   **ANSWER:**
   
   \[ 7 \ln 2 \]

6. \( 5 \ln 3 - 2 \ln 9 \)
   
   **SOLUTION:**
   
   \[ 5 \ln 3 - 2 \ln 9 = \ln 3^5 - \ln 9^2 \]
   
   \[ = \ln 243 - \ln 81 \]
   
   \[ = \ln \left( \frac{243}{81} \right) \]
   
   \[ = \ln 3 \]
   
   **ANSWER:**
   
   \[ \ln 3 \]

7. \( 3 \ln 6 + 2 \ln 9 \)
   
   **SOLUTION:**
   
   \[ 3 \ln 6 + 2 \ln 9 = \ln 6^3 + \ln 9^2 \]
   
   \[ = \ln 216 + \ln 81 \]
   
   \[ = \ln (216 \times 81) \]
   
   \[ = \ln 17496 \]
   
   **ANSWER:**
   
   \[ \ln 17496 \]

8. \( 3 \ln 5 + 4 \ln x \)
   
   **SOLUTION:**
   
   \[ 3 \ln 5 + 4 \ln x = \ln 5^3 + \ln x^4 \]
   
   \[ = \ln 125 + \ln x^4 \]
   
   \[ = \ln 125x^4 \]
   
   **ANSWER:**
   
   \[ \ln 125x^4 \]
Solve each equation. Round to the nearest ten-thousandth.

9. \(5e^x - 24 = 16\)

\[5e^x - 24 = 16\]
\[5e^x = 40\]
\[e^x = 8\]
\[\ln e^x = \ln 8\]
\[x = \ln 8\]
\[\approx 2.0794\]

**ANSWER:**
2.0794

10. \(-3e^x + 9 = 4\)

\[-3e^x + 9 = 4\]
\[-3e^x = -5\]
\[e^x = \frac{5}{3}\]
\[\ln e^x = \ln \frac{5}{3}\]
\[x = \ln \frac{5}{3}\]
\[\approx 0.5108\]

**ANSWER:**
0.5108

11. \(3e^{-3x} + 4 = 6\)

\[3e^{-3x} + 4 = 6\]
\[3e^{-3x} = 2\]
\[e^{-3x} = \frac{2}{3}\]
\[\ln e^{-3x} = \ln \frac{2}{3}\]
\[-3x = \ln \frac{2}{3}\]
\[x = -\frac{3}{\ln \frac{2}{3}}\]
\[\approx 0.1352\]

**ANSWER:**
0.1352

12. \(2e^{-x} - 3 = 8\)

\[2e^{-x} - 3 = 8\]
\[2e^{-x} = 11\]
\[e^{-x} = \frac{11}{2}\]
\[\ln e^{-x} = \ln \frac{11}{2}\]
\[-x = \ln \frac{11}{2}\]
\[x = -\ln \frac{11}{2}\]
\[\approx -1.7047\]

**ANSWER:**
-1.7047
13. \( \ln 3x = 8 \)

**SOLUTION:**

\[
\ln 3x = 8 \\
3x = e^8 \\
x = \frac{e^8}{3} \\
\approx 993.6527
\]

The solution is \( 993.6527 \).

**ANSWER:**

\( 993.6527 \)

14. \(-4 \ln 2x = -26 \)

**SOLUTION:**

\[
-4 \ln 2x = -26 \\
\ln 2x = \frac{-26}{-4} \\
\ln 2x = 6.5 \\
2x = e^{6.5} \\
x = \frac{e^{6.5}}{2} \\
\approx 332.5708
\]

The solution is \( 332.5708 \).

**ANSWER:**

\( 332.5708 \)

15. \( \ln (x + 5)^2 < 6 \)

**SOLUTION:**

\[
\ln (x + 5)^2 < 6 \\
e^{\ln(x+5)^2} < e^6 \\
(x + 5)^2 < e^6 \\
\sqrt{(x + 5)^2} < \sqrt{e^6} \\
x + 5 < \pm e^3 \\
-e^3 < x + 5 < e^3 \\
-e^3 - 5 < x < e^3 - 5 \\
-25.0855 < x < 15.0855
\]

The solution region is \( \{ x \mid -25.0855 < x < 15.0855, x \neq -5 \} \).

**ANSWER:**

\( \{ x \mid -25.0855 < x < 15.0855, x \neq -5 \} \)

16. \( \ln (x - 2)^3 > 15 \)

**SOLUTION:**

\[
\ln (x - 2)^3 > 15 \\
3 \ln (x - 2) > 15 \\
\ln (x - 2) > 5 \\
x - 2 > e^5 \\
x > e^5 + 2 \\
x > 150.4132
\]

The solution region is \( \{ x \mid x > 150.4132 \} \).

**ANSWER:**

\( \{ x \mid x > 150.4132 \} \)
17. \( e^x > 29 \)

**SOLUTION:**

\[
e^x > 29
\]

\[
\ln e^x > \ln 29
\]

\[
x > \ln 29
\]

\[
x > 3.3673
\]

The solution region is \( \{ x \mid x > 3.3673 \} \).

**ANSWER:**

\( \{ x \mid x > 3.3673 \} \)

18. \( 5 + e^{-x} > 14 \)

**SOLUTION:**

\[
5 + e^{-x} > 14
\]

\[
e^{-x} > 9
\]

\[
\ln e^{-x} > \ln 9
\]

\[
x > \ln 9
\]

\[
x < -\ln 9
\]

\[
x < -2.1972
\]

The solution region is \( \{ x \mid x < -2.1972 \} \).

**ANSWER:**

\( \{ x \mid x < -2.1972 \} \)

19. **SCIENCE** A virus is spreading through a computer network according to the formula

\[ v(t) = 30e^{0.1t}, \]

where \( v \) is the number of computers infected and \( t \) is the time in minutes. How long will it take the virus to infect 10,000 computers?

**SOLUTION:**

Substitute 10,000 for \( v(t) \) and solve for \( t \).

\[
30e^{0.1t} = 10000
\]

\[
e^{0.1t} = \frac{10000}{30}
\]

\[
0.1t = \ln \frac{10000}{30}
\]

\[
t = \frac{1}{0.1} \ln \frac{10000}{30}
\]

\[
\approx 58
\]

The virus will take about 58 min to infect 10,000 computers.

**ANSWER:**

about 58 min

**Write an equivalent exponential or logarithmic function.**

20. \( e^{-x} = 8 \)

**SOLUTION:**

\[
e^{-x} = 8
\]

\[
\log_e 8 = -x
\]

\[
\ln 8 = -x
\]

**ANSWER:**

\( \ln 8 = -x \)

21. \( e^{-5x} = 0.1 \)

**SOLUTION:**

\[
e^{-5x} = 0.1
\]

\[
\log_e 0.1 = -5x
\]

\[
\ln 0.1 = -5x
\]

**ANSWER:**

\( \ln 0.1 = -5x \)
22. ln 0.25 = x

**SOLUTION:**
ln 0.25 = x
log_e 0.25 = x
0.25 = e^x

**ANSWER:**
0.25 = e^x

23. ln 5.4 = x

**SOLUTION:**
ln 5.4 = x
log_e 5.4 = x
5.4 = e^x

**ANSWER:**
5.4 = e^x

24. e^{x-3} = 2

**SOLUTION:**
e^{x-3} = 2
log_e 2 = x - 3
ln 2 = x - 3

**ANSWER:**
ln 2 = x - 3

25. ln (x + 4) = 36

**SOLUTION:**
ln (x + 4) = 36
log_e (x + 4) = 36
x + 4 = e^{36}

**ANSWER:**
e^{36} = x + 4

26. \(e^{-2} = x^6\)

**SOLUTION:**
e^{-2} = x^6
log_e x^6 = -2
ln x^6 = -2
6 ln x = -2

**ANSWER:**
-2 = 6 ln x

27. ln \(e^x = 7\)

**SOLUTION:**
ln \(e^x = 7\)
log_e \(e^x = 7\)
e^7 = e^x

**ANSWER:**
e^7 = e^x

**Write each as a single logarithm.**

28. ln 125 - 2 ln 5

**SOLUTION:**
ln 125 - 2 ln 5 = ln 125 - ln 5^2
= ln 125 - ln 25
= ln \frac{125}{25}
= ln 5

**ANSWER:**
ln 5

29. 3 ln 10 + 2 ln 100

**SOLUTION:**
3 ln 10 + 2 ln 100 = ln 10^3 + ln 100^2
= ln (10^3 \times 100^2)
= ln 10^7
= 7 ln 10

**ANSWER:**
7 ln 10
30. \[4 \ln \frac{1}{3} - 6 \ln \frac{1}{9}\]

**SOLUTION:**

\[4 \ln \frac{1}{3} - 6 \ln \frac{1}{9} = \ln \left(\frac{1}{3}\right)^4 - \ln \left(\frac{1}{9}\right)^6\]

\[= \ln \frac{1}{3^4} - \ln \frac{1}{9^6}\]

\[= \ln \left(\frac{1}{3^4} \cdot \frac{1}{9^6}\right)\]

\[= \ln \frac{9^6}{3^4}\]

\[= \ln 3^{12}\]

\[= \ln \left(2^7 \cdot 3^5\right)\]

\[= \ln \left(\frac{1}{2^2}\right)^7\]

\[= \ln \frac{1}{2^{14}}\]

\[= \ln 2^{-14}\]

\[= -14 \ln 2\]

**ANSWER:**

\[-14 \ln 2\]

31. \[7 \ln \frac{1}{2} + 5 \ln 2\]

**SOLUTION:**

\[7 \ln \frac{1}{2} + 5 \ln 2 = \ln \left(\frac{1}{2}\right)^7 + \ln 2^5\]

\[= \ln \frac{1}{2^7} + \ln 2^5\]

\[= \ln \left(\frac{1}{2^7} \cdot 2^5\right)\]

\[= \ln \frac{1}{2^2}\]

\[= \ln 2^{-2}\]

\[= -2 \ln 2\]

**ANSWER:**

\[-2 \ln 2\]

32. \[8 \ln x - 4 \ln 5\]

**SOLUTION:**

\[8 \ln x - 4 \ln 5 = \ln x^8 - \ln 5^4\]

\[= \ln \frac{x^8}{5^4}\]

\[= \ln \frac{x^8}{625}\]

**ANSWER:**

\[\ln \frac{x^8}{625}\]

33. \[3 \ln x^2 + 4 \ln 3\]

**SOLUTION:**

\[3 \ln x^2 + 4 \ln 3 = \ln \left(x^2\right)^3 + \ln 3^4\]

\[= \ln x^6 + \ln 81\]

\[= \ln 81x^6\]

**ANSWER:**

\[\ln 81x^6\]

Solve each equation. Round to the nearest thousandth.

34. \[6e^x - 3 = 35\]

**SOLUTION:**

\[6e^x - 3 = 35\]

\[6e^x = 38\]

\[e^x = \frac{38}{6}\]

\[x = \ln \frac{38}{6}\]

\[\approx 1.8458\]

The solution is 1.8458.

**ANSWER:**

1.8458
35. \(4e^x + 2 = 180\)

**SOLUTION:**

\[
4e^x + 2 = 180
\]

\[
4e^x = 178
\]

\[
e^x = \frac{178}{4}
\]

\[
x = \ln \left(\frac{178}{4}\right)
\]

\[
\approx 3.7955
\]

The solution is 3.7955.

**ANSWER:**

3.7955

36. \(3e^{2x} - 5 = -4\)

**SOLUTION:**

\[
3e^{2x} - 5 = -4
\]

\[
3e^{2x} = 1
\]

\[
e^{2x} = \frac{1}{3}
\]

\[
2x = \ln \left(\frac{1}{3}\right)
\]

\[
x = \frac{\ln \left(\frac{1}{3}\right)}{2}
\]

\[
\approx -0.5493
\]

The solution is \(-0.5493\).

**ANSWER:**

\(-0.5493\)

37. \(-2e^{3x} + 19 = 3\)

**SOLUTION:**

\[
-2e^{3x} + 19 = 3
\]

\[
-2e^{3x} = -16
\]

\[
e^{3x} = 8
\]

\[
\ln e^{3x} = \ln 8
\]

\[
3x = \ln 8
\]

\[
x = \frac{\ln 8}{3}
\]

\[
x \approx 0.6931
\]

The solution is 0.6931.

**ANSWER:**

0.6931

38. \(6e^{4x} + 7 = 4\)

**SOLUTION:**

\[
6e^{4x} + 7 = 4
\]

\[
6e^{4x} = -3
\]

\[
e^{4x} = -\frac{1}{2}
\]

\[
\ln e^{4x} = \ln \left(-\frac{1}{2}\right)
\]

Logarithm is not defined for negative values.

Therefore, there is no solution.

**ANSWER:**

no solution
7-7 Base e and Natural Logarithms

39. \(-4e^{-x} + 9 = 2\)

**SOLUTION:**

\[-4e^{-x} + 9 = 2\]

\[-4e^{-x} = -7\]

\[e^{-x} = \frac{7}{4}\]

\[\ln e^{-x} = \ln \frac{7}{4}\]

\[-x = \ln \frac{7}{4}\]

\[x = -\ln \frac{7}{4}\]

\[x \approx -0.5596\]

The solution is \(-0.5596\)

**ANSWER:**

\(-0.5596\)

40. **CCSS SENSE-MAKING** The value of a certain car depreciates according to \(v(t) = 18500e^{-0.186t}\), where \(t\) is the number of years after the car is purchased new.

a. What will the car be worth in 18 months?

b. When will the car be worth half of its original value?

c. When will the car be worth less than $1000?

