8-1 Multiplying and Dividing Rational Expressions

Simplify each expression.

1. \[
\frac{x^2 - 5x - 24}{x^2 - 64}
\]

**SOLUTION:**

\[
\frac{x^2 - 5x - 24}{x^2 - 64} = \frac{(x - 8)(x + 3)}{(x + 8)(x - 8)} = \frac{x + 3}{x + 8}
\]

**ANSWER:**

\[
\frac{x + 3}{x + 8}
\]

2. \[
\frac{c + d}{3c^2 - 3d^2}
\]

**SOLUTION:**

\[
\frac{c + d}{3c^2 - 3d^2} = \frac{c + d}{3(c^2 - d^2)} = \frac{c + d}{3(c - d)(c + d)} = \frac{1}{3(c - d)}
\]

**ANSWER:**

\[
\frac{1}{3(c - d)}
\]

3. **MULTIPLE CHOICE** Identify all values of \(x\) for which \[
\frac{x + 7}{x^2 - 3x - 28}
\]
is undefined.

A \(-7, 4\)

B \(7, 4\)

C \(4, -7, 7\)

D \(-4, 7\)

**SOLUTION:**

\[
\frac{x + 7}{x^2 - 3x - 28} = \frac{x + 7}{(x - 7)(x + 4)}
\]

The function is undefined when the denominator tends to 0.

\[(x - 7)(x + 4) = 0\]

\[x = 7, -4\]

The correct choice is D.

**ANSWER:**

D
Simplify each expression.

4. \( \frac{y^2 + 3y - 40}{25 - y^2} \)

**SOLUTION:**
\[
\frac{y^2 + 3y - 40}{25 - y^2} = \frac{(y + 8)(y - 5)}{(5 - y)(5 + y)}
\]
\[
= \frac{(y + 8)(y - 5)}{(y - 5)(5 + y)}
\]
\[
= \frac{y + 8}{y + 5}
\]

**ANSWER:**
\[
\frac{y + 8}{y + 5}
\]

5. \( \frac{a^2 x - b^2 x}{by - ay} \)

**SOLUTION:**
\[
\frac{a^2 x - b^2 x}{by - ay} = \frac{x(a^2 - b^2)}{y(b - a)}
\]
\[
= \frac{x(a - b)(a + b)}{y(b - a)}
\]
\[
= -\frac{x(a - b)(a + b)}{y(b - a)}
\]
\[
= -\frac{x(a + b)}{y}
\]

**ANSWER:**
\[
-\frac{x(a + b)}{y}
\]

6. \( \frac{27x^3y^4}{16yz^3} \cdot \frac{8z}{9xy^3} \)

**SOLUTION:**
\[
\frac{27x^3y^4}{16yz^3} \cdot \frac{8z}{9xy^3} = \frac{3}{2z^3}
\]

**ANSWER:**
\[
\frac{3}{2z^3}
\]

7. \( \frac{12x^3 y}{13ab^2} + \frac{36xy^3}{26b} \)

**SOLUTION:**
Invert the second expression and multiply.
\[
\frac{12x^3 y}{13ab^2} + \frac{36xy^3}{26b} = \frac{12x^3 y}{13ab^2} \cdot \frac{26b}{36xy^3}
\]
\[
= \frac{2x^2}{3aby^2}
\]

**ANSWER:**
\[
\frac{2x^2}{3aby^2}
\]
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8. \( \frac{x^2 - 4x - 21}{x^2 - 6x + 8} \cdot \frac{x - 4}{x^2 - 2x - 35} \)

**SOLUTION:**

\[
\frac{x^2 - 4x - 21}{x^2 - 6x + 8} \cdot \frac{x - 4}{x^2 - 2x - 35} = \frac{(x - 7)(x + 3)}{(x - 2)(x + 2)} \cdot \frac{x - 4}{(x - 7)(x + 5)}
\]

\[
= \frac{(x + 3)}{(x - 2)(x + 5)}
\]

**ANSWER:**

\( \frac{x + 3}{(x - 2)(x + 5)} \)

9. \( \frac{a^2 - b^2}{3a^2 - 6a + 3} \cdot \frac{4a + 4b}{a^2 - 1} \)

**SOLUTION:**

Invert the second expressions and multiply.

\[
\frac{a^2 - b^2}{3a^2 - 6a + 3} \cdot \frac{4a + 4b}{a^2 - 1} = \frac{a^2 - b^2}{3a^2 - 6a + 3} \cdot \frac{4a + 4b}{a^2 - 1}
\]

\[
= \frac{(a + b)(a - b)}{3(a^2 - 2a + 1)} \cdot \frac{(a - 1)(a + 1)}{4(a + b)}
\]

\[
= \frac{(a + b)(a - b)}{3(a - 1)^2} \cdot \frac{(a - 1)(a + 1)}{4(a + b)}
\]

\[
= \frac{(a - b)(a + 1)}{12(a - 1)}
\]

**ANSWER:**

\( \frac{(a - b)(a + 1)}{12(a - 1)} \)

10. \( \frac{a^3 b^3}{xy^4} \cdot \frac{a^2 b}{x^2 y} \)

**SOLUTION:**

Invert the second expressions and multiply.

\[
\frac{a^3 b^3}{xy^4} \cdot \frac{a^2 b}{x^2 y} = \frac{a^3 b^3}{xy^4} \cdot \frac{x^2 y}{a^2 b}
\]

\[
= \frac{a b^2 x}{y^3}
\]

**ANSWER:**

\( \frac{a b^2 x}{y^3} \)

11. \( \frac{4x}{x + 6} \cdot \frac{x^2 + 3x - 18}{x^2 + 3x - 18} \)

**SOLUTION:**

Invert the second expressions and multiply.

\[
\frac{4x}{x + 6} \cdot \frac{x^2 + 3x - 18}{x^2 + 3x - 18} = \frac{4x}{x + 6} \cdot \frac{x(x + 3)(x - 3)}{x(x - 3)}
\]

\[
= 4
\]

**ANSWER:**

4

12. **CCSS SENSE-MAKING** The volume of a shipping container in the shape of a rectangular prism can be represented by the polynomial \( 6x^3 + 11x^2 + 4x \), where the height is \( x \).
Simplify each expression.

1. SOLUTION: 
ANSWER: 

2. SOLUTION: 
ANSWER: 

Thus, the equation for the decay of Carbon-14 is ________________

Substitute 0.005a for y in the equation.

Cubic centimeters. Find the height of ________________

Explain why this new ________________

The slick is about 0.0039 meters or 3.9 mm thick.

Trying to get into a train yard one evening, ________________

The length of ________________

Under what condition is the rational expression ________________

Will the ratio of the three dimensions be the same ________________

For all values of ________________

c. No because the expressions for height, length and width are different.

ANSWER: 

a. 2x + 1, 3x + 4 

b. 2 : 5 : 10 

c. no
15. \[
\frac{(x^2 - 9)(x^2 - z^2)}{4(x + z)(x - 3)}
\]

SOLUTION:

\[
\frac{(x^2 - 9)(x^2 - z^2)}{4(x + z)(x - 3)} = \frac{(x + 3)(x - 3)(x - z)(x + z)}{4(x + z)(x - 3)} \cdot \frac{1}{4} = \frac{(x + 3)(x - z)}{4}
\]

ANSWER:

\[
\frac{(x + 3)(x - z)}{4}
\]

16. \[
\frac{(x^2 - 16x + 64)(x + 2)}{(x^2 - 64)(x^2 - 6x - 16)}
\]

SOLUTION:

\[
\frac{(x^2 - 16x + 64)(x + 2)}{(x^2 - 64)(x^2 - 6x - 16)} = \frac{(x - 8)^2(x + 2)}{(x + 8)(x - 8)(x + 8)} \cdot \frac{1}{x + 8} = \frac{1}{x + 8}
\]

ANSWER:

\[
\frac{1}{x + 8}
\]

17. \[
\frac{x^2(x + 2)(x - 4)}{6x(x^2 + x - 20)}
\]

SOLUTION:

\[
\frac{x^2(x + 2)(x - 4)}{6x(x^2 + x - 20)} = \frac{x^3(x + 2)(x - 4)}{6x(x + 5)(x - 4)} \cdot \frac{x(x + 2)}{6(x + 5)} = \frac{x(x + 2)}{6(x + 5)}
\]

ANSWER:

\[
\frac{x(x + 2)}{6(x + 5)}
\]

18. \[
\frac{3y(y - 8)(y^2 + 2y - 24)}{15y^2(y^2 - 12y + 32)}
\]

SOLUTION:

\[
\frac{3y(y - 8)(y^2 + 2y - 24)}{15y^2(y^2 - 12y + 32)} = \frac{3y(y - 8)(y + 6)(y - 4)}{15y^2(y - 8)} \cdot \frac{1}{5y} = \frac{(y + 6)}{5y}
\]

ANSWER:

\[
\frac{(y + 6)}{5y}
\]

19. MULTIPLE CHOICE Identify all values of \(x\) for which \(\frac{(x - 3)(x + 6)}{(x^2 - 7x + 12)(x^2 - 36)}\) is undefined.

F 3, –6
G 4, 6
H –6, 6
J –6, 3, 4, 6

SOLUTION:

\[
\frac{(x - 3)(x + 6)}{(x^2 - 7x + 12)(x^2 - 36)} = \frac{(x - 3)(x + 6)}{(x - 4)(x - 3)(x - 6)(x + 6)}
\]

Therefore, the function is undefined for \(x = -6, 3, 4, 6\). So, the correct choice is J.

ANSWER:

J
8-1 Multiplying and Dividing Rational Expressions

Simplify each expression.

20. \( \frac{x^2 - 5x - 14}{28 + 3x - x^2} \)

**SOLUTION:**
\[
\frac{x^2 - 5x - 14}{28 + 3x - x^2} = \frac{(x - 7)(x + 2)}{(x^2 - 3x - 28)} = \frac{(x - 7)(x + 2)}{-(x + 4)(x - 7)} = \frac{x + 2}{x + 4}
\]

**ANSWER:**
\[
\frac{x + 2}{x + 4}
\]

21. \( \frac{x^3 - 9x^2}{x^2 - 3x - 54} \)

**SOLUTION:**
\[
\frac{x^3 - 9x^2}{x^2 - 3x - 54} = \frac{x^2(x - 9)}{(x - 9)(x + 6)} = \frac{x^2}{x + 6}
\]

**ANSWER:**
\[
\frac{x^2}{x + 6}
\]

22. \( \frac{(x - 4)(x^2 + 2x - 48)}{(36 - x^2)(x^2 + 4x - 32)} \)

**SOLUTION:**
\[
\frac{(x - 4)(x^2 + 2x - 48)}{(36 - x^2)(x^2 + 4x - 32)} = \frac{(x - 4)(x + 3)(x - 8)}{-(x - 6)(x + 6)(x - 8)(x - 4)} = -\frac{1}{x + 6}
\]

**ANSWER:**
\[
-\frac{1}{x + 6}
\]

23. \( \frac{16 - c^2}{c^2 + c - 20} \)

**SOLUTION:**
\[
\frac{16 - c^2}{c^2 + c - 20} = \frac{-(c - 4)(c + 4)}{(c + 5)(c - 4)} = \frac{-c + 4}{c + 5}
\]

**ANSWER:**
\[
\frac{-c + 4}{c + 5}
\]
24. **GEOMETRY** The cylinder has a volume of \((x + 3)(x^2 - 3x - 18)\pi\) cubic centimeters. Find the height of the cylinder.

![Cylinder Diagram]

**SOLUTION:**
\[
\pi r^2 h = (x + 3)(x^2 - 3x - 18)\pi
\]
\[
\pi (x + 3)^2 h = (x + 3)(x^2 - 3x - 18)\pi
\]
\[
h = \frac{x^2 - 3x - 18}{x + 3}
\]
\[
h = \frac{(x - 6)(x + 3)}{(x + 3)}
\]
\[
h = x - 6
\]

Therefore, the height of the cylinder is \((x - 6)\) centimeters.

**ANSWER:**
\[x - 6\text{ cm}\]

**Simplify each expression.**

25. \[
\frac{3ac^3f^3}{8a^2bcf^4} \cdot \frac{12ab^2c}{18ab^3c^2f}
\]

**SOLUTION:**
\[
\frac{3ac^3f^3}{8a^2bcf^4} \cdot \frac{12ab^2c}{18ab^3c^2f} = \frac{c}{4ab^2f^2}
\]

**ANSWER:**
\[\frac{c}{4ab^2f^2}\]

26. \[
\frac{14x^2z^3}{21w^4x^2yz} \div \frac{7wxyz}{12w^2y^3z}
\]

**SOLUTION:**
\[
\frac{14x^2z^3}{21w^4x^2yz} \div \frac{7wxyz}{12w^2y^3z} = \frac{7z^2}{18w^3y}
\]

**ANSWER:**
\[\frac{7z^2}{18w^3y}\]

27. \[
\frac{64a^2b^5}{35b^3c^3f^4} \div \frac{12a^4b^3c}{70abc^2f^2}
\]

**SOLUTION:**
Invert the second fraction and multiply.
\[
\frac{64a^2b^5}{35b^3c^3f^4} \div \frac{12a^4b^3c}{70abc^2f^2} = \frac{64a^2b^5}{35b^3c^3f^4} \times \frac{70abc^2f^2}{12a^4b^3c}
\]
\[
= \frac{32b}{3ac^3f^2}
\]

**ANSWER:**
\[\frac{32b}{3ac^3f^2}\]
Simplify each expression.

28. \[
\frac{9x^2y}{5z^4} + \frac{12x^4y^2}{50xy^4z^2}
\]

**SOLUTION:**

Invert the second fraction and multiply.

\[
\frac{9x^2y}{5z^4} + \frac{12x^4y^2}{50xy^4z^2} = \frac{9x^2y}{5z^4} \cdot \frac{50xy^4z^2}{12x^4y^2}
\]

\[
= \frac{15y^3}{2xz}
\]

**ANSWER:**

\[
\frac{15y^3}{2xz}
\]

29. \[
\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3}
\]

**SOLUTION:**

\[
\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3} = \frac{5a^4c}{3b}
\]

**ANSWER:**

\[
\frac{5a^4c}{3b}
\]

30. \[
\frac{14c^2f^5}{9a^2} \div \frac{35cf^4}{18ab^3}
\]

**SOLUTION:**

Invert the second fraction and multiply.

\[
\frac{14c^2f^5}{9a^2} \div \frac{35cf^4}{18ab^3} = \frac{14c^2f^5}{9a^2} \times \frac{18ab^3}{35cf^4}
\]

\[
= \frac{4c^2b^3}{5a}
\]

**ANSWER:**

\[
\frac{4c^2b^3}{5a}
\]

31. \[
\frac{y^2 + 8y + 15}{y - 6} \cdot \frac{y^2 - 9y + 18}{y^3 - 9}
\]

**SOLUTION:**

\[
\frac{y^2 + 8y + 15}{y - 6} \cdot \frac{y^2 - 9y + 18}{y^3 - 9} = \frac{(y + 5)(y + 3)}{y - 6} \cdot \frac{(y - 6)(y - 3)}{(y - 3)(y + 3)}
\]

\[
= y + 5
\]

**ANSWER:**

\[
y + 5
\]
8-1 Multiplying and Dividing Rational Expressions

32. \[
\frac{c^2 - 6c - 16}{c^2 - d^2} \div \frac{c^2 - 8c}{c + d}
\]

**SOLUTION:**
Invert the second fraction and multiply.

\[
\frac{c^2 - 6c - 16}{c^2 - d^2} \times \frac{c + d}{c^2 - 8c}
\]

\[
= \frac{(c - 8)(c + 2)}{(c - d)(c + d)} \times \frac{c + d}{c(c - 8)}
\]

\[
= \frac{(c + 2)}{c(c - d)}
\]

**ANSWER:**
\[
\frac{c + 2}{c(c - d)}
\]

33. \[
\frac{x^2 + 9x + 20}{8x + 16} \div \frac{4x^2 + 16x + 16}{x^2 - 25}
\]

**SOLUTION:**

\[
\frac{x^2 + 9x + 20}{8x + 16} \times \frac{x^2 - 25}{4x^2 + 16x + 16}
\]

\[
= \frac{(x + 4)(x + 5)}{4(x + 2)} \times \frac{(x - 5)(x + 5)}{4(x^2 + 4x + 4)}
\]

\[
= \frac{(x + 4)}{2} \times \frac{(x + 2)}{(x - 5)}
\]

\[
= \frac{(x + 4)(x + 2)}{2(x - 5)}
\]

**ANSWER:**
\[
\frac{(x + 4)(x + 2)}{2(x - 5)}
\]

34. \[
\frac{3a^2 + 6a + 3}{a^2 - 3a - 10} \div \frac{12a^2 - 12}{a^2 - 4}
\]

**SOLUTION:**
Invert the second fraction and multiply.

\[
\frac{3a^2 + 6a + 3}{a^2 - 3a - 10} \times \frac{a^2 - 4}{12a^2 - 12}
\]

\[
= \frac{3(a^2 + 2a + 1)}{a^2 - 3a - 10} \times \frac{a^2 - 4}{12(a^2 - 1)}
\]

\[
= \frac{3(a + 1)^2}{(a - 5)(a + 2)} \times \frac{(a - 2)(a + 2)}{12(a - 1)(a + 1)}
\]

\[
= \frac{(a + 1)}{(a - 5)} \times \frac{(a - 2)}{4(a - 1)}
\]

\[
= \frac{(a + 1)(a - 2)}{4(a - 5)(a - 1)}
\]

**ANSWER:**
\[
\frac{(a + 1)(a - 2)}{4(a - 5)(a - 1)}
\]
Simplify each expression.

35. \[ \frac{x^2 - 9}{6x - 12} \div \frac{x^2 + 10x + 21}{x^2 - x - 2} \]

**SOLUTION:**
Invert the fraction in the denominator and multiply.

\[ \frac{x^2 - 9}{6x - 12} \cdot \frac{x^2 - x - 2}{x^2 + 10x + 21} = \frac{(x+3)(x-3)}{6(x-2)} \cdot \frac{(x-2)(x+1)}{(x+3)(x+7)} = \frac{(x-3)(x+1)}{6(x+7)} \]

**ANSWER:**
\( \frac{(x-3)(x+1)}{6(x+7)} \)

36. \[ \frac{y-x}{x-y} \div \frac{z^3}{6z^2} \]

**SOLUTION:**
Invert the fraction in the denominator and multiply.

\[ \frac{y-x}{x-y} \cdot \frac{6z^2}{z^3} = \frac{y-x}{x-y} \cdot \frac{6}{z} = \frac{6}{z} \cdot \frac{y-x}{x-y} = -1 \]

**ANSWER:**
\( -\frac{1}{z} \)

37. \[ \frac{a^2 - b^2}{b^3} \div \frac{b^2 - ab}{a^2} \]

**SOLUTION:**
Invert the fraction in the denominator and multiply.

\[ \frac{a^2 - b^2}{b^3} \cdot \frac{a^2}{b^2 - ab} = \frac{(a-b)(a+b)}{b^3} \cdot \frac{a^2}{b(b-a)} = \frac{a^2(a+b)}{b^4} \]

**ANSWER:**
\( \frac{-a^2(a+b)}{b^4} \)

38. \[ \frac{x - y}{x^2 - y^2} \div \frac{a + b}{b^2 - a^2} \]

**SOLUTION:**
Invert the fraction in the denominator and multiply.

\[ \frac{x - y}{x^2 - y^2} \cdot \frac{b^2 - a^2}{a + b} = \frac{x - y}{x + y} \cdot \frac{b^2 - a^2}{a + b} \]

**ANSWER:**
\( \frac{x - y}{x + y} \cdot \frac{b^2 - a^2}{a + b} \)
8-1 Multiplying and Dividing Rational Expressions

39. CCSS REASONING At the end of her high school soccer career, Ashley had made 33 goals out of 121 attempts.

a. Write a ratio to represent the ratio of the number of goals made to goals attempted by Ashley at the end of her high school career.

b. Suppose Ashley attempted a goals and made m goals during her first year at college. Write a rational expression to represent the ratio of the number of career goals made to the number of career goals attempted at the end of her first year in college.

SOLUTION:

a. \( \frac{33}{121} \)

b. \( \frac{33 + m}{121 + a} \)

ANSWER:

a. \( \frac{33}{121} \)

b. \( \frac{33 + m}{121 + a} \)

40. GEOMETRY Parallelogram \( F \) has an area of \( 8x^2 + 10x - 3 \) square meters and a height of \( 2x + 3 \) meters. Parallelogram \( G \) has an area of \( 6x^2 + 13x - 5 \) square meters and a height of \( 3x - 1 \) meters. Find the area of right triangle \( H \).

\[
\frac{8x^2 + 10x - 3}{2x + 3} = \frac{(2x + 3)(4x - 1)}{(2x + 3)} = (4x - 1)
\]

The base of the parallelogram \( F \) is \( 4x - 1 \) meters.

Find the base of the parallelogram \( G \).

\[
\frac{6x^2 + 13x - 5}{3x - 1} = \frac{(2x + 5)(3x - 1)}{3x - 1} = (2x + 5)
\]

The base of the parallelogram \( G \) is \( 2x + 5 \) meters.

Find the area of the right triangle \( H \).

Area of the right triangle \( H \)

\[
\frac{1}{2}(2x + 5)(4x - 1)
\]

\[
\frac{1}{2}(8x^2 - 2x + 20x - 5)
\]

\[
\frac{1}{2}(8x^2 + 18x - 5)
\]

The area of the right triangle \( H \) is \( \frac{1}{2}(8x^2 + 18x - 5) \) m².

ANSWER:

\( \frac{1}{2}(8x^2 + 18x - 5) \) m²
41. **POLLUTION** The thickness of an oil spill from a ruptured pipe on a rig is modeled by the function

\[ T(x) = \frac{0.4(x^2 - 2x)}{x^3 + x^2 - 6x} \], where \( T \) is the thickness of the oil slick in meters and \( x \) is the distance from the rupture in meters.

**SOLUTION:**

a. Simplify the function.

\[ T(x) = \frac{0.4(x^2 - 2x)}{x^3 + x^2 - 6x} = \frac{0.4x(x - 2)}{x(x + 3)(x - 2)} = \frac{0.4}{x + 3} \]

b. How thick is the slick 100 meters from the rupture?

\[ T(100) = \frac{0.4}{100 + 3} = \frac{0.4}{103} \approx 0.0039 \]

The slick is about 0.0039 meters or 3.9 mm thick.

**ANSWER:**

a. \( T(x) = \frac{0.4}{x + 3} \)

b. about 3.9 mm thick

---

42. Simplify each expression.

\[ \frac{x^2 - 16}{3x^3 + 18x^2 + 24x} \cdot \frac{x^3 - 4x}{2x^2 - 7x - 4} \]

**SOLUTION:**

\[ \frac{x^2 - 16}{3x^3 + 18x^2 + 24x} \cdot \frac{x^3 - 4x}{2x^2 - 7x - 4} = \frac{(x + 4)(x - 4)}{3(x^2 + 6x + 8)} \cdot \frac{x(x - 4)}{2x^2 - 7x - 4} = \frac{(x + 4)(x - 4)}{3(x + 2)(x + 4)} \cdot \frac{x(x + 2)(x - 2)}{(2x + 1)(x - 4)} = \frac{(x - 2)}{3(2x + 1)} \]

**ANSWER:**

\[ \frac{x - 2}{3(2x + 1)} \]

43. Simplify each expression.

\[ \frac{3x^2 - 17x - 6}{4x^2 - 20x - 24} + \frac{6x^2 - 7x - 3}{2x^2 - x - 3} \]

**SOLUTION:**

\[ \frac{3x^2 - 17x - 6}{4x^2 - 20x - 24} + \frac{6x^2 - 7x - 3}{2x^2 - x - 3} = \frac{3x^2 - 17x - 6}{4(x^2 - 5x - 6)} + \frac{6x^2 - 7x - 3}{6x^2 - 7x - 3} = \frac{3x^2 - 17x - 6}{4(x - 6)} \cdot \frac{1}{6x^2 - 7x - 3} \]

\[ = \frac{(3x + 1)(x - 6)}{4(x - 6)(x + 1)} \cdot \frac{1}{(3x + 1)(2x - 3)} \]

\[ = \frac{1}{4} \]

**ANSWER:**

\[ \frac{1}{4} \]
44. \[
\frac{9 - x^2}{x^2 - 4x - 21} \left( \frac{2x^2 + 7x + 3}{2x^2 - 15x + 7} \right)^{-1}
\]

**SOLUTION:**
\[
\frac{9 - x^2}{x^2 - 4x - 21} \cdot \frac{2x^2 + 7x + 3}{2x^2 - 15x + 7}^{-1} = \frac{2x^2 + 7x + 3}{2x^2 - 15x + 7} \cdot \frac{9 - x^2}{x^2 - 4x - 21}
\]
\[
= (3-x)(3+x) \cdot (x-7)(x+3) \quad \text{and} \quad (2x-1)(x-7) \cdot (2x+1)(x+3)
\]
\[
= \frac{(3-x)(2x-1)}{(x+3)(2x+1)}
\]

**ANSWER:**
\[
(3 - x)(2x - 1)

(x + 3)(2x + 1)
\]

45. \[
\left( \frac{2x^2 + 2x - 12}{x^2 + 4x - 5} \right)^{-1} \cdot \frac{2x^3 - 8x}{x^2 - 2x - 35}
\]

**SOLUTION:**
\[
\left( \frac{2x^2 + 2x - 12}{x^2 + 4x - 5} \right)^{-1} \cdot \frac{2x^3 - 8x}{x^2 - 2x - 35} = \frac{2x^3 - 8x}{x^2 - 2x - 35} \cdot \frac{2x^2 + 2x - 12}{x^2 + 4x - 5}
\]
\[
= \frac{x^2 + 4x - 5}{2x^2 + 2x - 12} \cdot \frac{2x^3 - 8x}{x^2 - 2x - 35}
\]
\[
= \frac{x^2 + 4x - 5}{2(x^2 + x - 6)} \cdot \frac{2x(x^2 - 4)}{x^2 - 2x - 35}
\]
\[
= \frac{(x + 5)(x - 1)}{2(x + 3)(x - 2)} \cdot \frac{2x(x + 2)(x - 2)}{(x - 7)(x + 5)}
\]
\[
= \frac{x(x + 2)(x - 1)}{(x + 3)(x - 7)}
\]

**ANSWER:**
\[
\frac{x(x + 2)(x - 1)}{(x + 3)(x - 7)}
\]

46. \[
\left( \frac{3xy^3 z}{2a^2 bc^2} \right) \cdot 16a^4 b^3 c^5
\]

**SOLUTION:**
\[
\left( \frac{3xy^3 z}{2a^2 bc^2} \right) \cdot 16a^4 b^3 c^5 = \frac{3xy^3 z}{2a^2 bc^2} \cdot \frac{16a^4 b^3 c^5}{15x^3 yz^3}
\]
\[
= \frac{9y^3}{a^2 c} \cdot \frac{2}{5x^3}
\]
\[
= \frac{18y^3}{5a^2 cx^3}
\]

**ANSWER:**
\[
\frac{18y^3}{5a^2 cx^3}
\]

47. \[
\left( \frac{16x^3 y^3}{9aez} \right)^{-1}
\]

**SOLUTION:**
\[
\left( \frac{16x^3 y^3}{9aez} \right)^{-1} = \frac{20x^3 y^3 z^2}{16x^3 y^3} \cdot \frac{16x^3 y^3}{9aez}
\]
\[
= \frac{15y^3}{4a^2 cxz}
\]

**ANSWER:**
\[
\frac{15y^3}{4a^2 cxz}
\]
Simplify each expression.

48. \( \left( \frac{2xy^3}{3abc} \right)^2 \div \frac{6a^2b}{x^2y^4} \)

**SOLUTION:**

\[
\left( \frac{2xy^3}{3abc} \right)^2 \div \frac{6a^2b}{x^2y^4} = \frac{4x^2y^6}{9a^2b^2} \cdot \frac{x^2y^4}{6a^2b} = \frac{3bc^2}{8y^2}
\]

**ANSWER:**

\( \frac{3bc^2}{8y^2} \)

49. \( \frac{10x^2 + 35x - 20}{2x^2 + x - 6} \div \frac{4x^2 + 18x + 8}{4x^2 + 18x + 8} \)

**SOLUTION:**

\[
\frac{10x^2 + 35x - 20}{2x^2 + x - 6} \div \frac{4x^2 + 18x + 8}{4x^2 + 18x + 8} = \frac{5(2x - 1)(x + 4)}{2(4x + 1)(x + 2)}
\]

**ANSWER:**

\( \frac{5(2x - 1)(x + 4)}{2(4x + 1)(x + 2)} \)

50. \( \frac{2x^2 + 7x - 10}{6x^2 + 13x - 5} \div \frac{4x^2 + 12x - 72}{3x^2 - 11x - 4} \)

**SOLUTION:**

\[
\frac{2x^2 + 7x - 10}{6x^2 + 13x - 5} \div \frac{4x^2 + 12x - 72}{3x^2 - 11x - 4} = \frac{2x^2 + 7x - 10}{6x^2 + 13x - 5} \cdot \frac{3x^2 - 11x - 4}{4x^2 + 12x - 72}
\]

**ANSWER:**

\( \frac{x - 4}{-4(x - 3)} \)

51. \( \frac{3x^3 - 6x^2 - 42x}{12x^2 + 12x - 9} \div \frac{-2x^2 + 5x + 12}{-2x^2 + 5x + 12} \)

**SOLUTION:**

\[
\frac{3x^3 - 6x^2 - 24x}{12x^2 + 12x - 9} \div \frac{-2x^2 + 5x + 12}{-2x^2 + 5x + 12} = \frac{3x^3 - 6x^2 - 24x}{12x^2 + 12x - 9} \cdot \frac{-2x^2 + 5x + 12}{-2x^2 + 5x + 12}
\]

**ANSWER:**

\( \frac{2x + 1}{-9x(x + 2)} \)
52. **GEOMETRY** The area of the base of the rectangular prism at the right is 20 square centimeters.

a. Find the length of $BC$ in terms of $x$.

b. If $DC = 3BC$, determine the area of the shaded region in terms of $x$.

c. Determine the volume of the prism in terms of $x$.

**SOLUTION:**

a. The length of $BC$ is $\dfrac{20}{x}$ centimeters.

b. The length of $DC$ in terms of $x$ is $3 \left( \dfrac{20}{x} \right)$ or $\dfrac{60}{x}$.

Therefore, the area of the shaded region is $\dfrac{20 \cdot 60}{x \cdot x} = \dfrac{1200}{x^2}$ square centimeters.

c. The height of the prism is $\dfrac{60}{x}$ centimeters.

Volume of the prism = Base area $\times$ height

$$= 20 \cdot \dfrac{60}{x}$$

$$= \dfrac{1200}{x}$$

**ANSWER:**

a. $\dfrac{20}{x}$

b. $\dfrac{1200}{x^2}$

c. $\dfrac{1200}{x}$

53. Simplify each expression.

\[
\frac{x^2 + 4x - 32}{2x^2 + 9x - 5} \cdot \frac{3x^2 - 75}{3x^2 - 11x - 4} \div \frac{6x^2 - 18x - 60}{x^3 - 4x}
\]

**SOLUTION:**

\[
= \frac{x^2 + 4x - 32}{2x^2 + 9x - 5} \cdot \frac{3x^2 - 75}{3x^2 - 11x - 4} \div \frac{6x^2 - 18x - 60}{x^3 - 4x}
\]

\[
= \frac{(x+8)(x-4)}{2x^2 + 9x - 5} \cdot \frac{3(x-5)(x+5)}{(x-5)(x+5)} \div \frac{6(x-5)(x+2)}{x(x-2)(x+8)}
\]

\[
= \frac{x(x+8)(x-2)}{2(x+2)(2x-1)}
\]

**ANSWER:**

\[
\frac{x(x-2)(x+8)}{2(x-1)(3x+1)}
\]

54. Simplify each expression.

\[
\frac{8x^2 + 10x - 3}{3x^2 - 12x - 36} + \frac{2x^2 - 5x - 12}{3x^2 - 17x - 6} - \frac{4x^2 + 3x - 1}{4x^2 - 40x + 24}
\]

**SOLUTION:**

\[
= \frac{8x^2 + 10x - 3}{3x^2 - 12x - 36} + \frac{2x^2 - 5x - 12}{3x^2 - 17x - 6} - \frac{4x^2 + 3x - 1}{4x^2 - 40x + 24}
\]

\[
= \frac{8x^2 + 10x - 3}{3x^2 - 12x - 36} + \frac{2x^2 - 5x - 12}{3x^2 - 17x - 6} - \frac{4x^2 + 3x - 1}{4x^2 - 40x + 24}
\]

\[
= \frac{(4x-1)(2x+3)}{3(x-6)(x+2)} \cdot \frac{(x-6)(3x+1)}{(4x-5)(x+4)} \cdot \frac{(4x-1)(x+1)}{(2x+3)(x-4) 4(x^2 - 10x + 6)}
\]

\[
= \frac{(4x-1)^2(3x+1)(x+1)}{12(x+2)(x-4)(x^2 - 10x + 6)}
\]

**ANSWER:**

\[
\frac{(4x-1)^2(3x+1)(x+1)}{12(x+2)(x-4)(x^2 - 10x + 6)}
\]
Simplify each expression.

1. 
SOLUTION: 
ANSWER: 

2. 
SOLUTION: 
ANSWER: 

Thus, the equation for the decay of Carbon-14 is .
Substitute 0.005a for y in the equation .

55. \[ \frac{4x^2 - 9x - 9}{3x^2 + 6x - 18} \cdot \frac{-2x^2 + 5x + 3}{x^2 - 4x - 32} \cdot \frac{8x^2 + 10x + 3}{6x^2 - 6x - 12} \]

SOLUTION:
\[ \frac{4x^2 - 9x - 9}{3x^2 + 6x - 18} \cdot \frac{-2x^2 + 5x + 3}{x^2 - 4x - 32} \cdot \frac{8x^2 + 10x + 3}{6x^2 - 6x - 12} = \frac{4x^2 - 9x - 9}{3x^2 + 6x - 18} \cdot \frac{-2x^2 + 5x + 3}{x^2 - 4x - 32} \cdot \frac{8x^2 + 10x + 3}{6x^2 - 6x - 12} = \frac{4x^2 - 9x - 9}{3x^2 + 6x - 18} \cdot \frac{-2x^2 + 5x + 3}{x^2 - 4x - 32} \cdot \frac{8x^2 + 10x + 3}{6x^2 - 6x - 12} = \frac{(4x+3)(x-3)}{(x-8)(x+4)} \cdot \frac{6(x-2)(x+1)}{(2x+1)(4x+3)} \]
\[ = \frac{2(x^2-2x-6)}{(2x+1)^2(x^2+2x-6)} \]

ANSWER:
\[ \frac{-2(x-8)(x+4)(x-2)(x+1)}{(2x+1)^2(x^2+2x-6)} \]

56. CCSS PERSEVERANCE Use the formula \( d = rt \) and the following information.
An airplane is traveling at a rate \( r \) of 500 miles per hour for a time \( t \) of \((6 + x)\) hours. A second airplane travels at the rate of \((540 + 90x)\) miles per hour for a time \( t \) of 6 hours.

a. Write a rational expression to represent the ratio of the distance \( d \) traveled by the first airplane to the distance \( d \) traveled by the second airplane.

b. Simplify the rational expression. What does this expression tell you about the distances traveled by the two airplanes?

c. Under what condition is the rational expression undefined? Describe what this condition would tell you about the two airplanes.

SOLUTION:
a. \[ \frac{500(6 + x)}{(540 + 90x)(6)} \]
b. 

500(6 + x) \cdot 90(6 + x)(6)
\[ \frac{500}{540} = \frac{50}{54} = \frac{25}{27} \]
The second airplane travels a bit farther than the first airplane.

c. \( x = -6 \). Sample answer: When \( x = -6 \), the first airplane would travel for 0 hours and the second airplane would travel at a rate of 0 miles per hour.

ANSWER:
a. \[ \frac{(500)(6 + x)}{(540 + 90x)(6)} \]
b. \[ \frac{25}{27} \] : Sample answer: The second airplane travels a bit farther than the first airplane.
c. \( x = -6 \); Sample answer: When \( x = -6 \), the first airplane would travel for 0 hours and the second airplane would travel at a rate of 0 miles per hour.

57. TRAINS Trying to get into a train yard one evening, all of the trains are backed up for 2 miles along a system of tracks. Assume that each car occupies an average of 75 feet of space on a track and that the train yard has 5 tracks.

a. Write an expression that could be used to determine the number of train cars involved in the backup.

b. How many train cars are involved in the backup?

c. Suppose that there are 8 attendants doing safety checks on each car, and it takes each vehicle an average of 45 seconds for each check. Approximately how many hours will it take for all the vehicles in the backup to exit?
Simplify each expression.

1. SOLUTION:
   \[5 \text{ tracks} \cdot \frac{2 \text{ miles}}{1 \text{ track}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ car}}{75 \text{ feet}}\]

ANSWER:

2. SOLUTION:

ANSWER:

Thus, the equation for the decay of Carbon-14 is

Substitute 0.005\(a\) for \(y\) in the equation.

58. MULTIPLE REPRESENTATIONS In this problem, you will investigate the graph of a rational function.

a. ALGEBRAIC Simplify \(\frac{x^2 - 5x + 4}{x - 4}\).

b. TABULAR Let \(f(x) = \frac{x^2 - 5x + 4}{x - 4}\). Use the expression you wrote in part a to write the related function \(g(x)\). Use a graphing calculator to make a table for both functions for \(0 \leq x \leq 10\).

c. ANALYTICAL What are \(f(4)\) and \(g(4)\)? Explain the significance of these values.

d. GRAPHICAL Graph the functions on the graphing calculator. Use the TRACE function to investigate each graph, using the \(\downarrow\) and \(\uparrow\) keys to switch from one graph to the other. Compare and contrast the graphs.

e. VERBAL What conclusions can you draw about the expressions and the functions?

SOLUTION:

\[\frac{x^2 - 5x + 4}{x - 4} = \frac{(x - 4)(x - 1)}{x - 4} = x - 1\]

\[
\begin{array}{c|cccc}
\text{x} & 0 & 1 & 2 & 3 \\
\hline
f(x) & -1 & 0 & 1 & 2 \\
g(x) & -1 & 0 & 1 & 2 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{x} & 4 & 5 & 6 & 7 \\
\hline
f(x) & \text{ERR} & 4 & 5 & 6 \\
g(x) & 3 & 4 & 5 & 6 \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{x} & 8 & 9 & 10 \\
\hline
f(x) & 7 & 8 & 9 \\
g(x) & 7 & 8 & 9 \\
\end{array}
\]

c. \(f(4)\) results in an error because the function is undefined at \(x = 4\). \(g(4) = 3\)

d. The graphs appear to be the same on the graphing calculator. But \(f(x)\) is undefined for \(f(4)\) and \(g(4) = 3\).
8-1 Multiplying and Dividing Rational Expressions

e. The expressions and functions are equivalent except for \( x = 4 \).

**ANSWER:**
a. \( x - 1 \)

b. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>ERR</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

c. \( f(4) \) results in an error because the function is undefined at \( x = 4 \). \( g(4) = 3 \)

d. 

The graphs appear to be the same on the graphing calculator. But \( f(x) \) is undefined for \( f(4) \) and \( g(4) = 3 \).

e. The expressions and functions are equivalent except for \( x = 4 \).

59. **REASONING** Compare and contrast 
\[
\frac{(x - 6)(x + 2)(x + 3)}{x + 3} \quad \text{and} \quad (x - 6)(x + 2).
\]

**SOLUTION:**  
Sample answer: The two expressions are equivalent, except that the rational expression is undefined at \( x = 3 \).