**SOLUTION:**

a. 18 months is equal to 1.5 years. Substitute 1.5 for \(t\) and evaluate.

\[v(t) = 18500e^{-0.186(1.5)}\]

\[= 18500e^{-0.279}\]

\[\approx 13996\]

The car will be worth about 13,996 in 18 months.

b. Substitute 9250 for \(v(t)\) and solve for \(t\).

\[9250 = 18500e^{-0.186t}\]

\[
\frac{9250}{18500} = e^{-0.186t}
\]

\[e^{-0.186t} = \frac{1}{2}\]

\[\ln \frac{1}{2} = -0.186t\]

\[t = -\frac{\ln 0.5}{0.186}\]

\[\approx 3.73\]

The car will be worth half of its original value in about 3.73 years.

c. Substitute 1,000 for \(v(t)\) and solve for \(t\).

\[1000 = 18500e^{-0.186t}\]

\[e^{-0.186t} = \frac{1000}{18500}\]

\[\ln \frac{1000}{18500} = -0.186t\]

\[t = -\frac{\ln \frac{1000}{18500}}{0.186}\]

\[\approx 15.69\]

The car will be worth less than $1000 after 15.69 years.

**ANSWER:**

a. $13,996

b. about 3.73 yr

c. about 15.69 yr
7-7 Base e and Natural Logarithms

Solve each inequality. Round to the nearest thousandth.

41. \( e^x \leq 8.7 \)

**SOLUTION:**

\[ e^x \leq 8.7 \]

\[
\ln e^x \leq \ln 8.7 \\
x \leq \ln 8.7 \\
x \leq 2.1633 
\]

The solutions are \( \{ x \mid x \leq 2.1633 \} \).

**ANSWER:**

\( \{ x \mid x \leq 2.1633 \} \)

42. \( e^x \geq 42.1 \)

**SOLUTION:**

\[ e^x \geq 42.1 \]

\[
\ln e^x \geq \ln 42.1 \\
x \geq \ln 42.1 \\
x \geq 3.7400 
\]

The solutions are \( \{ x \mid x \geq 3.7400 \} \).

**ANSWER:**

\( \{ x \mid x \geq 3.7400 \} \)

43. \( \ln (3x + 4)^3 > 10 \)

**SOLUTION:**

\[
\ln (3x + 4)^3 > 10 \\
3 \ln (3x + 4) > 10 \\
\ln (3x + 4) > \frac{10}{3} \\
\log_e (3x + 4) > \frac{10}{3} \\
3x + 4 > e^{\frac{10}{3}} \\
3x > e^{\frac{10}{3}} - 4 \\
x > \frac{e^{\frac{10}{3}} - 4}{3} \\
x > 8.0105 
\]

The solutions are \( \{ x \mid x > 8.0105 \} \).

**ANSWER:**

\( \{ x \mid x > 8.0105 \} \)

44. \( 4 \ln x^2 < 72 \)

**SOLUTION:**

\[
4 \ln x^2 < 72 \\
\ln x^2 < 18 \\
e^{\ln x^2} < e^{18} \\
x^2 < e^{18} \\
\sqrt{x^2} < \sqrt{e^{18}} \\
x < \pm e^9 \\
-e^9 < x < e^9 \\
-8103.0839 < x < 8103.0839 
\]

The solutions are \( \{ x \mid -8103.0839 < x < 8103.0839 \} \).

**ANSWER:**

\( \{ x \mid -8103.0839 < x < 8103.0839, x \neq 0 \} \)
45. \( \ln (8x^4) > 24 \)

**SOLUTION:**

\[
\ln (8x^4) > 24
\]
\[
e^{\ln(8x^4)} > e^{24}
\]
\[
8x^4 > e^{24}
\]
\[
x^4 > \frac{e^{24}}{8}
\]
\[
\sqrt[4]{x^4} > \sqrt[8]{e^{24}}
\]
\[
x > \frac{e^6}{\sqrt[8]{8}} \quad \text{or} \quad x < -\frac{e^6}{\sqrt[8]{8}}
\]
\[
x > 239.8802 \quad \text{or} \quad x < -239.8802
\]

The solutions are \( \{ x : x > 239.8802 \text{ or } x < -239.8802 \} \).

**ANSWER:**

\( \{ x : x > 239.8802 \text{ or } x < -239.8802 \} \)

46. \(-2[\ln (x - 6)^{-1}] \leq 6\)

**SOLUTION:**

\(-2[\ln (x - 6)^{-1}] \leq 6\)
\[
2[\ln (x - 6)] \leq 6
\]
\[
\ln (x - 6) \leq 3
\]
\[
\log_e (x - 6) \leq 3
\]
\[
x - 6 \leq e^3
\]
\[
x \leq e^3 + 6
\]
\[
x \leq 26.0855
\]

Logarithms are not defined for negative values. So, the inequality is defined for \( x - 6 > 0 \).

Therefore, \( x > 6 \).

The solutions are \( \{ x : 6 < x \leq 26.0855 \} \).

**ANSWER:**

\( \{ x : 6 < x \leq 26.0855 \} \)

47. **FINANCIAL LITERACY** Use the formula for continuously compounded interest.

**a.** If you deposited $800 in an account paying 4.5% interest compounded continuously, how much money would be in the account in 5 years?

**b.** How long would it take you to double your money?

**c.** If you want to double your money in 9 years, what rate would you need?

**d.** If you want to open an account that pays 4.75% interest compounded continuously and have $10,000 in the account 12 years after your deposit, how much would you need to deposit?

**SOLUTION:**

**a.** Substitute 800, 0.045 and 5 for \( P, r \) and \( t \) in the continuously compounded interest.

\[
A = Pe^{rt}
\]
\[
= 800e^{0.045(5)}
\]
\[
= 800e^{0.225}
\]
\[
= 1001.86
\]

**b.** Substitute 1600, 800 and 0.045 for \( A, P \) and \( r \) in the continuously compounded interest.

\[
A = Pe^{rt}
\]
\[
1600 = 800e^{0.045t}
\]
\[
2 = e^{0.045t}
\]
\[
0.045t = \ln 2
\]
\[
t = \frac{\ln 2}{0.045}
\]
\[
\approx 15.4
\]

**c.** Substitute 1600, 800 and 9 for \( A, P \) and \( t \) in the continuously compounded interest.

\[
A = Pe^{rt}
\]
\[
1600 = 800e^{9r}
\]
\[
2 = e^{9r}
\]
\[
9r = \ln 2
\]
\[
r = \frac{\ln 2}{9}
\]
\[
\approx 0.077
\]
\[
= 7.7%
\]

**d.** Substitute 10000, 0.0475 and 12 for \( A, r \) and \( t \) in the continuously compounded interest.
50. \( \ln \sqrt[3]{x^3} \)

**SOLUTION:**

\[
\ln \sqrt[3]{x^3} = \ln \left( x^3 \right)^{\frac{1}{3}} = \frac{1}{3} \ln x^3 = \frac{3}{5} \ln x
\]

**ANSWER:**

\[
\frac{3}{5} \ln x
\]

51. \( \ln xy^4z^{-3} \)

**SOLUTION:**

\[
\ln xy^4z^{-3} = \ln x + \ln y^4 + \ln z^{-3} = \ln x + 4 \ln y - 3 \ln z
\]

**ANSWER:**

\[
\ln x + 4 \ln y - 3 \ln z
\]

Use the natural logarithm to solve each equation.

52. \( b^x = 24 \)

**SOLUTION:**

\[
b^x = 24
\]

\[
\ln b^x = \ln 24
\]

\[
x \ln b = \ln 24
\]

\[
x = \frac{\ln 24}{\ln b} \approx 1.5283
\]

The solution is about 1.5283.

**ANSWER:**

about 1.5283
53. $3^x = 0.4$

**SOLUTION:**

$$3^x = 0.4$$

$$\ln 3^x = \ln 0.4$$

$$x \ln 3 = \ln 0.4$$

$$x = \frac{\ln 0.4}{\ln 3}$$

$$x \approx -0.8340$$

The solution is about -0.8340.

**ANSWER:**

about -0.8340

54. $2^{3x} = 18$

**SOLUTION:**

$$2^{3x} = 18$$

$$\ln 2^{3x} = \ln 18$$

$$3x \ln 2 = \ln 18$$

$$3x = \frac{\ln 18}{\ln 2}$$

$$x = \frac{\ln 18}{3 \ln 2}$$

$$x \approx 1.3900$$

The solution is 1.3900.

**ANSWER:**

about 1.3900

55. $5^{2x} = 38$

**SOLUTION:**

$$5^{2x} = 38$$

$$\ln 5^{2x} = \ln 38$$

$$2x \ln 5 = \ln 38$$

$$x = \frac{\ln 38}{2 \ln 5}$$

$$x \approx 1.1301$$

The solution is 1.1301.

**ANSWER:**

about 1.1301

56. **CCSS MODELING** Newton’s Law of Cooling, which can be used to determine how fast an object will cool in given surroundings, is represented by $T(t) = T_s + (T_0 - T_s)e^{-kt}$, where $T_0$ is the initial temperature of the object, $T_s$ is the temperature of the surroundings, $t$ is the time in minutes, and $k$ is a constant value that depends on the type of object.

**a.** If a cup of coffee with an initial temperature of 180° is placed in a room with a temperature of 70°, then the coffee cools to 140° after 10 minutes, find the value of $k$.

**b.** Use this value of $k$ to determine the temperature of the coffee after 20 minutes.

**c.** When will the temperature of the coffee reach 75°?

**SOLUTION:**

**a.** Substitute 180, 70, 10 and 140 for $T_0$, $T_s$, $t$ and $T(t)$ respectively then solve for $k$. 
7-7 Base e and Natural Logarithms

\[ T(t) = T_s + (T_0 - T_s) e^{-kt} \]

140 = 70 + (180 - 70) e^{-10k}
140 = 70 + 110 e^{-10k}
70 = 110 e^{-10k}
\[
\frac{7}{11} = e^{-10k}
\]
\[
\ln \frac{7}{11} = -10k
\]
\[
\ln \frac{7}{11} = k
\]

The value of \( k \) is about 0.045.

b. Substitute 0.094446, 180, 70 and 20 for \( k, T_0, T_s \) and \( t \) respectively and simplify.

\[ T(t) = T_s + (T_0 - T_s) e^{-kt} \]
\[ T(t) = 70 + (180 - 70) e^{-0.045 \times 20} \]
\[ T(t) = 70 + 110 e^{-0.9} \]
\[ T(t) \approx 114.7 \]

The temperature of the coffee after 20 minutes is about 114.7°.

c. Substitute 0.094446, 180, 70 and 75 for \( k, T_0, T_s \) and \( T(t) \) respectively then solve for \( t \).

\[ T(t) = T_s + (T_0 - T_s) e^{-kt} \]
\[ 75 = 70 + (180 - 70) e^{-0.045t} \]
\[ 75 = 70 + 110 e^{-0.045t} \]
\[ 5 = 110 e^{-0.045t} \]
\[ \frac{1}{22} = e^{-0.045t} \]
\[ \ln \frac{1}{22} = -0.045t \]
\[ \ln \frac{1}{22} = t \]
\[ -0.045 \approx 68 \]

The temperature of the coffee will reach 75° in about 68 min.

ANSWER:

a. 0.045
b. about 114.7°
c. about 68 min

57. MULTIPLE REPRESENTATIONS In this problem, you will use \( f(x) = e^x \) and \( g(x) = \ln x \).

a. GRAPHICAL Graph both functions and their axis of symmetry, \( y = x \), for \(-5 \leq x \leq 5\). Then graph \( a(x) = e^{-x} \) on the same graph.

b. ANALYTICAL The graphs of \( a(x) \) and \( f(x) \) are reflections along which axis? What function would be a reflection of \( f(x) \) along the other axis?

c. LOGICAL Determine the two functions that are reflections of \( g(x) \). Graph these new functions.

d. VERBAL We know that \( f(x) \) and \( g(x) \) are inverses. Are any of the other functions that we have graphed inverses as well? Explain your reasoning.

SOLUTION:

SOLUTION:

b. \( y \)-axis; \( a(x) = -e^x \)

c. \( \ln (-x) \) is a reflection across the \( y \)-axis. \( -\ln x \) is a reflection across the \( x \)-axis.
Write an equivalent exponential or logarithmic function.

1. \(e^x = 30\)
   SOLUTION:
   
   \(\ln 30 = x\)
   
   ANSWER:
   

2. \(\ln x = 3\)
   SOLUTION:
   
   \(x = e^3\)
   
   ANSWER:
   

82. \(362^p = 216^{p - 1}\)
   SOLUTION:
   
   \(\log_{362} 216 = p - 1\)
   
   \(p = \log_{362} 216 + 1\)
   
   ANSWER:
   
   

58. CHALLENGE
   Solve \(4^x - 2^{x+1} = 15\) for \(x\).

   SOLUTION:
   
   \(4^x - 2^{x+1} = 15\)
   
   \((2^2)^x - 2^{x+1} = 15\)
   
   \((2^x)^2 - 2^x \cdot 2 = 15\)

   Let \(2^x = y\)

   \(y^2 - 2y = 15\)
   
   \(y^2 - 2y - 15 = 0\)
   
   \((y - 5)(y + 3) = 0\)

   By the Zero Product Property:

   \(y - 5 = 0\) or \(y + 3 = 0\)

   \(y = 5\) or \(y = -3\)

   \(2^x = 5\) or \(2^x = -3\)

   \(x \log 2 = \log 5\) or \(x \log 2 = \log (-3)\)

   Logarithms are not defined for negative values.

   Therefore, \(x = \frac{\log 5}{\log 2} \approx 2.3219\)

   ANSWER:
   
   2.3219
59. PROOF Prove $\ln ab = \ln a + \ln b$ for natural logarithms.

**SOLUTION:**
Let $p = \ln a$ and $q = \ln b$.

That means that $e^p = a$ and $e^q = b$.

\[
ab = e^p \times e^q
\]

\[
ab = e^{p+q}
\]

\[
\ln(ab) = (p + q)
\]

\[
\ln(ab) = \ln a + \ln b
\]

**ANSWER:**
Let $p = \ln a$ and $q = \ln b$.

That means that $e^p = a$ and $e^q = b$.

\[
ab = e^p \times e^q
\]

\[
ab = e^{p+q}
\]

\[
\ln(ab) = (p + q)
\]

\[
\ln(ab) = \ln a + \ln b
\]

60. REASONING Determine whether $x > \ln x$ is sometimes, always, or never true. Explain your reasoning.

**SOLUTION:**
Sample answer: Always; the graph of $y = x$ is always greater than the graph of $y = \ln x$ and the graphs never intersect.

**ANSWER:**
Sample answer: Always; the graph of $y = x$ is always greater than the graph of $y = \ln x$ and the graphs never intersect.

61. OPEN ENDED Express the value 3 using $e^x$ and the natural log.

**SOLUTION:**
Sample answer: $e^{\ln 3}$

**ANSWER:**
Sample answer: $e^{\ln 3}$

62. WRITING IN MATH Explain how the natural log can be used to solve a natural base exponential function.

**SOLUTION:**
Sample answer: The natural log and natural base are inverse functions, so taking the natural log of a natural base will undo the natural base and make the problem easier to solve.

**ANSWER:**
Sample answer: The natural log and natural base are inverse functions, so taking the natural log of a natural base will undo the natural base and make the problem easier to solve.

63. Given the function $y = 2.34x + 11.33$, which statement best describes the effect of moving the graph down two units?

A The $x$-intercept decreases.
B The $y$-intercept decreases.
C The $x$-intercept remains the same.
D The $y$-intercept remains the same.

**SOLUTION:**
The $y$-intercept decreases if the graph moves down two units. Therefore, option B is the correct answer.

**ANSWER:**
B
7-7 Base e and Natural Logarithms

64. **GRIDDED RESPONSE** Aidan sells wooden picture frames over the Internet. He purchases supplies for $85 and pays $19.95 for his website. If he charges $15 for each frame, how many will he need to sell in order to make a profit of at least $270.

**SOLUTION:**
Let \( x \) be the number of frames.
She will earn $15x for each necklace that she sells but will need to subtract from that her fixed costs of supplies ($85) and website fee ($19.95)

\[
270 < 15x - 85 - 19.95 \\
270 < 15x - 104.95 \\
374.95 < 15x \\
24.997 < x
\]

Therefore, she needs to sell 25 frames to make a profit of at least $270.

**ANSWER:**
25

65. Solve \(|2x - 5| = 17\).
\( F \) -6, -11  \\
\( G \) -6, 11  \\
\( H \) 6, -11  \\
\( J \) 6, 11

**SOLUTION:**
\[|2x - 5| = 17\]
\[2x - 5 = 17 \quad \text{and} \quad -(2x - 5) = 17\]
\[2x = 22 \quad \text{and} \quad -2x + 5 = 17\]
\[x = 11 \quad \text{and} \quad -2x = 12\]
\[x = 11 \quad \text{and} \quad x = -6\]

The solutions are -6 and 11.
Therefore, option \( G \) is the correct answer.

**ANSWER:**
G

66. A local pet store sells rabbit food. The cost of two 5-pound bags is $7.99. The total cost \( c \) of purchasing \( n \) bags can be found by—
\( A \) multiplying \( n \) by \( c \).  \\
\( B \) multiplying \( n \) by 5.  \\
\( C \) multiplying \( n \) by the cost of 1 bag.  \\
\( D \) dividing \( n \) by the cost of 1 bag.

**SOLUTION:**
The total cost \( c \) of purchasing \( n \) bags can be found by multiplying \( n \) by the cost of 1 bag.
Therefore, option \( C \) is the correct answer.

**ANSWER:**
C

Solve each equation or inequality. Round to the nearest ten-thousandth

67. \( 2^x = 53 \)

**SOLUTION:**
\[2^x = 53\]
\[\ln 2^x = \ln 53\]
\[x \ln 2 = \ln 53\]
\[x = \frac{\ln 53}{\ln 2}\]
\[\approx 5.7279\]

**ANSWER:**
5.7279

68. \( 2.3^{x^2} = 66.6 \)

**SOLUTION:**
\[2.3^{x^2} = 66.6\]
\[\ln 2.3^{x^2} = \ln 66.6\]
\[x^2 \ln 2.3 = \ln 66.6\]
\[x^2 = \frac{\ln 66.6}{\ln 2.3}\]
\[x = \pm \sqrt{\frac{\ln 66.6}{\ln 2.3}}\]
\[\approx \pm 2.2452\]

**ANSWER:**
\( \pm 2.2452 \)
69. $3^{4x-7} < 4^{2x} + 3$

**SOLUTION:**

\[
\begin{align*}
3^{4x-7} &< 4^{2x} + 3 \\
\ln 3^{4x-7} &< \ln 4^{2x} + 3 \\
(4x-7) \ln 3 &< (2x+3) \ln 4 \\
(4x-7) &< (2x+3) \cdot 1.3863 \\
4.3944491x - 7.6902860 &< 2.7725887x + 4.1588831 \\
x &< 7.3059
\end{align*}
\]

The solution region is \( \{ x \mid x < 7.3059 \} \).

**ANSWER:**

\( x < 7.3059 \)

70. $6^{3y} = 8^{y-1}$

**SOLUTION:**

\[
\begin{align*}
6^{3y} &= 8^{y-1} \\
\log 6^{3y} &= \log 8^{y-1} \\
3y \log 6 &= (y-1) \log 8 \\
\frac{3y}{\log 8} &= y-1 \\
3y(0.8617) &= y-1 \\
2.5851y &= y-1 \\
1.5851y &= -1 \\
y &= -0.6309
\end{align*}
\]

The solution is \(-0.6309\).

**ANSWER:**

\(-0.6309\)

71. $12^{x-5} \geq 9.32$

**SOLUTION:**

\[
\begin{align*}
12^{x-5} &\geq 9.32 \\
\log 12^{x-5} &\geq \log 9.32 \\
(x-5) \log 12 &\geq \log 9.32 \\
x-5 &\geq \frac{\log 9.32}{\log 12} \\
x &\geq \frac{\log 9.32}{\log 12} + 5 \\
x &\geq 8.0086
\end{align*}
\]

The solution is \( x \geq 8.0086 \).

**ANSWER:**

\( 8.0086 \)
73. **SOUND** Use the formula \( L = 10 \log_{10} R \), where \( L \) is the loudness of a sound and \( R \) is the sound’s relative intensity. Suppose the sound of one alarm clock is 80 decibels. Find out how much louder 10 alarm clocks would be than one alarm clock.

**SOLUTION:**
Substitute 80 for \( L \) and solve for \( R \).

\[
80 = 10 \log_{10} R \\
8 = \log_{10} R \\
R = 10^8
\]

If 10 alarm clocks ring at a time, the relative velocity of the sound is \( 10 \times 10^8 \). Substitute \( 10 \times 10^8 \) for \( R \) and solve for \( L \).

\[
L = 10 \log_{10} \left( 10 \times 10^8 \right) \\
= 10 \log_{10} 10^9 \\
= 90
\]

The loudness would be increased by 10 decibels.

**ANSWER:**
10 decibels

---

74. **Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.**

\( x^3 + 5x^2 + 8x + 4; x + 1 \)

**SOLUTION:**
Divide the polynomial \( x^3 + 5x^2 + 8x + 4 \) by \( x + 1 \).

\[
\begin{array}{c|ccccc}
 & x^3 & +5x^2 & +8x & +4 \\
\hline
x+1 & x^3 & +x^2 \\
 & \text{(-)}4x^2 & +4x & \\
\hline & 4x & +4 & \\
\hline
\end{array}
\]

Factor the quotient \( x^2 + 4x + 4 \).

\[
x^2 + 4x + 4 = (x + 2)(x + 2)
\]

Therefore, the factors are \( x + 2 \) and \( x + 2 \).

**ANSWER:**
\( x + 2, x + 2 \)
7-7 Base e and Natural Logarithms

75. \(x^3 + 4x^2 + 7x + 6; x + 2\)

**SOLUTION:**

Divide the polynomial \(x^3 + 4x^2 + 7x + 6\) by \(x + 2\).

\[
\begin{array}{c|cccc}
  & x^3 & +4x^2 & +7x & +6 \\
\hline
x & -x^2 & -2x & -4 & -8 \\
\hline
& x^2 & +2x & +3 \\
\hline
\end{array}
\]

The quotient is a prime. So the factor is \(x^2 + 2x + 3\).

**ANSWER:**

\(x^2 + 2x + 3\)

76. **CRAFTS** Mrs. Hall is selling crocheted items. She sells large afghans for $60, baby blankets for $40, doilies for $25, and pot holders for $5. She takes the following number of items to the fair: 12 afghans, 25 baby blankets, 45 doilies, and 50 pot holders.

a. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.

b. Suppose Mrs. Hall sells all of the items. Find her total income as a matrix.

**SOLUTION:**

a. \[
\begin{bmatrix}
12 & 25 & 45 & 50 \\
60 & 40 & 25 & 5
\end{bmatrix}
\]

b. Multiply the matrixes.

\[
\begin{bmatrix}
12 & 25 & 45 & 50 \\
60 & 40 & 25 & 5
\end{bmatrix}
\begin{bmatrix}
60 \\
40 \\
25 \\
5
\end{bmatrix}
=[ 720 + 1000 + 1155 + 250 ]
\]

**ANSWER:**

\[
\begin{bmatrix}
3095
\end{bmatrix}
\]
7-7 Base e and Natural Logarithms

Solve each equation.

77. \(2^{3x} + 5 = 128\)

**SOLUTION:**
\[
2^{3x} + 5 = 128 \\
\log 2^{3x} + 5 = \log 128 \\
(3x + 5) \log 2 = \log 128 \\
3x + 5 = \frac{\log 128}{\log 2} \\
3x = \frac{\log 128}{\log 2} - 5 \\
\log 2 = \log 128 \\
x = \frac{\log 128 - 5}{3} \\
= \frac{2}{3}
\]

The solution is \(\frac{2}{3}\).

**ANSWER:**
\(\frac{2}{3}\)

78. \(5^{n-3} = \frac{1}{25}\)

**SOLUTION:**
\[
5^{n-3} = \frac{1}{25} \\
\log 5^{n-3} = \log \frac{1}{25} \\
(n-3) \log 5 = \log 5^{-2} \\
(n-3) \log 5 = -2 \log 5 \\
n - 3 = -2 \\
n = 1
\]

The solution is 1.

**ANSWER:**
1

79. \(\left(\frac{1}{9}\right)^m = 81^{m+4}\)

**SOLUTION:**
\[
\left(\frac{1}{9}\right)^m = 81^{m+4} \\
\log \left(\frac{1}{9}\right)^m = \log 81^{m+4} \\
\log 1_{9}^m = \log \left(9^2\right)^{m+4} \\
\log 9^{-m} = \log 9^{2m+8} \\
-m \log 9 = (2m + 8) \log 9 \\
-m = 2m + 8 \\
m = -\frac{8}{3}
\]

The solution is \(-\frac{8}{3}\).

**ANSWER:**
\(-\frac{8}{3}\)

80. \(\left(\frac{1}{7}\right)^{y-3} = 343\)

**SOLUTION:**
\[
\left(\frac{1}{7}\right)^{y-3} = 343 \\
\log \left(\frac{1}{7}\right)^{y-3} = \log 343 \\
\log \frac{1}{7}^{y-3} = \log 7^3 \\
\log 7^{-(y-3)} = \log 7^3 \\
-(y-3) \log 7 = 3 \log 7 \\
-(y-3) = 3 \\
y = 0
\]

The solution is 0.

**ANSWER:**
0
7-7 Base e and Natural Logarithms

81. \(10^{x-1} = 100^{2x-3}\)

**SOLUTION:**
\[
\begin{align*}
10^{x-1} &= 100^{2x-3} \\
10^{x-1} &= (10^2)^{2x-3} \\
10^{x-1} &= 10^{4x-6} \\
\log 10^{x-1} &= \log 10^{4x-6} \\
(x-1)\log 10 &= (4x-6)\log 10 \\
x-1 &= 4x-6 \\
3x &= 5 \\
x &= \frac{5}{3}
\end{align*}
\]

The solution is \(\frac{5}{3}\).

**ANSWER:**
\[
\frac{5}{3}
\]

82. \(36^{2p} = 216^{p-1}\)

**SOLUTION:**
\[
\begin{align*}
36^{2p} &= 216^{p-1} \\
\left(6^2\right)^{2p} &= \left(6^3\right)^{p-1} \\
6^{4p} &= 6^{3p-3} \\
4p &= 3p - 3 \\
p &= -3
\end{align*}
\]

The solution is 3.

**ANSWER:**
\[-3\]
1. **PALEONTOLOGY** The half-life of Potassium-40 is about 1.25 billion years.
   
   a. Determine the value of \( k \) and the equation of decay for Potassium-40.
   b. A specimen currently contains 36 milligrams of Potassium-40. How long will it take the specimen to decay to only 15 milligrams of Potassium-40?
   c. How many milligrams of Potassium-40 will be left after 300 million years?
   d. How long will it take Potassium-40 to decay to one eighth of its original amount?

**SOLUTION:**

a. Exponential decay can be modeled by the function
   \[ f(x) = ae^{-kt}, \]
   where \( a \) is the initial value, \( t \) is time in years, and \( k \) is a constant representing the rate of continuous decay.

   Substitute \( 1.25 \times 10^9 \), and \( 0.5a \) for \( t \) and \( f(x) \) respectively, then solve for \( k \).

   \[
   0.5a = ae^{-k1.25 \times 10^9} \\
   e^{-k1.25 \times 10^9} = 0.5 \\
   -k1.25 \times 10^9 = \ln 0.5 \\
   k = \frac{-\ln 0.5}{1.25 \times 10^9} \\
   = 5.545 \times 10^{-10}
   \]

b. Substitute 36, 15 and \( 5.545 \times 10^{-10} \) for \( a, f(x) \) and \( k \) respectively then solve for \( t \).

   \[
   15 = 36e^{-5.545 \times 10^{-10}t} \\
   e^{-5.545 \times 10^{-10}t} = \frac{15}{36} \\
   -5.545 \times 10^{-10}t = \ln \frac{15}{36} \\
   t = \frac{\ln 15 - \ln 36}{-5.545 \times 10^{-10}} \\
   = 3750120003
   \]

   It will take, \( 3,750,120,003 \) years.