**ANSWER:**  
Sample answer: The two expressions are equivalent, except that the rational expression is undefined at \( x = 3 \).

60. **CCSS CRITIQUE** Troy and Beverly are simplifying \[
\frac{x + y}{x - y} \div \frac{4}{y - x}.
\] Is either of them correct? Explain your reasoning.

**SOLUTION:**  
Sample answer: Beverly: Troy’s mistake was multiplying by the reciprocal of the dividend instead of the divisor.

**ANSWER:**  
Sample answer: Beverly: Troy’s mistake was multiplying by the reciprocal of the dividend instead of the divisor.
8-1 Multiplying and Dividing Rational Expressions

61. **CHALLENGE** Find the value that makes the following statement true.

\[
\frac{x - 6}{x + 3} \cdot \frac{x^2 + x - 6}{x - 6} = x - 2
\]

**SOLUTION:**

\[
\frac{x - 6}{x + 3} \cdot \frac{x^2 + x - 6}{x - 6} = \frac{x^2 + x - 6}{x + 3}
\]

\[
= \frac{(x + 3)(x - 2)}{x + 3}
\]

\[
= x - 2
\]

Therefore, the expression \(x^2 + x - 6\) makes the statement true.

**ANSWER:**

\(x^2 + x - 6\)

---

62. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

\[
\begin{align*}
\frac{1}{x - 1} \\
\frac{x^2 + 3x + 2}{x - 5} \\
\frac{x + 1}{\sqrt{x + 3}} \\
x^2 + \frac{1}{3}
\end{align*}
\]

**SOLUTION:**

\(\frac{1}{x - 1}\) does not belong with the other three. The other three expressions are rational expressions.

Since the denominator of \(\frac{x + 1}{\sqrt{x + 3}}\) is not a polynomial, \(\frac{x + 1}{\sqrt{x + 3}}\) is not a rational expression.

**ANSWER:**

\(\frac{x + 1}{\sqrt{x + 3}}\) does not belong with the other three. The other three expressions are rational expressions.

Since the denominator of \(\frac{x + 1}{\sqrt{x + 3}}\) is not a polynomial, \(\frac{x + 1}{\sqrt{x + 3}}\) is not a rational expression.
63. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

*A rational function that has a variable in the denominator is defined for all real values of x.*

**SOLUTION:**
Sample answer: Sometimes; with a denominator like $x^2 + 2$, in which the denominator cannot equal 0, the rational expression can be defined for all values of $x$.

**ANSWER:**
Sample answer: Sometimes; with a denominator like $x^2 + 2$, in which the denominator cannot equal 0, the rational expression can be defined for all values of $x$.

64. **OPEN ENDED** Write a rational expression that simplifies to $\frac{x-1}{x+4}$.

**SOLUTION:**
Sample answer: $\frac{x^2 - 1}{x^2 + 5x + 4}$

**ANSWER:**
Sample answer: $\frac{x^2 - 1}{x^2 + 5x + 4}$

65. **WRITING IN MATH** The rational expression $\frac{x^2 + 3x}{4x}$ is simplified to $\frac{x+3}{4}$. Explain why this new expression is not defined for all values of $x$.

**SOLUTION:**
Sample answer: When the original expression was simplified, a factor of $x$ was taken out of the denominator. If $x$ were to equal 0, then this expression would be undefined. So, the simplified expression is also undefined for $x$.

**ANSWER:**
Sample answer: When the original expression was simplified, a factor of $x$ was taken out of the denominator. If $x$ were to equal 0, then this expression would be undefined. So, the simplified expression is also undefined for $x$. 

---

**GEOMETRY**

66. **Volumes of Prisms and Cylinders**

- **Volume of a Rectangular Prism:** The volume of a rectangular prism is given by the formula $V = l \times w \times h$, where $l$ is the length, $w$ is the width, and $h$ is the height.

- **Volume of a Cylinder:** The volume of a cylinder is given by the formula $V = \pi r^2 h$, where $r$ is the radius of the base and $h$ is the height.

---

**SAT/ACT**

67. **Mathematics**

- **Problem:** Calculate the total area of the rectangle if the width is 10 units and the height is 5 units.

**Solution:**
The area of a rectangle is given by the formula $A = l \times w$, where $l$ is the length and $w$ is the width.

**Answer:**
The total area of the rectangle is $10 \times 5 = 50$ square units.

---

**Algebra**

68. **Expressions and Functions**

- **Expression:** Simplify the expression $\frac{x^2 + 3x}{4x}$.

**Solution:**
Simplify the expression by factoring out the common factor from the numerator.

**Answer:**
The simplified expression is $\frac{x+3}{4}$.

---

**Simplification of Expressions**

69. **Simplification Problem:** Simplify the expression $\frac{2x^2 - 3x + 1}{x^2 - 4}$.

**Solution:**
Factor the numerator and denominator, then cancel out common factors.

**Answer:**
The simplified expression is $\frac{2x-1}{x+2}$.

---

**Multi-Step Problem**

70. **Multi-Step Problem:** Solve the equation $2x + 3 = 7$ for $x$.

**Solution:**
Solve the equation by isolating $x$.

**Answer:**
The solution is $x = 2$.
66. SAT/ACT The Mason family wants to drive an average of 250 miles per day on their vacation. On the first five days, they travel 220 miles, 300 miles, 210 miles, 275 miles, and 240 miles. How many miles must they travel on the sixth day to meet their goal?

A 235 miles  
B 251 miles  
C 255 miles  
D 275 miles  
E 315 miles

**SOLUTION:**
Let \( x \) miles must they travel on the sixth day to meet their goal.

\[
\frac{220 + 300 + 210 + 275 + 240 + x}{6} = 250
\]

\[
\frac{1245 + x}{6} = 250
\]

\[
1245 + x = 1500
\]

\[
x = 1500 - 1245
\]

\[
x = 255
\]

So, they must travel 255 miles on the sixth day to meet their goal. The correct choice is C.

**ANSWER:**
C

67. Which of the following equations gives the relationship between \( N \) and \( T \) in the table?

<table>
<thead>
<tr>
<th>( N )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

F \( T = 2 - N \)  
G \( T = 4 - 3N \)  
H \( T = 3N + 1 \)  
J \( T = 3N - 2 \)

**SOLUTION:**
The relationship between \( N \) and \( T \) in the table is \( T = 3N - 2 \). The correct choice is J.

**ANSWER:**
J

68. A monthly cell phone plan costs $39.99 for up to 300 minutes and 20 cents per minute thereafter. Which of the following represents the total monthly bill (in dollars) to talk for \( x \) minutes if \( x \) is greater than 300?

A \( 39.99 + 0.20(300 - x) \)  
B \( 39.99 + 0.20(x - 300) \)  
C \( 39.99 + 0.20x \)  
D \( 39.99 + 20x \)

**SOLUTION:**
If \( x > 300 \), then \( x - 300 \) represents the amount of minutes beyond the time included in the initial $39.99 charge. So, the total monthly bill (in dollars) to talk for \( x \) minutes is represented by the expression \( 39.99 + 0.20(x - 300) \). So, the correct choice is B.

**ANSWER:**
B
69. **SHORT RESPONSE** The area of a circle 6 meters in diameter exceeds the combined areas of a circle 4 meters in diameter and a circle 2 meters in diameter by how many square meters?

**SOLUTION:**
Area of the circle of diameter 6 meters is $9\pi$ square meters.
Combined area of the circle of diameter 4 meters and the circle of 2 meters in diameter is $4\pi + \pi$ or $5\pi$ square meters.
Therefore, the area exceeds by $4\pi$ square meters.

**ANSWER:** $4\pi$

70. **ANTHROPOLOGY** An anthropologist studying the bones of a prehistoric person finds there is so little remaining Carbon-14 in the bones that instruments cannot measure it. This means that there is less than 0.5% of the amount of Carbon-14 the bones would have contained when the person was alive. The half-life of Carbon-14 is 5760 years. How long ago did the person die?

**SOLUTION:**

\[
y = ae^{-kt}
\]
\[
0.5a = ae^{-k(5760)}
\]
\[
0.5 = e^{-5760k}
\]
\[
\ln 0.5 = \ln(e^{-5760k})
\]
\[
\ln 0.5 = -5760k
\]
\[
k = \frac{\ln 0.5}{-5760}
\]
\[
k \approx 0.00012
\]

Thus, the equation for the decay of Carbon-14 is 
\[
y = ae^{-0.00012t}
\]

Substitute $0.005a$ for $y$ in the equation $y = ae^{-0.00012t}$.

\[
0.005a = ae^{-0.00012t}
\]
\[
0.005 = e^{-0.00012t}
\]
\[
\ln 0.005 = \ln(e^{-0.00012t})
\]
\[
\ln 0.005 = -0.00012t
\]
\[
\frac{\ln 0.005}{-0.00012} = t
\]
\[
t \approx 44153
\]

Therefore, the person died more than 44,000 years ago.

**ANSWER:** more than 44,000 years ago
8-1 Multiplying and Dividing Rational Expressions

Solve each equation. Round to the nearest ten thousandth.

71. \(3e^x + 1 = 5\)

**SOLUTION:**

\[
3e^x + 1 = 5
\]

\[
3e^x = 4
\]

\[
e^x = \frac{4}{3}
\]

\[
\ln e^x = \ln \frac{4}{3}
\]

\[
x = \ln \frac{4}{3}
\]

\[
x \approx 0.2877
\]

**ANSWER:**

0.2877

73. \(-3e^{4x} + 11 = 2\)

**SOLUTION:**

\[-3e^{4x} + 11 = 2\]

\[-3e^{4x} = -9\]

\[e^{4x} = 3\]

\[\ln e^{4x} = \ln 3\]

\[4x = \ln 3\]

\[x = \frac{\ln 3}{4}\]

\[x \approx 0.2747\]

**ANSWER:**

0.2747

74. \(8 + 3e^{3x} = 26\)

**SOLUTION:**

\[8 + 3e^{3x} = 26\]

\[3e^{3x} = 18\]

\[e^{3x} = 6\]

\[\ln e^{3x} = \ln 6\]

\[3x = \ln 6\]

\[x = \frac{\ln 6}{3}\]

\[x \approx 0.5973\]

**ANSWER:**

0.5973
**8-1 Multiplying and Dividing Rational Expressions**

75. **NOISE ORDINANCE** A proposed city ordinance will make it illegal in a residential area to create sound that exceeds 72 decibels during the day and 55 decibels during the night. How many times as intense is the noise level allowed during the day as at night?

**SOLUTION:**
The loudness $L$, in decibels, of a sound is

$$L = 10 \log \frac{I}{m},$$

where $I$ is the intensity of the sound and $m$ is the minimum intensity of sound detectable by the human ear.

Substitute 72 for $L$, $m = 1$ in $L = 10 \log \frac{I}{m}$.

$$72 = 10 \log \frac{I}{1}$$

Now, substitute 55 for $L$, $m = 1$ in $L = 10 \log \frac{I}{m}$.

$$55 = 10 \log \frac{I}{1}$$

Therefore, the noise level allowed during the day is $10^{1.7}$ or about 50 times the noise level allowed during the night.

**ANSWER:**
$10^{1.7}$ or about 50 times

---

76. $\sqrt{50x^4}$

**SOLUTION:**

$$\sqrt{50x^4} = \sqrt{2 \cdot (25x^4)}$$

$$= \sqrt{2 \cdot (5x^2)^2}$$

$$= 5x^2 \sqrt{2}$$

**ANSWER:**
$5x^2 \sqrt{2}$

77. $\sqrt[3]{16y^3}$

**SOLUTION:**

$$\sqrt[3]{16y^3} = \sqrt[3]{2(8y^3)}$$

$$= \sqrt[3]{2y^3}$$

$$= 2y \sqrt[3]{2}$$

**ANSWER:**
$2y \sqrt[3]{2}$

78. $\sqrt{18x^2y^3}$

**SOLUTION:**

$$\sqrt{18x^2y^3} = \sqrt{2y(9x^2y^2)}$$

$$= \sqrt{2y(3xy)^2}$$

$$= 3|xy| y \sqrt{2y}$$

**ANSWER:**
$3|xy| y \sqrt{2y}$
8-1 Multiplying and Dividing Rational Expressions

79. \( \sqrt[4]{40a^3b^4} \)

**SOLUTION:**

\[ \sqrt[4]{40a^3b^4} = \sqrt[4]{4 \cdot 10a^3b^4} = 2ab^2\sqrt[4]{10a} \]

**ANSWER:**

\[ 2ab^2\sqrt{10a} \]

80. **AUTOMOBILES** The length of the cargo space in a sport-utility vehicle is 4 inches greater than the height of the space. The width is 16 inches less than twice the height. The cargo space has a total volume of 55,296 cubic inches.

a. Write a polynomial function that represents the volume of the cargo space.

b. Will a package 34 inches long, 44 inches wide, and 34 inches tall fit in the cargo space? Explain.

**SOLUTION:**

a. Let \( h \) be the height of the cargo space. Therefore, the length of the space is \( h + 4 \) and the width of the space is \( 2h - 16 \).

\[ V = (h + 4)(2h - 16)h \]

\[ = (2h^2 - 16h + 8h - 64)h \]

\[ = (2h^2 - 8h - 64)h \]

\[ = 2h^3 - 8h^2 - 64h \]

A polynomial function that represents the volume of the cargo space is \( V = 2h^3 - 8h^2 - 64h \).

b. Substitute 55296 for \( V \) in the equation

\[ 55296 = 2h^3 - 8h^2 - 64h \]

\[ 2h^3 - 8h^2 - 64h - 55296 = 0 \]

\[ \frac{2}{32} \]

\[ \frac{1}{0} \]

\[ \frac{0}{64} \]

\[ \frac{1792}{55296} \]

\[ \frac{55296}{0} \]

\[ \frac{1728}{2} \]

\[ \frac{64}{0} \]

So, \( h = 32 \) is a root of the equation \( 2h^3 - 8h^2 - 64h - 55296 = 0 \). The depressed polynomial is \( (2h^2 + 56h + 1728) \).

Therefore, \( 2h^2 - 8h^2 - 64h - 55296 = (h - 32)(2h^2 + 56h + 1728) \).

Use the Quadratic formula to find the roots of \( 2h^2 + 56h + 1728 = 0 \).

\[ h = \frac{-56 \pm \sqrt{56^2 - 4(2)(1728)}}{2(2)} \]

\[ \frac{-56 \pm \sqrt{3136 - 13824}}{4} \]

\[ Therefore, the other two roots of the equation \( 2h^3 - 8h^2 - 64h - 55296 = 0 \) are imaginary. The length of the space is 36 inches. The width of the space is 48 inches. So, the package is too tall to fit.

**ANSWER:**

a. \( V = 2h^3 - 8h^2 - 64h \)

b. No; the dimensions of the space are \( \ell = 36 \text{ in.}, w = 48 \text{ in.}, h = 32 \text{ in.} \), so the package is too tall to fit.
8-1 Multiplying and Dividing Rational Expressions

Simplify.

81. \((2a + 3b) + (8a - 5b)\)

\[\text{SOLUTION:}\]
\[2a + 3b + 8a - 5b = 10a - 2b\]

\[\text{ANSWER:}\]
\[10a - 2b\]

82. \((x^2 - 4x + 3) - (4x^2 + 3x - 5)\)

\[\text{SOLUTION:}\]
\[x^2 - 4x + 3 - 4x^2 - 3x + 5 = -3x^2 - 7x + 8\]

\[\text{ANSWER:}\]
\[-3x^2 - 7x + 8\]

83. \((5y + 3y^2) + (-8y - 6y^2)\)

\[\text{SOLUTION:}\]
\[5y + 3y^2 - 8y - 6y^2 = -3y - 3y^2\]

\[\text{ANSWER:}\]
\[-3y - 3y^2\]

84. \(2x(3y + 9)\)

\[\text{SOLUTION:}\]
\[2x(3y + 9) = 6xy + 18x\]

\[\text{ANSWER:}\]
\[6xy + 18x\]

85. \((x + 6)(x + 3)\)

\[\text{SOLUTION:}\]
\[(x + 6)(x + 3)\]
\[= x^2 + 6x + 3x + 18\]
\[= x^2 + 9x + 18\]

\[\text{ANSWER:}\]
\[x^2 + 9x + 18\]

86. \((x + 1)(x^2 - 2x + 3)\)

\[\text{SOLUTION:}\]
\[(x + 1)(x^2 - 2x + 3)\]
\[= x^3 - 2x^2 + 3x + x^2 - 2x + 3\]
\[= x^3 - x^2 + x + 3\]

\[\text{ANSWER:}\]
\[x^3 - x^2 + x + 3\]
8-2 Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. \(16x, 8x^2y^3, 5x^3y\)

**SOLUTION:**
\[
16x = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \\
8x^2y^3 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \\
5x^3y = 5 \cdot x \cdot x \cdot x \cdot y \\
LCM = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y \\
= 80x^3y^2
\]

**ANSWER:** 
\(80x^3y^2\)

2. \(7a^2, 9ab^3, 21abc^4\)

**SOLUTION:**
\[
7a^2 = 7 \cdot a \cdot a \\
9ab^3 = 3 \cdot 3 \cdot a \cdot b \cdot b \cdot b \\
21abc^4 = 3 \cdot 7 \cdot a \cdot b \cdot c \cdot c \cdot c \cdot c \\
LCM = 3 \cdot 7 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \\
= 63a^2b^3c^4
\]

**ANSWER:** 
\(63a^2b^3c^4\)

3. \(3y^2 - 9y, y^2 - 8y + 15\)

**SOLUTION:**
\[
3y^2 - 9y = 3 \cdot y \cdot (y - 3) \\
y^2 - 8y + 15 = (y - 5)(y - 3) \\
LCM = 3 \cdot y \cdot (y - 5)(y - 3) \\
= 3y(y - 5)(y - 3)
\]

**ANSWER:** 
\(3y(y - 3)(y - 5)\)

4. \(x^3 - 6x^2 - 16x, x^2 - 4\)

**SOLUTION:**
\[
x^3 - 6x^2 - 16x = x \cdot (x - 8) \cdot (x + 2) \\
x^2 - 4 = (x + 2)(x - 2) \\
LCM = x \cdot (x - 8) \cdot (x + 2) \cdot (x - 2) \\
= x(x + 2)(x - 2)(x - 8)
\]

**ANSWER:** 
\(x(x + 2)(x - 2)(x - 8)\)

5. \(\frac{12y}{5x} + \frac{5x}{4y^3}\)

**SOLUTION:**
The LCD is \(20xy^3\).
\[
\frac{12y}{5x} + \frac{5x}{4y^3} = \frac{12y}{5x} \cdot \frac{4y^3}{4y^3} + \frac{5x}{4y^3} \cdot \frac{5x}{5x} \\
= \frac{48y^4 + 25x^2}{20xy^3}
\]

**ANSWER:** 
\(\frac{48y^4 + 25x^2}{20xy^3}\)
6. \( \frac{5}{6ab} + \frac{3b^3}{14a^3} \)

\[ \text{SOLUTION:} \]

The LCD is \( 42a^3b \).

\[ \frac{5}{6ab} + \frac{3b^3}{14a^3} = \frac{5}{6ab} \cdot \frac{2b}{2b} + \frac{3b^3}{14a^3} \cdot \frac{3ab}{3ab} \]

\[ = \frac{10b + 9b^4}{42a^3b} \]

\[ \text{ANSWER:} \quad \frac{10b + 9b^4}{42a^3b} \]

7. \( \frac{7b}{12a} - \frac{1}{18ab^3} \)

\[ \text{SOLUTION:} \]

The LCD is \( 36ab^3 \).

\[ \frac{7b}{12a} - \frac{1}{18ab^3} = \frac{7b}{12a} \cdot \frac{3b^2}{3b^2} - \frac{1}{18ab^3} \cdot \frac{2a}{2a} \]

\[ = \frac{21b^4 - 2}{36ab^3} \]

\[ \text{ANSWER:} \quad \frac{21b^4 - 2}{36ab^3} \]

8. \( \frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d} \)

\[ \text{SOLUTION:} \]

The LCD is \( 56c^4d^2 \).

\[ \frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d} = \frac{y^2}{8c^2d^2} \cdot \frac{7c^2}{7c^2} - \frac{3x}{14c^4d} \cdot \frac{4cd}{4cd} \]

\[ = \frac{7c^2y^2 - 12dx}{56c^4d^2} \]

\[ \text{ANSWER:} \quad \frac{7c^2y^2 - 12dx}{56c^4d^2} \]

9. \( \frac{4x}{x^2 + 9x + 18} + \frac{5}{x + 6} \)

\[ \text{SOLUTION:} \]

The LCD is \((x + 3)(x + 6)\).

\[ \frac{4x}{x^2 + 9x + 18} + \frac{5}{x + 6} = \frac{4x}{(x + 3)(x + 6)} + \frac{5}{x + 6} \]

\[ = \frac{4x + 5(x + 3)}{(x + 3)(x + 6)} + \frac{5(x + 3)}{(x + 3)(x + 6)} \]

\[ = \frac{9x + 15}{(x + 3)(x + 6)} \]

\[ \text{ANSWER:} \quad \frac{9x + 15}{(x + 3)(x + 6)} \]
Find the LCM of each set of polynomials.

1. \(16x, 8x^2y^3, 5x^3y\)

SOLUTION:

\[
\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27} = \frac{8}{y - 3} + \frac{2y - 5}{(y - 3)(y - 9)}
\]

The LCD is \((y - 3)(y - 9)\).

\[
\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27} = \frac{8}{y - 3} + \frac{2y - 5}{(y - 3)(y - 9)}
= \frac{8y - 72 + 2y - 5}{(y - 3)(y - 9)}
= \frac{10y - 77}{(y - 3)(y - 9)}
\]

ANSWER:

\[
\frac{10y - 77}{(y - 3)(y - 9)}
\]

2. \(7a^2, \ldots \)

x be the number of books he had originally.

Therefore, the correct choice is F.

ANSWER:

F

8-2 Adding and Subtracting Rational Expressions

10. \(\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27}\)

SOLUTION:

\[
\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27} = \frac{8}{y - 3} + \frac{2y - 5}{(y - 3)(y - 9)}
\]

The LCD is \((y - 3)(y - 9)\).

\[
\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27} = \frac{8}{y - 3} + \frac{2y - 5}{(y - 3)(y - 9)}
= \frac{8y - 72 + 2y - 5}{(y - 3)(y - 9)}
= \frac{10y - 77}{(y - 3)(y - 9)}
\]

ANSWER:

\[
\frac{10y - 77}{(y - 3)(y - 9)}
\]

11. \(\frac{4}{3x + 6} - \frac{x + 1}{x^2 - 4}\)

SOLUTION:

\[
\frac{4}{3x + 6} - \frac{x + 1}{x^2 - 4} = \frac{4}{3(x + 2)} - \frac{x + 1}{(x + 2)(x - 2)}
\]

The LCD is \(3(x + 2)(x - 2)\).

\[
\frac{4}{3x + 6} - \frac{x + 1}{x^2 - 4} = \frac{4}{3(x + 2)} - \frac{x + 1}{(x + 2)(x - 2)}
= \frac{4x + 8 - 3x - 3}{3(x + 2)(x - 2)}
= \frac{x - 11}{3(x + 2)(x - 2)}
\]

ANSWER:

\[
\frac{x - 11}{3(x + 2)(x - 2)}
\]

12. \(\frac{3a + 2}{a^2 - 16} - \frac{7}{6a + 24}\)

SOLUTION:

\[
\frac{3a + 2}{a^2 - 16} - \frac{7}{6a + 24} = \frac{3a + 2}{(a + 4)(a - 4)} - \frac{7}{6(a + 4)}
\]

The LCD is \(6(a + 4)(a - 4)\).

\[
\frac{3a + 2}{a^2 - 16} - \frac{7}{6a + 24} = \frac{3a + 2}{(a + 4)(a - 4)} - \frac{7}{6(a + 4)}
= \frac{18a + 12 - 7a + 28}{6(a + 4)(a - 4)}
= \frac{11a + 40}{6(a + 4)(a - 4)}
\]

ANSWER:

\[
\frac{11a + 40}{6(a + 4)(a - 4)}
\]
13. GEOMETRY Find the perimeter of the rectangle.

\[
\frac{3}{x-2} + \frac{4}{x+1}
\]

\[\text{SOLUTION:}\]
The perimeter \( P \) of the rectangle is:

\[ P = 2\left(\frac{4}{x+1}\right) + 2\left(\frac{3}{x-2}\right) \]

The LCD is \((x+1)(x-2)\).

\[
\frac{4}{x+1} + \frac{3}{x-2} = \frac{8}{x+1} \cdot \frac{(x-2)}{(x-2)} + \frac{6}{x-2} \cdot \frac{(x+1)}{(x+1)}
\]

\[
= \frac{8x - 16 + 6x + 6}{(x+1)(x-2)}
\]

\[
= \frac{14x - 10}{(x+1)(x-2)}
\]

\[\text{ANSWER:}\]

\[
\frac{14x - 10}{(x+1)(x-2)}
\]

---

14. \( \frac{4+2}{x} \)

\[\text{SOLUTION:}\]

\[
\frac{4}{x} + \frac{2}{x} = \frac{4x + 2}{x}
\]

\[\text{ANSWER:}\]

\[
\frac{4x + 2}{3x - 2}
\]

15. \( \frac{y}{2} + \frac{6}{y} \)

\[\text{SOLUTION:}\]

\[
\frac{6 + 4}{y} = \frac{6y + 4}{y^2}
\]

\[
= \frac{2y + 6}{y + 3}
\]

\[\text{ANSWER:}\]

\[
\frac{3y + 2}{y + 3}
\]
Find the LCM of each set of polynomials.

18. $24cd, 40a^2c^3d^4, 15abd^3$

\[
\text{SOLUTION:} \\
24cd = 2 \cdot 2 \cdot 2 \cdot 3 \cdot c \cdot d \\
40a^2c^3d^4 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot c \cdot c \cdot d \cdot d \cdot d \\
15abd^3 = 3 \cdot 5 \cdot a \cdot b \cdot d \cdot d \\
\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot a \cdot a \cdot b \cdot c \cdot c \cdot d \cdot d \cdot d \cdot d \\
= 120a^2bc^3d^4
\]

\[
\text{ANSWER:} \\
120a^2bc^3d^4
\]

19. $4x^2y, 18xy^4, 10xz^2$

\[
\text{SOLUTION:} \\
4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \\
18xy^4 = 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y \\
10xz^2 = 2 \cdot 5 \cdot x \cdot z \cdot z \\
\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \\
= 180x^2y^4z^2
\]

\[
\text{ANSWER:} \\
180x^2y^4z^2
\]

20. $x^2 - 9x + 20, x^2 + x - 30$

\[
\text{SOLUTION:} \\
x^2 - 9x + 20 = (x - 4)(x - 5) \\
x^2 + x - 30 = (x + 6)(x - 5) \\
\text{LCM} = (x - 4)(x - 5)(x + 6)
\]

\[
\text{ANSWER:} \\
(x - 4)(x - 5)(x + 6)
\]
21. \(6x^2 + 21x - 12, 4x^2 + 22x + 24\)

**SOLUTION:**

\[
\begin{align*}
6x^2 + 21x - 12 &= 3(2x^2 + 7x - 4) \\
&= 3(x + 4)(2x - 1) \\
4x^2 + 22x + 24 &= 2(2x^2 + 11x + 12) \\
&= 2(2x + 3)(x + 4) \\
\text{LCM} &= 2 \cdot 3 \cdot (x + 4)(2x - 1)(2x + 3) \\
&= 6(x + 4)(2x - 1)(2x + 3)
\end{align*}
\]

**ANSWER:**

\(6(x + 4)(2x - 1)(2x + 3)\)

**CCSS PERSEVERANCE** Simplify each expression.

22. \(\frac{5a}{24cf^4} + \frac{a}{36bc^4f^3}\)

**SOLUTION:**

The LCD is \(72bc^4f^4\).

\[
\begin{align*}
\frac{5a}{24cf^4} + \frac{a}{36bc^4f^3} &= \frac{5a}{24cf^4} \cdot \frac{3bc^3}{3bc^3} + \frac{a}{36bc^4f^3} \cdot \frac{2f}{2f} \\
&= \frac{15abc^3 + 2af}{72bc^4f^4}
\end{align*}
\]

**ANSWER:**

\(\frac{15abc^3 + 2af}{72bc^4f^4}\)

23. \(\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z}\)

**SOLUTION:**

The LCD is \(105x^3y^4z\).

\[
\begin{align*}
\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z} &= \frac{4b \cdot 7y^2z}{15x^3y^2 \cdot 7y^2z} - \frac{3b \cdot 3x}{35x^2y^4z \cdot 3x} \\
&= \frac{28by^2z - 9bx}{105x^3y^4z}
\end{align*}
\]

**ANSWER:**

\(\frac{28by^2z - 9bx}{105x^3y^4z}\)

24. \(\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2}\)

**SOLUTION:**

The LCD is \(30a^2b^2\).

\[
\begin{align*}
\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2} &= \frac{5b \cdot 5a}{6a \cdot 5a} + \frac{3b \cdot 3ab^2}{10a^2 \cdot 3ab^2} + \frac{2 \cdot 30a}{ab^2 \cdot 30a} \\
&= \frac{25ab^3 + 9b^3 + 60a}{30a^2b^2}
\end{align*}
\]

**ANSWER:**

\(\frac{25ab^3 + 9b^3 + 60a}{30a^2b^2}\)
25. \( \frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy} \)

**SOLUTION:**
The LCD is \(15x^3y\).

\[
\frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy} = \frac{4\cdot 5x^2y}{15x^3y} + \frac{8\cdot 15y}{15x^3y} + \frac{3\cdot 2x^2}{3x^2} \cdot \frac{5x^2y}{15x^3y} = \frac{20x^2y + 120y + 6x^2}{15x^3y}
\]

**ANSWER:**
\( \frac{20x^2y + 120y + 6x^2}{15x^3y} \)

26. \( \frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2} \)

**SOLUTION:**
The LCD is \(90y^2\).

\[
\frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2} = \frac{8\cdot 30y}{90y^2} + \frac{2\cdot 10y^2}{10y^2} - \frac{3\cdot 9}{90y^2} = \frac{240y + 20y^2 - 27}{90y^2}
\]

**ANSWER:**
\( \frac{240y + 20y^2 - 27}{90y^2} \)

27. \( \frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3} \)

**SOLUTION:**
The LCD is \(240ab^3\).

\[
\frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3} = \frac{15b^3}{15b^3 \cdot 16a} + \frac{10a \cdot b^2}{10a \cdot b^2 \cdot 12b} - \frac{9 \cdot 12b}{9 \cdot 12b \cdot 10b^3} = \frac{15b^3 + 100ab^2 - 216a}{240ab^3}
\]

**ANSWER:**
\( \frac{15b^3 + 100ab^2 - 216a}{240ab^3} \)

28. \( \frac{8}{x^2 - 6x - 16} + \frac{9}{x^2 - 3x - 40} \)

**SOLUTION:**

\[
\frac{8}{x^2 - 6x - 16} + \frac{9}{x^2 - 3x - 40} = \frac{8}{(x-8)(x+2)} + \frac{9}{(x-8)(x+5)}
\]

The LCD is \((x-8)(x+2)(x+5)\).

\[
\frac{8}{x^2 - 6x - 16} + \frac{9}{x^2 - 3x - 40} = \frac{8(x+5) + 9(x+2)}{(x-8)(x+2)(x+5)} = \frac{8x + 40 + 9x + 18}{(x-8)(x+2)(x+5)} = \frac{17x + 58}{(x-8)(x+2)(x+5)}
\]

**ANSWER:**
\( \frac{17x + 58}{(x-8)(x+2)(x+5)} \)
29. \( \frac{6}{y^2 - 2y - 35} + \frac{4}{y^2 + 9y + 20} \)

\[ \text{SOLUTION:} \quad \frac{6}{y^2 - 2y - 35} + \frac{4}{y^2 + 9y + 20} = \frac{6}{(y - 7)(y + 5)} + \frac{4}{(y + 5)(y + 4)} \]

The LCD is \((y - 7)(y + 5)(y + 4)\).

\[ \frac{6}{y^2 - 2y - 35} + \frac{4}{y^2 + 9y + 20} = \frac{6(y + 4) + 4(y - 7)}{(y - 7)(y + 5)(y + 4)} \]

\[ = \frac{6y + 24 + 4y - 28}{(y - 7)(y + 5)(y + 4)} \]

\[ = \frac{10y - 4}{(y - 7)(y + 5)(y + 4)} \]

\[ \text{ANSWER:} \quad \frac{10y - 4}{(y - 7)(y + 5)(y + 4)} \]

30. \( \frac{12}{3y^2 - 10y - 8} - \frac{3}{y^2 - 6y + 8} \)

\[ \text{SOLUTION:} \quad \frac{12}{3y^2 - 10y - 8} - \frac{3}{y^2 - 6y + 8} = \frac{12}{(y - 4)(3y + 2)} - \frac{3}{(y - 2)(y - 4)} \]

The LCD is \((y - 2)(y - 4)(3y + 2)\).

\[ \frac{12}{3y^2 - 10y - 8} - \frac{3}{y^2 - 6y + 8} = \frac{12(y - 2) - 3(3y + 2)}{(y - 2)(y - 4)(3y + 2)} \]

\[ = \frac{12y - 24 - 9y - 6}{(y - 2)(y - 4)(3y + 2)} \]

\[ = \frac{3y - 30}{(y - 2)(y - 4)(3y + 2)} \]

\[ \text{ANSWER:} \quad \frac{3y - 30}{(3y + 2)(y - 4)(y - 2)} \]
31. \[
\frac{6}{2x^2 + 11x - 6} - \frac{8}{x^2 + 3x - 18}
\]

**SOLUTION:**
\[
\frac{6}{2x^2 + 11x - 6} - \frac{8}{x^2 + 3x - 18} = \frac{6}{(2x - 1)(x + 6)} - \frac{8}{(x + 6)(x - 3)}
\]
The LCD is \((2x - 1)(x + 6)(x - 3)\).

\[
\frac{6(2x - 1) - 8(x - 3)}{(2x - 1)(x + 6)(x - 3)} = \frac{6x - 3 - 8x + 24}{(2x - 1)(x + 6)(x - 3)} = \frac{-10x + 21}{(2x - 1)(x + 6)(x - 3)}
\]

**ANSWER:**
\[
\frac{-10x + 21}{(2x - 1)(x + 6)(x - 3)}
\]

32. \[
\frac{2x}{4x^2 + 9x + 2} + \frac{3}{2x^2 - 8x - 24}
\]

**SOLUTION:**
\[
\frac{2x}{4x^2 + 9x + 2} + \frac{3}{2x^2 - 8x - 24} = \frac{2x}{(x + 2)(4x + 1)} + \frac{3}{2(x + 2)(x - 6)}
\]
The LCD is \(2(x + 2)(4x + 1)(x - 6)\).

\[
\frac{2x[2(x - 6)] + 3(4x + 1)}{2(x + 2)(4x + 1)(x - 6)} = \frac{4x^2 - 24x + 12x + 3}{2(x + 2)(4x + 1)(x - 6)} = \frac{4x^2 - 12x + 3}{2(x + 2)(4x + 1)(x - 6)}
\]

**ANSWER:**
\[
\frac{4x^2 - 12x + 3}{2(x - 6)(4x + 1)(x + 2)}
\]
33. \( \frac{4x}{3x^2 + 3x - 18} - \frac{2x}{2x^2 + 11x + 15} \)

**SOLUTION:**

\[
\frac{4x}{3(x-2)(x+3)} - \frac{2x}{(x+3)(2x+5)}
\]

\[
= \frac{4x(2x+5) - 2x[3(x-2)]}{3(x-2)(x+3)(2x+5)}
\]

\[
= \frac{8x^2 + 20x - 6x^2 + 12x}{3(x-2)(x+3)(2x+5)}
\]

\[
= \frac{2x^2 + 32x}{3(x-2)(x+3)(2x+5)}
\]

The LCD is \( 3(x-2)(x+3)(2x+5) \).

**ANSWER:**

\[
\frac{2x^2 + 32x}{3(x-2)(x+3)(2x+5)}
\]

34. **BIOLOGY** After a person eats something, the pH or acid level \( A \) of his or her mouth can be determined by the formula

\[
A = \frac{20.4t}{t^2 + 36} + 6.5,
\]

where \( t \) is the number of minutes that have elapsed since the food was eaten.

a. Simplify the equation.

b. What would the acid level be after 30 minutes?

**SOLUTION:**

a. The LCD is \( t^2 + 36 \).

\[
A = \frac{20.4t + 6.5(t^2 + 36)}{t^2 + 36} = \frac{20.4t + 6.5t^2 + 234}{t^2 + 36}
\]

\[
= \frac{6.5t^2 + 20.4t + 234}{t^2 + 36}
\]

b. Substitute \( t = 30 \) minutes in the expression for \( A \).

\[
A = \frac{6.5(30)^2 + 20.4(30) + 234}{(30)^2 + 36}
\]

\[
= \frac{6696}{936}
\]

\[
= 7.2
\]

**ANSWER:**

a. \( A = \frac{6.5t^2 + 20.4t + 234}{t^2 + 36} \)

b. \( \approx 7.2 \)
35. **GEOMETRY** Both triangles in the figure at the right are equilateral. If the area of the smaller triangle is 200 square centimeters and the area of the larger triangle is 300 square centimeters, find the minimum distance from \( A \) to \( B \) in terms of \( x \) and \( y \) and simplify.

**SOLUTION:**
From the figure, the base length of the larger triangle is \( x + 2y \) and the base length of the smaller triangle is \( x \).