**ANSWER:**

a. \( 5.545 \times 10^{-10} \)

b. \( 1,578,843,530 \) yr

c. About \( 30.48 \) mg

d. \( 3,750,120,003 \) yr

\[ f(x) = 36e^{-5.545 \times 10^{-10}(300000000)} = 36e^{-0.16035} \approx 30.48 \]

After 300 million years, it will have about 30.48 mg of Potassium-40.
2. SCIENCE A certain food is dropped on the floor and is growing bacteria exponentially according to the model \( y = 2e^{kt} \), where \( t \) is the time in seconds.

a. If there are 2 cells initially and 8 cells after 20 seconds, find the value of \( k \) for the bacteria.

b. The “5-second rule” says that if a person who drops food on the floor eats it within 5 seconds, there will be no harm. How much bacteria is on the food after 5 seconds?

c. Would you eat food that had been on the floor for 5 seconds? Why or why not? Do you think that the information you obtained in this exercise is reasonable? Explain.

**SOLUTION:**

a. Substitute 8 and 20 for \( y \) and \( t \) respectively, then solve for \( k \).

\[
y = 2e^{kt} \\
8 = 2e^{20k} \\
e^{20k} = 4 \\
20k = \ln 4 \\
k = \frac{\ln 4}{20} \\
\approx 0.0693
\]

b. Substitute 5 and 0.0693 for \( t \) and \( k \) respectively then solve for \( y \).

\[
y = 2e^{0.0693(5)} \\
= 2e^{0.3465} \\
= 2.828
\]

It has 2.828 cells.

c. Sample answer: Yes; it has not even grown 1 cell in 5 seconds. There are many factors that affect this equation, such as how clean the floor is and what type of food was dropped.

**ANSWER:**

a. \( k \approx 0.0693 \)

b. about 2.828 cells

c. Sample answer: Yes; it has not even grown 1 cell in 5 seconds. There are many factors that affect this equation, such as how clean the floor is and what type of food was dropped.

3. ZOOLOGY Suppose the red fox population in a restricted habitat follows the function

\[
P(t) = \frac{16,500}{1 + 18e^{-0.085t}}
\]

where \( t \) represents the time in years.

a. Graph the function for \( 0 \leq t \leq 200 \).

b. What is the horizontal asymptote?

c. What is the maximum population?

d. When does the population reach 16,450?

**SOLUTION:**

b. The horizontal asymptote is \( y = 16,500 \).

c. The maximum population is 16,500.

d. Substitute 16450 for \( P(t) \) and solve for \( t \).

\[
16450 = \frac{16,500}{1 + 18e^{-0.085t}} \\
1 + 18e^{-0.085t} = \frac{16,500}{16450} \\
18e^{-0.085t} = \frac{16,500}{16450} - 1 \\
e^{-0.085t} = \frac{50}{16450 \times 18} \\
-0.085t = \ln \left( \frac{50}{16450 \times 18} \right) \\
t = -\frac{\ln \left( \frac{50}{16450 \times 18} \right)}{-0.085} \\
\approx 102
\]

The population reaches 16,450 in about 102 years.

**ANSWER:**

a.
1. PALEONTOLOGY The half-life of Potassium-40 is about 1.25 billion years.

a. Determine the value of $k$ and the equation of decay for Potassium-40.

b. $y = 16,500$

c. 16,500

d. about 102 years

4. CCSS PERSEVERANCE The half-life of Rubidium-87 is about 48.8 billion years.

a. Determine the value of $k$ and the equation of decay for Rubidium-87.

b. A specimen currently contains 50 milligrams of Rubidium-87. How long will it take the specimen to decay to only 18 milligrams of Rubidium-87?

c. How many milligrams of Rubidium-87 will be left after 800 million years?

d. How long will it take Rubidium-87 to decay to one-sixteenth its original amount?

SOLUTION:

a. Exponential decay can be modeled by the function $f(x) = ae^{-kt}$, where $a$ is the initial value, $t$ is time in years, and $k$ is a constant representing the rate of continuous decay.

Substitute $4.8 \times 10^9$, and $0.5a$ for $t$ and $f(x)$ respectively, then solve for $k$.

$$0.5a = ae^{-k \cdot 48.8 \times 10^9}$$

$$e^{-k \cdot 48.8 \times 10^9} = 0.5$$

$$-k \cdot 48.8 \times 10^9 = \ln 0.5$$

$$k = -\frac{\ln 0.5}{48.8 \times 10^9}$$

$$\approx 1.42 \times 10^{-11}$$

b. Substitute 50, 18 and $1.42 \times 10^{-11}$ for $a, f(x)$ and $k$ respectively then solve for $t$.

$$18 = 50e^{-1.42 \times 10^{-11}t}$$

$$e^{-1.42 \times 10^{-11}t} = \frac{18}{50}$$

$$-1.42 \times 10^{-11}t = \ln \frac{18}{50}$$

$$t = \frac{\ln 18 - \ln 50}{-1.42 \times 10^{-11}}$$

$$\approx 71,947,270,950$$

It will take 71,947,270,950 years.

c. Substitute 50, 800,000,000 and $1.42 \times 10^{-11}$ for $a, t$ and $k$ respectively then evaluate.

$$f(x) = 50e^{-1.42 \times 10^{-11}(800,000,000)}$$

$$= 50e^{-0.01136}$$

$$\approx 49.4$$

After 800 million years, it will have about 49.4 mg of Rubidium-87.

d. Substitute 50, $\frac{50}{16}$ and $1.42 \times 10^{-10}$ for $a, f(x)$ and $k$ respectively then solve for $t$.

$$\frac{50}{16} = 50e^{-1.42 \times 10^{-11}t}$$

$$e^{-1.42 \times 10^{-11}t} = \frac{1}{16}$$

$$-1.42 \times 10^{-11}t = \ln \frac{1}{16}$$

$$t = \frac{\ln 1 - \ln 16}{-1.42 \times 10^{-11}}$$

$$\approx 195,252,726,918$$

It will take 195.3 billion years.

ANSWER:

a. $k \approx 1.42 \times 10^{-11}$

b. 71,947,270,950 yr

c. about 49.4 mg

d. 195.3 billion yr

5. BIOLOGY A certain bacteria is growing exponentially according to the model $y = 80e^{kt}$, where $t$ is the time in minutes.
7-8 Using Exponential and Logarithmic Functions

a. If there are 80 cells initially and 675 cells after 30 minutes, find the value of $k$ for the bacteria.

b. When will the bacteria reach a population of 6000 cells?

c. If a second type of bacteria is growing exponentially according to the model $y = 35e^{0.0978t}$, determine how long it will be before the number of cells of this bacteria exceed the number of cells in the other bacteria.

**SOLUTION:**

a. Substitute 675 and 30 for $y$ and $t$ respectively, then solve for $k$.

$$y = 80e^{kt}$$

$$675 = 80e^{30k}$$

$$e^{30k} = \frac{675}{80}$$

$$30k = \ln \frac{675}{80}$$

$$k = \frac{\ln 675 - \ln 80}{30}$$

$$\approx 0.071$$

b. Substitute 6000 and 0.071 for $y$ and $k$ respectively then solve for $t$.

$$6000 = 80e^{0.071t}$$

$$e^{0.071t} = \frac{6000}{80}$$

$$0.071t = \ln 75$$

$$t = \frac{\ln 75}{0.071}$$

$$\approx 60.8$$

It will take about 60.8 min.

**ANSWER:**

a. $k \approx 0.071$

b. about 60.8 min

c. about 30.85 min

6. **FORESTRY** The population of trees in a certain forest follows the function

$$f(t) = \frac{18000}{1 + 16e^{-0.084t}}$$

where $t$ is the time in years.

a. Graph the function for $0 \leq t \leq 100$.

b. When does the population reach 17500 trees?

**SOLUTION:**

a. Substitute 17500 for $f(t)$ then solve for $t$. 

It will take about 30.85 min.
7. **PALEONTOLOGY** A paleontologist finds a human bone and determines that the Carbon-14 found in the bone is 85% of that found in living bone tissue. How old is the bone?

**SOLUTION:**
The formula for the decay of Carbon-14 is \( y = a e^{-kt} \). Let \( a \) be the initial amount of Carbon-14 in the animal’s body. The amount \( y \) that remains after \( t \) years is 85% of \( a \) or 0.85\( a \). The rate of continues growth (\( k \)) is 0.00012. Substitute corresponding values.

\[
0.85a = ae^{-0.00012t} \\
0.85 = e^{-0.00012t} \\
\ln 0.85 = \ln e^{-0.00012t} \\
-0.00012t = \ln 0.85 \\
t = \frac{\ln 0.85}{-0.00012} \\
\approx 1354
\]

The bone is about 1354 yr old.

**ANSWER:**
about 1354 yr old

8. **ANTHROPOLOGY** An anthropologist has determined that a newly discovered human bone is 8000 years old. How much of the original amount of Carbon-14 is in the bone?

**SOLUTION:**
The formula for the decay of Carbon-14 is \( y = a e^{-kt} \). Let 100 be the initial amount of Carbon-14 in the animal’s body. Substitute 100, 8000 and 0.00012 for \( a, t \) and \( k \) then solve for \( y \).

\[
y = 100e^{-0.00012(8000)} \\
= 100e^{-0.96} \\
\approx 38
\]

The original amount of Carbon-14 in the bone is about 38%.

**ANSWER:**
about 38%
9. **RADIOACTIVE DECAY** 100 milligrams of Uranium-238 are stored in a container. If Uranium-238 has a half-life of about 4.47 billion years, after how many years will only 10 milligrams be present?

**SOLUTION:**

The formula for the decay of Carbon-14 is \( y = a e^{-kt} \).

Substitute 4.47 and 0.5a for \( t \) and \( y \) then solve for \( k \).

\[
0.5a = ae^{-k(4.47)}
\]

\[
0.5 = e^{-4.47k}
\]

\[
\ln 0.5 = -4.47k
\]

\[
k = \frac{1}{-4.47} \cdot \ln 0.5
\]

\[
\approx 0.155066
\]

Substitute 10, 100 and 0.155066 for \( y \), \( a \) and \( k \) the solve for \( t \).

\[
10 = 100e^{-0.155066t}
\]

\[
0.1 = e^{-0.155066t}
\]

\[
\ln 0.1 = -0.155066t
\]

\[
-0.155066t = \ln 0.1
\]

\[
t = \frac{\ln 0.1}{-0.155066}
\]

\[
\approx 14.85
\]

10 milligrams Uranium-238 will be present after about 14.85 billion years.

**ANSWER:**

about 14.85 billion yr

10. **POPULATION GROWTH** The population of the state of Oregon has grown from 3.4 million in 2000 to 3.7 million in 2006.

**a.** Write an exponential growth equation of the form \( y = ae^{kt} \) for Oregon, where \( t \) is the number of years after 2000.

**b.** Use your equation to predict the population of Oregon in 2020.

**c.** According to the equation, when will Oregon reach 6 million people?

**SOLUTION:**

\[
y = 3.4e^{0.014t}
\]

\[
3.7 = 3.4e^{kt}
\]

\[
e^{kt} = \frac{3.7}{3.4}
\]

\[
k = \frac{1}{t} \ln \frac{3.7}{3.4}
\]

\[
\approx -0.014
\]

The exponential growth equation is \( y = 3.4e^{0.014t} \).

**b.** Substitute 20 for \( t \) and evaluate.

\[
y = 3.4e^{0.014(20)}
\]

\[
= 4.5
\]

The population will be 4.5 million in 2020.

**c.** Substitute 6 for \( y \) and solve for \( t \).

\[
6 = 3.4e^{0.014t}
\]

\[
e^{0.014t} = \frac{6}{3.4}
\]

\[
0.014t = \ln \frac{6}{3.4}
\]

\[
t = \frac{\ln 6 - \ln 3.4}{0.014}
\]

\[
\approx 41
\]

The population will be 6 million in about 2041.
11. **HALF-LIFE** A substance decays 99.9% of its total mass after 200 years. Determine the half-life of the substance.

**SOLUTION:**
Substitute 0.001\(a\) and 200 and for \(y\), and \(t\) in the exponential decay formula then solve for \(k\).

\[
y = ae^{-kt} \\
0.001a = ae^{-200k} \\
0.001 = e^{-200k} \\
-200k = \ln 0.001 \\
k = \frac{\ln 0.001}{-200} \\
\approx 0.0345
\]

Substitute 0.0345 and 0.5\(a\) for \(k\) and \(y\) respectively then solve for \(t\).

\[
0.5a = ae^{-0.0345t} \\
0.5 = e^{-0.0345t} \\
\ln 0.5 = -0.0345t \\
t = \frac{\ln 0.5}{-0.0345} \\
\approx 20.1
\]

The half-life of the substance is about 20.1 years.

**ANSWER:**
about 20.1 yr

12. **LOGISTIC GROWTH** The population in millions of the state of Ohio after 1900 can be modeled by \(P(t) = \frac{12.95}{1 + 2.4e^{-kt}}\), where \(t\) is the number of years after 1900 and \(k\) is a constant.

a. If Ohio had a population of 10 million in 1970, find the value of \(k\).

b. According to the equation, when will the population of Ohio reach 12 million?

**SOLUTION:**
a. Substitute 10 and 70 for \(P(t)\) and \(t\) respectively then solve for \(k\).

\[
10 = \frac{12.95}{1 + 2.4e^{-kt}} \\
10 + 24e^{-70k} = 12.95 \\
24e^{-70k} = 2.95 \\
e^{-70k} = \frac{2.95}{24} \\
-70k = \ln \frac{2.95}{24} \\
k = \frac{\ln 2.95 - \ln 24}{-70} \\
\approx 0.0299464
\]

b. Substitute 12 and 0.0299464 for \(P(t)\) and \(k\) respectively then solve for \(t\).

\[
12 = \frac{12.95}{1 + 2.4e^{-0.0299464t}} \\
12 + 28.8e^{-0.0299464t} = 12.95 \\
28.8e^{-0.0299464t} = 0.95 \\
e^{-0.0299464t} = \frac{0.95}{28.8} \\
-0.0299464t = \ln \frac{0.95}{28.8} \\
t = \frac{\ln 0.95 - \ln 28.8}{-0.0299464} \\
\approx 114
\]

The population will reach 12 million in 2014.