The minimum distance from \( A \) to \( B \) is the sum of the heights of the larger triangle and the smaller triangle. The height of the larger triangle is
\[
\frac{2(300)}{x + 2y} = \frac{600}{x + 2y}
\]
and the height of the smaller triangle is
\[
\frac{2(200)}{x} = \frac{400}{x}
\]
So, the distance between \( A \) and \( B \) is:

\[
A = \frac{600}{x + 2y} + \frac{400}{x} = \frac{600x + 400(x+2y)}{x(x+2y)} = \frac{600x + 400x + 800y}{x(x+2y)} = \frac{1000x + 800y}{x(x+2y)}
\]

**ANSWER:**
\[
\frac{1000x + 800y}{x(x+2y)}
\]

---

8-2 Adding and Subtracting Rational Expressions

36. **Simplify each expression.**

\[
\frac{2}{x-3} + \frac{3x}{x^2-9} = \frac{2}{x-3} + \frac{3x}{(x-3)(x+3)}
\]

**SOLUTION:**
\[
\begin{align*}
\frac{2}{x-3} + \frac{3x}{x+3} & = \frac{2(x+3)}{(x-3)(x+3)} + \frac{3x}{x+3} \\
& = \frac{2(x+3) + 3x}{(x-3)(x+3)} + \frac{3(x-3) - 4x}{(x-3)(x+3)} \\
& = \frac{2x + 6 + 3x}{3x - 9 - 4x} \\
& = \frac{5x + 6}{-x - 9}
\end{align*}
\]

**ANSWER:**
\[
\frac{5x + 6}{-x - 9}
\]
8-2 Adding and Subtracting Rational Expressions

37. \[
\frac{4}{x+5} + \frac{9}{x-6} - \frac{5}{x-6} - \frac{8}{x+5}
\]

**SOLUTION:**

\[
\begin{align*}
\frac{4}{x+5} + \frac{9}{x-6} - \frac{5}{x-6} & - \frac{8}{x+5} \\
&= \frac{4(x-6) + 9(x+5)}{(x+5)(x-6)} - \frac{5(x+5) - 8(x-6)}{(x-6)(x+5)} \\
&= \frac{4x - 24 + 9x + 45}{5x + 25 - 8x + 48} \\
&= \frac{13x + 21}{-3x + 73}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\frac{13x + 21}{-3x + 73}
\end{align*}
\]

38. \[
\frac{5}{x+6} - \frac{2x}{2x-1} - \frac{2x-1}{x} + \frac{4}{2x-1} - \frac{2x}{x+6}
\]

**SOLUTION:**

\[
\begin{align*}
\frac{5}{x+6} - \frac{2x}{2x-1} & - \frac{2x-1}{x} + \frac{4}{2x-1} - \frac{2x}{x+6} \\
&= \frac{5(2x-1) - 2x(x+6)}{(x+6)(2x-1)} - \frac{x(x+6) + 4(2x-1)}{(2x-1)(x+6)} \\
&= \frac{10x - 5 - 2x^2 - 12x}{x^2 + 6x + 8x - 4} \\
&= \frac{-2x^2 - 2x - 5}{x^2 + 14x - 4}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\frac{-2x^2 - 2x - 5}{x^2 + 14x - 4}
\end{align*}
\]
8-2 Adding and Subtracting Rational Expressions

39. \[
\frac{8}{x-9} - \frac{x}{3x+2} = \frac{8(3x+2) - x(x-9)}{(x-9)(3x+2)} = \frac{24x + 16 - x^2 + 9x}{3x - 27 + 12x^2 + 8x} = \frac{-x^2 + 33x + 16}{12x^2 + 11x - 27}
\]

**SOLUTION:**

\[
\frac{8}{x-9} - \frac{x}{3x+2} = \frac{8(3x+2) - x(x-9)}{(x-9)(3x+2)} = \frac{24x + 16 - x^2 + 9x}{3x - 27 + 12x^2 + 8x} = \frac{-x^2 + 33x + 16}{12x^2 + 11x - 27}
\]

**ANSWER:**

\[
\frac{-x^2 + 33x + 16}{12x^2 + 11x - 27}
\]

40. **OIL PRODUCTION**

Managers of an oil company have estimated that oil will be pumped from a certain well at a rate based on the function

\[R(x) = \frac{20}{x} + \frac{200x}{3x^2 + 20},\]

where \(R(x)\) is the rate of production in thousands of barrels per year \(x\) years after pumping begins.

**a.** Simplify \(R(x)\).

**b.** At what rate will oil be pumping from the well in 50 years?

**SOLUTION:**

**a.**

\[R(x) = \frac{20}{x} + \frac{200x}{3x^2 + 20} = \frac{20(3x^2 + 20) + 200x(x)}{x(3x^2 + 20)} = \frac{60x^2 + 400 + 200x^2}{3x^2 + 20x} = \frac{260x^2 + 400}{3x^2 + 20x} \]

**b.** Substitute \(x = 50\) in \(R(x)\).

\[
R(50) = \frac{260(50)^2 + 400}{3(50)^2 + 20(50)} = \frac{260(2500) + 400}{3(125000) + 1000} = \frac{650000 + 400}{375000 + 1000} = \frac{650400}{376000} \approx 1.73
\]

Therefore, the rate of oil pumping from the well in 50 years is about 1730 barrels per year.

**ANSWER:**

a. \[R(x) = \frac{260x^2 + 400}{3x^2 + 20x} \]

b. about 1730 barrels/yr

Find the LCM of each set of polynomials.

41. \(12xy^4, 14x^4y^2, 5xyz^3, 15x^5y^3\)

**SOLUTION:**

\[
\begin{align*}
12xy^4 &= 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y \\
14x^4y^2 &= 2 \cdot 7 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \\
5xyz^3 &= 5 \cdot x \cdot y \cdot z \cdot z \\
15x^5y^3 &= 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \\
\text{LCM} &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \\
&= 420x^5y^4z^3
\end{align*}
\]

**ANSWER:**

\[420x^5y^4z^3\]
8-2 Adding and Subtracting Rational Expressions

42. $-6abc^2, 18a^2b^2, 15a^4c, 8b^3$

**SOLUTION:**

$-6abc^2 = -2 \cdot 3 \cdot a \cdot b \cdot c \cdot c$

$18a^2b^2 = 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b$

$15a^4c = 3 \cdot 5 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c$

$8b^3 = 2 \cdot 2 \cdot b \cdot b \cdot b$

$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c$

$\text{LCM} = -360a^3b^3c^2$

**ANSWER:**

$-360a^3b^3c^2$

43. $x^2 - 3x - 28, 2x^2 + 9x + 4, x^2 - 16$

**SOLUTION:**

$x^2 - 3x - 28 = (x - 7)(x + 4)$

$2x^2 + 9x + 4 = (x + 4)(2x + 1)$

$x^2 - 16 = (x + 4)(x - 4)$

$\text{LCM} = (x - 7)(x + 4)(x - 4)(2x + 1)$

**ANSWER:**

$(x + 4)(x - 4)(2x + 1)(x - 7)$

44. $x^2 - 5x - 24, x^2 - 9, 3x^2 + 8x - 3$

**SOLUTION:**

$x^2 - 5x - 24 = (x - 8)(x + 3)$

$x^2 - 9 = (x + 3)(x - 3)$

$3x^2 + 8x - 3 = (x + 3)(3x - 1)$

$\text{LCM} = (x - 8)(x + 3)(x - 3)(3x - 1)$

**ANSWER:**

$(x + 3)(x - 3)(x - 8)(3x - 1)$

Simplify each expression.

45. $\frac{1}{12a} + 6 - \frac{3}{5a^2}$

**SOLUTION:**

The LCD is $60a^2$.

$\frac{1}{12a} - 6 - \frac{3}{5a^2} = \frac{1}{12a} \cdot \frac{5a^2}{5a^2} + 6 \cdot \frac{5a^2}{60a^2} - \frac{3}{5a^2} \cdot \frac{12}{12} = \frac{5a + 360a^2}{60a^2} - \frac{360a^2 + 5a - 36}{60a^2}$

**ANSWER:**

$\frac{360a^2 + 5a - 36}{60a^2}$

46. $\frac{5}{16y^2} - 4 - \frac{8}{3x^2y}$

**SOLUTION:**

The LCD is $48x^2y^2$.

$\frac{5}{16y^2} - 4 - \frac{8}{3x^2y}$

$= \frac{5}{16y^2} \cdot \frac{3x^2}{3x^2} - 4 \cdot \frac{48x^2y^2}{48x^2y^2} - \frac{8}{3x^2y} \cdot \frac{16y}{16y}$

$= \frac{15x^2 - 192x^2y^2 - 128y}{48x^2y^2}$

**ANSWER:**

$\frac{15x^2 - 192x^2y^2 - 128y}{48x^2y^2}$
47. \[
\frac{5}{6x^2 + 46x - 16} + \frac{2}{6x^2 + 57x + 72}
\]

**SOLUTION:**
\[
\frac{5}{6x^2 + 46x - 16} + \frac{2}{6x^2 + 57x + 72} = \frac{5}{2(x + 8)(3x - 1)} + \frac{2}{3(2x + 3)(x + 8)}
\]
The LCD is \(6(x + 8)(3x - 1)(2x + 3)\).
\[
= \frac{5(2x + 3) + 2(3x - 1)}{6(x + 8)(3x - 1)(2x + 3)}
\]
\[
= \frac{30x + 45 + 12x - 4}{6(x + 8)(3x - 1)(2x + 3)}
\]
\[
= \frac{42x + 41}{6(x + 8)(3x - 1)(2x + 3)}
\]

**ANSWER:**
\[
\frac{42x + 41}{6(3x - 1)(x + 8)(2x + 3)}
\]

48. \[
\frac{1}{8x^2 - 20x - 12} + \frac{4}{6x^2 + 27x + 12}
\]

**SOLUTION:**
\[
\frac{1}{8x^2 - 20x - 12} + \frac{4}{6x^2 + 27x + 12} = \frac{1}{4(2x + 1)(x - 3)} + \frac{4}{3(x + 4)(2x + 1)}
\]
The LCD is \(12(2x + 1)(x - 3)(x + 4)\).
\[
= \frac{1(3x + 12 + 16x - 48)}{12(2x + 1)(x - 3)(x + 4)}
\]
\[
= \frac{19x - 36}{12(2x + 1)(x - 3)(x + 4)}
\]

**ANSWER:**
\[
\frac{19x - 36}{12(2x + 1)(x - 3)(x + 4)}
\]
49. \[ \frac{x^2 + y^2}{x^2 - y^2} + \frac{y}{x+y} - \frac{x}{x-y} \]

\[ \text{SOLUTION:} \]
\[ \frac{x^2 + y^2}{x^2 - y^2} + \frac{y}{x+y} - \frac{x}{x-y} \]
\[ = \frac{x^2 + y^2 + y(x-y) - x(x+y)}{(x-y)(x+y)} \]
\[ = \frac{x^2 + y^2 + y^2 - x^2 - xy}{(x-y)(x+y)} \]
\[ = 0 \]

\[ \text{ANSWER:} \]
\[ 0 \]

50. \[ \frac{x^2 + x}{x^2 - 9x + 8} + \frac{4}{x-1} - \frac{3}{x-8} \]

\[ \text{SOLUTION:} \]
\[ \frac{x^2 + x}{x^2 - 9x + 8} + \frac{4}{x-1} - \frac{3}{x-8} \]
\[ = \frac{x^2 + x + 4(x-8) - 3(x-1)}{(x-8)(x-1)} \]
\[ = \frac{x^2 + x + 4x - 32 - 3x + 3}{(x-8)(x-1)} \]
\[ = \frac{x^2 + 2x - 29}{x^2 - 9x + 8} \]

\[ \text{ANSWER:} \]
\[ \frac{x^2 + 2x - 29}{x^2 - 9x + 8} \]
53. **GEOMETRY** An expression for the length of one rectangle is \( \frac{x^2 - 9}{x - 2} \). The length of a similar rectangle is expressed as \( \frac{x + 3}{x^2 - 4} \). What is the scale factor of the two rectangles? Write in simplest form.

**SOLUTION:**
Divide the expressions to find the scale factor.

\[
\text{The scale factor of the two triangles} = \frac{\frac{x^2 - 9}{x - 2}}{\frac{x + 3}{x^2 - 4}} = \frac{(x+2)(x-3)}{(x+2)(x-2)} = \frac{x-3}{x+2}
\]

That is, the scale factor is \( (x - 3)(x + 2) \) to 1.

**ANSWER:**
\((x - 3)(x + 2)\) to 1
### 8-2 Adding and Subtracting Rational Expressions

54. **CCSS MODELING** Cameron is taking a 20-mile kayaking trip. He travels half the distance at one rate. The rest of the distance he travels 2 miles per hour slower.

   **a.** If \( x \) represents the faster pace in miles per hour, write an expression that represents the time spent at that pace.

   **b.** Write an expression for the amount of time spent at the slower pace.

   **c.** Write an expression for the amount of time Cameron needed to complete the trip.

**SOLUTION:**

**a.** \( \frac{10}{x} \)

**b.** \( \frac{10}{x - 2} \)

**c.**

\[
\frac{10}{x} + \frac{10}{x - 2} = \frac{10(x - 2) + 10x}{x(x - 2)}
= \frac{10x - 20 + 10x}{x(x - 2)}
= \frac{20x - 20}{x(x - 2)}
= \frac{20(x - 1)}{x(x - 2)}
\]

**ANSWER:**

**a.** \( \frac{10}{x} \)

**b.** \( \frac{10}{x - 2} \)

**c.** \( \frac{20(x - 1)}{x(x - 2)} \)

---

55. **Find the slope of the line that passes through each pair of points.**

**Problem:**

Find the slope of the line \( AB \) is:

\[
m = \frac{\frac{3}{p} - 1}{\frac{1}{2} - \frac{3}{p}}
= \frac{\frac{6 - p}{2p}}{\frac{p - 6}{3p}}
= \frac{3}{2}
\]

**ANSWER:**

\( \frac{3}{2} \)
8-2 Adding and Subtracting Rational Expressions

56. \( C\left(\frac{1}{4}, \frac{4}{q}\right) \text{ and } D\left(\frac{5}{q}, \frac{1}{5}\right) \)

**SOLUTION:**
The slope of the line \(CD\) is:

\[
m = \frac{\frac{1}{q} - \frac{4}{q}}{\frac{5}{q} - \frac{1}{5}} = \frac{20 - q}{5q} - \frac{4q}{5q} = -\frac{4}{5}
\]

**ANSWER:**
\[-\frac{4}{5}\]

57. \( E\left(\frac{7}{w}, \frac{1}{7}\right) \text{ and } F\left(\frac{1}{7}, \frac{7}{w}\right) \)

**SOLUTION:**
The slope of the line \(EF\) is:

\[
m = \frac{\frac{1}{7} - \frac{7}{w}}{\frac{1}{7} - \frac{1}{7}} = \frac{49 - w}{7w} - \frac{-w}{7w} = -1
\]

**ANSWER:**
\[-1\]

58. \( G\left(\frac{6}{n}, \frac{1}{6}\right) \text{ and } H\left(\frac{1}{6}, \frac{6}{n}\right) \)

**SOLUTION:**
The slope of the line \(GH\) is:

\[
m = \frac{\frac{6}{n} - \frac{1}{6}}{\frac{1}{6} - \frac{6}{n}} = \frac{36 - n}{6n} - \frac{-n - 36}{6n} = -1
\]

**ANSWER:**
\[-1\]
59. **PHOTOGRAPHY** The focal length of a lens establishes the field of view of the camera. The shorter the focal length is, the larger the field of view. For a camera with a fixed focal length of 70 mm to focus on an object $x$ mm from the lens, the film must be placed a distance $y$ from the lens. This is represented by \[
\frac{1}{x} + \frac{1}{y} = \frac{1}{70}.
\]

**a.** Express $y$ as a function of $x$.

**b.** What happens to the focusing distance when the object is 70 mm away?

**SOLUTION:**

\[
\begin{align*}
\frac{1}{x} + \frac{1}{y} &= \frac{1}{70} \\
\frac{1}{y} &= \frac{1}{70} - \frac{1}{x} \\
y &= \frac{70x}{x - 70}
\end{align*}
\]

**b.** Sample answer: When the object is 70 mm away, $y$ needs to be 0, which is impossible.

**ANSWER:**

\[
\begin{align*}
a. \quad y &= \frac{70x}{x - 70}
\end{align*}
\]

**b.** Sample answer: When the object is 70 mm away, $y$ needs to be 0, which is impossible.

60. **PHARMACOLOGY** Two drugs are administered to a patient. The concentrations in the bloodstream of each are given by 

\[
f(t) = \frac{2t}{3t^2 + 9t + 6} \quad \text{and} \quad g(t) = \frac{3t}{2t^2 + 6t + 4}
\]

where $t$ is the time, in hours, after the drugs are administered.

**a.** Add the two functions together to determine a function for the total concentration of drugs in the patient’s bloodstream.

**b.** What is the concentration of drugs after 8 hours?

**SOLUTION:**

**a.** The total concentration of drugs in the patient’s bloodstream is:

\[
h(t) = \frac{2t}{3t^2 + 9t + 6} + \frac{3t}{2t^2 + 6t + 4}
\]

\[
= \frac{2t}{3(t^2 + 3t + 2)} + \frac{3t}{2(t^2 + 3t + 2)}
\]

\[
= \frac{4t + 9t}{6(t^2 + 3t + 2)}
\]

\[
= \frac{13t}{6t^2 + 18t + 12}
\]

**b.** Substitute $t = 8$ in the expression for $h(t)$. The concentration of drugs after 8 hours is:

\[
h(8) = \frac{13(8)}{6(8)^2 + 18(8) + 12}
\]

\[
= 0.19
\]

**ANSWER:**

**a.** $h(t) = \frac{13t}{6t^2 + 18t + 12}$

**b.** About 0.19

61. **DOPPLER EFFECT** Refer to the application at the beginning of the lesson. George is equidistant from two fire engines traveling toward him from opposite directions.
### 8-2 Adding and Subtracting Rational Expressions

**a.** Let \( x \) be the speed of the faster fire engine and \( y \) be the speed of the slower fire engine. Write and simplify a rational expression representing the difference in pitch between the two sirens according to George.

**b.** If one is traveling at 45 meters per second and the other is traveling at 70 meters per second, what is the difference in their pitches according to George? The speed of sound in air is 332 meters per second, and both engines have a siren with a pitch of 500 Hz.

**SOLUTION:**

**a.** Doppler effect of the faster fire engine is represented by the rational expression \( \frac{P_0 S_0}{(S_0 - x)} \).

Doppler effect of the slower fire engine is represented by the rational expression \( \frac{P_0 S_0}{(S_0 - y)} \).

\[
\frac{P_0 S_0}{(S_0 - x)} - \frac{P_0 S_0}{(S_0 - y)} = \frac{P_0 S_0 (S_0 - y) - P_0 S_0 (S_0 - x)}{(S_0 - x)(S_0 - y)}
\]

\[
= \frac{P_0 S_0 [S_0 - y - S_0 + x]}{(S_0 - x)(S_0 - y)}
\]

\[
= \frac{P_0 S_0 (x - y)}{(S_0 - x)(S_0 - y)}
\]

\[
= \frac{P_0 S_0 x - P_0 S_0 y}{(S_0 - x)(S_0 - y)}
\]

**b.** Substitute \( x = 70 \), \( y = 45 \), \( S_0 = 332 \), and \( P_0 = 500 \) in the expression \( \frac{P_0 S_0 x - P_0 S_0 y}{(S_0 - x)(S_0 - y)} \).

\[
\frac{P_0 S_0 x - P_0 S_0 y}{(S_0 - x)(S_0 - y)} = \frac{500(332)(70) - 500(332)(45)}{(332 - 70)(332 - 45)}
\]

\[
= \frac{11620000 - 7470000}{(262)(287)}
\]

\[
= \frac{4150000}{75194}
\]

\[
\approx 55.2
\]

According to George, the difference in their pitches is about 55.2 Hz.

**ANSWER:**

8. If the object is moving at a constant speed \( v \) and the observer is moving towards the object at a constant speed \( u \), the Doppler effect for the frequency of sound is given by the formula:

\[
\begin{align*}
\text{Frequency observed} &= \text{Frequency emitted} 
\times \frac{v + u}{v - u}, \quad \text{if } v + u > v - u \\
&= \text{Frequency emitted} \times \frac{v - u}{v + u}, \quad \text{if } v - u > v + u.
\end{align*}
\]

### 62. RESEARCH A student studying learning behavior performed an experiment in which a rat was repeatedly sent through a maze. It was determined that the time it took the rat to complete the maze followed the rational function \( T(x) = 4 + \frac{10}{x} \), where \( x \) represented the number of trials.

**a.** What is the domain of the function?

**b.** Graph the function for \( 0 \leq x \leq 10 \).

**c.** Make a table of the function for \( x = 20, 50, 100, 200, \) and 400.

**d.** If it were possible to have an infinite number of trials, what do you think would be the rat’s best time? Explain your reasoning.

**SOLUTION:**

**a.** Domain: \( \{x \mid x \neq 0\} \)

**b.**

![Graph of T(x)](image)

**c.**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( T(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.5</td>
</tr>
<tr>
<td>50</td>
<td>4.2</td>
</tr>
<tr>
<td>100</td>
<td>4.1</td>
</tr>
<tr>
<td>200</td>
<td>4.05</td>
</tr>
<tr>
<td>400</td>
<td>4.025</td>
</tr>
</tbody>
</table>

**d.** Sample answer: 4; The fraction approaches 0 as \( x \) approaches infinity; \( 4 + 0 = 4 \).
8-2 Adding and Subtracting Rational Expressions

ANSWER:
a. \( x \neq 0 \)

b. 

![Graph](image)

c. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( T(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
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</tr>
<tr>
<td>200</td>
<td>4.05</td>
</tr>
<tr>
<td>400</td>
<td>4.025</td>
</tr>
</tbody>
</table>

d. Sample answer: 4; The fraction approaches 0 as \( x \) approaches infinity; \( 4 + 0 = 4 \).

63. CHALLENGE Simplify \( \frac{5x^2 - \frac{x+1}{x}}{\frac{4}{3-x^{-1}} + 6x^{-1}} \).

SOLUTION:

\[
\frac{5x^2 - \frac{x+1}{x}}{\frac{4}{3-x^{-1}} + 6x^{-1}} = \frac{5}{\frac{x^2}{x} - \frac{x}{x}} = \frac{5}{\frac{4}{3-x^{-1}} + 6x^{-1}} \frac{4}{\frac{3-x^{-1}}{x} + 6x^{-1}}
\]

\[
= \frac{5 - x^2 - x}{4x(x) + 6(3x - 1)}
\]

\[
= \frac{(5 - x^2 - x)(3x - 1)}{x(4x^2 + 18x - 6)}
\]

\[
= \frac{15x - 3x^3 - 3x^2 - 5 + x^2 + x}{4x^3 + 18x^2 - 6x}
\]

\[
= \frac{-3x^3 - 2x^2 + 16x - 5}{4x^3 + 18x^2 - 6x}
\]

ANSWER:

\[
\frac{-3x^3 - 2x^2 + 16x - 5}{4x^3 + 18x^2 - 6x}
\]
64. CCSS ARGUMENTS  The sum of any two rational numbers is always a rational number. So, the set of rational numbers is said to be closed under addition. Determine whether the set of rational expressions is closed under addition, subtraction, multiplication, and division by a nonzero rational expression. Justify your reasoning.

**SOLUTION:**
Sample answer: The set of rational expressions is closed under all of these operations because the sum, difference, product, and quotient of two rational expressions is a rational expression.

**ANSWER:**
Sample answer: The set of rational expressions is closed under all of these operations because the sum, difference, product, and quotient of two rational expressions is a rational expression.

65. OPEN ENDED Write three monomials with an LCM of 180 $a^4 b^6 c$.

**SOLUTION:**
Sample answer: $20a^4 b^2 c$, $15ab^6$, $9abc$

**ANSWER:**
Sample answer: $20a^4 b^2 c$, $15ab^6$, $9abc$

66. WRITING IN MATH Write a how-to manual for adding rational expressions that have unlike denominators.

**SOLUTION:**
Sample answer: First, factor the denominators of all of the expressions. Find the LCD of the denominators. Convert each expression so they all have the LCD. Add or subtract the numerators. Then simplify. It is the same.

**ANSWER:**
Sample answer: First, factor the denominators of all of the expressions. Find the LCD of the denominators. Convert each expression so they all have the LCD. Add or subtract the numerators. Then simplify. It is the same.
67. **PROBABILITY** A drawing is to be held to select the winner of a new bike. There are 100 seniors, 150 juniors, and 200 sophomores who had correct entries. The drawing will contain 3 tickets for each senior name, 2 for each junior, and 1 for each sophomore. What is the probability that a senior’s ticket will be chosen?

A  \( \frac{1}{8} \)  

B  \( \frac{2}{9} \)  

C  \( \frac{2}{7} \)  

D  \( \frac{3}{8} \)

**SOLUTION:**
Number of senior tickets = 100(3) or 300.
Number of junior tickets = 150(2) or 300.
Number of sophomore tickets = 200(1) or 200.

\[
P(\text{senior ticket}) = \frac{300}{300 + 300 + 200} = \frac{300}{800} = \frac{3}{8}
\]

So, the correct choice is D.

**ANSWER:**
D

68. **SHORT RESPONSE** Find the area of the figure.

![Figure](attachment:figure.png)

**SOLUTION:**
\[
A = \text{Area of the triangle} + \text{Area of the semi circle}
\]

\[
= \frac{1}{2} \times 8 \times 6 + \frac{1}{2} \pi \left( \frac{\sqrt{8^2 + 6^2}}{2} \right)^2
\]

\[
= 24 + \frac{1}{2} \pi (25)
\]

\[
= 24 + 12.5 \pi \text{ cm}^2
\]

**ANSWER:**
\( 24 + 12.5 \pi \text{ cm}^2 \)
69. SAT/ACT If Mauricio receives $b$ books in addition to the number of books he had, he will have $t$ times as many as he had originally. In terms of $b$ and $t$, how many books did Mauricio have at the beginning?

\[
\begin{align*}
F & \quad \frac{b}{t-1} \\
G & \quad \frac{b}{t+1} \\
H & \quad \frac{t+1}{b} \\
J & \quad \frac{b}{t} \\
K & \quad \frac{t}{b}
\end{align*}
\]

**SOLUTION:**
Let $x$ be the number of books he had originally.

Therefore, $x + b = tx$.

\[
\begin{align*}
tx - x &= b \\
x &= \frac{b}{t-1}
\end{align*}
\]

The correct choice is F.

**ANSWER:**
F

70. If \( \frac{2a}{a} + \frac{1}{a} = 4 \), then $a = \_\_\_$. 

\[
\begin{align*}
A & \quad \frac{1}{8} \\
B & \quad \frac{1}{2} \\
C & \quad \frac{1}{8} \\
D & \quad 2
\end{align*}
\]

**SOLUTION:**
\[
\begin{align*}
\frac{2a}{a} + \frac{1}{a} &= 4 \\
2 + \frac{1}{a} &= 4 \\
\frac{1}{a} &= 2 \\
a &= \frac{1}{2}
\end{align*}
\]

The correct choice is B.

**ANSWER:**
B
Simplify each expression.

71. \( \frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2} \)

**SOLUTION:**
\[ \frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2} = -4bc \]

**ANSWER:**
\[ \frac{-4bc}{33a} \]

72. \( \frac{x^2 - y^2}{6y} + \frac{x + y}{36y^2} \)

**SOLUTION:**
Flip the second expression and multiply.
\[ \frac{x^2 - y^2}{6y} + \frac{x + y}{36y^2} = \frac{x^2 - y^2}{6y} \cdot \frac{36y^2}{x + y} \]
\[ = \frac{(x+y)(x-y)}{6y} \cdot \frac{36y^2}{x + y} \]
\[ = 6y(x - y) \]

**ANSWER:**
\[ 6y(x - y) \]

73. \( \frac{n^2 - n - 12}{n+2} \div \frac{n-4}{n^2 - 4n - 12} \)

**SOLUTION:**
Flip the second expression and multiply.
\[ \frac{n^2 - n - 12}{n+2} \div \frac{n-4}{n^2 - 4n - 12} = \frac{n^2 - n - 12}{n+2} \cdot \frac{n^2 - 4n - 12}{n-4} \]
\[ = \frac{(n-4)(n+3)}{n+2} \cdot \frac{(n-6)(n+2)}{n-4} \]
\[ = (n + 3)(n - 6) \]

**ANSWER:**
\[ (n + 3)(n - 6) \]
8-2 Adding and Subtracting Rational Expressions

74. **BIOLOGY** Bacteria usually reproduce by a process known as *binary fission*. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal conditions, some bacteria reproduce every 20 minutes.

a. Find the constant *k* for this type of bacterium under ideal conditions.

b. Write the equation for modeling the exponential growth of this bacterium.

**SOLUTION:**

a. Substitute 2 for *y*, 1 for *a*, 20 for *t* in the equation

\[ y = ae^{kt} \]

\[ 2 = 1 \left( e^{k(20)} \right) \]

\[ \ln 2 = \ln \left( e^{20k} \right) \]

\[ \ln 2 = 20k \]

\[ k = \frac{\ln 2}{20} \]

\[ k \approx 0.0347 \]

b. Substitute *k* = 0.0347 in the equation

\[ y = ae^{kt} \]

\[ y = ae^{0.0347t} \]

**ANSWER:**

a. about 0.0347

b. \[ y = ae^{0.0347t} \]

Graph each function. State the domain and range of each function.

75. \[ y = -\sqrt{2x + 1} \]

**SOLUTION:**

\[ D = \{ x \mid x \geq -0.5 \} \]

\[ R = \{ y \mid y \leq 0 \} \]

**ANSWER:**

\[ D = \{ x \mid x \geq -0.5 \} \]

\[ R = \{ y \mid y \leq 0 \} \]
76. \( y = \sqrt{5x - 3} \)

**SOLUTION:**
\[ D = \{ x \mid x \geq 0.6 \}, \quad R = \{ y \mid y \geq 0 \} \]

**ANSWER:**
\[ D = \{ x \mid x \geq 0.6 \}, \quad R = \{ y \mid y \geq 0 \} \]

77. \( y = \sqrt{x + 6 - 3} \)

**SOLUTION:**
\[ D = \{ x \mid x \geq -6 \}, \quad R = \{ y \mid y \geq -3 \} \]

**ANSWER:**
\[ D = \{ x \mid x \geq -6 \}, \quad R = \{ y \mid y \geq -3 \} \]
70. If \( y = 5 - \sqrt{x + 4} \) then \( a = \boxed{\text{B}} \).

**SOLUTION:**
\[ D = \{ x \mid x \geq -4 \}, \quad R = \{ y \mid y \leq 5 \} \]

**ANSWER:**
\[ D = \{ x \mid x \geq -4 \}, \quad R = \{ y \mid y \leq 5 \} \]
8.2 Adding and Subtracting Rational Expressions

80. \( y = 2\sqrt{3} - 4x + 3 \)

**SOLUTION:**
\[ D = \{ x | x \leq 0.75 \}, \ R = \{ y | y \geq 3 \} \]

**ANSWER:**
\[ D = \{ x | x \leq 0.75 \}, \ R = \{ y | y \geq 3 \} \]

---

81. \( 3x + 8 = 0 \)

**SOLUTION:**
\[ 3x + 8 = 0 \]
\[ 3x = -8 \]
\[ x = -\frac{8}{3} \]

There is only one real root.

**ANSWER:**
\[-\frac{8}{3} : 1 \text{ real}\]

---

82. \( 2x^2 - 5x + 12 = 0 \)

**SOLUTION:**
Use the quadratic formula.
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(12)}}{2(2)} \]
\[ x = \frac{5 \pm \sqrt{25 - 96}}{4} \]
\[ x = \frac{5 \pm \sqrt{-71}}{4} \]

There are two imaginary roots.

**ANSWER:**
\[ \frac{5 \pm i\sqrt{71}}{4} : 2 \text{ imaginary} \]
8-2 Adding and Subtracting Rational Expressions

83. \(x^3 + 9x = 0\)

\[SOLUTION:\]
\[x(x^2 + 9) = 0\]
\[x = 0 \quad \text{or} \quad x^2 = -9\]
\[x = 0 \quad \text{or} \quad x = \pm 3i\]

There is one real root and two imaginary roots.

\[ANSWER:\]
0, 3i, −3i; 1 real, 2 imaginary

84. \(x^4 - 81 = 0\)

\[SOLUTION:\]
\[(x^2 + 9)(x^2 - 9) = 0\]
\[x^2 + 9 = 0 \quad \text{or} \quad x^2 - 9 = 0\]
\[x = \pm 3i \quad \text{or} \quad x = \pm 3\]

There are two real roots and two imaginary roots.

\[ANSWER:\]
3, −3, 3i, and −3i; 2 real, 2 imaginary

Graph each function.

85. \(y = 4(x + 3)^2 + 1\)

\[SOLUTION:\]

\[ANSWER:\]
8-2 Adding and Subtracting Rational Expressions

86. \( y = -(x - 5)^2 - 3 \)

**SOLUTION:**

\[ y = -(x - 5)^2 - 3 \]

**ANSWER:**

\[ y = -(x - 5)^2 - 3 \]

87. \( y = \frac{1}{4}(x - 2)^2 + 4 \)

**SOLUTION:**

\[ y = \frac{1}{4}(x - 2)^2 + 4 \]

**ANSWER:**

\[ y = \frac{1}{4}(x - 2)^2 + 4 \]
88. \[ y = \frac{1}{2}(x-3)^2 - 5 \]

**SOLUTION:**

\[ y = \frac{1}{2}(x-3)^2 - 5 \]

**ANSWER:**

\[ y = \frac{1}{2}(x-3)^2 - 5 \]

89. \[ y = x^2 + 6x + 2 \]

**SOLUTION:**

\[ y = x^2 + 6x + 2 \]

**ANSWER:**

\[ y = x^2 + 6x + 2 \]
8-2 Adding and Subtracting Rational Expressions

90. \( y = x^2 - 8x + 18 \)

**SOLUTION:**

\[
\begin{array}{c}
\text{Graph of } y = x^2 - 8x + 18 \\
O \quad x
\end{array}
\]

**ANSWER:**

\[
\begin{array}{c}
\text{Graph of } y = x^2 - 8x + 18 \\
O \quad x
\end{array}
\]
8-3 Graphing Reciprocal Functions

Identify the asymptotes, domain, and range of each function.

1. SOLUTION:
The vertical asymptote is

\[ x - 1 = 0 \]
\[ x = 1 \]

The horizontal asymptote is \( f(x) = 0 \).

Domain: \( D = \{ x \mid x \neq 1 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq 0 \} \);

**ANSWER:**
\( x = 1, f(x) = 0; D = \{ x \mid x \neq 1 \}; R = \{ f(x) \mid f(x) \neq 0 \} \)

2. SOLUTION:
Vertical Asymptote:
\[ x + 2 = 0 \]
\[ x = -2 \]

Horizontal Asymptote:
\[ f(x) = \frac{3}{x+2} + 1 \]
\[ = \frac{3 + x + 2}{x + 2} \]
\[ = \frac{x + 5}{x + 2} \]

The degrees of the numerator and the denominator are same. Therefore, the horizontal asymptote is \( f(x) = 1 \).

Domain: \( D = \{ x \mid x \neq -2 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq 1 \} \);

**ANSWER:**
\( x = -2, f(x) = 1; D = \{ x \mid x \neq -2 \}; R = \{ f(x) \mid f(x) \neq 1 \} \)
8-3 Graphing Reciprocal Functions

Graph each function. State the domain and range.

3. \( f(x) = \frac{5}{x} \)

**SOLUTION:**

\[
\begin{align*}
D &= \{x \mid x \neq 0\}; \\
R &= \{f(x) \mid f(x) \neq 0\}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
D &= \{x \mid x \neq 0\}; \\
R &= \{f(x) \mid f(x) \neq 0\}
\end{align*}
\]

4. \( f(x) = \frac{2}{x + 3} \)

**SOLUTION:**

\[
\begin{align*}
D &= \{x \mid x \neq -3\}; \\
R &= \{f(x) \mid f(x) \neq 0\}
\end{align*}
\]
5. \( f(x) = \frac{-1}{x-2} + 4 \)

**SOLUTION:**

\[
\begin{align*}
D &= \{x \mid x \neq 2\}; \quad R = \{f(x) \mid f(x) \neq 4\}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
D &= \{x \mid x \neq 2\}; \quad R = \{y \mid y \neq 4\}
\end{align*}
\]

6. **CCSS SENSE-MAKING** A group of friends plans to get their youth group leader a gift certificate for a day at a spa. The certificate costs $150.

   a. If \( c \) represents the cost for each friend and \( f \) represents the number of friends, write an equation to represent the cost to each friend as a function of how many friends give.

   b. Graph the function.

   c. Explain any limitations to the range or domain in this situation.

   **SOLUTION:**

   a. \( c = \frac{150}{f} \)
Identify the asymptotes, domain, and range of each function.

7.

\[ f(x) = \frac{5}{x+4} \]

**SOLUTION:**
Asymptotes: 
Vertical asymptote: 
\[ x = -4 \] or \( f(x) = 0 \).

Domain: \( D = \{ x \mid x \neq -4 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq 0 \} \);

**ANSWER:** 
\( x = -4, f(x) = 0; D = \{ x \mid x \neq -4 \}; R = \{ f(x) \mid f(x) \neq 0 \} \)

8.

\[ f(x) = \frac{6}{x-3} \]

**SOLUTION:**
Asymptotes: 
The graph is translated 3 units down. Therefore, the asymptotes are \( x = 0 \) and \( f(x) = -3 \).

Domain: \( D = \{ x \mid x \neq 0 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq -3 \} \);

**ANSWER:** 
\( x = 0, f(x) = -3; D = \{ x \mid x \neq 0 \}; R = \{ f(x) \mid f(x) \neq -3 \} \)

9.

\[ f(x) = \frac{2}{x+6} - 2 \]

**SOLUTION:**
Asymptotes: 
The graph is translated 6 units left and 2 units down. Therefore, the asymptotes are \( x = -6 \) and \( f(x) = -2 \).

Domain: \( D = \{ x \mid x \neq -6 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq -2 \} \);

**ANSWER:** 
\( x = -6, f(x) = -2; D = \{ x \mid x \neq -6 \}; R = \{ f(x) \mid f(x) \neq -2 \} \)

10.

\[ f(x) = \frac{-3}{x-1} + 5 \]

**SOLUTION:**
Asymptotes: 
The graph is translated 1 unit right and 5 units up. Therefore, the asymptotes are \( x = 1 \) and \( f(x) = 5 \).

Domain: \( D = \{ x \mid x \neq 1 \} \);
Range: \( R = \{ f(x) \mid f(x) \neq 5 \} \);

**ANSWER:** 
\( x = 1, f(x) = 5; D = \{ x \mid x \neq 1 \}; R = \{ f(x) \mid f(x) \neq 5 \} \)
8-3 Graphing Reciprocal Functions

Graph each function. State the domain and range.

11. \( f(x) = \frac{3}{x} \)

**SOLUTION:**

\[ f(x) = \frac{3}{x} \]

D = \{x | x ≠ 0\}; R = \{f(x) | f(x) ≠ 0\}

**ANSWER:**

\[ f(x) = \frac{3}{x} \]

D = \{x | x ≠ 0\}; R = \{f(x) | f(x) ≠ 0\}

12. \( f(x) = \frac{-4}{x+2} \)

**SOLUTION:**

\[ f(x) = \frac{-4}{x+2} \]

D = \{x | x ≠ -2\}; R = \{f(x) | f(x) ≠ 0\}

**ANSWER:**

\[ f(x) = \frac{-4}{x+2} \]

D = \{x | x ≠ -2\}; R = \{f(x) | f(x) ≠ 0\}
13. \( f(x) = \frac{2}{x-6} \)

**SOLUTION:**

\[ \text{D} = \{ x \mid x \neq 6 \}; \text{R} = \{ f(x) \mid f(x) \neq 0 \} \]

**ANSWER:**

\[ \text{D} = \{ x \mid x \neq 6 \}; \text{R} = \{ f(x) \mid f(x) \neq 0 \} \]

14. \( f(x) = \frac{6}{x} - 5 \)

**SOLUTION:**

\[ \text{D} = \{ x \mid x \neq 0 \}; \text{R} = \{ f(x) \mid f(x) \neq -5 \} \]

**ANSWER:**

\[ \text{D} = \{ x \mid x \neq 0 \}; \text{R} = \{ f(x) \mid f(x) \neq -5 \} \]
8-3 Graphing Reciprocal Functions

15. \( f(x) = \frac{2}{x} + 3 \)

**SOLUTION:**

\[ f(x) = \frac{2}{x} + 3 \]

D = \( \{ x \mid x \neq 0 \} \); R = \( \{ f(x) \mid f(x) \neq 3 \} \)

**ANSWER:**

\[ f(x) = \frac{2}{x} + 3 \]

D = \( \{ x \mid x \neq 0 \} \); R = \( \{ f(x) \mid f(x) \neq 3 \} \)

16. \( f(x) = \frac{8}{x} \)

**SOLUTION:**

\[ f(x) = \frac{8}{x} \]

D = \( \{ x \mid x \neq 0 \} \); R = \( \{ f(x) \mid f(x) \neq 0 \} \)

**ANSWER:**

\[ f(x) = \frac{8}{x} \]

D = \( \{ x \mid x \neq 0 \} \); R = \( \{ f(x) \mid f(x) \neq 0 \} \)
17. \( f(x) = \frac{-2}{x - 5} \)

**SOLUTION:**

\[
\begin{array}{c}
\text{Graph:}
\hline
x & f(x) = \frac{-2}{x - 5} \\
\hline
-4 & \text{axis} \\
4 & \text{asymptote} \\
0 & \text{zero} \\
12 & \text{zero} \\
\hline
\end{array}
\]

\( \text{D} = \{x \mid x \neq 5\}; \text{R} = \{f(x) \mid f(x) \neq 0\} \)

**ANSWER:**

\[
\begin{array}{c}
\text{Graph:}
\hline
x & f(x) = \frac{-2}{x - 5} \\
\hline
-4 & \text{axis} \\
4 & \text{asymptote} \\
0 & \text{zero} \\
12 & \text{zero} \\
\hline
\end{array}
\]

\( \text{D} = \{x \mid x \neq 5\}; \text{R} = \{f(x) \mid f(x) \neq 0\} \)

18. \( f(x) = \frac{3}{x - 7} - 8 \)

**SOLUTION:**

\[
\begin{array}{c}
\text{Graph:}
\hline
x & f(x) = \frac{3}{x - 7} - 8 \\
\hline
-12 & \text{asymptote} \\
12 & \text{horizontal asymptote} \\
\hline
\end{array}
\]

\( \text{D} = \{x \mid x \neq 7\}; \text{R} = \{f(x) \mid f(x) \neq -8\} \)

**ANSWER:**

\[
\begin{array}{c}
\text{Graph:}
\hline
x & f(x) = \frac{3}{x - 7} - 8 \\
\hline
-12 & \text{asymptote} \\
12 & \text{horizontal asymptote} \\
\hline
\end{array}
\]

\( \text{D} = \{x \mid x \neq 7\}; \text{R} = \{f(x) \mid f(x) \neq -8\} \)
Identify the asymptotes, domain, and range of each function.

1. SOLUTION:
The vertical asymptote is $x = -3$; Range: $R = \{y | y \neq 6\}$

ANSWER:

20. SOLUTION:
The vertical asymptote is $x = -3$; Range: $R = \{f(x) | f(x) \neq 0\}$

ANSWER:
8-3 Graphing Reciprocal Functions

21. \( f(x) = \frac{-6}{x+4} - 2 \)

**SOLUTION:**

\[ f(x) = \frac{-6}{x+4} - 2 \]

\( D = \{ x \mid x \neq -4 \}; \quad R = \{ f(x) \mid f(x) \neq -2 \} \)

**ANSWER:**

\( D = \{ x \mid x \neq -4 \}; \quad R = \{ f(x) \mid f(x) \neq -2 \} \)

22. \( f(x) = \frac{-5}{x-2} + 2 \)

**SOLUTION:**

\[ f(x) = \frac{-5}{x-2} + 2 \]

\( D = \{ x \mid x \neq 2 \}; \quad R = \{ f(x) \mid f(x) \neq 2 \} \)

**ANSWER:**

\( D = \{ x \mid x \neq 2 \}; \quad R = \{ f(x) \mid f(x) \neq 2 \} \)

23. **CYCLING** Marina’s New Year’s resolution is to ride her bike 5000 miles.

a. If \( m \) represents the mileage Marina rides each day and \( d \) represents the number of days, write an equation to represent the mileage each day as a function of the number of days that she rides.

b. Graph the function.

c. If she rides her bike every day of the year, how many miles should she ride each day to meet her goal?

**SOLUTION:**

a. An equation to represent the mileage each day as a function of the number of days is \( m = \frac{5000}{d} \).
8-3 Graphing Reciprocal Functions

b. 

![Graph](image)

c. Substitute \( d = 365 \) in the equation \( m = \frac{5000}{d} \).

\[
m = \frac{5000}{365} \\
\approx 13.7
\]

Therefore, she should ride about 13.7 miles each day to meet her goal.

**ANSWER:**

a. \( m = \frac{5000}{d} \)

b.  

![Graph](image)

c. 13.7 mi

24. CCSS MODELING Parker has 200 grams of an unknown liquid. Knowing the density will help him discover what type of liquid this is.

a. Density of a liquid is found by dividing the mass by the volume. Write an equation to represent the density of this unknown as a function of volume.

b. Graph the function.

c. From the graph, identify the asymptotes, domain, and range of the function.

**SOLUTION:**

a. \( d = \frac{200}{v} \)

b. 

![Graph](image)

c. Vertical asymptote is at \( v = 0 \). Horizontal asymptote is at \( d = 0 \).

\[
D = \{v \mid v \neq 0\} \\
R = \{v \mid v \neq 0\}
\]

**ANSWER:**

a. \( d = \frac{200}{v} \)

b.  

![Graph](image)

c. \( v = 0, d = 0; D = \{v \mid v \neq 0\}; R = \{d \mid d \neq 0\} \)
25. \( f(x) = \frac{3}{2x - 4} \)

**SOLUTION:**

\[
\begin{align*}
\text{D} &= \{x \mid x \neq 2\}; \\
\text{R} &= \{y \mid y \neq 0\}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\text{D} &= \{x \mid x \neq 2\}; \\
\text{R} &= \{y \mid y \neq 0\}
\end{align*}
\]

26. \( f(x) = \frac{5}{3x} \)

**SOLUTION:**

\[
\begin{align*}
\text{D} &= \{x \mid x \neq 0\}; \\
\text{R} &= \{y \mid y \neq 0\}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\text{D} &= \{x \mid x \neq 0\}; \\
\text{R} &= \{y \mid y \neq 0\}
\end{align*}
\]
27. \( f(x) = \frac{2}{4x + 1} \)

**SOLUTION:**

\[
D = \left\{ x \mid x \neq -\frac{1}{4} \right\}; \quad R = \{ f(x) \mid f(x) \neq 0 \}
\]

**ANSWER:**

\[
D = \left\{ x \mid x \neq -\frac{1}{4} \right\}; \quad R = \{ f(x) \mid f(x) \neq 0 \}
\]

28. \( f(x) = \frac{1}{2x + 3} \)

**SOLUTION:**

\[
D = \left\{ x \mid x \neq -\frac{3}{2} \right\}; \quad R = \{ f(x) \mid f(x) \neq 0 \}
\]

**ANSWER:**

\[
D = \left\{ x \mid x \neq -\frac{3}{2} \right\}; \quad R = \{ f(x) \mid f(x) \neq 0 \}
\]

29. **BASEBALL** The distance from the pitcher’s mound to home plate is 60.5 feet.

   a. If \( r \) represents the speed of the pitch and \( t \) represents the time it takes the ball to get to the plate, write an equation to represent the speed as a function of time.

   b. Graph the function.

   c. If a two-seam fastball reaches the plate in 0.48 second, what was its speed?

**SOLUTION:**

   a. The equation to represent the speed as a function
8-3 Graphing Reciprocal Functions

Graph each function. State the domain and range, and identify the asymptotes.

30. \( f(x) = \frac{-3}{x+7} - 1 \)

**SOLUTION:**
Domain: \( D = \{x \mid x \neq -7\} \);
Range: \( R = \{f(x) \mid f(x) \neq -1\} \);
The graph is translated 7 units left and 1 unit down. Therefore, the asymptotes are \( x = -7 \) and \( f(x) = -1 \).

ANSWER:
\( D = \{x \mid x \neq -7\} \); \( R = \{f(x) \mid f(x) \neq -1\} \); \( x = -7, f(x) = -1 \)
8-3 Graphing Reciprocal Functions

31. \( f(x) = \frac{-4}{x+2} - 5 \)

**SOLUTION:**
Domain: \( D = \{ x \mid x \neq -2 \} \); 
Range: \( R = \{ f(x) \mid f(x) \neq -5 \} \);

The graph is translated 2 units left and 5 units down. Therefore, the asymptotes are \( x = -2 \) and \( f(x) = -5 \).

**ANSWER:**
\( D = \{ x \mid x \neq -2 \} \); \( R = \{ f(x) \mid f(x) \neq -5 \} \); \( x = -2, f(x) = -5 \)

32. \( f(x) = \frac{6}{x-1} + 2 \)

**SOLUTION:**
Domain: \( D = \{ x \mid x \neq 1 \} \); 
Range: \( R = \{ f(x) \mid f(x) \neq 2 \} \);

The graph is translated 1 unit right and 2 units up. Therefore, the asymptotes are \( x = 1 \) and \( f(x) = 2 \).

**ANSWER:**
\( D = \{ x \mid x \neq 1 \} \); \( R = \{ f(x) \mid f(x) \neq 2 \} \); \( x = 1, f(x) = 2 \)
8-3 Graphing Reciprocal Functions

33. \( f(x) = \frac{2}{x-4} + 3 \)

**SOLUTION:**
Domain: \( D = \{x \mid x \neq 4\} \);
Range: \( R = \{f(x) \mid f(x) \neq 3\} \);

The graph is translated 4 units right and 3 units up. Therefore, the asymptotes are \( x = 4 \) and \( f(x) = 3 \).

**ANSWER:**
\( D = \{x \mid x \neq 4\} \);
\( R = \{f(x) \mid f(x) \neq 3\} \);
\( x = 4, f(x) = 3 \)

34. \( f(x) = \frac{-7}{x-8} - 9 \)

**SOLUTION:**
Domain: \( D = \{x \mid x \neq 8\} \);
Range: \( R = \{f(x) \mid f(x) \neq -9\} \);

The graph is translated 8 units right and 9 units down. Therefore, the asymptotes are \( x = 8 \) and \( f(x) = -9 \).

**ANSWER:**
\( D = \{x \mid x \neq 8\} \);
\( R = \{f(x) \mid f(x) \neq -9\} \);
\( x = 8, f(x) = -9 \)
35. \( f(x) = \frac{-6}{x-7} - 8 \)

**SOLUTION:**

Domain: \( D = \{x \mid x \neq 7\}; \)

Range: \( R = \{f(x) \mid f(x) \neq -8\}; \)

The graph is translated 7 units right and 8 units down. Therefore, the asymptotes are \( x = 7 \) and \( f(x) = -8. \)

**ANSWER:**

\( D = \{x \mid x \neq 7\}; \) \( R = \{f(x) \mid f(x) \neq -8\}; \) \( x = 7, f(x) = -8 \)

36. **AUTOMOBILES** Lawanda’s car went 440 miles on one tank of gas.

a. If \( g \) represents the number of miles to the gallon that the car gets and \( t \) represents the size of the gas tank, write an equation to represent the miles to the gallon as a function of tank size.

b. Graph the function.

c. How many miles does the car get per gallon if it has a 15-gallon tank?

**SOLUTION:**

a. The equation to represent the miles to the gallon as a function of tank size is \( g = \frac{440}{t}. \)

b. 

\[
\begin{align*}
g &= \frac{440}{t} \\
\frac{440}{15} &= 29 \frac{1}{3} \text{ mi/gal}
\end{align*}
\]

**ANSWER:**

\( g = \frac{440}{t} \)

b. 

\[
\begin{align*}
g &= \frac{440}{t} \\
\frac{440}{15} &= 29 \frac{1}{3} \text{ mi/gal}
\end{align*}
\]
8-3 Graphing Reciprocal Functions

37. MULTIPLE REPRESENTATIONS Consider the functions \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{1}{x^2} \).

a. TABULAR Make a table of values comparing the two functions.

b. GRAPHICAL Use the table of values to graph both functions.

c. VERBAL Compare and contrast the two graphs.

d. ANALYTICAL Make a conjecture about the difference between the graphs of functions of the form \( f(x) = \frac{1}{x^n} \) with an even exponent in the denominator and those with an odd exponent in the denominator.

SOLUTION:

\[ f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = \frac{1}{x^2} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-\frac{1}{3}</td>
<td>-\frac{1}{9}</td>
</tr>
<tr>
<td>-2</td>
<td>-\frac{1}{2}</td>
<td>-\frac{1}{4}</td>
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<td>-1</td>
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</tr>
<tr>
<td>3</td>
<td>\frac{1}{3}</td>
<td>\frac{1}{9}</td>
</tr>
</tbody>
</table>

b. Graph the functions with the given information.

c. The positive portion of \( g(x) = \frac{1}{x^2} \) is similar to the graph of \( f(x) = \frac{1}{x} \). Positive values of \( x \) produce positive values of \( f(x) \). The negative portion of \( g(x) = \frac{1}{x^2} \) appears to be a reflection of \( f(x) = \frac{1}{x} \) over the x-axis. Negative values of \( x \) produce positive values of \( g(x) \).

d. Sample answer: When \( n \) is even, the graph will show symmetry with respect to the y-axis. When the \( n \) is odd, the graph will show symmetry with respect to the origin.

**ANSWER:**

<table>
<thead>
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</table>
c. The positive portion of \( g(x) = \frac{1}{x^2} \) is similar to the graph of \( f(x) = \frac{1}{x} \). Positive values of \( x \) produce positive values of \( f(x) \). The negative portion of \( g(x) = \frac{1}{x^2} \) appears to be a reflection of \( f(x) = \frac{1}{x} \over \text{over the} x\text{-axis}. Negative values of \( x \) produce positive values of \( g(x) \).

d. Sample answer: When \( n \) is even, the graph will show symmetry with respect to the \( y\)-axis. When the \( n \) is odd, the graph will show symmetry with respect to the origin.

38. OPEN ENDED Write a reciprocal function for which the graph has a vertical asymptote at \( x = -4 \) and a horizontal asymptote at \( f(x) = 6 \).

**SOLUTION:**
Sample answer: \( f(x) = \frac{1}{x + 4} + 6 \)

**ANSWER:**
Sample answer: \( f(x) = \frac{1}{x + 4} + 6 \)

39. REASONING Compare and contrast the graphs of each pair of equations.

a. \( y = \frac{1}{x} \) and \( y - 7 = \frac{1}{x} \)

b. \( y = \frac{1}{x} \) and \( y = 4 \left( \frac{1}{x} \right) \)

c. \( y = \frac{1}{x} \) and \( y = \frac{1}{x + 5} \)

d. Without making a table of values, use what you observed in parts a-c to sketch a graph of \( y - 7 = 4 \left( \frac{1}{x + 5} \right) \).

**ANSWER:**
Sample answer: \( f(x) = \frac{1}{x + 4} + 6 \)

**SOLUTION:**

a. The first graph has a vertical asymptote at \( x = 0 \) and a horizontal asymptote at \( y = 0 \). The second graph is translated 7 units up and has a vertical asymptote at \( x = 0 \) and a horizontal asymptote at \( y = 7 \).

b. Both graphs have a vertical asymptote at \( x = 0 \) and a horizontal asymptote at \( y = 0 \). The second graph is stretched by a factor of 4.

c. The first graph has a vertical asymptote at \( x = 0 \) and a horizontal asymptote at \( y = 0 \). The second graph is translated 5 units to the left and has a vertical asymptote at \( x = -5 \) and a horizontal asymptote at \( y = 0 \).

d.
40. CCSS ARGUMENTS Find the function that does not belong. Explain.

\[
\begin{align*}
\quad f(x) &= \frac{3}{x + 1} \\
\quad g(x) &= \frac{x + 2}{x^2 + 1} \\
\quad h(x) &= \frac{5}{x^2 + 2x + 1} \\
\quad j(x) &= \frac{20}{x - 7}
\end{align*}
\]

**SOLUTION:**
\[g(x) = \frac{x + 2}{x^2 + 1}, \text{all other choices have unknowns only in the denominator.}\]

**ANSWER:**
\[g(x), \text{all other choices have unknowns only in the denominator.}\]

41. CHALLENGE Write two different reciprocal functions with graphs having the same vertical and horizontal asymptotes. Then graph the functions.

**SOLUTION:**
Sample answer: \( f(x) = \frac{2}{x - 3} + 4 \) and \( g(x) = \frac{5}{x - 3} + 4 \).

**ANSWER:**
Sample answer: \( f(x) = \frac{2}{x - 3} + 4 \) and \( g(x) = \frac{5}{x - 3} + 4 \).
42. **WRITING IN MATH** Refer to the beginning of the lesson. Explain how rational functions can be used in fundraising. Explain why only part of the graph is meaningful in the context of the problem.

**SOLUTION:**
A rational function can be used to determine how much each person needs to sell if the cost of the trip and the number of people going on the trip and selling the candy is \( s \). The number of people selling and the number of bars sold per person must both be non-negative they also must both be integers. So only the first quadrant part of the graph is meaningful in the context of the problem.

**ANSWER:**
A rational function can be used to determine how much each person needs to sell if the cost of the trip and the number of people going on the trip and selling the candy is \( s \). The number of people selling and the number of bars sold per person must both be non-negative they also must both be integers. So only the first quadrant part of the graph is meaningful in the context of the problem.

43. **SHORT RESPONSE** What is the value of \((x + y)(x + y)\) if \(xy = -3\) and \(x^2 + y^2 = 10\)?

**SOLUTION:**
\[
(x + y)(x + y) = x^2 + y^2 + 2(xy)
\]
\[
= 10 + 2(-3)
\]
\[
= 4
\]

**ANSWER:**
4

44. **GRIDDED RESPONSE** If \(x = 2y, y = 4z, 2z = w\), and \(w \neq 0\), then \(\frac{x}{w} = \) ___.

**SOLUTION:**
\[
\frac{x}{w} = \frac{2y}{2z} = \frac{4z}{w}
\]

**ANSWER:**
4

45. If \(c = 1 + \frac{1}{d}\) and \(d > 1\), then \(c\) could equal ___.

A \(\frac{5}{7}\)

B \(\frac{9}{7}\)

C \(\frac{15}{7}\)

D \(\frac{19}{7}\)

**SOLUTION:**
When \(d > 1\), \(c \approx 1\).

Therefore, the correct choice is B.

**ANSWER:**
B
46. SAT/ACT A car travels $m$ miles at the rate of $t$ miles per hour. How many hours does the trip take?

\[
F \quad \frac{m}{t}
\]

\[
G \quad m - t
\]

\[
H \quad mt
\]

\[
J \quad \frac{t}{m}
\]

\[
K \quad t - m
\]

**SOLUTION:**

\[
\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{m}{t}
\]

The correct choice is $F$.

**ANSWER:**

$F$

47. If $-1 < a < b < 0$, then which of the following has the greatest value?

A. $a - b$

B. $b - a$

C. $a + b$

D. $2b - a$

**SOLUTION:**

The expression $b - a$ has the greatest value. Therefore, the correct choice is $B$.

**ANSWER:**

$B$

48. **BUSINESS** A small corporation decides that 8% of its profits will be divided among its six managers. There are two sales managers and four nonsales managers. Fifty percent will be split equally among all six managers. The other 50% will be split among the four nonsales managers. Let $p$ represent the profits.

a. Write an expression to represent the share of the profits each nonsales manager will receive.

b. Simplify this expression.

c. Write an expression in simplest form to represent the share of the profits each sales manager will receive.

**SOLUTION:**

a. The expression to represent the share of the profits of each nonsales manager is:

\[
\frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4}
\]

b. Simplified expression:

\[
\frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4} = \frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4} = \frac{0.05p}{3}
\]

C. The expression to represent the share of the profits of each sales manager will receive is:

\[
\frac{0.5(0.08p)}{6}
\]

**ANSWER:**

a. $\frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4}$

b. $\frac{0.05p}{3}$

c. $\frac{0.5(0.08p)}{6}$
Simplifying each expression.

49. \( \frac{p^3}{2n} - \frac{p^2}{4n} \)

\[ \begin{align*}
\text{SOLUTION:} \quad & \quad \frac{p^3}{2n} - \frac{p^2}{4n} \\
& = \frac{p^3}{2n} - \frac{p^2}{4n} \\
& = \frac{p^3 - p^2}{4n} \\
& = \frac{p^2(p - 1)}{4n}
\end{align*} \]

\[ \text{ANSWER:} \quad \frac{p^2(p - 1)}{4n} \]

50. \( \frac{m + q}{m^2 + q^2} \)

\[ \begin{align*}
\text{SOLUTION:} \quad & \quad \frac{m + q}{m^2 + q^2} \\
& = \frac{m + q}{m^2 + q^2} \\
& = \frac{m + q}{m^2 + q^2}
\end{align*} \]

\[ \text{ANSWER:} \quad \frac{m + q}{m^2 + q^2} \]

51. \( \frac{x + y}{x + y} \)

\[ \begin{align*}
\text{SOLUTION:} \quad & \quad \frac{x + y}{x + y} \\
& = \frac{x + y}{x + y} \\
& = \frac{2x + y}{2x + y}
\end{align*} \]

\[ \text{ANSWER:} \quad \frac{2x + y}{2x + y} \]
Graph each function. State the domain and range.

52. \( y = 2(3)^x \)

**SOLUTION:**
\( D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \)

![Graph of \( y = 2(3)^x \)](image)

**ANSWER:**
\( D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \)

![Graph of \( y = 2(3)^x \)](image)

53. \( y = 5(2)^x \)

**SOLUTION:**
\( D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \)

![Graph of \( y = 5(2)^x \)](image)

**ANSWER:**
\( D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \)

![Graph of \( y = 5(2)^x \)](image)
54. \( y = 0.5(4)^x \)

**SOLUTION:**
\[ D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \]

**ANSWER:**
\[ D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \]

55. \( y = 4 \left( \frac{1}{3} \right)^x \)

**SOLUTION:**
\[ D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \]

**ANSWER:**
\[ D = \{ x \mid x \text{ is all real numbers} \}, \ R = \{ y \mid y > 0 \} \]
Find \( f + g \)(x), \( f - g \)(x), \( f \cdot g \)(x), and \( \left( \frac{f}{g} \right) \) for each \( f \)(x) and \( g \)(x).

56. \( f(x) = x + 9 \)
   \( g(x) = x - 9 \)

**SOLUTION:**
\[
(f + g)x = f(x) + g(x) \\
= x + 9 + x - 9 \\
= 2x \\
(f - g)x = f(x) - g(x) \\
= x + 9 - x + 9 \\
= 18 \\
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (x + 9)(x - 9) \\
= x^2 - 81 \\
\left( \frac{f}{g} \right) x = \frac{f(x)}{g(x)} \\
= \frac{x^2 + 9}{x - 9}, \quad x \neq 9.
\]

**ANSWER:**
\( (f + g)(x) = 2x; \)
\( (f - g)(x) = 18; \)
\( (f \cdot g)(x) = x^2 - 81; \)
\( \left( \frac{f}{g} \right)(x) = \frac{x + 9}{x - 9}, \quad x \neq 9 \)

57. \( f(x) = 2x - 3 \)
   \( g(x) = 4x + 9 \)

**SOLUTION:**
\[
(f + g)x = f(x) + g(x) \\
= 2x - 3 + 4x + 9 \\
= 6x + 6 \\
(f - g)x = f(x) - g(x) \\
= 2x - 3 - 4x - 9 \\
= -2x - 12 \\
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (2x - 3)(4x + 9) \\
= 8x^2 + 18x - 12x - 27 \\
= 8x^2 + 6x - 27 \\
\left( \frac{f}{g} \right) x = \frac{f(x)}{g(x)} \\
= \frac{2x - 3}{4x + 9}, \quad x \neq -\frac{9}{4}.
\]

**ANSWER:**
\( (f + g)(x) = 6x + 6; \)
\( (f - g)(x) = -2x - 12; \)
\( (f \cdot g)(x) = 8x^2 + 6x - 27; \)
\( \left( \frac{f}{g} \right)(x) = \frac{2x - 3}{4x + 9}, \quad x \neq -\frac{9}{4} \)
8-3 Graphing Reciprocal Functions

58. \( f(x) = 2x^2 \)
   \( g(x) = 8 - x \)

**SOLUTION:**

\[
(f + g)x = f(x) + g(x) \\
= 2x^2 + 8 - x \\
= 2x^2 - x + 8 \\
(f - g)x = f(x) - g(x) \\
= 2x^2 - 8 + x \\
= 2x^2 + x - 8 \\
(f \cdot g)(x) = f(x) \cdot g(x) \\
= (2x^2)(8-x) \\
= 16x^2 - 2x^3 \\
= -2x^3 + 16x^2 \\
\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} \\
= \frac{2x^2}{8-x}, \ x \neq 8.
\]

**ANSWER:**

\( (f + g)(x) = 2x^2 - x + 8; \)
\( (f - g)(x) = 2x^2 + x - 8; \)
\( (f \cdot g)(x) = -2x^3 + 16x^2; \)
\( \left(\frac{f}{g}\right)(x) = \frac{2x^2}{8-x}, \ x \neq 8 \)

59. **GEOMETRY** The width of a rectangular prism is \( w \) centimeters. The height is 2 centimeters less than the width. The length is 4 centimeters more than the width. If the volume of the prism is 8 times the measure of the length, find the dimensions of the prism.

![Diagram of a rectangular prism](image)

**SOLUTION:**

\[ V = \ell wh \]
\[ 8(w+4) = (w+4)(w)(w-2) \]
\[ 8 = w(w-2) \]
\[ 8 = w^2 - 2w \]
\[ w^2 - 2w - 8 = 0 \]
\[ (w-4)(w+2) = 0 \]
\[ w - 4 = 0 \text{ or } w + 2 = 0 \]
\[ w = 4 \text{ or } w = -2 \]

The width of the prism can not be negative. So, the width of the prism is 4 cm. The dimensions of the prism is \( w = 4 \text{ cm}, \ell = 8 \text{ cm}, h = 2 \text{ cm} \).

**ANSWER:**

\( w = 4 \text{ cm}, \ell = 8 \text{ cm}, h = 2 \text{ cm} \)
8-3 Graphing Reciprocal Functions

Graph each polynomial function. Estimate the x-coordinates at which the relative maxima and relative minima occur. State the domain and range for each function.

60. \( f(x) = x^3 + 2x^2 - 3x - 5 \)

**SOLUTION:**

Sample answer = rel. max. at \( x = -2 \), rel. min. at \( x = 0.5 \); \( D = \{ \text{all real numbers} \} \), \( R = \{ \text{all real numbers} \} \)

**ANSWER:**

Sample answer = rel. max. at \( x = -2 \), rel. min. at \( x = 0.5 \); \( D = \{ \text{all real numbers} \} \), \( R = \{ \text{all real numbers} \} \)

61. \( f(x) = x^4 - 8x^2 + 10 \)

**SOLUTION:**

Sample answer = rel. max. at \( x = 0 \), rel. min. at \( x = -2 \) and at \( x = 2 \); \( D = \{ \text{all real numbers} \} \), \( R = \{ f(x) | f(x) \geq -6 \} \)

**ANSWER:**

Sample answer = rel. max. at \( x = 0 \), rel. min. at \( x = -2 \) and at \( x = 2 \); \( D = \{ \text{all real numbers} \} \), \( R = \{ f(x) | f(x) \geq -6 \} \)
8-4 Graphing Rational Functions

Graph each function.

1. \( f(x) = \frac{x^4 - 2}{x^2 - 1} \)

\[ \text{SOLUTION:} \]
\[ x^2 - 1 = 0 \]
\[ (x + 1)(x - 1) = 0 \]
\[ x = -1 \text{ or } x = 1 \]

The vertical asymptotes are at \( x = -1 \) and \( x = 1 \).

\[ \text{ANSWER:} \]

\[ f(x) = \frac{x^4 - 2}{x^2 - 1} \]

\[ \text{ANSWER:} \]

\[ f(x) = \frac{x^4 - 2}{x^2 - 1} \]

2. \( f(x) = \frac{x^3}{x + 2} \)

\[ \text{SOLUTION:} \]
The vertical asymptote is at \( x = -2 \).

\[ \text{ANSWER:} \]

\[ f(x) = \frac{x^3}{x + 2} \]

\[ \text{ANSWER:} \]

3. CCSS REASONING  Eduardo is a kicker for his high school football team. So far this season, he has made 7 out of 11 field goals. He would like to improve his field goal percentage. If he can make \( x \) consecutive field goals, his field goal percentage can be determined using the function \( P(x) = \frac{7 + x}{11 + x} \).

a. Graph the function.

b. What part of the graph is meaningful in the context of this problem?

c. Describe the meaning of the intercept of the vertical axis.

d. What is the equation of the horizontal asymptote? Explain its meaning with respect to Eduardo’s field
8-4 Graphing Rational Functions

Graph each function.

4. \( f(x) = \frac{6x^2 - 3x + 2}{x} \)

**SOLUTION:**

The vertical asymptote is at \( x = 0 \).

\[
\frac{6x - 3}{x} \quad \frac{6x^2 - 3x + 2}{x} \\
\frac{6x^2}{x} - \frac{3x + 2}{x} \\
\frac{-3x + 2}{x} \\
\frac{-3x}{2}
\]

The oblique asymptote is \( f(x) = 6x - 3 \).

**ANSWER:**

- The part in the first quadrant

- It represents his original field goal percentage of 63.6%.

- \( y = 1 \); this represents 100% which he cannot achieve because he has already missed 4 field goals.
5. \( f(x) = \frac{x^2 + 8x + 20}{x + 2} \)

**SOLUTION:**
The vertical asymptote is at \( x = -2 \).

\[
\begin{align*}
&= \frac{x + 6}{x + 2} \frac{x^2 + 8x + 20}{x^2 + 2x} \\
&= \frac{6x + 20}{6x + 12} \\
&= \frac{6}{8}
\end{align*}
\]

The oblique asymptote is \( f(x) = x + 6 \).

![Graph of \( f(x) = \frac{x^2 + 8x + 20}{x + 2} \)]

**ANSWER:**

6. \( f(x) = \frac{x^2 - 4x - 5}{x + 1} \)

**SOLUTION:**

\[
f(x) = \frac{(x + 1)(x - 5)}{x + 1} = x - 5 \quad x \neq -1
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 4x - 5}{x + 1} \) is the graph of \( f(x) = x - 5 \) with a hole at \( x = -1 \).

![Graph of \( f(x) = \frac{x^2 - 4x - 5}{x + 1} \)]

**ANSWER:**
SOLUTION:
\[ f(x) = \frac{x^2 + x - 12}{x + 4} \]

Therefore, the graph of \( f(x) = \frac{x^2 + x - 12}{x + 4} \) is the graph of \( f(x) = x - 3 \) with a hole at \( x = -4 \).

ANSWER:

Graph each function.

8. \( f(x) = \frac{x^4}{6x + 12} \)

SOLUTION:

\[ 6x + 12 = 0 \]
\[ 6x = -12 \]
\[ x = -2 \]

The vertical asymptote is at \( x = -2 \).
9. \( f(x) = \frac{x^3}{8x - 4} \)

**SOLUTION:**

\[
8x - 4 = 0 \\
8x = 4 \\
x = \frac{4}{8} \\
x = \frac{1}{2}
\]

The vertical asymptote is at \( x = \frac{1}{2} \).

**ANSWER:**

\[
\begin{array}{c}
\text{Graph of } f(x) = \frac{x^3}{8x - 4} \\
\end{array}
\]

10. \( f(x) = \frac{x^4 - 16}{x^2 - 1} \)

**SOLUTION:**

\[
x^2 - 1 = 0 \\
x^2 = 1 \\
x = \pm 1
\]

The vertical asymptotes are at \( x = 1 \) and \( x = -1 \).

**ANSWER:**

\[
\begin{array}{c}
\text{Graph of } f(x) = \frac{x^4 - 16}{x^2 - 1} \\
\end{array}
\]
11. \( f(x) = \frac{x^3 + 64}{16x - 24} \)

**SOLUTION:**

\[
16x - 24 = 0 \\
16x = 24 \\
x = \frac{24}{16} \\
x = \frac{3}{2}
\]

The vertical asymptote is at \( x = \frac{3}{2} \).

**ANSWER:**

![Graph of f(x)](image)

12. **SCHOOL SPIRIT** As president of Student Council, Brandy is getting T-shirts made for a pep rally. Each T-shirt costs $9.50, and there is a set-up fee of $75. The student council plans to sell the shirts, but each of the 15 council members will get one for free.

- **a.** Write a function for the average cost of a T-shirt to be sold. Graph the function.

- **b.** What is the average cost if 200 shirts are ordered? if 500 shirts are ordered?

- **c.** How many T-shirts must be ordered to bring the average cost under $9.75?

**SOLUTION:**

**b.** The average cost for 200 shirts is:

\[
c(200) = \frac{9.5(200) + 75}{200 - 15} \\
\approx $10.68
\]

The average cost for 500 shirts is:

\[
c(500) = \frac{9.5(500) + 75}{500 - 15} \\
\approx $9.95
\]

**c.**

\[
9.75 > \frac{9.5t + 75}{t - 15} \\
9.75t - 146.25 > 9.5t + 75 \\
t > 885
\]

The number of T-shirts ordered to bring the average cost under $9.75 is more than 885.

**ANSWER:**

- **a.** \( c(t) = \frac{9.5t - 75}{t - 15} \);
Graph each function.

13. \( f(x) = \frac{x}{x + 2} \)

**SOLUTION:**

\[ x + 2 = 0 \]
\[ x = -2 \]

The vertical asymptote is at \( x = -2 \).

The degrees of the numerator and denominator expressions are same. Therefore, the horizontal asymptote is at \( y = 1 \).

**ANSWER:**
14. \[ f(x) = \frac{5}{(x-1)(x+4)} \]

**SOLUTION:**
\[(x-1)(x+4) = 0 \]
\[x - 1 = 0 \text{ or } x + 4 = 0 \]
\[x = 1 \text{ or } x = -4 \]

The vertical asymptotes are at \(x = 1\) and \(x = -4\).

The horizontal asymptote is at \(y = 0\).

**ANSWER:**

15. \[ f(x) = \frac{4}{(x-2)^2} \]

**SOLUTION:**
\[(x-2)^2 = 0 \]
\[x - 2 = 0 \]
\[x = 2 \]

The vertical asymptote is at \(x = 2\).

The horizontal asymptote is at \(y = 0\).

**ANSWER:**
16. \( f(x) = \frac{x-3}{x+1} \)

**SOLUTION:**
\[ x + 1 = 0 \]
\[ x = -1 \]

The vertical asymptote is at \( x = -1 \).

The horizontal asymptote is at \( y = 1 \).

**ANSWER:**

\[ f(x) = \frac{x-3}{x+1} \]

17. \( f(x) = \frac{1}{(x+4)^2} \)

**SOLUTION:**
\[ (x + 4)^2 = 0 \]
\[ x + 4 = 0 \]
\[ x = -4 \]

The vertical asymptote is at \( x = -4 \).

The horizontal asymptote is at \( y = 0 \).

**ANSWER:**
18. \( f(x) = \frac{2x}{(x+2)(x-5)} \)

**SOLUTION:**
\[
(x+2)(x-5) = 0
\]
\[x + 2 = 0 \text{ or } x - 5 = 0\]
\[x = -2 \text{ or } x = 5\]

The vertical asymptotes are at \( x = -2 \) and \( x = 5 \).

**ANSWER:**

19. \( f(x) = \frac{(x-4)^2}{x+2} \)

**SOLUTION:**
\[
x + 2 = 0
\]
\[x = -2\]

The vertical asymptote is at \( x = -2 \).

20. \( f(x) = \frac{(x+3)^2}{x-5} \)

**SOLUTION:**
\[
x - 5 = 0
\]
\[x = 5\]
The vertical asymptote is at \(x = 5\).

\[
f(x) = \frac{(x+3)^2}{x - 5} = \frac{x^2 + 6x + 9}{x - 5}
\]

\[
x - 5 \cdot x^2 + 6x + 9
\]

\[
\frac{x+11}{x - 5} \cdot x^2 - 5x
\]

\[
11x + 9
\]

\[
\frac{11x - 55}{64}
\]

The oblique asymptote is \(f(x) = x + 11\).

ANSWER:

\[
f(x) = \frac{(x+3)^2}{x - 5}
\]

\[
\begin{array}{c|c}
\hline
x & f(x) \\
\hline
-5 & \infty \\
-4 & 20 \\
-3 & 60 \\
-2 & 140 \\
-1 & 240 \\
0 & 300 \\
1 & 300 \\
2 & 240 \\
3 & 140 \\
4 & 60 \\
5 & 20 \\
6 & \infty \\
\hline
\end{array}
\]

\[
f(x) = x + 11
\]

\[
\begin{array}{c|c}
\hline
x & f(x) \\
\hline
-5 & 6 \\
-4 & 11 \\
-3 & 16 \\
-2 & 21 \\
-1 & 26 \\
0 & 31 \\
1 & 36 \\
2 & 41 \\
3 & 46 \\
4 & 51 \\
5 & 56 \\
6 & \infty \\
\hline
\end{array}
\]

\[
21. \quad f(x) = \frac{x^3 + 1}{x^2 - 4}
\]

SOLUTION:

\[
x^2 - 4 = 0
\]

\[
x^2 = 4
\]

\[
x = \pm 2
\]

The vertical asymptotes are at \(x = 2\) and \(x = -2\).

\[
x^3 - 4 \cdot x^3 + 0x^2 + 0x + 1
\]

\[
x^3 - 4x
\]

\[
4x + 1
\]

The oblique asymptote is \(f(x) = x\).

\[
22. \quad f(x) = \frac{4x^3}{2x^2 + x - 1}
\]
Graph each function.

SOLUTION:

\[ 2x^2 + x - 1 = 0 \]
\[ 2x^2 + 2x - x - 1 = 0 \]
\[ 2x(x+1) - 1(x+1) = 0 \]
\[ (x+1)(2x-1) = 0 \]
\[ x+1 = 0 \text{ or } 2x-1 = 0 \]
\[ x = -1 \text{ or } x = 1 \]
\[ x = \frac{1}{2} \]

\[ f(x) = \frac{2x-1}{2x^2 + x - 1} \]

The oblique asymptote is \( f(x) = 2x - 1 \).

ANSWER:

SOLUTION:

\[ f(x) = \frac{3x^2 + 8}{2x - 1} \]

\[ 2x - 1 = 0 \]
\[ 2x = 1 \]
\[ x = \frac{1}{2} \]

The vertical asymptote is at \( x = \frac{1}{2} \).

\[ f(x) = \frac{\frac{3}{2}x + \frac{3}{4}}{2x - 1} \]

\[ 2x - 1 = 3x^2 + 8 \]
\[ 3x^2 - \frac{3}{2}x \]
\[ \frac{3}{2}x + 8 \]
\[ \frac{3}{2}x - \frac{3}{4} \]

\[ \left( \frac{35}{4} \right) \]

The oblique asymptote is \( f(x) = \frac{3}{2}x + \frac{3}{4} \).

ANSWER:
Graph each function.

1. 

SOLUTION: 

The vertical asymptotes are at $x = -1$ and $x = 1$.

40. 

SOLUTION: 

Therefore, the graph of $f(x)$ is the graph of $g(x)$ with the holes at $x = -2$.

24. $f(x) = \frac{2x^2 + 5}{3x + 4}$

**SOLUTION:**

$$3x + 4 = 0$$

$$3x = -4$$

$$x = -\frac{4}{3}$$

$$\frac{2}{3}x - \frac{8}{9}$$

$$3x + 4 \left(\frac{2}{3}x - \frac{8}{9}\right)$$

$$\frac{2}{3}x^2 + \frac{8}{3}x$$

$$\frac{-8}{3}x + 5$$

$$\frac{-8}{3}x - \frac{32}{9}$$

The oblique asymptote is $f(x) = \frac{2}{3}x - \frac{8}{9}$.
8-4 Graphing Rational Functions

25. \( f(x) = \frac{x^4 - 2x^2 + 1}{x^3 + 2} \)

SOLUTION:
The vertical asymptote is at about \( x = -1.26 \).

\[
x^3 + 2 \left( x^4 - 2x^2 + 1 \right)
\]
\[
x^4 + 2x
\]
\[-2x^2 - 2x
\]

The oblique asymptote is \( f(x) = x \).