**ANSWER:**
a. \(k \approx 0.0299464\)
b. 2014
13. **MULTIPLE REPRESENTATIONS** In this problem, you will explore population growth. The population growth of a country follows the exponential function \( f(t) = 8e^{0.075t} \) or the logistic function \( g(t) = \frac{400}{1 + 16e^{-0.025t}} \). The population is measured in millions and \( t \) is time in years.

a. **GRAPHICAL** Graph both functions for \( 0 \leq t \leq 100 \).

b. **ANALYTICAL** Determine the intersection of the graphs. What is the significance of this intersection?

c. **ANALYTICAL** Which function is a more accurate estimate of the country’s population 100 years from now? Explain your reasoning.

**SOLUTION:**

b. The graphs intersect at \( t = 20.79 \). Sample answer: This intersection indicates the point at which both functions determine the same population at the same time.

c. Sample answer: The logistic function \( g(t) \) is a more accurate estimate of the country’s population since \( f(t) \) will continue to grow exponentially and \( g(t) \) considers limitations on population growth such as food supply.

14. **OPEN ENDED** Give an example of a quantity that grows or decays at a fixed rate. Write a real-world problem involving the rate and solve by using logarithms.

**SOLUTION:**

Sample answer: money in a bank; See students’ work.

**ANSWER:**

Sample answer: money in a bank; See students’ work.

15. **CHALLENGE** Solve \( \frac{120,000}{1 + 48e^{-0.015t}} = 24e^{0.055t} \) for \( t \).

**SOLUTION:**

Use a graphing calculator to solve the equation by graphing.

Enter each expression separately.

\[
Y_1 = \frac{120,000}{1 + 48e^{-0.015x}}
\]

\[
Y_2 = 24e^{0.055x}
\]
Graph the equations on the same coordinate plane.

Calculate the intersection point of the graphs.

The $x$-value of the intersection point is the solution of the original equation. So, $t \approx 113.45$.

**ANSWER:**
$t \approx 113.45$

16. **CCSS ARGUMENTS** Explain mathematically why

\[
f(t) = \frac{c}{1 + 60e^{-0.5t}}\]

approaches, but never reaches

the value of $c$ as $t \to +\infty$.

**SOLUTION:**

Sample answer: As $t \to +\infty$, $e^{-t} \to 0$. So, the
denominator approaches $1 + 0$ or 1. As the
denominator approaches 1, $f(t) \to \frac{c}{1}$ or c.

However, since $e^{-t}$ never reaches 0, $f(t)$ can never reach c.

**ANSWER:**

Sample answer: As $t \to +\infty$, $e^{-t} \to 0$. So, the
denominator approaches $1 + 0$ or 1. As the
denominator approaches 1, $f(t) \to \frac{c}{1}$ or c.

However, since $e^{-t}$ never reaches 0, $f(t)$ can never reach c.

17. **OPEN ENDED** Give an example of a quantity that grows logistically and has limitations to growth.

Explain why the quantity grows in this manner.

**SOLUTION:**

Sample answer: The spread of the flu throughout a small town. The growth of this is limited to the population of the town itself.

**ANSWER:**

Sample answer: The spread of the flu throughout a small town. The growth of this is limited to the population of the town itself.
18. **WRITING IN MATH** How are exponential, continuous exponential, and logistic functions used to model different real-world situations?

**SOLUTION:**
Sample answer: Exponential functions can be used to model situations that incorporate a percentage of growth or decay for a specific number of times per year. Continuous exponential functions can be used to model situations that incorporate a percentage of growth or decay continuously. Logistic functions can be used to model situations that incorporate a percentage of growth or decay continuously and consist of a limiting factor.

**ANSWER:**
Sample answer: Exponential functions can be used to model situations that incorporate a percentage of growth or decay for a specific number of times per year. Continuous exponential functions can be used to model situations that incorporate a percentage of growth or decay continuously. Logistic functions can be used to model situations that incorporate a percentage of growth or decay continuously and consist of a limiting factor.

19. Kareem is making a circle graph showing the favorite ice cream flavors of customers at his store. The table summarizes the data. What central angle should Kareem use for the section representing chocolate?

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>chocolate</td>
<td>35</td>
</tr>
<tr>
<td>vanilla</td>
<td>42</td>
</tr>
<tr>
<td>strawberry</td>
<td>7</td>
</tr>
<tr>
<td>mint chip</td>
<td>12</td>
</tr>
<tr>
<td>butter pecan</td>
<td>4</td>
</tr>
</tbody>
</table>

A 35°  
B 63°  
C 126°  
D 150°  

**SOLUTION:**
The total number of customers is 35 + 42 + 7 + 12 + 4 = 100.

The central angle for the chocolate section is \( \frac{35}{100} \times 360 \).

\[ \frac{35}{100} \times 360 = 126 \]

Option C is the correct answer.

**ANSWER:**
C
20. **PROBABILITY** Lydia has 6 books on her bookshelf. Two are literature books, one is a science book, two are math books, and one is a dictionary. What is the probability that she randomly chooses a science book and the dictionary?

F 1/3

G 1/12

H 1/4

J 1/15

**SOLUTION:**
The probability of getting a science book and the dictionary is 1/3.

The probability of getting a dictionary and a science book is 1/12.

Here, the order is not a matter. Therefore, the probability of getting a science book and the dictionary is

\[
\frac{1}{30} + \frac{1}{30} = \frac{1}{15}.
\]

Option J is the correct answer.

**ANSWER:**

J

21. **SAT/ACT** Peter has made a game for his daughter’s birthday party. The playing board is a circle divided evenly into 8 sectors. If the circle has a radius of 18 inches, what is the approximate area of one of the sectors?

A 4 in²

B 14 in²

C 32 in²

D 127 in²

E 254 in²

**SOLUTION:**
The area of a circle is \(\pi r^2\).

The area of one sector is \(\frac{324\pi}{8} = 40.5\pi \approx 127\) in².

Therefore option D is the correct answer.

**ANSWER:**

D

22. **STATISTICS** In a survey of 90 physical trainers, 15 said they went for a run at least 5 times per week. Of that group, 5 said they also swim during the week, and at least 25% all of the trainers run and swim every week. Which conclusion is valid based on the information given?

F The report is accurate because 15 out of 90 is 25%.

G The report is accurate because 5 out of 15 is 33%, which is at least 25%.

H The report is inaccurate because 5 out of 90 is only 5.6%.

J The report is inaccurate because no one knows if swimming is really exercising.

**SOLUTION:**

Option H is the correct answer.

The report is inaccurate because 5 out of 90 is only 5.6%.

**ANSWER:**

H

**Write an equivalent exponential or logarithmic equation**

23. \(e^7 = y\)

**SOLUTION:**

\(e^7 = y\)

\(\log_e e^7 = \log_e y\)

\(7 = \ln y\)

**ANSWER:**

\(\ln y = 7\)

24. \(e^{2n-4} = 36\)

**SOLUTION:**

\(e^{2n-4} = 36\)

\(\log_e e^{2n-4} = \log_e 36\)

\(2n - 4 = \ln 36\)

**ANSWER:**

\(\ln 36 = 2n - 4\)
25. \( \ln 5 + 4 \ln x = 9 \)

**SOLUTION:**

\[
\ln 5 + 4 \ln x = 9 \\
\ln 5 + \ln x^4 = 9 \\
\ln 5x^4 = 9 \\
\ln e^{5x^4} = 9 \\
5x^4 = e^9
\]

**ANSWER:**

\[5x^4 = e^9\]

26. **EARTHQUAKES** The table shows the magnitude of some major earthquakes. (*Hint:* A magnitude \( x \) earthquake has an intensity of \( 10^x \).)

<table>
<thead>
<tr>
<th>Year</th>
<th>Location</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963</td>
<td>Yugoslavia</td>
<td>6.0</td>
</tr>
<tr>
<td>1970</td>
<td>Peru</td>
<td>7.8</td>
</tr>
<tr>
<td>1988</td>
<td>Armenia</td>
<td>7.0</td>
</tr>
<tr>
<td>2004</td>
<td>Morocco</td>
<td>6.4</td>
</tr>
<tr>
<td>2007</td>
<td>Indonesia</td>
<td>8.4</td>
</tr>
<tr>
<td>2010</td>
<td>Haiti</td>
<td>7.0</td>
</tr>
</tbody>
</table>

**a.** For which two earthquakes was the intensity of one 10 times that of the other? For which two was the intensity of one 100 times that of the other?

**b.** What would be the magnitude of an earthquake that is 1000 times as intense as the 1963 earthquake in Yugoslavia?

**c.** Suppose you know that \( \log_7 2 \approx 0.3562 \) and \( \log_7 3 \approx 0.5646 \). Describe two different methods that you could use to approximate \( \log_7 2.5 \). (You may use a calculator, of course.) Then describe how you can check your result.

**SOLUTION:**

**a.** The Richter scale magnitude reading \( m \) is given by \( m = \log_{10} x \), where \( x \) represents the amplitude of the seismic wave causing ground motion.

Find the amplitude of the seismic wave each magnitude.

Armenia and Yugoslavia, or Haiti and Yugoslavia; Morocco and Indonesia

**b.** Substitute \( 10^9 \) for \( x \) and evaluate.

\[
m = \log_{10} 10^6 = 9.0
\]

The magnitude of the earthquake is 9.0.

**c.** Sample answer:

Method 1: Use the Change of Base Formula and find that the value is about 0.4709.

Method 2: Use the values \( \log_7 2 \approx 0.3562 \) and \( \log_7 3 \approx 0.5646 \). First, average the values and then guess and check by raising 7 to the various powers. Continue until you get the desired closeness to 2.5. To check find \( 7^{0.4709} \) which is very close to 2.5.

**ANSWER:**

**a.** Armenia and Yugoslavia, or Haiti and Yugoslavia; Morocco and Indonesia

**b.** 9.0

**c.** Sample answer: Method 1: Use the Change of Base Formula and find that the value is about 0.4709.

Method 2: Use the values \( \log_7 2 \approx 0.3562 \) and \( \log_7 3 \approx 0.5646 \). First, average the values and then guess and check by raising 7 to the various powers. Continue until you get the desired closeness to 2.5. To check find \( 7^{0.4709} \) which is very close to 2.5.
7-8 Using Exponential and Logarithmic Functions

Solve each equation. Write in simplest form.

27. \( \frac{8}{5}x = \frac{4}{15} \)

**SOLUTION:**

\[
8 \times x = 4 \\
5 \quad 15 \\

x = \frac{4 \times 5}{15} \\
\quad = \frac{1}{6}
\]

**ANSWER:**

\[ \frac{1}{6} \]

30. \( \frac{6}{7} = 9p \)

**SOLUTION:**

\[
6 = 9p \\
\frac{6}{9} = p \\
\frac{2}{3} = p
\]

**ANSWER:**

\[ \frac{2}{3} \]

31. \( \frac{9}{8} = b \)

**SOLUTION:**

\[
9 = b \\
\frac{9}{8} = b
\]

**ANSWER:**

\[ \frac{9}{8} \]

32. \( \frac{6}{7} = \frac{3}{4} \)

**SOLUTION:**

\[
6 = 3 \\
7 = \frac{4}{4} \\
\frac{3}{4} = \frac{7}{4} \\
\frac{7}{8} = \frac{7}{8}
\]

**ANSWER:**

\[ \frac{7}{8} \]
33. \( \frac{1}{3} z = \frac{5}{6} \)

**SOLUTION:**

\[
\frac{1}{3} z = \frac{5}{6} \\
\times 3 \quad \times 3 \\
\hline
\quad z = \frac{15}{6} \\
\quad = \frac{5}{2} \\
\quad = 2 \frac{1}{2}
\]

**ANSWER:**

\( 2 \frac{1}{2} \)

34. \( \frac{2}{3} q = 7 \)

**SOLUTION:**

\[
\frac{2}{3} q = 7 \\
\times 3 \quad \times 3 \\
\hline
\quad q = \frac{21}{2} \\
\quad = 10 \frac{1}{2}
\]

**ANSWER:**

\( 10 \frac{1}{2} \)
Graph each function. State the domain and range.

1. \( f(x) = 3^x - 3 + 2 \)

**SOLUTION:**

The function is defined for all values of \( x \). Therefore, the domain is set of all real numbers.

The value of \( f(x) \) tends to 2 as \( x \) tends to \(-\infty\).

The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \( \infty \).

Therefore, the range of the function is \( \{ f(x) | f(x) > 2 \} \).

**ANSWER:**

D: all real numbers
R: \( \{ f(x) | f(x) > 2 \} \)

2. \( f(x) = 2 \left( \frac{3}{4} \right)^{x+1} - 3 \)

**SOLUTION:**

The function is defined for all values of \( x \). Therefore, the domain is set of all real numbers.

The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \(-\infty\).

The value of \( f(x) \) tends to 3 as \( x \) tends to \( \infty \).

Therefore, the range of the function is \( \{ f(x) | f(x) > -3 \} \).

**ANSWER:**

D: all real numbers
R: \( \{ f(x) | f(x) > -3 \} \)
Solve each equation or inequality. Round to four decimal places if necessary.