ANSWER:

\[
\begin{array}{c}
\text{f(x)} \\
\hline
\text{x} \\
\hline
\text{f(x) = x} \\
\hline
\text{f(x) = x^4 - 2x^2 + 1} \\
\hline
\text{x^3 + 2}
\end{array}
\]

26. \( f(x) = \frac{x^4 - x^3 - 12}{x^3 - 6} \)

SOLUTION:
The vertical asymptote is at about \( x = 1.82 \).

\[
x^3 - 6 \left( x^4 - x^3 - 12 \right)
\]
\[
x^4 - 6x
\]
\[-x^2 + 6x
\]

The oblique asymptote is \( f(x) = x \).

ANSWER:

\[
\begin{array}{c}
\text{f(x)} \\
\hline
\text{x} \\
\hline
\text{f(x) = x} \\
\hline
\text{f(x) = x^4 - x^3 - 12} \\
\hline
\text{x^3 - 6}
\end{array}
\]

27. CCSS PERSEVERANCE  The current in amperes in an electrical circuit with three resistors in a series is given by the equation \( I = \frac{V}{R_1 + R_2 + R_3} \), where \( V \) is the voltage in volts in a the circuit and \( R_1, R_2, \) and \( R_3 \) are the resistances in ohms of the three resistors.

a. Let \( R_1 \) be the independent variable, and let \( I \) be the dependent variable. Graph the function if \( V = 120 \)
8-4 Graphing Rational Functions

volts, $R_2 = 25$ ohms, and $R_3 = 75$ ohms.

b. Give the equation of the vertical asymptote and the $R_1$- and $I$-intercepts of the graph.

c. Find the value of $I$ when the value of $R_1$ is 140 ohms.

d. What domain and range values are meaningful in the context of the problem?

**SOLUTION:**

a. Graph the function

\[ I(R_1) = \frac{120}{R_1 + 25 + 75} \quad \text{or} \quad I(R_1) = \frac{120}{R_1 + 100}. \]

\[ \text{b. The vertical Asymptote is:} \]

\[ R_1 + 100 = 0 \]

\[ R_1 = -100 \]

$R_1$- Intercept: No

$I$-intercept: 1.2

\[ \text{c. Substitute 140 for } R_1. \]

\[ I(140) = \frac{120}{140 + 100} \]

\[ = 0.5 \text{ amperes} \]

\[ \text{d.} \]

\[ \text{Domain: } R_1 \geq 0; \]

\[ \text{Range: } 0 < I \leq 1.2; \]

\[ \text{ANSWER:} \]

\[ \text{a.} \]

\[ \text{b. } R_1 = -100; \text{ no } R_1 \text{-intercept; 1.2} \]

\[ \text{c. 0.5 amperes} \]

\[ \text{d. } R_1 \geq 0 \text{ and } 0 < I \leq 1.2 \]
Graph each function.

28. \( f(x) = \frac{x^2 - 2x - 8}{x - 4} \)

**SOLUTION:**
\[
f(x) = \frac{(x + 2)(x - 4)}{x - 4} = x + 2, \quad x \neq 4
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 2x - 8}{x - 4} \) is the graph of \( f(x) = x + 2 \) with a hole at \( x = 4 \).

![Graph of f(x)](image)

**ANSWER:**

29. \( f(x) = \frac{x^2 + 4x - 12}{x - 2} \)

**SOLUTION:**
\[
f(x) = \frac{(x + 6)(x - 2)}{x - 2} = x + 6, \quad x \neq 2
\]

Therefore, the graph of \( f(x) = \frac{x^2 + 4x - 12}{x - 2} \) is the graph of \( f(x) = x + 6 \) with a hole at \( x = 2 \).

![Graph of f(x)](image)

**ANSWER:**
30. \( f(x) = \frac{x^2 - 25}{x + 5} \)

**SOLUTION:**

\[
f(x) = \frac{(x + 5)(x - 5)}{x + 5} = x - 5, \ x = -5
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 25}{x + 5} \) is the graph of \( f(x) = x - 5 \) with a hole at \( x = -5 \).

**ANSWER:**

31. \( f(x) = \frac{x^2 - 64}{x - 8} \)

**SOLUTION:**

\[
f(x) = \frac{(x - 8)(x + 8)}{x - 8} = x + 8, \ x = 8
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 64}{x - 8} \) is the graph of \( f(x) = x + 8 \) with a hole at \( x = 8 \).

**ANSWER:**
32. \( f(x) = \frac{(x-4)(x^2-4)}{x^2-6x+8} \)

**SOLUTION:**
\[
f(x) = \frac{(x-4)(x-2)(x+2)}{(x-4)(x-2)} = x + 2, \quad x \neq 4, 2
\]
Therefore, the graph of \( f(x) = \frac{(x-4)(x^2-4)}{x^2-6x+8} \) is the graph of \( f(x) = x + 2 \) with the holes at \( x = 4 \) and \( 2 \).

33. \( f(x) = \frac{(x+5)(x^2 + 2x - 3)}{x^2 + 8x + 15} \)

**SOLUTION:**
\[
f(x) = \frac{(x+5)(x+3)(x-1)}{(x+3)(x+5)} = x - 1, \quad x = -3, -5
\]
Therefore, the graph of \( f(x) = \frac{(x+5)(x^2 + 2x - 3)}{x^2 + 8x + 15} \) is the graph of \( f(x) = x - 1 \) with the holes at \( x = -3 \) and \( -5 \).
34. \( f(x) = \frac{3x^4 + 6x^3 + 3x^2}{x^2 + 2x + 1} \)

**SOLUTION:**

\[
f(x) = \frac{3x^2(x^2 + 2x + 1)}{x^2 + 2x + 1}
= 3x^2, \quad x^2 + 2x + 1 \neq 0
= 3x^2, \quad x \neq -1
\]

Therefore, the graph of \( f(x) = \frac{3x^4 + 6x^3 + 3x^2}{x^2 + 2x + 1} \) is the graph of \( f(x) = 3x^2 \) with a hole at \( x = -1 \).

**ANSWER:**

\[
\begin{align*}
\text{Graph of } & \quad f(x) = \frac{3x^4 + 6x^3 + 3x^2}{x^2 + 2x + 1} \\
\text{Input } & \quad x = -1 \\
\text{Output } & \quad f(-1) = 3(-1)^2 = 3
\end{align*}
\]

35. \( f(x) = \frac{2x^4 + 10x^3 + 12x^2}{x^2 + 5x + 6} \)

**SOLUTION:**

\[
f(x) = \frac{2x^2(x^2 + 5x + 6)}{x^2 + 5x + 6}
= 2x^2, \quad x^2 + 5x + 6 \neq 0
= 2x^2, \quad x \neq -3, -2
\]

Therefore, the graph of \( f(x) = \frac{2x^4 + 10x^3 + 12x^2}{x^2 + 5x + 6} \) is the graph of \( f(x) = 2x^2 \) with the holes at \( x = -3 \) and \( x = -1 \).

**ANSWER:**

\[
\begin{align*}
\text{Graph of } & \quad f(x) = \frac{2x^4 + 10x^3 + 12x^2}{x^2 + 5x + 6} \\
\text{Input } & \quad x = -3 \\
\text{Output } & \quad f(-3) = 2(-3)^2 = 18
\end{align*}
\]

36. **BUSINESS** Liam purchased a snow plow for $4500 and plows the parking lots of local businesses. Each time he plows a parking lot, he incurs a cost of $50 for gas and maintenance.

a. Write and graph the rational function representing his average cost per customer as a function of the number of parking lots.
b. What are the asymptotes of the graph?

c. Why is the first quadrant in the graph the only relevant quadrant?

d. How many total parking lots does Liam need to plow for his average cost per parking lot to be less than $80?

**SOLUTION:**

a. Let \( x \) be the number of parking lots. Therefore, the function that represents the average cost per customer is 
\[
f(x) = \frac{4500 + 50x}{x}.
\]

b. The vertical asymptote is \( x = 0 \) and the horizontal asymptote is \( f(x) = 50 \).

c. Sample answer: The number of parking lots and the average cost cannot be negative.

d. Substitute 80 for \( f(x) \).

\[
80 = \frac{4500 + 50x}{x}
\]

\[
30x = 4500
\]

\[
x = 150
\]

**ANSWER:**

a. \( f(x) = \frac{4500 + 50x}{x} \)

b. \( x = 0 \) and \( f(x) = 50 \)

c. Sample answer: The number of parking lots and the average cost cannot be negative.

d. 150

37. **FINANCIAL LITERACY** Kristina bought a new cell phone with Internet access. The phone cost $150, and her monthly usage charge is $30 plus $10 for the Internet access.

a. Write and graph the rational function representing her average monthly cost as a function of the number of months Kristina uses the phone.

b. What are the asymptotes of the graph?

c. Why is the first quadrant in the graph the only relevant quadrant?

d. After how many months will the average monthly charge be $45?

**SOLUTION:**

a. Let \( x \) be the number of months Kristina uses the phone.

Therefore, the function that represents the average monthly costs is 
\[
f(x) = \frac{150 + 40x}{x}.
\]
### 8-4 Graphing Rational Functions

#### Graph each function.

1. **Solution:**
   - The vertical asymptotes are at $x = -1$ and $x = 1$.

30. **Solution:**
   - Therefore, the graph of $f$ is the graph of $f(x)$ with the holes at $x = -2$. 

#### 38. CCSS SENSE-MAKING

Alana plays softball for Centerville High School. So far this season she has gotten a hit 4 out of 12 times at bat. She is determined to improve her batting average. If she can get $x$ consecutive hits, her batting average can be determined using $B(x) = \frac{4 + x}{12 + x}$.

- **a.** Graph the function.

- **b.** What part of the graph is meaningful in the context of the problem?

- **c.** Describe the meaning of the intercept of the vertical axis.

- **d.** What is the equation of the horizontal asymptote? Explain its meaning with respect to Alana’s batting average.

**Solution:**

- **a.**

- **b.** The part in the first quadrant

- **c.** It represents her original batting average of .333.

- **d.** $y = 1$; This represents 100%, which she can never achieve because she has already missed getting a hit 8 times.

**Answer:**

- **a.**
Graph each function.

39. \( f(x) = \frac{x+1}{x^2 + 6x + 5} \)

**SOLUTION:**

\[
\begin{align*}
  f(x) &= \frac{x+1}{(x+5)(x+1)} \\
  &= \frac{1}{x+5}, \quad x \neq -1
\end{align*}
\]

Therefore, the graph of \( f(x) = \frac{x+1}{x^2 + 6x + 5} \) is the graph of \( f(x) = \frac{1}{x+5} \) with the holes at \( x = -1 \).

---

8-4 Graphing Rational Functions

b. the part in the first quadrant

c. It represents her original batting average of .333.

d. \( y = 1 \); This represents 100%, which she can never achieve because she has already missed getting a hit 8 times.
8-4 Graphing Rational Functions

40. \( f(x) = \frac{x^2 - 10x - 24}{x + 2} \)

**SOLUTION:**

\[
\begin{align*}
  f(x) & = \frac{(x + 2)(x - 12)}{x + 2} \\
        & = x - 12, \ x \neq -2
\end{align*}
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 10x - 24}{x + 2} \) is the graph of \( f(x) = x - 12 \) with the holes at \( x = -2 \).

![](image1.png)

ANSWER:

41. \( f(x) = \frac{6x^2 + 4x + 2}{x + 2} \)

**SOLUTION:**

Graph the function.

![](image2.png)

ANSWER:
42. **OPEN ENDED** Sketch the graph of a rational function with a horizontal asymptote \( y = 1 \) and a vertical asymptote \( x = -2 \).

**SOLUTION:**
Sample graph:

![Graph of a rational function with horizontal asymptote y = 1 and vertical asymptote x = -2](image)

**ANSWER:**
Sample graph:

![Sample graph of a rational function](image)

43. **CHALLENGE** Compare and contrast \( g(x) = \frac{x^2-1}{x(x^2-2)} \) and \( f(x) \) shown.

**SOLUTION:**
Similarities: Both have vertical asymptotes at \( x = 0 \).
Both approach 0 as \( x \) approaches \( -\infty \) and approach 0 as \( x \) approaches \( \infty \). Differences: \( f(x) \) has holes at \( x = 1 \) and \( x = -1 \), while \( g(x) \) has vertical asymptotes at \( x = \sqrt{2} \) and \( x = -\sqrt{2} \). \( f(x) \) has no zeros, but \( g(x) \) has zeros at \( x = 1 \) and \( x = -1 \).

**ANSWER:**
Similarities: Both have vertical asymptotes at \( x = 0 \).
Both approach 0 as \( x \) approaches \( -\infty \) and approach 0 as \( x \) approaches \( \infty \). Differences: \( f(x) \) has holes at \( x = 1 \) and \( x = -1 \), while \( g(x) \) has vertical asymptotes at \( x = \sqrt{2} \) and \( x = -\sqrt{2} \). \( f(x) \) has no zeros, but \( g(x) \) has zeros at \( x = 1 \) and \( x = -1 \).

44. **REASONING** What is the difference between the graphs of \( f(x) = x - 2 \) and \( g(x) = \frac{(x + 3)(x - 2)}{x + 3} \)?

**SOLUTION:**
The graph of \( g(x) \) has a hole in it at \(-3\).

**ANSWER:**
The graph of \( g(x) \) has a hole in it at \(-3\).
45. **PROOF** A rational function has an equation of the form \( f(x) = \frac{a(x)}{b(x)} \), where \( a(x) \) and \( b(x) \) are polynomial functions and \( b(x) \neq 0 \). Show that \( f(x) = \frac{x}{a - b} + c \) is a rational function.

**SOLUTION:**
\[
f(x) = \frac{x}{a - b} + c
\]

Find the common denominator.

\[
f(x) = \frac{x}{a - b} + \frac{c(a - b)}{(a - b)}
\]

\[
= \frac{x + ca - cb}{a - b}
\]

Since \( a(x) \) and \( b(x) \) are polynomial functions, \( f(x) \) is a rational function.

**ANSWER:**
\[
f(x) = \frac{x}{a - b} + \frac{c(a - b)}{(a - b)} \rightarrow \frac{x + ca - cb}{a - b}
\]

46. **WRITING IN MATH** How can factoring be used to determine the vertical asymptotes or point discontinuity of a rational function?

**SOLUTION:**
Sample answer: By factoring the denominator of a rational function and determining the values that cause each factor to equal zero you can determine the asymptotes of a rational function. After factoring the numerator and denominator of a rational function, if there is a common factor \( x - c \), then there is point discontinuity at \( x = c \).

**ANSWER:**
Sample answer: By factoring the denominator of a rational function and determining the values that cause each factor to equal zero you can determine the asymptotes of a rational function. After factoring the numerator and denominator of a rational function, if there is a common factor \( x - c \), then there is point discontinuity at \( x = c \).
47. **PROBABILITY** Of the 6 courses offered by the music department at her school, Kaila must choose exactly 2 of them. How many different combinations of 2 courses are possible for Kaila if there are no restrictions on which 2 courses she can choose?

A 48  
B 18  
C 15  
D 12  

**SOLUTION:**

\[ 6 \binom{n}{2} = \frac{6 \cdot 5}{2 \cdot 1} = 15 \]

The correct choice is C.

**ANSWER:**  
C

48. The projected sales of a game cartridge is given by the function \[ S(p) = \frac{3000}{2p + a} \], where \( S(p) \) is the number of cartridges sold, in thousands, \( p \) is the price per cartridge, in dollars, and \( a \) is a constant.

If 100,000 cartridges are sold at $10 per cartridge, how many cartridges will be sold at $20 per cartridge?

F 20,000  
G 50,000  
H 60,000  
J 150,000  

**SOLUTION:**

Find the value of the constant \( a \).

\[ 100 = \frac{3000}{2(10) + a} \]

\[ 20 + a = 30 \]

\[ a = 10 \]

Now,  

\[ S(p) = \frac{3000}{2(20) + 10} = \frac{3000}{50} = 60 \]

The correct choice is H.

**ANSWER:**  
H
49. GRIDDED RESPONSE Five distinct points lie in a plane such that 3 of the points are on line \( \ell \) and 3 of the points are on a different line \( m \). What is the total number of lines that can be drawn so that each line passes through exactly 2 of these 5 points?

**SOLUTION:**

From the figure, 4 lines that can be drawn so that each line passes through exactly 2 points.

**ANSWER:**

4

50. GEOMETRY In the figure below, what is the value of \( w + x + y + z \)?

![Diagram](https://example.com/diagram.png)

**SOLUTION:**

By the Triangle Angle-Sum Theorem, \( y + z + 40 = 180 \) and \( w + x + 40 = 180 \).

Therefore,

\[
y + z = 140;
w + x = 140;
\]

Now,

\[
w + x + y + z = 140 + 140 = 280
\]

The correct choice B.

**ANSWER:**

B
8-4 Graphing Rational Functions

Graph each function. State the domain and range.

51. \( f(x) = \frac{-5}{x + 2} \)

**SOLUTION:**
The vertical asymptote is at \( x = -2 \).
The horizontal asymptote is at \( y = 0 \).

**ANSWER:**

\[ D = \{ x \mid x \neq -2 \}, \quad R = \{ f(x) \mid f(x) \neq 0 \} \]

52. \( f(x) = \frac{4}{x - 1} - 3 \)

**SOLUTION:**
The vertical asymptote is at \( x = 1 \).
The horizontal asymptote is at \( y = -3 \).

**ANSWER:**

\[ D = \{ x \mid x \neq 1 \}, \quad R = \{ f(x) \mid f(x) \neq -3 \} \]
53. \( f(x) = \frac{1}{x + 6} + 1 \)

**SOLUTION:**
The vertical asymptote is at \( x = -6 \).
The horizontal asymptote is at \( y = 1 \).

\[
\begin{align*}
D &= \{ x \mid x \neq -6 \}, \\
R &= \{ f(x) \mid f(x) \neq 1 \}
\end{align*}
\]

**ANSWER:**

\[
\begin{align*}
\text{Simplify each expression.}
\end{align*}
\]

54. \( \frac{m}{m^2 - 4} + \frac{2}{3m + 6} \)

**SOLUTION:**

\[
\frac{m}{m^2 - 4} + \frac{2}{3m + 6} = \frac{m}{(m-2)(m+2)} + \frac{2}{3(m+2)}
\]
The LCD is \( 3(m-2)(m+2) \).

\[
\frac{m}{(m-2)(m+2)} + \frac{2}{3(m+2)} = \frac{3m + 2(m-2)}{3(m-2)(m+2)} = \frac{5m-4}{3(m-2)(m+2)}
\]

**ANSWER:**

\[
\frac{5m-4}{3(m+2)(m-2)}
\]

55. \( \frac{y}{y+3} - \frac{6y}{y^2 - 9} \)

**SOLUTION:**

\[
\frac{y}{y+3} - \frac{6y}{y^2 - 9} = \frac{y}{y+3} - \frac{6y}{(y+3)(y-3)}
\]
The LCD is \( (y+3)(y-3) \).

\[
\frac{y}{y+3} - \frac{6y}{(y+3)(y-3)} = \frac{y(y-3) - 6y}{(y+3)(y-3)} = \frac{y(y-9)}{(y+3)(y-3)}
\]

**ANSWER:**

\[
\frac{y(y-9)}{(y+3)(y-3)}
\]
56. \( \frac{5}{x^2-3x-28} + \frac{7}{2x-14} \)

**SOLUTION:**

\[
\frac{5}{x^2-3x-28} + \frac{7}{2x-14} = \frac{5}{(x-7)(x+4)} + \frac{7}{2(x-7)}
\]

The LCD is \(2(x-7)(x+4)\).

\[
\frac{5}{(x-7)(x+4)} + \frac{7}{2(x-7)} = \frac{2(5) + 7(x+4)}{2(x-7)(x+4)}
\]

\[
= \frac{7x+38}{2(x-7)(x+4)}
\]

**ANSWER:**

\[
\frac{7x+38}{2(x-7)(x+4)}
\]

57. \( \frac{d-4}{d^2+2d-8} - \frac{d+2}{d^2-16} \)

**SOLUTION:**

\[
\frac{d-4}{d^2+2d-8} - \frac{d+2}{d^2-16} = \frac{d-4}{(d+4)(d-2)} - \frac{d+2}{(d-4)(d+4)}
\]

The LCD is \((d-4)(d+4)(d+2)\).

\[
\frac{d-4}{(d+4)(d-2)} - \frac{d+2}{(d-4)(d+4)} = \frac{(d-4)(d-4) - (d+2)(d-2)}{(d-4)(d+4)(d-2)}
\]

\[
= \frac{d^2 - 8d + 16 - d^2 + 4}{(d-4)(d+4)(d-2)}
\]

\[
= \frac{-8d + 20}{(d-4)(d+4)(d-2)}
\]

**ANSWER:**

\[
\frac{-8d + 20}{(d-4)(d+4)(d-2)}
\]

58. \( y^3 \cdot y^3 \)

**SOLUTION:**

\[
y^3 \cdot y^3 = y^3 + 3 = y^9
\]

**ANSWER:**

\[
y^9
\]

59. \( x^4 \cdot x^4 \)

**SOLUTION:**

\[
x^4 \cdot x^4 = x^{4+4} = x^8
\]

**ANSWER:**

\[
x^8
\]
62. **TRAVEL** Mr. and Mrs. Wells are taking their daughter to college. The table shows their distances from home after various amounts of time.

a. Find the average rate of change in their distances from home between 1 and 3 hours after leaving home.

b. Find the average rate of change in their distances from home between 0 and 5 hours after leaving home.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>165</td>
</tr>
<tr>
<td>4</td>
<td>165</td>
</tr>
<tr>
<td>5</td>
<td>225</td>
</tr>
</tbody>
</table>

**SOLUTION:**
a. Average rate of change between 1 and 3 hours:

\[
\frac{165 - 55}{3 - 1} = \frac{110}{2} = 55 \text{mph}
\]

b. Average rate of change between 0 and 5 hours:

\[
\frac{225 - 0}{5 - 0} = \frac{225}{5} = 45 \text{mph}
\]

**ANSWER:**

a. 55 mph

b. 45 mph
8-5 Variation Functions

1. If \( y \) varies directly as \( x \) and \( y = 12 \) when \( x = 8 \), find \( y \) when \( x = 14 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{12}{8} = \frac{y_2}{14}
\]

\[
y_2 = 21
\]

**ANSWER:**
21

2. Suppose \( y \) varies jointly as \( x \) and \( z \). Find \( y \) when \( x = 9 \) and \( z = -3 \), if \( y = -50 \) when \( z \) is 5 and \( x \) is -10.

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{y_1}{9 \cdot (-3)} = \frac{-50}{5 \cdot (-10)}
\]

\[
y_1 = -27
\]

**ANSWER:**
-27

3. If \( y \) varies inversely as \( x \) and \( y = -18 \) when \( x = 16 \), find \( x \) when \( y = 9 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{16}{9} = \frac{x_2}{-18}
\]

\[
x_2 = -32
\]

**ANSWER:**
-32

4. **TRAVEL** A map of Illinois is scaled so that 2 inches represents 15 miles. How far apart are Chicago and Rockford if they are 12 inches apart on the map?

**SOLUTION:**
Let \( x \) be the distance between Chicago and Rockford in miles.

\[
\frac{15}{2} = \frac{x}{12}
\]

\[
x = 90
\]

Answer: 90 miles

5. Suppose \( a \) varies directly as \( b \), and \( a \) varies inversely as \( c \). Find \( b \) when \( a = 8 \) and \( c = -3 \), if \( b = 16 \) when \( c = 2 \) and \( a = 4 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{8 \cdot (-3)}{b_1} = \frac{2 \cdot 4}{16}
\]

\[
b_1 = -48
\]

**ANSWER:**
-48
8-5 Variation Functions

6. Suppose \( d \) varies directly as \( f \), and \( d \) varies inversely as \( g \). Find \( g \) when \( d = 6 \) and \( f = -7 \), if \( g = 12 \) when \( d = 9 \) and \( f = 3 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{6 \cdot g}{-7} = \frac{9 \cdot 12}{3}
\]

\[
g = \frac{-42}{1} = -42
\]

**ANSWER:**

\(-42\)

If \( x \) varies directly as \( y \), find \( x \) when \( y \) = 8.

7. \( x = 6 \) when \( y = 32 \)

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{8}{x_1} = \frac{32}{6}
\]

\[
x_1 = \frac{3}{2} = 1.5
\]

**ANSWER:**

\(1.5\)

8. \( x = 11 \) when \( y = -3 \)

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{8}{x_1} = \frac{-3}{11}
\]

\[
x_1 = \frac{88}{3}
\]

**ANSWER:**

\(\frac{88}{3}\)

9. \( x = 14 \) when \( y = -2 \)

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{8}{x_1} = \frac{-2}{14}
\]

\[
x_1 = -56
\]

**ANSWER:**

\(-56\)

10. \( x = -4 \) when \( y = 10 \)

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{8}{x_1} = \frac{10}{-4}
\]

\[
x_1 = \frac{-16}{5} = -3.2
\]

**ANSWER:**

\(-3.2\)
8-5 Variation Functions

11. MOON Astronaut Neil Armstrong, the first man on the Moon, weighed 360 pounds on Earth with all his equipment on, but weighed only 60 pounds on the Moon. Write an equation that relates weight on the Moon m with weight on Earth w.

SOLUTION: The equation that relates weight on the Moon m with weight on Earth w is:

\[
\frac{m}{w} = \frac{60}{360} = \frac{1}{6}
\]

\[
m = \frac{1}{6}w
\]

ANSWER:

\[
m = \frac{1}{6}w
\]

If \( a \) varies jointly as \( b \) and \( c \), find \( a \) when \( b = 4 \) and \( c = -3 \).

12. \( a = -96 \) when \( b = 3 \) and \( c = -8 \)

SOLUTION: Use a proportion that relates the values.

\[
\frac{a}{b \cdot c} = \frac{96}{3 \cdot (-8)} = \frac{-96}{-24} = 4
\]

\[
a = 4 \cdot 3 \cdot (-8) = -96
\]

ANSWER:

\[
a = -96
\]

13. \( a = -60 \) when \( b = -5 \) and \( c = 4 \)

SOLUTION: Use a proportion that relates the values.

\[
\frac{a}{b \cdot c} = \frac{-60}{(-5) \cdot 4} = \frac{-60}{-20} = 3
\]

\[
a = 3 \cdot 4 \cdot (-5) = -60
\]

ANSWER:

\[
a = -60
\]

14. \( a = -108 \) when \( b = 2 \) and \( c = 9 \)

SOLUTION: Use a proportion that relates the values.

\[
\frac{a}{b \cdot c} = \frac{-108}{2 \cdot 9} = \frac{-108}{18} = -6
\]

\[
a = -6 \cdot 4 \cdot (-3) = -72
\]

ANSWER:

\[
a = -72
\]

15. \( a = 24 \) when \( b = 8 \) and \( c = 12 \)

SOLUTION: Use a proportion that relates the values.

\[
\frac{a}{b \cdot c} = \frac{24}{8 \cdot 12} = \frac{24}{96} = \frac{1}{4}
\]

\[
a = \frac{1}{4} \cdot 4 \cdot (-3) = -3
\]

ANSWER:

\[
a = -3
\]
8-5 Variation Functions

16. CCSS MODELING According to the A.C. Nielsen Company, the average American watches 4 hours of television a day.

a. Write an equation to represent the average number of hours spent watching television by $m$ household members during a period of $d$ days.

$$t = 4 \cdot m \cdot d$$
$$= 4md$$

b. Assume that members of your household watch the same amount of television each day as the average American. How many hours of television would the members of your household watch in a week?

**SOLUTION:**

a. Let $t$ be the average number of hours spent watching television.

$$t = 4 \cdot m \cdot d$$

$$= 4md$$

b. Sample answer for four members:

$$t = 4 \cdot 4 \cdot 7$$
$$= 112 \text{ hours}$$

**ANSWER:**

a. $t = 4 \cdot m \cdot d$

b. Sample answer for four household members: 112 hours

17. $f$ varies inversely as $g$, find $f$ when $g = -6$.

17. $f = 15$ when $g = 9$

**SOLUTION:**

Use a proportion that relates the values.

$$\frac{15}{-6} = \frac{f_2}{9}$$

$$f_2 = -22.5$$

**ANSWER:**

$-22.5$

18. $f = 4$ when $g = 28$

**SOLUTION:**

Use a proportion that relates the values.

$$\frac{4}{-6} = \frac{f_2}{28}$$

$$f_2 = -\frac{56}{3}$$

**ANSWER:**

$-\frac{56}{3}$

19. $f = -12$ when $g = 19$

**SOLUTION:**

Use a proportion that relates the values.

$$\frac{-12}{-6} = \frac{f_2}{19}$$

$$f_2 = 38$$

**ANSWER:**

38
8-5 Variation Functions

20. \( f = 0.6 \) when \( g = -21 \)

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{0.6}{-6} = \frac{f_2}{-21}
\]

\[
f_2 = 2.1
\]

**ANSWER:**
2.1

21. **COMMUNITY SERVICE** Every year students at West High School collect canned goods for a local food pantry. They plan to distribute flyers to homes in the community asking for donations. Last year, 12 students were able to distribute 1000 flyers in four hours.

a. Write an equation that relates the number of students \( s \) to the amount of time \( t \) it takes to distribute 1000 flyers.

b. How long would it take 15 students to hand out the same number of flyers this year?

**SOLUTION:**

a. The number of students \( s \) varies directly as the number of flyers distributed and inversely as the amount of time \( t \).

Therefore, \( \frac{12 \cdot 4}{1000} = \frac{s \cdot t}{1000} \).

\[
\Rightarrow s = \frac{48}{t}
\]

b. Substitute \( t = 15 \) in the expression.

\[
15 = \frac{48}{t}
\]

\[
t = 3.2 \text{ hours}
\]

**ANSWER:**

a. \( s = \frac{48}{t} \)

b. 3.2 hours
22. **BIRDS** When a group of snow geese migrate, the distance that they fly varies directly with the amount of time they are in the air.

a. A group of snow geese migrated 375 miles in 7.5 hours. Write a direct variation equation that represents this situation.

b. Every year, geese migrate 3000 miles from their winter home in the southwest United States to their summer home in the Canadian Arctic. Estimate the number of hours of flying time that it takes for the geese to migrate.

**SOLUTION:**

a. Let \( d \) be the distance that snow geese fly and \( t \) be the amount of time they are in the air.

   Since it is direct variation, \( \frac{375}{7.5} = \frac{d}{t} \).

   That is, \( d = 50t \).

b. Substitute \( d = 3000 \) miles in the expression.

   \[ 3000 = 50t \]
   \[ t = 60 \text{ hours} \]

**ANSWER:**

a. \( d = 50t \)

b. 60 hours

23. Suppose \( a \) varies directly as \( b \), and \( a \) varies inversely as \( c \). Find \( b \) when \( a = 5 \) and \( c = -4 \), if \( b = 12 \) when \( c = 3 \) and \( a = 8 \).

**SOLUTION:**

Use a proportion that relates the values.

\[
\frac{5 \cdot (-4)}{b_1} = \frac{3 \cdot 8}{12}
\]

\[
b_1 = -10
\]

**ANSWER:**

\(-10\)

24. Suppose \( x \) varies directly as \( y \), and \( x \) varies inversely as \( z \). Find \( z \) when \( x = 10 \) and \( y = -7 \), if \( z = 20 \) when \( x = 6 \) and \( y = 14 \).

**SOLUTION:**

Use a proportion that relates the values.

\[
\frac{10 \cdot z_1}{-7} = \frac{6 \cdot 20}{14}
\]

\[
z_1 = -6
\]

**ANSWER:**

\(-6\)
Determine whether each relation shows direct or inverse variation, or neither.

**25.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( \frac{y}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>48</td>
<td>3</td>
</tr>
<tr>
<td>32</td>
<td>96</td>
<td>3</td>
</tr>
</tbody>
</table>

**SOLUTION:**

Since \( \frac{y}{x} = 3 \) (a constant), the relation shows direct variation.

**ANSWER:**

direct

**26.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>xy</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>-2</td>
<td>-8</td>
<td>16</td>
</tr>
<tr>
<td>-8</td>
<td>-2</td>
<td>16</td>
</tr>
</tbody>
</table>

Since \( xy = 16 \) (a constant), the relation shows inverse variation.

**ANSWER:**

inverse

**27.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( \frac{y}{x} )</th>
<th>xy</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>5</td>
<td>125</td>
</tr>
</tbody>
</table>

Since neither \( xy \) nor \( \frac{y}{x} \) is constant, the relation is neither direct nor inverse.

**ANSWER:**

neither
28. If \( y \) varies inversely as \( x \) and \( y = 6 \) when \( x = 19 \), find \( y \) when \( x = 2 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{y_1}{x_2} = \frac{y_2}{x_1}
\]

\[
\frac{6}{2} = \frac{y_2}{19}
\]

\( y_2 = 57 \)

**ANSWER:**
57

29. If \( x \) varies inversely as \( y \) and \( x = 16 \) when \( y = 5 \), find \( x \) when \( y = 20 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{x_2}{y_1} = \frac{x_1}{y_2}
\]

\[
\frac{16}{5} = \frac{x_1}{20}
\]

\( x_1 = 4 \)

**ANSWER:**
4

30. Suppose \( a \) varies directly as \( b \), and \( a \) varies inversely as \( c \). Find \( b \) when \( a = 7 \) and \( c = -8 \), if \( b = 15 \) when \( c = 2 \) and \( a = 4 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{a_1c_1}{b_1} = \frac{a_2c_2}{b_2}
\]

\[
\frac{7 \cdot (-8)}{2 \cdot 4} = \frac{2 \cdot 4}{15}
\]

\( b_1 = -105 \)

**ANSWER:**
-105

31. Suppose \( x \) varies directly as \( y \), and \( x \) varies inversely as \( z \). Find \( z \) when \( x = 8 \) and \( y = -6 \), if \( z = 26 \) when \( x = 8 \) and \( y = 13 \).

**SOLUTION:**
Use a proportion that relates the values.

\[
\frac{x_1y_1}{z_1} = \frac{x_2y_2}{z_2}
\]

\[
\frac{8 \cdot (-6)}{z_1} = \frac{8 \cdot 13}{26}
\]

\( z_1 = -12 \)

**ANSWER:**
-12
8-5 Variation Functions

State whether each equation represents a direct, joint, inverse, or combined variation. Then name the constant of variation.

32. \( \frac{x}{y} = 2.75 \)

**SOLUTION:**
\[ x = 2.75y. \]
Direct variation;
Constant of variation = 2.75.

**ANSWER:**
direct; 2.75

33. \( fg = -2 \)

**SOLUTION:**
\[ f = \frac{-2}{g}. \]
Inverse variation;
Constant of variation = -2.

**ANSWER:**
inverse; -2

34. \( a = 3bc \)

**SOLUTION:**
Joint variation; Constant of variation = 3.

**ANSWER:**
joint; 3

35. \( 10 = \frac{xy^2}{z} \)

**SOLUTION:**
\[ x = \frac{10z}{y^2}. \] Combined variation; Constant of variation = 10.

**ANSWER:**
combined; 10

36. \( y = -11x \)

**SOLUTION:**
Direct variation; Constant of variation = -11.

**ANSWER:**
direct; -11

37. \( \frac{n}{p} = 4 \)

**SOLUTION:**
\[ n = 4p. \] Direct variation;
Constant of variation = 4.

**ANSWER:**
direct; 4
**8-5 Variation Functions**

38. \(9n = pr\)

**SOLUTION:**
\[
p = \frac{9n}{r}
\]
Combined variation;

Constant of variation = 9.

**ANSWER:**
combined; 9

39. \(-2y = z\)

**SOLUTION:**
Direct variation;

Constant of variation = –2.

**ANSWER:**
direct; –2

40. \(a = 27b\)

**SOLUTION:**
Direct variation;

Constant of variation = 27.

**ANSWER:**
direct; 27

41. \(c = \frac{7}{d}\)

**SOLUTION:**
Inverse variation;

Constant of variation = 7.

**ANSWER:**
inverse; 7

42. \(-10 = gh\)

**SOLUTION:**
\[
h = -\frac{10}{g}
\]
Inverse variation;

Constant of variation = –10.

**ANSWER:**
inverse; –10

43. \(m = 20cd\)

**SOLUTION:**
Joint variation Constant of variation = 20

**ANSWER:**
joint; 20

44. **CCSS PRECISION** The volume of a gas \(v\) varies inversely as the pressure \(p\) and directly as the temperature \(t\).

**a.** Write an equation to represent the volume of a gas in terms of pressure and temperature. Is your equation a direct, joint, inverse, or combined variation?

**b.** A certain gas has a volume of 8 liters, a temperature of 275 Kelvin, and a pressure of 1.25 atmospheres. If the gas is compressed to a volume of 6 liters and is heated to 300 Kelvin, what will the new pressure be?

**c.** If the volume stays the same, but the pressure drops by half, then what must have happened to the temperature?

**SOLUTION:**
\[
v = \frac{k}{p}t
\]
Because volume varies with respect to two quantities, the situation represents a combined variation.
8-5 Variation Functions

b. Write a proportion that relates the volume, pressure and temperature.

\[
\frac{V_1}{T_1} p_1 = \frac{V_2}{T_2} p_2
\]

\[
\frac{8 \cdot (1.25)}{275} = \frac{6 \cdot p_2}{300}
\]

\[
p_2 = \frac{20}{11} \approx 1.82 \text{ atmospheres}
\]

c. If the volume remains constant and the pressure drops by half, then the temperature must drop by half also.

\textit{ANSWER:}

a. \( v = \frac{kt}{p} \); combined

b. approximately 1.82 atmospheres or \( \frac{20}{11} \) atm

c. It dropped by half.

45. VACATION The time it takes the Levensteins to reach Lake Tahoe varies inversely with their average rate of speed.

a. If they are 800 miles away, write and graph an equation relating their travel time to their average rate of speed.

b. Their goal is to arrive within 18 hours. What minimum average speed will accomplish this goal?

\textit{SOLUTION:}

a. Let \( r \) be the average rate of speed and \( t \) be the time taken. Therefore, the equation that relates the average speed and their travel time is \( 800 = rt \).

Graph the function.

b. Substitute \( t = 18 \) in the equation to find the minimum average speed.

\[
r = \frac{800}{18} \approx 44.4 \text{ mph}
\]

\textit{ANSWER:}

a.

b. 44.4 mph

46. MUSIC The maximum number of songs that a digital audio player can hold depends on the lengths and the quality of the songs that are recorded. A song will take up more space on the player if it is recorded at a higher quality, like from a CD, than at a lower quality, like from the Internet.

a. If a certain player has 5400 megabytes of storage space, write a function that represents the number of songs the player can hold as a function of the average size of the songs.

b. Is your function a \textit{direct}, \textit{joint}, \textit{inverse}, or
8-5 Variation Functions

combined variation?

c. Suppose the average file size for a high-quality song is 8 megabytes and the average size for a low-quality song is 5 megabytes. Determine how many more songs the player can hold if they are low quality than if they are high quality.

**SOLUTION:**

a. \( f(x) = \frac{5400}{x} \)

b. Inverse variation

c. 
\[
f(8) = \frac{5400}{8} = 675
\]

Thus, the player can hold 675 high quality songs.

\[
f(5) = \frac{5400}{5} = 1080
\]

Thus, the player can hold 1080 low quality songs.

Therefore, the player can hold 1080 – 675 or 405 more low quality songs than if they are high quality.

**ANSWER:**

a. \( f(x) = \frac{5400}{x} \)

b. inverse

c. 405

47. **GRAVITY** According to the Law of Universal Gravitation, the attractive force \( F \) in newtons between any two bodies in the universe is directly proportional to the product of the masses \( m_1 \) and \( m_2 \) in kilograms of the two bodies and inversely proportional to the square of the distance \( d \) in meters between the bodies. That is, \( F = \frac{Gm_1m_2}{d^2} \). \( G \) is the universal gravitational constant. Its value is \( 6.67 \times 10^{-11} \) Nm\(^2\)/kg\(^2\).

a. The distance between Earth and the Moon is about \( 3.84 \times 10^8 \) meters. The mass of the Moon is \( 7.36 \times 10^{22} \) kilograms. The mass of Earth is \( 5.97 \times 10^{24} \) kilograms. What is the gravitational force that the Moon and Earth exert upon each other?

b. The distance between Earth and the Sun is about \( 1.5 \times 10^{11} \) meters. The mass of the Sun is about \( 1.99 \times 10^{30} \) kilograms. What is the gravitational force that the Sun and Earth exert upon each other?

c. Find the gravitational force exerted on each other by two 1000-kilogram iron balls at a distance of 0.1 meter apart.