3. \(8^c + 1 = 16^{2c+3}\)

**SOLUTION:**

\[
8^c + 1 = 16^{2c+3} \\
(2^3)^c + 1 = (2^4)^{2c+3} \\
2^{3c+3} = 2^{8c+12} \\
3c + 3 = 8c + 12 \\
5c = 9 \\
c = \frac{9}{5}
\]

**ANSWER:**

\(c = \frac{9}{5}\)

4. \(9^{x-2} > \left(\frac{1}{27}\right)^x\)

**SOLUTION:**

\[
9^{x-2} > \left(\frac{1}{27}\right)^x \\
(3^2)^{x-2} > \frac{1}{(3^3)^x} \\
3^{2x-4} > 3^{-3x} \\
2x - 4 > -3x \\
5x > 4 \\
x > \frac{4}{5}
\]

**ANSWER:**

\(\left\{ x \mid x > \frac{4}{5}\right\}\)

5. \(2^a + 3 = 3^{2a-1}\)

**SOLUTION:**

\[
2^a + 3 = 3^{2a-1} \\
\log 2^a + 3 = \log 3^{2a-1} \\
(a + 3)\log 2 = (2a - 1)\log 3 \\
a + 3 = (2a - 1)\cdot \frac{\log 3}{\log 2} \\
= 2a\frac{\log 3}{\log 2} - \frac{\log 3}{\log 2} \\
3 + \frac{\log 3}{\log 2} = a\left(2\frac{\log 3}{\log 2} - 1\right) \\
a \approx 2.1130
\]

**ANSWER:**

\(a \approx 2.1130\)

6. \(\log_2 (x^2 - 7) = \log_2 6x\)

**SOLUTION:**

\[
\log_2 (x^2 - 7) = \log_2 6x \\
x^2 - 7 = 6x \\
x^2 - 6x - 7 = 0 \\
(x - 7)(x + 1) = 0
\]

By Zero Product Property:

\(x - 7 = 0\) or \(x + 1 = 0\)

\(x = 7\) or \(x = -1\)

The \(x\)-value \(-1\) makes the argument negative. Logarithm is not defined for negative numbers. Therefore, the solution is 7.

**ANSWER:**

\(x = 7\)
7. \( \log_5 x > 2 \)

**SOLUTION:**
\[
\log_5 x > 2 \\
x > 5^2 \\
x > 25
\]

**ANSWER:**
\( \{x | x > 25\} \)

8. \( \log_3 x + \log_3 (x - 3) = \log_3 4 \)

**SOLUTION:**
\[
\log_3 x + \log_3 (x - 3) = \log_3 4 \\
\log_3 (x(x - 3)) = \log_3 4 \\
\log_3 (x^2 - 3x) = \log_3 4 \\
x^2 - 3x = 4 \\
x^2 - 3x - 4 = 0 \\
(x - 4)(x + 1) = 0
\]

By Zero Product Property:
\[
x - 4 = 0 \text{ or } x + 1 = 0 \\
x = 4 \text{ or } x = -1
\]

The \( x \)-value \(-1 \) makes the argument negative. Logarithm is not defined for negative numbers. Therefore, the solution is 4.

**ANSWER:**
\( x = 4 \)

9. \( 6^n - 1 \leq 11^n \)

**SOLUTION:**
\[
6^n - 1 \leq 11^n \\
\log 6^n - 1 \leq \log 11^n \\
(n - 1) \log 6 \leq n \log 11 \\
\frac{n - 1}{n} \leq \frac{\log 11}{\log 6} \\
1 - \frac{1}{n} \leq \frac{\log 11}{\log 6} \\
\frac{1}{n} \geq 1 - \frac{\log 11}{\log 6} \\
n \leq \frac{1}{1 - \frac{\log 11}{\log 6}} \\
n \leq -2.9560
\]

**ANSWER:**
\( \{n | n \leq -2.9560\} \)

10. \( 4e^{2x} - 1 = 5 \)

**SOLUTION:**
\[
4e^{2x} - 1 = 5 \\
4e^{2x} = 6 \\
e^{2x} = \frac{6}{4} \\
\ln e^{2x} = \ln \frac{6}{4} \\
2x = \ln \frac{6}{4} \\
x = \frac{1}{2} \cdot \ln \frac{6}{4} \\
\approx 0.2027
\]

**ANSWER:**
\( x \approx 0.2027 \)
11. \( \ln (x + 2)^2 > 2 \)

**SOLUTION:**

\[
\ln (x+2)^2 > 2
\]

\[
(e^{\ln (x+2)})^2 > e^2
\]

\[
(x+2)^2 > e^2
\]

\[
\sqrt{(x+2)^2} > \sqrt{e^2}
\]

\[
x + 2 < -e \text{ or } x + 2 > e
\]

\[
x < -e - 2 \text{ or } x > e - 2
\]

\[
x < -4.7183 \text{ or } x > 0.7183
\]

\[
\{x \mid x < -4.7183 \text{ or } x > 0.7183, x \neq -2\}
\]

**ANSWER:**

\[
\{x \mid x < -4.7183 \text{ or } x > 0.7183, x \neq -2\}
\]

**Use log₅ 11 ≈ 1.4899 and log₅ 2 ≈ 0.4307 to approximate the value of each expression.**

12. log₅ 44

**SOLUTION:**

\[
\log_5 44 = \log_5 (2 \cdot 2 \cdot 11)
\]

\[
= \log_5 2 + \log_5 2 + \log_5 11
\]

\[
\approx 0.4307 + 0.4307 + 1.4899
\]

\[
= 2.3513
\]

**ANSWER:**

2.3513

13. log₅ \( \frac{11}{2} \)

**SOLUTION:**

\[
\log_5 \left( \frac{11}{2} \right) = \log_5 11 - \log_5 2
\]

\[
\approx 1.4899 - 0.4307
\]

\[
= 1.0592
\]

**ANSWER:**

1.0592

14. **POPULATION** The population of a city 10 years ago was 150,000. Since then, the population has increased at a steady rate each year. The population is currently 185,000.

a. Write an exponential function that could be used to model the population after \( x \) years if the population changes at the same rate.

b. What will the population be in 25 years?

**SOLUTION:**

a. Substitute 185000, 10 and 150000 for \( y, t \) and \( a \) in the equation \( y = a(1 + r)^t \) then solve for \( r \).

\[
y = a(1 + r)^t
\]

\[
185000 = 150000(1 + r)^{10}
\]

\[
(1 + r)^{10} = \frac{185000}{150000}
\]

\[
\sqrt[10]{(1 + r)^{10}} = \sqrt[10]{\frac{37}{30}}
\]

\[
1 + r = 1.0212
\]

\[
r \approx 0.0212
\]

The annual rate of growth for this city is about 0.0212.

The exponential function that models the population after \( x \) years from the current date is \( y = 185,000 (1.0212)^x \).

b. Substitute 185,000 for \( a \), 0.0212 for \( r \), and 25 for \( x \) in \( y = a(1 + r)^t \) then evaluate for \( y \)

\[
y = 185000 (1 + 0.0212)^{25}
\]

\[
y = 185000 (1.0212)^{25}
\]

\[
y \approx 312566
\]

The population will be about 312,566 in 25 years.

**ANSWER:**

a. \( y = 150,000(1.0212)^x \)

b. about 253,432
15. Write \( \log_9 27 = \frac{3}{2} \) in exponential form.

SOLUTION:

\[
\log_9 27 = \frac{3}{2} \\
9^{\frac{3}{2}} = 27
\]

ANSWER:

\[
9^{\frac{3}{2}} = 27
\]

16. AGRICULTURE An equation that models the decline in the number of U.S. farms is \( y = 3,962,520 (0.98)^x \), where \( x \) is the number of years since 1960 and \( y \) is the number of farms.

a. How can you tell that the number is declining?

b. By what annual rate is the number declining?

c. Predict when the number of farms will be less than 1 million.

SOLUTION:

a. Since the base of the exponential is less than one \((b < 1)\), the number is declining.

b. 
\[
1 - r = 0.98 \\
r = 1 - 0.98 \\
= 0.02
\]

The number is declining by 0.02 or 2%.

c. Substitute 1,000,000 for \( y \) and solve for \( x \).

\[
1000000 = 3962520 (0.98)^x \\
\frac{1000000}{3962520} = 0.98^x \\
\log \frac{1000000}{3962520} = \log 0.98^x \\
x \log 0.98 = \log 1000000 - \log 3962520 \\
x = \frac{\log 1000000 - \log 3962520}{\log 0.98} \\
x \approx 68
\]

Therefore, the number of farms will be less than 1 million in about 2028(1960 + 68).

ANSWER:

a. \( \{ b \mid b < 1 \} \)

b. 2%

c. in about 2028
17. **MULTIPLE CHOICE** What is the value of
\[
\log_4 \frac{1}{64}
\]
A $-3$
B $\frac{1}{3}$
C $\frac{1}{3}$
D 3

**SOLUTION:**
\[
\log_4 \frac{1}{64} = \log_4 4^{-3} = -3 \log_4 4 = -3
\]

Option A is the correct answer.

**ANSWER:**
A

18. **SAVINGS** You put $7500 in a savings account paying 3% interest compounded continuously.

a. Assuming there are no deposits or withdrawals from the account, what is the balance after 5 years?
b. How long will it take your savings to double?
c. In how many years will you have $10,000 in your account?

**SOLUTION:**
a. Substitute 7500, 0.03 and 5 for $P, r$ and $t$ respectively in the continuous compound interest formula then evaluate.

\[
A = Pe^{rt}
\]
\[
= 7500e^{(0.03)5}
\]
\[
= 7500e^{0.15}
\]
\[
\approx 8713.76
\]

b. Substitute 15000, 7500 and 0.03 for $A, P$ and $r$ then solve for $t$.

\[
15000 = 7500e^{0.03t}
\]
\[
2 = e^{0.03t}
\]
\[
\ln 2 = \ln e^{0.03t}
\]
\[
\ln 2 = 0.03t
\]
\[
t = \frac{\ln 2}{0.03}
\]
\[
\approx 23.1
\]

The principal will take approximately 23.1 years to double.

c. Substitute 10000, 7500 and 0.03 for $A, P$ and $r$ then solve for $t$.

\[
10000 = 7500e^{0.03t}
\]
\[
\frac{10000}{7500} = e^{0.03t}
\]
\[
\ln \frac{4}{3} = \ln e^{0.03t}
\]
\[
\ln 4 - \ln 3 = 0.03t
\]
\[
t = \frac{\ln 4 - \ln 3}{0.03}
\]
\[
\approx 9.6
\]

You will have $10,000 in your account about 9.6 years.

**ANSWER:**
a. $8713.76$
b. $\approx 23.1$ years
c. about 9.6 years
19. MULTIPLE CHOICE What is the solution of \[ \log_4 16 - \log_4 x = \log_4 8 \]?

- F \( \frac{1}{2} \)
- G 2
- H 4
- J 8

**SOLUTION:**

\[ \log_4 \frac{16}{x} = \log_4 8 \]
\[ \frac{16}{x} = 8 \]
\[ x = \frac{16}{8} \]
\[ x = 2 \]

Option G is the correct answer.

**ANSWER:**

G

20. MULTIPLE CHOICE Which function is graphed below?

- A \( y = \log_{10} (x - 5) \)
- B \( y = 5 \log_{10} x \)
- C \( y = \log_{10} (x + 5) \)
- D \( y = -5 \log_{10} x \)

**SOLUTION:**

Function of the given graph is \( y = \log_{10} (x + 5) \)

Option C is the correct answer.

**ANSWER:**

C

21. Write \( 2 \ln 6 + 3 \ln 4 - 5 \ln \left( \frac{1}{3} \right) \) as a single logarithm.

**SOLUTION:**

\[
2 \ln 6 + 3 \ln 4 - 5 \ln \left( \frac{1}{3} \right) = \ln 6^2 + \ln 4^3 - \ln \left( \frac{1}{3} \right)^5
\]
\[
= \ln (6^2 \cdot 4^3) - \ln \left( \frac{1}{3} \right)^5
\]
\[
= \ln \left( \frac{6^2 \cdot 4^3}{\left( \frac{1}{3} \right)^5} \right)
\]
\[
= \ln 559,872
\]

**ANSWER:**

\[ \ln \left( \frac{6^2 \cdot 4^3}{\left( \frac{1}{3} \right)^5} \right) \] or \( \ln 559,872 \)
Choose a word or term from the list above that best completes each statement or phrase.

1. A function of the form \( f(x) = b^x \) where \( b > 1 \) is an(n) ________ function.

   SOLUTION:
   A function of the form \( f(x) = b^x \) where \( b > 1 \) is an exponential growth function.

   ANSWER: exponential growth

2. In \( x = b^y \), the variable \( y \) is called the ________ of \( x \).

   SOLUTION:
   In \( x = b^y \), the variable \( y \) is called the logarithm of \( x \).

   ANSWER: logarithm

3. Base 10 logarithms are called ________.

   SOLUTION:
   Base 10 logarithms are called common logarithms.

   ANSWER: common logarithms

4. An(n) ________ is an equation in which variables occur as exponents.

   SOLUTION:
   An(n) exponential equation is an equation in which variables occur as exponents.

   ANSWER: exponential equation

5. The ________ allows you to write equivalent logarithmic expressions that have different bases.

   SOLUTION:
   The change of base formula allows you to write equivalent logarithmic expressions that have different bases.

   ANSWER: change of base formula

6. The base of the exponential function, \( A(t) = a(1 - r)t \), \( 1 - r \) is called the ________.

   SOLUTION:
   The base of the exponential function, \( A(t) = a(1 - r)t \), \( 1 - r \) is called the decay factor.

   ANSWER: decay factor

7. The function \( y = \log_b x \), where \( b > 0 \) and \( b \neq 1 \), is called an(n) ________.

   SOLUTION:
   The function \( y = \log_b x \), where \( b > 0 \) and \( b \neq 1 \), is called an logarithmic function.

   ANSWER: logarithmic function

8. An exponential function with base \( e \) is called the ________.

   SOLUTION:
   An exponential function with base \( e \) is called the natural base exponential function.

   ANSWER: natural base exponential function

9. The logarithm with base \( e \) is called the ________.

   SOLUTION:
   The logarithm with base \( e \) is called the natural logarithm.

   ANSWER: natural logarithm

10. The number \( e \) is referred to as the ________.

    SOLUTION:
    The number \( e \) is referred to as the natural base.

    ANSWER: natural base
Graph each function. State the domain and range.

11. \( f(x) = 3^x \)

\[ \text{SOLUTION:} \]

\[ f(x) = 3^x \]

The function is defined for all values of \( x \). Therefore, the domain is the set of all real numbers. The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \( \infty \). The value of \( f(x) \) tends to 0 as \( x \) tends to \( -\infty \). Therefore, the range of the function is \( \{ f(x) | f(x) > 0 \} \).

\[ \text{A N S W E R:} \]

\[ f(x) = 3^x \]

\[ D = \{ \text{all real numbers} \} \quad R = \{ f(x) | f(x) > 0 \} \]

12. \( f(x) = -5(2)^x \)

\[ \text{SOLUTION:} \]

\[ f(x) = -5(2)^x \]

The function is defined for all values of \( x \). Therefore, the domain is the set of all real numbers. The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \( \infty \). The value of \( f(x) \) tends to 0 as \( x \) tends to \( -\infty \). Therefore, the range of the function is \( \{ f(x) | f(x) < 0 \} \).

\[ \text{A N S W E R:} \]

\[ f(x) = -5(2)^x \]

\[ D = \{ \text{all real numbers} \} \quad R = \{ f(x) | f(x) < 0 \} \]
13. $f(x) = 3(4)^x - 6$

**SOLUTION:**

The function is defined for all values of $x$. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to $\infty$ as $x$ tends to $\infty$.
The value of $f(x)$ tends to $-6$ as $x$ tends to $-\infty$.
Therefore, the range of the function is \{f(x) \mid f(x) > -6\}.

**ANSWER:**

$$D = \{\text{all real numbers}\} \quad R = \{f(x) \mid f(x) > -6\}$$

---

14. $f(x) = 3^{2x} + 5$

**SOLUTION:**

The function is defined for all values of $x$. Therefore, the domain is set of all real numbers.
The value of $f(x)$ tends to $\infty$ as $x$ tends to $\infty$.
The value of $f(x)$ tends to 5 as $x$ tends to $-\infty$.
Therefore, the range of the function is \{f(x) \mid f(x) > 5\}.

**ANSWER:**

$$D = \{\text{all real numbers}\} \quad R = \{f(x) \mid f(x) > 5\}$$
15.  \( f(x) = 3\left(\frac{1}{4}\right)^{x+3} - 1 \)

**SOLUTION:**

The function is defined for all values of \( x \). Therefore, the domain is set of all real numbers.

The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \( -\infty \).
The value of \( f(x) \) tends to \(-1 \) as \( x \) tends to \( \infty \).
Therefore, the range of the function is \([f(x) \mid f(x) > -1]\).

**ANSWER:**

\[ D = \{ \text{all real numbers} \} \quad R = \{ f(x) \mid f(x) > -1 \} \]

16.  \( f(x) = \frac{3}{5}\left(\frac{2}{3}\right)^{x-2} + 3 \)

**SOLUTION:**

The function is defined for all values of \( x \). Therefore, the domain is set of all real numbers.
The value of \( f(x) \) tends to \( \infty \) as \( x \) tends to \( -\infty \).
The value of \( f(x) \) tends to \( 3 \) as \( x \) tends to \( \infty \).
Therefore, the range of the function is \([f(x) \mid f(x) > 3]\).

**ANSWER:**

\[ D = \{ \text{all real numbers} \} \quad R = \{ f(x) \mid f(x) > 3 \} \]
17. **POPULATION** A city with a population of 120,000 decreases at a rate of 3% annually.
   a. Write the function that represents this situation.
   b. What will the population be in 10 years?

   **SOLUTION:**
   a. Exponential decay with a constant percent increase over specific time periods is modeled by \( f(x) = a(1 - r)^x \).
   Substitute 120,000 for \( a \) and 0.03 for \( r \).

   \[
   f(x) = 120000(1 - 0.03)^x \\
   f(x) = 120000(0.97)^x 
   \]

   b. Substitute 10 for \( x \) and evaluate.

   \[
   f(10) = 120000(0.97)^{10} \\
   \approx 88491
   \]

   The population will be about 88,491.

   **ANSWER:**
   a. \( f(x) = 120,000(0.97)^x \)
   b. about 88,491

---

18. Solve each equation or inequality.

18. \( 16^x = \frac{1}{64} \)

   **SOLUTION:**

   \[
   16^x = \frac{1}{64} \\
   (2^4)^x = \left( \frac{1}{2} \right)^6 \\
   2^{4x} = 2^{-6} \\
   \log 2^{4x} = \log 2^{-6} \\
   4x \log 2 = -6 \log 2 \\
   x = \frac{-6 \log 2}{4 \log 2} \\
   x = -\frac{3}{2}
   \]

   The solution is \(-\frac{3}{2}\).

   **ANSWER:**
   \(-\frac{3}{2}\)

19. \( 3^{4x} = 9^{3x+7} \)

   **SOLUTION:**

   \[
   3^{4x} = 9^{3x+7} \\
   3^{4x} = \left(3^2\right)^{3x+7} \\
   3^{4x} = 3^{6x+14} \\
   \log 3^{4x} = \log 3^{6x+14} \\
   4x \log 3 = (6x + 14) \log 3 \\
   4x = 6x + 14 \\
   2x = -14 \\
   x = -7
   \]

   The solution is \(-7\).

   **ANSWER:**
   \(-7\)
20. \(64^3n = 8^2n - 3\)

**SOLUTION:**

\[
64^3n = 8^2n - 3 \\
(8^3)^n = 8^2n - 3 \\
8^{6n} = 8^{2n - 3} \\
\log 8^{6n} = \log 8^{2n - 3} \\
6n \log 8 = (2n - 3) \log 8 \\
6n = 2n - 3 \\
4n = -3 \\
\frac{n}{4} = -\frac{3}{4}
\]

The solution is \(\frac{-3}{4}\).

**ANSWER:**

\[-\frac{3}{4}\]

21. \(8^{3 - 3y} = 256^{4y}\)

**SOLUTION:**

\[
8^{3 - 3y} = 256^{4y} \\
(2^3)^{3 - 3y} = (2^8)^{4y} \\
2^{9 - 9y} = 2^{32y} \\
\log 2^{9 - 9y} = \log 2^{32y} \\
(9 - 9y) \log 2 = 32y \log 2 \\
9 - 9y = 32y \\
9 = 41y \\
y = \frac{9}{41}
\]

The solution is \(\frac{9}{41}\).

**ANSWER:**

\[
\frac{9}{41}
\]

22. \(9^{x - 2} < \left(\frac{1}{81}\right)^{x - 2}\)

**SOLUTION:**

\[
9^{x - 2} < \left(\frac{1}{81}\right)^{x - 2} \\
9^{x - 2} < \left(\frac{1}{9^2}\right)^{x - 2} \\
9^{x - 2} > \frac{1}{9^{2x - 4}} \\
\log 9^{x - 2} > \log 9^{2x - 4} \\
(x - 2) \log 9 > (2x - 4) \log 9 \\
x - 2 > 2x - 4 \\
3x > -2 \\
x > -\frac{2}{3}
\]

The solution is \(\{x | x > -\frac{2}{3}\}\).

**ANSWER:**

\[
x > -\frac{2}{3}
\]
24. **Bacteria** A bacteria population started with 5000 bacteria. After 8 hours there were 28,000 in the sample.

**a.** Write an exponential function that could be used to model the number of bacteria after $x$ hours if the number of bacteria changes at the same rate.

**b.** How many bacteria can be expected in the sample after 32 hours?

**SOLUTION:**

**a.** At the beginning of the experiment, the time is 0 hours and there are 5000 bacteria cells. Thus, the $y$-intercept, and the value of $a$, is 5000. When $x = 8$, the number of bacteria cells is 28,000. Substitute these values into an exponential function to determine the value of $b$.

\[
y = ab^x
\]

\[
28000 = 5000b^8
\]

\[
5.6 = b^8
\]

\[
b = \sqrt[8]{5.6} 
\]

\[
\approx 1.240
\]

An equation that models the number of bacteria is $y = 5000(1.240)^x$.

**b.** Substitute 32 for $x$ and evaluate.

\[
y = 5000(1.240)^{32}
\]

\[
\approx 4880496
\]

There will be approximately 4,880,496 bacteria cells after 32 hours.

**Answer:**

**a.** $y = 5000(1.240)^x$

**b.** about 4,880,496
25. Write $\log_2 \frac{1}{16} = -4$ in exponential form.

**SOLUTION:**

$$\log_2 \frac{1}{16} = -4$$

$$\frac{1}{16} = 2^{-4}$$

**ANSWER:**

$$2^{-4} = \frac{1}{16}$$

26. Write $10^2 = 100$ in logarithmic form.

**SOLUTION:**

$$10^2 = 100$$

$$\log_{10} 10^2 = \log_{10} 100$$

$$2 = \log_{10} 100$$

**ANSWER:**

$$\log_{10} 100 = 2$$

27. $\log_4 256$

**SOLUTION:**

$$\log_4 256 = \log_4 4^4$$

$$= 4 \log_4 4$$

$$= 4$$

**ANSWER:**

4

28. $\log_2 \frac{1}{8}$

**SOLUTION:**

$$\log_2 \frac{1}{8} = \log_2 \frac{1}{2^3}$$

$$= \log_2 2^{-3}$$

$$= -3 \log_2 2$$

$$= -3$$

**ANSWER:**

-3
30. \( f(x) = \frac{1}{6} \log_3(x - 2) \)

**SOLUTION:**

\[
\begin{align*}
\text{SOLUTION:} \\
\end{align*}
\]

ANSWER:

\[
\begin{align*}
\text{SOLUTION:} \\
\end{align*}
\]

Solve each equation or inequality.

31. \( \log_4 x = \frac{3}{2} \)

**SOLUTION:**

\[
\begin{align*}
\log_4 x &= \frac{3}{2} \\
x &= 4^{\frac{3}{2}} \\
x &= (\sqrt{4})^3 \\
x &= 8
\end{align*}
\]

The solution is 8.

**ANSWER:** 8

32. \( \log_2 \frac{1}{64} = x \)

**SOLUTION:**

\[
\begin{align*}
\log_2 \frac{1}{64} &= x \\
\log_2 2^{-6} &= x \\
-6 &= x \\
\end{align*}
\]

The solution is -6.

**ANSWER:** -6

33. \( \log_4 x < 3 \)

**SOLUTION:**

If \( b > 1, \ x > 0, \) and \( \log_b x < y, \) then \( 0 < x < b^y \) so

\[
\begin{align*}
\log_4 x < 3 \\
0 < x < 4^3 \\
0 < x < 64
\end{align*}
\]

**ANSWER:** \( \{x \mid 0 < x < 64\} \)

34. \( \log_5 x < -3 \)

**SOLUTION:**

If \( b > 1, \ x > 0, \) and \( \log_b x < y, \) then \( 0 < x < b^y \) so

\[
\begin{align*}
\log_5 x < -3 \\
0 < x < 5^{-3} \\
0 < x < \frac{1}{125}
\end{align*}
\]

**ANSWER:** \( \{x \mid 0 < x < \frac{1}{125}\} \)
35. \( \log_y (3x - 1) = \log_y (4x) \)

**SOLUTION:**
\[
\log_y (3x - 1) = \log_y (4x) \\
3x - 1 = 4x \\
x = -1
\]

The value of \( x \) makes the argument negative.
Logarithms are not defined for negative numbers.
Therefore, there is no solution.

**ANSWER:**
no solution

36. \( \log_2 (x^2 - 18) = \log_2 (-3x) \)

**SOLUTION:**
\[
\log_2 (x^2 - 18) = \log_2 (-3x) \\
x^2 - 18 = -3x \\
x^2 + 3x - 18 = 0 \\
(x + 6)(x - 3) = 0
\]

By Zero Product Property:
\[
x + 6 = 0 \quad \text{or} \quad x - 3 = 0 \\
x = -6 \quad \text{or} \quad x = 3
\]

The \( x \)-value 3 makes the argument negative.
Logarithms are not defined for negative numbers.
Therefore, the solution is \(-6\).

**ANSWER:**
\(-6\)

37. \( \log_3 (3x + 4) \leq \log_3 (x - 2) \)

**SOLUTION:**
\[
\log_3 (3x + 4) \leq \log_3 (x - 2) \\
3x + 4 \leq x - 2 \\
2x \leq -6 \\
x \leq -3
\]

The value of \( x \) makes the argument negative.
Logarithms are not defined for negative numbers.
Therefore, there is no solution.

**ANSWER:**
no solution

38. **EARTHQUAKE** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude \( M \) is given by \( M = \log_{10} x \),
where \( x \) represents the amplitude of the seismic wave causing ground motion. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 10 as an aftershock with a Richter scale rating of 7?

**SOLUTION:**
Substitute 10 and 7 for \( M \) and find the value of \( x \).
\[
M = \log_{10} x \\
10 = \log_{10} x \\
x = 10^{10} \\
7 = \log_{10} x \\
x = 10^7
\]

The ratio between the amplitude is \( 10^3 \).

**ANSWER:**
1000
Choose a word or term from the list above that best completes each statement or phrase.

1. A function of the form $f(x) = k^x$ can be used to model this situation. Solve for $k$.

The annual rate of growth for this city is about 0.041 or about 4.1%.

ANSWER: about 4.1%

39. $\log_5 8$

**SOLUTION:**

$$\log_5 8 = \log_5 \left( \frac{16}{2} \right)$$

$$= \log_5 16 - \log_5 2$$

$$\approx 1.7227 - 0.4307$$

$$\approx 1.2920$$

**ANSWER:**

1.2920

40. $\log_5 64$

**SOLUTION:**

$$\log_5 64 = \log_5 (16 \cdot 2 \cdot 2)$$

$$= \log_5 16 + \log_5 2 + \log_5 2$$

$$\approx 1.7227 + 0.4307 + 0.4307$$

$$\approx 2.5841$$

**ANSWER:**

2.5841

41. $\log_5 4$

**SOLUTION:**

$$\log_5 4 = \log_5 2^2$$

$$= 2\log_5 2$$

$$\approx 2(0.4307)$$

$$= 0.8614$$

**ANSWER:**

0.8614

42. $\log_5 \frac{1}{8}$

**SOLUTION:**

$$\log_5 \frac{1}{8} = \log_5 \frac{1}{2^3}$$

$$= \log_2 3$$

$$= -3\log_5 2$$

$$\approx -3(0.4307)$$

$$= -1.2921$$

**ANSWER:**

-1.2921

43. $\log_5 \frac{1}{2}$

**SOLUTION:**

$$\log_5 \frac{1}{2} = \log_5 2^{-1}$$

$$= -1\log_5 2$$

$$\approx -1(0.4307)$$

$$= -0.4307$$

**ANSWER:**

-0.4307
Solve each equation. Check your solution.