**SOLUTION:**

a. 
\[
F = \frac{Gm_1m_2}{d^2}
= \frac{(6.67 \times 10^{-11}) (7.36 \times 10^{22}) (5.97 \times 10^{24})}{(3.84 \times 10^8)^2}
\approx \frac{293.07 \times 10^{-11+22+24}}{14.7456 \times 10^{15}}
= \frac{293.07 \times 10^{35}}{14.7456 \times 10^{16}}
\approx 19.88 \times 10^{19} \times 10
\approx 1.988 \times 10^{26}
\approx 2 \times 10^{26}
\]

The gravitational force that the Moon and Earth exert upon each other is about \( 2 \times 10^{26} \) newtons.

b. 

\[
\]
### 8-5 Variation Functions

The gravitational force that the Sun and Earth exert upon each other is about $3.5 \times 10^{22}$ newtons.

**c.**

$$F = \frac{\left(6.67 \times 10^{-11}\right)\left(10^3\right)^2}{(0.1)^2}$$

$$= \frac{6.67 \times 10^{-11+3+3}}{0.01}$$

$$= \frac{6.67 \times 10^{-5}}{10^{-2}}$$

$$= 6.67 \times 10^{-3}$$

The gravitational force exerted on each other by two 1000-kilogram iron balls at a distance of 0.1 meter apart is $6.67 \times 10^{-3}$ newtons.

**ANSWER:**

- **a.** about $2 \times 10^{20}$ newtons
- **b.** about $3.5 \times 10^{22}$ newtons
- **c.** $6.67 \times 10^{-3}$ newtons

---

**48. CCSS CRITIQUE** Jamil and Savannah are setting up a proportion to begin solving the combined variation in which $z$ varies directly as $x$ and $z$ varies inversely as $y$. Who has set up the correct proportion? Explain your reasoning.

**SOLUTION:**

Jamil; Savannah multiplied when she should have divided and divided when she should have multiplied.

**ANSWER:**

Jamil; Savannah multiplied when she should have divided and divided when she should have multiplied.

---

**49. CHALLENGE** If $a$ varies inversely as $b$, $c$ varies jointly as $b$ and $f$, and $f$ varies directly as $g$, how are $a$ and $g$ related?

**SOLUTION:**

$a$ and $g$ are directly related.

**ANSWER:**

$a$ and $g$ are directly related.
50. **REASONING** Explain why some mathematicians consider every joint variation a combined variation, but not every combined variation a joint variation.

**SOLUTION:**
Sample answer: Every joint variation is a combined variation because there are two combined direct variations. However, a combined variation can have a combination of a direct and an inverse variation, so it cannot be considered as a joint variation.

**ANSWER:**
Sample answer: Every joint variation is a combined variation because there are two combined direct variations. However, a combined variation can have a combination of a direct and an inverse variation, so it cannot be considered as a joint variation.

51. **OPEN ENDED** Describe three real-life quantities that vary jointly with each other.

**SOLUTION:**
Sample answer: The force of an object varies jointly as its mass and acceleration.

**ANSWER:**
Sample answer: The force of an object varies jointly as its mass and acceleration.

52. **WRITING IN MATH** Determine the type(s) of variation(s) for which 0 cannot be one of the values. Explain your reasoning.

**SOLUTION:**
Sample answer: Inverse and some types of combined variation functions cannot have a value of 0 in the domain because division by zero is undefined.

**ANSWER:**
Sample answer: Inverse and some types of combined variation functions cannot have a value of 0 in the domain because division by zero is undefined.

53. **SAT/ACT** Rafael left the dorm and drove toward the cabin at an average speed of 40 km/h. Monica left some time later driving in the same direction at an average speed of 48 km/h. After driving for five hours, Monica caught up with Rafael. How long did Rafael drive before Monica caught up?

<table>
<thead>
<tr>
<th>Option</th>
<th>Time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
</tr>
</tbody>
</table>

**SOLUTION:**

\[
\begin{array}{|c|c|c|}
\hline
\text{Rafael} & \text{Speed} & \text{Time} \\
\hline
40 \text{km/h} & x + 5 & \\
\hline
\text{Monica} & 48 \text{km/h} & 5 \\
\hline
\end{array}
\]

\[40(x + 5) = 48(5)\]
\[40x + 200 = 240\]
\[40x = 40\]
\[x = 1\]

Therefore, Rafael started 1 hour before Monica. So, Rafael drives for 6 hours before Monica caught up. The correct choice is D.

**ANSWER:**
D
8-5 Variation Functions

54. 75% of 88 is the same as 60% of what number?

F 100
G 105
H 108
J 110

**SOLUTION:**

\[
\frac{75}{100} \times 88 = \frac{60}{100} \times x
\]

\[
75(88) = 60x
\]

\[
x = \frac{75(88)}{60}
\]

\[
x = \frac{6600}{60}
\]

\[
x = 110
\]

Therefore, the correct choice is J.

**ANSWER:**

J

55. **EXTENDED RESPONSE** Audrey’s hair is 7 inches long and is expected to grow at an average rate of 3 inches per year.

a. Make a table that shows the expected length of Audrey’s hair after each of the first 4 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
</tr>
</tbody>
</table>

b. \( f(x) = 3x + 7 \)

c. Substitute 9 for \( x \) in the function \( f(x) = 3x + 7 \).

\[
f(9) = 3(9) + 7
\]

\[
= 27 + 7
\]

\[
= 34
\]

The length of Audrey’s hair would be 34 inches after 9 years.

**ANSWER:**

a.

b. \( f(x) = 3x + 7 \)

c. 34 in.
56. Which of the following is equal to the sum of two consecutive even integers?

A 144

B 146

C 147

D 148

**SOLUTION:**
Let $x$ be an even integer. Then the sum of two consecutive even integers is \( x + (x + 2) \) or \( 2x + 2 \).

\[
2x + 2 = 144 \\
2x = 142 \\
x = 71
\]

But 71 is not an even integer.

So, option A is incorrect.

\[
2x + 2 = 146 \\
2x = 144 \\
x = 72
\]

Therefore, the two even integers are 72 and 74. So, the correct choice is B.

**ANSWER:**
B

---

57. \( f(x) = \frac{1}{x^2 + 5x + 6} \)

**SOLUTION:**
\[
x^2 + 5x + 6 = 0 \\
(x + 3)(x + 2) = 0
\]

\[x = -3 \text{ or } x = -2\]

The function is undefined when \( x = -2 \) or \( x = -3 \).

The vertical asymptotes are at \( x = -3 \) and \( x = -2 \).

**ANSWER:**
asymptotes: \( x = -2, x = -3 \)

58. \( f(x) = \frac{x + 2}{x^3 + 3x - 4} \)

**SOLUTION:**
\[
f(x) = \frac{x + 2}{x^3 + 3x - 4} = \frac{x + 2}{(x + 4)(x - 1)}
\]

The function is undefined when \( x = -4 \) or \( x = 1 \).
Therefore, the vertical asymptotes are at \( x = -4 \) and \( x = 1 \).

**ANSWER:**
asymptotes: \( x = -4, x = 1 \)

---

8-5 Variation Functions
8-5 Variation Functions

59. \( f(x) = \frac{x^2 + 4x + 3}{x + 3} \)

**SOLUTION:**

\[
\begin{align*}
f(x) &= \frac{x^2 + 4x + 3}{x + 3} \\
&= \frac{(x + 3)(x + 1)}{x + 3} \\
&= x + 1, \quad (x \neq -3)
\end{align*}
\]

The function \( f(x) \) has a hole at \( x = -3 \).

**ANSWER:**
hole: \( x = -3 \)

60. **PHOTOGRAPHY** The formula \( \frac{1}{p} = \frac{1}{q} - \frac{1}{f} \) can be used to determine how far the film should be placed from the lens of a camera to create a perfect photograph. The variable \( q \) represents the distance from the lens to the film, \( f \) represents the focal length of the lens, and \( p \) represents the distance from the object to the lens.

- **a.** Solve the formula for \( \frac{1}{p} \).

- **b.** Write the expression containing \( f \) and \( q \) as a single rational expression.

- **c.** If a camera has a focal length of 8 centimeters and the lens is 10 centimeters from the film, how far should an object be from the lens so that the picture will be in focus?

**SOLUTION:**

- **a.**

\[
\begin{align*}
\frac{1}{p} &= \frac{1}{q} - \frac{1}{f} \\
\frac{1}{q} &= \frac{1}{f} - \frac{1}{p} \\
\frac{1}{p} &= \frac{1}{f} - \frac{1}{q}
\end{align*}
\]

- **b.**

\[
q - f
\]

\[
fq
\]

- **c.** Substitute 8 for \( f \), 10 for \( q \) in the equation

\[
\frac{1}{p} = \frac{10 - 8}{10} = \frac{2}{10} = \frac{1}{5}
\]

\[
p = 40
\]

The object should be 40 cm from the lens.

**ANSWER:**

- **a.**

\[
\frac{1}{p} = \frac{1}{q} - \frac{1}{f}
\]

- **b.**

\[
\frac{q - f}{fq}
\]

- **c.** 40 cm
8-5 Variation Functions

Solve each equation. Check your solutions.

61. \( \log_3 42 - \log_3 n = \log_3 7 \)

**SOLUTION:**
\[
\log_3 42 - \log_3 n = \log_3 7 \\
\log_3 \left( \frac{42}{n} \right) = \log_3 7 \\
\Rightarrow \frac{42}{n} = 7 \\
\Rightarrow n = \frac{42}{7} \\
\Rightarrow n = 6 \\
\]
Check:
\[
\log_3 42 - \log_3 6 = \log_3 7 \\
\log_3 \left( \frac{42}{6} \right) = \log_3 7 \\
\log_3 7 = \log_3 7 \checkmark \\
\]
The solution is 6.

**ANSWER:**
6

62. \( \log_2 (3x) + \log_2 5 = \log_2 30 \)

**SOLUTION:**
\[
\log_2 (3x) + \log_2 5 = \log_2 30 \\
\log_2 [3x \cdot 5] = \log_2 30 \\
\log_2 (15x) = \log_2 30 \\
15x = 30 \\
x = 2 \\
\]
Check:
\[
\log_2 6 + \log_2 5 = \log_2 30 \\
\log_2 (6 \cdot 5) = \log_2 30 \\
\log_2 30 = \log_2 30 \checkmark \\
\]
The solution is 2.

**ANSWER:**
2

63. \( 2 \log_5 x = \log_5 9 \)

**SOLUTION:**
\[
2 \log_5 x = \log_5 9 \\
\log_5 x^2 = \log_5 9 \\
x^2 = 9 \\
x = \sqrt{9} \\
x = 3 \\
\]
Check:
\[
2 \log_5 3 = \log_5 9 \\
\log_5 3^2 = \log_5 9 \\
\log_5 9 = \log_5 9 \checkmark \\
\]
The solution is 3.

**ANSWER:**
3
8-5 Variation Functions

64. \( \log_{10} a + \log_{10} (a + 21) = 2 \)

**SOLUTION:**
\[
\log_{10} a + \log_{10} (a + 21) = 2
\]
\[
\log_{10}[a(a + 21)] = 2
\]
\[
\log_{10}[a^2 + 21a] = 2
\]
\[
\Rightarrow a^2 + 21a = 10^2
\]
\[
a^2 + 21a - 100 = 0
\]
\[
(a + 25)(a - 4) = 0
\]
\[
a = -25 \text{ or } a = 4
\]

Check:
\[
\log_{10} 4 + \log_{10} (25) = 2
\]
\[
\log_{10} 100 = 2
\]
\[
\log_{10} 10^2 = 2
\]
\[
2 \log_{10} 10 = 2
\]
\[
2(1) = 2
\]
\[
2 = 2 \checkmark
\]

The solution is \( a = 4 \).

**ANSWER:**

4

---

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

65. \( 2x^3 - 5x^2 - 28x + 15; x - 5 \)

**SOLUTION:**

\[
\begin{array}{ccc}
2 & -5 & -28 & 15 \\
10 & 25 & -15 \\
2 & 5 & -3 & 0 \\
\end{array}
\]

The polynomial can be factored as \( (x - 5)(2x^2 + 5x - 3) \).

Factor the depressed polynomial \( 2x^2 + 5x - 3 \).

\[
2x^2 + 5x - 3 = 0
\]
\[
2x^2 + 6x - x - 3 = 0
\]
\[
2x(x + 3) - 1(x + 3) = 0
\]
\[
(x + 3)(2x - 1) = 0
\]
\[
x + 3 = 0 \text{ or } 2x - 1 = 0
\]
\[
x = -3 \text{ or } x = \frac{1}{2}
\]

The remaining factors of the given polynomial are \( x + 3, x - \frac{1}{2} \).

**ANSWER:**

\( x + 3, x - \frac{1}{2}, \text{ or } 2x - 1 \)
8-5 Variation Functions

66. \(3x^3 + 10x^2 - x - 12; x + 3\)

**SOLUTION:**

\[
\begin{array}{c|cccc}
-3 & 3 & 10 & -1 & -12 \\
-9 & 9 & -30 & 27 & 108 \\
3 & 1 & -4 & 0 & 0
\end{array}
\]

The polynomial can be factored as \((x+3)(3x^2 + x - 4)\).

Factor the depressed polynomial \(3x^2 + x - 4\).

\[
3x^2 + x - 4 = 0 \\
3x^2 - 3x + 4x - 4 = 0 \\
3x(x-1) + 4(x-1) = 0 \\
(x-1)(3x + 4) = 0
\]

\(x-1 = 0\) or \(3x + 4 = 0\)

\(x = 1\) or \(x = -\frac{4}{3}\)

The remaining factors of the given polynomial are \(x-1, x + \frac{4}{3}\).

**ANSWER:**

\(x-1, x + \frac{4}{3}, \text{ or } 3x + 4\)

**Find the LCM of each set of polynomials.**

67. \(a, 2a, a + 1\)

**SOLUTION:**

\[2a(a + 1)\]

**ANSWER:**

\[2a(a + 1)\]

68. \(x, 4y, x - y\)

**SOLUTION:**

\[4xy(x - y)\]

**ANSWER:**

\[4xy(x - y)\]

69. \(8, 24x, 12\)

**SOLUTION:**

\[8 = 2 \cdot 2 \cdot 2 \\
24x = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \\
12 = 2 \cdot 2 \cdot 3\]

Therefore, the LCM is \(2 \cdot 2 \cdot 2 \cdot 3 \cdot x = 24x\).

**ANSWER:**

\(24x\)

70. \(x^4, 3x^2, 2xy\)

**SOLUTION:**

\[x^4 = x \cdot x \cdot x \cdot x \\
3x^2 = 3 \cdot x \cdot x \\
2xy = 2 \cdot x \cdot y\]

Therefore, the LCM is \(3 \cdot 2 \cdot x \cdot x \cdot x \cdot y = 6x^3y\).

**ANSWER:**

\(6x^4y\)
8-5 Variation Functions

71. 12a, 15, 4b²

**SOLUTION:**

\[ 12a = 2 \cdot 2 \cdot 3 \cdot a \]
\[ 15 = 3 \cdot 5 \]
\[ 4b² = 2 \cdot 2 \cdot b \cdot b \]

Therefore, the LCM is \( 2 \cdot 2 \cdot 3 \cdot 5 \cdot a \cdot b \cdot b \) or \( 60ab² \).

**ANSWER:**

\[ 60ab² \]

72. \( x + 2, x - 3, x² - x - 6 \)

**SOLUTION:**

\[ x² - x - 6 = (x - 3)(x + 2) \]

Therefore, the LCM is \( x² - x - 6 \).

**ANSWER:**

\[ x² - x - 6 \]
8-6 Solving Rational Equations and Inequalities

Solve each equation. Check your solution.

1. \( \frac{4}{7} + \frac{3}{x-3} = \frac{53}{56} \)

SOLUTION:

\[
\frac{4}{7} + \frac{3}{x-3} = \frac{53}{56} \\
56(x-3)\left(\frac{4}{7}\right) + 56(x-3)\left(\frac{3}{x-3}\right) = 56(x-3)\left(\frac{53}{56}\right) \\
32(x-3) + 56(3) = 53(x-3) \\
32x - 96 + 168 = 53x - 159 \\
32x - 53x = -159 + 96 - 168 \\
-21x = -231 \\
x = 11
\]

Check:

\[
\frac{4}{7} + \frac{3}{11-3} = \frac{53}{56} \\
\frac{4}{7} + \frac{3}{8} = \frac{53}{56} \\
4(8) + 3(7) ? 53 \\
56 \quad 56 \\
32 + 21 ? 53 \\
56 \quad 56 \\
53 \quad 53 \quad \checkmark
\]

The solution is 11.

ANSWER:

11

2. \( \frac{7}{3} - \frac{3}{x-5} = \frac{19}{12} \)

SOLUTION:

\[
\frac{7}{3} - \frac{3}{x-5} = \frac{19}{12} \\
12(x-5)\left(\frac{7}{3}\right) - 12(x-5)\left(\frac{3}{x-5}\right) = 12(x-5)\left(\frac{19}{12}\right) \\
28(x-5) - 36 = 19x - 95 \\
28x - 140 - 36 = 19x - 95 \\
28x - 19x = -95 + 140 + 36 \\
9x = 81 \\
x = 9
\]

Check:

\[
\frac{7}{3} - \frac{3}{9-5} = \frac{19}{12} \\
\frac{7}{3} - \frac{3}{4} = \frac{19}{12} \\
\frac{7(4) - 3(3)}{12} = \frac{19}{12} \\
\frac{28 - 9}{12} = \frac{19}{12} \\
\frac{19}{19} \checkmark \\
\frac{12}{12} \checkmark
\]

The solution is 9.

ANSWER:

9
Solve each equation. Check your solution.

3. \[ \frac{10}{2x+1} + \frac{4}{3} = 2 \]

**SOLUTION:**

\[
\frac{10}{2x+1} + \frac{4}{3} = 2 \\
10(3) + 4(2x+1) \\
3(2x+1) \\
30 + 8x + 4 \\
3(2x+1) \\
8x + 34 = 6(2x + 1) \\
8x + 34 = 12x + 6 \\
-4x = 6 - 34 \\
-4x = -28 \\
x = 7 \\
\]

Check:

\[
\frac{10}{2(7)+1} + \frac{4}{3} = 2 \\
\frac{10}{14+1} + \frac{4}{3} = 2 \\
\frac{10}{15} + \frac{4}{3} = 2 \\
\frac{2}{3} + \frac{4}{3} = 2 \\
\frac{2+4}{3} = 2 \\
\frac{6}{3} = 2 \\
3 = 3 \checkmark \\
\]

The solution is 7.

**ANSWER:**

7

4. \[ \frac{11}{4} - \frac{5}{y+3} = \frac{23}{12} \]

**SOLUTION:**

\[
\frac{11}{4} - \frac{5}{y+3} = \frac{23}{12} \\
12(y+3)\frac{11}{4} - 12(y+3)\frac{5}{(y+3)} = 12(y+3)\frac{23}{12} \\
33y + 99 - 60 = 23(y + 3) \\
33y + 99 - 60 = 23y + 69 \\
33y - 23y = 69 - 99 \\
10y = 30 \\
y = 3 \\
\]

Check:

\[
\frac{11}{4} - \frac{5}{3+3} = \frac{23}{12} \\
\frac{11}{4} - \frac{5}{6} = \frac{23}{12} \\
\frac{11(6)}{24} - \frac{5(4)}{12} = \frac{23}{12} \\
\frac{66}{24} - \frac{20}{12} = \frac{23}{12} \\
\frac{46}{24} = \frac{23}{12} \\
\frac{23}{24} = \frac{23}{12} \checkmark \\
\]

The solution is 3.

**ANSWER:**

3
8-6 Solving Rational Equations and Inequalities

5. \( \frac{8}{x-5} - \frac{9}{x-4} = \frac{5}{x^2 - 9x + 20} \)

**SOLUTION:**
\[
\frac{8}{x-5} - \frac{9}{x-4} = \frac{5}{x^2 - 9x + 20} \\
8(x-4) - 9(x-5) = 5 \\
(x-5)(x-4) = x^2 - 9x + 20 \\
8x - 32 - 9x + 45 = 5 \\
x^2 - 9x + 20 = 5 \\
-x + 13 = 5 \\
x = 8
\]

Check:
\[
\frac{8}{8-5} - \frac{9}{3} = \frac{5}{64 - 72 + 20} \\
\frac{8}{3} - \frac{9}{12} = \frac{5}{64} \\
\frac{12 - 27}{12} = 5 \\
\frac{5}{12} = 5 \checkmark
\]

The solution is 8.

**ANSWER:**
8

6. \( \frac{14}{x+3} + \frac{10}{x-2} = \frac{122}{x^2 + x - 6} \)

**SOLUTION:**
\[
\frac{14}{x+3} + \frac{10}{x-2} = \frac{122}{x^2 + x - 6} \\
14(x-2) + 10(x+3) = 122(x-2)(x+3) \\
14x - 28 + 10x + 30 = 122x^2 + x - 6 \\
24x + 2 = 122x^2 + x - 6 \\
24x + 2 = 122x^2 + x - 6 \\
24x = 122 - 2 \\
x = 120 \\
x = 5
\]

Check:
\[
\frac{14}{5+3} + \frac{10}{5-2} = \frac{122}{5^2 + 5 - 6} \\
\frac{14}{8} + \frac{10}{3} = \frac{122}{24} \\
14(3) + 10(8) = 122 \\
42 + 80 = 122 \\
122 = 122 \checkmark
\]

The solution is \( x = 5 \).

**ANSWER:**
5
Solve each equation. Check your solution.

7. \( \frac{14}{x-8} - \frac{5}{x-6} = \frac{82}{x^2-14x+48} \)

**SOLUTION:**

\[
\begin{align*}
\frac{14}{x-8} - \frac{5}{x-6} &= \frac{82}{x^2-14x+48} \\
14(x-6) - 5(x-8) &= 82 \\
(2x)(x-6) &= x^2-14x+48 \\
14x - 84 - 5x + 40 &= x^2-14x+48 \\
x^2 - 14x + 48 &= x^2-14x+48 \\
9x - 44 + 82 &= 82 \\
x &= \frac{126}{9} \\
x &= 14
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{14}{14-8} - \frac{5}{14-6} &= ? \frac{82}{14^2-14(14)+48} \\
14 - \frac{5}{8} &= \frac{82}{196-196+48} \\
14 - \frac{5}{8} &= \frac{82}{48} \\
112 - 30 &= 82 \\
82 &= 82 \checkmark
\end{align*}
\]

The solution is 14.

**ANSWER:**

14

---

8. \( \frac{5}{x+2} - \frac{3}{x-2} = \frac{12}{x^2-4} \)

**SOLUTION:**

\[
\begin{align*}
\frac{5}{x+2} - \frac{3}{x-2} &= \frac{12}{x^2-4} \\
5(x-2) - 3(x+2) &= 12 \\
(x+2)(x-2) &= x^2-4 \\
5x - 10 - 3x - 6 &= 12 \\
x^2 - 4 &= x^2-4 \\
2x - 16 &= 12 \\
x^2 - 4 &= x^2-4 \\
2x - 16 &= 12 \\
2x &= 28 \\
x &= 14
\end{align*}
\]

Check:

\[
\begin{align*}
\frac{5}{14+2} - \frac{3}{14-2} &= ? \frac{12}{14^2-4} \\
5 - \frac{3}{16} &= \frac{12}{196-196+48} \\
5 - \frac{3}{16} &= \frac{12}{192} \\
60 - 48 &= 12 \\
12 &= 12 \checkmark
\end{align*}
\]

The solution is 14.

**ANSWER:**

14

---

9. **CCSS STRUCTURE** Sara has 10 pounds of dried fruit selling for $6.25 per pound. She wants to know how many pounds of mixed nuts selling for $4.50 per pound she needs to make a trail mix selling for $5 per pound.

a. Let \( m \) = the number of pounds of mixed nuts.

Complete the following table.
8-6 Solving Rational Equations and Inequalities

<table>
<thead>
<tr>
<th>Pounds</th>
<th>Price per Pound</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dried Fruit</td>
<td>10</td>
<td>$6.25</td>
</tr>
<tr>
<td>Mixed Nuts</td>
<td>m</td>
<td>$4.50</td>
</tr>
<tr>
<td>Trail Mix</td>
<td>10 + m</td>
<td>$5.00</td>
</tr>
</tbody>
</table>

b. Write a rational equation using the last column of the table.

c. Solve the equation to determine how many pounds of mixed nuts are needed.

SOLUTION:

a. 

<table>
<thead>
<tr>
<th>Pounds</th>
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<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Mixed Nuts</td>
<td>m</td>
<td>$4.50</td>
</tr>
<tr>
<td>Trail Mix</td>
<td>10 + m</td>
<td>$5.00</td>
</tr>
</tbody>
</table>

b. 

6.25(10) + 4.5m = 5(10 + m) 

62.5 + 4.5m = 50 + 5m

c. 62.5 + 4.5m = 50 + 5m

4.5m - 5m = 50 - 62.5

-0.5m = -12.5

m = 25

Therefore, 25 pounds of mixed nuts are needed.

ANSWER:

b. 62.5 + 4.5m = 50 + 5m

c. 25

10. DISTANCE Alicia’s average speed riding her bike is 11.5 miles per hour. She takes a round trip of 40 miles. It takes her 1 hour and 20 minutes with the wind and 2 hours and 30 minutes against the wind.

a. Write an expression for Alicia’s time with the wind.

b. Write an expression for Alicia’s time against the wind.

c. How long does it take to complete the trip?

d. Write and solve the rational equation to determine the speed of the wind.

SOLUTION:

a. Let x be the speed of the wind.
The expression for Alicia’s time with the wind is 

\[
\frac{20}{11.5 + x}
\]

b. The expression for Alicia’s time against the wind is 

\[
\frac{20}{11.5 - x}
\]

c. 

1 hr 20 minutes + 2 hr 30 minutes = 3 hr 50 minutes

d. 

\[
\frac{20}{11.5 + x} + \frac{20}{11.5 - x} = \frac{3}{6}
\]

\[
\frac{20}{11.5 + x} + \frac{20}{11.5 - x} = \frac{23}{6}
\]

\[
20(11.5 - x) + 20(11.5 + x) = \frac{23(11.5 + x)(11.5 - x)}{6}
\]

\[
230 - 20x + 230 + 20x = \frac{23(132.25 - x^2)}{6}
\]

\[
\frac{23(132.25 - x^2)}{6}
\]

\[
\frac{460}{132.25 - x^2} = \frac{23}{6}
\]

\[
460(6) = 23(132.25 - x^2)
\]

\[
2760 = 3041.75 - 23x^2
\]

\[
23x^2 = 3041.75 - 2760
\]

\[
23x^2 = 281.75
\]

\[
x^2 = 12.25
\]

\[
x = 3.5
\]

The speed of the wind is 3.5 mph.
ANSWER:

11. WORK Kendal and Chandi wax cars. Kendal can wax a particular car in 60 minutes and Chandi can wax the same car in 80 minutes. They plan on waxing the same car together and want to know how long it will take.

a. How much will Kendal complete in 1 minute?

b. How much will Kendal complete in \( x \) minutes?

c. How much will Chandi complete in 1 minute?

d. How much will Chandi complete in \( x \) minutes?

e. Write a rational equation representing Kendal and Chandi working together on the car.

f. Solve the equation to determine how long it will take them to finish the car.

SOLUTION:

a. \( \frac{1}{60} \)

b. \( \frac{x}{60} = \frac{x}{60} \)

c. \( \frac{1}{80} \)

d. \( \frac{x}{80} = \frac{x}{80} \)

e. \( \frac{x}{60} + \frac{x}{80} = \frac{1}{60} + \frac{1}{80} = \frac{2}{3} \)

f. about 34.3 min

Solve each inequality. Check your solutions.

12. \( \frac{3}{5x} + \frac{1}{6x} > \frac{2}{3} \)

SOLUTION:
The excluded value for this inequality is 0.

Solve the related equation \( \frac{3}{5x} + \frac{1}{6x} = \frac{2}{3} \).
8-6 Solving Rational Equations and Inequalities

\[
\frac{3}{5x} + \frac{1}{6x} = \frac{2}{3} \\
-\frac{18x + 5x}{30x^2} = \frac{2}{3} \\
\frac{23x}{30x^2} = \frac{2}{3} \\
\frac{23}{30x} = \frac{2}{3} \\
x = \frac{3(23)}{30(2)} \\
x = \frac{23}{20} \\
x = 1.15
\]

Divide the real line into three intervals as shown.

![Interval Diagram]

Test \( x = -1 \).

\[
\frac{3}{5(-1)} + \frac{1}{6(-1)} > \frac{2}{3} \\
\frac{-3}{5} + \frac{1}{6} > \frac{2}{3} \\
\frac{-18 - 5}{30} > \frac{2}{3} \\
\frac{-23}{30} > \frac{2}{3} \\
\]

Test \( x = 1 \).

\[
\frac{3}{5(1)} + \frac{1}{6(1)} > \frac{2}{3} \\
\frac{3}{5} + \frac{1}{6} > \frac{2}{3} \\
\frac{18 + 5}{30} > \frac{2}{3} \\
\frac{23}{30} > \frac{2}{3} \\
\]

Test \( x = 2 \).

\[
\frac{3}{5(2)} + \frac{1}{6(2)} > \frac{2}{3} \\
\frac{3}{10} + \frac{1}{12} > \frac{2}{3} \\
\frac{36 + 10}{120} > \frac{2}{3} \\
\frac{46}{120} > \frac{2}{3} \\
\frac{23}{60} > \frac{2}{3} \\
\]

Therefore, the solution is \( 0 < x < 1.15 \).

**ANSWER:**

\( 0 < x < 1.15 \)

13. \( \frac{1}{4c} + \frac{1}{9c} \leq \frac{1}{2} \)

**SOLUTION:**

The excluded value for this inequality is 0.

Solve the related equation \( \frac{1}{4c} + \frac{1}{9c} = \frac{1}{2} \).

\[
\frac{1}{4c} + \frac{1}{9c} = \frac{1}{2} \\
\frac{9c + 4c}{36c^2} = \frac{1}{2} \\
\frac{13c}{36c^2} = \frac{1}{2} \\
\frac{c}{36} = \frac{13}{18} \\
\]

Divide the real line into three intervals as shown.
Solve each equation. Check your solution.

1. SOLUTION:

Check:

The solution is 11.

Solve each equation. Check your solutions.

34.

SOLUTION:

Check:

Solve the related equation \( \frac{4}{3y} + \frac{2}{5y} = \frac{3}{2} \).

The excluded value for this inequality is \( y = 0 \).

Solve the related equation \( \frac{4}{3y} + \frac{2}{5y} = \frac{3}{2} \).

\[
\frac{4}{3y} + \frac{2}{5y} = \frac{3}{2}
\]

Divide the real line in to three intervals as shown.

\[
\frac{x}{2} + \frac{1}{4} < 0 \quad \text{or} \quad \frac{x}{2} - \frac{1}{4} > 0 \quad \text{or} \quad \frac{x}{2} + \frac{1}{4} > 0
\]

Test \( y = -1 \).

\[
\frac{4}{3(-1)} + \frac{2}{5(-1)} < \frac{3}{2}
\]

Test \( y = 1 \).

\[
\frac{4}{3(1)} + \frac{2}{5(1)} < \frac{3}{2}
\]

Therefore, the solution is \( c < 0 \) or \( \frac{13}{18} < c \).

ANSWER:

\( c < 0 \), or \( \frac{13}{18} < c \)

14. \( \frac{4}{3y} + \frac{2}{5y} < \frac{3}{2} \)

SOLUTION:

Therefore, the solution is extraneous.

Test \( y = 2 \).

\[
\frac{26}{15} < 2
\]

ANSWER:

\( y = 2 \)
Solve each equation. Check your solution.

1. \[ \frac{4}{3(2)} + \frac{2}{5(2)} < \frac{3}{2} \]

\[ \frac{4}{6} + \frac{2}{10} < \frac{3}{2} \]

\[ \frac{2}{3} + \frac{1}{5} < \frac{3}{2} \]

\[ \frac{10 + 3}{15} < \frac{3}{2} \]

\[ \frac{13}{15} < \frac{3}{2} \]

\[ \frac{13}{15} < \sqrt{3} \]

Therefore, the solution is \( y < 0 \) or \( y > \frac{52}{45} \).

**ANSWER:**
\( y > \frac{52}{45} \), or \( y < 0 \)

15. \[ \frac{1}{3b} + \frac{1}{4b} < \frac{1}{5} \]

**SOLUTION:**
The excluded value of this inequality is \( b = 0 \).

Solve the related equation \( \frac{1}{3b} + \frac{1}{4b} = \frac{1}{5} \).

\[ \frac{1}{3b} + \frac{1}{4b} = \frac{1}{5} \]

\[ 4b + 3b = 1 \]

\[ 12b^2 = 5 \]

\[ 7b = \frac{1}{2} \]

\[ 12b = 5 \]

\[ b = \frac{35}{12} \]

Divide the real line in to three intervals as shown.

Test \( b = -1 \).

\[ \frac{1}{3(-1)} + \frac{1}{4(-1)} < \frac{1}{5} \]

\[ \frac{1}{3} - \frac{1}{4} < \frac{1}{5} \]

\[ \frac{-4 - 3}{12} < \frac{1}{5} \]

\[ \frac{-7}{12} < \sqrt{3} \]

Therefore, the solution is \( b < 0 \), or \( \frac{35}{12} < b \).

**ANSWER:**
\( b < 0 \), or \( \frac{35}{12} < b \)
Solve each equation. Check your solutions.

16. \( \frac{9}{x-7} - \frac{7}{x-6} = \frac{13}{x^2-13x+42} \)

**SOLUTION:**
\[
\begin{align*}
\frac{9}{x-7} - \frac{7}{x-6} &= \frac{13}{x^2-13x+42} \\
9(x-6) - 7(x-7) &= 13(x-7)(x-6) \\
9x-54 - 7x + 49 &= 13x^2 - 13x + 42 \\
2x &= 13 \\
x &= 9
\end{align*}
\]

Check:
\[
\begin{align*}
\frac{9}{9-7} - \frac{7}{9-6} &= \frac{13}{9^2 - 13(9) + 42} \\
9 - 7 &= \frac{13}{9^2 - 13(9) + 42} \\
\frac{2}{3} &= \frac{13}{81 - 117 + 42} \\
\frac{27-14}{9} &= \frac{13}{6} \\
\frac{13}{6} &= \frac{13}{6} \checkmark
\end{align*}
\]

The solution is 9.

**ANSWER:**
9

17. \( \frac{13}{y+3} - \frac{12}{y+4} = \frac{18}{y^2 + 7y + 12} \)

**SOLUTION:**
\[
\begin{align*}
\frac{13}{y+3} - \frac{12}{y+4} &= \frac{18}{y^2 + 7y + 12} \\
13(y+4) - 12(y+3) &= 18(y^2 + 7y + 12) \\
13y + 52 - 12y - 36 &= 18y^2 + 13y + 52 - 36 \\
y^2 + 7y + 12 &= 18y^2 + 13y + 52 - 36 \\
y^2 + 7y + 12 &= 18y^2 + 13y + 16 \\
y + 16 &= 18y^2 + 13y + 16 \\
y &= 2
\end{align*}
\]

Check:
\[
\begin{align*}
\frac{13}{2+3} - \frac{12}{2+4} &= \frac{18}{2^2 + 7(2) + 12} \\
\frac{13}{5} - \frac{12}{6} &= \frac{18}{4 + 14 + 12} \\
\frac{78 - 60}{30} &= \frac{18}{30} \checkmark
\end{align*}
\]

The solution is 2.

**ANSWER:**
2
18. \( \frac{14}{x - 2} - \frac{18}{x + 1} = \frac{22}{x^2 - x - 2} \)

**SOLUTION:**

\[
\frac{14}{x - 2} - \frac{18}{x + 1} = \frac{22}{x^2 - x - 2} \\
14(x + 1) - 18(x - 2) = 22(x^2 - x - 2) \\
14x + 14 - 18x + 36 = 22x^2 - 22x - 44 \\
-4x + 50 = 22x^2 - 22x - 44 \\
-4x + 50 = 22 \\
-4x = -28 \\
x = 7
\]

Check:

\[
\frac{14}{7 - 2} - \frac{18}{5} = \frac{22}{49 - 7 - 2} \\
\frac{14}{5} - \frac{18}{49 - 7 - 2} = \frac{22}{40} \\
\frac{112 - 90}{40} = \frac{22}{40} \\
22 = 22 \checkmark
\]

The solution is 7.

**ANSWER:**

7

19. \( \frac{11}{a+2} - \frac{10}{a+5} = \frac{36}{a^2 + 7a + 10} \)

**SOLUTION:**

\[
\frac{11}{a+2} - \frac{10}{a+5} = \frac{36}{a^2 + 7a + 10} \\
11(a+5) - 10(a+2) = 36(a+2)(a+5) \\
11a + 55 - 10a - 20 = 36a^2 + 7a + 10 \\
11a + 35 = 36a^2 + 7a + 10 \\
a = 1
\]

Check:

\[
\frac{11}{1+2} - \frac{10}{1+5} = \frac{36}{1^2 + 7 + 10} \\
\frac{11}{3} - \frac{10}{6} = \frac{36}{18} \\
\frac{11}{3} - \frac{10}{6} = \frac{36}{18} \\
\frac{12 \cdot 6}{6} = 2 \\
2 = 2 \checkmark
\]

The solution is 1.

**ANSWER:**

1

20. \( \frac{x}{2x-1} + \frac{3}{x+4} = \frac{21}{2x^2 + 7x - 4} \)

**SOLUTION:**
Solve each equation. Check your solution.

1.

SOLUTION:

Check:

The solution is 11.

Solve each equation. Check your solutions.

34.

SOLUTION:

Check:

ANSWER:

21. \( \frac{y - 1}{y - 5} + \frac{2}{2y + 1} = \frac{2}{y^2 - 9y - 5} \)

SOLUTION:

\( \frac{2}{y - 5} + \frac{y - 1}{y - 5} \left( \frac{y - 5}{2y + 1} \right) = \frac{2}{2y^2 - 9y - 5} \)

\( \frac{4y + 2 + y^2 - 5y - y + 5}{2y^2 + y - 10y - 5} = \frac{2}{2y^2 - 9y - 5} \)

\( \frac{y^2 - 2y + 7}{2y^2 - 9y - 5} = \frac{2}{y^2 - 2y + 5} = 0 \)

\( y^2 - 2y + 5 = 0 \)

Use the Quadratic formula to solve \( y^2 - 2y + 5 = 0 \).

\( y = \frac{2 \pm \sqrt{4 - 20}}{2} \) (imaginary)

There is no real solution for the quadratic equation \( y^2 - 2y + 5 = 0 \). Therefore, the solution for the given rational equation is \( \emptyset \).

ANSWER:

\( \emptyset \)
22. **CHEMISTRY** How many milliliters of a 20% acid solution must be added to 40 milliliters of a 75% acid solution to create a 30% acid solution?