44. \( \log_5 x - \log_5 2 = \log_5 15 \)

\textbf{SOLUTION:}
\[
\log_5 \left( \frac{x}{2} \right) = \log_5 15
\]
\[
\frac{x}{2} = 15
\]
\[
x = 30
\]
Substitute 30 for \( x \) and check the solution.
\[
\log_5 30 - \log_5 2 = \log_5 15
\]
\[
15 \cdot 2 - \log_5 2 = \log_5 15
\]
\[
\log_5 15 + \log_5 2 - \log_5 2 = \log_5 15
\]
\[
\log_5 15 = \log_5 15 \checkmark
\]
The solution checks.

\textbf{ANSWER:}
30

45. \( 3 \log_4 a = \log_4 27 \)

\textbf{SOLUTION:}
\[
3 \log_4 a = \log_4 27
\]
\[
\log_4 a^3 = \log_4 3^3
\]
\[
a^3 = 3^3
\]
\[
a = 3
\]
Substitute 3 for \( a \) and check the solution.
\[
3 \log_4 3 = \log_4 27
\]
\[
\log_4 3^3 = \log_4 27
\]
\[
\log_4 27 = \log_4 27
\]
\[
27 = 27 \checkmark
\]
The solution checks.

\textbf{ANSWER:}
3

46. \( 2 \log_3 x + \log_3 3 = \log_3 36 \)

\textbf{SOLUTION:}
\[
2 \log_3 x + \log_3 3 = \log_3 36
\]
\[
\log_3 x^2 + \log_3 3 = \log_3 36
\]
\[
\log_3 3x^2 = \log_3 36
\]
\[
3x^2 = 36
\]
\[
x^2 = 12
\]
\[
x = 2 \sqrt{3}
\]
Substitute \( 2 \sqrt{3} \) for \( x \) and check the solution.
\[
2 \log_3 2 \sqrt{3} + \log_3 3 = \log_3 36
\]
\[
\log_3 \left( 2 \sqrt{3} \right)^2 + \log_3 3 = \log_3 36
\]
\[
\log_3 12 + \log_3 3 = \log_3 36
\]
\[
\log_3 36 = \log_3 36
\]
\[
36 = 36 \checkmark
\]
The solution checks.

\textbf{ANSWER:}
\( 2 \sqrt{3} \)
47. \( \log_4 n + \log_4 (n - 4) = \log_4 5 \)

**SOLUTION:**
\[
\log_4 n + \log_4 (n - 4) = \log_4 5 \\
\log_4 (n(n - 4)) = \log_4 5 \\
\log_4 (n^2 - 4n) = \log_4 5 \\
\frac{1}{4} (n^2 - 4n) = 5 \\
\frac{1}{4} n^2 - n = 5 \\
\frac{1}{4} n^2 - n - 5 = 0 \\
(n - 5)(n + 1) = 0 \\
\]
By zero Product property:
\[
n - 5 = 0 \text{ or } n + 1 = 0 \\
n = 5 \text{ or } n = -1 \\
\]
The \( x \)-value \(-1 \) makes the argument negative. Logarithm is not defined for negative numbers. Therefore, the solution is 5.

Substitute 5 and \(-1 \) for \( x \) and check the solution.
\[
\log_4 5 + \log_4 (5 - 4) = \log_4 5 \\
\log_4 5 + \log_4 1 = \log_4 5 \\
\log_4 5 + 0 = \log_4 5 \\
\log_4 5 = \log_4 5 \checkmark \\
\log_4 (-1) + \log_4 (-1 - 4) = \log_4 5 \\
\log_4 (-1) + \log_4 (-5) = \log_4 5 \times \\
\]
Therefore, the solution is 5.

**ANSWER:**
5

48. **SOUND** Use the formula \( L = 10 \log_{10} R \), where \( L \) is the loudness of a sound and \( R \) is the sound’s relative intensity, to find out how much louder 20 people talking would be than one person talking. Suppose the sound of one person talking has a relative intensity of 80 decibels.

**SOLUTION:**
Substitute 80 for \( R \) and solve for \( L \).
\[
L = 10 \log_{10} 80 \\
\approx 19.03090 \\
\]
If 20 people are talking at a time, the relative intensity of the sound is
\[
20 \times 19.031 \approx 380.62 \\
\]
Subtract the loudness of one person talking.
\[
380.62 - 19.031 \approx 361.6 \\
\]

**ANSWER:**
361.6

Solve each equation or inequality. Round to the nearest ten-thousandth.

49. \( 3^x = 15 \)

**SOLUTION:**
\[
3^x = 15 \\
\log 3^x = \log 15 \\
x \log 3 = \log 15 \\
x = \frac{\log 15}{\log 3} \\
\approx 2.4650 \\
\]

**ANSWER:**
\( x \approx 2.4650 \)
50. \(6^2 = 28\)

**SOLUTION:**

\[
6^2 = 28 \\
\log_6 6^2 = \log_6 28 \\
x^2 \log_6 6 = \log_6 28 \\
x^2 = \frac{\log_6 28}{\log_6 6} \\
x = \sqrt{\frac{\log_6 28}{\log_6 6}} \\
\approx \pm 1.3637
\]

**ANSWER:**

\(x \approx \pm 1.3637\)

51. \(8^m + 1 = 30\)

**SOLUTION:**

\[
8^m + 1 = 30 \\
\log_8 (8^m + 1) = \log_8 30 \\
(m + 1) \log_8 8 = \log_8 30 \\
m + 1 = \frac{\log_8 30}{\log_8 8} \\
m = \frac{\log_8 30}{\log_8 8} - 1 \\
\approx 0.6356
\]

**ANSWER:**

\(m \approx 0.6356\)

52. \(12^{r-1} = 7^r\)

**SOLUTION:**

\[
12^{r-1} = 7^r \\
\log_{12} 12^{r-1} = \log_{12} 7^r \\
(r - 1) \log_{12} 12 = r \log_{12} 7 \\
r - 1 = \frac{r \log_{12} 7}{\log_{12} 12} \\
r - 1 = \frac{\log_{12} 7}{\log_{12} 12} \\
r = 1 - \frac{\log_{12} 7}{\log_{12} 12} \\
r = \frac{\log_{12} 12 - \log_{12} 7}{\log_{12} 12} \\
r \approx \frac{0}{\log_{12} 12} \\
r \approx 4.6102
\]

**ANSWER:**

\(r \approx 4.6102\)

53. \(3^{5n} > 24\)

**SOLUTION:**

\[
3^{5n} > 24 \\
\log_{10} 3^{5n} > \log_{10} 24 \\
5n \log_{10} 3 > \log_{10} 24 \\
5n > \frac{\log_{10} 24}{\log_{10} 3} \\
\frac{1}{5} \cdot \frac{\log_{10} 24}{\log_{10} 3} > n \\
0.5786 > n
\]

**ANSWER:**

\(\{n | n < 0.5786\}\)
54. \(5^x + 2 \leq 3^x\)

**SOLUTION:**

\[
5^{x+2} \leq 3^x \\
\log 5^{x+2} \leq \log 3^x \\
(x + 2) \log 5 \leq x \log 3 \\
\frac{x + 2}{x} \leq \frac{\log 3}{\log 5} \\
1 + \frac{2}{x} \leq \frac{\log 3}{\log 5} \\
\frac{2}{x} \leq \frac{\log 3}{\log 5} - 1 \\
\frac{2}{x} \leq \frac{\log 3 - \log 5}{\log 5} \\
x \geq \frac{2 \log 5}{\log 3 - \log 5} \\
\leq -6.3013
\]

**ANSWER:**

\(\{x | x \leq -6.3013\}\)

55. **SAVINGS** You deposited $1000 into an account that pays an annual interest rate \(r\) of 5% compounded quarterly. Use \(A = P \left(1 + \frac{r}{n}\right)^n\).

a. How long will it take until you have $1500 in your account?

b. How long it will take for your money to double?

**SOLUTION:**

a. Substitute 1500, 1000, 0.05 and 4 for \(A, P, r\) and \(n\) then solve for \(t\).

\[
1500 = 1000 \left(1 + \frac{0.05}{4}\right)^t \\
1.5 = \left(1 + \frac{0.05}{4}\right)^t \\
1.5 = 1.0125^t \\
\log 1.5 = \log 1.0125^t \\
4t \log 1.0125 = \log 1.5 \\
t = \frac{1}{4} \frac{\log 1.5}{\log 1.0125} \\
\approx 8.2
\]

It will take about 8.2 years.

b. Substitute 2000, 1000, 0.05 and 4 for \(A, P, r\) and \(n\) then solve for \(t\).

\[
2000 = 1000 \left(1 + \frac{0.05}{4}\right)^t \\
2 = \left(1 + \frac{0.05}{4}\right)^t \\
2 = 1.0125^t \\
\log 2 = \log 1.0125^t \\
4t \log 1.0125 = \log 2 \\
t = \frac{1}{4} \frac{\log 2}{\log 1.0125} \\
\approx 13.9
\]

It will take about 13.9 years.

**ANSWER:**

a. about 8.2 years

b. about 13.9 years
Solve each equation or inequality. Round to the nearest ten-thousandth.

56. $4e^x - 11 = 17$

**SOLUTION:**

$4e^x - 11 = 17$

$4e^x = 28$

$e^x = 7$

$\ln e^x = \ln 7$

$x = \ln 7$

$x \approx 1.9459$

**ANSWER:**

$x \approx 1.9459$

57. $2e^{-x} + 1 = 15$

**SOLUTION:**

$2e^{-x} + 1 = 15$

$2e^{-x} = 14$

$e^{-x} = 7$

$\ln e^{-x} = \ln 7$

$-x = \ln 7$

$x = -\ln 7$

$x \approx -1.9459$

**ANSWER:**

$x \approx -1.9459$

58. $\ln 2x = 6$

**SOLUTION:**

$\ln 2x = 6$

$2x = e^6$

$x = \frac{e^6}{2}$

$x \approx 201.7144$

**ANSWER:**

$x \approx 201.7144$

59. $2 + e^x > 9$

**SOLUTION:**

$2 + e^x > 9$

$e^x > 7$

$\ln e^x > \ln 7$

$x > 1.9459$

**ANSWER:**

$x \in \{x | x > 1.9459\}$

60. $\ln (x + 3)^5 < 5$

**SOLUTION:**

$\ln (x + 3)^5 < 5$

$5 \ln (x + 3) < 5$

$\ln (x + 3) < 1$

$x + 3 < e^1$

$x < e^1 - 3$

$x < -0.2817$

**ANSWER:**

$x \in \{x | -3 < x < -0.2817\}$

61. $e^{-x} > 18$

**SOLUTION:**

$e^{-x} > 18$

$\ln e^{-x} > \ln 18$

$-x > \ln 18$

$x < -\ln 18$

$x < -2.8904$

**ANSWER:**

$x \in \{x | x < -2.8904\}$
62. **Savings** If you deposit $2000 in an account paying 6.4% interest compounded continuously, how long will it take for your money to triple? Use $A = Pe^{rt}$.

**Solution:**
Substitute 6000, 2000 and 0.064 for $A$, $P$ and $r$ in the equation $A = Pe^{rt}$ then solve for $t$.

\[
6000 = 2000e^{0.064t} \\
3 = e^{0.064t} \\
\ln 3 = \ln e^{0.064t} \\
0.064t = \ln 3 \\
t = \frac{\ln 3}{0.064} \\
\approx 17.2
\]

It will take about 17.2 years.

**Answer:**
about 17.2 years

63. **Cars** Abe bought a used car for $2500. It is expected to depreciate at a rate of 25% per year. What will be the value of the car in 3 years?

**Solution:**
Substitute 2500, 3 and 0.25 for $a$, $t$ and $r$ in the equation $y = a(1-r)^t$ then evaluate.

\[
y = a(1-r)^t \\
y = 2500(1-0.25)^3 \\
= 2500(0.75)^3 \\
\approx 1054.69
\]

The value of the car will be about $1054.69.

**Answer:**
Sample answer: $1054.69

64. **Biology** For a certain strain of bacteria, $k$ is 0.728 when $t$ is measured in days. Using the formula $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria?

**Solution:**
Substitute 0.728, 10 and 675 for $k$, $a$ and $y$ in the equation $y = ae^{kt}$ then solve for $t$.

\[
y = ae^{kt} \\
675 = 10e^{0.728t} \\
67.5 = e^{0.728t} \\
\ln 67.5 = \ln e^{0.728t} \\
0.728t = \ln 67.5 \\
t = \frac{\ln 67.5}{0.728} \\
\approx 5.8
\]

It will take about 5.8 days.

**Answer:**
$\approx 5.8$ days
65. **POPULATION** The population of a city 20 years ago was 24,330. Since then, the population has increased at a steady rate each year. If the population is currently 55,250, find the annual rate of growth for this city.

**SOLUTION:**
Substitute 24330, 20 and 55250 for \( y, t \) and \( a \) in the equation \( y = ae^{kt} \) then solve for \( k \).

\[
\begin{align*}
y &= ae^{kt} \\
55250 &= 24330e^{k \cdot 20} \\
e^{k \cdot 20} &= \frac{55250}{24330} \\
20k &= \ln \left( \frac{55250}{24330} \right) \\
k &= \frac{\ln \left( \frac{55250}{24330} \right)}{20} \\
&\approx 0.041
\end{align*}
\]

The annual rate of growth for this city is about 0.041 or about 4.1%.

**ANSWER:**
about 4.1%