**SOLUTION:**
Let $x$ milliliters of a 20% acid solution is added to 40 milliliters of a 75% acid solution.

<table>
<thead>
<tr>
<th>Amount of Acid</th>
<th>Original</th>
<th>Added</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.75(40)</td>
<td>0.2(x)</td>
<td>0.75(40) + 0.2(x)</td>
</tr>
<tr>
<td>Total Solution</td>
<td>40</td>
<td>$x$</td>
<td>$40 + x$</td>
</tr>
</tbody>
</table>

percent $\frac{\text{amount of acid}}{\text{total solution}}$  
$\frac{30}{100} = \frac{0.75(40) + 0.2(x)}{40 + x}$  
$\frac{3}{10} = \frac{7.5(4) + 0.2(x)}{40 + x}$  
$3(40 + x) = 10[7.5(4) + 0.2(x)]$  
$120 + 3x = 300 + 2x$  
$x = 300 - 120$  
$x = 180$

Check:

$\frac{30}{100} = \frac{0.75(40) + 0.2(180)}{40 + 180}$  
$\frac{30}{100} = \frac{30 + 36}{220}$  
$\frac{30}{100} = \frac{66}{220}$  
$0.3 = 0.3$ ✓

Therefore, 180 milliliters of a 20% acid solution must be added to 40 milliliters of a 75% acid solution to create a 30% acid solution.

**ANSWER:**
180 mL

23. **GROCERIES** Ellen bought 3 pounds of bananas for $0.90 per pound. How many pounds of apples costing $1.25 per pound must she purchase so that the total cost for fruit is $1 per pound?

**SOLUTION:**
Let Ellen bought $x$ pounds of apples.

$3(0.90) + x(1.25) = (3 + x)(1)$  
$2.7 + 1.25x = 3 + x$  
$1.25x - x = 3 - 2.7$  
$0.25x = 0.3$  
$x = \frac{0.3}{0.25}$  
$x = 1.2$

She needs to purchase 1.2 pounds of apples.

**ANSWER:**
1.2 lb
24. **BUILDING** Bryan’s volunteer group can build a garage in 12 hours. Sequoia’s group can build it in 16 hours. How long would it take them if they worked together?

**SOLUTION:**
The rate for Bryan’s volunteer group is $\frac{1}{12}$.

The rate for Sequoia’s group is $\frac{1}{16}$.

Let their combined rate is $\frac{1}{x}$.

\[
\frac{\frac{1}{12} + \frac{1}{16}}{\frac{1}{x}} = \frac{\frac{1}{16}}{\frac{1}{x}}
\]

\[
\frac{192}{x} \cdot x = \frac{1}{12} \cdot 16 + \frac{1}{16} \cdot 12 = 1
\]

\[
\frac{2}{x} = 2
\]

\[
x = 192
\]

Therefore, it would take about 6.86 hours to build a garage if they worked together.

**ANSWER:**
about 6.86 hours

**Solve each inequality. Check your solutions.**

25. \[\frac{3 - \frac{4}{x}}{4x} > \frac{5}{4x}\]

**SOLUTION:**
The excluded value for this inequality is $x = 0$.

\[
3 - \frac{4}{x} > \frac{5}{4x}
\]

\[
\frac{3x - 4}{4x} > \frac{5}{4x}
\]

\[
3x - 4 > 5
\]

\[
3x > 9
\]

\[
x > 3
\]

Divide the real line in to three intervals as shown.

Test $x = -1$.

\[
3 - \frac{4}{(-1)} > \frac{5}{4(-1)}
\]

\[
3 + 4 > \frac{5}{4}
\]

\[
7 > \frac{5}{4}
\]

Test $x = 1$.

\[
3 - \frac{4}{1} > \frac{5}{4(1)}
\]

\[
3 - 4 > \frac{5}{4}
\]

\[
-1 > \frac{5}{4}
\]

Test $x = 2$.

\[
3 - \frac{4}{2} > \frac{5}{4(2)}
\]

\[
6 - 4 > \frac{5}{8}
\]

\[
2 > \frac{5}{8}
\]

\[
1 > \frac{5}{8}
\]

The solution for the inequality is $x < 0$ or $x > 1.75$. 

8-6 Solving Rational Equations and Inequalities

ANSWER:
\[ x < 0 \text{ or } x > 1.75 \]

26. \[ \frac{5}{3a} - \frac{3}{4a} > \frac{5}{6} \]

SOLUTION:
The excluded value for this inequality is \( a = 0 \).

\[
\begin{align*}
\frac{5}{3a} - \frac{3}{4a} &> \frac{5}{6} \\
\frac{20a - 9a}{12a^2} &> \frac{5}{6} \\
\frac{11a}{12a^2} &> \frac{5}{6} \\
\frac{11}{12a} &> \frac{5}{6} \\
66 &> 60a \\
60a &< 66 \\
a &< \frac{66}{60} \\
a &< \frac{11}{10} \\
a &< 1.1
\end{align*}
\]

Divide the inequality in to three intervals as shown.

Test \( a = -1 \).

\[
\begin{align*}
\frac{5}{-3} - \frac{3}{4(-1)} &> \frac{5}{6} \\
\frac{-5}{4} + \frac{3}{4} &> \frac{5}{6} \\
\frac{-20 + 9}{12} &> \frac{5}{6} \\
\frac{-11}{12} &> \frac{5}{6}
\end{align*}
\]

Test \( a = 1 \).

\[
\begin{align*}
\frac{5}{3} - \frac{3}{4} &> \frac{5}{6} \\
\frac{20 - 9}{12} &> \frac{5}{6} \\
\frac{11}{12} &> \frac{5}{6}
\end{align*}
\]

Test \( a = 2 \).

\[
\begin{align*}
\frac{5}{6} - \frac{3}{8} &< \frac{5}{6} \\
\frac{20 - 9}{24} &< \frac{5}{6} \\
\frac{11}{24} &< \frac{5}{6}
\end{align*}
\]

Therefore, the solution set is \( 0 < a < 1.1 \).

ANSWER:
\[ 0 < a < 1.1 \]

27. \[ \frac{x - 2}{x + 2} + \frac{1}{x - 2} > \frac{x - 4}{x - 2} \]

SOLUTION:
The excluded values for this inequality is \( x = -2 \) and \( x = 2 \).

Solve the related equation \[ \frac{x - 2}{x + 2} + \frac{1}{x - 2} = \frac{x - 4}{x - 2} \].

\[
\begin{align*}
\frac{x - 2}{x + 2} + \frac{1}{x - 2} & = \frac{x - 4}{x - 2} \\
(x - 2)(x - 2) + (x + 2) & = x - 4 \\
(x + 2)(x - 2) & = x - 2 \\
x^2 - 4x + 4 + x + 2 & = x - 4 \\
(x + 2) & = x - 4 \\
x^2 - 3x + 6 & = (x - 4)(x + 2) \\
x^2 - 3x + 6 & = x^2 + 2x - 4x - 8 \\
x & = -8 - 6 \\
x & = -14 \\
x & = 14
\end{align*}
\]
8-6 Solving Rational Equations and Inequalities

Divide the real line in to four intervals as shown.

Test \( x = -4 \).
\[
\begin{align*}
\frac{-4 - 2}{-4 + 2} + \frac{1}{4 - 2} < & -4 - 4 \\
\frac{-6}{-2} + \frac{1}{2} & < -6
\end{align*}
\]
\[
\begin{align*}
\frac{3}{-2} & < 0
\end{align*}
\]
\[
\begin{align*}
\frac{18 - 1}{6} & > 3 \\
\frac{17}{6} & > 3
\end{align*}
\]
\[
\begin{align*}
\frac{17}{6} & > 3 \\
\frac{5}{6} & > 0 \checkmark
\end{align*}
\]

Test \( x = 0 \).
\[
\begin{align*}
\frac{-2 + 1}{2} < & -4 \\
-1 & < 2
\end{align*}
\]
\[
\begin{align*}
\frac{-2 - 1}{2} & > 2 \\
-3 & > 2
\end{align*}
\]
\[
\begin{align*}
\frac{-3}{2} & > 2 \\
-1.5 & > 2
\end{align*}
\]

Test \( x = 4 \).
\[
\begin{align*}
\frac{4 - 2}{4 + 2} + \frac{1}{4 - 2} < & 4 - 4 \\
\frac{2}{6} + \frac{1}{2} & < 0 \\
\frac{2 + 3}{6} & > 0
\end{align*}
\]
\[
\begin{align*}
\frac{5}{6} & > 0 \checkmark
\end{align*}
\]

Test \( x = 16 \).
\[
\begin{align*}
\frac{16 - 2}{16 + 2} + \frac{1}{16 - 2} > & 16 - 4 \\
\frac{14}{18} + \frac{1}{14} & > 14
\end{align*}
\]
\[
\begin{align*}
\frac{7}{9} + \frac{1}{14} & > 7 \\
\frac{98 + 9}{126} & > 7
\]
\[
\begin{align*}
\frac{107}{126} & > 7
\end{align*}
\]

Therefore, the solution set for the inequality is \( x < -2 \) or \( 2 < x < 14 \).

\textbf{Answer:}

\( x < -2 \), or \( 2 < x < 14 \)

28. \( \frac{3}{4} - \frac{1}{x - 3} > \frac{x}{x + 4} \)

\textbf{Solution:}

The excluded value for this inequality is \( x = 3 \) and \( x = -4 \).

Solve the related equation \( \frac{3}{4} - \frac{1}{x - 3} = \frac{x}{x + 4} \).

\[
\begin{align*}
\frac{3}{4} - \frac{1}{x - 3} = \frac{x}{x + 4}
\end{align*}
\]
\[
\begin{align*}
3(x - 3) - 4 & = x(x + 4) \\
(3x - 9 - 4)(x + 4) & = x \\
(3x - 13)(x + 4) & = x(4x - 12) \\
3x^2 + 12x - 13x - 52 & = 4x^2 - 12x \\
-x^2 + 11x - 52 & = 0 \\
x^2 - 11x + 52 & = 0
\end{align*}
\]

There exists no real solution for the quadratic equation \( x^2 - 11x + 52 = 0 \).

Divide the real line in to three intervals as shown.
Solve each equation. Check your solution.

1. 

SOLUTION: 

Check: 

The solution is 11. 

Solve each equation. Check your solutions.

34. 

SOLUTION: 

Check:

The excluded value for this inequality is $x = 4$.

Solve the related equation 

\[
\frac{x}{5} + \frac{2}{3} = \frac{3}{x - 4}.
\]

\[
\frac{x}{5} + \frac{2}{3} = \frac{3}{x - 4}
\]

\[
\frac{3x + 10}{15} = \frac{3}{x - 4}
\]

\[
(3x + 10)(x - 4) = 45
\]

\[
3x^2 - 12x + 10x - 40 = 45
\]

\[
3x^2 - 2x - 85 = 0
\]

Solve the quadratic equation using the Quadratic formula.

\[
x = \frac{2 \pm \sqrt{4 - 4(3)(-85)}}{6}
\]

\[
= \frac{2 \pm \sqrt{1024}}{6}
\]

\[
= \frac{2 \pm 32}{6}
\]

\[
x = \frac{2 + 32}{6} \text{ or } x = \frac{2 - 32}{6}
\]

\[
x = \frac{34}{6} \text{ or } x = -\frac{30}{6}
\]

\[
x = \frac{17}{3} \text{ or } x = -5
\]

Divide the real line in to 4 intervals as shown.

Test $x = -6$. 

The solution set is 

\[-4 < x < 3 \]

ANSWER: 

\[-4 < x < 3 \]

29. 

\[
\frac{x}{5} + \frac{2}{3} < \frac{3}{x - 4}
\]
8-6 Solving Rational Equations and Inequalities

\[-6 + \frac{2}{3} < \frac{3}{-6 - 4} \]
\[-6 + \frac{2}{3} < -\frac{3}{10} \]
\[-18 + 10 \frac{8}{3} < \frac{3}{10} \]
\[-18 + 10 \frac{8}{15} < \frac{3}{10} \]

Test \(x = 0\).
\[0 + \frac{2}{3} < -\frac{3}{4} \]
\[\frac{2}{3} < -\frac{3}{4} \]

Test \(x = 5\).
\[5 + \frac{2}{3} < \frac{3}{5} \]
\[\frac{2}{3} < \frac{3}{5} \]
\[1 + \frac{2}{3} < \frac{3}{1} \]
\[\frac{3}{2} < 1 \]
\[\frac{5}{3} < 1 \]

Test \(x = 6\).
\[6 + \frac{2}{3} < \frac{3}{6 - 4} \]
\[\frac{6}{3} < \frac{3}{3} \]
\[18 + 10 \frac{8}{15} < \frac{3}{1} \]
\[\frac{28}{15} < 1 \]

The solution set for the inequality is \(x < -5\) or \(4 < x < \frac{17}{3}\).

**ANSWER:**
\(x < -5\) or \(4 < x < \frac{17}{3}\)

30. \(\frac{x}{x + 2} + \frac{1}{x - 1} < \frac{3}{2} \)

**SOLUTION:**
The excluded values for this inequality are \(x = -2\) and \(x = 1\).

Solve the related equation \(\frac{x}{x + 2} + \frac{1}{x - 1} = \frac{3}{2}\).

\[\frac{x}{x + 2} + \frac{1}{x - 1} = \frac{3}{2} \]
\[\frac{x(x - 1) + (x + 2)}{(x + 2)(x - 1)} = \frac{3}{2} \]
\[\frac{x^2 - x + x + 2}{x^2 - x + 2x - 2} = \frac{3}{2} \]
\[\frac{x^2 + 2}{x^2 + x - 2} = \frac{3}{2} \]
\[\frac{2x^2 + 4}{x^2 + x - 2} = 3(x^2 + x - 2) \]
\[2x^2 + 4 = 3x^2 + 3x - 6 \]
\[x^2 - 3x + 10 = 0 \]
\[x^2 + 3x - 10 = 0 \]
\[(x + 5)(x - 2) = 0 \]
\[x + 5 = 0 \text{ or } x - 2 = 0 \]
\[x = -5 \text{ or } x = 2 \]

Divide the real line in to 5 intervals as shown.

\[\begin{array}{cccccc}
-6 & -4 & -3 & -1 & 0 & 2 & 3 & 4 & 5 & 6 \\
\end{array} \]

Test \(x = -6\).
\[\frac{-6 + 1}{-6 + 2} < \frac{3}{2} \]
\[\frac{-6 + 1}{-6 + 2} < \frac{3}{2} \]
\[\frac{-6}{-4} < \frac{3}{2} \]
\[\frac{3}{2} < \frac{3}{2} \]
\[\frac{21 - 2}{14} < \frac{3}{2} \]
\[\frac{19}{14} < \frac{3}{2} \]
\[19 < 3 \]
\[14 < \frac{3}{2} \]
Test $x = -4$. 
\[
\begin{align*}
\frac{-4}{-4 + 2} + \frac{1}{-4 - 1} & \leq \frac{3}{2} \\
\frac{-4}{-2} + \frac{1}{5} & \leq \frac{3}{2} \\
2 - \frac{1}{5} & \leq \frac{3}{2} \\
\frac{10 - 1}{5} & \leq \frac{3}{2} \\
\frac{9}{5} & \leq \frac{3}{2} \\
\end{align*}
\]
Test $x = 0$. 
\[
\begin{align*}
\frac{0}{0 + 2} + \frac{1}{0 - 1} & \leq \frac{3}{2} \\
-1 & < \frac{3}{2} \\
\end{align*}
\]
Test $x = \frac{3}{2}$. 
\[
\begin{align*}
\frac{\frac{3}{2}}{\frac{3}{2} + 2} + \frac{1}{\frac{3}{2} - 1} & \leq \frac{3}{2} \\
\frac{\frac{3}{2}}{\frac{3}{2}} + \frac{1}{\frac{1}{2}} & \leq \frac{3}{2} \\
\frac{3}{2} + 2 & \leq \frac{3}{2} \\
\frac{9}{14} + \frac{2}{2} & \leq \frac{3}{2} \\
\frac{9 + 28}{14} & \leq \frac{3}{2} \\
\frac{37}{14} & \leq \frac{3}{2} \\
\frac{2}{5} & \leq \frac{3}{2} \\
\end{align*}
\]
Test $x = 4$. 
\[
\begin{align*}
\frac{4}{4 + 2} + \frac{1}{4 - 1} & < \frac{3}{2} \\
\frac{4}{6} + \frac{1}{3} & < \frac{3}{2} \\
\frac{2}{3} + \frac{1}{3} & < \frac{3}{2} \\
1 & < \frac{3}{2} \\
\end{align*}
\]
The solution set for the inequality is $x < -5$ or $-2 < x < 1$ or $x > 2$. 

**ANSWER:** 
$2 < x, -2 < x < 1, x < -5$

31. **AIR TRAVEL** It takes a plane 20 hours to fly to its destination against the wind. The return trip takes 16 hours. If the plane’s average speed in still air is 500 miles per hour, what is the average speed of the wind during the flight?

**SOLUTION:**

<table>
<thead>
<tr>
<th>Distance traveled with the wind</th>
<th>Distance traveled against the wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(500 + w)(16)$</td>
<td>$(500 - w)(20)$</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(500 + w)(16) & = (500 - w)(20) \\
(500 + w)(4) & = (500 - w)(5) \\
2000 + 4w & = 2500 - 5w \\
9w & = 500 \\
w & = \frac{500}{9} \\
w & \approx 55.56 \\
\end{align*}
\]
The average speed of the wind during the flight is about 55.56 miles per hour.

**ANSWER:** 
55.56 mph
32. **FINANCIAL LITERACY** Judie wants to invest $10,000 in two different accounts. The risky account earns 9% interest, while the other account earns 5% interest. She wants to earn $750 interest for the year. Of tables, graphs, or equations, choose the best representation needed and determine how much should be invested in each account.

**SOLUTION:**
Judie invests $x$ dollars in the account earns 9% interest and $(10000 - x)$ dollars in the account earns 5% interest.

\[
0.09x + 0.05(10000 - x) = 750
\]
\[
0.09x + 500 - 0.05x = 750
\]
\[
0.04x = 250
\]
\[
x = \frac{250}{0.04}
\]
\[
x = 6250
\]

Thus, Judie should invest $6250 at 9% account and $3750 at 5% account.

**ANSWER:**
$6250 at 9% and $3750 at 5%.

33. **MULTIPLE REPRESENTATIONS** Consider \[
\frac{2}{x - 3} + \frac{1}{x} = \frac{x - 1}{x - 3}.
\]

**a. ALGEBRAIC** Solve the equation for $x$. Were any values of $x$ extraneous?

\[
\frac{2}{x - 3} + \frac{1}{x} = \frac{x - 1}{x - 3}
\]
\[
2x + x - 3 = x - 1
\]
\[
x(x - 3) = x - 3
\]
\[
3x - 3 = \frac{x - 1}{x - 3}
\]
\[
3x - 3 = x(x - 1)
\]
\[
x^2 - 4x + 3 = 0
\]
\[
(x - 3)(x - 1) = 0
\]
\[
x = 3 \text{ or } x = 1
\]

Check: $x = 1$
\[
\frac{2}{1 - 3} + \frac{1}{1} = \frac{1 - 1}{1 - 3}
\]
\[
-1 + 1 = 0
\]
\[
0 = 0 \checkmark
\]

$x = 3$ is the excluded value for the equation. Therefore, $x = 3$ is the extraneous solution and $x = 1$ is the solution for the equation.

**b.**

\[
\begin{align*}
y_1 &= \frac{2}{x - 3} + \frac{1}{x} \\
y_2 &= \frac{x - 1}{x + 3}
\end{align*}
\]

c. Two graphs intersect at $x = 1$ and they do not intersect at the extraneous solution $x = 3$.]

**d.** Graph both sides of the equation. Where the graphs intersect, there is a solution. If they do not, then the possible solution is extraneous.

**ANSWER:**
Solve each equation. Check your solutions.

34. \( \frac{2}{y+3} - \frac{3}{4-y} = \frac{2y-2}{y^2-y-12} \)

**SOLUTION:**

\[
\begin{align*}
\frac{2}{y+3} - \frac{3}{4-y} &= \frac{2y-2}{y^2-y-12} \\
\frac{2}{y+3} + \frac{3}{y-4} &= \frac{2y-2}{y^2-y-12} \\
2(y-4) + 3(y+3) &= 2y-2 \\
(y+3)(y-4) &= y^2-y-12 \\
2y-8 + 3y+9 &= 2y-2 \\
y^2 - y-12 &= y^2 - y-12 \\
5y+1 &= 2y-2 \\
3y &= -3 \\
y &= -1
\end{align*}
\]

test:

\[
\begin{align*}
2 - 3 &= -2-2 \\
5 &= 1+1-12 \\
1 - 3 &= -4 \\
5 &= -10 \\
5 - 3 &= 2 \\
5 &= 5 \\
2 &= \sqrt{5} \\
5 &= 5
\end{align*}
\]

The solution is \( y = -1 \).

**ANSWER:**

\(-1\)
35. \[ \frac{2}{y+2} - \frac{y}{2-y} = \frac{y^2 + 4}{y^2 - 4} \]

**SOLUTION:**

\[
\frac{2}{y+2} + \frac{y}{y-2} = \frac{y^2 + 4}{y^2 - 4} \\
2(y+2) + y(y+2) = (y+2)(y-2) \\
2y + 4 + y^2 + 2y = y^2 - 4 \\
y^2 + 4y + 4 = y^2 + 4 \\
4y = 0 \\
y = 0
\]

Check:

\[
\frac{2}{2} - 0 = \frac{4}{4} \\
2 - 0 = 2
\]

1 ≠ -1

The solution set is \(\emptyset\).

**ANSWER:**

\(\emptyset\)

36. **OPEN ENDED** Give an example of a rational equation that can be solved by multiplying each side of the equation by \(4(x + 3)(x - 4)\).

**SOLUTION:**

Sample answer:

\[ \frac{4}{x+3} = \frac{x}{x-4} + \frac{7}{4} \]

**ANSWER:**

Sample answer: \[ \frac{4}{x+3} = \frac{x}{x-4} + \frac{7}{4} \]

37. **CHALLENGE** Solve \[ \frac{9x + 20}{x^2 - 25} = \frac{x + 4}{x - 5} \]

**SOLUTION:**

\[
\frac{1 + \frac{9}{x} + \frac{20}{x^2}}{1 - \frac{25}{x^2}} = \frac{x + 4}{x - 5} \\
\frac{x^2 + 9x + 20}{x^2 - 25} = \frac{x + 4}{x - 5} \\
\frac{(x + 5)(x - 5)}{(x + 5)(x - 5)} = \frac{x + 4}{x - 5} \\
\frac{x^2 + 9x + 20}{x + 5} = \frac{x + 4}{x - 5} \\
x^2 + 9x + 20 = (x + 4)(x + 5) \\
x^2 + 9x + 20 = x^2 + 9x + 20
\]

Therefore, the solution is all real numbers except 5, -5, and 0.

**ANSWER:**

all real numbers except 5, -5, 0
38. **CCSS TOOLS** While using the table feature on the graphing calculator to explore \( f(x) = \frac{1}{x^2 - x - 6} \), the values \(-2\) and \(3\) say “**ERROR**.” Explain its meaning.

**SOLUTION:**
Sample answer:

\[
f(x) = \frac{1}{x^2 - x - 6} = \frac{1}{(x - 3)(x + 2)}
\]

The denominator will equal 0 when \( x = -2 \) or \( x = 3 \). The values \(-2\) and \(3\) are undefined values. On the graph of \(f(x)\) these values would be vertical asymptotes at these values.

**ANSWER:**
Sample answer: The values \(-2\) and \(3\) are undefined values. On the graph of \(f(x)\) these values would be vertical asymptotes at these values.

39. **WRITING IN MATH** Why should you check solutions of rational equations and inequalities?

**SOLUTION:**
Sample answer: Multiplying each side of a rational equation or inequality by the LCD can result in extraneous solutions. Therefore, you should check all solutions to make sure that they satisfy the original equation or inequality.

**ANSWER:**
Sample answer: Multiplying each side of a rational equation or inequality by the LCD can result in extraneous solutions. Therefore, you should check all solutions to make sure that they satisfy the original equation or inequality.

40. Nine pounds of mixed nuts containing 55% peanuts were mixed with 6 pounds of another kind of mixed nuts that contain 40% peanuts. What percent of the new mixture is peanuts?

A 58%

B 51%

C 49%

D 47%

**SOLUTION:**
Let the new mixture contains \( x \) percent of peanuts.

\[
0.55(9) + 0.4(6) = x(9 + 6)
\]

\[
4.95 + 2.4 = 15x
\]

\[
7.35 = 15x
\]

\[
x = \frac{7.35}{15}
\]

\[
x = 0.49
\]

So, the new mixture contains 0.49 or 49% percent of peanuts. The correct choice is C.

**ANSWER:**
C
41. Working alone, Dato can dig a 10-foot by 10-foot hole in five hours. Pedro can dig the same hole in six hours. How long would it take them if they worked together?

\[ \frac{1}{5} + \frac{1}{6} = \frac{1}{x} \]
\[ \frac{6+5}{30} = \frac{1}{x} \]
\[ \frac{11}{30} = \frac{1}{x} \]
\[ x = \frac{30}{11} \]
\[ x \approx 2.73 \]

It would take about 2.73 hours to dig the hole if they worked together. The correct choice is J.

**ANSWER:** J

42. An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged 6 kilometers per hour on the return trip. Find the average speed of the trip to Guam.

\[ \text{Speed} = \frac{\text{distance}}{\text{time}} \]
\[ 6 = \frac{x}{4} \]
\[ x = 24 \]

Therefore, the distance of the trip to Guam is 24 kilometers.

\[ \text{Speed} = \frac{\text{distance}}{\text{time}} \]
\[ = \frac{24}{3} \]
\[ = 8 \]

The average speed of the trip to Guam is 8 km/h. So, the correct choice is B.

**ANSWER:** B
43. SHORT RESPONSE If a line $\ell$ is perpendicular to a segment $CD$ at point $F$ and $CF = FD$, how many points on line $\ell$ are the same distance from point $C$ as from point $D$?

**SOLUTION:**

all of the points

**ANSWER:**

all of the points

Determine whether each relation shows direct or inverse variation, or neither.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>1.5</td>
</tr>
<tr>
<td>56</td>
<td>0.75</td>
</tr>
<tr>
<td>112</td>
<td>0.375</td>
</tr>
</tbody>
</table>

44. **SOLUTION:**

$$xy = 42$$ (a constant) for all $x$ and $y$ values in the relation, the relation is an inverse variation.

**ANSWER:**

inverse

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$\frac{y}{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td>216</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>648</td>
<td></td>
</tr>
</tbody>
</table>

45. **SOLUTION:**

$$\frac{y}{x} = 120$$ (a constant) for all $x$ and $y$ values in the relation, the relation is a direct variation.

**ANSWER:**

direct

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td>72</td>
<td>9</td>
</tr>
</tbody>
</table>

46. **SOLUTION:**

Since neither $xy$ or $\frac{y}{x}$ are constant, the relation is neither direct nor inverse variation.

**ANSWER:**

neither
8-6 Solving Rational Equations and Inequalities

Graph each function.

47. \( f(x) = \frac{x + 4}{x^2 + 7x + 12} \)

**SOLUTION:**

\[
 f(x) = \frac{x + 4}{x^2 + 7x + 12} = \frac{x + 4}{(x + 4)(x + 3)} = \frac{1}{x + 3}
\]

The vertical asymptote is at \( x = -3 \) and there is a hole at \( x = -4 \). The horizontal asymptote is at \( y = 0 \).

48. \( f(x) = \frac{x^2 - 5x - 14}{x - 7} \)

**SOLUTION:**

\[
 f(x) = \frac{x^2 - 5x - 14}{x - 7} = \frac{(x - 7)(x + 2)}{x - 7} = x + 2
\]

Therefore, the graph of \( f(x) = \frac{x^2 - 5x - 14}{x - 7} \) is same as the graph of \( f(x) = x + 2 \) with a hole at \( x = 7 \).
49. \( f(x) = \frac{x^3 + 3x - 6}{x - 2} \)

**SOLUTION:**
The vertical asymptote is at \( x = 2 \). Since the degree of the numerator is greater than the denominator, there is no horizontal asymptote.

\[
\frac{x + 5}{x - 2} \cdot \frac{x^2 + 3x - 6}{x^2 - 2x} = \frac{5x - 6}{5x - 10}
\]

Therefore, the oblique asymptote is \( y = x + 5 \).

**ANSWER:**

50. **WEATHER** The atmospheric pressure \( P \), in bars, of a given height on Earth is given by using the formula

\[ P = a \cdot e^{-\frac{k}{H}}. \]

In the formula, \( a \) is the surface pressure on Earth, which is approximately 1 bar, \( k \) is the altitude for which you want to find the pressure in kilometers, and \( H \) is always 7 kilometers.

**a.** Find the pressure for 2, 4, and 7 kilometers.

**b.** What do you notice about the pressure as altitude increases?

**SOLUTION:**

**a.**

<table>
<thead>
<tr>
<th>( k )</th>
<th>( P = e^{\frac{k}{7}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( P = e^{\frac{2}{7}} )</td>
</tr>
<tr>
<td></td>
<td>( \approx 0.75 )</td>
</tr>
<tr>
<td>4</td>
<td>( P = e^{\frac{4}{7}} )</td>
</tr>
<tr>
<td></td>
<td>( \approx 0.56 )</td>
</tr>
<tr>
<td>7</td>
<td>( P = e^{\frac{7}{7}} )</td>
</tr>
<tr>
<td></td>
<td>( = e^1 )</td>
</tr>
<tr>
<td></td>
<td>( \approx 0.37 )</td>
</tr>
</tbody>
</table>

**b.** The pressure decreases as the altitude increases.

**ANSWER:**

**a.** 0.75 bars; 0.56 bars; 0.37 bars

**b.** The pressure decreases as the altitude increases.
51. **COMPUTERS** Since computers have been invented, computational speed has multiplied by a factor of 4 about every three years.

a. If a typical computer operates with a computational speed $s$ today, write an expression for the speed at which you can expect an equivalent computer to operate after $x$ three-year periods.

b. Suppose your computer operates with a processor speed of 2.8 gigahertz and you want a computer that can operate at 5.6 gigahertz. If a computer with that speed is currently unavailable for home use, how long can you expect to wait until you can buy such a computer?

**SOLUTION:**

a. $s \cdot 4^x$

b. Substitute 2.8 for $s$ and 5.6 for $f(x)$ in the equation $f(x) = s \cdot 4^x$.

$$f(x) = s \cdot 4^x$$

$$5.6 = 2.8(4)^x$$

$$2 = 4^x$$

$$2 = 2^{2x}$$

$$1 = 2^x$$

$$x = 0.5$$

So, we can expect 0.5 three-year periods or 1.5 year to buy 5.6 gigahertz speed home computers.

**ANSWER:**

a. $s \cdot 4^x$

b. 0.5 three-yr periods or 1.5 yr

---

52. Determine whether the following are possible lengths of the sides of a right triangle.

52. 5, 12, 13

**SOLUTION:**

Because the longest side is 13 units, use 13 as $c$, the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$

$$13^2 = 5^2 + 12^2$$

$$169 = 25 + 144$$

$$169 = 169 \checkmark$$

Because $c^2 = a^2 + b^2$, the triangle is a right triangle.

**ANSWER:**

yes

---

53. 60, 80, 100

**SOLUTION:**

Because the longest side is 100 units, use 100 as $c$, the measure of the hypotenuse.

$$c^2 = a^2 + b^2$$

$$100^2 = 60^2 + 80^2$$

$$10000 = 3600 + 6400$$

$$10000 = 10000 \checkmark$$

Because $c^2 = a^2 + b^2$, the triangle is a right triangle.

**ANSWER:**

yes
54. 7, 24, 25

**SOLUTION:**
Because the longest side is 25 units, use 25 as \( c \), the measure of the hypotenuse.

\[
c^2 = a^2 + b^2
\]

\[
25^2 = 24^2 + 7^2
\]

\[
625 = 576 + 49
\]

\[
625 = 625 \checkmark
\]

Because \( c^2 = a^2 + b^2 \), the triangle is a right triangle.

**ANSWER:**
yes
Simplify each expression.

1. \( \frac{r^2 + rs}{2r} \div \frac{r + s}{16r^2} \)

   **SOLUTION:**
   \[
   \frac{r^2 + rs}{2r} \div \frac{r + s}{16r^2} = \frac{r^2 + rs}{2r} \cdot \frac{16r^2}{r + s} = \frac{r(r + s)}{2r} \cdot \frac{16r^2}{r + s} = 8r^2
   \]

   **ANSWER:**
   \( 8r^2 \)

2. \( \frac{m^2 - 4 \cdot 6m}{3m^2 \cdot 2 - m} \)

   **SOLUTION:**
   \[
   \frac{m^2 - 4 \cdot 6m}{3m^2 \cdot 2 - m} = \frac{(m + 2)(m - 2)}{3m^2 \cdot 2 - m} \cdot \frac{6m}{(m - 2)} = \frac{6m(m + 2)(m - 2)}{-3m^2(m - 2)} = \frac{2(m + 2)}{m}
   \]

   **ANSWER:**
   \( \frac{2(m + 2)}{m} \)

3. \( \frac{m^2 + m - 6}{n^2 - 9} \div \frac{m - 2}{n + 3} \)

   **SOLUTION:**
   \[
   \frac{m^2 + m - 6}{n^2 - 9} \div \frac{m - 2}{n + 3} = \frac{(m + 3)(m - 2)}{(n + 3)(n - 3)(m - 2)} = \frac{n + 3}{n - 3}
   \]

   **ANSWER:**
   \( \frac{n + 3}{n - 3} \)

4. \( \frac{x^2 + 4x + 3}{x^2 - x - 20} \)

   **SOLUTION:**
   \[
   \frac{x^2 + 4x + 3}{x^2 - x - 20} = \frac{x^2 + 4x + 3}{x^2 - 2x - 15 \cdot x^2 - x - 20} = \frac{(x + 3)(x + 1)(x - 5)(x + 4)}{(x + 3)(x + 1)(x - 5)(x + 4)} = \frac{x + 4}{x - 1}
   \]

   **ANSWER:**
   \( \frac{x + 4}{x - 1} \)
5. \[ \frac{x + 4}{6x + 3} + \frac{1}{2x + 1} \]

**SOLUTION:**

\[
\frac{x + 4}{6x + 3} + \frac{1}{2x + 1} = \frac{x + 4}{3(2x + 1)} + \frac{1}{2x + 1} \\
= \frac{x + 4 + 3}{3(2x + 1)} \\
= \frac{x + 7}{3(2x + 1)}
\]

**ANSWER:**

\[
\frac{x + 7}{3(2x + 1)}
\]

6. \[ \frac{x}{x^2 - 1} - \frac{3}{2x + 2} \]

**SOLUTION:**

\[
\frac{x}{x^2 - 1} - \frac{3}{2x + 2} = \frac{x}{(x + 1)(x - 1)} - \frac{3}{2(x + 1)} \\
= \frac{x}{(x + 1)(x - 1)} - \frac{3}{2(x + 1)}(x - 1) \\
= \frac{2x - 3(x - 1)}{2(x + 1)(x - 1)} \\
= \frac{-x + 3}{3(x + 1)(x - 1)}
\]

**ANSWER:**

\[
\frac{-x + 3}{2(x - 1)(x + 1)}
\]

7. \[ \frac{1}{y} + \frac{2}{7} - \frac{3}{2y^2} \]

**SOLUTION:**

\[
\frac{1}{y} + \frac{2}{7} - \frac{3}{2y^2} = \frac{1(14y)}{14y^2} + \frac{2(2y^2)}{2y^2} - \frac{3(7)}{2y^2} \\
= \frac{14y + 4y^2 - 3 \cdot 7}{14y^2} \\
= \frac{4y^2 + 14y - 21}{14y^2}
\]

**ANSWER:**

\[
\frac{4y^2 + 14y - 21}{14y^2}
\]

8. \[ \frac{2 + \frac{1}{x}}{5 - \frac{1}{x}} \]

**SOLUTION:**

\[
\frac{2 + \frac{1}{x}}{5 - \frac{1}{x}} = \frac{\left(\frac{2x + 1}{x}\right)}{\left(\frac{5x - 1}{x}\right)} \\
= \frac{2x + 1}{x} \cdot \frac{x}{5x - 1} \\
= \frac{2x + 1}{5x - 1}
\]

**ANSWER:**

\[
\frac{2x + 1}{5x - 1}
\]
9. Identify the asymptotes, domain, and range of the function graphed.

\[ f(x) = \frac{6}{x + 2} - 5 \]

**SOLUTION:**
Vertical asymptote: \( x = -2 \)

Horizontal asymptote: \( f(x) = -5 \)

\[ D = \{ x | x \neq -2 \} \]

\[ R = \{ f(x) | f(x) \neq -5 \} \]

**ANSWER:**
\( x = -2; f(x) = -5; D = \{ x | x \neq -2 \}, R = \{ f(x) | f(x) \neq -5 \} \)

10. **MULTIPLE CHOICE** What is the equation for the vertical asymptote of the rational function

\[ f(x) = \frac{x + 1}{x^2 + 3x + 2} ? \]

A \( x = -2 \)

B \( x = -1 \)

C \( x = 1 \)

D \( x = 2 \)

**SOLUTION:**

\[ f(x) = \frac{x + 1}{x^2 + 3x + 2} = \frac{x + 1}{(x + 1)(x + 2)} = \frac{1}{x + 2} \]

Equate the denominator expression to 0 and solve for \( x \).

\[ x + 2 = 0 \]
\[ x = -2 \]

The vertical asymptote of the rational function is \( x = -2 \).

The correct choice is A.

**ANSWER:**
A

**Graph each function.**

11. \( f(x) = -\frac{8}{x} - 9 \)

**SOLUTION:**
Simplify each expression.

1. \( \frac{8}{x} - 9 = \frac{8 - 9x}{x} \)

\[-8 - 9x = 0 \]
\[9x = -8 \]
\[x = -\frac{8}{9} \]

There is a zero at \( x = -\frac{8}{9} \). There is a vertical asymptote at \( x = 0 \).

The degree of the numerator and denominator polynomial is one. Therefore, there is a horizontal asymptote at \( y = -\frac{9}{1} \) or \( y = -9 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-17</td>
</tr>
<tr>
<td>2</td>
<td>-13</td>
</tr>
<tr>
<td>4</td>
<td>-11</td>
</tr>
<tr>
<td>5</td>
<td>-10.6</td>
</tr>
<tr>
<td>8</td>
<td>-10</td>
</tr>
</tbody>
</table>

Draw the asymptotes, and then use the table of values to graph the function.

\( f(x) = \frac{8}{x} - 9 \)

ANSWER:

12. \( f(x) = \frac{2}{x + 4} \)

\( x + 4 = 0 \)
\( x = -4 \)

There is a vertical asymptote at \( x = -4 \).

The degree of the numerator is less than the degree of the denominator, the horizontal asymptote is the line \( y = 0 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-1</td>
</tr>
<tr>
<td>-5</td>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Draw the asymptotes, and then use the table of values to graph the function.
13. \( f(x) = \frac{3}{x-1} + 8 \)

**SOLUTION:**
\[
f(x) = \frac{3}{x-1} + 8 \\
= \frac{3 + 8(x-1)}{x-1} \\
= \frac{3 + 8x - 8}{x-1} \\
= \frac{8x - 5}{x-1} \\
\]
\(8x - 5 = 0\)
\(8x = 5\)
\(x = \frac{5}{8}\)

There is a zero at \( x = \frac{5}{8} \).

\( x - 1 = 0 \)
\( x = 1 \)

There is a vertical asymptote at \( x = 1 \).

Since the degree of the numerator and denominator polynomials are equal, there is a horizontal asymptote at \( y = 8 \).

14. \( f(x) = \frac{5x}{x+1} \)

**SOLUTION:**
\(5x = 0\)
\(x = 0\)

There is a zero at \( x = 0 \).

\(x + 1 = 0\)
\(x = -1\)
There is a vertical asymptote at \( x = -1 \).

Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at \( y = 5 \).

\[
\begin{array}{|c|c|}
\hline
x & f(x) \\
\hline
-5 & 6.25 \\
-3 & 7.5 \\
-2 & 10 \\
0 & 0 \\
1 & 2.5 \\
4 & 4 \\
\hline
\end{array}
\]

Draw the asymptotes, and then use the table of values to graph the function.

\[
f(x) = \frac{5x}{x+1}
\]

**ANSWER:**

\[
f(x) = \frac{5x}{x+1}
\]

\[
15. \ f(x) = \frac{x}{x-5}
\]

**SOLUTION:**

There is a zero at \( x = 0 \).

\[
x - 5 = 0
\]

\[
x = 5
\]

There is a vertical asymptote at \( x = 5 \).

Since the degree of the numerator and denominator polynomials are equal, there is a horizontal asymptote at \( y = 1 \).

\[
\begin{array}{|c|c|}
\hline
x & f(x) \\
\hline
-5 & 0.5 \\
0 & 0 \\
1 & -0.25 \\
3 & -1.5 \\
4 & -4 \\
6 & 6 \\
\hline
\end{array}
\]

Draw the asymptotes, and then use the table of values to graph the function.

Simplify each expression.

1. \( f(x) = \frac{x^2 + 5x - 6}{x - 1} \)

**SOLUTION:**

\[ f(x) = \frac{x^2 + 5x - 6}{x - 1} = \frac{(x + 6)(x - 1)}{x - 1} = x + 6 \]

Therefore, the graph of \( f(x) = \frac{x^2 + 5x - 6}{x - 1} \) is the graph of \( f(x) = x + 6 \) with a hole at \( x = 1 \).

17. Determine the equations of any vertical asymptotes and the values of \( x \) for any holes in the graph of the function \( f(x) = \frac{x + 5}{x^2 - 2x - 35} \).

**SOLUTION:**

\[ f(x) = \frac{x + 5}{x^2 - 2x - 35} = \frac{x + 5}{(x - 7)(x + 5)} = \frac{1}{x - 7} \]

Therefore, there is a vertical asymptote at \( x = 7 \) and a hole at \( x = -5 \).

**ANSWER:**

vertical asymptote: \( x = 7 \); hole: \( x = -5 \)

18. Determine the equations of any oblique asymptotes in the graph of the function \( f(x) = \frac{x^2 + x - 5}{x + 3} \).

**SOLUTION:**

The difference between the degree of the numerator and the degree of the denominator is 1, so there is an oblique asymptote.

Divide the numerator by the denominator to determine the equation of the oblique asymptote.

\[ f(x) = \frac{x^2 + x - 5}{x + 3} = \frac{(x + 3)(x - 2)}{x + 3} = x - 2 \]

Therefore, the equation of the oblique asymptote is \( f(x) = x - 2 \).

**ANSWER:**

\( f(x) = x - 2 \)
Solve each equation or inequality.

19. \( \frac{-1}{x + 4} = 6 - \frac{x}{x + 4} \)

**SOLUTION:**
\[
\frac{-1}{x + 4} = 6 - \frac{x}{x + 4} \\
\frac{-1}{x + 4} = \frac{6(x + 4) - x}{x + 4} \\
-1 = 6x + 24 - x \\
-1 = 5x + 24 \\
5x = -25 \\
x = -5
\]

Check:
\[
\frac{-1}{-5 + 4} = \frac{6}{-5 + 4} \\
\frac{-1}{-1} = \frac{6 + 5}{-1} \\
1 = 6 - 5 \\
1 = 1 \checkmark
\]

The solution is \( x = -5 \).

**ANSWER:**
\( x = -5 \)

---

20. \( \frac{1}{3} = \frac{5}{m + 3} + \frac{8}{21} \)

**SOLUTION:**
\[
\frac{1}{3} = \frac{5}{m + 3} + \frac{8}{21} \\
\frac{1}{3} = \frac{5(21) + 8(m + 3)}{21(m + 3)} \\
\frac{1}{3} = \frac{105 + 8m + 24}{21(m + 3)} \\
\frac{1}{3} = \frac{8m + 129}{21(m + 3)} \\
21(m + 3) = 3(8m + 129) \\
21m + 63 = 24m + 387 \\
-3m = 324 \\
m = -108
\]

Check:
\[
\frac{1}{3} = \frac{5}{-108 + 3} + \frac{8}{21} \\
\frac{1}{3} = \frac{5}{-105} + \frac{8}{21} \\
\frac{1}{3} = \frac{1}{21} + \frac{8}{21} \\
\frac{1}{3} = \frac{-1 + 8}{21} \\
\frac{1}{3} = \frac{7}{21} \\
\frac{1}{3} = \frac{1}{3} \checkmark
\]

The solution is \( m = -108 \).

**ANSWER:**
\( m = -108 \)

---

21. \( 7 + \frac{2}{x} < -\frac{5}{x} \)

**SOLUTION:**
The excluded value for this inequality is \( x = 0 \).
Solve the related equation \( \frac{7 + \frac{2}{x}}{x} = -\frac{5}{x} \).

\[
\begin{align*}
7 + \frac{2}{x} &= -\frac{5}{x} \\
7x + 2 &= -5 \\
x &= -\frac{7}{2}
\end{align*}
\]

Divide the real line into three intervals as shown.

Test \( x = -2 \).

\[
\begin{align*}
7 + \frac{2}{(-2)} &< -\frac{5}{(-2)} \\
7 - 1 &< \frac{5}{2} \\
6 &< \frac{5}{2}
\end{align*}
\]

Test \( x = -\frac{1}{2} \).

\[
\begin{align*}
7 + \frac{2}{-\frac{1}{2}} &< -\frac{5}{-\frac{1}{2}} \\
7 - 4 &< 10 \\
3 &< 10 \checkmark
\end{align*}
\]

Test \( x = 1 \).

\[
\begin{align*}
7 + \frac{2}{1} &< -\frac{5}{1} \\
9 &< -5
\end{align*}
\]

The solution of the inequality is \(-1 < x < 0\).

ANSWER:
\(-1 < x < 0\)
24. \( \frac{r+2}{3r} = \frac{r+4-2}{r-2} \)

**SOLUTION:**

\[
\frac{r+2}{3r} = \frac{r+4-2}{r-2} \\
\frac{r+2}{3r} = \frac{3(r+4)-2(r-2)}{3(r-2)} \\
\frac{r+2}{3r} = \frac{3r+12-2r+4}{3(r-2)} \\
(r+2)(r-2) = r^2 + 16r \\
r^2 - 4 = r^2 + 16r \\
r = \frac{4}{16} \\
r = \frac{1}{4}
\]

The solution is \( r = \frac{1}{4} \).

**ANSWER:**

\( r = \frac{1}{4} \)
25. If \( y \) varies inversely as \( x \) and \( y = 18 \) when \( x = 2 \), find \( x \) when \( y = -10 \).

**SOLUTION:**

\[
y = \frac{k}{x}
\]

\[18 = \frac{k}{2}\]

\[k = 18 \times \frac{2}{1} = 36\]

Therefore, \( y = -\frac{9}{x} \).

Substitute \( y = -10 \) in the equation \( y = -\frac{9}{x} \).

\[-10 = -\frac{9}{x}\]

\[9 = 10x\]

\[x = \frac{9}{10}\]

**ANSWER:**  
\[\frac{9}{10}\]

26. If \( m \) varies directly as \( n \) and \( m = 24 \) when \( n = -3 \), find \( n \) when \( m = 30 \).

**SOLUTION:**

\[m = kn\]

\[24 = k(-3)\]

\[k = -8\]

Substitute \( m = 30 \) in \( m = -8n \).

\[m = -8n\]

\[30 = -8n\]

\[n = \frac{-30}{8}\]

\[= \frac{-15}{4}\]

**ANSWER:**  
\[\frac{-15}{4}\]
27. Suppose $r$ varies jointly as $s$ and $t$. If $r = 20$ when $r = 140$ and $t = -5$, find $s$ when $r = 7$ and $t = 2.5$.

**SOLUTION:**

$$r = kst$$

$$140 = k(20)(-5)$$

$$140 = -100k$$

$$k = \frac{-140}{100}$$

$$k = \frac{-7}{5}$$

Substitute $r = 7$, $t = 2.5$ in the relation $r = \frac{-7}{5}st$.

$$7 = \frac{-7}{5}s(2.5)$$

$$7 = \frac{-7}{2}s$$

$$14 = -7s$$

$$s = \frac{14}{-7}$$

$$s = -2$$

**ANSWER:**

$-2$

---

28. **BICYCLING** When Susan rides her bike, the distance that she travels varies directly with the amount of time she is biking. Suppose she bikes 50 miles in 2.5 hours. At this rate, how many hours would it take her to bike 80 miles?

**SOLUTION:**

$$m = kt$$

$$50 = k(2.5)$$

$$k = \frac{50}{2.5}$$

$$k = 20$$

Substitute $m = 80$ in the relation $m = 20t$.

$$m = 20t$$

$$80 = 20t$$

$$t = 4$$

So, it would take 4 hours to bike 80 miles.

**ANSWER:**

4 hours
29. **PAINTING** Peter can paint a house in 10 hours. Melanie can paint the same house in 9 hours. How long would it take if they worked together?

**SOLUTION:**

\[
\frac{x}{10} + \frac{x}{9} = 1
\]

\[
\frac{9x + 10x}{90} = 1
\]

\[
\frac{19x}{90} = 1
\]

\[
19x = 90
\]

\[
x = \frac{90}{19}
\]

\[
x \approx 4.7
\]

It would take about 4.7 hours if they worked together.

**ANSWER:** about 4.7 hours

30. **MULTIPLE CHOICE** How many liters of a 25% acid solution must be added to 30 liters of an 80% acid solution to create a 50% acid solution?

F 18

G 30

H 36

J 66

**SOLUTION:**

\[
x \cdot 25\% = 0.25x
\]

\[
30 \cdot 80\% = 24
\]

\[
(30 + x) \cdot 50\% = 15 + 0.5x
\]

\[
24 + 0.25x = 15 + 0.5x
\]

\[
9 = 0.25x
\]

\[
36 = x
\]

**ANSWER:** H
31. What is the volume of the rectangular prism?

\[
\frac{1}{x+2} \cdot \frac{x+2}{x^2+6x+5} \cdot \frac{x+5}{x+6}\]

**SOLUTION:**

\[
\frac{1}{x+2} \cdot \frac{x+2}{x^2+6x+5} \cdot \frac{x+5}{x+6} = \frac{(x+2)(x+5)}{(x+2)(x^2+6x+5)} = \frac{(x+2)(x+5)}{(x+2)(x+5)(x+1)} = \frac{1}{(x+1)}
\]

The volume of the prism is \(\frac{1}{x+1}\) cubic units.

**ANSWER:**

\(\frac{1}{x+1}\) cubic units
Choose a term from the list above that best completes each statement or phrase.

1. A(n) ______ is a rational expression whose numerator and/or denominator contains a rational expression.

   SOLUTION: complex fraction

   ANSWER: complex fraction

2. If two quantities show ______, their product is equal to a constant $k$.

   SOLUTION: inverse variation

   ANSWER: inverse variation

3. A(n) ______ asymptote is a linear asymptote that is neither horizontal nor vertical.

   SOLUTION: oblique

   ANSWER: oblique

4. A(n) ______ can be expressed in the form $y = kx$.

   SOLUTION: direct variation

   ANSWER: direct variation

5. Equations that contain one or more rational expressions are called ______.

   SOLUTION: rational equations

   ANSWER: rational equations

6. The graph of $y = \frac{x}{x + 2}$ has a(n) ______ at $x = -2$.

   SOLUTION: vertical asymptote

   ANSWER: vertical asymptote

7. ______ occurs when one quantity varies directly as the product of two or more other quantities.

   SOLUTION: Joint variation

   ANSWER: Joint variation
8. A ratio of two polynomial expressions is called a(n) 
   ________.

   SOLUTION:
   rational expression

   ANSWER:
   rational expression

9. ________ occurs when one quantity varies directly 
   and/or inversely as two or more other quantities.

   SOLUTION:
   Combined variation

   ANSWER:
   Combined variation

10. ________ looks like a hole in a graph because the 
    graph is undefined at that point.

   SOLUTION:
   Point discontinuity

   ANSWER:
   Point discontinuity

11. Simplify each expression.

    \[
    \frac{-16xy \cdot 15z^3}{27z \cdot 8x^2}
    \]

    SOLUTION:
    \[
    \frac{-16xy \cdot 15z^3}{27z \cdot 8x^2} = \frac{(-16)(15)xyz^3}{(27)(8)x^2z}
    \]
    \[
    = \frac{(-2)(5)yz^2}{(9)x}
    \]
    \[
    = \frac{10yz^2}{9x}
    \]

    ANSWER:
    \[
    \frac{10yz^2}{9x}
    \]

12. \[
    \frac{x^2 - 2x - 8}{x^2 + x - 12} \cdot \frac{x^2 + 2x - 15}{x^2 + 7x + 10}
    \]

    SOLUTION:
    \[
    \frac{x^2 - 2x - 8}{x^2 + x - 12} \cdot \frac{x^2 + 2x - 15}{x^2 + 7x + 10}
    \]
    \[
    = \frac{(x - 4)(x + 2)}{(x + 4)(x - 3)} \cdot \frac{(x + 5)(x - 3)}{(x + 5)(x + 2)}
    \]
    \[
    = \frac{x - 4}{x + 4}
    \]

    ANSWER:
    \[
    \frac{x - 4}{x + 4}
    \]
13. \[
\frac{x^2 - 1}{x^2 - 4} \cdot \frac{x^2 - 5x - 14}{x^2 - 6x - 7}
\]

**SOLUTION:**
\[
\frac{x^2 - 1}{x^2 - 4} \cdot \frac{x^2 - 5x - 14}{x^2 - 6x - 7} = \frac{(x + 1)(x - 1)}{(x + 2)(x - 2)} \cdot \frac{(x - 7)(x + 2)}{(x - 7)(x + 1)} = \frac{x - 1}{x - 2}
\]

**ANSWER:**
\[
\frac{x - 1}{x - 2}
\]

14. \[
\frac{x + y}{15x} + \frac{x^2 - y^2}{3x^2}
\]

**SOLUTION:**
\[
\frac{x + y}{15x} + \frac{x^2 - y^2}{3x^2} = \frac{x + y}{15x} \cdot \frac{3x^2}{x^2 - y^2} = \frac{x + y}{15x} \cdot \frac{3x^2}{(x - y)(x + y)} = \frac{x}{5(x - y)}
\]

**ANSWER:**
\[
\frac{x}{5(x - y)}
\]

15. \[
\frac{x^2 + 3x - 18}{x^2 + 7x + 6}
\]

**SOLUTION:**
\[
\frac{x^2 + 3x - 18}{x^2 + 7x + 6} = \frac{x^2 + 3x - 18}{x + 4} \cdot \frac{x + 4}{x^2 + 7x + 6} = \frac{(x + 6)(x - 3)}{(x + 6)(x + 1)} = \frac{x - 3}{x + 1}
\]

**ANSWER:**
\[
\frac{x - 3}{x + 1}
\]
16. GEOMETRY A triangle has an area of $3x^2 + 9x - 54$ square centimeters. If the height of the triangle is $x + 6$ centimeters, find the length of the base.

**SOLUTION:**
Let $b =$ length of the base of the triangle.

\[3x^2 + 9x - 54 = \frac{1}{2}b(x + 6)\]

\[3\left(\frac{x^2 + 3x - 18}{x + 6}\right) = \frac{1}{2}b\]

\[6(x + 6)(x - 3) = b\]

\[6(x - 3) = b\]

\[6x - 18 = b\]

The length of the base of the triangle is $6x - 18$ centimeters.

**ANSWER:**
$6x - 18$ cm

Simplify each expression.

17. $\frac{9}{4ab} + \frac{5a}{6b^2}$

**SOLUTION:**

\[\frac{9}{4ab} + \frac{5a}{6b^2} = \frac{9}{4ab}\left(\frac{3b}{3b}\right) + \frac{5a}{6b^2}\left(\frac{2a}{2a}\right)\]

\[= \frac{27b}{12ab^2} + \frac{10a^2}{12ab^2}\]

\[= \frac{27b + 10a^2}{12ab^2}\]

**ANSWER:**
$\frac{27b + 10a^2}{12ab^2}$

18. $\frac{3}{4x - 8} - \frac{x - 1}{x^2 - 4}$

**SOLUTION:**

\[\frac{3}{4(x - 2)} - \frac{x - 1}{(x + 2)(x - 2)}\]

\[= \frac{3}{4(x - 2)} - \frac{(x - 1)\cdot 4}{4(x - 2)(x + 2)(x - 2)}\]

\[= \frac{3(x + 6)}{4(x - 2)(x + 2)} - \frac{4x - 1}{4(x - 2)(x + 2)}\]

\[= \frac{3x + 6 - 4x + 4}{4(x - 2)(x + 2)}\]

\[= \frac{-x + 10}{4(x - 2)(x + 2)}\]

**ANSWER:**
$\frac{-x + 10}{4(x - 2)(x + 2)}$

19. $\frac{y}{2x} + \frac{4y}{3x^2} - \frac{5}{6xy^2}$

**SOLUTION:**

\[\frac{y}{2x} + \frac{4y}{3x^2} - \frac{5}{6xy^2}\]

\[= \frac{y\cdot 3xy^2}{2x\cdot 3xy^2} + \frac{4y\cdot 6xy^2}{3x^2\cdot 6xy^2} - \frac{5\cdot x}{6x^2y^2}\]

\[= \frac{3xy^3}{6x^2y^2} + \frac{8y^3}{6x^2y^2} - \frac{5x}{6x^2y^2}\]

\[= \frac{3xy^3 + 8y^3 - 5x}{6x^2y^2}\]

**ANSWER:**
$\frac{3xy^3 + 8y^3 - 5x}{6x^2y^2}$
20. \[
\frac{2}{x^2 - 3x - 10} - \frac{6}{x^2 - 8x + 15}
\]

**SOLUTION:**
\[
\frac{2}{x^2 - 3x - 10} - \frac{6}{x^2 - 8x + 15} = \frac{2}{(x - 5)(x + 2)} - \frac{6}{(x - 5)(x - 3)} = \frac{2}{(x - 5)(x + 2)} \cdot \frac{x - 3}{x - 3} - \frac{6}{(x - 5)(x - 3)} \cdot \frac{x + 2}{x + 2} = \frac{2(x - 3)}{(x - 5)(x + 2)(x - 3)} - \frac{6(x + 2)}{(x - 5)(x - 3)(x + 2)} = \frac{2x - 6 - 6x - 12}{(x - 5)(x + 2)(x - 3)} = \frac{-4x - 18}{(x - 5)(x + 2)(x - 3)}
\]

**ANSWER:**
\[
-\frac{4x - 18}{(x - 5)(x + 2)(x - 3)}
\]

21. \[
\frac{3}{3x^2 + 2x - 8} + \frac{4x}{2x^2 + 6x + 4}
\]

**SOLUTION:**
\[
\frac{3}{3x^2 + 2x - 8} + \frac{4x}{2x^2 + 6x + 4} = \frac{3}{(3x - 4)(x + 2)} + \frac{4x}{(x + 2)(2x + 2)} = \frac{3(2x + 2) + 4x(3x - 4)}{(3x - 4)(x + 2)(2x + 2)} = \frac{6x + 6 + 12x^2 - 16x}{2(3x - 4)(x + 2)(x + 1)} = \frac{12x^2 - 10x + 6}{2(x + 2)(x + 1)(3x - 4)} = \frac{12x^2 - 10x + 6}{2(x + 2)(x + 1)(3x - 4)}
\]

**ANSWER:**
\[
\frac{12x^2 - 10x + 6}{2(x + 2)(3x - 4)(x + 1)}
\]

22. \[
\frac{3}{2x + 3} - \frac{x + 1}{x + 1} = \frac{2x + 3}{2x + 3} - 1 = \frac{3(x + 1) - x(2x + 3)}{(2x + 3)(x + 1)}
\]

**SOLUTION:**
\[
\frac{3}{2x + 3} - \frac{x + 1}{x + 1} = \frac{3(x + 1) - x(2x + 3)}{(2x + 3)(x + 1)} = \frac{3x + 3 - 2x^2 - 3x}{4x^2 + 6x + 5x + 5} = \frac{-2x^2 + 3}{4x^2 + 11x + 5}
\]

**ANSWER:**
\[
-\frac{2x^2 + 3}{4x^2 + 11x + 5}
\]

23. **GEOMETRY** What is the perimeter of the rectangle?

**SOLUTION:**
\[
2 \left( \frac{1}{x + 1} \right) + 2 \left( \frac{4}{x + 6} \right) = \frac{2}{x + 1} + \frac{8}{x + 6}
\]

**ANSWER:**
\[
\frac{2x + 12 + 8x + 8}{(x + 1)(x + 6)} = \frac{10x + 20}{(x + 1)(x + 6)}
\]
Graph each function. State the domain and range.

24. \( f(x) = -\frac{12}{x} + 2 \)

**SOLUTION:**

The graph of \( f(x) = -\frac{12}{x} + 2 \) represents a transformation of the graph of \( f(x) = \frac{1}{x} \).

\( a = -12 \): The graph is expanded and is reflected across the \( x \)-axis.

\( k = 2 \): The graph is translated 2 units up. There is an asymptote at \( f(x) = 2 \).

\[ D = \{ x | x \neq 0 \}, \quad R = \{ f(x) | f(x) \neq 2 \} \]

**ANSWER:**

\[ D = \{ x | x \neq 0 \}, \quad R = \{ f(x) | f(x) \neq 2 \} \]
26. \( f(x) = \frac{3}{x + 5} \)

**SOLUTION:**

The graph of \( f(x) = \frac{3}{x + 5} \) represents a transformation of the graph of \( f(x) = \frac{1}{x} \).

\( a = 3 \): The graph is expanded.

\( h = -5 \): The graph is translated 5 units left. There is an asymptote at \( x = -5 \).

\[ D = \{ x | x \neq -5 \}, \quad R = \{ f(x) | f(x) \neq 0 \} \]

**ANSWER:**

\[ D = \{ x | x \neq -5 \}, \quad R = \{ f(x) | f(x) \neq 0 \} \]

---

27. \( f(x) = \frac{6}{x - 9} \)

**SOLUTION:**

The graph of \( f(x) = \frac{6}{x - 9} \) represents a transformation of the graph of \( f(x) = \frac{1}{x} \).

\( a = 6 \): The graph is expanded.

\( h = 9 \): The graph is translated 9 units right. There is an asymptote at \( x = 9 \).

\[ D = \{ x | x \neq 9 \}, \quad R = \{ f(x) | f(x) \neq 0 \} \]

**ANSWER:**

\[ D = \{ x | x \neq 9 \}, \quad R = \{ f(x) | f(x) \neq 0 \} \]
28. \( f(x) = \frac{7}{x-2} + 3 \)

**SOLUTION:**

The graph of \( f(x) = \frac{7}{x-2} + 3 \) represents a transformation of the graph of \( f(x) = \frac{1}{x} \).

\( a = 7 \): The graph is expanded.

\( h = 2 \): The graph is translated 2 units right. There is an asymptote at \( x = 2 \).

\( k = 3 \): The graph is translated 3 units up. There is an asymptote at \( f(x) = 3 \).

\[ D = \{x \mid x \neq 2\}, \quad R = \{f(x) \mid f(x) \neq 3\} \]

ANSWER:

29. \( f(x) = -\frac{4}{x+4} - 8 \)

**SOLUTION:**

The graph of \( f(x) = -\frac{4}{x+4} - 8 \) represents a transformation of the graph of \( f(x) = \frac{1}{x} \).

\( a = -4 \): Since \( a < 0 \), the graph is reflected across the \( x \)-axis.

\( h = -4 \): The graph is translated 4 units left. There is an asymptote at \( x = -4 \).

\( k = -8 \): The graph is translated 8 units down. There is an asymptote at \( f(x) = -8 \).

Since \( |\text{-}4| < 1 \), the graph is stretched vertically.

\[ D = \{x \mid x \neq -4\}, \quad R = \{f(x) \mid f(x) \neq -8\} \]

ANSWER:

30. **CONSERVATION** The student council is planting 28 trees for a service project. The number of trees each person plants depends on the number of student council members.

a. Write a function to represent this situation.

b. Graph the function.
Choose a term from the list above that best completes each statement or phrase.

SOLUTION:

1. A(n) occurs when one quantity varies directly as another. Therefore, it will take to plant the garden if they work together. 

ANSWER:

2. The excluded value for this inequality is . Solve the related equation and .

ANSWER:

3. If a is , then . Substitute .

ANSWER:

4. The graph of . Therefore, there is a vertical asymptote at .

ANSWER:

5. If there is a horizontal asymptote when the degree of the numerator and the degree of the denominator are equal, there is a horizontal asymptote at .

ANSWER:

6. Let be the number of student council members. The function representing the situation is .

b. The graph of represents a transformation of the graph of . Here , the graph is stretched vertically.

ANSWER:

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

31. \( f(x) = \frac{3}{x^2 + 4x} \)

SOLUTION:

\( x^2 + 4x = 0 \)
\( x(x + 4) = 0 \)
\( x = 0 \) or \( x = -4 \)

Therefore, the vertical asymptotes are at \( x = 0 \) and \( x = -4 \).

ANSWER:

\( x = -4, x = 0 \)

32. \( f(x) = \frac{x + 2}{x^2 + 6x + 8} \)

SOLUTION:

\( f(x) = \frac{x + 2}{x^2 + 6x + 8} = \frac{x + 2}{(x + 4)(x + 2)} = \frac{1}{x + 4} \)

\( x + 4 = 0 \)
\( x = -4 \)

Therefore, there is a vertical asymptote at \( x = -4 \). There is a hole at \( x = -2 \).

ANSWER:

\( x = -4 \); hole: \( x = -2 \)
33. \( f(x) = \frac{x^2 - 9}{x^2 - 5x - 24} \)

**SOLUTION:**

\[
\begin{align*}
f(x) &= \frac{x^2 - 9}{x^2 - 5x - 24} \\
&= \frac{(x + 3)(x - 3)}{(x - 8)(x + 3)} \\
&= \frac{x - 3}{x - 8}
\end{align*}
\]

Therefore, there is a vertical asymptote at \( x = 8 \). There is a hole at \( x = -3 \).

**Answer:**

\( x = 8 \); hole: \( x = -3 \)

Graph each rational function.

34. \( f(x) = \frac{x + 2}{(x + 5)^2} \)

**SOLUTION:**

There is a zero at \( x = -2 \).

\[
(x + 5)^2 = 0 \\
x + 5 = 0 \\
x = -5
\]

There is a vertical asymptote at \( x = -5 \).

Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is at \( y = 0 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-1.25</td>
</tr>
<tr>
<td>-6</td>
<td>-4</td>
</tr>
<tr>
<td>-4</td>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
<td>-0.25</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

35. \( f(x) = \frac{x}{x + 1} \)

**SOLUTION:**

There is a zero at \( x = 0 \).

\[
x + 1 = 0 \\
x = -1
\]

There is a vertical asymptote at \( x = -1 \).

Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at \( y = 1 \).
<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>1.2</td>
</tr>
<tr>
<td>−5</td>
<td>1.25</td>
</tr>
<tr>
<td>−3</td>
<td>1.5</td>
</tr>
<tr>
<td>−2</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Draw the asymptotes, and then use a table of values to graph the function.

**SOLUTION:**

$$f(x) = \frac{x^2 + 4x + 4}{x + 2}$$

The graph of $f(x) = \frac{x^2 + 4x + 4}{x + 2}$ is same as the graph of $f(x) = x + 2$ with a hole at $x = −2$.

**ANSWER:**

$$f(x) = \frac{x - 1}{x^2 + 5x + 6}$$

**SOLUTION:**

$$f(x) = \frac{x - 1}{x^2 + 5x + 6}$$

There is a zero at $x = 1$. 
38. **SALES** Aliyah is selling magazine subscriptions. Out of the first 15 houses, she sold subscriptions to 10 of them. Suppose Aliyah goes to \( x \) more houses and sells subscriptions to all of them. The percentage of houses that she sold to out of the total houses can be determined using

\[
P(x) = \frac{10 + x}{15 + x}.
\]

a. Graph the function.

b. What domain and range values are meaningful in the context of the problem?

**SOLUTION:**

a. There is a vertical asymptote at \( x = -15 \). Since the degree of the numerator and denominator are equal, there is a horizontal asymptote at \( y = 1 \).

Graph the function.

\[
P(x) = \frac{10 + x}{15 + x}
\]

b. \( D = \{x \geq 0\}; R = \{0 \leq P(x) \leq 1.0\} \)

**ANSWER:**

b. \( D = \{x \geq 0\}; R = \{0 \leq P(x) \leq 1.0\} \)
39. If \( a \) varies directly as \( b \) and \( b = 18 \) when \( a = 27 \), find \( a \) when \( b = 10 \).

**SOLUTION:**
\[
a = kb
\]
\[
27 = k(18)
\]
\[
k = \frac{27}{18}
\]
\[
k = \frac{3}{2}
\]

Substitute \( b = 10 \) in the relation \( a = \frac{3}{2}b \).

\[
a = \frac{3}{2}(10)
\]
\[
a = 15
\]

**ANSWER:**
\( a = 15 \)

40. If \( y \) varies inversely as \( x \) and \( y = 15 \) when \( x = 3.5 \), find \( y \) when \( x = -5 \).

**SOLUTION:**
\[
y = \frac{k}{x}
\]
\[
15 = \frac{k}{3.5}
\]
\[
k = 15(3.5)
\]
\[
k = 52.5
\]

Substitute \( x = -5 \) in the relation \( y = \frac{52.5}{x} \).

\[
y = \frac{52.5}{-5}
\]
\[
y = -10.5
\]

**ANSWER:**
\( y = -10.5 \)
41. If $y$ varies inversely as $x$ and $y = -3$ when $x = 9$, find $y$ when $x = 81$.

**SOLUTION:**

\[
y = \frac{k}{x}
\]

\[-3 = \frac{k}{9}
\]

$k = -27$

Substitute $x = 81$ in the relation $y = \frac{-27}{x}$.

\[
y = \frac{-27}{81}
\]

\[
y = \frac{-1}{3}
\]

**ANSWER:**

\[y = \frac{-1}{3}\]

42. If $y$ varies jointly as $x$ and $z$, and $x = 8$ and $z = 3$ when $y = 72$, find $y$ when $x = -2$ and $z = -5$.

**SOLUTION:**

\[y = kxz\]

\[72 = k(8)(3)\]

\[72 = 24k\]

\[k = \frac{72}{24}\]

\[k = 3\]

Substitute $x = -2$ and $z = -5$ in the relation $y = 3xz$.

\[y = 3(-2)(-5)\]

\[y = 30\]

**ANSWER:**

\[y = 30\]
43. If $y$ varies jointly as $x$ and $z$, and $y = 18$ when $x = 6$ and $z = 15$, find $y$ when $x = 12$ and $z = 4$.

**SOLUTION:**

$$y = kxz$$

$$18 = k(6)(15)$$

$$18 = 90k$$

$$k = \frac{18}{90}$$

$$k = \frac{1}{5}$$

Substitute $x = 12$ and $z = 4$ in the relation $y = \frac{1}{5}xz$.

$$y = \frac{1}{5}(12)(4)$$

$$y = \frac{48}{5}$$

**ANSWER:**

$$y = \frac{48}{5}$$

44. **JOBS** Lisa’s earnings vary directly with how many hours she babysits. If she earns $68 for 8 hours of babysitting, find her earnings after 5 hours of babysitting.

**SOLUTION:**

Let $x$ = number of hours.

Let $y$ = Lisa’s earnings.

$$y = kx$$

$$68 = k(8)$$

$$k = \frac{68}{8}$$

$$k = \frac{17}{2}$$

Substitute $x = 5$ in the relation $y = \frac{17}{2}x$.

$$y = \frac{17}{2}x$$

$$y = \frac{17}{2}(5)$$

$$y = \frac{85}{2}$$

$$y = 42.50$$

After 5 hours of babysitting, Lisa’s earnings is $42.50$.

**ANSWER:**

$42.50
Solve each equation or inequality. Check your solutions.

45. \( \frac{1}{3} + \frac{4}{x-2} = 6 \)

**SOLUTION:**

\[
\frac{1}{3} + \frac{4}{x-2} = 6
\]

\[
x + 10 = 18(x - 2)
\]

\[
x + 10 = 18x - 36
\]

\[
-17x = -46
\]

\[
x = \frac{46}{17}
\]

Check:

\[
\frac{1}{3} + \frac{4}{\left(\frac{46}{17} + 2\right)} = 6
\]

\[
\frac{1}{3} + \frac{4}{\left(\frac{46}{17} - 34\right)} = 6
\]

\[
\frac{1}{3} + \frac{12}{68} = 6
\]

\[
4 + \frac{68}{12} = 6
\]

\[
\frac{72}{12} = 6
\]

\[
6 = 6 \checkmark
\]

The solution is \( x = \frac{46}{17} \).

**ANSWER:**

\( x = \frac{46}{17} \)

---

46. \( \frac{6}{x+5} - \frac{3}{x-3} = \frac{6}{x^2 + 2x - 15} \)

**SOLUTION:**

\[
\frac{6}{x+5} - \frac{3}{x-3} = \frac{6}{x^2 + 2x - 15}
\]

\[
6(x-3) - 3(x+5) = 6
\]

\[
(x+5)(x-3) = x^2 + 2x - 15
\]

\[
6x - 18 - 3x - 15 = 6
\]

\[
x^2 + 2x - 15 = x^2 + 2x - 15
\]

\[
3x - 33 = 6
\]

\[
3x = 39
\]

\[
x = 13
\]

Check:

\[
\frac{6}{13+5} - \frac{3}{13-3} = \frac{6}{13^2 + 2(13) - 15}
\]

\[
\frac{6}{18} - \frac{3}{10} = \frac{6}{169 + 26 - 15}
\]

\[
\frac{60 - 54}{180} = \frac{6}{180}
\]

\[
\frac{6}{180} = \frac{6}{180} \checkmark
\]

The solution is \( x = 13 \).

**ANSWER:**

\( x = 13 \)
47. \( \frac{2}{x^2 - 9} = \frac{3}{x^2 - 2x - 3} \)

**SOLUTION:**

\[
\frac{2}{x^2 - 9} = \frac{3}{x^2 - 2x - 3} = \frac{2}{(x+3)(x-3)} = \frac{3}{(x+3)(x+1)}
\]

\[
\frac{2}{2(x+1)} = \frac{3}{3(x+3)}
\]

\[
\frac{2}{(x+3)(x-3)}(x+1) = \frac{3}{(x+3)(x+1)(x+3)}
\]

\[
2x + 2 = 3x + 9
\]

\[-x = 7
\]

\[
x = -7
\]

Check:

\[
\frac{2}{(-7)^2 - 9} = \frac{3}{(-7)^2 - 2(-7) - 3}
\]

\[
\frac{2}{49 - 9} = \frac{3}{49 + 14 - 3}
\]

\[
\frac{2}{40} = \frac{3}{60}
\]

\[
\frac{1}{20} = \frac{1}{20} \checkmark
\]

The solution is \( x = -7 \).

**ANSWER:**

\( x = -7 \)

48. \( \frac{4}{2x-3} + \frac{x}{x+1} = \frac{-8x}{2x^2 - x - 3} \)

**SOLUTION:**

\[
\frac{4}{2x-3} + \frac{x}{x+1} = \frac{-8x}{2x^2 - x - 3}
\]

\[
4(2x+1) + x(2x-3) = -8x
\]

\[
\frac{4(2x+1)}{(2x-3)(x+1)} = \frac{-8x}{2x^2 - x - 3}
\]

\[
\frac{4x + 4 + 2x^2 - 3x}{2x^2 - x - 3} = \frac{-8x}{2x^2 - x - 3}
\]

\[
\frac{2x^2 + x + 4}{2x^2 - x - 3} = \frac{-8x}{2x^2 - x - 3}
\]

\[
2x^2 + x + 4 = -8x
\]

\[
2x^2 + 9x + 4 = 0
\]

\[
2x^2 + x + 8x + 4 = 0
\]

\[
x(2x+1) + 4(2x+1) = 0
\]

\[
(2x+1)(x+4) = 0
\]

\[
x = \frac{-1}{2} \text{ or } x = -4
\]

Check: \( x = -\frac{1}{2} \)

\[
\frac{4}{2\left(-\frac{1}{2}\right)-3} + \frac{-\frac{1}{2} + 1}{2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)-3}
\]

\[
\frac{4}{-1 - \frac{1}{2}} + \frac{2}{\frac{1}{2} + 1 - 3}
\]

\[
\frac{-1 - \frac{1}{2}}{\frac{4}{1-3}} + \frac{2}{-2}
\]

\[
-2 = 2
\]

Check: \( x = -4 \)
Choose a term from the list above that best completes each statement or phrase.

1. A(n) ...

ANSWER:

Therefore, it will take...

SOLUTION:

\[
\frac{4}{x+4} - \frac{28}{x^2 + x - 12} = \frac{1}{x-3}
\]

\[
\frac{x}{x+4} - \frac{28}{(x+4)(x-3)} = \frac{1}{x-3}
\]

\[
\frac{x(x-3)-28}{(x+4)(x-3)} = \frac{1}{x-3}
\]

\[
x^2 - 3x - 28 = x + 4
\]

\[
x^2 - 3x - 28 = x + 4
\]

\[
x^2 - 3x - 28 = 0
\]

\[
x^2 - 4x - 32 = 0
\]

\[
(x - 8)(x + 4) = 0
\]

x = 8 or x = -4

ANSWER:

Check: x = 8

When x = -4, left side expression of the rational equation becomes undefined. Therefore, the solution of the equation is x = 8.

ANSWER:

x = 8
Therefore, the solution is \( x < 1 \).

**ANSWER:**
\( x < 1 \)

51. \( \frac{1}{2x} - \frac{4}{5x} > \frac{1}{3} \)

**SOLUTION:**
The excluded value for this inequality is \( x = 0 \).

Solve the related equation \( \frac{1}{2x} - \frac{4}{5x} = \frac{1}{3} \).
The LCD is \( 30x \).

\[
\frac{1}{2x} - \frac{4}{5x} = \frac{1}{3} \\
30x \cdot \frac{1}{2x} - 30x \cdot \frac{4}{5x} = 30x \cdot \frac{1}{3} \\
15 - 24 = 10x \\
-9 = 10x \\
\frac{-9}{10} = x
\]

Draw vertical lines at the excluded value and at the solution to separate the number line into intervals.

Now test a sample value in each interval to determine whether the values in the interval satisfy the inequality.

Test \( x = -1 \).
\[
\frac{1}{2(-1)} - \frac{4}{5(-1)} > \frac{1}{3} \\
\frac{1}{2} + \frac{4}{5} > \frac{1}{3} \\
\frac{3}{10} > \frac{1}{3} \\
\frac{9}{30} \neq \frac{1}{3}
\]

Test \( x = -0.5 \).
52. **YARD WORK** Lana can plant a garden in 3 hours. Milo can plant the same garden in 4 hours. How long will it take them if they work together?

**SOLUTION:**
\[
\frac{x}{3} + \frac{x}{4} = 1 \\
\frac{4x + 3x}{12} = 1 \\
7x = 12 \\
x = \frac{12}{7} \\
x = 1 \frac{5}{7}
\]

Therefore, it will take \(1 \frac{5}{7}\) hr to plant the garden if they work together.

**ANSWER:**
\[1 \frac{5}{7}\]